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# Can omitted carbon abatement explain productivity stagnation? Quantile shadow-price Fisher index applied to OECD countries

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**Abstract**

Explaining the secular stagnation of productivity growth is a widely recognized challenge to economists and policymakers. One potentially important explanation without much attention concerns the ongoing low-carbon transition. This paper explores whether considering greenhouse gas emissions can explain productivity stagnation in OECD countries. We propose a quantile shadow-price Fisher index to gauge green total factor productivity (TFP) based on the newly developed penalized convex quantile regression approach. The quantile shadow-price Fisher index requires neither the real price data nor an *ad hoc* choice of quantiles and allows the quantiles to move in the inter-period sample. An empirical application to 38 OECD countries during 1990–2019 demonstrates that the measured productivity growth is considerably higher when the GHG emissions are accounted for. For countries that have reduced GHG emissions most actively, the average green TFP growth rate could double the conventional TFP growth. The impacts of ignoring human capital and different representations of fixed capital on green TFP growth are also discussed explicitly.

**Keywords**: Green TFP, Undesirable output, Shadow price, Fisher index, Quantile estimation

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# 1 Introduction

The secular stagnation of productivity growth has occurred in virtually all Western countries since the financial crisis that started in the US in 2008 and subsequently led to the European debt crises and the period known as the Great Recession (see, e.g., Syverson, 2017; Crafts, 2018). Several possible explanations for productivity stagnation have been suggested in economics.

Firstly, since technological progress is traditionally seen as the main driver of productivity growth, it seems natural that the recent stagnation may be due to the slowdown of innovations. Most notably, Gordon (2012) and Bloom et al. (2020) have suggested that new ideas get harder to find over time. As previous innovations have already been utilized, it is increasingly more difficult to generate genuinely new innovations which would further boost productivity growth. Bloom et al. (2020) provide evidence that links declining innovation to productivity stagnation.

Secondly, since most countries have unemployment and underutilized productive capacity, aggregate productivity slowdown could also relate to inefficient allocation of resources in the economy. Empirical work in the US and in Europe suggests that business dynamism (e.g., firm entry, job creation, or job turnover) has been declining (see, e.g., Decker et al., 2016; Grossman et al., 2017), which can slow down productivity growth. Further, De Loecker et al. (2020) suggest that there have been rising markups, which further suggest market power of firms has been increasing. Such increasing market power is connected to productivity slowdown. Both declining business dynamism and rising markups can contribute to the misallocation of resources. Previous misallocation studies (e.g., Hsieh and Klenow, 2009; Restuccia and Rogerson, 2017) focused on comparing developing countries with the US, but during the current productivity stagnation it has been suggested that misallocation of resources might have something to do with the declining productivity and it might be related to the previous two explanations. For example, increasing market power and markups can lead to inefficient allocation of resources.

The third type of explanation refers to measurement challenges in total factor productivity (TFP). For example, the digital economy provides new goods such as information and entertainment services free of charge (e.g., Brynjolfsson et al., 2021). Since free digital services and improved quality of services are not included in the conventional national accounts, measured productivity growth can be downward biased due to mismeasurement.

One potentially important explanation for productivity stagnation, which relates to the broader theme of mismeasurement but has thus far attracted little attention, relates to the ongoing transition to mitigate climate change. Specifically, reducing greenhouse gas (GHG) emissions requires massive capital investments, which are included in the measured capital stock (or capital services). However, such capital investments do not contribute to the measured GDP. Since the conventional TFP measures ignore the social benefits of GHG abatement, the measured TFP can slow down when the inputs of the GHG abatement are included, but the outputs are excluded.

The purpose of this paper is to explore whether considering GHG emissions can explain productivity stagnation in OECD countries. Our first contribution is to empirically investigate the impacts of GHG emissions, fixed capital, and human capital on productivity growth. We measure productivity growth with and without GHG emissions, compare green TFP growth based on either capital stocks or capital services, and calculate green TFP growth with and without human capital. The results confirm that the measured productivity growth is considerably higher when the GHG emissions are accounted for. For countries that have reduced GHG emissions most actively, the average green TFP growth rate could double the conventional TFP growth. Further, the choice of fixed capital and human capital would also have nonnegligible impacts on green TFP growth.

To achieve our main purpose, the second contribution of this paper is to construct a novel quantile shadow-price Fisher index to gauge green TFP growth. This index is more relevant and useful in the presence of undesirable outputs. The proposed quantile shadow-price Fisher index does not require the real price data for input-output variables and can avoid an *ad hoc* choice of quantiles which may lead to different productivity estimates and allow quantiles to move in the inter-period sample.

To operationalize the proposed index, the third contribution of this paper is to develop a penalized convex quantile regression (CQR) approach to estimate shadow prices. In doing so, we regularize the CQR approach by adding an extra regularization term on subgradients to increase the convexity of the objective function. Compared to the conventional full frontier approach, penalized CQR can guarantee the uniqueness of estimated shadow prices and take inefficiency into account explicitly. Furthermore, the proposed approach is more robust to outliers and heterogeneity by inheriting the appealing features from quantile regression.

The rest of the paper is organized as follows. The next section critically reviews the existing literature on TFP measurement. Section 3 introduces the Fisher ideal TFP index and the shadow-price Fisher index and proposes the quantile shadow-price Fisher index. The newly developed penalized CQR approach is presented in Section 4. Section 5 describes the data and variables and discusses the impact of GHG emissions on productivity estimates. Section 6 further discusses green productivity estimates with alternative capital and labor specifications. Section 7 concludes this paper with future research avenues. Formal proof and additional figures and tables are provided in Appendix A–B.

# 2 Literature review

TFP growth is a key engine for sustainable economic growth. A proper TFP measurement is critically important to economists and policymakers in terms of monitoring the quality of economic growth and devising economic policies. Since the pioneering work of Solow (1957), a vast body of literature on theoretical and empirical studies of TFP has emerged (see Van Beveren, 2012 for a detailed review). However, the conventional TFP has been criticized for ignoring the contribution of natural capital in providing different ecosystem services. With the exhaustion of natural resources and the deterioration of the ecological environment worldwide, the conventional TFP may not provide precise policy decisions on sustainable economic development. Extensions to the conventional TFP such as green TFP (see, e.g., Kuosmanen, 2013; Shen et al., 2017) or environmental TFP (see, e.g., Hoang and Coelli, 2011; Wang et al., 2018) are recently proposed to integrate eco-environmental factors into the conventional TFP framework.

Measuring (green) TFP at either micro- or macro-levels requires aggregation of various inputs and outputs in one way or another such that TFP is measured as the ratio of aggregate output to aggregate input. In practice, however, the aggregation of multiple inputs or outputs poses a major challenge. In general, there exist two competing approaches in the literature on productivity analysis: the axiomatic (or test) approach and the economic (or exact index number) approach (Diewert and Nakamura, 2003).[1]

[1] In the (semi-)parametric stream of literature, other approaches such as the generalized method of mo-

The axiomatic index number approach (e.g., the Fisher index and the Törnquist index) postulates a number of axiomatic properties that any meaningful index number should satisfy and then tries to construct an ideal index number that meets all the properties. The appeal of the axiomatic approach lies in the absence of estimation requirements and assumptions regarding optimizing behavior. It thus has been widely used for, e.g., the national-level productivity measure (Feenstra et al., 2015). However, the axiomatic approach requires precise information on inputs and output prices, which may not always be available in real-world applications, especially when dealing with non-market goods and services such as GHG emissions. Even if the market prices of undesirable outputs are available in some rare cases, they do not reflect the true social cost of undesirable outputs. This is the main motivation to resort to shadow pricing.

By contrast, the economic approach (e.g., the Malmquist index) relies on economic theories and behavioral assumptions. For example, when applying the Malmquist index (Caves et al., 1982; Nishimizu and Page, 1982; Färe et al., 1994), it is necessary to make assumptions on the benchmark technology to estimate production, cost, or (directional) distance functions. While the Malmquist index solely demands quantity data of inputs and outputs, a longstanding trade-off exists regarding the benchmark technology specification between constant returns to scale (CRS) and variable returns to scale (VRS). More specifically, the technical change component of the CRS Malmquist index may not accurately identify the true technical changes, while the use of VRS benchmark technology may result in an infeasible solution. Therefore, the question of which specification should be selected remains open. Moreover, the Malmquist index exhibits other noteworthy limitations, including inconsistent measures of inter-period observations and being a non-circular index (Wang et al., 2018).

When estimating green TFP in practice, the Malmquist index and its extensions (e.g., the Malmquist-Luenberger index) are the most frequently employed techniques due to the absence or unavailability of price data for environmental factors. The calculation of the Malmquist index relies on the estimation of production, cost, or (directional) distance functions, which are typically derived through data envelopment analysis (DEA) or stochastic frontier analysis (SFA) (see, e.g., Hoang and Coelli, 2011; Shen et al., 2017; Wang et al.,

ments and semi-parametric estimation can also be used to estimate productivity growth (Van Biesebroeck, 2007).

2018; Odeck and Schøyen, 2020). However, DEA ignores any stochastic noise in the data, whereas SFA requires an *ex-ante* specification of the functional form (e.g., translog function) (Kuosmanen and Johnson, 2010). Recently, stochastic nonparametric envelopment of data (StoNED) has been developed as a unified framework that integrates the strengths of both DEA and SFA, effectively bridging the gap between these two approaches (Kuosmanen and Kortelainen, 2012). StoNED has been used to estimate the Malmquist index (e.g., Kuosmanen, 2013; Zhou, 2018). However, these full frontier approaches ignore the impact of inefficiency and are sensitive to outliers and the choice of the direction vector (Kuosmanen and Zhou, 2021), which may yield an inaccurate Malmquist green TFP estimate.

To leverage the information of inefficient observations and avoid the need for an arbitrary choice of the direction vector, partial frontier estimation is an emerging and promising approach. Notably, order-$\alpha$ (see, e.g., Aragon et al., 2005; Daouia and Simar, 2007), a non-convex partial frontier estimator, has been applied to estimate the Malmquist index in the banking sector (see, e.g., Wheelock and Wilson, 2009; Wheelock and Wilson, 2013), with the constructed index relying on an *a priori* quantile. In practice, however, observations can shift from one quantile to another over multi-period samples due to efficiency changes. Therefore, the quantile Malmquist index with a fixed quantile may not effectively utilize all available information.

The quantile shadow-price Fisher index is an alternative to measure productivity change. When the condition of allocative efficiency holds, one can recover economic prices from the quantity data. That is, the shadow prices can exactly represent the economic prices of inputs and outputs (Balk, 1993, 1998, Kuosmanen et al., 2004). However, the conventional frontier-based approaches, including the partial frontier approaches, can not guarantee the uniqueness of shadow prices. The shadow prices estimated by the quantile-based approaches, i.e., CQR (Wang et al., 2014) and convex expectile regression (CER) (Kuosmanen and Zhou, 2021), also not necessarily be unique. While Kuosmanen et al. (2004) propose an interval Fisher index for the cases with non-unique shadow prices, a better strategy would be to derive the quantile Fisher index directly using the unique shadow prices.

In the present paper, we propose a new quantile shadow-price Fisher index to perform green growth accounting, which is applied to OECD counties from 1990 to 2019. The unique shadow prices are estimated by developing a penalized CQR approach.

# 3 Quantile shadow-price Fisher index

This section starts by reviewing the Fisher ideal TFP index, the Malmquist TFP index, and the shadow-price TFP index. Subsequently, a new quantile shadow-price Fisher index is proposed. Suppose there are $I$ observations, indexed by $i = 1, \ldots, I$. For each observation $i$, $\boldsymbol{y} = (y_{i1}, \ldots, y_{in})' \in \mathbb{R}^n_{++}$, $\boldsymbol{b} = (b_{i1}, \ldots, b_{ij})' \in \mathbb{R}^j_{++}$, and $\boldsymbol{x} = (x_{i1}, \ldots, x_{im})' \in \mathbb{R}^m_{++}$ denote the desirable output, undesirable output, and input quantity vectors, respectively; $\boldsymbol{p} = (p_{i1}, \ldots, p_{in})'$ and $\boldsymbol{w} = (w_{i1}, \ldots, w_{im})'$ are the associated desirable output and input price vectors, respectively. For the sake of inter-period comparison, notations 0 and 1 indicate the base period and target period, respectively.

A TFP index is typically defined as the ratio of the output quantity index and input quantity index. Accordingly, the Fisher ideal TFP index can be stated as

$$F(\boldsymbol{p}^{0,1}, \boldsymbol{w}^{0,1}, \boldsymbol{y}^{0,1}, \boldsymbol{x}^{0,1}) \equiv \frac{F_o(\boldsymbol{p}^{0,1}, \boldsymbol{y}^{0,1})}{F_i(\boldsymbol{w}^{0,1}, \boldsymbol{x}^{0,1})} \tag{1}$$

$$F_o(\boldsymbol{p}^{0,1}, \boldsymbol{y}^{0,1}) \equiv \left[\frac{\boldsymbol{p}^0\boldsymbol{y}^1}{\boldsymbol{p}^0\boldsymbol{y}^0} \times \frac{\boldsymbol{p}^1\boldsymbol{y}^1}{\boldsymbol{p}^1\boldsymbol{y}^0}\right]^{1/2} \tag{2}$$

$$F_i(\boldsymbol{w}^{0,1}, \boldsymbol{x}^{0,1}) \equiv \left[\frac{\boldsymbol{w}^0\boldsymbol{x}^1}{\boldsymbol{w}^0\boldsymbol{x}^0} \times \frac{\boldsymbol{w}^1\boldsymbol{x}^1}{\boldsymbol{w}^1\boldsymbol{x}^0}\right]^{1/2} \tag{3}$$

where $F_o(\boldsymbol{p}^{0,1}, \boldsymbol{y}^{0,1})$ and $F_i(\boldsymbol{w}^{0,1}, \boldsymbol{x}^{0,1})$ are the Fisher ideal output and input quantity indices, respectively.

The Fisher ideal TFP index nowadays remains widely used because it requires neither estimation nor assumption on optimizing behavior, particularly at the macro-level (e.g., countries, regions). In practice, however, the Fisher ideal TFP index requires market prices of all inputs and outputs (i.e., $\boldsymbol{p}$ and $\boldsymbol{w}$) that are not always reliable or available. For example, if a market faces imperfect competition (e.g., natural monopolies) or government interventions (e.g., taxes, subsidies, tariffs), the prices of inputs or outputs would greatly deviate from their actual market prices. Notably, in the case of measuring green productivity (or environmental productivity), the non-market goods and services are modeled as inputs or outputs, but their market prices are notoriously hard to measure.

Alternatively, the Malmquist TFP index is also widely used to measure TFP growth. For instance, the standard input-oriented Malmquist TFP index is defined as

$$M(\boldsymbol{y}^{0,1}, \boldsymbol{x}^{0,1}) \equiv \left[\frac{D_i^0(\boldsymbol{y}^0, \boldsymbol{x}^0)}{D_i^0(\boldsymbol{y}^1, \boldsymbol{x}^1)} \times \frac{D_i^1(\boldsymbol{y}^0, \boldsymbol{x}^0)}{D_i^1(\boldsymbol{y}^1, \boldsymbol{x}^1)}\right]^{1/2} \tag{4}$$

where $D_i^t(\boldsymbol{y}, \boldsymbol{x}) = \sup\{\theta > 0 : (\boldsymbol{x}/\theta) \in L(\boldsymbol{y})\}$, where $L(\boldsymbol{y})$ is the input requirement set, denotes the input distance function characterizing the production technology of period $t$ ($t \in \{0, 1\}$).

Färe and Grosskopf (1992) demonstrate that if CRS, profit maximization, and allocative efficiency of inputs $\boldsymbol{x}$ and outputs $\boldsymbol{y}$ in both periods are assumed, the Malmquist TFP index (4) equals the Fisher ideal TFP index (1). However, Balk (1993) convincingly argues that these conditions are so strong that they are unlikely to be fulfilled in practice, and hence the equivalence relation between the Fisher and the Malmquist TFP indices cannot be held. Under slightly milder conditions, the Malmquist TFP index can reasonably approximate the Fisher ideal TFP index, and vice versa, even if the prices and technology change (Balk, 1993).

To minimize the assumptions on the economic behavior of production units and their production technology, Kuosmanen et al. (2004) propose an intermediate route between these two indices and introduce the following shadow-price Fisher TFP index.

$$F_s(\boldsymbol{\rho}^{0,1}, \boldsymbol{\omega}^{0,1}, \boldsymbol{y}^{0,1}, \boldsymbol{x}^{0,1}) \equiv \left[\frac{\boldsymbol{\rho}^0\boldsymbol{y}^1}{\boldsymbol{\rho}^0\boldsymbol{y}^0} \times \frac{\boldsymbol{\rho}^1\boldsymbol{y}^1}{\boldsymbol{\rho}^1\boldsymbol{y}^0}\right]^{1/2} \Big/ \left[\frac{\boldsymbol{\omega}^0\boldsymbol{x}^1}{\boldsymbol{\omega}^0\boldsymbol{x}^0} \times \frac{\boldsymbol{\omega}^1\boldsymbol{x}^1}{\boldsymbol{\omega}^1\boldsymbol{x}^0}\right]^{1/2} \tag{5}$$

where $\boldsymbol{\rho} = (p_{i1}, \ldots, p_{in})'$ and $\boldsymbol{\omega} = (w_{i1}, \ldots, w_{im})'$ are the desirable output and input shadow-price vectors. Formally, we have the following equivalence relation between the Fisher ideal TFP index and the shadow-price Fisher TFP index.

**Theorem 1.** *The shadow-price Fisher TFP index* (5) *and the Fisher ideal TFP index* (1) *are equivalent, if the shadow prices are unique and the allocative efficiency condition is held.*

*Proof.* See proof in Kuosmanen et al. (2004). ■

Compared to the Fisher ideal TFP index (1), the shadow-price Fisher TFP index (5) can be utilized to measure productivity growth in the absence of economically relevant prices or cost/revenue shares. However, several drawbacks are noted in practice. First, the present shadow-price Fisher TFP index can not be directly applied to measure green TFP growth when considering environmental bads. Second, the shadow prices $\boldsymbol{\rho}$ and $\boldsymbol{\omega}$ are generally non-unique in the conventional frontier estimation, leading to an inaccurate approximation from the shadow-price TFP index to the Fisher ideal TFP index (Balk, 1993; Kuosmanen et al., 2004). Third, while the shadow-price Fisher index is easy to compute whilst remaining consistent with the economic theory, it may be sensitive to random noise, heteroscedasticity, and outliers. This is because the estimated shadow prices merely rely on the conventional full frontier (e.g., the DEA frontier). Moreover, the information on inefficiency is usually neglected in shadow pricing environmental bads.

To mitigate the effects of these potential biases on green TFP measure, we extend the shadow-price Fisher index (5) to a more generalized setting, develop an approach to ensure the uniqueness of shadow prices estimates, and take the inefficiency explicitly into account. Specifically, we propose the following quantile shadow-price Fisher index.[2]

$$F_s^b(\tilde{\boldsymbol{\rho}}^{0,1}, \tilde{\boldsymbol{\delta}}^{0,1}, \tilde{\boldsymbol{\omega}}^{0,1}, \boldsymbol{y}^{0,1}, \boldsymbol{x}^{0,1}, \boldsymbol{b}^{0,1}) \equiv \left[\frac{\tilde{\boldsymbol{\rho}}^0\boldsymbol{y}^1 - \tilde{\boldsymbol{\delta}}^0\boldsymbol{b}^1}{\tilde{\boldsymbol{\rho}}^0\boldsymbol{y}^0 - \tilde{\boldsymbol{\delta}}^0\boldsymbol{b}^0} \times \frac{\tilde{\boldsymbol{\rho}}^1\boldsymbol{y}^1 - \tilde{\boldsymbol{\delta}}^1\boldsymbol{b}^1}{\tilde{\boldsymbol{\rho}}^1\boldsymbol{y}^0 - \tilde{\boldsymbol{\delta}}^1\boldsymbol{b}^0}\right]^{1/2} \Bigg/ \left[\frac{\tilde{\boldsymbol{\omega}}^0\boldsymbol{x}^1}{\tilde{\boldsymbol{\omega}}^0\boldsymbol{x}^0} \times \frac{\tilde{\boldsymbol{\omega}}^1\boldsymbol{x}^1}{\tilde{\boldsymbol{\omega}}^1\boldsymbol{x}^0}\right]^{1/2} \tag{6}$$

where $\tilde{\boldsymbol{\rho}}$, $\tilde{\boldsymbol{\delta}}$, and $\tilde{\boldsymbol{\omega}}$ are the quantile-based, locally estimated shadow prices for desirable outputs, undesirable outputs, and inputs, respectively. The value of $F_s^b$ above (below) unity reveals green TFP growth (decline). Note that when introducing the undesirable outputs in productivity measure, their effects are subtracted from the desirable outputs in the quantile shadow-price Fisher index (6), leading to a higher productivity estimate in comparison with a situation where undesirable outputs $\boldsymbol{b}$ are omitted. It is because even though the adjusted $\tilde{\boldsymbol{\rho}}^t\boldsymbol{y}^t - \tilde{\boldsymbol{\delta}}^t\boldsymbol{b}^t$ ($t \in \{0, 1\}$) is obviously smaller than $\tilde{\boldsymbol{\rho}}^t\boldsymbol{y}^t$, the change $(\tilde{\boldsymbol{\rho}}^t\boldsymbol{y}^t - \tilde{\boldsymbol{\delta}}^t\boldsymbol{b}^t)/(\tilde{\boldsymbol{\rho}}^t\boldsymbol{y}^t - \tilde{\boldsymbol{\delta}}^t\boldsymbol{b}^t)$ tends to higher than $\tilde{\boldsymbol{\rho}}^t\boldsymbol{y}^t/\tilde{\boldsymbol{\rho}}^t\boldsymbol{y}^t$ when the undesirable outputs $\boldsymbol{b}$ decrease over time, i.e., $\boldsymbol{b}^0 > \boldsymbol{b}^1$.

To calculate the quantile shadow-price Fisher index (6), we develop penalized CQR to obtain the robust and unique shadow prices at each quantile and then apply this local estimation strategy to derive $\tilde{\boldsymbol{\rho}}$, $\tilde{\boldsymbol{\delta}}$, and $\tilde{\boldsymbol{\omega}}$. The quantile shadow-price Fisher index (6) thus can enable shadow pricing environmental bads with the efficiency level of each observation accounted.

Note that when inputs $\boldsymbol{x}$ and outputs $\boldsymbol{y}$ are not allocated efficiently, the price-based Fisher index (e.g., the Fisher ideal TFP index) may not accurately reflect the true cost of producing the desired output level, as it does not consider the opportunity cost of allocating resources away from their best alternative use. This is even more relevant when environment bads $\boldsymbol{b}$ with negative externalities are introduced. In this respect, when $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{b}$ are not

[2] Our approach is effectively a hybrid of the index number and the economic approaches.

allocated efficiently (from the societal point of view), the shadow-price Fisher TFP index is more meaningful than the price-based Fisher ideal TFP index. Furthermore, if prices are distorted by market failures, the shadow prices seem more relevant.

# 4 Quantile function estimation

Consider the general multiple-input single-output production function model

$$y_i = f(\boldsymbol{x}_i, \boldsymbol{b}_i) + \varepsilon_i \tag{7}$$

where $y_i$, $\boldsymbol{b}_i$, and $\boldsymbol{x}_i$ are desirable output, undesirable outputs, and inputs, respectively, and $\varepsilon_i$ is the error term. To empirically estimate quantile production functions, we do not assume an *a priori* functional form or smoothness for $f$ but rather assume certain shape constraints such as monotonicity and concavity. That is, the function $f$ is supposed to be a family of continuous, monotonic increasing, and/or globally concave, and/or homogeneous functions.

While the undesirable outputs $\boldsymbol{b}$ are modeled like inputs $\boldsymbol{x}$ in (7), we would stress that they are outputs due to the fact that the function $f$ can be equivalently represented as the directional distance function with a specific direction vector that defines the boundary of the production possibility set (Kuosmanen et al., 2020).

We can transform the nonparametric production function (7) to the following conditional nonparametric quantile function

$$Q_y[\tau \mid (\boldsymbol{x}, \boldsymbol{b})] = f(\boldsymbol{x}, \boldsymbol{b}) + F_\varepsilon^{-1}(\tau) \tag{8}$$

where the quantile $\tau$ $(0 < \tau < 1)$ denotes that $Q_y$ splits the observed data into proportions $\tau\%$ below and $(1-\tau)\%$ above, and $F_{\varepsilon_i}$ is the cumulative distribution function of the error term $\varepsilon_i$.

Following Kuosmanen and Zhou (2021), for a given quantile $\tau$, we can differentiate $Q_y$ with respect to $\boldsymbol{b}$ or $\boldsymbol{x}$ to obtain

$$\begin{aligned} \frac{\partial Q_{y_i}}{\partial \boldsymbol{b}_i} &= \frac{\partial Q_y[\tau \mid (\boldsymbol{x}, \boldsymbol{b})]}{\partial \boldsymbol{b}_i} \\ \frac{\partial Q_{y_i}}{\partial \boldsymbol{x}_i} &= \frac{\partial Q_y[\tau \mid (\boldsymbol{x}, \boldsymbol{b})]}{\partial \boldsymbol{x}_i} \end{aligned} \tag{9}$$

where the first part of (9) is referred to as the marginal rates of transformation (MRTs) between desirable output $Q_y$ and undesirable outputs $\boldsymbol{b}$ and the second part as the marginal products (MPs) of inputs $\boldsymbol{x}$ on desirable output $Q_y$. Such MRTs and MPs locally estimated at the level of $\tau 100\%$ can be denoted as the quantile-based shadow prices of inputs and undesirable outputs, respectively (cf. Färe et al., 1993; Dai et al., 2020).

To estimate the shadow prices of inputs and undesirable outputs at a given quantile $\tau$, we can solve the following CQR model (Wang et al., 2014)

$$
\begin{aligned}
\min_{\alpha,\boldsymbol{\beta},\boldsymbol{\theta},\varepsilon^-,\varepsilon^+} \quad & (1-\tau)\sum_{i=1}^{N}\varepsilon_i^- + \tau\sum_{i=1}^{N}\varepsilon_i^+ && \\
\text{s.t.} \quad & y_i = \alpha_i + \boldsymbol{\beta}_i'\boldsymbol{x}_i + \boldsymbol{\theta}_i'\boldsymbol{b}_i + \varepsilon_i^+ - \varepsilon_i^- && \forall i \\
& \alpha_i + \boldsymbol{\beta}_i'\boldsymbol{x}_i + \boldsymbol{\theta}_i'\boldsymbol{b}_i \le \alpha_h + \boldsymbol{\beta}_h'\boldsymbol{x}_i + \boldsymbol{\theta}_h'\boldsymbol{b}_i && \forall i, h \\
& \boldsymbol{\beta}_i \ge \mathbf{0}, \boldsymbol{\theta}_i \ge \mathbf{0} && \forall i \\
& \varepsilon_i^+ \ge 0,\ \varepsilon_i^- \ge 0 && \forall i
\end{aligned}
\tag{10}
$$

where the estimated $\hat{\boldsymbol{\theta}}$ (i.e., MRTs) and $\hat{\boldsymbol{\beta}}$ (i.e., MPs) are the shadow prices of undesirable outputs and inputs at the level of $\tau 100\%$, respectively. In the CQR model (10), the first set of constraints can be interpreted as multivariate regression equations. The second set of constraints denotes a system of Afriat inequalities that impose concavity. The third set of constraints imposes monotonicity, and the last refers to the sign constraints of the error terms. Note that the error term $\varepsilon_i$ in (7) is decomposed into two non-negative components $\varepsilon_i^+$ and $\varepsilon_i^-$ in (10), which capture the asymmetric deviations from the quantile production function.

Note that the sign constraint imposed on undesirable outputs (i.e., $\boldsymbol{\theta}_i \ge \mathbf{0}$) guarantees nonnegative shadow prices for undesirable outputs, which follows a normative interpretation of the quantile production function as benchmark technology (cf. Hailu and Veeman, 2001; Kuosmanen and Zhou, 2021). Of course, this constraint can be relaxed to allow for the weak disposability of undesirable outputs. Furthermore, problem (10) presents a VRS specification of the quantile production function through the intercept term $\alpha_i$, which is a free variable. An additional constraint that forces $\alpha_i$ to be zero leads to a CRS model.

However, the estimated shadow prices $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\beta}}$ by CQR (10) are not necessarily unique (Dai et al., 2023). To obtain the unique estimates, a natural way is to regularize the CQR problem by imposing an $L_2$-norm regularization on the subgradients $\boldsymbol{\theta}_i$ and $\boldsymbol{\beta}_i$. Given a prespecified regularization parameter $\gamma \geq 0$, the penalized CQR approach is formulated as

$$\begin{aligned}
\min_{\alpha,\boldsymbol{\beta},\boldsymbol{\theta},\varepsilon^-,\varepsilon^+} \quad & (1-\tau)\sum_{i=1}^{N}\varepsilon_i^- + \tau\sum_{i=1}^{N}\varepsilon_i^+ + \frac{\gamma}{2}\sum_{i=1}^{N}\left(||\boldsymbol{\beta}_i||^2 + ||\boldsymbol{\theta}_i||^2\right) && (11)\\
\text{s.t.} \quad & y_i = \alpha_i + \boldsymbol{\beta}_i'\boldsymbol{x}_i + \boldsymbol{\theta}_i'\boldsymbol{b}_i + \varepsilon_i^+ - \varepsilon_i^- && \forall i\\
& \alpha_i + \boldsymbol{\beta}_i'\boldsymbol{x}_i + \boldsymbol{\theta}_i'\boldsymbol{b}_i \leq \alpha_h + \boldsymbol{\beta}_h'\boldsymbol{x}_i + \boldsymbol{\theta}_h'\boldsymbol{b}_i && \forall i,h\\
& \boldsymbol{\beta}_i \geq \mathbf{0}, \boldsymbol{\theta}_i \geq \mathbf{0} && \forall i\\
& \varepsilon_i^+ \geq 0,\ \varepsilon_i^- \geq 0 && \forall i
\end{aligned}$$

Problem (11) has a strongly convex objective function such that the subgradients $\boldsymbol{\beta}_i$ and $\boldsymbol{\theta}_i$ cannot take any value for a given objective function and feasibility. For any given $\gamma > 0$, the penalized CQR approach can ensure the uniqueness of subgradients (Theorem 2) and even avoid the quantile crossing problem (cf. Dai et al., 2022).

**Theorem 2.** *The quantile shadow prices $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\theta}}$ estimated by penalized CQR* (11) *are unique for all $\gamma > 0$.*

*Proof.* See Appendix A. ■

To operationalize the proposed penalized CQR approach, the value of tuning parameter $\gamma$ needs to be prespecified. If one has no prior knowledge of $\gamma$, the standard approaches in machine learning, such as cross-validation and Stein's unbiased risk estimate, can be used to determine the optimal value of $\gamma$ (see, e.g., Mazumder et al., 2019; Dai, 2023). However, as $\gamma \to \infty$, the estimated shadow prices $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\theta}}$ will flatten out to 0. This will deteriorate the measurement of the quantile shadow-price Fisher index (6). For our purpose, we simply set $\gamma^* = 0.01$ to slightly restrict the subgradients but still can obtain the unique shadow prices.

Note that the subgradients estimated by CQR may be unbounded at the domain boundary of a convex hull, resulting in the overfitting problem in CQR (see, e.g., Mazumder et al., 2019; Dai, 2023). Penalized CQR, on the other hand, avoids this issue effectively through the regularization in problem (11), which presents another appealing feature of penalized CQR over CQR. Alternatively, the overfitting problem can be addressed by incorporating Lipschitz regularization into convex regression (Mazumder et al., 2019), where an additional boundedness constraint is imposed on subgradients (e.g., $||\cdot||_2 \leq L$ and $||\cdot||_\infty \leq L$, where $L$ is the tuning parameter). Nevertheless, the comparative effectiveness in addressing overfitting between penalized CQR and Lipschitz CQR warrants further scrutiny.

In practice, we employ a local estimation strategy to determine the quantile-based shadow prices $\tilde{\boldsymbol{\rho}}$, $\tilde{\boldsymbol{\delta}}$, and $\tilde{\boldsymbol{\omega}}$ when computing the quantile shadow-price Fisher index $F_s^b$. Note that $\tilde{\boldsymbol{\rho}}$ always equals unity by construction. For each observation, we solve problem (11) for a given number of quantiles and then use the geometric mean of the shadow prices ($\tilde{\boldsymbol{\delta}}$ or $\tilde{\boldsymbol{\omega}}$) estimated on the two quantiles nearest to the observation as its shadow price. However, for those observations that fall below the lowest quantile or above the highest quantile, the shadow prices of the nearest quantile (i.e., the lowest or highest quantile) are directly used. Following Kuosmanen and Zhou (2021), we consider here 10 equidistant quantiles, i.e., $\tau = (0.05, 0.15, \ldots, 0.85, 0.95)$. Formally, the quantile-based shadow prices $\tilde{\boldsymbol{\delta}}$ and $\tilde{\boldsymbol{\omega}}$ for each observation are calculated as

$$\tilde{\boldsymbol{\delta}} = \begin{cases} (\hat{\boldsymbol{\theta}}_i^{\tau^*-0.1} \times \hat{\boldsymbol{\theta}}_i^{\tau^*})^{1/2} & \text{if } 0.05 < \tau^* < 0.95, \\ \hat{\boldsymbol{\theta}}_i^{\tau^*} & \text{otherwise.} \end{cases}$$

$$\tilde{\boldsymbol{\omega}} = \begin{cases} (\hat{\boldsymbol{\beta}}_i^{\tau^*-0.1} \times \hat{\boldsymbol{\beta}}_i^{\tau^*})^{1/2} & \text{if } 0.05 < \tau^* < 0.95, \\ \hat{\boldsymbol{\beta}}_i^{\tau^*} & \text{otherwise.} \end{cases}$$

where $\tau^*$ denotes the nearest quantile above the observation and $\tau^* - 0.1$ denotes the nearest quantile below the observation, as determined by the difference between $\hat{\varepsilon}_i^+$ and $\hat{\varepsilon}_i^-$. Such a local estimation strategy can make full use of the information of each observation and takes the inefficiency explicitly into account. The estimation of the shadow prices $\hat{\boldsymbol{\theta}}_i^\tau$ and $\hat{\boldsymbol{\beta}}_i^\tau$ can be implemented in Python using the pyStoNED package (Dai et al., 2021) with the standard solver Mosek (10.0.40).

We notice that the nonconvex technology may yield different estimated quantile production functions, probably affecting the green TFP measure. In so doing, one might apply the isotonic CQR approach (Dai et al., 2023) or the unconditional order-$\alpha$ approach (Daouia et al., 2017) to estimate the quantile-based shadow prices. However, the idea of using step functions (i.e., the estimated nonconvex quantile functions) for shadow pricing tends to be unreliable in the present setting. For instance, if the isotonic CQR problem is solved to the optimum, then the optimal $\hat{\boldsymbol{\beta}}_i^\tau$ and $\hat{\boldsymbol{\theta}}_i^\tau$ should go to zero, and the step functions are based on $\hat{\alpha}_i^\tau$ only. When inputs have zero shadow prices, the calculation of the quantile shadow-price Fisher index will be challenged. To model nonconvexity in the estimation of quantile functions, one possibility is to consider conditional convexity, which leads to a piece-wise linear function (see, e.g., Kuosmanen, 2001). We leave it for future work to develop alternative CQR preserving conditional convexity.

# 5 Quantile productivity measure

## 5.1 Data and variables

We proceed to apply the quantile shadow-price Fisher index to empirically estimate productivity growth in 38 OECD countries from 1990 to 2019. This empirical application focuses on quantifying productivity growth estimated by the quantile shadow-price Fisher index and identifying the impacts of GHG emissions on productivity measures.

We consider a baseline model specification with the following inputs and outputs:

- Capital input: Capital stocks at constant 2017 national prices (in millions, 2017 US$).
- Labor input: Number of persons engaged (in millions).
- Desirable output: Real GDP at constant 2017 national prices (in millions, 2017 US$).
- Undesirable output: Total GHG emissions excluding Land Use Change and Forestry (in million tonnes of $CO_2$ equivalents).

The source data on capital, labor, and GDP were collected from the Penn World Table 10.01 (PWT10) (Feenstra et al., 2015)[3] and the data on GHG emissions were from the World Bank database.[4] The descriptive statistics of the inputs and outputs are reported in Table B1.

## 5.2 Quantile productivity analysis

Since the United Nations Framework Convention on Climate Change was established in earlier 1990s, the international community has been committed to tackling climate change and proposed a series of climate actions (Kuosmanen et al., 2020; Dai et al., 2020). Consequently, a large share of capital investments over the recent decades has been devoted to reducing

[3]The corresponding selected variables in PWT10 are *rnna*, *emp*, and *rgdpna*, respectively.

[4]*DataBank*: `https://databank.worldbank.org/source/world-development-indicators/Series/EN.ATM.GHGT.KT.CE`, accessed 1 February 2023.

GHG emissions instead of increasing GDP, while productivity slowdown has been reported in certain developed countries (see, e.g., Syverson, 2017; Crafts, 2018). A natural question arises as to whether the productivity slowdown is a mismeasurement due to the omission of undesirable outputs in traditional TFP measures. Therefore, we proceed to investigate the impact of GHG emissions on productivity growth by applying the proposed quantile shadow-price Fisher index to the traditional TFP and green TFP measures.

Fig. 1 presents the cumulative TFP and green TFP measures for the OECD aggregate. Given thus constructed country-specific inputs and outputs data, we can generate a new artificial observation as the OECD aggregate (i.e., $\boldsymbol{X} = \sum \boldsymbol{x}_i$ and $\boldsymbol{Y} = \sum \boldsymbol{y}_i$) to measure the productivity growth at the OECD level. Using panel data of 39 observations during 1990–2019, we calculate the CRS models (11) and subsequently obtain the quantile shadow-price Fisher index (6).

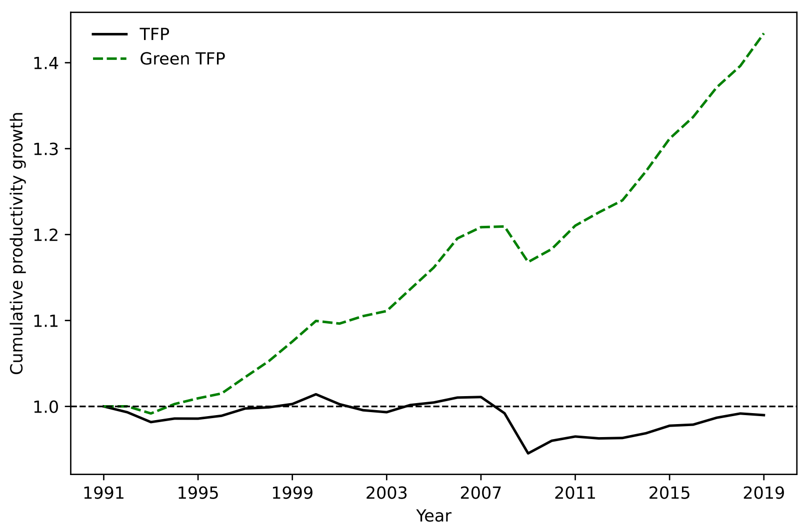
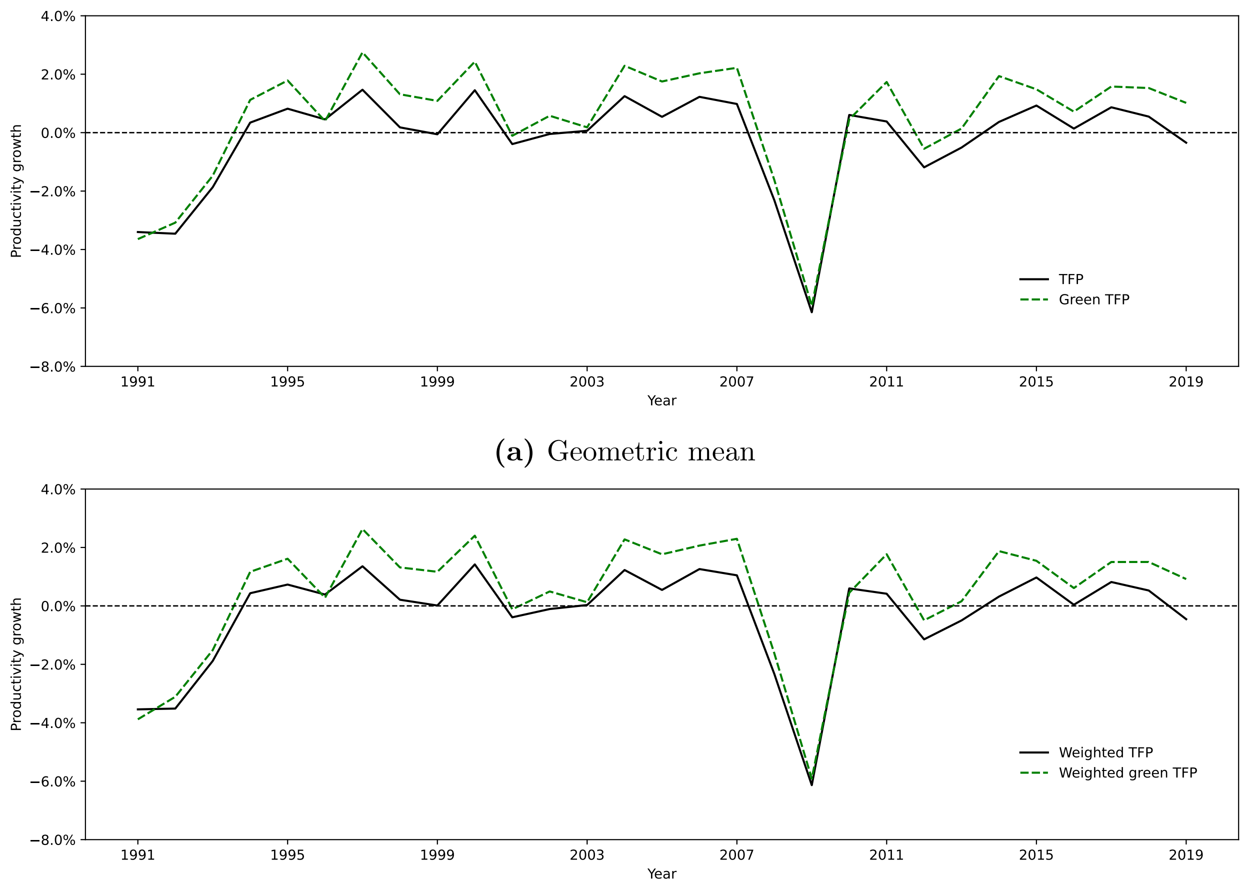



**Fig. 1.** The cumulative TFP and green TFP measures.

As shown in Fig. 1, green TFP growth is faster than the TFP growth at the entire OECD level. After the global financial crisis, the cumulative TFP growth is below unity, indicating a negative growth and confirming the existence of the secular stagnation of productivity growth in Western countries. However, the cumulative green TFP shows strong growth for almost all the periods, except for the financial crisis in 2008-2009. This result strongly suggests that ignoring the massive investments in carbon abatement can indeed help to explain why the measured TFP growth has ground to a halt, as in contrast the green TFP exhibits strong cumulative growth.

Fig. 2 demonstrates the geometric average productivity growth across OECD countries during 1990–2019 with and without GHG emissions.[5] It is evident that the average TFP growth and green TFP growth are not identical, yet they show a similar evolution path. The green TFP growth is generally higher than the TFP growth over the time period 1990–2019. The largest difference between these two productivity measures occurred in 2014, with an absolute difference of 1.57 percentage points. The average absolute difference over the period amounts to 0.76 percentage points, which cannot be omitted from the productivity measure. After considering the GDP heterogeneity across OECD countries, the productivity change in terms of the weighted geometric mean becomes a little bit smaller than that in terms of the geometric mean but can conclude the same finding.




**(a)** Geometric mean

**(b)** Weighted geometric mean

**Fig. 2.** Productivity growth per year: with and without GHG emissions.

[5]All empirical results exclusively present TFP and green TFP estimated by the CRS models unless otherwise stated.

It is worth noting that the financial crisis has greatly affected productivity growth for OECD countries. This is because the economic crisis results in a serious problem for economic growth, which further affects both the factor inputs, especially capital investment, and GHG emissions. For instance, during the 1997 Asian financial crisis, the green TFP of Japan and South Korea plummeted by 2.6% and 5.9% from 1997–1998, respectively. Moreover, the 2008 global financial crisis was accompanied by an average of 5.93% decline in green TFP for the entire OECD countries during 2008–2009.

Fig. 3 describes the average productivity growth with and without GHG emissions at the country level. Several interesting findings are worth noting. First, the traditional TFP measure tends to underestimate productivity growth, even during a period of carbon reduction or the financial crisis. There is a noticeable difference in productivity growth between the TFP measures for all countries (0.73 percentage points; the absolute difference between green TFP and TFP), especially for transition economies such as Slovakia (1.97) and Lithuania (1.49). After the Kyoto Protocol came into effect in 2005, most countries reduced GHG emissions according to their commitments. Notably, the GHG emissions of Denmark, Finland, and the United Kingdom decreased by 42%, 36%, and 34%, respectively, during 2006–2019. However, this period still witnessed varying degrees of upward difference between the green TFP and traditional TFP measures, indicating that neglecting the impact of GHG emissions will overestimate the contribution from conventional factor inputs and, in turn, underestimate productivity growth. This finding is consistent with Yörük and Zaim (2005) and Shen et al. (2017), which also note that the traditional TFP index undervalues the Luenberger green TFP index for OECD countries. Overall, the productivity growth slowdown may partly be explained by the carbon reduction efforts by OECD countries.

Second, productivity growth depends on the GHG emissions reduction relative to the other two factor inputs. Under the Kyoto Protocol, OECD countries invest much more capital in facilitating low-carbon transition by utilizing cleaner production technologies, switching to cleaner fuels, or establishing market-based instruments (e.g., emissions trading systems). That is, if a country is willing to reduce GHG emissions even by a small proportion, then more conventional inputs are needed and they will increase far faster than the GHG emissions abatement. In this case, the green TFP growth will be higher than the TFP growth; in other words, the GHG emissions reduction leads to a greater enhancement in productivity growth than the other two conventional inputs.

Third, the countries from the transition economies can serve as the benchmark in terms of green development. The transition economies have relatively higher productivity growth than other OECD countries, particularly when introducing the GHG emissions in the entire sample period, indicating that these countries utilize the resources more efficiently but emit relatively lower GHGs. A similar finding has been detected in Kuosmanen et al. (2020). However, these advantages would fade away under the double constraints of the global financial crisis and the GHG emissions reduction targets, as reflected by the low green TFP growth from 2006 to 2019 in transition economies.

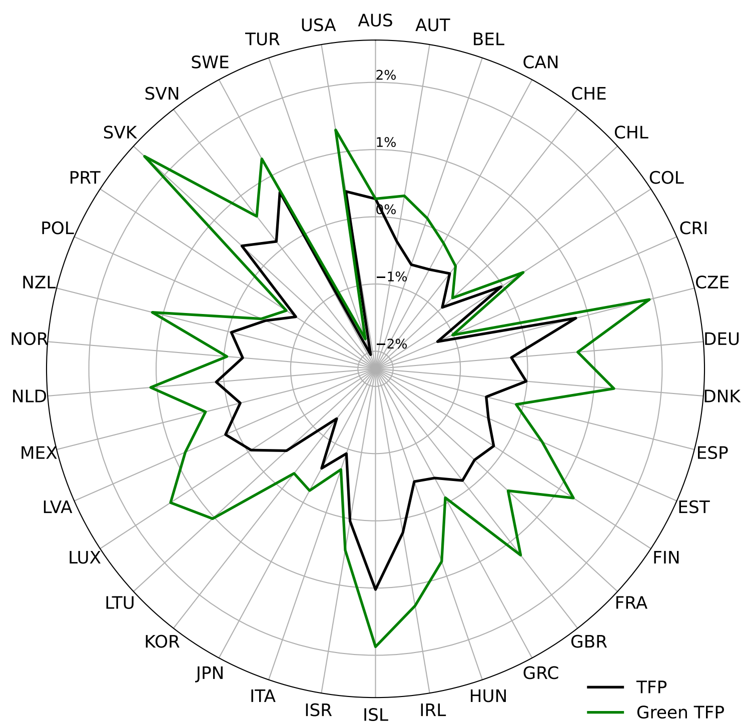



**Fig. 3.** Productivity growth per country: with and without GHG emissions.

Fourth, for countries that have reduced GHG emissions most actively, the average green TFP growth rate could double the conventional TFP growth. For example, Estonia reduced more than twice GHG emissions (the largest decrease in percentage values) during the sample period, and the productivity growth became positive from -0.43% (TFP growth) to 0.44% (green TFP growth). However, for the countries that are continuously increasing GHG emissions, the difference between TFP growth and green TFP growth is relatively small. For example, Australia's GHG emissions increased by 16.9%, and the country saw productivity growth of around 0.27% whether measured by TFP (0.267%) or green TFP (0.268%).

Finally, there exists a large variance in both green TFP and TFP growth among OECD countries (cf. Dai, 2023), suggesting that the GHG emissions reduction actions are implemented inefficiently in OECD countries and that the relationship between economic growth and environmental protection and resources utilization is less coordinated. Therefore, the current policy to improve productivity growth is inefficient, and resource misallocation for the entire sample might also exist. However, the magnitude of resource misallocation and its effect on green TFP growth need to be further investigated. We leave this as an interesting question for future research.

Fig. 4 further demonstrates the average green TFP growth measured by the quantile shadow-price Fisher index under the CRS and VRS specifications. It reveals that the productivity growth estimates are robust regardless of CRS or VRS. Specifically, the median absolute difference between the CRS- and VRS-based green TFP growth is 0.2 percentage points. The largest difference appears in Estonia (13, 2018–2019), and the smallest difference is close to zero for 8.56% of all the observations with 4-digit decimal accuracy. Further, there are only 44 cases (4%) where the absolute value is greater than 2 percentage points, and there are 84.3% of the sample where the difference is less than 1 percentage point (see Fig. B1).

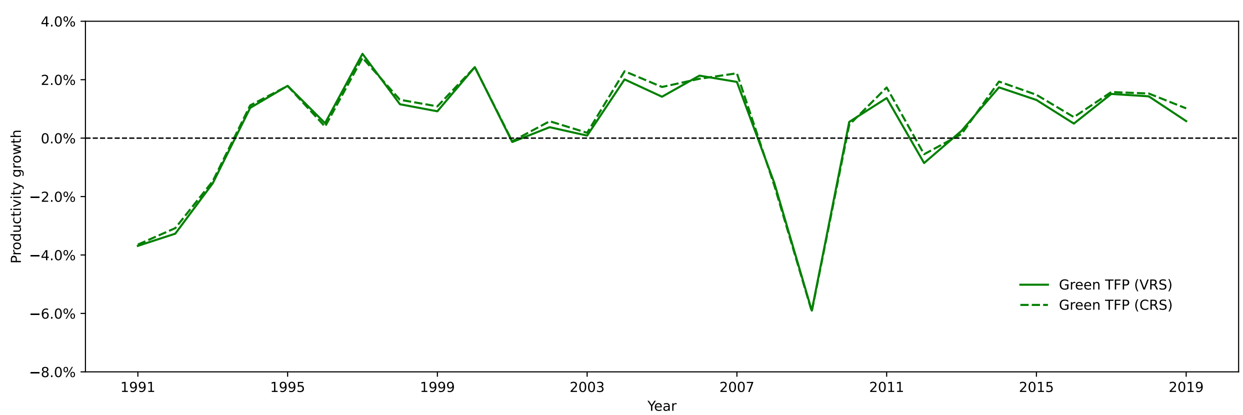



**Fig. 4.** Green productivity growth per year: CRS and VRS models.

# 6 Alternative model specifications

To examine the impact of different forms of physical capital inputs, we further compare the baseline model with an alternative specification where the capital input is represented by capital services. We also compare the baseline model with an extended specification that includes human capital to examine the impact of human capital accumulation.

## 6.1 Capital services

Reliable figures for fixed capital are crucial for a better understanding of productivity growth, but they are often denoted by two interrelated but distinct concepts: capital stocks and capital services. The former refers to the stock of physical assets at a point in time, whereas the latter represents the flow of services created by these assets in a period. Compared to capital services, capital stocks are less relevant for measuring productivity growth and are more likely to overestimate TFP growth (see, e.g., Schreyer, 2001; Schreyer, 2004). In this subsection, we are primarily interested in the impact of alternative representations of fixed capital (capital stocks versus capital services) on the green TFP growth estimates obtained from the quantile shadow-price Fisher index approach.

In practice, we calculate the capital services per country per year ($rkna2_{i,t}$) by using

$$rkna2_{i,t} = \begin{cases} (1 - labsh_{i,t}) \times rgdpna_{i,t} & t = 2017; \\ [(1 - labsh_{i,2017}) \times rgdpna_{i,2017}] \times rkna_{i,t} & t \neq 2017. \end{cases}$$

where $labsh$ denotes the share of labor compensation in GDP at current national prices, $rgdpna$ represents the real GDP at constant 2017 national prices (in millions, 2017US$), and $rkna$ is the capital services at constant 2017 national prices (2017=1). The data of the above variables were also collected from PWT10. Fig. B2 illustrates the difference between the capital services and the capital stocks for all OECD countries, where capital services grew significantly faster than capital stocks during the sample period.

Fig. 5 depicts the scatter plot of green TFP growth estimated with capital services versus capital stocks. If an observation (i.e., an OECD country) is located below the 45-degree line (i.e., the red dot line), then the green TFP growth with capital services is smaller in terms of the average value than that with capital stocks. Obviously, the green TFP growth with capital stocks is generally higher than that with capital services from 1990 to 2019. This suggests that if the fixed capital input is denoted by capital stocks, the growth of green TFP could be overestimated and the contribution of capital assets to economic growth may be underestimated. This finding is in line with the traditional TFP growth analysis (see, e.g., Schreyer, 2001; Schreyer, 2004).

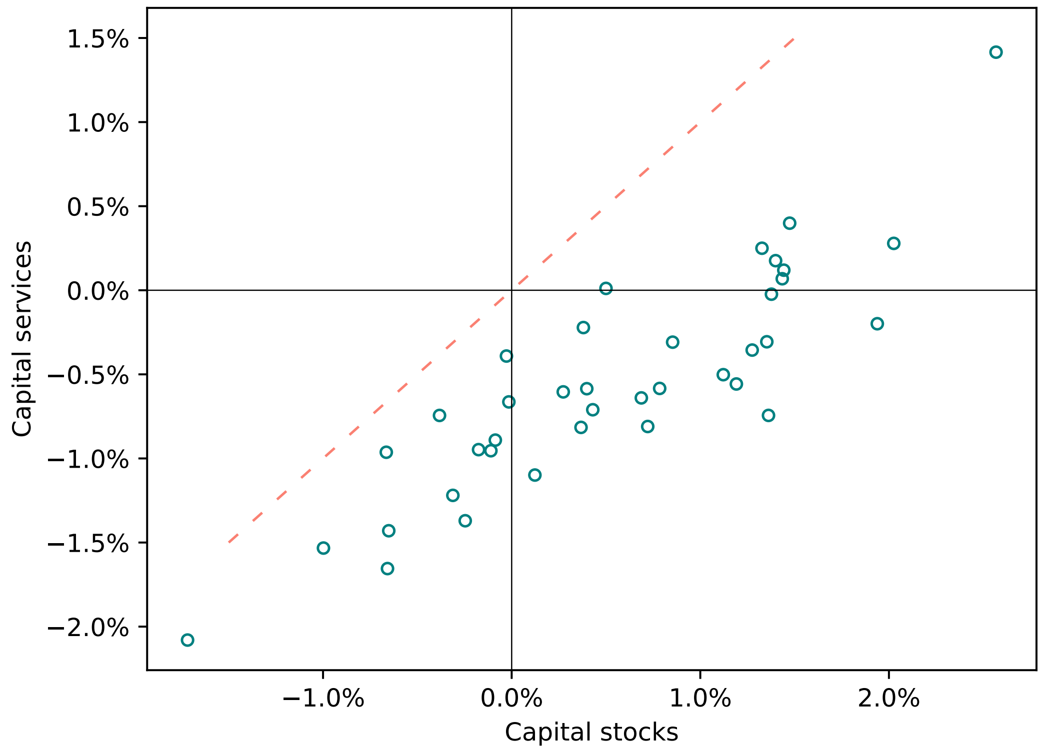



**Fig. 5.** Green TFP growth estimated with capital services or capital stocks.

At the country level, we also observe that the proxy of fixed capital has nonnegligible impacts on the green TFP measure. Table 1 reports the average green TFP growth and economic growth for three selected countries, with the smallest absolute difference in green TFP growth between the use of capital stocks and capital services. For instance, Italy's annual green TFP declined by 0.1% from 2010 to 2019 when estimated with capital services, but grew by 0.04% when estimated with capital stocks. Over the same period, the capital services of Italy grew by 0.66% per year, which was higher than the increase in capital stocks (0.59%). The resulting 0.7 percentage points difference translated to the 0.14 percentage points adjustment to the green TFP measure. Therefore, the green TFP growth of Italy was overestimated with capital stocks. The overestimation can also be observed in all other countries or periods.

**Table 1.** Green TFP growth and economic growth.

| | | Italy | Poland | Turkey |
|---|---|---|---|---|
| Output | 1990–99 | 1.41 | 3.42 | 3.12 |
| | 2000–09 | 0.48 | 3.83 | 3.61 |
| | 2010–19 | 0.24 | 3.50 | 5.47 |
| Capital services | 1990–99 | 2.57 | 4.69 | 6.47 |
| | 2000–09 | 2.39 | 4.89 | 6.07 |
| | 2010–19 | 0.66 | 4.15 | 6.60 |
| Capital stocks | 1990–99 | 2.22 | 3.95 | 6.15 |
| | 2000–09 | 2.12 | 4.30 | 5.79 |
| | 2010–19 | 0.59 | 3.96 | 6.49 |
| Green TFP with capital services | 1990–99 | -1.06 | -1.22 | -3.31 |
| | 2000–09 | -1.74 | -1.03 | -2.24 |
| | 2010–19 | -0.10 | -0.33 | -0.82 |
| Green TFP with capital stocks | 1990–99 | -0.65 | -0.43 | -2.82 |
| | 2000–09 | -1.39 | -0.47 | -1.99 |
| | 2010–19 | 0.04 | -0.26 | -0.45 |

## 6.2 Human capital

Inspired by endogenous growth models, extensive empirical studies have demonstrated the positive externalities of human capital on TFP growth (see, e.g., Barro, 2001; Henderson and Russell, 2005; Bowlus and Robinson, 2012). Recently, several studies have highlighted various potential environmental benefits of human capital accumulation (see, e.g., Yao et al., 2020; Angrist et al., 2023), which may help to reduce GHG emissions and affect green productivity. We thus, in this subsection, incorporate human capital into quantile production functions to investigate the impact of human capital on green TFP measures. Note that human capital is proxied by the average years of schooling in the population aged 25 years and older.[6]

Fig. 6 shows the estimated densities of green TFP growth with human capital for 1991, 2001, 2011, and 2019. A relatively larger discrepancy in the overall distribution appeared in

[6] The data on average years of schooling were collected from PWT10, and the corresponding selected variable in PWT10 is *yr_sch*.

2019, indicating that the cross-country differences in green TFP growth have become larger. Furthermore, the center of the four distributions does not shift rightward. That is, the green TFP does not always grow with time in OECD countries, even though human capital is considered in the production model.

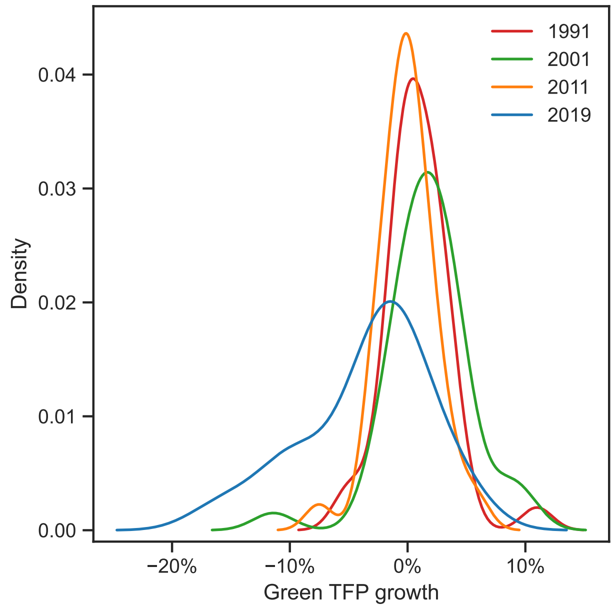



**Fig. 6.** Estimated densities of green TFP growth with human capital.

Fig. 7 depicts the average green TFP growth with and without human capital for each OECD country. If an observation is located below the 45-degree line (i.e., the red dot line), then the green TFP growth with human capital is larger than that without human capital. Furthermore, if human capital is reasonably well measured, an increase in green TFP growth after considering human capital in the quantile production function indicates a relative shortage of human capital (or measurement error in labor input, equivalently). Similarly, a decrease in green TFP growth can be interpreted that there exists sufficient or even redundant human capital (Henderson and Russell, 2005).

In the earlier period, most points locate below the 45-degree line, indicating that the green TFP growth can increase after incorporating human capital. In fact, there was a 0.02% average increase in green TFP growth for OECD countries due to the inclusion of human capital in 1991. This suggests that the neglect of human capital or the mismeasurement of labor input in production models underestimated green productivity. Notably, the largest green TFP growth improvements driven by considering human capital in 1991 occurred in relatively developed countries such as Australia, Chile, Germany, Finland, and the United Kingdom, as well as the transition economies (e.g., Lithuania and Slovakia).

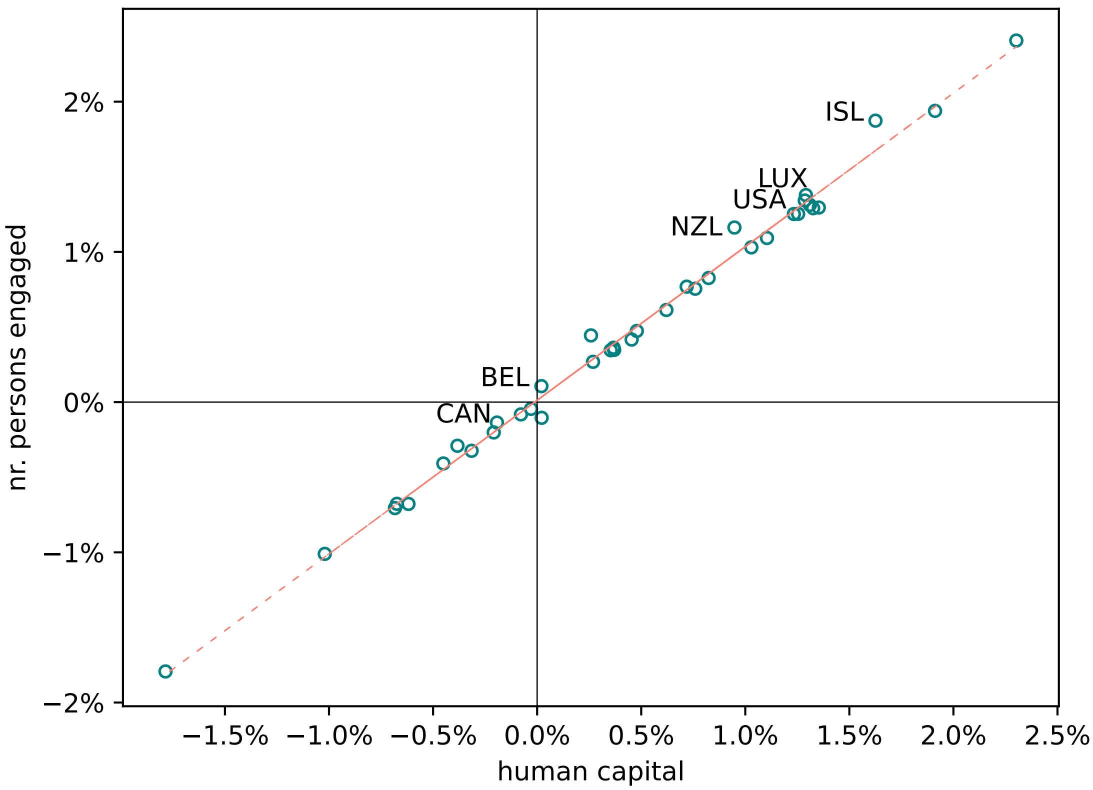



**Fig. 7.** Green TFP growth with and without human capital.

In the recent year of the sample, there is an increasing number of countries located below the red dot line, but the mean value of green TFP growth does not explicitly increase after the inclusion of human capital. This implies that although the average effect of human capital becomes less pronounced over time, the neglect of human capital remains to affect the accuracy of green productivity measurement for more countries. Note that recent years have witnessed a relatively large difference in the quantity of human capital among OECD countries, which may lead to the average negative impact of human capital.

Overall, human capital accumulation positively influences the green TFP growth for more than half of OECD countries (Fig. 7). This finding is consistent with Henderson and Russell (2005), which investigates the impact of human capital on country-level efficiency estimation. We also observe that several developed countries such as Canada, Luxembourg, and the United States have decreased green TFP growth after the inclusion of human capital, indicating that the human capital growth in these countries is poised to be saturated and sufficient.

# 7 Conclusions

In this paper, we have shown that the measured productivity growth is considerably higher when the GHG emissions are accounted for. For countries that have reduced GHG emissions most actively, the average green TFP growth rate could double the conventional TFP growth. Green productivity growth depends on GHG emissions reduction relative to the traditional factor inputs. If the fixed capital input is denoted by capital stocks, then the growth of green TFP can be overestimated, and the contribution from capital assets to economic growth may be underestimated. Furthermore, the positive impact of human capital accumulation on green TFP growth has been confirmed.

Our methodological contribution is to develop a new quantile shadow-price Fisher index to measure green productivity growth based on quantile production functions estimated by the developed penalized CQR approach. We then apply the quantile shadow-price Fisher index to calculate productivity growth for 38 OECD countries over the period 1990–2019 and empirically explore the impacts of GHG emissions, fixed capital, and human capital on productivity measures.

The proposed estimation and index approaches offer four major advantages over conventional methods for measuring green TFP growth. First, the penalized approach has unique quantile shadow price estimates for the inputs and the undesirable output. Second, this approach takes inefficiency explicitly into account and is more robust to outliers and heterogeneity. The estimated shadow prices can reflect the full information of all observations. Third, the quantile shadow-price Fisher index does not require the real price data for input-output vectors, which is necessary for calculating the Fisher ideal index. Therefore, such environmental factors as GHG emissions can be integrated into productivity growth accounting using the Fisher index. Finally, the quantile shadow-price Fisher index can avoid an *ad hoc* choice of quantiles which might lead to different estimations of productivity growth and allow the quantiles to move in the inter-period sample.

While the findings drawn from this study provide insights into the quantile shadow-price Fisher index, the possible future research avenues are also highlighted. Decomposing the quantile shadow-price Fisher index to its components (e.g., efficiency change, quantile change, and technological change) is a fascinating avenue. Such decomposition can help better understand the driving forces of green TFP growth. In addition, the resource misallocation effect at the country level deserves further scrutiny.

## Statements and Declarations

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# Appendix

## A Proof of Theorem 2

We can prove the uniqueness of $\boldsymbol{\beta}_i$ and $\boldsymbol{\theta}_i$ from the perspective of the strong convexity argument. For the sake of illustration, the penalized CQR problem is rephrased as

$$\begin{aligned} \min \quad & Q_\gamma(\varepsilon^+, \varepsilon^-, \boldsymbol{\beta}, \boldsymbol{\theta}) = f_1(\varepsilon^+) + f_2(\varepsilon^-) + \frac{\gamma}{2}\sum_{i=1}^{n}\|\boldsymbol{\beta}_i\|^2 + \frac{\gamma}{2}\sum_{i=1}^{n}\|\boldsymbol{\theta}_i\|^2 \\ s.t. \quad & x \in \Omega \end{aligned} \tag{A1}$$

where $f_1(\varepsilon^+) = \tau\sum_{i=1}^{n}\varepsilon_i^+$, $f_2(\varepsilon^-) = (1-\tau)\sum_{i=1}^{n}\varepsilon_i^-$, and $\Omega$ is a convex set characterized by the Afriat inequalities. Note that the function of the squared Euclidean norm is strongly convex.

Suppose that there are two distinct optimal solutions to (A1), $(\varepsilon_1^+, \varepsilon_1^-, \boldsymbol{\beta}_1, \boldsymbol{\theta}_1) \in \Omega$ and $(\varepsilon_2^+, \varepsilon_2^-, \boldsymbol{\beta}_2, \boldsymbol{\theta}_2) \in \Omega$. This implies that

$$Q_\gamma(\varepsilon_1^+, \varepsilon_1^-, \boldsymbol{\beta}_1, \boldsymbol{\theta}_1) = Q_\gamma(\varepsilon_2^+, \varepsilon_2^-, \boldsymbol{\beta}_2, \boldsymbol{\theta}_2) \le Q_\gamma(\varepsilon_m^+, \varepsilon_m^-, \boldsymbol{\beta}_m, \boldsymbol{\theta}_m), \forall(\varepsilon_m^+, \varepsilon_m^-, \boldsymbol{\beta}_m, \boldsymbol{\theta}_m) \in \Omega. \tag{A2}$$

Then, consider $(\varepsilon_m^+, \varepsilon_m^-, \boldsymbol{\beta}_m, \boldsymbol{\theta}_m) = ((\varepsilon_1^+ + \varepsilon_2^+)/2, (\varepsilon_1^- + \varepsilon_2^-)/2, (\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2)/2, (\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2)/2)$. By convexity of $\Omega$, we have $(\varepsilon_m^+, \varepsilon_m^-, \boldsymbol{\beta}_m, \boldsymbol{\theta}_m) \in \Omega$. By strong convexity of $\|\boldsymbol{\beta}\|^2$ and $\|\boldsymbol{\theta}\|^2$, we have

$$\begin{aligned} Q_\gamma(\varepsilon_m^+, \varepsilon_m^-, \boldsymbol{\beta}_m, \boldsymbol{\theta}_m) =& \frac{\gamma}{2}\sum_{i=1}^{n}\left\|\frac{\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2}{2}\right\|^2 + \frac{\gamma}{2}\sum_{i=1}^{n}\left\|\frac{\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2}{2}\right\|^2 + f_1\left(\frac{\varepsilon_1^+ + \varepsilon_2^+}{2}\right) + f_2\left(\frac{\varepsilon_1^- + \varepsilon_2^-}{2}\right) \\ <& \frac{\gamma}{4}\sum_{i=1}^{n}\|\boldsymbol{\beta}_1\|^2 + \frac{\gamma}{4}\sum_{i=1}^{n}\|\boldsymbol{\theta}_1\|^2 + \frac{1}{2}f_1(\varepsilon_1^+) + \frac{1}{2}f_1(\varepsilon_1^-) \\ &+ \frac{\gamma}{4}\sum_{i=1}^{n}\|\boldsymbol{\beta}_2\|^2 + \frac{\gamma}{4}\sum_{i=1}^{n}\|\boldsymbol{\theta}_2\|^2 + \frac{1}{2}f_2(\varepsilon_2^+) + \frac{1}{2}f_2(\varepsilon_2^-) \\ =& \frac{1}{2}Q_\gamma(\varepsilon_1^+, \varepsilon_1^-, \boldsymbol{\beta}_1, \boldsymbol{\theta}_1) + \frac{1}{2}Q_\gamma(\varepsilon_2^+, \varepsilon_2^-, \boldsymbol{\beta}_2, \boldsymbol{\theta}_2) = Q_\gamma(\varepsilon_1^+, \varepsilon_1^-, \boldsymbol{\beta}_1, \boldsymbol{\theta}_1) \end{aligned}$$

which contradicts (A2). The penalized CQR problem (A1) thus has optimal and unique components $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$. ∎

# B Additional figure and table

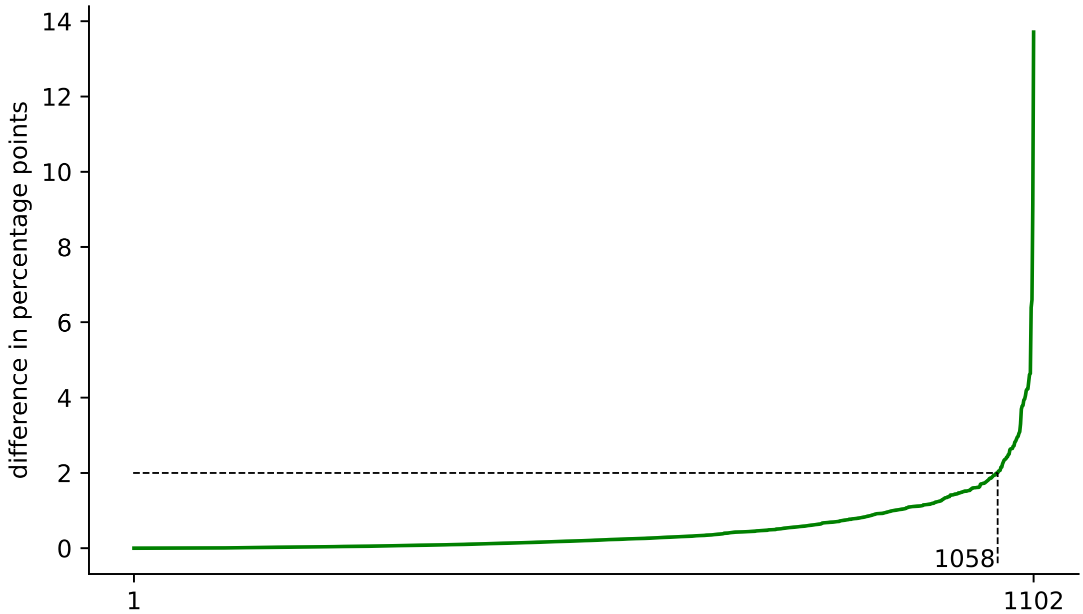


**Fig. B1.** Distribution of the absolute difference of the CRS- and VRS-based quantile shadow-price Fish index (i.e., green TFP growth). The horizontal axis presents the observations sorted in ascending order ($N = 1102$).

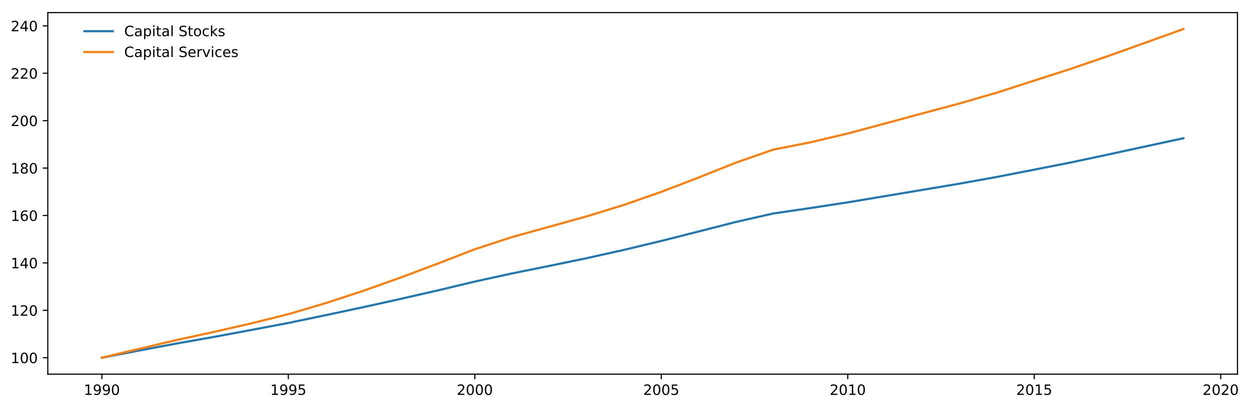


**Fig. B2.** Capital services and capital stocks, all OECD countries, 1990 = 100.

**Table B1.** Descriptive statistics of the input and output variables.

| Variable | Unit | Mean | Std. Dev. | Min. | Max. | Obs. |
|---|---|---|---|---|---|---|
| Labor | million | 15 | 25 | 0.1 | 158 | 1140 |
| Capital stocks | million 2017US$ | 5174738 | 9683973 | 45410 | 69059464 | 1140 |
| GDP | million 2017US$ | 1193039 | 2581171 | 7374 | 20563592 | 1140 |
| GHG | million tonnes | 399 | 1016 | 3 | 6787 | 1140 |
| Capital services | million 2017US$ | 479156 | 1017760 | 2480 | 8255854 | 1140 |
| Human capital | years | 11 | 2 | 5 | 14 | 1140 |