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Feedback Optimizing MPC for Load Frequency Control and Economic Dispatch

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Abstract: With increasing renewable generation, demand response, and deregulation, power networks are becoming more uncertain, time-varying, and strongly coupled. As a result, the conventional approach of performing separate economic dispatch (ED) and load-frequency control (LFC) operations may no longer guarantee smooth and cost-efficient regulation of frequency across interconnected power networks. To address this, we present a tracking model predictive control (MPC) algorithm which simultaneously achieves economic dispatch and secondary frequency control in a multi-area power network. A unique feature of the proposed algorithm is that it exploits the implicit feedback in MPC to regulate the interconnected power system towards steady-state equilibria that solve a multi-area economic dispatch problem, without explicitly computing the latter as a reference to be followed or estimating the unknown disturbances. This feedback-based optimization approach endows the algorithm with inherent robustness to uncertainty (such as unknown step changes in the demand). Simulation results for a two-area power network show improved steady-state economic performance compared to standard MPC-based frequency control schemes, and better dynamic performance compared to other feedback-based optimization schemes.

Keywords: Control and optimization of power systems; Renewable energy sources; Smart grids; Control of energy storage systems; Power system planning and operation.

1. INTRODUCTION

A fundamental objective in power system operation is the supply of uninterrupted power at rated quality and least possible operating cost/disutility. To achieve this, a combination of load-frequency control (LFC) and economic dispatch (ED) is used in a hierarchical control framework. The ED acts at a much slower timescale and determines the optimal equilibrium that maximizes the social welfare of the power system for an expected/projected value of the demand. In real-time, unknown fluctuations in the predicted demand are assumed subtle, hence the equilibrium from the most recent ED to a large extent remains economically optimal. As a result, conventional LFC schemes such as tie-line bias control are designed to reject these fluctuations in projected demand and return the power system to the most recent ED setpoint. This is usually achieved by driving an area control error (ACE) signal to zero.

The problem with this conventional LFC strategy is that with larger and faster demand fluctuations, the previous ED setpoints may no longer be optimal at the current time. Studies have shown that under significant variability and uncertainty in the demand, conventional LFC based on tie-line bias control can be severely inefficient (Li et al., 2016).

With higher penetration of renewable generation and demand responsive loads, future power systems will have lower inertia with faster, larger, and more uncertain fluctuation in the net predicted demand (uncontrollable load minus intermittent generation). This can result in frequent violation of tie-line thermal constraints, large frequency deviation and a worsening of the economic performance of LFC. As a result, the conventional approach of separately considering ED and LFC at different timescales may no longer achieve economic optimality in frequency control. To address this problem, it is common practice to dispatch generation and demand response resources closer to the timescale of frequency control. However, this can be computationally intensive, non-robust, and may cause stability issues in the frequency dynamics (Hauswirth et al., 2020). Therefore, novel LFC schemes capable of achieving ED in real-time while guaranteeing closed-loop stability and optimal transient performance will be useful in power networks with high penetration of intermittent generation. Also, achieving real-time ED in LFC will improve the accuracy of real-time pricing schemes required for implementing demand response programs (Zhang and Papachristodoulou, 2015).

Recently, the interest in developing LFC schemes with autonomous ED capabilities has increased significantly (Dörfler et al., 2019). In recent studies, the goal has been to autonomously solve the economic dispatch problem within the feedback loop of LFC (Dörfler et al., 2019; Molzahn

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et al., 2017). Most of the currently proposed algorithms remodel conventional LFC after optimization algorithms that take the form of dynamical systems which converge asymptotically to the optimal ED solutions in closed-loop (Molzahn et al., 2017; Li et al., 2016; Miao and Fan, 2017; Dörfler et al., 2019). One limitation of these algorithms is that the power system dynamics are not considered in the control design, assuming a pre-stabilized power system. Therefore, these controllers lack transient performance guarantees and can be difficult to tune, often yielding an oscillatory frequency response.

In an attempt to address the above limitations, (Köhler et al., 2017; Jia et al., 2020) have proposed a distributed economic MPC approach to the LFC problem. Although convergence to the ED solution was obtained, the algorithm in (Köhler et al., 2017; Jia et al., 2020) required explicit estimation of the unknown disturbances and also to guarantee closed-loop stability, the power system dynamics had to be passive (which is hard to satisfy with second-order turbine-governor dynamics (Trip and De Persis, 2017)). These limitations make it hard to implement these algorithms on real power system networks. To circumvent the limitations of economic MPC, a standard tracking MPC formulation of the LFC problem can be adopted. However, current formulations of tracking MPC lack autonomous ED capabilities. Efforts to integrate economic performance into a tracking MPC formulation of the LFC problem rely on scaling the penalty on the control input by the economic cost/price of activating LFC (Ersdal et al., 2016; Mc Namara and Milano, 2018), or adopting a multi-objective function approach incorporating both the ED and the LFC performance objectives (Sokoler et al., 2015). Both of these approaches improve the economic performance of LFC but do not guarantee convergence to the ED setpoints in steady-state.

In this paper, we propose an LFC algorithm based on the feedback-optimizing MPC framework in (Asuk and Trodden, 2021), to address the limitations of currently available MPC-based solutions for simultaneous ED and LFC. The main contribution is a novel tracking MPC formulation of the LFC problem that guarantees convergence to the 'true' ED setpoints without estimating the unknown disturbances (when they are piecewise constant), while retaining the complexity of standard tracking MPC. Because most commercially available MPC packages are based on the standard tracking MPC formulation (Mc Namara and Milano, 2018), it can be argued that the proposed formulation is easier to implement in practice compared to an economic MPC formulation. Also, the feedback approach to steady-state optimization endows the proposed algorithm with an inherent robustness to model uncertainty.

The paper is structured as follows: Section 2 presents the power system model and the economic dispatch problem. Section 3 formulates the control problem as a feedback-optimizing load-frequency control problem and Section 4 presents a MPC solution to the control problem with theoretical performance guarantees. Section 5 presents numerical simulations of the algorithm for a two-area power system and compares the performance with other solutions in the literature. Section 6 concludes the paper and discusses further research questions.

2. POWER SYSTEM MODEL, LOAD FREQUENCY CONTROL AND ECONOMIC RE-DISPATCH

Consider a transmission level network with arbitrary topology described by a weighted directed graph $\mathscr{G} = (\mathcal{N}, \mathscr{E}, \mathscr{W})$, where $\mathcal{N} = \{1, \dots, i, \dots N_a\}$ is the set of nodes or control areas and $\mathscr{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of edges or tie-line interconnection between the nodes. The set $\mathscr{W}$ contains the weights of the edges $\mathscr{E}$ i.e $\mathscr{W} = \{T_{ij},\ \forall (i,j) \in \mathscr{E}\}$ where T_{ij} $[pu\ MW/Hz]$ is the synchronizing coefficient of the edge/tie-line $(i,j) \in \mathscr{E}$. The neighbourhood set of the node i is denoted by $\mathcal{N}_i$. For the transmission graph $\mathscr{G}$, the graph Laplacian $\mathscr{L}_{\mathscr{G}}$ contains the topology information in $\mathscr{G}$. For simplicity, we model the renewable generation as a negative demand, i.e. if P_i^{RES} is the renewable generation and P_i^L the uncontrollable load in node i, then the net-demand (or net-load), $w_i = P_i^L - P_i^{RES}$. Each node is assumed to have a single generator, a demand responsive load, and a net-load.

Let the power network be operating around a nominal equilibrium determined by an ED problem at a slower timescale. As common in power system control at the transmission level, the following assumptions are made (see Li et al. (2016) for details).

Assumption 1. The voltage magnitudes are fixed at all nodes, i.e. $v_i\ \forall i \in \mathcal{N}$ are constant. The transmission lines are of negligible resistance and reactive power injections and flows are omitted.

With Assumption 1 in place, each control area $i \in \mathcal{N}$, is modelled by the following dynamic linear differential equation (Wang et al., 2017).

$$H_i \dot{f}_i = P_i^m - P_i^{dr} - D_i f_i - P_i^{tie} - w_i, \tag{1a}$$

$$\dot{P}_i^{tie} = \sum_{j \in \mathcal{N}_i} T_{ij}(f_i - f_j), \tag{1b}$$

$$\tau_{t,i} \dot{P}_i^m = -P_i^m + P_i^v \tag{1c}$$

$$\tau_{v,i} \dot{P}_i^v = -P_i^v + \alpha_i^m u_i - (1/r_i) f_i \tag{1d}$$

$$\tau_i^{dr} \dot{P}_i^{dr} = -P_i^{dr} + \alpha_i^{dr} u_i \tag{1e}$$

where f_i $[Hz]$, P_i^m $[pu\ MW]$, P_i^{dr} $[pu\ MW]$, P_i^{tie}, P_i^v $[pu\ MW]$, are respectively the deviations in frequency, mechanical power input, responsive demand, tie-line power flow, and governor valve position. $\tau_{t,i}$, r_i, τ_i^v and u_i are respectively the turbine charging time constant, the droop coefficient, the speed-governor time constant, and the load-reference set-point for node/control area i. The control inputs u_i^m and u_i^{dr} applied to the governor and demand response loads respectively, are derived from u_i via the generation and demand participation factors, α_i^m and α_i^{dr} respectively.

State-space dynamics The (centralized) state space model for the complete power system is

$$\dot{x} = A_c x + B_c u + E_c w,\ y = C_c x \tag{2}$$

where

$$x := \begin{bmatrix} P^{tie} & f & P^m & P^v & P^{dr} \end{bmatrix}^\top \in \mathbb{R}^n,$$

$$u := \begin{bmatrix} u^m & u^{dr} \end{bmatrix}^\top \in \mathbb{R}^m,\ y := f \in \mathbb{R}^p,$$

and where a variable without subscripts denotes the vector of variables corresponding to each node; for example, $f := [f_i]_{i\in\mathcal{N}}$. Finally, the power system is subject to inequality constraints on the generation, demand response and line flows which can be expressed as the following state and input polyhedral constraints:

$$u \in \mathbb{U} \subseteq \mathbb{R}^m,\ x \in \mathbb{X} \subseteq \mathbb{R}^n. \tag{3}$$

2.1 Load-Frequency Control and Economic Dispatch

Let a constant but unknown disturbance w_i, $\forall i \in \mathcal{N}$ occur in real-time, say due to variation in renewable generation or unpredicted load changes. Then by means of LFC, the generators and controllable loads are made to adjust their power generation P_i^m, $\forall i \in \mathcal{N}$ and consumption P_i^{dr}, $\forall i \in \mathcal{N}$ respectively in order to restore the grid frequency in the most economically efficient manner. To improve the economic performance of conventional tie-line bias control, the non-interaction constraint will be relaxed allowing for the coordination of control resources across the power network in steady-state. This is achieved by regulating the frequency deviation, f rather than the area control error, ACE to zero. Towards this goal, we define the following multi-area ED problem for the LFC,

$$\begin{aligned} \min_{P^m,P^{dr}} \quad & -\Phi(P^m,P^{dr}) = \sum_{i\in\mathcal{N}} C_i(P_i^m) - \sum_{i\in\mathcal{N}} U_i(P_i^{dr}) \\ \text{s.t.} \quad & P_i^m - P_i^{dr} - P_i^{tie} - w_i = \mathbf{0}, \quad \forall i \in \mathcal{N} \end{aligned} \tag{4}$$

where $C_i(P_i^m) = \frac{1}{2}q_i(P_i^m)^2 + r_iP_i^m + s_i$ is the generator cost function and $U_i(P_i^{dr}) = \frac{1}{2}\tilde{q}_i(P_i^{dr})^2 + \tilde{r}_iP_i^{dr} + \tilde{s}_i$ the utility function for controllable loads in node i. The cost function $\Phi(P^m,P^{dr})$ is the social welfare of the power network defined as $\Phi(P^m,P^{dr}) = U_i(P_i^{dr}) - C_i(P_i^m)$. We make the following assumption about problem (4).

Assumption 2. Each $C_i(P_i^m)$ is a strictly convex function in P_i^m and each $U_i(P_i^m)$ is a strictly concave function in P_i^{dr}.

Remark 3. Assumption 2 is not restrictive as generator cost and demand utility can be fitted to quadratic functions that meet the assumption.

3. PROBLEM FORMULATION

In order to apply MPC to the LFC problem, a discrete-time model of the power system is required. We discretize the continuous-time model (2) using standard techniques to obtain the discrete time model,

$$x_{k+1} = Ax_k + Bu_k + Ew_k;\ y_k = Cx_k \tag{5}$$

where A, B, E and C are the discrete-time equivalents of the continuous-time state-space matrices A_c, B_c, E_c and C_c respectively. Given the power system dynamics (5) and a step disturbance $w_k = \bar{w}$, a forced steady-state equilibrium is given by,

$$A\bar{x} + B\bar{u} + E\bar{w} = \bar{x};\ \bar{y} = C\bar{x}. \tag{6}$$

with the steady-state input–output map

$$\bar{y} = G_u\bar{u} + G_w\bar{w} \tag{7}$$

where $G_u := C(I_n - A)^{-1}B$ and $G_w := C(I_n - A)^{-1}E$ are the DC gains of the discrete-time system power system network (5) and the quantities with an over-bar represent steady-state values. We make the following assumptions about the discrete-time dynamics (5).

Assumption 4. (Basic assumptions).

(1) the state x_k is measurable at every sampling instant.
(2) the number of outputs, p is less than or equal to the number of inputs, m.

For the multi-area ED problem (4), the constraint is the power flow balance for all nodes and is satisfied at any steady-state equilibrium of the power system network. The cost function in (4) can also be expressed compactly as,

$$-\Phi(\bar{u}) = \frac{1}{2}\bar{u}^\top Q\bar{u} + R^\top\bar{u} + s \tag{8}$$

where,
$Q = \Gamma_m^\top Q_m\Gamma_m - \Gamma_{dr}^\top Q_{dr}\Gamma_{dr}$, $R = \Gamma_m^\top R_m - \Gamma_{dr}^\top R_{dr}$, $s = \mathbb{1}_N^\top s_m - \mathbb{1}_N^\top s_{dr}$, $\Gamma_m = blkdiag(\alpha_1^m, \dots \alpha_N^m)$, $\Gamma_{dr} = blkdiag(\alpha_1^{dr}, \dots \alpha_N^{dr})$, $s_m = [s_i]_{\forall i\in\mathcal{N}}$ $s_{dr} = [\tilde{s}_i]_{\forall i\in\mathcal{N}}$, $R_m = [r_i]_{\forall i\in\mathcal{N}}$, $R_{dr} = [\tilde{r}_i]_{\forall i\in\mathcal{N}}$, $Q_m = blkdiag(q_1, \dots q_N)$, and $Q_{dr} = blkdiag(\tilde{q}_1, \dots \tilde{q}_N)$.
In order to design the LFC algorithms for the power system models presented above, we begin by defining the following optimal load-frequency control (OLFC)problem.

Problem 5. (The OLFC Problem). Design for the linear time-invariant power system (2), a state feedback load-frequency control law

$$u_k = \kappa(x_k, u_{k-1}) \tag{9}$$

such that for any unknown step change in $\bar{w}$ with $\bar{w}$ constrained to some bounded set $\mathbb{W}$:

a) the frequency deviation, f_i is regulated to zero and the tie-line flow deviation, P_i^{tie} is driven to economically optimal values for all $i \in \mathcal{N}$,
b) the steady-state values of P^m and P^{dr} maximizes the social welfare $\Phi(P^m,P^{dr})$ in real time,
c) the feedback policy $\kappa(\cdot,\cdot)$ minimizes a transient performance criterion and (3) are satisfied $\forall k \geq 0$.

Parts (a) and (b) of the OLFC problem are solved via the static optimization formulation,

$$\min_{\bar{u}} \quad -\Phi(\bar{u}) \text{ s.t. } \quad \bar{y} - G_u\bar{u} - G_w\bar{w} = \mathbf{0}. \tag{10}$$

If Assumption 2 is satisfied, then problem (10) is convex and feasible, and a unique minimizer $\bar{u}^*$ exists for every disturbance $\bar{w}$. The third control goal is achieved by designing a model predictive load-frequency control algorithm to track the optimal solution of problem (10) while rejecting the unknown step disturbance $\bar{w}$.

3.1 Karush-Kuhn-Tucker (KKT) Optimality Conditions

To design the model predictive load frequency controller that solves the optimization problem (10) in feedback, we first examine the necessary conditions for optimality of the problem. Problem (10) is a convex optimization problem and strong duality holds (Boyd et al., 2004). Therefore, the Karush–Kuhn–Tucker (KKT) conditions are necessary and sufficient for optimality. We obtain the KKT conditions for (10) by forming the corresponding Lagrangian,

$$\mathcal{L}(\bar{u},\lambda) = -\Phi(\bar{u}) + \lambda^\top(\bar{y} - G_u\bar{u} - G_w\bar{w}) \tag{11}$$

where λ is a multiplier of appropriate dimension. The corresponding KKT optimality conditions are

$$\nabla\mathcal{L}(\bar{u},\lambda) = \begin{bmatrix} -\nabla\Phi(\bar{u}) - G_u^\top\lambda \\ \bar{y} - G_u\bar{u} - G_w\bar{w} \end{bmatrix} = \mathbf{0}_{m+p}. \tag{12}$$

The optimum $\bar{u}^*$ of problem (10) must satisfy the KKT system of equations in (12). Solving (12) however requires a knowledge of $\bar{w}$ which is assumed unknown a priori. To circumvent this, we express (12) in the following subspace form (Bertsekas, 1997):

$$\nabla\mathcal{L}(\bar{u},\lambda) = \mathbf{0}_{m+p} \iff \begin{bmatrix} \nabla\Phi(\bar{u}) \in \mathrm{range}(G_u)^\top \\ \bar{y} - G_u\bar{u} - G_w\bar{w} = \mathbf{0}_p \end{bmatrix} \quad (13)$$

By a fundamental theorem of linear algebra, $\mathrm{range}(G_u)^\top = \mathrm{null}(G_u)^\perp$, and therefore

$$\nabla\Phi(\bar{u}) \in \mathrm{range}(G_u)^\top \iff \nabla\Phi(\bar{u}) \in \mathrm{null}(G_u)^\perp. \quad (14)$$

Therefore, let $\tilde{G}$ be any full-rank matrix such that,

$$\tilde{G}G_u^\top = \mathbf{0} \quad \text{or} \quad \mathrm{range}(\tilde{G})^\top = \mathrm{null}(G_u). \quad (15)$$

Remark 6. For the model (5) with the steady-state input-output map (7) (*i.e.,* the case of non-singular $(I_n - A)$, the matrix

$$\tilde{G} = (G_u^\top)^\dagger \quad (16)$$

satisfies (15) where $(G_u^\top)^\dagger$ is the pseudo-inverse of $G_u^\top$.

Remark 7. For the power system (5), $(G_u^\top)^\dagger$ depends on the connection topology of the nodes/control areas in the power network which is encapsulated in the Laplacian matrix ($\mathscr{L}_\mathscr{G}$) of the power network graph $\mathcal{G}$.

With this, the KKT optimality condition (12) becomes

$$\nabla\mathcal{L}(\bar{u},\lambda) = \begin{bmatrix} \tilde{G}\nabla\Phi(\bar{u}) \\ \bar{y} - G_u\bar{u} - G_w\bar{w} \end{bmatrix} = \mathbf{0}_{2p} \quad (17)$$

It follows that $\bar{u}$ is optimal with respect to problem (10) if, and only if, it satisfies (17). This establishes the following result, which—similar to Lawrence et al. (2018)—allows the steady-state equilibrium optimization problem to be posed as a stabilization problem.

Proposition 8. Parts (a) and (b) of the OLFC problem is solved if, from any initial state x_0 and any disturbance $\bar{w}$, the control law

$$u_k = \kappa(x_k, u_{k-1}) \quad (18)$$

is such that u_k:

(1) is regulated to a steady-state equilibrium, **and**,
(2) satisfies $\lim_{k\to\infty} \tilde{G}\nabla\Phi(u_k) = \mathbf{0}_p$.

Proof. Condition (1) is satisfied if and only if $\bar{y} - G_u\bar{u} - G_w\bar{w} = \mathbf{0}$, which is necessary and sufficient for equilibrium. Condition (2) implies, and is implied by, the KKT conditions (17) being met in the limit, which is necessary and sufficient for optimality.

4. FEEDBACK OPTIMIZING MODEL PREDICTIVE LOAD FREQUENCY CONTROL (MPLFC)

In this section, based on the results of Proposition 8, we construct an MPC controller to regulate the tracking error, $\tilde{G}\nabla\Phi(u_k)$ to zero, and consequently solve Problem 5 without knowledge of $\bar{u}^*$ or $\bar{w}$. Using the velocity model form of the linear quadratic optimal control problem (Pannocchia and Rawlings, 2001), we develop an MPC formulation that steers the power system network asymptotically and admissibly to the economically optimal steady-state equilibrium, without knowledge of this equilibrium and while minimizing a linear-quadratic (LQ) transient performance criterion. We call this controller the feedback optimizing model predictive load-frequency control (MPLFC) algorithm.

Instead of defining and regulating the tracking error as the difference between u_k and the unknown optimum $\bar{u}^*$, we define the tracking error as the residual of the KKT optimality condition (17). For the cost function $\Phi(\bar{u})$, under inactive steady-state inequality constraints and non-singular $(I_n - A)$, the tracking error $\tilde{G}\nabla\Phi(u_k)$ is an affine function of the measured input and is given by,

$$e_k := \tilde{G}\nabla\Phi(u_k) = \Lambda_u u_k + r \quad (19)$$

where $\Lambda_u = (G_u^\top)^\dagger Q$ and $r = (G_u^\top)^\dagger R$. The tracking error, (19), is related to the marginal cost differences between neighbouring control areas and is computed directly from the input u_k, provided the steady-state cost, Φ and the input–output DC gain matrix G_u are known. This choice therefore eliminates the need for knowledge of the optimal equilibrium $\bar{u}^*$ and the disturbance $\bar{w}$.

In order to achieve economic dispatch and frequency regulation simultaneously, the power system frequency dynamics (2) is regulated to steady-state equilibria such that $e = 0$ and $\bar{y} = 0$. To achieve this, we adopt the following velocity form of (5),

$$\epsilon_{k+1} = A_\epsilon \epsilon_k + B_\epsilon \delta u_k \quad (20a)$$
$$\tilde{e}_k = C_\epsilon \epsilon_k + D_\epsilon \delta u_k \quad (20b)$$

where

$$\epsilon_k := \begin{bmatrix} \delta x_k \\ \tilde{e}_{k-1} \end{bmatrix} \quad \text{with} \quad \begin{matrix} \delta x_k := x_k - x_{k-1}, \delta u_k := u_k - u_{k-1}, \\ \tilde{e}_k := e_k + \Pi y_k \end{matrix} \quad (21)$$

and

$$A_\epsilon = \begin{bmatrix} A & \mathbf{0}_{n\times p} \\ \Pi C & I_p \end{bmatrix}, \qquad B_\epsilon = \begin{bmatrix} B \\ \Lambda_u \end{bmatrix}, \quad (22a)$$
$$C_\epsilon = \begin{bmatrix} \Pi C & I_p \end{bmatrix}, \qquad D_\epsilon = \Lambda_u. \quad (22b)$$

The constant Π is a penalty on the output y, and $\tilde{e}$ is a weighted sum of e and y. We make the following assumption about the system (5) and cost $\Phi(\bar{u})$.

Assumption 9. The matrix

$$S := \begin{bmatrix} A - I_n & B \\ \Pi C & \Lambda_u \end{bmatrix} \quad (23)$$

has full row rank, *i.e.,* $\mathrm{rank}(S) = p + m$.

The following proposition can be easily proved.

Proposition 10. (Reachability). The pair (A_ϵ, B_ϵ) is reachable if and only if (A, B) is reachable and Assumption 9 is satisfied.

Given the tracking error and velocity dynamics, the feedback optimizing model predictive load frequency control (MPLFC) problem is defined, for a state ϵ_k, as

$$\begin{aligned} \min_{\delta\mathbf{u}_k} V_N(\epsilon_k, \delta\mathbf{u}_k) &= V_f(\epsilon_{k+N}) + \frac{1}{2}\sum_{i=0}^{N-1} l(\tilde{e}_{k+i}, \delta u_{k+i}) \\ \text{s.t.} \quad \epsilon_{k+i+1} &= A_\epsilon \epsilon_{k+i} + B_\epsilon \delta u_{k+i} \ \forall i \in \mathbb{I}_{[0,N-1]}, \\ \tilde{e}_{k+i} &= C_\epsilon \epsilon_{k+i} + D_\epsilon \delta u_{k+i} \ \forall i \in \mathbb{I}_{[0,N-1]}, \\ &(\delta\mathbf{u}_k, \boldsymbol{\epsilon}_k) \in \mathbb{G}, \ \epsilon_{k+N} \in \mathbb{E}_f. \end{aligned} \quad (24)$$

In this problem, $\mathbb{I}_{[0,N-1]}$ is the set of positive integers from 0 to $N-1$, and the decision variable is the sequence of control increments over the N-step prediction horizon:

$$\delta\mathbf{u}_k := \{\delta u_k, \delta u_{k+1} \ldots, \delta u_{k+N-1}\}, \tag{25}$$

and $\boldsymbol{\epsilon}_k$ is the associated sequence of state predictions:

$$\boldsymbol{\epsilon}_k := \{\epsilon_k, \epsilon_{k+1} \ldots, \epsilon_{k+N-1}\}. \tag{26}$$

These sequences are chosen to minimize the objective $V_N(\epsilon_k, \delta\mathbf{u}_k)$, which consists of a stage cost

$$l(\tilde{e}_k, \delta u_k) := \tilde{e}_k^\top Q_e \tilde{e}_k + \delta u_k^\top R \delta u_k \tag{27}$$

and a terminal cost

$$V_f(\epsilon_{k+N}) := (1/2)\epsilon_{k+N}^\top P_\epsilon \epsilon_{k+N}. \tag{28}$$

It is simple to verify that the following assumption ensures positive definiteness of this cost.

Assumption 11. The matrices R, Q_e and P_ϵ satisfy $R \succ 0$, $P_\epsilon \succeq 0$ and

$$Q_e - Q_e D_\epsilon (R + D_\epsilon^\top Q_e D_\epsilon)^{-1} D_\epsilon^\top Q_e^\top \succeq 0 \tag{29}$$

The terminal cost is employed, in the usual way (Rawlings and Mayne, 2009), towards guaranteeing stability.

Assumption 12. The matrix $P_\epsilon \succeq 0$ satisfies the Lyapunov equation

$$\begin{aligned}(A_\epsilon + B_\epsilon K_\epsilon)^\top P_\epsilon (A_\epsilon + B_\epsilon K_\epsilon) - P_\epsilon = -[C_\epsilon^\top Q_e C_\epsilon + \\ 2C_\epsilon^\top Q_e D_\epsilon K_\epsilon + K_\epsilon^\top R_\epsilon K_\epsilon]\end{aligned} \tag{30}$$

where K_ϵ is such that $A_\epsilon + B_\epsilon K_\epsilon$ is Schur.

Finally, the constraint set $\mathbb{G}$ enforces the constraints (3):

$$\mathbb{G} := \left\{(\delta\mathbf{u}_k, \boldsymbol{\epsilon}_k) | (u_{k+i}, x_{k+i}) \in \mathbb{U} \times \mathbb{X}, i = 0, \ldots, N-1\right\}.$$

and the set $\mathbb{E}_f$ is a terminal set constructed such that

$$\begin{aligned}\epsilon_{k+N} \in \mathbb{E}_f \implies (\epsilon_{k+N}, K_\epsilon \epsilon_{k+N}) \in \mathbb{X} \times \mathbb{U} \text{ and} \\ (A_\epsilon + B_\epsilon K_\epsilon)\epsilon_{k+N} \in \mathbb{E}_f\end{aligned} \tag{31}$$

The main challenge is reformulating the constraints on u and x in terms of the optimization variables δu and ϵ. Inspired by Betti et al. (2013), we write the relation between these variables as the dynamic system

$$\begin{bmatrix}\epsilon_{k+i+1} \\ r\end{bmatrix} = \mathcal{A}\begin{bmatrix}\epsilon_{k+i} \\ r\end{bmatrix} + \mathcal{B}\delta u_{k+i} \tag{32}$$

$$\begin{bmatrix}x_{k+i} \\ u_{k+i-1}\end{bmatrix} = \mathcal{C}\begin{bmatrix}\epsilon_{k+i} \\ r\end{bmatrix} + \mathcal{D}\bar{w}. \tag{33}$$

where

$$\mathcal{A} := \begin{bmatrix}A_\epsilon & \mathbf{0}_{(n+p)\times p} \\ \mathbf{0}_{p\times(n+p)} & I_p\end{bmatrix}, \quad \mathcal{B} := \begin{bmatrix}B_\epsilon \\ \mathbf{0}_{p\times m}\end{bmatrix},$$

and

$$\mathcal{C} := \begin{bmatrix}A & B \\ \mathbf{0}_{m\times n} & I_m\end{bmatrix} S^{-1} \begin{bmatrix}I_n & \mathbf{0}_{n\times p} & \mathbf{0}_{n\times p} \\ \mathbf{0}_{p\times n} & I_p & -I_p\end{bmatrix},$$

$$\mathcal{D} := \begin{bmatrix}E \\ \mathbf{0}_{m\times n}\end{bmatrix} - \begin{bmatrix}A & B \\ \mathbf{0}_{m\times n} & I_m\end{bmatrix} S^{-1} \begin{bmatrix}E \\ \mathbf{0}_{p\times n}\end{bmatrix}.$$

Imposition of constraints $\mathbb{G}$ is then realized by imposing

$$\mathcal{C}\begin{bmatrix}\epsilon_{k+i} \\ r\end{bmatrix} \in (\mathbb{X} \times \mathbb{U}) \ominus \mathcal{D}\mathbb{W}, \ \forall i \in [0, \ldots, N-1]. \tag{34}$$

The terminal set $\mathbb{E}_f$ is defined as the (projection of the) maximal constraint admissible set $\mathcal{O}_\infty$ (Rawlings and Mayne, 2009) for the system (32) under the terminal control law $\delta u_{k+i} = K_\epsilon \epsilon_{k+i}$, $i \geq N$:

$$\mathbb{E}_f := \left\{\epsilon : \begin{bmatrix}\epsilon \\ r\end{bmatrix} \in \mathcal{O}_\infty\right\}.$$

Solution of this optimal control problem, followed by the application of the first control in the optimized sequence, yields the MPLFC control law,

$$\delta u_k = \kappa(\epsilon_k, u_{k-1}).$$

4.1 Stability and Performance Guarantees

For the following analysis, we assume that the disturbance $\bar{w}$ stays constant (otherwise steady-state operation is not well defined). The following result summarizes the stability and recursive feasibility of the MPLFC algorithm, and follows directly from well established results on conventional linear MPC (Rawlings and Mayne, 2009).

Theorem 13. (Stability and feasibility). The control law $u_k = u_{k-1} + \kappa(\epsilon_k, u_{k-1})$ solves the MPLFC problem (5).

5. NUMERICAL SIMULATION

To illustrate the performance of the MPLFC algorithm, we perform numerical simulations of the controller on the two-area power system (See Fig. 1) from Example 12.4 in (Saadat, 1999) with parameters: $H_i = \{4, 3\}$, $D_i =$

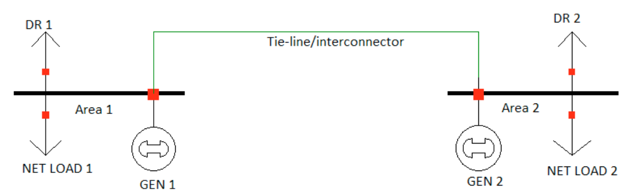

Fig. 1. two-area power system network

$\{0.6, 0.9\}$, $\tau_{v,i} = \{0.2, 0.3\}$, $\tau_{t,i} = \{0.5, 0.6\}, \tau_i^{dr} = \{1, 1\}$, $\alpha_i^{dr} = \{0.5, 0.5\}$, $\alpha_i^m = \{0.5, 0.5\}$, $q_i = \{1, 0.5\}$, $r_i = \{0.5, 0.8\}$, $s_i = \tilde{s}_i = \mathbf{0}$, $\tilde{q}_i = \{0.1, 0.5\}$, $\tilde{r}_i = \{0.1, 0.1\}$, $T_{12} = T_{21} = 2$, where $i \in \{1, 2\}$. The system is stabilizable and observable, and subject to the following inequality constraints on the input, output and disturbances:

$$\mathbb{U} := \left\{-5I_2 \leq u \leq 5I_2\right\}, \mathbb{Y} := \left\{-2I_2 \leq y \leq 2I_2\right\}, \\ \text{and } \mathbb{W} := \left\{-2I_2 \leq w \leq 2I_2\right\}$$

The disturbance, $w(t)$, is unknown but slowly-varying as

$$w(t) = \begin{cases}\begin{bmatrix}0.4 & 0\end{bmatrix}^\top & 5 \leq t < 55 \\ \begin{bmatrix}0 & 0.9\end{bmatrix}^\top & 55 \leq t < 105 \\ \begin{bmatrix}1.5 & 0.1\end{bmatrix}^\top & t \geq 105\end{cases} \tag{35}$$

The objective function for the steady-state ED problem is derived from the cost parameters and a penalty of $\Pi = 400I_2$ on each output of the power system. To design the MPLFC, the system is discretized using zero-order hold with a sampling time of 0.1 seconds. The transient performance criterion is chosen with $Q_e = 0.1 \times I_2$ and $R_\delta = 500I_2$. The MPLFC control law is then designed with the prediction horizon, $N = 5$. We compare the performance of MPLFC to a conventional MPC algorithm, and a centralized version of the distributed averaging PI (DAPI) controller from (Molzahn et al., 2017). Figure 2 show the performance of the proposed MPLFC algorithm in comparison to DAPI and a standard MPC algorithm in the second control area. From the figures, the proposed MPLFC shows much improved transient performance compared to DAPI with less oscillations, a faster convergence to the economic dispatch setpoints and guaranteed constraint satisfaction. Also, the conventional tracking MPC-based LFC can be seen to regulate the frequency and tie-line deviations to zero for both control areas. However,

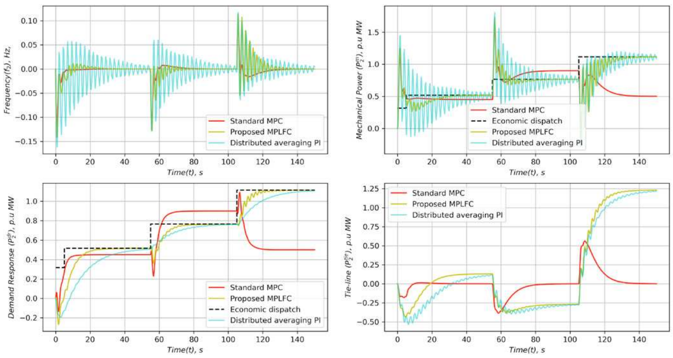

Fig. 2. LFC Performance in Area/Node 2

it fails to achieve this using inputs that are economically dispatching. This is evident as the inputs can be seen to not track the economic dispatch values shown in dashed lines.

6. CONCLUSION

An MPC-based approach to the combined ED and LFC in a multi-area power system has been presented. The approach uses a form of MPC that combines steady-state optimization and tracking to provide a LFC law that drives the power system to the ED equilibria, without explicitly computing this and using it as an explicit setpoint. Recursive feasibility and stability of the closed-loop were established under mild conditions on the system, cost, and constraints. Simulation results on a two-area network have demonstrated the capability of the approach for frequency restoration while tracking a changing economic equilibrium. Comparisons with standard approaches in the literature have also been made. Results show that the MPLFC algorithm developed shows superior performance to the other two controllers (DAPI and tracking MPC). Also, because MPLFC uses the same assumptions as a standard tracking MPC, it is easier to implement on real systems, and as shown in (Mc Namara and Milano, 2018) is applicable to systems with high renewable generation and low inertia. Future work will consider decentralized and distributed solutions, more detailed implementation using more realistic models, data communication issues such as delays, and also the more realistic setting of output measurements with state estimation.

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