Low Complexity Hybrid Precoding Algorithm for THz Ultra-massive MIMO Systems

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***Abstract* - The hybrid precoding problem in point-to-point TeraHertz (THz) multiple-input multiple-output (MIMO) systems can be solved by minimizing the Frobenius norm between the digital precoder and the hybrid precoder. To achieve this, we developed an alternating minimization algorithm, the MBCD-HP for mmWave massive MIMO systems. However, the complexity can be very high when the number of antennas increases, as required in THz transmission. In this paper, we improve MBCD-HP algorithm by utilizing the structural properties of the optimization matrix. The simulation results demonstrate that the proposed method significantly reduces the computational complexity and improves the performance.**

Keywords – TeraHertz MIMO; hybrid precoding; trace optimization; alternating minimization; block coordinate descent.

I. INTRODUCTION

Research on TeraHertz (THz) communication has attracted significant attention due to its vast unused bandwidth for wireless communication [1]. However, their shorter wavelengths cause high attenuation, necessitating the use of ultra-massive antenna arrays to compensate for the loss [2]. Meanwhile, the high cost and power consumption of radio frequency (RF) chains make fully digital precoding in THz MIMO challenging [3]. Hybrid precoding can support ultra-massive MIMO with fewer RF chains, making it a promising technology for THz communications [4].

The hybrid precoder can be obtained by minimizing its Euclidean distance from the optimal fully digital precoder [5]. To achieve this in mmWave massive MIMO, we translated the Frobenius norm minimization problem into a trace minimization problem in [6], and developed an alternating minimization algorithm called MBCD-HP, which uses the Block Coordinate Descent (BCD) algorithm [7] and Lagrange's method to solve the analog and digital precoding subproblems respectively. However, when the number of transmitter antennas increases, as in the ultra-massive MIMO applied in THz, the MBCD algorithm can become computationally complex due to the growing size of the optimization matrix and the increasing number of loops.

This paper modifies the MBCD-HP algorithm to reduce its complexity, and improve its performance. The proposed method utilizes the optimization matrix's special structure and only involves the first few columns in the computation instead of the entire matrix, reducing the number of operations in a single loop to the number of RF chains instead of the number of transmitter antennas. This significantly reduces the algorithm's runtime, particularly in ultra-massive MIMO scenarios, and diminishes the overwriting of variables in the objective matrix, leading to better minimization performance. The simulation results demonstrate that the proposed algorithm outperforms high-performance precoders such as MO-AltMin [8], HD-LSR [9], OMP [5], IPM [6] and MBCD-HP, while requiring much lower complexity than MO-AltMin and MBCD-HP algorithms.

II. SYSTEM MODEL

We consider a single user THz MIMO downlink system with fully-connected architecture as in [6]:

- The channel matrix: $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$
- The optimal fully digital precoder: $\mathbf{F}_{opt} \in \mathbb{C}^{N_t \times N_s}$
- The hybrid precoder: $\mathbf{F} = \mathbf{F}_R \mathbf{F}_D$, s.t. $\|\mathbf{F}\|_F^2 = N_s$
- The baseband precoder: $\mathbf{F}_D \in \mathbb{C}^{M_t \times N_s}$
- The analog precoder: $\mathbf{F}_R \in \mathbb{C}^{N_t \times M_t}$, s.t. $|\mathbf{F}_{R\,i,j}| = 1$
- The number of transmit antennas: N_t
- The number of receive antennas: N_r
- The number of transmitter RF chains: M_t
- The number of transmitted streams: N_s

III. PROBLEM STATEMENT

From [5], it follows that the hybrid precoder is given by

$$\begin{aligned} &\underset{\mathbf{F}_R, \mathbf{F}_D}{\arg\min} \| \mathbf{F}_{opt} - \mathbf{F}_R \mathbf{F}_D \|_F^2 \\ &\text{s.t. } \| \mathbf{F}_R \mathbf{F}_D \|_F^2 = N_s, \ |\mathbf{F}_{R_{i,j}}| = 1, \forall i, j \end{aligned} \tag{1}$$

The optimal fully digital precoder $\mathbf{F}_{\text{opt}}$ contains the leading N_s columns of right singular vectors of the channel matrix $\mathbf{H}$. [6] expresses the objective function in (1) in terms of trace as

$$\mathrm{Tr}[\mathbf{F}_{opt}\mathbf{F}_{opt}^H - \mathbf{F}_R\mathbf{F}_D\mathbf{F}_{opt}^H - \mathbf{F}_{opt}\mathbf{F}_D^H\mathbf{F}_R^H + \mathbf{F}_R\mathbf{F}_D\mathbf{F}_D^H\mathbf{F}_R^H] \tag{2}$$

The analog precoding subproblem is given by [6]

$$\begin{aligned} &\underset{\mathbf{F}_R}{\arg\min}\, \mathrm{Tr}[\mathbf{F}_R\tilde{\mathbf{F}}_D\tilde{\mathbf{F}}_D^H\mathbf{F}_R^H - \mathbf{F}_R\tilde{\mathbf{F}}_D\mathbf{F}_{opt}^H - \mathbf{F}_{opt}\tilde{\mathbf{F}}_D^H\mathbf{F}_R^H] \\ &\text{s.t. } |\mathbf{F}_{R_{i,j}}| = 1, \forall i, j \end{aligned} \tag{3}$$

where $\tilde{\mathbf{F}}_D$ is the un-normalized digital precoder discarding the power constraint in (1). The normalization of the digital precoder can be performed afterwards, which still makes the Euclidean distance sufficiently small [10].

IV. PROPOSED IMPROVED MBCD-HP ALGORITHM

To solve the optimization problem in (3), [6] redefines the problem (3) as a semi-definite programming problem (SDP) in $\mathbf{X}$ as

$$\begin{aligned} &\underset{\mathbf{X} \in \mathbf{H}_m}{\arg\min}\, \mathrm{Tr}[\mathbf{M}\mathbf{X}], \text{ s.t. } \mathbf{X} \succeq 0 \\ &\mathbf{M} = \begin{bmatrix} \tilde{\mathbf{F}}_D\tilde{\mathbf{F}}_D^H & -\tilde{\mathbf{F}}_D\mathbf{F}_{opt}^H \\ -\mathbf{F}_{opt}\tilde{\mathbf{F}}_D^H & \mathbf{0} \end{bmatrix}, \ \mathbf{X} = \begin{bmatrix} \mathbf{F}_R^H\mathbf{F}_R & \mathbf{F}_R^H \\ \mathbf{F}_R & \mathbf{I}_{N_t} \end{bmatrix} \end{aligned} \tag{4}$$

We notice in matrix $\mathbf{X}$ that the last N_t columns are the Hermitian of the last N_t rows, and this structure can be utilized to reduce computation in MBCD. The improved MBCD is described in Algorithm 1. As shown in Algorithm 1, we scan only the first M_t columns of $\mathbf{X}$ in the second *for* loop instead of all the columns as in the MBCD algorithm in [6], and use the Hermitian to compute the last N_t columns. In this way, the computations in the second *for* loop are reduced to M_t columns instead of $M_t + N_t$ columns. And owning to the fact that N_t is significantly larger than M_t, especially in ultra-massive MIMO scenario, the improvement can substantially reduce the runtime of the MBCD algorithm. Furthermore, since the last N_t rows of

X will not be overwritten by computing the last N_t columns, the improved algorithm shows better performance.

Algorithm 1 Improved MBCD Algorithm

Require: **M**, M_t, feasible initial $n \times n$ matrix **X**.

Choose $v > 0$ such that $v \to 0$ and integer $N_{it} > 1$.
for $k = 1, \ldots, N_{it}$ **do**
  for $i = 1, \ldots, M_t$ **do**
    Compute $\mathbf{z} = [1, \ldots, i\text{-}1, i\text{+}1, \ldots, n]$.
    Compute $\mathbf{Y} = \mathbf{X}_{\mathbf{z},\mathbf{z}}\mathbf{M}_{\mathbf{z},i}$.
    Compute $\zeta = \mathbf{Re}(\mathbf{Y}^H\mathbf{M}_{\mathbf{z},i})$.
    if $\zeta > 0$ **then**
$$\mathbf{X}_{\mathbf{z},i} = \mathbf{X}_{i,\mathbf{z}}^H = -\sqrt{\frac{1-v}{\zeta}}\mathbf{Y}\cdot$$
    else
$$\mathbf{X}_{\mathbf{z},i} = \mathbf{X}_{i,\mathbf{z}}^H = 0\cdot$$
    end if
    $\mathbf{X}_{i,i} = 1$.
  end for
end for
return X

V. SIMULATION RESULTS

To evaluate the performance, we adopt narrow-band block-fading channel model in this paper similar to [6] and other existing precoding algorithms. The values of different parameters considered in simulations are $N_s = 4$, $M_t = N_s$, $N_r = 16$, $N_t = 64$, except in Figure 1 where N_t is variable. It should be noted that values of N_t and N_r used in simulations are smaller than the typical values used in THz ultra-massive MIMO to save simulation time. The parameters in Algorithm 1 are used with $v = 0.01$ and $N_{it} = 12$.

Figure 1 shows the average runtime of different precoding algorithms as a function of the number of transmit antennas. Figure 2 and Figure 3 show the performance in terms of spectral efficiency and bit error rate as a function of SNR respectively. The complexity of Improved MBCD-HP is much smaller than MO-Altmin and existing MBCD-HP as proven by Figure 1. The complexities of MO-Altmin and existing MBCD-HP are going to be worse when operating in THz MIMO with higher N_t and N_r. Improved MBCD-HP exhibits the best spectral and BER performances among all the hybrid precoders. Moreover, OMP and HD-LSR can only work in narrowband channel, unlike the proposed Improved MBCD-HP.

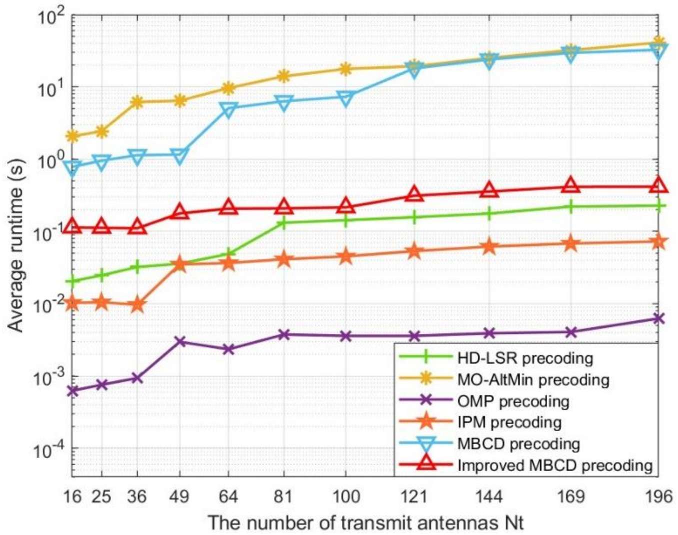

Figure 1. Average runtime as a function of N_t

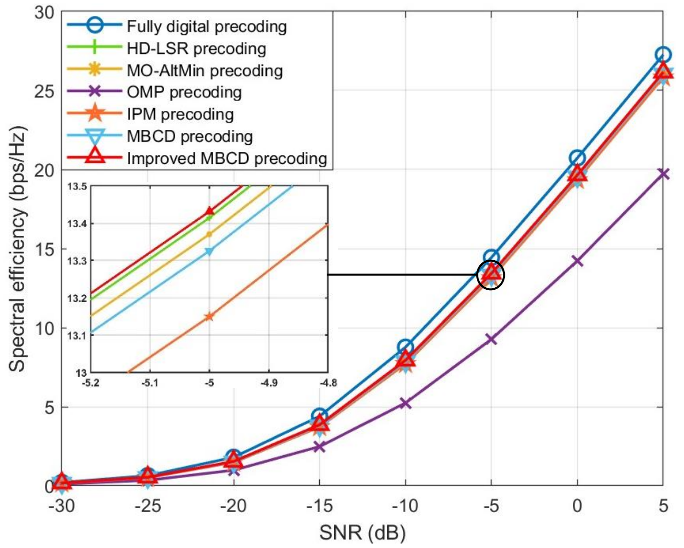

Figure 2. Spectral efficiency as a function of SNR

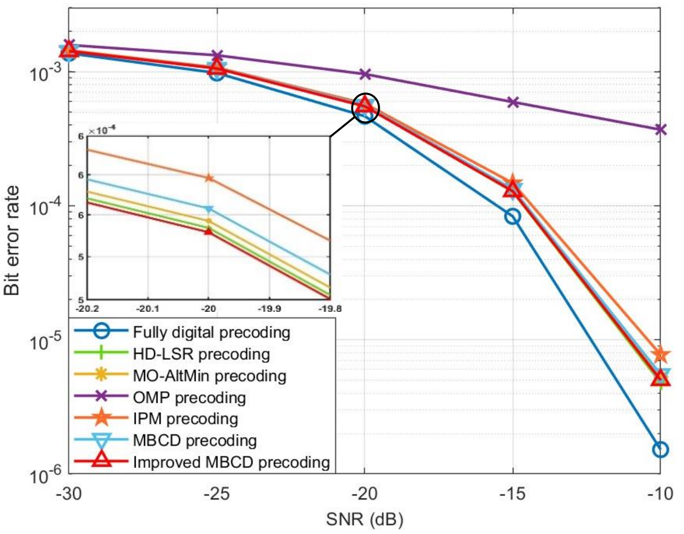

Figure 3. Bit error rate as a function of SNR

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