
Article:

Scerif, G., Blakey, E., Gattas, S. et al. (2023) Making the executive ‘function’ for the foundations of mathematics: the need for explicit theories of change for early interventions. Educational Psychology Review, 35 (4). 110. ISSN: 1040-726X

https://doi.org/10.1007/s10648-023-09824-3

Educational Psychology Review (2023) 35:110
https://doi.org/10.1007/s10648-023-09824-3

REVIEW ARTICLE

Making the Executive 'Function' for the Foundations of Mathematics: the Need for Explicit Theories of Change for Early Interventions

Gaia Scerif[1] · Emma Blakey[2] · Sylvia Gattas[1] · Zachary Hawes[3] · Steven Howard[4] · Rebecca Merkley[5] · Rosemary O'Connor[1] · Victoria Simms[6]

Accepted: 12 October 2023

Abstract

A vast body of work highlights executive functions (EFs) as robust correlates of mathematics achievement over the primary and preschool years. Yet, despite such correlational evidence, there is limited evidence that EF interventions yield improvements in early years mathematics. As intervention studies are a powerful tool to move beyond correlation to causality, failures of transfer from executive functions interventions are, we argue, highly problematic for both applied and theoretical reasons. We review the existing correlational and intervention literature at complementary neuroscientific, cognitive, developmental and educational levels. We appraise distinct theories of change underpinning the correlations between EF and early mathematics, as well as explicit or implicit theories of change for different types of EF interventions. We find that isolated EF interventions are less likely to transfer to improvements in mathematics than integrated interventions. Via this conceptual piece, we highlight that the field of EF development is in need of (1) a clearer framework for the mechanisms underpinning the relationships between early EF and other developing domains, such as mathematical cognition; (2) clearer putative theories of change for how interventions of different kinds operate in the context of EF and such domains; (3) and greater clarity on the developmental and educational contexts that influence these causal associations. Our synthesis of the evidence emphasises the need to consider the dynamic development of EFs

---

Highlights
- Relationships between EF and mathematics during the preschool years are robust.
- However, many EF interventions have not resulted in mathematics improvements.
- Greater clarity is needed on mechanisms underpinning relationships and interventions.
- We appraise theories of change for EF and mathematics interventions.
- Implications for theory, intervention design and outcomes are discussed.

The first author acknowledges responsibility for positioning this opinion piece. All other authors agreed to be listed alphabetically in recognition of diverse but equally important contributions.

Extended author information available on the last page of the article

with co-developing cognitive functions, such as early math skills, when designing education environments. [234 words].

Keywords Executive functions · Mathematics · Early intervention · Causal hypotheses · Theories of change

Executive functions (EFs henceforth) are thought to encompass separable but interacting processes often referred to as inhibitory control, working memory and cognitive flexibility (Miyake et al., 2000; Miyake & Friedman, 2012). Inhibitory control is one's ability to focus attention on information that is relevant to our current goals, while ignoring or inhibiting distracting stimuli or actions that are currently not goal relevant. Working memory is one's ability to hold information in mind and actively manipulate it. Cognitive flexibility is one's ability to shift attention between features, rules, goals, or tasks at hand. Although separable via tasks that demand primarily one of these skills, in everyday situations and complex tasks, these EFs operate in concert to enable planning, multi-tasking and flexible goal-oriented behaviour. Given the role of EFs for controlling and organising attention and thinking, it is unsurprising they have been implicated as essential to early learning (e.g. for school readiness, Bierman et al., 2008). Indeed, multiple empirical studies point to concurrent and longitudinal relationships between EFs and mathematics before the onset of formal mathematics education, but interventions that have focused on executive functions have thus far failed to transfer to improvements in early mathematics. In this conceptual piece, our key goals are to (a) review the correlational and intervention evidence, (b) propose that putative causal mechanisms and theories of change underpinning early years executive functions interventions need to be much more explicit and (c) discuss previous failures and successes of transfer onto mathematics outcomes, in the context of evidence from developmental cognitive neuroscience and education. We close by proposing possible ways to optimise transfer in future research and practice.

Understanding Relationships Between Early EFs and Mathematics: a Framework

Why Should EFs Be Relevant to Mathematics Prior to the Onset of Formal Education?

Mathematics Demands EF EFs have been posited as a key contributor to the learning and performance of mathematics. For example, solving word problems requires focus on relevant details and ignoring distractors, while integrating across its multiple informational elements. Even simple mental arithmetic requires maintaining and manipulating numerical information in mind, and cognitive flexibility may be required when switching between numerical information, rules and operations across a range of mathematical tasks. Indeed, a large amount of evidence point to complex cognitive mappings from EF to mathematics during the primary school years (e.g. Cragg & Gilmore, 2014; EF+Math Program, n.d.; Gilmore, 2023) and correlations do not entail concordance (Mazzocco et al., 2017).

From a neuroscientific point of view, it has long been known that cognitive control circuits are involved in mathematical performance (e.g. Menon et al., 2000). However, in comparison to adults, children recruit frontoparietal and frontostriatal circuits to a greater extent (e.g. Bellon et al., 2020; Holloway & Ansari, 2010; Jolles et al., 2016) suggesting a more active role of EFs when mathematical processing is required for children compared to adults. These differences in neural activation may be due to less specialised nodes of the networks involved in mathematical operations and less efficient communication across these nodes. This greater involvement of control circuits associated with EFs in school-aged children compared to adults when performing mathematical operations has been ascribed to less well specialised nodes of the networks involved in mathematical operations and less efficient communication across these nodes (Engelhardt et al., 2019; Wilkey & Price, 2019). Of note, these child–adult differences do not mean that EF do not play a key role for mathematics in adulthood: these reported differences may be due to the difficulty and/or novelty of the mathematics in question, because adults show similar high levels of involvement of frontoparietal circuits to children when the mathematical tasks are novel and challenging (e.g. for negative number representations, Blair et al., 2012; or across numerical and spatial cognitive skills, Hawes et al., 2019). In turn, similarities between children and adults when challenging tasks are presented suggest at least a partial alignment between the development of expertise and developmental changes in how the brain tackles novel and challenging mathematical problems. Indeed, the crucial interplay between cognitive control functions and content knowledge has long been acknowledged for adults (Halford et al., 2007; Unsworth et al., 2009).

Early Mathematics Demands EF Cognitive control circuits appear crucial to early mathematical learning and therefore flag a key role for the cognitive control skills in early mathematics (Houdé et al., 2010). Should the evidence from developmental cognitive neuroscience influence our models of EF and early mathematics development? And should it also shape our models of early executive functions interventions targeting early mathematics? These questions call for a closer evaluation of the specific putative mechanisms underpinning the relationships between EFs and mathematics, in particular with regards to early childhood, as this literature has been less frequently synthesised prior to the onset of primary school. EF skills may be important to build foundations of numeracy in young children for overlapping, but also distinguishable, cognitive reasons to the reasons that underpin these relationships in school aged children or adults. It is very likely that simple but foundational mathematical activities such as demonstrating cardinality understanding, displaying accurate one-to-one correspondence and other key preschool numerical skills require EFs.

EF as Early Cognitive Contributors to Mathematics Performance and Learning One way of conceptualizing this contribution of EF to early mathematics may be to consider it a performance limiting factor for the extent to which young children can demonstrate their early mathematical skills. For example, even as simple a skill as counting objects requires EFs: working memory is necessary when keeping track of a count word list and demonstrating cardinal knowledge. Therefore, we shall refer to this mechanism as a 'performance limiting' contribution of EFs to early mathematics. For more complex skills in the early years, such as addition, EF may be a performance limiting factor because efficient identification of goals and strategic deployment are needed to add accurately and efficiently. Flexible strategy identification is a central shared element of executive functions and fluid intelligence in both adults (Duncan, 2013; Engle et al., 1999; Heitz et al., 2006) and young children (de Abreu et al., 2010). Therefore, goal setting and strategic deployment may be construed as a limiting factor for performing early mathematical tasks. A second and non-mutually exclusive mechanism is that EFs may provide an additional learning foundation to the acquisition, and not only to the performance, of these early mathematical skills in preschool, as they have been suggested to do for primary school (Gilmore & Cragg, 2014; Ribner, 2020). Under this account, EFs play a "learning facilitator" role for early mathematics learning, and we will refer to this as such. For example, with appropriate inhibitory engagement, it may be easier to ignore non-numerical information embedded in stimuli that are mixed in their numerical and non-numerical dimensions when learning about number. With good working memory skills, it may be easier to operate on and deepen one's knowledge of numerical concepts. It may also be easier to shift between numerical rules and representations, such as digit, number and quantity, to practice and learn less familiar ones with strong cognitive flexibility. Whether EFs are solely performance limiting, or are also facilitating new mathematical learning, it remains unclear whether these cognitive functions play a greater role in pre-schoolers compared to older children. In addition, there is uncertainty as to whether EFs play a consistent or changing role in mathematical learning across the pre- and primary years. Consistent executive demands may depend on the fact that mathematics is continually changing throughout the curriculum (as detailed by Gilmore, 2023) and therefore consistently demanding on the executive. Thus, when automaticity is attained for some skills, further learning and / or manipulation of those skills is required.

Early EF and Developmental Cognitive Neuroscience In addition to these cognitive reasons for the importance of considering mathematics and EF in unison during the preschool years, evidence from developmental cognitive neuroscience also suggests that a particular focus on the early years is necessary: circuits that are foundational to arousal, self-regulation and executive control undergo striking development from birth to five years of age (Fiske & Holmboe, 2019; Hendry et al., 2019). At the same time networks involved in linguistic and referential processing also specialise rapidly over this initial period of life (Johnson, 2001, 2011). These data suggest that developing executive networks and mathematical expertise need to be studied in concert, and not only in isolation, if we are to understand the acquisition of mathematical knowledge and ability, but also to be able to better understand and theorise about EFs. Of note, here we do not claim that EFs are not relevant to mathematical learning later in education, as they may in fact continue to be involved when new mathematical skills are acquired. However, specifically in the early years, evidence suggests that *both* prior mathematical understanding and EFs are emerging. In contrast, EFs are relatively expert functions in adulthood. So, adults may draw on developed and well established EFs differently from young children, when they, as adults, learn new mathematical content. The developmental interplay between developing, not developed, EFs *and* mathematics is crucial to understanding this early period.

Early EF and Educational Contexts In addition to cognitively and neurobiologically distinct reasons why EFs may contribute to the learning foundations of preschool mathematics, educational factors may also matter. The diversity of approaches to developing EF and mathematics in preschool settings may contribute to the variability in mathematics competencies upon starting school. Preschool curricula and educational practices vary in the depth, breadth and quality of activities relevant to early mathematics (e.g. Hodgen et al., 2020), but also because of the highly variable range of explicit and implicit foci on developing EF skills by different educators and educational settings prior to the onset of compulsory education. For example, in the UK, the statutory framework for the early years foundation stage (EYFS, Department of Education, 2021) provides detailed guidance on expectations for foundational mathematical skills for children before they reach the end of the first year in compulsory education (when they are aged between 4 and 5 years of age). This framework also provides limited guidance on cognitive regulatory functions such as EFs. This is an improvement on previous versions of this statutory framework, because EFs remain poorly understood even by primary school educators (Gilmore & Cragg, 2014). However, these recent changes in statutory guidance does not explicitly integrate EFs and early mathematics, leaving this integrative element to individual practitioners and settings. An interesting future direction will be to review systematically how EFs and related constructs such as self-regulation are integrated into early years learning for mathematics across countries and programmes.

What Is the Extant Empirical Evidence That EFs and Mathematics Relate to Each Other, Prior to the Onset of Compulsory and Curricularised Mathematics Education?

Early EFs and Mathematics—the Evidence Mounting empirical data has highlighted preschool EF as a concurrent but also longer-term predictor of both foundational numeracy skills. EFs also predict performance on multi-componential standardised assessments of mathematics over the period from 3 to 5 years of age and later elementary years (e.g. Clark et al., 2013; Coolen et al., 2021; Fuhs et al., 2014, 2016; Mulder et al., 2017; Ribner et al., 2017, 2018; Welsh et al., 2010). In these studies, mathematical achievement is operationalised as standardised performance scores on mixed mathematical assessment tools, and foundational skills as indexed by classic cardinality or other componential skill tasks. EFs have also been identified as a key factor underpinning attainment gaps in early mathematical skills (Blakey et al., 2020). Several key issues emerge from this growing literature, with some paralleling the larger body of work in elementary school children, and some specific to this age range. First, there is converging evidence that individual differences in early EFs, starting from as early as 2 years of age, predicts emerging numeracy (e.g. Mulder et al., 2017). Second, in contrast with the literature on testing the relationship between specific EFs and specific mathematical skills in older children (Cragg & Gilmore, 2014), the majority of research thus far has focused on latent EF factors as predictors of mathematical outcomes on omnibus mathematics achievement measures, with fewer studies potentially investigating distinct EF or componential mathematical skills (e.g. Chan & Scalise, 2022; Coolen et al., 2021; Fuhs et al., 2016; Purpura et al., 2017; Ribner et al., 2018). Third, fewer studies have been designed to investigate dynamic bidirectional relations between mathematics and EF (e.g. Fuhs et al., 2014; Coolen et al., 2021). While correlations between EFs and mathematics in preschool are consistent with the extant literature on mathematics in elementary school children, measurement issues and unidirectional models appear to be issues unique to the preschool age literature.

As an early example of how EF predicts early mathematics, Clark et al. (2013) found that a single latent EF factor measured at 3 years of age predicted both formal and informal indices of mathematics achievement measured via the Test of Early Mathematics Achievement at 5 years of age for Canadian preschoolers. Fuhs et al. (2014) also found that EFs measured at the beginning and end of prekindergarten (4 to 5 years of age) in the USA predicted gains in standardised mathematics achievement a year later. In the UK, EF predicted foundational symbolic mathematics skills (such as cardinality, counting, symbol identification) that were clustered into a single factor in the year preceding the onset of elementary education (Coolen et al., 2021). This finding converges with previous research that indicated that EF predicting standardised mathematics outcomes in Scottish pre-schoolers (Bull et al., 2008). In terms of even further long-term longitudinal predictions from the preschool years, another large-scale study of children growing up in very low-income settings in the USA found that EF skills at 5 years old predicted mathematics achievement over 5 years later. In addition, participants with poor mathematics knowledge but high EF at school entry caught up those who had higher levels of mathematical skills at school entry (Ribner et al., 2017).

Early EFs and Mathematics—the Caveats Some caveats to this body of literature should be highlighted, in view of implications for specifying mechanisms of intervention. While most of these studies found that EFs clustered into a single latent construct, which was then used to predict a single aspect or index of mathematical performance, there was some evidence of differentiation as to which specific EFs might longitudinally predict single aspects of mathematical performance. This point is interesting to consider in the light of theoretical and empirical evidence for differentiation across distinct EFs and later distinct mathematical skills for elementary school children and beyond (Cragg & Gilmore, 2014; Cragg et al., 2017). It is also really important to consider differentiation across EFs in their relationships to developing mathematics, because even primary school mathematics has recently been highlighted as highly multi-componential and hierarchical (Gilmore, 2023).

Differentiation might mean that distinct EFs may be more crucial for particular components of mathematics, and/or when particular components of mathematics build on each other to progress from more familiar to more complex or novel mathematical skills. Indeed, Ribner et al. (2018) found that, although exploratory factor analyses clustered EF skills into a single factor, and mathematical skills were clustered into conceptual and procedural skills, when individual elements of EF were considered, inhibitory skills related to conceptual mathematics skills, whereas working memory related to procedural mathematics skills. In addition, multiple studies (McKinnon & Blair, 2019; Schmitt et al., 2017; Gunzenhauser & Nuckles, 2021) have reported both direct and indirect effects of individual differences in EFs in predicting mathematical skills in the pre-elementary school years. Direct effects were indexed by predictive relationships that survived controlling for possible moderating factors, whereas indirect effects were via mediating factors. Direct and indirect effects have also been reported for older children (e.g. Cragg et al., 2017) and may suggest multiple routes through which early EF-focused interventions may operate to improve early mathematics. As discussed by Blakey et al. (2020), interventions that focus on specific EF skills will allow researchers to draw firmer causal inferences about what EFs support which early mathematical skills. However, interventions that take more of a holistic approach and target multiple EFs are more likely to show stronger and synergistic intervention effects. As we discuss mechanisms of change later, we will return to open questions with regards to specific or general EF skills engendering transfer to improved mathematics skills as a future direction for investigation by intervention specialists. The multiple routes through which early EFs may improve early mathematics have important implications for understanding causality, as they may guide which EFs to embed in which specific mathematical skills to ensure optimal intervention outcomes.

As a second caveat, many of these studies were designed to investigate EF as a predictor of mathematics outcomes, but not to model the possible *bidirectional* role of early mathematical skills as predictors of later EFs. Why is it important to consider whether there are bidirectional relationships between early EFs and mathematics? Bidirectionality may have implications for how changes in both EFs and mathematics play out over time: if it is not only the case that better EFs predict better mathematics, but also that better mathematics predict later better EF, improvements across the two domains may engender a virtuous cycle of improvement. For the few studies employing a fully balanced longitudinal design (i.e. measuring EF and mathematics with the same level of specification at multiple longitudinal time-points), bi-directionality has been reported (e.g. Cameron et al., 2019; Clements et al., 2016; Fuhs et al., 2014; McKinnon et al., 2019; Miller-Cotto & Byrnes, 2020), with early mathematics achievement predicting later EF and, vice versa. However, there is also some indication of asymmetries in early in childhood, with at least one study reporting concurrent and longitudinal relationships between EF and foundational mathematics skills in 4-year olds, but only EF surviving as a predictor of growth for these skills once baseline levels of mathematics skills were taken into account (Coolen et al., 2021). An earlier study with pre-schoolers from lower-income settings found a similar asymmetry for children who were a year older, whereas bidirectional relationships between EF and mathematics preceded this pattern (Schmitt et al., 2017), suggesting potential changes in directionality over time. Of note, this literature is mixed and complicated by conflicting findings. For example, Ellis et al. (2021) failed to replicate Schmitt et al. (2017)'s findings: in two samples, EF best predicted later mathematics but not the other way around. The authors suggest that this might be due to distinct studies measuring EF differently. Importantly, the interpretation of research on bidirectional relationships between EF and math is further complicated by the lack of clear consensus on how to model developmental change longitudinally (Curran & Hancock, 2021). Specifically, cross-lagged panel models, which are commonly used (e.g. Coolen et al., 2021; Miller-Cotto et al., 2022) have been criticised for confounding within-person variance with between-person variance. Therefore, conclusions based on these analyses may over-estimate the strength of reciprocal relationships between cognitive processes. However, bidirectional relationships between working memory and mathematics were also found in a longitudinal study of students from kindergarten to grade 5 when fixed effects were modelled to better account for within-person variance over time (Zhang et al., 2023). Further longitudinal studies and their meta-analytic synthesis will be necessary to understand the bidirectionality of these relationships: future work needs to pinpoint whether unidirectional or bidirectional findings depend on differences in sample characteristics, EF measurement, or analysis techniques.

Clarifying this literature is important, as bidirectional relationships point to dynamic mechanisms of change for EF-focused interventions, in favour of meaningfully combined, as opposed to isolated interventions. In both unidirectional and bidirectional models, the best fitting measurement models for both EF and mathematics is often specified a priori, but this is in and of itself a debated question, both for EF and for mathematics (e.g. is a single latent factor via confirmatory factor analysis a better fit to mathematics and EF data, or should we consider individual components for children of different ages, see Nguyen, Duncan & Bailey (2019) for a treatment of this point). For simplicity, here we consider EFs as closely related over the preschool years, even if greater differentiation may emerge later (e.g. Wiebe et al., 2011, but see Karr et al., 2018) and we also consider mathematical skills as a multicomponential but highly related set of skills, although they clearly differentiate later (Gilmore, 2023). We note, however, that early differentiation is possible, but so far not as widely tested for both EFs and mathematics. Where possible, if distinguishable elements have been measured, we shall point the readers to component-specific relationships (e.g. between working memory and mental arithmetic, e.g. Cragg & Gilmore, 2014).

Finally, an important caveat to the correlational and longitudinal literature is its under-representation of pre-schoolers from low-income settings (cf. Welsh et al., 2010; Ribner et al., 2017) and from non-Western Industrialized Rich and Democratic Countries (WEIRD, Henrich et al., 2010). Thus far, this more varied literature has typically used a 'deficit' framework, i.e. it has predicted poorer mathematics and EFs in low-income settings, but a hypothesis that remains to be tested empirically is whether pre-schoolers learning in under-resourced schools or from lower SES households are simply not provided the same type or frequency of opportunities to develop EFs during mathematics learning, compared to peers from more privileged backgrounds (as suggested by, e.g. Lawson et al., 2018) or whether instead different

EF and mathematics-enhancing activities take place in these diverse settings, to support their development or co-development.

Overall, the extant literature converges on preschool EF as a robust predictor of mathematical skills, both during preschool and later in development. However, this literature currently suffers from non-balanced longitudinal designs, from the inability to fully capture both latent factors and individual components of EF and mathematics, their possible bidirectional relationships, and from under-representation of diversity.

Moving Beyond Correlations by Investigating Causality: Evaluating Theories of Change of Early EF Interventions

Understanding Preschool EF Interventions and Failures of Transfer to Mathematics

Theories of Change of 'Isolated' EF Interventions We refer to EF focused interventions that do not explicitly focus on integrating mathematical content as 'isolated' EF interventions. How do we reconcile robust correlations and longitudinal data with EF interventions' failures to transfer to mathematics? Despite the convincing correlational and longitudinal data reviewed above, multiple developmentalists have argued that these provide weak tests of theoretical models, and that interventions are very important tools to test causality claims (e.g. Bailey et al., 2018). Early proposals of transfer of EF training (with a focus on computerised working memory training alone; Klingberg, 2010; Constantinidis & Klingber, 2016) posited that training EF would transfer to untrained functions in so far as those functions are gated by better executive control skills. These suggestions were grounded in the neurobiology of the interactions between cognitive control systems, such as those supporting EFs, and specialised systems, such as those dedicated to mathematical processing.

However, multiple systematic reviews and meta-analyses have now exposed the failures of transfer of many EF intervention programmes of this kind across childhood and adulthood. We refer the reader to these existing reviews for a holistic treatment of EF transfer failures to other untrained cognitive functions (e.g. Melby-Lervag & Hulme, 2013; Melby-Lervaget al., 2016; Kassai et al., 2019; Takacs & Kassai, 2019). Failures of EF interventions have been recently reviewed particularly in relation to working memory, and in the context of school mathematics. Interestingly, there has also been a recent emphasis on greater successes of domain-general training (including EF interventions) when specific transfer target domains are integrated with EF training regimes (Peng & Swanson, 2022). A recent systematic review instead reports on greater successes of EF interventions that integrate domain-specific contents (Peng & Swanson, 2022), to which we refer as 'integrated EF interventions'. However, only one of the studies in this recent review of integrated EF interventions included young children.

Here, we add to this literature by summarizing key features that are relevant to EF interventions when they aim to improve mathematics skills for children prior to commencement of elementary school. First, while evidence for the immediate efficacy of isolated EF training on the trained EF seems robust even in pre-schoolers, there is poor evidence of even near transfer (from one EF to another, for example, from inhibitory training to working memory outcomes; e.g. Thorell et al., 2009). Second, there is limited evidence of transfer from EF interventions to untrained functions such as mathematics achievement in the early years, particularly when the EF training regime is computerised in nature and does not involve EF-enhancing interactions with peers or adults (Diamond & Ling, 2019). For example, Blakey et al. (2020) found that while EFs did correlate with preschoolers' mathematical skills, computerised EF training did not improve mathematical skills. A third point of note, extending in particular to our target age group, is that more promise has been attained by curriculum-based EF-focused preschool interventions in demonstrating efficacy of EF improvement (e.g. Diamond et al., 2007; Blair & Raver, 2014). Evidence of transfer of this improvement to untrained pre-academic skills seems to be variable and moderated by the characteristics of the target sample, with more socio-economically disadvantaged children (e.g. Blair & Raver, 2014) or inattentive children (Solomon et al., 2018) benefiting most. Despite this greater promise for curriculum-based interventions, even the efficacy of some previously successful curriculum-based EF interventions has been called into question in a recent large scale randomised control trial (Nesbitt & Farran, 2021). Nesbitt and Farran detail how multiple moderator variables, including children's characteristics and the pre-existing quality of preschool settings, may be key reasons for variable efficacy and limited transfer. So far, the state of the evidence suggests that curriculum-based interventions may be a more effective approach in improving EFs compared to targeted computerised training. However, even with this approach, transfer to skills that are otherwise very clearly correlated to EFs (early mathematics) has been very difficult to engender.

A plausible mechanism of action for 'isolated' EF interventions is perhaps that they engender transfer by providing better generic regulation skills in preschool classroom situations (e.g. sitting still, waiting one's turn, sustaining attention toward a learning experience), and that those in turn might foster better focus on learning all new materials, including emerging mathematical skills. Like with mathematics content, for these benefits to accrue, EF might need to be practiced in these specific contexts (e.g. sustaining attention toward a learning task, applying and sustaining strategies to persist when tasks are challenging, overcoming 'big' emotional reactions to instead interact constructively). While perhaps foundational perhaps to the youngest children, the evidence in pre-schoolers so far suggests that this general type of mechanism of change is not sufficient for engendering transfer to mathematical skills. Beyond the negative evidence so far, isolated EF interventions may simply be theoretically flawed in suggesting that they should transfer to mathematics without mathematical content, because they treat EF as independent of developing mathematics functions, which is not appropriate given the ample developmental evidence reviewed above. Isolated EF interventions may also fail to transfer if they ignore the differing levels of knowledge and understanding of basic mathematics by individual learners, or differing mathematics provisions by their educators, and so they may be doomed to fail not because of their EF component, but because they interact with prior mathematics skills differently for different children and different educational provisions. In this context, we hypothesise instead that very explicit and intentional boosting of EF demands embedded in mathematics learning will be more beneficial than either focusing on EF alone, or mathematics alone, precisely because of their early co-development and interactions.

Alternatives to 'Isolated' EF Interventions: Integrated EF/Mathematics Interventions

Theories of Change of Integrated EF Interventions Increasingly, education scientists and policy makers have proposed a much more specific need to embed EF into the context of the target transfer domain, in this context mathematics (EF + Math, n.d.; Mulcahy et al., 2021; Niebaum & Munakata, 2020). In parallel, cognitive neuroscientists have also strongly argued that differential recruitment of circuits involved in mathematics-specific EFs (e.g. attention to number, working memory for numerical material, inhibition of and flexibility across numerical dimensions) deserve further investigation, because they significantly predict mathematics achievement over and above recruitment of more generalised networks (Wilkey & Price, 2019). The proposal to combine EF and mathematics in preschool intervention programmes seems really intuitive. A comprehensive primer of the rationale for combined intervention programmes in primary school (US Years 3–8) has been articulated by the EF + Math Program team (n.d.). Over the preschool years, combined approaches are also being increasingly championed (e.g. Joswick et al., 2019). For example, Joswick et al. (2019) proposed practical and enjoyable ways of modifying a number sense activity to develop executive functions, and provided general strategies for modifying multiple mathematics activities to foster simultaneously early mathematics and EFs. In addition, transfer from optimised high-quality preschool mathematics interventions to EF has been demonstrated (e.g. Clements et al., 2016, 2020; Day-Hess & Clements, 2017; Scalise et al., 2020). While here we focus primarily on why combining EF and mathematics should improve transfer to mathematics (unlike EF-alone programmes), it has also been pointed out elsewhere that high quality mathematics interventions should and can transfer to EF improvements. For example, recent conceptual papers have argued that EF can be improved by optimal mathematics activities, i.e. they have suggested that good mathematics pedagogy can help develop both mathematics and EF (Mulcahy et al., 2021). The authors suggest that this is because good mathematics pedagogy provides young children with opportunities for reflection and it embeds challenge to EFs across a variety of mathematical contexts. From a cognitive point of view, transfer from high quality mathematics interventions to EF is highly plausible, given the growing evidence for bidirectional relationships between EF and mathematics (as detailed above). Mulcahy and colleagues also reflect on the additional element provided by good mathematics pedagogy, which is peer-based, play-based and provides adult-scaffolded support, elements that may be excluded from computer-based intervention regimes (Diamond & Ling, 2019). Moreover, EFs improve via school attendance, even when they are not specifically targeted by interventions, because increases in executive function skills are correlated with time spent in school (e.g. Finch, 2019; Brod et al., 2017), suggesting that the classroom is a great place to target EFs.

To date, the evidence on the efficacy of integrated EF and mathematics interventions is growing, but it is mixed. McClelland et al. (2019) embedded numeracy content in a pre-existing EF intervention and found improvements in both EF and numeracy. Furthermore, Kroesbergen et al. (2014) found that numerical working memory showed greater improvement in counting than children who were trained in non-numerical working memory. Furthermore, Clements et al. (2020) did not find evidence of improved numeracy after an intervention that combined numeracy and EF elements. Furthermore, a small-scale but pioneering proof-of-principle study (Prager et al., 2023) contrasted brief training sessions of EF training alone with comparable sessions of numeracy training alone, and an integrated EF and numeracy training condition. Training in EF alone improved EF, but not general mathematics skills, whereas integrated EF and mathematics training improved mathematics, but not to a significantly greater degree than the numeracy condition only. Of note, these integrated EF and mathematics interventions varied widely in their duration, format of intervention, and in precisely how EF and mathematics were integrated with each other. As a whole, the evidence thus far suggests some potential for EF and mathematics interventions in improving mathematics, but this evidence is mixed. Here we argue that, in addition to gathering further empirical evidence, it is key that researchers are clearer about the ingredients of the hypothesised mechanisms of these integrated interventions.

Integrated EF Interventions: Hypotheses Operationalizing clearly the cognitive, educational and biological elements of EF interventions that integrate with mathematical contents may help us understand why transfer from EF to mathematics is more likely to happen in integrated interventions. Here, we focus in particular on mechanistic predictions for how adding EF to mathematics interventions would facilitate reciprocal and recursive influences between EFs and mathematics, continuing to drive both forward. We argue that by developing EFs and mathematical skills in tandem, integrated interventions encourage the use of EFs in the service of a given mathematical goal. When the EF demands are being pushed, stretched and/or engaged, so too is the need to think more strategically. For example, a common early years activity is to flash dots presented on cardboard plates and ask children to estimate how many dots they see. Without a time limit, children are free to count all of the dots in a one-to-one fashion (which for some children is a high enough EF challenge). However, for other children, imposing the time limit, and arguably increasing EF demands, leads to new ways of approaching the task (e.g. ‘groupitizing’ and multiplicative strategies, pushing them to think in terms of multiple sets… ‘I saw three threes!’). By manipulating the EF demands (and doing so in a way that is adaptive and responsive to the learner(s) in question) one is also able to influence what the learner(s) is/are able to attend to.

Why are integrated approaches more likely to be successful, in comparison to EF interventions that do not focus on mathematical content in preschool? What are the precise mechanisms of change by which integrated interventions succeed in engendering transfer, as opposed to isolated ones that target EF alone, or mathematics alone? We argue that early EF training programmes can improve EF, but that this process needs to be contextualised and integrated into mathematical content in order to foster deep mathematical learning. The absence or limited focus on appropriate mathematical content is problematic. Conversely, we argue that some mathematic instruction programmes focus on breaking down steps, stripping out challenge not associated with the mathematics content. This is not leveraging EFs (and EF-related strategies that have been acquired) to support mathematics learning. Similarly, by leveraging EF for mathematics learning there is opportunity to acquire new EF strategies (at a minimum) related to mathematics. Indeed, proponents of combined intervention approaches advocate the need to embed EF in the context of well-designed activities that involve the target content of transfer, because optimised mathematics content (e.g. Clements et al., 2016, 2020), particularly in the context of preschool play-based activities (e.g. Scalise et al., 2020) is crucial. They also emphasise the need to consider individual differences in children involved in the interventions, as children's level of EF and/or mathematics may moderate intervention success (Dong et al., 2022).

Integrated EF Interventions: Mechanisms Two key elements of successful combined interventions will be, first, the nature of the mathematical content of preschool play-based activities that are appropriate to both pre-schoolers' mathematics proficiency and EF skills, and, second, an understanding of individual differences in both mathematics skill and EF. Of note, here we stress the need to ensure, scale and sometimes even boost the EF challenge in mathematics learning, rather than focusing on mathematics content alone. A third important factor for theories of change of combined EF and mathematics interventions is, we propose, that they are much more explicit in operationalizing the mechanisms by which EF and mathematics *interact* while beginning learners approach new mathematical content and develop or deepen their mathematical skills. Operationalizing mechanisms by which EF and mathematics interplay during learning is key. It is key for both proposing mechanisms of change and in proposing detailed characteristics of combined interventions that are likely to be effective, because differing relationships might lead to different intervention designs. Numerous interactions between EFs and early mathematics are possible, because of the complex ways in which children's EF skills and mathematical knowledge interact when we ask them to perform mathematical tasks (e.g. in assessments) as opposed to when we ask them to engage in learning of new mathematical concepts. For example:

a. If EF represents a constraint or a limitation on performing mathematics tasks, EF may be construed as 'a *performance* foe' for mathematics (for example because doing well on mathematics assessments is EF demanding and requires fluid strategic deployment). If this were to be the key mechanism underpinning EF/mathematics relations, a direction for intervention may be to reduce or remove EF demands from mathematical activities. As we argue below, this may indeed be what underpins relationships between EF and mathematics for older struggling learners, but the evidence from pre-schoolers does not point in this direction.

b. If EF demands while learning mathematics pose a challenge to beginning learners, EF may act as ‘a *learning* foe’ for mathematics (i.e. learning mathematics is hard and demands EF in order to be learnt well). Again, this kind of approach might be key for older children with pre-existing difficulties or specific areas for remediation.
c. A third possibility is that EF acts as ‘*a learning ally*’ to the learning of new mathematical concepts, rather than an enemy, because greater EF involvement and fluid strategic thinking mean not only generalised better classroom learning (although this matters), but deeper processing of mathematical concepts, the opportunity to extend skills via challenge to mastery, facilitating both the ability to work with more (and more complex) information at once, while also resisting contrary impulses and remain engaged with the challenge, thereby leaving capacity for re-processing and going beyond the basics.
d. A fourth possibility, that builds on the third and is not mutually exclusive of it, is that guided and explicit embedding of EF challenge by adult practitioners in play-based mathematics activities may be ‘an explicit learning ally’. The role of learning ally would not only depend on the additional deeper processing of mathematical concepts that comes with greater EF challenge, but also because EF challenge embedded in mathematics may begin to build meta-cognitive evaluation by young children of why facing challenge/exploring challenging situations is good. If this is the case, as long as appropriate levels of mathematical materials are selected (to avoid catastrophic failure or misunderstanding), a moderate, well-calibrated, increasing and intentional element of executive challenge should facilitate the highest level of transfer to improvements in mathematics in beginning learners.

We acknowledge that EF demands may contribute to the correlations between EF and mathematics abilities in older children who struggle with mathematics, but propose that it is important to keep in mind the distinction between remediation for later difficulties for a subset of learners, in contrast with laying robust foundations of learning for all learners. Reducing EF demands of mathematics activities may not be the best way to combine EF and mathematics for young children who will be expected to engage with diverse forms of numeracy as they grow, if an optimal level of EF challenge is best to ensure deep processing, explicit reflection and therefore better learning. In contrast, EF and mathematics educational interventions that simplify and reduce EF load, and assessments that limit the impact of EF load may be really important for children who already struggle. There is indeed evidence of the positive impact of reducing working memory demands for struggling learners (e.g. Gathercole & Alloway, 2008). However, an appropriate and well-modulated level of EF challenge may be helpful in fostering improvements/deeper understanding and extension, especially in beginning learners, because both their EFs and mathematical representations are emerging, as so manipulating them in concert will be especially beneficial. This same principle may apply to older learners who are approaching wholly novel mathematical problems (e.g. negative numbers, algebra), but at that point the efficiency of their EF processes will already be better honed than in younger learners.

Integrated EF Interventions and Practical Suggestions To provide a tangible example, let us return to the 'estimate/count the dots game' we introduced earlier. It is easy to imagine different scenarios in which the EF demands are increased or kept constant (e.g. time to estimate number of dots presented on a paper plate) in contrast with changing the mathematics at hand (e.g. the number of dots on the plate). For a child who is just learning or struggling to enumerate dot arrays between 5 and 10 (and maybe they struggle to count/label sets beyond 10), increasing the mathematical content will not only overwhelm the EF system, but also be beyond the limits of their current mathematical capacity. By reducing the mathematical challenge (or keeping it within the child's current 'comfortable' zone), but adapting the EF demands (e.g. by gradually decreasing the amount of time available for estimation, or by presenting dots in mixed colours), so that they are appropriate and adaptive to the learner, we hypothesise that EF demands will facilitate deeper processing of the mathematical content at hand (i.e. they will be 'a learning ally'). Extending the example further, presenting dots in mixed colours will initially engage inhibitory control ('not the two blue dots or the two green dots, but the two red dots'), but it may also encourage fluent processing of items in groups (two, two and two, six dots!). They may even engender early meta-cognitive reflections about efficient strategies and enjoyment of success in the face of challenge ('an enhanced learning ally'). Joswick et al. (2019) provide multiple similar practical suggestions for early years educators, to help modifying early years mathematics activities to embed greater EF challenge.

Our key proposal with regards to cognitive underpinnings of integrating mathematics and EF challenge is that, to capitalise on executive challenge within mathematics as a mechanism of change, one must understand where the child is at/set age-appropriate mathematics content, but then also work to stretch EF demands adaptively and intentionally, not reduce them. The key is therefore to capture the appropriate level of content (or start from very basic but engaging and fun content with the youngest children), but then build deeper expertise via EF challenge, and then in turn scale mathematics challenge, which will in turn bring increased EF demands. A flexible mathematics programme that has embedded EF challenge must also be designed to identify any struggling learners and, at that point, flexibly track down to a more appropriate level of either mathematics content, EF challenge or both, before returning to stretching this again. An example of this flexible approach to introducing EF challenge into interventions has been implemented and evaluated in the context of self-regulation focused interventions (e.g. Howard et al., 2018, 2020a, b).

Why Would Integrated EF/Mathematics Interventions Be More Effective Than Isolated Interventions? Insights from Developmental Cognitive Neuroscience and Education

From the point of view of cognitive and developmental psychology, the failures of isolated attempts to train EF and improve mathematics and the greater likelihood of success when EF and mathematics are integrated perhaps should not be surprising. EFs can be measured in non-mathematical contexts and separated from mathematical content in adult cognitive systems. However, in children, the target systems for these early interventions have not yet fully developed either control functions or mathematical concepts and skills. Therefore, targeting EFs in isolation and devoid of mathematical content, misses key and bidirectional dynamic interactions that characterise these two sets of processes. EF training in isolation may improve EF itself, but without exposure to mathematical content at the appropriate, sustained and increasing level of EF challenge, authentically embedded into mathematics, there may not be a reason for the developing system to be pushed to co-ordinate the processing, deeper encoding, manipulation and retrieval of such mathematical contents (Merkleyet al. 2018).

EF Interventions and Developmental Cognitive Neuroscience Taking a developmental cognitive neuroscience perspective to transfer also offers potential reason for intervention failures and successes. The starting point for predicting that isolated EF training would transfer to other untrained functions derives from the role of frontoparietal and frontostriatal circuits as a key factor in controlling the functions of (already developed) expert systems that, in adult brains, have come to be dedicated to, for example, mathematical skills. They may also extend to the role that these (again, already developed) circuits may play when adults face new or challenging mathematical learning (Hawes et al., 2019). For example, Klingberg (2010; Constantinidis & Klingberg, 2016) proposed that the intensive and explicit training of working memory functions subserved by frontoparietal networks should transfer to untrained functions. Under this model, training the efficiency of control networks (e.g. frontal networks) might transfer in the extent to and efficiency of communication with expert and specialised nodes (e.g. parietal and hippocampal nodes), and therefore result in improvements in the functions that they subserve, either via sustained communication, or because novel strategies/approaches are now available. However, evidence from developmental cognitive neuroscience questions the applicability of these (adult) models of transfer to the developing brain. If both control networks (involved in EFs) and expert systems (involved in mathematical expertise) are not yet as efficient and specialised as in their adult counterparts, training them in isolation may not be the most efficient way of engendering improvements.

Of note, if the proposed mechanism by which EF training leads to improvements in the functions they subserve is sustained communication, then we would expect that the more training should result in more improvements. In contrast, when looking at computerised EF training interventions, number of sessions is not a significant moderator for improvements (Kassai et al., 2019; Sala & Gobet, 2017). This suggests that computerised cognitive training may actually just be improving task specific strategies, which in turn would explain why we do not see far transfer, and strategic learning is unrelated to duration of training because strategies can be picked up quickly. Training-related changes in strategy use in general do not necessarily reflect fundamental changes to the underlying cognitive/neural system, but integrated EF and mathematics activities may facilitate the establishment of strategies that have positive effects on both EF and mathematics, as we discuss later. Even more problematically for the adult transfer view, a key mechanisms for successful EF interventions for developing brains may not simply be to control the interaction of frontoparietal and frontostriatal networks with expert networks, but rather to co-ordinate with them so that they specialise most efficiently. Their specialisation itself engages cognitive control more effectively via interactive specialisation processes (e.g. Johnson, 2011) and newly acquired knowledge influences the efficiency of EFs (Amso & Scerif, 2015), so that intervening on developing EFs in isolation is much less likely to achieve transfer to developing mathematical expertise than a synergistic approach that integrates them.

In contrast, the approach of (introducing and modulating carefully) executive challenge in concert with mathematics, rather than removing challenge entirely, is also supported by a focus on neurobiological mechanisms: optimal alerting and arousal are key to the effective engagement of frontoparietal systems while learning (Abrahamse et al., 2016; Aston-Jones & Cohen, 2005; Braem & Egner, 2018), whereas suboptimal levels of challenge leave a learning system either hypoaroused or too stressed to achieve good learning. These neurobiological models emphasise that hitting a sweet spot of attentional arousal and engagement is key for fostering most efficient cognitive control, and for learning associated with such control. Returning again to the interaction between effective executive control mechanisms and interactive specialisation of increasingly expert systems of numerical cognition, combining optimal EF challenge with well-chosen mathematical content is therefore also supported by an understanding of the role of EF in interactive specialisation processes (Johnson, 2011; Amso & Scerif, 2015).

EF Interventions and the Educational Context In addition to considerations about the cognitive and neural underpinnings of optimal interactions between mathematics and EF challenge, a comprehensive theory of change for EF interventions must not dismiss the educational context. Integrated and curriculum-based interventions depend not only on their content, but also on the role that educators' understanding and expectations can have in fostering EF challenge in the context of mathematics learning (EF+Math, n.d.). The complex interplay that we have reviewed between EF and mathematics suggests that educators are faced with some pretty important decisions; decisions that require knowledge of EFs, mathematics, their interaction, as well as their own expectations of individual characteristics of the learner(s). For example, the EF+Math program (n.d.) reports on a significant literature on how expectations may be lower of young pupils from disadvantaged backgrounds. These lower expectations may be reflected in ways that quite precisely fail to leverage the mechanisms of change we discuss above: if EF challenge is a contributor to developing strong mathematics foundations and some children are less challenged than others because of lower expectations, this might initiate a vicious circle of lower opportunities for challenge, rather than a virtuous one. Additionally, quantitatively reduced expectations may be compounded by qualitatively different approaches to teaching early mathematics, with a focus away from concepts of 'productive struggles' in learning and 'embracing challenge' to simpler 'drill and practice' learning (EF+Math, n.d.). As the formal evaluation of integrated EF and mathematics programmes is still in progress, the body of empirical evidence on whether the integration of mathematics and EF is more beneficial for disadvantaged children than for all children for now is not as large as it could be. However, the EF+Math programme recently published a formal insights report, that summarised the preliminary findings from 10 programmes integrating mathematics and EF in middle school grades (EF+Math, 2023). The report pointed to benefits for children from low-income communities. Evidence for differential benefits from the early years remains limited. Preliminary evidence from one such integrated EF+mathematics intervention for pre-schoolers in the UK has suggested greater benefits for low-income children than for other children (Scerif et al., 2023), but this requires larger scale replication, currently underway.

At the same time, considering the interplay between EF and mathematics learning across diverse environments also highlights how good EFs alone are not all it takes to support emerging mathematical learning: for example, recent work in very low-income South African settings suggests strong EFs compared to preschool EFs in higher income countries (Howard et al., 2020a, b), but these EF strengths may not yet be coupled with mathematical content that then results in strong early mathematical skills (Merkley et al., in preparation). It is also worth reflecting back to the different intervention steps that may be taken by educators working with children from lower vs higher socio-economic backgrounds. Children from lower socio-economic backgrounds might have relatively strong EFs in relation to their skills in curriculum areas, such as mathematics. Thus, EFs (even if lower than their higher SES peers), might represent a relative strength and one that might be better leveraged to improve their mathematical skills, for example. Ignoring EFs as a relative strength and instead treating EF as an obstacle to learning (i.e. treating it as 'a learning foe') might result in educators reducing EF demands from mathematics. This in turn may actually be much more harmful than helpful in this group of learners. In contrast, across diverse socio-economic and socio-cultural environments, all children should be given the opportunity to develop their propensities for active learning of mathematics. An important outcome of well-designed mathematics interventions that include EF challenge embedded in mathematics may be to reduce inequalities by giving all young children opportunities to learn and opportunities to be challenged in mathematics (Byrnes & Miller-Cotto, 2016; Byrnes & Wasik, 2009; Byrnes et al., 2019; EF+Math, n.d.).

Another important consideration for interventions targeting early EF and mathematics is the role of teacher training and teacher knowledge of EFs. Teachers have observed that EFs are important for mathematics learning based on their experience in the classroom (Gilmore & Cragg, 2014). Importantly, many teachers are not familiar with the EF terminology used by cognitive scientists, but may use different ways to describe these cognitive skills, such as ignoring distractions or holding information in mind (Gilmore & Cragg, 2014). Teachers continue to learn and improve over their careers, and the number of years of experience a teacher has is positively related to their students' academic achievement (Podolsky et al., 2019). One key factor influencing teacher effectiveness is subject content knowledge and pedagogical knowledge—the more of these skills teachers have, the better their students learn (Baumert, et al., 2010; Hill et al, 2005; Kelcey et al., 2019; Voss et al., 2011). There is a clear need for intervention studies to target educators, not only children, as many early childhood educators do not receive training in mathematics pedagogy and thus have no specific math-related qualifications (Ginsburg et al., 2008; von Spreckelsen et al., 2019). Many researchers in neuroscience and education have hypothesised that training teachers on general principles of how students learn, based on learning science research, can help them adapt and improve their instruction (e.g. Ansari et al, 2017; Willingham, 2017). Thus, if combined EF and mathematics interventions in early years classrooms are successful, teachers are likely key drivers of change. In order to successfully introduce and modulate executive challenge in early years math lessons, educators should be trained to recognise when their students are using EFs. Previous intervention studies that have targeted children as well as educators, ideally co-developing materials around educators' prior expertise show most evidence of promise in this respect (Hawes et al., 2021; Howard et al., 2020a, 2020b), because teacher implementation fidelity, teacher engagement and uptake are key factors in the effectiveness of many education interventions (Hill & Erickson, 2019).

Future Directions and Practical Solutions: the Orchestrating Numeracy and the Executive (The ONE) Programme and Its Theory of Change as an Example

Many open questions at the level of children, educators, cognitive and educational mechanisms remain unaddressed, but we provide a road map for further intervention research integrating EF and mathematics (Future Directions Box). Future studies will need to investigate whether integrated interventions are more or less successful than isolated ones across the spectrum of individual differences, from children who have poorer EF or mathematics to begin with, to different profiles of EF and mathematics in neurodivergent children. Future research will also need to investigate whether the integration of specific EFs (e.g. inhibition or maintenance, rather their combination) into specific mathematical activities is as beneficial to young children as it has been proposed for older children, or whether a mixed diet of EF challenge combining all three core EFs is best. Finally, in addition to understanding cognitive changes in children, future EF interventions that are integrated in the curriculum will need to find good ways of measuring educators' understanding of EF and their integration with mathematics. Indeed, a needed future direction, extending beyond early EF interventions and mathematics, is to test whether integrated EF and mathematics interventions are just as beneficial for adults as they are for young children, or whether isolated interventions are more successful for adult and specialised systems.

Future Directions Box. Open Questions for researchers investigating integrated EF and mathematics interventions:

1. A focus on neurodiversity: How do EF interventions operate for neurodivergent children who may control their attention to mathematics differently? How do integrated EF interventions fair, compared to isolated ones?
2. A further focus on individual differences: What are the differences in gains from interventions, given wide baseline differences across children? For example, Dong and colleagues (Dong et al., 2022) found that baseline EF competencies have a larger positive relationship to mathematics for children with low math competence than for mid to high math competence. They propose that this is because children with lower baseline mathematics competence are learning nearly all new mathematical material, whereas children with higher competence already have some skills and strategies that reduce the need for good EF in their mathematical learning
3. Understanding specific EFs and their mechanisms for transfer: What specific EFs are key for the development of which early mathematical skill? How are they involved in transfer? Some evidence of specificity has been discussed in older children, but this work remains less frequently explored in pre-schoolers (Blakey et al., 2020 for an exception)
4. Measurement issues at the child and practitioner level: How can we measure teachers and practitioners' self-efficacy in presenting and understanding EF and mathematics? This knowledge may vary and there is a dearth of measures for this construct or related set of constructs (Bardack and Obradović, 2019)

We have highlighted weaknesses of isolated EF interventions, and the promise of integrated EF and mathematics intervention. Empirical evidence supporting our hypothesis is currently limited but growing. For example, excellent examples integrating mathematics and EF are the DREME network projects (e.g. Day-Hess & Clements, 2017), and the EF + Math programme for older children (EF + Math, 2023). In combination with co-developing interventions with teachers and gathering more empirical evidence of efficacy, particularly in the early years, we urge cognitive and education scientists in this area to state as clearly as possible what theory of change underpins their EF interventions, be they isolated or integrated.

We therefore close with a brief outline of a practical example from our collaborative work, to illustrate the need to consider and detail the target mechanisms of any intervention. The Orchestrating Numeracy and the Executive (The "ONE") Programme is an Early Years intervention aiming to improve young children's numeracy by integrating age appropriate and broad mathematical content and executive functions challenge into educator-led play based activities. The proposed mechanism of change is the greater exposure to integrated mathematics and EF challenge for preschool children. In turn, this greater exposure is enabled by a focus on two sets of agents: (1) at the level of children, combined mathematics and EF play-based activities that can easily embed (rather than remove) EF challenge at its sweet-spot into mathematics, and (2) at the level of educators supporting children, professional development about the integration between early mathematics and executive functions. The hypothesised mechanisms, targeted levels and intervention elements for The ONE are detailed elsewhere (Scerif et al., 2023). In this programme, EF is operationalised as a learning ally of early mathematics, as described earlier in this review. The testable hypothesis emerging from the model is that early mathematics skills will improve more in children exposed to the activities compared to practice as usual, because of the enhanced opportunity to practice mathematical content at the sweet-spot of EF challenge. In addition, the focus on multiple levels of change (children and educators) hypothesises that changes will be larger for educators for whom and settings where the quality of the EF and mathematics scaffolding is lower. Preliminary evidence from a small scale randomised controlled trial of The ONE supports this hypothesis (Scerif et al., 2023). Clarity of such models is also crucial for the collaboration with independent evaluators and larger scale evaluations of any such intervention (e.g. Brown et al., in prep.). Other champions of integrated EF and mathematics intervention programmes (e.g. Day-Hess & Clements, 2017; EF + Math, 2023) describe their proposed theory of change and testable hypotheses.

Conclusions

Multiple existing reviews and empirical evidence present a striking paradox: on the one hand, correlational and longitudinal evidence point to robust interrelations between early EF and mathematics; on the other hand, EF focused interventions have failed to consistently improve mathematics. We have contributed to this growing literature by identifying reasons why isolated EF interventions have failed to transfer to improvements in early mathematics. We have also highlighted key proposed agents of change for integrated EF and mathematics interventions, both in theory and with a practical example of an integrated EF and mathematics early years intervention. First, we have suggested the careful consideration of how and why to embed EF challenge into mathematics learning. Second, we have highlighted the need to train early years practitioners in their potential role as scaffolding EF challenge in mathematics. Finally, we have advocated the development of child–practitioner activities that embed (rather than remove) executive challenge in well selected mathematical content.

Acknowledgements A Project Grant by the Nuffield Foundation (to Scerif [PI], Hawes, Howard, and Merkley, "Fostering resilience by injecting executive challenge into early mathematics") supported discussions around and evaluation of theories of change for integrated executive functions and mathematics interventions. We are very grateful to all Advisory Board members for the Project Grant, for their intellectual contributions to these broader discussions: Jennie Challender, Aleisha Clarke, Keely Cook, Amy Godfrey, Ruth Maisey, Gill Mason, Joanne Mason, Edward Melhuish, Fionnuala O'Reilly, Jelena Sugevic, Kathy Sylva, and Ellen Wright.

References

Abrahamse, E., Braem, S., Notebaert, W., & Verguts, T. (2016). Grounding cognitive control in associative learning. *Psychological Bulletin, 142*(7), 693–728. https://doi.org/10.1037/bul0000047

Amso, D., & Scerif, G. (2015). The attentive brain: insights from developmental cognitive neuroscience. *Nature Reviews. Neuroscience, 16*(10), 606–619. https://doi.org/10.1038/nrn4025

Ansari, D., König, J., Leask, M., & Tokuhama-Espinosa, T. (2017). Developmental cognitive neuroscience: Implications for teachers' pedagogical knowledge. In S. Guerriero (Ed.), *Pedagogical Knowledge and the Changing Nature of the Teaching Profession* (pp. 195–222). OECD Publishing.

Aston-Jones, G., & Cohen, J. D. (2005). Adaptive gain and the role of the locus coeruleus-norepinephrine system in optimal performance. *The Journal of Comparative Neurology, 493*(1), 99–110. https://doi.org/10.1002/cne.20723

Bailey, D. H., Duncan, G. J., Watts, T., Clements, D. H., & Sarama, J. (2018). Risky business: Correlation and causation in longitudinal studies of skill development. *American Psychologist, 73*(1), 81.

Bardack, S., & Obradović, J. (2019). Observing teachers' displays and scaffolding of executive functioning in the classroom context. *Journal of Applied Developmental Psychology, 62*, 205–219.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*, 133–180.

Bellon, E., Fias, W., Ansari, D., & De Smedt, B. (2020). The neural basis of metacognitive monitoring during arithmetic in the developing brain. *Human Brain Mapping, 41*(16), 4562–4573. https://doi.org/10.1002/hbm.25142

Bierman, K. L., Nix, R. L., Greenberg, M. T., Blair, C., & Domitrovich, C. E. (2008). Executive functions and school readiness intervention: Impact, moderation, and mediation in the Head Start REDI program. *Development and Psychopathology, 20*(3), 821–843. https://doi.org/10.1017/S0954579408000394

Blair, C., & Raver, C. C. (2014). Closing the achievement gap through modification of neurocognitive and neuroendocrine function: Results from a cluster randomized controlled trial of an innovative approach to the education of children in kindergarten. *PLoS One, 9*(11), e112393.

Blair, K. P., Rosenberg-Lee, M., Tsang, J. M., Schwartz, D. L., & Menon, V. (2012). Beyond natural numbers: Negative number representation in parietal cortex. *Frontiers in Human Neuroscience, 6*, 7. https://doi.org/10.3389/fnhum.2012.00007

Blakey, E., Matthews, D., Cragg, L., Buck, J., Cameron, D., Higgins, B., Pepper, L., Ridley, E., Sullivan, E., & Carroll, D. J. (2020). The role of executive functions in socioeconomic attainment gaps: Results from a randomized controlled trial. *Child Development, 91*(5), 1594–1614. https://doi.org/10.1111/cdev.13358

Braem, S., & Egner, T. (2018). Getting a grip on cognitive flexibility. *Current Directions in Psychological Science, 27*(6), 470–476. https://doi.org/10.1177/0963721418787475

Brod, G., Bunge, S. A., & Shing, Y. L. (2017). Does one year of schooling improve children's cognitive control and alter associated brain activation? *Psychological Science, 28*(7), 967–978. https://doi.org/10.1177/0956797617699838

Brown, E. R., Groom, M., & Anstell, S. (in preparation). Embedding Executive Challenge into Early Maths: A two-arm cluster randomised controlled trial. Education Endowment Foundation Evaluation Protocol.

Byrnes, J. P., & Miller-Cotto, D. (2016). The growth of mathematics and reading skills in segregated and diverse schools: An opportunity-propensity analysis of a national database. *Contemporary Educational Psychology, 46*, 34–51.

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: longitudinal predictors of mathematical achievement at age 7 years. *Developmental Neuropsychology, 33*(3), 205–228. https://doi.org/10.1080/87565640801982312

Byrnes, J. P., & Wasik, B. A. (2009). Factors predictive of mathematics achievement in kindergarten, first and third grades: An opportunity–propensity analysis. *Contemporary Educational Psychology, 34*(2), 167–183.

Byrnes, J. P., Wang, A., & Miller-Cotto, D. (2019). Children as mediators of their own cognitive development in kindergarten. *Cognitive Development, 50*, 80–97.

Cameron, C. E., Kim, H., Duncan, R. J., Becker, D. R., & McClelland, M. M. (2019). Bidirectional and co-developing associations of cognitive, mathematics, and literacy skills during kindergarten. *Journal of Applied Developmental Psychology, 62*, 135–144.

Chan, J. Y.-C., & Scalise, N. R. (2022). Numeracy skills mediate the relation between executive function and mathematics achievement in early childhood. *Cognitive Development*. https://doi.org/10.1016/j.cogdev.2022.101154

Clark, C. A., Sheffield, T. D., Wiebe, S. A., & Espy, K. A. (2013). Longitudinal associations between executive control and developing mathematical competence in preschool boys and girls. *Child Development, 84*(2), 662–677. https://doi.org/10.1111/j.1467-8624.2012.01854.x

Clements, D. H., Sarama, J., & Germeroth, C. (2016). Learning executive function and early mathematics: Directions of causal relations. *Early Childhood Research Quarterly, 36*, 79–90.

Clements, D. H., Sarama, J., Layzer, C., Unlu, F., & Fesler, L. (2020). Effects on mathematics and executive function of a mathematics and play intervention versus mathematics alone. *Journal for Research in Mathematics Education, 51*(3), 301–333.

Constantinidis, C., & Klingberg, T. (2016). The neuroscience of working memory capacity and training. *Nature Reviews. Neuroscience, 17*(7), 438–449. https://doi.org/10.1038/nrn.2016.43

Coolen, I., Merkley, R., Ansari, D., Dove, E., Dowker, A., Mills, A., Murphy, V., von Spreckelsen, M., & Scerif, G. (2021). Domain-general and domain-specific influences on emerging numerical cognition: Contrasting uni-and bidirectional prediction models. *Cognition, 215*, 104816. https://doi.org/10.1016/j.cognition.2021.104816

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education, 3*(2), 63–68.

Cragg, L., Keeble, S., Richardson, S., Roome, H. E., & Gilmore, C. (2017). Direct and indirect influences of executive functions on mathematics achievement. *Cognition, 162*, 12–26.

Curran, P. J., & Hancock, G. R. (2021). The challenge of modeling co-developmental processes over time. *Child Development Perspectives, 15*(2), 67–75.

Day-Hess, C., & Clements, D. H. (2017). The DREME network: Research and interventions in early childhood mathematics. *Advances in Child Development and Behavior, 53*, 1–41. https://doi.org/10.1016/bs.acdb.2017.03.002

de Abreu, P. M. E., Conway, A. R., & Gathercole, S. E. (2010). Working memory and fluid intelligence in young children. *Intelligence, 38*(6), 552–561.

Department of Education. (2021). Develoment Matters Guidance, United Kingdom Department for Education, https://www.gov.uk/government/publications/development-matters--2/development-matters

Diamond, A., Barnett, W. S., Thomas, J., & Munro, S. (2007). Preschool program improves cognitive control. Science (New York, N.Y.), 318(5855), 1387–1388. https://doi.org/10.1126/science.1151148

Diamond, A. & Ling, DS. (2019). Review of the evidence on, and fundamental questions about, efforts to improve executive functions, including working memory. *Cognitive and working memory training: Perspectives from psychology, neuroscience, and human development.*

Dong, Y., Clements, D. H., Sarama, J., Dumas, D., Banse, H. W., & Day-Hess, C. (2022). Mathematics and executive function competencies in the context of interventions: A quantile regression analysis. *The Journal of Experimental Education, 90*(2), 297–318.

Duncan, J. (2013). The structure of cognition: Attentional episodes in mind and brain. *Neuron, 80*(1), 35–50. https://doi.org/10.1016/j.neuron.2013.09.015

EF + Math. (n.d.). Executive functions, mathematics, and equity: A primer. EF+Math Program. Retrieved from: https://static1.squarespace.com/static/5d0922cc7057e60001fb066c/t/5dd2f002db60447f5cf40581/1574105092451/EF%2BMath_Primer.pdf. Accessed 22 Oct 2023

EF + Math. (2023). Strengthening executive function skills to improve mathematics learning. Advanced Education and Development Research Fund. Retrieved from: https://aerdf.dspacedirect.org/items/0af77bf3-2a6f-415a-a1f7-c5c2c053ba21. Accessed 22 Oct 2023

Ellis, A., Ahmed, S. F., Zeytinoglu, S., Isbell, E., Calkins, S. D., Leerkes, E. M., ... & Davis-Kean, P. E. (2021). Reciprocal associations between executive function and academic achievement: A conceptual replication of Schmitt et al. *Journal of Numerical Cognition, 7*(3), 453–472.

Engelhardt, L. E., Harden, K. P., Tucker-Drob, E. M., & Church, J. A. (2019). The neural architecture of executive functions is established by middle childhood. *NeuroImage, 185*, 479–489.

Engle, R. W., Tuholski, S. W., Laughlin, J. E., & Conway, A. R. A. (1999). Working memory, short-term memory, and general fluid intelligence: a latent-variable approach. *Journal of Experimental Psychology. General, 128*(3), 309–331. https://doi.org/10.1037//0096-3445.128.3.309

Finch, J. E. (2019). Do schools promote executive functions? Differential working memory growth across school-year and summer months. *AERA Open, 5*(2), 2332858419848443.

Fiske, A., & Holmboe, K. (2019). Neural substrates of early executive function development. *Developmental Review : DR, 52*, 42–62. https://doi.org/10.1016/j.dr.2019.100866

Fuhs, M. W., Hornburg, C. B., & McNeil, N. M. (2016). Specific early number skills mediate the association between executive functioning skills and mathematics achievement. *Developmental Psychology, 52*(8), 1217–1235. https://doi.org/10.1037/dev0000145

Fuhs, M. W., Nesbitt, K. T., Farran, D. C., & Dong, N. (2014). Longitudinal associations between executive functioning and academic skills across content areas. *Developmental Psychology, 50*(6), 1698–1709. https://doi.org/10.1037/a0036633

Gathercole, S., & Alloway, T. P. (2008). *Working memory and learning: A practical guide for teachers*. Sage.

Gilmore, C. (2023). Understanding the complexities of mathematical cognition: A multi-level framework. *Quarterly Journal of Experimental Psychology, 76*(9), 1953–1972. https://doi.org/10.1177/17470218231175325

Gilmore, C., & Cragg, L. (2014). Teachers' understanding of the role of executive functions in mathematics learning. *Mind, Brain and Education: The Official Journal of the International Mind, Brain, and Education Society, 8*(3), 132–136. https://doi.org/10.1111/mbe.12050

Ginsburg, H. P., Lee, J. S., & Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. Social Policy Report. *Society for Research in Child Development, 22*(1), 1–24.

Gunzenhauser, C., & Nückles, M. (2021). Training Executive Functions to Improve Academic Achievement: Tackling Avenues to Far Transfer. *Frontiers in Psychology, 12*, 624008. https://doi.org/10.3389/fpsyg.2021.624008

Halford, G. S., Cowan, N., & Andrews, G. (2007). Separating cognitive capacity from knowledge: A new hypothesis. *Trends in Cognitive Sciences, 11*(6), 236–242.

Hawes, Z., Merkley, R., Stager, C. L., & Ansari, D. (2021). Integrating numerical cognition research and mathematics education to strengthen the teaching and learning of early number. *The British Journal of Educational Psychology, 91*(4), 1073–1109. https://doi.org/10.1111/bjep.12421

Hawes, Z., Sokolowski, H. M., Ononye, C. B., & Ansari, D. (2019). Neural underpinnings of numerical and spatial cognition: An fMRI meta-analysis of brain regions associated with symbolic number, arithmetic, and mental rotation. *Neuroscience and Biobehavioral Reviews, 103*, 316–336. https://doi.org/10.1016/j.neubiorev.2019.05.007

Heitz, R., Redick, T., Hambrick, D., Kane, M., Conway, A., & Engle, R. (2006). Working memory, executive function, and general fluid intelligence are not the same. *Behavioral and Brain Sciences, 29*(2), 135–136. https://doi.org/10.1017/S0140525X06319036

Hendry, A., Johnson, M. H., & Holmboe, K. (2019). Early development of visual attention: Change, stability, and longitudinal associations. *Annual Review of Developmental Psychology, 1*, 251–275.

Henrich, J., Heine, S. J., & Norenzayan, A. (2010). The weirdest people in the world? *Behavioral and Brain Sciences, 33*(2–3), 61–83.

Hill, H. C., & Erickson, A. (2019). Using implementation fidelity to aid in interpreting program impacts: A brief review. *Educational Researcher, 48*(9), 590–598.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*, 371–406.

Hodgen, J., Barclay, N., Foster, C., Gilmore, C., Marks, R., & Simms, V. (2020). Early years and key stage 1 mathematics teaching. *Evidence Review. Education Endowment Foundation.*

Holloway, I. D., & Ansari, D. (2010). Developmental specialization in the right intraparietal sulcus for the abstract representation of numerical magnitude. *Journal of Cognitive Neuroscience, 22*(11), 2627–2637. https://doi.org/10.1162/jocn.2009.21399

Houdé, O., Rossi, S., Lubin, A., & Joliot, M. (2010). Mapping numerical processing, reading, and executive functions in the developing brain: An fMRI meta-analysis of 52 studies including 842 children. *Developmental Science, 13*(6), 876–885.

Howard, S. J., Vasseleu, E., Neilsen-Hewett, C., & Cliff, K. (2018). Evaluation of the Preschool Situational Self-Regulation Toolkit (PRSIST) Program for Supporting children's early self-regulation development: Study protocol for a cluster randomized controlled trial. *Trials, 19*(1), 64. https://doi.org/10.1186/s13063-018-2455-4

Howard, S. J., Vasseleu, E., Batterham, M., & Neilsen-Hewett, C. (2020a). Everyday practices and activities to improve pre-school self-regulation: Cluster RCT evaluation of the PRSIST program. *Frontiers in Psychology, 11*, 137. https://doi.org/10.3389/fpsyg.2020.00137

Howard, S. J., Cook, C. J., Everts, L., Melhuish, E., Scerif, G., Norris, S., Twine, R., Kahn, K., & Draper, C. E. (2020b). Challenging socioeconomic status: A cross-cultural comparison of early executive function. *Developmental Science, 23*(1), e12854. https://doi.org/10.1111/desc.12854

Johnson, M. H. (2001). Functional brain development in humans. *Nature reviews. Neuroscience, 2*(7), 475–483. https://doi.org/10.1038/35081509

Johnson, M. H. (2011). Interactive specialization: a domain-general framework for human functional brain development? *Developmental Cognitive Neuroscience, 1*(1), 7–21. https://doi.org/10.1016/j.dcn.2010.07.003

Jolles, D., Wassermann, D., Chokhani, R., Richardson, J., Tenison, C., Bammer, R., Fuchs, L., Supekar, K., & Menon, V. (2016). Plasticity of left perisylvian white-matter tracts is associated with individual differences in math learning. *Brain Structure & Function, 221*(3), 1337–1351. https://doi.org/10.1007/s00429-014-0975-6

Joswick, C., Clements, D. H., Sarama, J., Banse, H. W., & Day-Hess, C. A. (2019). Double impact: Mathematics and executive function. *Teaching Children Mathematics, 25*(7), 416–426.

Karr, J. E., Areshenkoff, C. N., Rast, P., Hofer, S. M., Iverson, G. L., & Garcia-Barrera, M. A. (2018). The unity and diversity of executive functions: A systematic review and re-analysis of latent variable studies. *Psychological bulletin, 144*(11), 1147–1185. https://doi.org/10.1037/bul0000160

Kassai, R., Futo, J., Demetrovics, Z., & Takacs, Z. K. (2019). A meta-analysis of the experimental evidence on the near- and far-transfer effects among children's executive function skills. *Psychological Bulletin, 145*(2), 165–188. https://doi.org/10.1037/bul0000180

Kelcey, B., Hill, H. C., & Chin, M. J. (2019). Teacher mathematical knowledge, instructional quality, and student outcomes: A multilevel quantile mediation analysis. *School Effectiveness and School Improvement, 30*(4), 398–431.

Klingberg, T. (2010). Training and plasticity of working memory. *Trends in Cognitive Sciences, 14*(7), 317–324.

Kroesbergen, E. H., van't Noordende, J. E., & Kolkman, M. E. (2014). Training working memory in kindergarten children: Effects on working memory and early numeracy. *Child Neuropsychology, 20*(1), 23–37. https://doi.org/10.1080/09297049.2012.736483

Lawson, G. M., Hook, C. J., & Farah, M. J. (2018). A meta-analysis of the relationship between socioeconomic status and executive function performance among children. *Developmental Science, 21*(2), e12529.

Mazzocco, M. M. M., Chan, J. Y., & Bock, A. M. (2017). Early Executive Function and Mathematics Relations: Correlation Does Not Ensure Concordance. *Adv Child Dev Behav, 53*, 289–307. https://doi.org/10.1016/bs.acdb.2017.05.001

McKinnon, R. D., & Blair, C. (2019). Bidirectional relations among executive function, teacher–child relationships, and early reading and math achievement: A cross-lagged panel analysis. *Early Childhood Research Quarterly, 46*, 152–165.

Menon, V., Rivera, S. M., White, C. D., Glover, G. H., & Reiss, A. L. (2000). Dissociating prefrontal and parietal cortex activation during arithmetic processing. *NeuroImage, 12*(4), 357–365. https://doi.org/10.1006/nimg.2000.0613

Merkley, R., Matusz, P. J., & Scerif, G. (2018). The control of selective attention and emerging mathematical cognition: Beyond unidirectional influences. In *Heterogeneity of function in numerical cognition* (pp. 111–126). Academic Press.

Miller-Cotto, D., & Byrnes, J. P. (2020). What's the best way to characterize the relationship between working memory and achievement?: An initial examination of competing theories. *Journal of Educational Psychology, 112*(5), 1074.

Miller-Cotto, D., Smith, L. V., Wang, A. H., & Ribner, A. D. (2022). Changing the conversation: A culturally responsive perspective on executive functions, minoritized children and their families. *Infant and Child Development, 31*(1), e2286.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology, 41*(1), 49–100.

Mulcahy, C., Day Hess, C. A., Clements, D. H., Ernst, J. R., Pan, S. E., Mazzocco, M. M., & Sarama, J. (2021). Supporting young children's development of executive function through early mathematics. *Policy Insights from the Behavioral and Brain Sciences, 8*(2), 192–199.

McClelland, M. M., Tominey, S. L., Schmitt, S. A., Hatfield, B. E., Purpura, D. J., Gonzales, C. R., & Tracy, A. N. (2019). Red light, purple light! results of an intervention to promote school readiness for children from low-income backgrounds. *Frontiers in Psychology, 10*, 2365. https://doi.org/10.3389/fpsyg.2019.02365

Melby-Lervåg, M., & Hulme, C. (2013). Is working memory training effective? *A meta-analytic review. Developmental psychology, 49*(2), 270–291. https://doi.org/10.1037/a0028228

Melby-Lervåg, M., Redick, T. S., & Hulme, C. (2016). Working memory training does not improve performance on measures of intelligence or other measures of "far transfer": evidence from a meta-analytic review. *Perspectives on Psychological Science : a journal of the Association for Psychological Science, 11*(4), 512–534. https://doi.org/10.1177/1745691616635612

Miyake, A., & Friedman, N. P. (2012). The nature and organization of individual differences in executive functions: four general conclusions. *Current Directions in Psychological Science, 21*(1), 8–14. https://doi.org/10.1177/0963721411429458

Mulder, H., Verhagen, J., Van der Ven, S. H., Slot, P. L., & Leseman, P. P. (2017). Early executive function at age two predicts emergent mathematics and literacy at age five. *Frontiers in Psychology, 8*, 1706.

Nesbitt, K. T., & Farran, D. C. (2021). Effects of prekindergarten curricula: Tools of the mind as a case study. *Monographs of the Society for Research in Child Development, 86*(1), 7–119. https://doi.org/10.1111/mono.12425

Nguyen, T., Duncan, R. J., & Bailey, D. H. (2019). Theoretical and Methodological Implications of Associations between Executive Function and Mathematics in Early Childhood. *Contemporary Educational Psychology, 58*, 276–287. https://doi.org/10.1016/j.cedpsych.2019.04.002

Niebaum, J., & Munakata, Y. (2020). Deciding what to do: Developments in children's spontaneous monitoring of cognitive demands. *Child Development Perspectives, 14*(4), 202–207. https://doi.org/10.1111/cdep.12383

Peng, P., & Swanson, H. L. (2022). The domain-specific approach of working memory training. *Developmental Review, 65*, 101035.

Podolsky, A., Kini, T., & Darling-Hammond, L. (2019). Does teaching experience increase teacher effectiveness? A review of US research. *Journal of Professional Capital and Community, 4*(4), 286–308.

Prager, E. O., Ernst, J. R., Mazzocco, M. M. M., & Carlson, S. M. (2023). Executive function and mathematics in preschool children: Training and transfer effects. *Journal of Experimental Child Psychology, 232*, 105663. https://doi.org/10.1016/j.jecp.2023.105663

Purpura, D. J., Schmitt, S. A., & Ganley, C. M. (2017). Foundations of mathematics and literacy: The role of executive functioning components. *Journal of Experimental Child Psychology, 153*, 15–34.

Ribner, A. D. (2020). Executive function facilitates learning from math instruction in kindergarten: Evidence from the ECLS-K. *Learning and Instruction, 65*, 101251.

Ribner, A., Moeller, K., Willougby, M., & Blair, C. (2018). Cognitive abilities and mathematical competencies at school entry. *Mind, Brain and Education : the official journal of the International Mind, Brain, and Education Society, 12*(4), 175–185. https://doi.org/10.1111/mbe.12160

Ribner, A. D., Willoughby, M. T., Blair, C. B., & Family Life Project Key Investigators. (2017). executive function buffers the association between early math and later academic skills. *Frontiers in Psychology, 8*, 869. https://doi.org/10.3389/fpsyg.2017.00869

Sala, G., & Gobet, F. (2017). Does far transfer exist? Negative evidence from chess, music, and working memory training. *Current Directions in Psychological Science, 26*(6), 515–520. https://doi.org/10.1177/0963721417712760

Scalise, N. R., Daubert, E. N., & Ramani, G. B. (2020). Benefits of playing numerical card games on head start children's mathematical skills. *Journal of Experimental Education, 88*(2), 200–220. https://doi.org/10.1080/00220973.2019.1581721

Scerif, G., Gattas, S., Hawes, Z., Howard, S., Merkley, R., & O'Connor, R. (2023). Orchestrating numeracy and the executive: The One Programme. https://doi.org/10.31234/osf.io/2gxzv

Schmitt, S. A., Geldhof, G. J., Purpura, D. J., Duncan, R., & McClelland, M. M. (2017). Examining the relations between executive function, math, and literacy during the transition to kindergarten: A multi-analytic approach. *Journal of Educational Psychology, 109*(8), 1120.

Solomon, T., Plamondon, A., O'Hara, A., Finch, H., Goco, G., Chaban, P., Huggins, L., Ferguson, B., & Tannock, R. (2018). A cluster randomized-controlled trial of the impact of the tools of the mind curriculum on self-regulation in Canadian preschoolers. *Frontiers in Psychology, 8*, 2366. https://doi.org/10.3389/fpsyg.2017.02366

Takacs, Z. K., & Kassai, R. (2019). The efficacy of different interventions to foster children's executive function skills: A series of meta-analyses. *Psychological Bulletin, 145*(7), 653.

Thorell, L. B., Lindqvist, S., Bergman Nutley, S., Bohlin, G., & Klingberg, T. (2009). Training and transfer effects of executive functions in preschool children. *Developmental Science, 12*(1), 106–113. https://doi.org/10.1111/j.1467-7687.2008.00745.x

Unsworth, N., Redick, T. S., Heitz, R. P., Broadway, J. M., & Engle, R. W. (2009). Complex working memory span tasks and higher-order cognition: A latent-variable analysis of the relationship between processing and storage. *Memory, 17*(6), 635–654.

von Spreckelsen, M., Dove, E., Coolen, I., Mills, A., Dowker, A., Sylva, K., ... & Scerif, G. (2019). Let's talk about mathematics: The role of observed "mathematics-talk" and mathematics provisions in preschoolers' numeracy. *Mind, Brain, and Education*, *13*(4), 326–340.

Voss, T., Kunter, M., & Baumert, J. (2011). Assessing teacher candidates' general pedagogical/ psychological knowledge: Test construction and validation. *Journal of Educational Psychology, 103*, 952–969.

Welsh, J. A., Nix, R. L., Blair, C., Bierman, K. L., & Nelson, K. E. (2010). The development of cognitive skills and gains in academic school readiness for children from low-income families. *Journal of Educational Psychology, 102*(1), 43–53. https://doi.org/10.1037/a0016738

Wiebe, S. A., Sheffield, T., Nelson, J. M., Clark, C. A., Chevalier, N., & Espy, K. A. (2011). The structure of executive function in 3-year-olds. *Journal of Experimental Child Psychology, 108*(3), 436–452. https://doi.org/10.1016/j.jecp.2010.08.008

Wilkey, E. D., & Price, G. R. (2019). Attention to number: The convergence of numerical magnitude processing, attention, and mathematics in the inferior frontal gyrus. *Human Brain Mapping, 40*(3), 928–943. https://doi.org/10.1002/hbm.24422

Willingham, D. T. (2017). A mental model of the learner: Teaching the basic science of educational psychology to future teachers. *Mind, Brain, and Education, 11*(4), 166–175.

Zhang, H., Miller-Cotto, D., & Jordan, N. C. (2023). Estimating the co-development of executive functions and math achievement throughout the elementary grades using a cross-lagged panel model with fixed effects. *Contemporary Educational Psychology, 72*, 102126.

Authors and Affiliations

Gaia Scerif[1] · Emma Blakey[2] · Sylvia Gattas[1] · Zachary Hawes[3] · Steven Howard[4] · Rebecca Merkley[5] · Rosemary O'Connor[1] · Victoria Simms[6]

✉ Gaia Scerif
gaia.scerif@psy.ox.ac.uk

[1] Attention, Brain and Cognitive Development Group Department of Experimental Psychology, University of Oxford, Oxford, UK

[2] Department of Psychology, University of Sheffield, Sheffield, UK

[3] Department of Applied Psychology and Human Development, University of Toronto, Toronto, Canada

[4] School of Education, University of Wollongong, Wollongong, Australia

[5] Department of Cognitive Science, Carleton University, Ottawa, Canada

[6] School of Psychology, Ulster University, Coleraine, UK