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Chowdhury, S.M., Kovenock, D., Rojo Arjona, D. et al. (1 more author) (2021) Focality and asymmetry in multi-battle contests. The Economic Journal, 131 (636). pp. 1593-1619. ISSN 0013-0133

https://doi.org/10.1093/ej/ueaa130

Published as: Chowdhury, S.M., Kovenock, D., Rojo-Arjona, D., & Wilcox, N. (2020). Focality and Asymmetry in Multi-battle Contests. Economic Journal, Forthcoming.

Focality and Asymmetry in Multi-battle Contests

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September 30, 2020

Abstract

This article examines the influence of focality in Colonel Blotto games with a lottery contest success function, where the equilibrium is unique and in pure strategies. We hypothesize that the salience of battlefields affects strategic behaviour (the salient target hypothesis) and present a controlled test of this hypothesis against Nash predictions, checking the robustness of equilibrium play. When the sources of salience come from asymmetries in battlefield values or labels (as in Schelling, 1960), subjects over-allocate the resource to the salient battlefields relative to the Nash prediction. However, the effect is stronger with salient values. In the absence of salience, we find support for the Nash prediction.

JEL: C72, C91; D74

Keywords: Conflict; Experiment; Colonel Blotto; Focal point; Asymmetry

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We have benefited from the comments of Sebastian Cortes-Corrales, Jason Shachat, Roman Sheremeta, Robert Sugden, Daniel Zizzo, participants at the Workshop on Strategic Aspects of Terrorism, Security and Espionage at Stony Brook University, the Munich Workshop on Behavior and Terrorism, the 2016 Latin American Meetings of the Econometric Society in Medellin and seminar participants at the Max Planck Institute for Tax Law and Public Finance, Calcutta, Chapman, Durham/Newcastle, East Anglia, Middlesex, Sangkyunkwan and Ajou. David Rojo Arjona acknowledges generous funding from the Economic Science Institute. Any remaining errors are our own.

1 Introduction

This article experimentally examines the influence of "focality" or "salience" on individual behaviour in a class of games in which the Nash equilibrium is unique and in pure strategies.[1] Specifically, we investigate the robustness of equilibrium in a class of multi-battle contests known as the Colonel Blotto game. In this game, two players simultaneously allocate their respective endowments, or "budgets", of a one-dimensional resource (such as effort, money, time, or troops) across a set of "battlefields". Each player's allocation to a given battlefield determines the likelihood that the player wins the battlefield, with the probability of winning given by the player's own expenditure divided by the sum of the two players' expenditures (hence, the term "lottery").[2]

Our approach lies at the intersection of two distinct literatures, the literature on contests (in particular, multi-battle contests and Colonel Blotto games) and the literature on salience and focality. Both topics originated with work from renown scholars (Borel, 1921 and Schelling, 1960, respectively) and have been the focus of substantial recent activity.

Colonel Blotto games and other closely related multi-battle contests are widely applied in the formal modelling of strategic behaviour in the context of military operations, network and infrastructure security, and anti-terrorism defence.[3] In these environments, the salience of specific battlefields or targets often influences the strategic allocation of resources in the conflict. We call this observation the *salient target hypothesis*. For instance, during the US military invasion of Iraq in 2003, the capture of high-profile targets such as Saddam Hussein and the killing of his sons, Uday and Qusay were viewed as having significant symbolic value. Similarly, the killing of Abu Musab al-Zarqawi was viewed as having higher symbolic than tactical benefit. In the context of terrorism, Nemeth (2010)(p.30) argues that the 2000 suicide attack on the USS Cole in the Yemeni port of Aden involved a well-defended target that had high symbolic value despite the relatively few casualties involved. More well-known attacks on salient targets include the Black September terrorist attack on the Israeli Olympic team at the 1972 Munich Olympics and the September 11, 2001 al-Qaeda terrorist attacks on the World Trade Center and the Pentagon. In the context of information network security, young hackers often gain fame by attacking salient targets. In 1999, Jonathan James became notorious for his hack of NASA and The US Department of Defense. In 2000, Michael Calce (a.k.a. MafiaBoy) became famous for his distributed denial of service attacks on Amazon, CNN, eBay and Yahoo!

In each of these cases it can be argued that the salience of a target by itself induces a higher value to successful attack than less salient targets. For instance, in the case of military campaigns a high-profile victory may have positive implications for morale at home and induce an enemy population to become discouraged. High profile terrorist attacks may induce fear in the intended target population, place pressure on the target governments, and aid in both fundraising and recruiting among potential supporters. Despite this *perceived* positive association between the salience of a target and higher value, it is not clear whether

[1]The terms focality, salience, prominence are used almost interchangeably in the literature. There is a tendency to refer to focal points or focality when referring to behaviour and salience or prominence when describing an aspect or element of the game that generates that behaviour. In this paper we will conform to this usage.

[2]Randomization is independent across battlefields and in the event of zero aggregate expenditure to a battlefield, the probability that each player wins is $\frac{1}{2}$.

[3]See, for instance, Shubik and Weber (1981), Powell (2007b,a, 2009), Powers and Shen (2009), Kovenock *et al.* (2019), Chia and Chuang (2011), Arce *et al.* (2011), Fuchs and Khargonekar (2012), Nikoofal and Zhuang (2012), Arce *et al.* (2012), Bachrach *et al.* (2013), Gupta *et al.* (2014a,b), Hausken (2014), Goyal and Vigier (2014). Other applications include the optimal allocation of advertising budgets across markets or marketing budgets across channels (Friedman, 1958), the allocation of political campaign funds across different primaries or state races (Snyder, 1989; Laslier, 2002; Klumpp and Polborn, 2006; Kovenock and Roberson, 2009a), redistributive politics (Myerson, 1993; Lizzeri and Persico, 2001; Roberson, 2008; Kovenock and Roberson, 2008, 2009b; Crutzen and Sahuguet, 2009), and the allocation R&D budgets across several potential classes of innovations or projects (Clark and Konrad, 2008). See Kovenock and Roberson (2012) for a review.

the levels of resources allocated to attack such targets are optimal given the induced value of the target, or whether the salience of the target itself has an influence independent of its induced value. In fact, it appears very difficult to even address this question with case studies or other data from the field, since real-world combatants' actual target values are unobservable. The purpose of this article is to investigate the salient target hypothesis by carrying out a laboratory experiment that separates out the positive effects of target value and salience in the determination of resource allocation in multi-battle contests. To our knowledge, this article is the first to examine the joint role that value and salience play in influencing strategic behaviour in such games.

The theoretical origin of the Colonel Blotto game traces back to Borel (1921), who anticipated its future application, particularly to military strategy.[4] McDonald and Tukey (1949) refer to the use of Colonel Blotto games in military research at Princeton during World War II and a substantial amount of work on the game was undertaken in the late 40s and 50s in the context of military operations research (Gross and Wagner, 1950; Gross, 1950; Blackett, 1953, 1954, 1958). Since that time, work on the problem has been sporadic (including Bellman, 1969; Shubik and Weber, 1981), until very recently, when a resurgence of interest has taken place.[5]

In addition to this extensive theoretical literature, there is a young and rapidly growing experimental literature on Blotto games. Several existing contributions employ an auction contest success function (CSF) to determine the winner of each battlefield. Some of these reward subjects based on the total number or value of battlefields won (Avrahami and Kareev, 2009; Arad and Rubinstein, 2012; Cinar *et al.*, 2012; Chowdhury *et al.*, 2013) or reward the winner of a randomly selected battlefield (Avrahami *et al.*, 2014). Others reward subjects who win either a majority of the battlefields or more than half of the total value of battlefields (Arad, 2012; Montero *et al.*, 2016; Mago and Sheremeta, 2017). Experimental evidence on multi-battle contests employing a lottery CSF is still limited. Chowdhury *et al.* (2013) examine Colonel Blotto games with a lottery CSF in which subjects are rewarded based on the number of battlefields won. Mago and Sheremeta (2019) carry out an analysis of the case where the reward is based on attaining a majority of the battlefields. Duffy and Matros (2017) examine Colonel Blotto games with values that are heterogeneous across battlefields but symmetric across players and treat both a reward structure linear in the value of battlefields won and one that rewards winning critical values of more than half of the total value of the battlefields.[6]

There is also an experimental literature investigating the power of focal points. Starting with Schelling (1960), successful coordination in games with multiple equilibria has been hypothesized to arise because players' beliefs go through a process of "mutual accommodation" anchored by the focality of some equilibria (e.g., Grand Central Station seems a natural place to meet in New York). This mutual accommodation is partially supported by experimental evidence (see Mehta *et al.*, 1994; Bardsley *et al.*, 2010; Crawford *et al.*, 2008; Dugar and Shahriar, 2012; Isoni *et al.*, 2013). Schelling (1960)(p.84) also claims that a lack of common interest should prevent such mutual accommodation in games of pure conflict. Unfortunately, the only evidence to date on games of pure conflict comes exclusively from Hide-and-Seek games, which have a unique equilibrium in nondegenerate mixed-strategies (Rubinstein and Tversky, 1993; Rubinstein *et al.*, 1997;

[4] The original problem posed by Borel was not solved until Borel and Ville (1938).

[5] See, for instance, Robson (2005), Roberson (2006, 2008), Clark and Konrad (2007), Kvasov (2007), Powell (2007b,a), Hart (2008), Golman and Page (2009), Adamo and Matros (2009), Powell (2009), Kovenock *et al.* (2019), Kovenock and Roberson (2011), Hortala-Vallve and Llorente-Saguer (2012), Kovenock and Roberson (2012), Roberson and Kvasov (2012), Rinott *et al.* (2012), Thomas (2017), Weinstein (2012), Rietzke and Roberson (2013), Hernández and Zanette (2013), Washburn (2013), Osorio (2013), McBride and Hewitt (2013), Sela and Erez (2013), Dziubiński (2013), Barelli *et al.* (2014), Gupta *et al.* (2014a,b), Goyal and Vigier (2014), Hart (2015), Schwartz *et al.* (2014), Macdonell and Mastronardi (2015), and Kovenock and Roberson (2020).

[6] Kovenock *et al.* (2019) also examine, under both the auction and lottery CSFs, the case in which players have asymmetric objectives; one player has a best-shot objective and the other a weakest-link objective.

Rubinstein, 1999; Hargreaves Heap *et al.*, 2014). In this type of game, there is already evidence of the inability of subjects to play the equilibrium in mixed strategies, even in the absence of salience (Palacios-Huerta and Volij, 2008; Levitt *et al.*, 2010; Wooders, 2010). Therefore, it is not surprising that subjects do not play the equilibrium mixed strategies in Hide-and-Seek games in the presence of salience. Given this confound, it is not clear whether, contrary to Schelling's expectations, salience matters in the absence of common interest. However, the Colonel Blotto games examined in this article have a unique equilibrium in pure strategies. Consequently, our game allows us to test for the effect of salience, without important confounding factors and allowing control for other experimental details. This provides a clean test of Schelling's hypothesis in a prototype game of pure conflict.

In this article we examine two-person, four-battlefield, constant-sum Colonel Blotto games in which each player earns the sum of the values of the battlefields that he wins, budgets are use-it-or-lose-it, and a lottery CSF (Tullock, 1980) is employed in each battlefield. In this class of games, the unique Nash equilibrium is for the allocation to each battlefield to be proportional to the value of the battlefield and for the sum of the allocations to exactly exhaust the player's budget (Friedman, 1958). The recent experimental research on such games provides substantial support for Nash equilibrium behaviour in games with values that are symmetric across both players and battlefields. Chowdhury *et al.* (2013) find support for the equilibrium prediction that each player should divide his budget equally across battlefields.[7] Duffy and Matros (2017) claim qualitative but not quantitative support for the Nash equilibrium prediction in games in which battlefield values are symmetric across players, but not across battlefields. In light of this previous work, we believe that support for the equilibrium prediction in these studies, to the extent that it exists, arises from the absence of asymmetries in the battlefields that yield focal behaviour. In line with the salient target hypothesis, deviations from the Nash prediction may arise in the presence of certain asymmetries in the primitives or visual representation of the game, such as battlefield values or labels.[8]

We test this hypothesis by employing an experimental design in which subjects are randomly matched and face such asymmetries in each of these features, while keeping the other features symmetric. In the case of an asymmetric label, a single battlefield is coloured differently from the others (black instead of white). In the case of an asymmetric value, one battlefield has a higher value than the others. We single out one battlefield (target) from the others to address a concern that arises in the econometric analysis of the game: because of the constraint that the sum of battlefield allocations must equal the subject's budget, a negative correlation of bids is induced across battlefields. In our experimental setting, all the information about bidding behaviour in the Blotto game collapses into statements about the bidding behaviour in the target battlefield. This avoids the improper assumption of independence of allocations across battlefields.

In the completely symmetric case, we find strong support for Nash behaviour. However, consistent with the salient target hypothesis, subjects deviate from Nash behaviour in the presence of asymmetries in labels and battlefield values. With asymmetries, subjects over-allocate relative to the Nash equilibrium prediction in the saliently labelled and high value battlefields. Moreover, as might be expected given its payoff irrelevance, asymmetry in labels has a weaker effect than in values. With asymmetric values, the high value battlefield

[7] Aberrations from the normative recommendations are explained by behavioural anomalies. A subset of these may disappear with a more controlled design (e.g., order effects due to the graphical representation of the battlefields) while others represent a minor but intrinsic tendency (e.g., natural inclination to choose round numbers). Our experimental design provides a tighter test removing successfully the first subset of anomalies and controlling for the second.

[8] Avrahami *et al.* (2014) find that subjects allocate zero to some battlefields and overbid in battlefields with a higher probability to be selected to determine the final winner. One could interpret this result as evidence for the salient target hypothesis in multibattle contests with an auction CSF. However, this effect also takes place in completely symmetric treatments, where salience should be absent. Therefore, the salient target hypothesis cannot be driving the results across all treatments. In fact, Avrahami *et al.* (2014) explain all their results exclusively in terms of behavioural effects related to the relative budgets.

receives a significantly higher allocation than in the unique Nash equilibrium, even though the Nash allocation for that battlefield is already substantially higher than the allocation to the other battlefields. Finally, we explore the robustness of our results supporting the salient target hypothesis along two dimensions. The first dimension deals with a lower asymmetric value and its implication that, under the salient target hypothesis, we should find - and actually do find - a lower allocation than in the Nash equilibrium prediction. In the second dimension, we replicate the main treatments but include an asymmetry in budgets. High and low budget subjects follow the same pattern of allocations as subject pairs with symmetric budgets. Allocations are consistent with Nash equilibrium in the case of symmetric battlefields but not with asymmetric battlefields; and the deviation from equilibrium is more pronounced in the case of asymmetric values. Thus, the major results in the main treatments appear robust to an asymmetry in budgets.

The rest of the paper is organized as follows. Section 2 presents the basic model examined and the underlying theoretical predictions. Section 3 describes the experimental design and concrete hypotheses; and section 4 presents the results. Section 5 discusses implications for the examination of multi-battle contests and concludes.

2 Basic model and theoretical predictions

The Colonel Blotto game is a two-player constant-sum game. Two players, A and B, simultaneously allocate their corresponding budget X_A and X_B of a continuous (sunk) resource across the n independent battlefields, such that the allocation to each battlefield is non-negative. Any part of the budget not allocated is lost, so we will assume that each player i's pure strategy space is the set of non-negative n-tuples $\mathbf{x_i} = (x_{i1}, x_{i2}, \ldots, x_{in})$ satisfying $\sum_{j=1}^{n} x_{ij} = X_i$. For the purposes of our experiment we set $n = 4$ and assume that each battlefield j has a symmetric value across players v_j that may differ across battlefields (see Figure 1).[9] Each player's

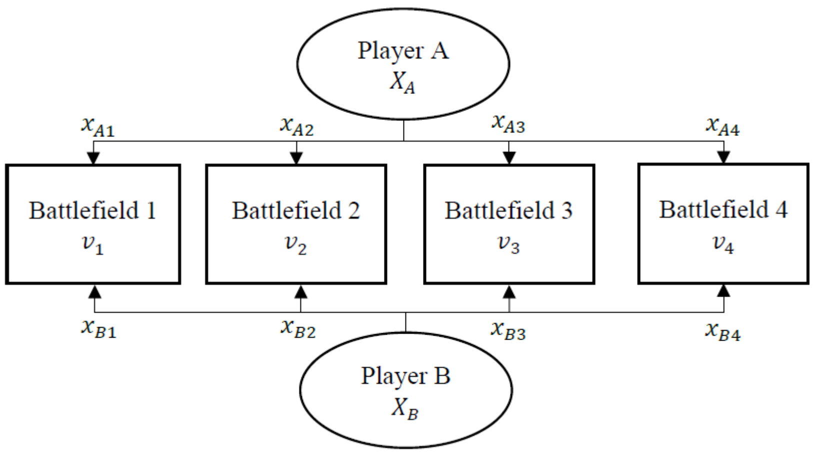


Figure 1: **Colonel Blotto game**

objective is to maximize the expected sum of the values of battlefields won. The winner in each battlefield j

[9] We chose $n = 4$ because it is the smallest number of battlefields that allows us to define the salience of a single battlefield with respect to its larger complement within the set of battlefields and at the same time assures that there is one non-salient battlefield that is interior whenever the salient battlefield is interior in the horizontal positioning of battlefields illustrated in Figure 1. (The former requirement rules out $n = 2$ and the latter rules out $n = 3$.) Both Arad and Rubinstein (2012) and Chowdhury *et al.* (2013) raise the possibility that allocations may be influenced by the horizontal positioning of battlefields.

is determined by a lottery CSF (Tullock, 1980) and, in the event that both players $i = A, B$ allocate $x_{ij} = 0$ to battlefield j, a fair randomizing device determines the winner:

$$p_{ij}(x_{ij}, x_{-ij}) = \begin{cases} \frac{x_{ij}}{(x_{ij}+x_{-ij})} & \text{if } x_{ij} + x_{-ij} \neq 0 \\ \frac{1}{2} & \text{if } x_{ij} = x_{-ij} = 0 \end{cases}$$

The solution to this problem can be found in a result due to Friedman (1958).

Theorem 1. *The pair of n-tuples x_A^* and x_B^* is a Nash equilibrium of the Colonel Blotto game with a linear count objective, battlefield valuations $\{v_j\}_{j=1}^n$ symmetric across players, budget-constrained use-it-or-lose-it costs with budget constraints X_A and X_B, and a lottery CSF in each battlefield, if $\forall j \;\; x_{Aj}^* = X_A \frac{v_j}{\sum_{k=1}^n v_k}$ and $x_{Bj}^* = X_B \frac{v_j}{\sum_{k=1}^n v_k}$. The equilibrium expected payoff of player i is $\frac{X_i}{X_A+X_B} \sum_{j=1}^n v_j$.*

For the purposes of this study, two special cases of this theorem are relevant. In the case where battlefields are symmetric in value, so that $v_j = v$ for every j, the optimal allocation across the four battlefields is to set $x_{ij} = X_i \frac{v}{4v} = \frac{X_i}{4}$. That is, the optimal allocation requires that each player spreads his budget equally across the battlefields. In the case where all battlefields have identical values except battlefield j, the optimal allocation of player i to battlefields j and $m \neq j$ are $x_{ij} = X_i \frac{v_j}{V}$ and $x_{im} = X_i \frac{v_m}{V}$, where $V = \sum_{k=1}^4 v_k$. If the value of battlefield j is higher, $\frac{x_{ij}}{x_{im}} = \frac{v_j}{v_m} > 1$, so that battlefield j receives a proportionally higher allocation than the common allocation to the other battlefields. Alternatively, if the value of battlefield j is lower, battlefield j receives a proportionally lower allocation than the common allocation to the other battlefields.

2.1 Robustness of predictions to out-of-equilibrium behaviour

It is also of interest in the examination of our experimental data to understand the optimal allocation of player i, $\mathbf{x_i} = (x_{i1}, \ldots, x_{in})$, in the event that player $-i$ chooses, or is expected to choose, an allocation, $\mathbf{x_{-i}} = (x_{-i1}, \ldots, x_{-in})$, that is not an equilibrium allocation. Friedman (1958)(p.703) also proposes a characterisation of the best response function (expression 1). However, his proposed characterisation is not general but rather restricted to the special case where the optimal allocation to every battlefield is positive.[10] The actual full characterisation of the best-response function appears in Kovenock and Rojo Arjona (2019). The parameters of all our experimental treatments belong to the especial case where Friedman's characterisation is correct.[11] For each feasible vector of allocations of player $-i$, $\mathbf{x_{-i}} \gg \mathbf{0}$, player i's best response to $\mathbf{x_{-i}}$ is to set for each battlefield $j = 1, \ldots, n$.

$$x_{ij} = r_{ij}(\mathbf{x_{-i}}) \equiv \frac{(x_{-ij} v_j)^{\frac{1}{2}}}{\sum_{k=1}^n (x_{-ik} v_k)^{\frac{1}{2}}} [X_i + X_{-i}] - x_{-ij} \tag{1}$$

Note that player i's best response has the property that the best response in battlefield j to the n-tuple $\mathbf{x_{-i}}$ depends on the allocations of player $-i$ to battlefields other than battlefield j. This arises because the opportunity cost of a unit allocated by player i to battlefield j depends upon $-i$'s allocations to the other battlefields.

In understanding out of equilibrium behaviour in this game, one special case of the rival player $-i$'s allocation that is of specific interest for illustrative purposes is that in which every v_k $\forall k \neq j$ is identical, and the

[10]We thank an anonymous referee for pointing this out.
[11]The proof of this statement appears in the Online Appendix.

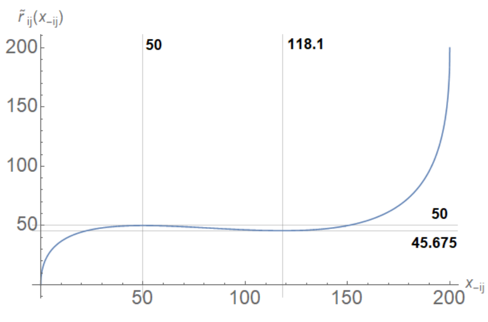
allocation of $-i$ to battlefield j, x_{-ij}, varies, holding the other $(n-1)$ battlefield allocations equal to $1/(n-1)$ of the remaining budget. In this case, for a given x_{-ij} and $x_{-ik} = (X_{-i} - x_{-ij})/(n-1)$ for all $k \neq j$, equation (1) above may be written as

$$x_{ij} = \tilde{r}_{ij}(x_{-ij}) \equiv \frac{(x_{-ij}v_j)^{\frac{1}{2}}}{(x_{-ij}v_j)^{\frac{1}{2}} + (n-1)[v_k(\frac{X_{-i}-x_{-ij}}{n-1})]^{\frac{1}{2}}}[X_i + X_{-i}] - x_{-ij} \tag{2}$$

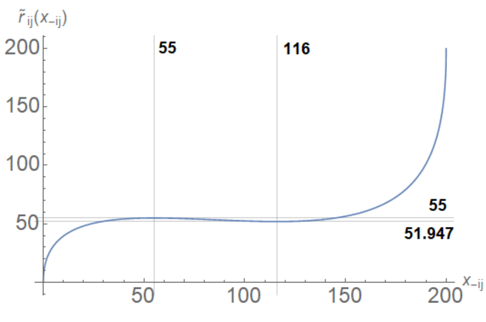
This formulation of the best response of player i in battlefield j, obtained by equating player $-i$'s allocations to all battlefields $k \neq j$, yields x_{ij} as a function of x_{-ij} and the parameters of the game X_A,X_B, and $(v_1, \ldots, v_n)$. We refer to the function $\tilde{r}_{ij}(x_{-ij})$ as i's conditional best response to x_{-ij} in battlefield j. Two combinations of parameters are relevant for our main experimental design. Both involve $n = 4$ and $X_A = X_B = 200$. The first combination, that of symmetric valuations, has $v_k = 15$ for every k. The other combination, that of asymmetric valuations, sets the value of the salient battlefield $v_j = 16.5$ and the remaining battlefields $v_k = 14.5$, $k \neq j$.

Figures 2a and 2b below indicate these two conditional best response functions for the symmetric case and asymmetric case with a single high value battlefield.[12] For both configurations of valuations, $\tilde{r}_{ij}(x_{-ij})$ is initially increasing in x_{-ij} (holding $-i$'s allocations to the remaining battlefields equal) up to the equilibrium levels of x_{-ij} and x_{ij} for the corresponding parameters. In the symmetric case, this is $x_{-ij} = x_{ij} = 50$ and, in the asymmetric case, this is $x_{-ij} = x_{ij} = 55$. After these values of x_{-ij}, player i's conditional best response to x_{-ij} decreases slowly in both configurations until x_{-ij} reaches approximately 118.1 (at which the optimal response $\tilde{r}_{ij}(x_{-ij})$ is approximately 45.675) in the symmetric case and 116 (at which the optimal response $\tilde{r}_{ij}(x_{-ij})$ is approximately 51.947) in the asymmetric case. For larger values of x_{-ij}, $\tilde{r}_{ij}(x_{-ij})$ increases in x_{-ij}, approaching 200 as x_{-ij} approaches the complete budget $X_{-i} = 200$. For values of x_{-ij} close to 200, player $-i$ is placing so little on the other battlefields $k \neq j$ that player i is able to win those battlefields with near certainty with a very small allocation. Consequently, his optimal response is to place almost all of this budget in battlefield j.

Figure 2: Best response functions



(a) Symmetric case



(b) Asymmetric case

An important implication of this analysis of out-of-equilibrium behaviour is that the optimal allocation of player i to battlefield j depends on the complete vector of battlefield allocations of the rival player, $\mathbf{x_{-i}}$. However, if as x_{-ij} changes player $-i$ maintains identical allocations across all battlefields other than battlefield j (so that $-i$'s total expenditure equals X_{-i}), we may examine the optimal response $\tilde{r}_{ij}(x_{-ij})$ of

[12]The asymmetric case with a single low value battlefield can be found in Appendix A.

player i in battlefield j as a function of x_{-ij} alone. From (2), deviations of x_{-ij} away from the equilibrium level, but still in a relatively large neighbourhood about the equilibrium level ($x_{-ij} \in (0; 118.1)$ in the symmetric case and $x_{-ij} \in (0; 116)$ in the asymmetric case) lead to a reduction in the optimal response $\tilde{r}_{ij}(x_{-ij})$. As a consequence, a player anticipating a rival's over- or under-allocation to a battlefield relative to equilibrium (due, perhaps, to the salience of that battlefield) will best respond by reducing his own allocation, albeit possibly very slightly. An increase in the allocation x_{ij} is not a best response to a deviation from the equilibrium level of x_{-ij} for a substantial neighbourhood about the equilibrium level of x_{-ij}.[13]

Finally, because equilibrium allocations are proportional to a player's budget, in the remainder of the paper, equilibrium and observed allocations are normalized with respect to the player's budget, so that we will refer to the share of a player's budget allocated to a battlefield. The allocation share of player $i \in \{A, B\}$ to battlefield $j \in \{1, 2, 3, 4\}$ is defined as $s_{ij} = \frac{x_{ij}}{X_i}$.

3 Experimental design and hypotheses

3.1 Experimental design

We have three main treatments: the symmetric treatment, the focal treatment, and the asymmetric treatment. In the symmetric treatment, subjects play 20 periods of a Colonel Blotto game with the same parameters throughout the session. Both subjects have a budget of $X_i = 200$ tokens that they are required to allocate across the different battlefields.[14] The four battlefields are represented by a row of four boxes, each containing a white circle. Every box is valued at $v_j = 15$ points.

The other two main treatments present alternative methods to single out one box (the target box) from the others. In the focal treatment, the game is identical to the symmetric treatment except that a *label asymmetry* is added: one box contains a black circle and the remaining boxes a white circle. Thus, following other focal point experiments, we induce the salience of a target while maintaining minimal asymmetry to test the salient target hypothesis. In the asymmetric treatment, the game is identical to the symmetric treatment but a *value asymmetry* is induced: one box is assigned a value of $v_T = 16.5$ points while the

[13] Our discussion of player i's optimal response to out of equilibrium behaviour by player $-i$ in a salient battlefield j assumes that in increasing x_{-ij} above its equilibrium level, the allocations x_{-ik}, $k \neq j$, are reduced symmetrically. Similarly, in decreasing x_{-ij} below its equilibrium level, the x_{-ik}, $k \neq j$, are increased symmetrically. Due to our symmetric treatment of the non-salient battlefields, this seems a reasonable benchmark. However, note that in the non-salient battlefields $k \neq j$, v_k is constant and the allocations x_{-ik}, $k \neq j$, appear in the best response function $x_{ij} = r_{ij}(\mathbf{x_{-i}})$ in (1) only in the denominator, where $(x_{-ik})^{1/2}$ is concave in x_{-ik}. As a result, for a fixed x_{-ij}, from Jensen's inequality, the denominator in (1) is maximized over all x_{-ik}, $k \neq j$, satisfying player $-i$'s budget constraint by setting $x_{-ik} = (X_{-i} - x_{-ij})/3$. Consequently, setting $x_{-ik} = (X_{-i} - x_{-ij})/3$ minimizes the value of $x_{ij} = r_{ij}(\mathbf{x_{-i}})$ for a fixed value of x_{-ij}. Nonetheless, it is possible to show that if one perturbs $\mathbf{x_{-i}}$ in a neighbourhood of its equilibrium level by raising the allocation to the salient battlefield x_{-ij} and symmetrically reducing the allocations of only a subset of the remaining battlefields (while holding the allocations to the other battlefields constant at the equilibrium levels) one obtains a qualitatively similar result – player i's optimal response to the deviation from equilibrium behaviour is to reduce his allocation $x_{ij} = r_{ij}(\mathbf{x_{-i}})$. This includes the extreme case, in which the allocation to only one of the battlefields $k \neq j$ is reduced with the increase in x_{-ij} (while the others are kept constant at the equilibrium level). Similarly, if one perturbs $\mathbf{x_{-i}}$ in a neighbourhood of its equilibrium level by lowering the allocation to the salient battlefield x_{-ij} and symmetrically increasing the allocations of only a subset of the remaining battlefields (while holding the allocations to the other battlefields constant at the equilibrium levels), player i's optimal response to the deviation from equilibrium behaviour is again to reduce his allocation $x_{ij} = r_{ij}(\mathbf{x_{-i}})$. Consequently, the best response behaviour of player i to a range of deviations from the equilibrium that involve misallocation of $-i$ to the salient battlefield j and misallocation in (weakly) the opposite direction for battlefields $k \neq j$ yields a reduction of i' s allocation x_{ij} to the salient battlefield j. It is, however, possible to obtain an increase in the best response $x_{ij} = r_{ij}(\mathbf{x_{-i}})$ to a perturbation of $\mathbf{x_{-i}}$ in a neighbourhood of its equilibrium level if an increase in x_{-ij} is accompanied by an increase in x_{-ik}, for some $k \neq j$.

[14] Because we employ a lottery CSF and budgets are use-it-or-lose-it, any optimal allocation must exhaust the budget. In addition, continuity in the space of allocations is approximated by allowing subjects to include one decimal in their allocations. Actually, only 5.71 percent of all allocation decisions included decimals.

others are valued at $v_{-T} = 14.5$ points. Hence, the sum of the values of the battlefields is identical across the three main treatments (i.e., 60 points). Although there is no compelling reason why one specific asymmetric configuration of values should be selected, we chose these specific values because they represent the (common) subjective values of each battlefield that are implied if the mean bid across all subjects in the focal treatment is generated by equilibrium behaviour. That is, subjects conforming to equilibrium behaviour in the asymmetric treatment generate the same mean bid in each battlefield as arises in the data in the focal treatment.

An inherent characteristic of multi-battle contests with a fixed budget is the negative correlation between a player's allocations to individual battlefields. In contrast, most attempts to analyse data in multi-battle experimental contests have implicitly assumed independence between battlefields.[15] To deal with the dependence of allocations arising from the budget constraint, we collapse all of the information concerning a subject's allocation into one single number: the allocation share to the target box (s_{iT}). To justify this approach, note that our games have the following features. First, subjects can only distinguish the boxes either by position or by being the target box.[16] Second, the positions of the boxes are randomized across pairs every period; and subjects are informed of this. This randomization allows potential position effects of the boxes to be controlled. If the position of the box has no significant effect, the implication is that subjects can only distinguish the target box from the others. In that case, all relevant information about a subject's allocation is divided into the allocation share to the target box (s_{iT}) and the remaining joint share that the other boxes receive, $1 - s_{iT}$. If successful, this method addresses the concern about dependence in our data. In section 4.1., we report the absence of evidence for position effects and proceed under the understanding that the relevant information to test our hypotheses resides in the allocation share to the target box.

In addition, we design four other treatments to check for the validity and robustness of our results. First, the motivating applications in the introduction suggest a positive relationship between salience and allocations to the target box. For an increased external validity of the experiment, the parameters in the main treatments are selected to be informative about the potential positive relationship between salience and allocations. However, salience can also affect behaviour negatively. For example, previous experiments on coordination report a lower rate of coordination when subjects have to choose between four labels – one salient frowning face and three smiley faces – than when there are one salient smiley face and three frowning faces (Rubinstein and Tversky, 1993; Hargreaves Heap *et al.*, 2014). Similarly, in a game with a target box valued at 13.5 and others at 15.5, we should expect that, contrary to the case with a single high value, salience will induce negative psychological payoffs and, therefore, an under-allocation to the target box. We investigate this game in our *low value asymmetry* treatment. Second, we examine the same three main treatments implementing a *budget asymmetry*: one subject (the strong player) receives a budget of $X_s = 200$ tokens and the other one (the weak player) receives a budget of $X_w = 160$ tokens. Thus, we have 7 treatments. Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box. For treatments with identical (asymmetric) budgets, we run two (four) sessions with a total of 32 subjects with 200 tokens (and 32 subjects with 160 tokens) – see Table 1 for a summary.

Each subject is allowed to take part in only one session. Within a session, two groups are formed; and, in every period, subjects are matched randomly and anonymously with a subject from the other group.

[15] An exception is Arad and Rubinstein (2012), who present exclusively summary statistics.

[16] The experimenter, however, can identify each box by an ID number from 1 to 4. The box with ID 1 is always the target box in each treatment (even in the symmetric treatment where boxes cannot be distinguished otherwise).

Table 1: Experimental design

Treatment	**Budget** (X_A; X_B)	**Label** (target; others)	**Value** (target; others)	**# Sessions**	**# Subjects**
		Main treatments			
IS	(200; 200)	(White; White)	(15; 15)	2	32
IF	(200; 200)	(Black; White)	(15; 15)	2	32
IV	(200; 200)	(White; White)	(16.5;14.5)		
		Robustness checks			
IL	(200; 200)	(White; White)	(13.5; 15.5)	2	32
AS	(200; 160)	(White; White)	(15; 15)	4	64
AF	(200; 160)	(Black; White)	(15; 15)	4	64
AV	(200; 160)	(White; White)	(16.5;14.5)	4	64

Note: Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box.

Each subject is informed of their own allocations, their opponent's allocations, the winner of each box and earnings at the end of each period. To show how a winner is determined in each battlefield, subjects are shown a roulette wheel divided into two portions corresponding to each player's proportional allocation for that battlefield. An animated spinning arrow determines the winner randomly by stopping in one of the subjects' portions (see the Online Appendix for screenshots). At the end of the experiment, subjects receive a payment based on the points they accumulate in five randomly selected periods, with an exchange rate of $1 for every 20 points.

The experiment was conducted at the Economic Science Institute Laboratory at Chapman University. Subjects were recruited from the common subject pool – excluding subjects who had previous experience in contest experiments – and participated anonymously at computer workstations. The show-up fee was $7. Instructions (reproduced in Appendix B) were presented on a separate piece of paper and subjects were informed that the same instructions were given to every subject in the session. After reading the instructions, subjects were required to complete a quiz (see Online Appendix). Sessions lasted for approximately 65 minutes – including 10 minutes for instructions. Subjects are bound to receive average earnings of $14.5.

3.2 Hypotheses

First, we restrict our attention to the treatments with identical budgets. The null hypothesis across treatments is that behaviour is consistent with the Nash Equilibrium (NE). The NE prediction is different in games with battlefields with symmetric values (treatments IS and IF) and games with a battlefield of asymmetric value (IV and IL). The prediction in the former treatments should not change the optimal allocation of the NE because labels are payoff irrelevant. For the values selected in our experimental design, the null hypothesis of NE produces two statements:

H1 (NE-*symmetry*): *In the IS and IF treatments, the mean share allocated to the target box T is* $\bar{s}_{iT} = 0.25$.

In games with asymmetric values, the NE prediction changes slightly:

H2 (NE-*asymmetry*): *In the IV and IL treatments, the mean shares allocated to the target box T are* $\bar{s}_{iT} = 0.275$ *and* $\bar{s}_{iT} = 0.225$*, respectively.*

The salient target hypothesis leads to a different behavioural pattern across treatments and provides an alternative hypothesis to evaluate the robustness of the equilibrium. In the completely symmetric battlefield case (treatment IS), this hypothesis and Nash behaviour should produce the same results, as there is no salient target. In treatments with battlefield asymmetries, however, a difference in behaviour should arise according to the salient target hypothesis. Looking at the evidence in Hide-and-Seek games, Crawford and Iriberri (2007) have suggested that the salient connotations of the labels affect psychological payoffs (e.g., a label of a happy face will translate into a positive payoff). In line with that suggestion, we expect that our connotations in the main treatments should induce positive payoffs. As a consequence, subjects should increase correspondingly their allocations in the salient target. Thus, the corresponding alternative prediction is:

H3 (Label salience): *In the IF treatment, the mean share allocated to the target box T is* $\bar{s}_{iT} > 0.25$.

A similar argument implies that positive (psychological) payoffs are derived from winning the target box with the highest value in the IV treatment and, therefore, higher allocations than the NE prediction are expected:

H4 (Value salience): *In the IV treatment, the mean share allocated to target box T is* $\bar{s}_{iT} > 0.275$.

A natural question arises regarding the relative salience of labels and values. Labels do not have an explicit value ex-ante. Given the explicit payoff relevance of a higher value, it is reasonable to expect that the effect of value salience should be stronger. We examine this empirical question in the next section.

To check how general the salience target hypothesis is, we move beyond the positive relationship between salience and allocations and test if a lower value of the target box will induce under-allocation. A mirror argument of the one leading to *H4* implies that negative (psychological) payoffs are derived from winning the target box with the lowest value in the IL treatment and, therefore, lower allocations than the NE prediction are expected:

H5 (Low value salience): *In the IL treatment, the mean share allocated to target box T is* $\bar{s}_{iT} < 0.225$.

Finally, we check the robustness of our main results by including asymmetric budgets. Determining the predictions for the treatments with a budget asymmetry is straightforward. Because optimal allocation shares are invariant to the size of the budgets, each treatment with a budget asymmetry should produce the same NE prediction as their counterpart with identical budgets. Thus, if our results supporting the salient target hypothesis were considered just a spurious coincidence, finding the same results with budget asymmetries would be unlikely. Alternatively, budget asymmetries could induce more awareness about equilibrium recommendations. For example, Avrahami and Kareev (2009)(p.948) find that, especially, the weaker players "approximate the sophisticated game-theoretic solution". Given this prior in favour of equilibrium in the presence of budget asymmetries, if we find evidence for the salient target hypothesis in treatments with identical budgets as well as asymmetric budgets, we can claim strong support for our results.

H6 (Robust results): *In each of our main treatments, the pattern in the identical budget version (either* $\bar{s}^I_{iT} = \bar{s}_{NE}$ *or* $\bar{s}^I_{iT} > \bar{s}_{NE}$*) should be the same as in the asymmetric budget version (*$\bar{s}^A_{iT} = \bar{s}_{NE}$ *or* $\bar{s}^A_{iT} > \bar{s}_{NE}$*) where superscript I and A refer to the identical budget version of the treatment and the asymmetric budget version, respectively, and* $\bar{s}_{NE}$ *refers to the equilibrium share.*

A summary of the predictions for each treatment by the two competing hypotheses is presented in Table 2. This summary makes clear that our design separates the Nash Equilibrium and the salient target hypothesis.

Table 2: Summary of predictions

	$\bar{s}_{iT}$ **in Treatments**						
Predictions	IS	IF	IV	IL	AS	AF	AV
Nash Equilibrium hypothesis	$= 0.25$	$= 0.25$	$= 0.275$	$= 0.225$	$= 0.25$	$= 0.25$	$= 0.275$
Salience Target hypothesis	$= 0.25$	> 0.25	> 0.275	< 0.225	$= 0.25$	> 0.25	> 0.275

Note: Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box.

4 Results

4.1 Data overview

Our variable of interest is the within-subject mean share allocated to the target box (s_{iT}).[17] As noted in section 3.1., the share allocated to the target box is a better measure of the effect of salience if the position of the boxes has no significant effect. A Friedman test (Friedman, 1937) reveals that the individual average allocation is not significantly different across locations ($p-value = 1.00$). This result is also robust across treatments (see Appendix C for the associated summary statistics). To support our claim of lack of position effects and justify our assumption of equal allocation to the non-salient battlefields in our out-of-equilibrium analysis, we also test whether the distribution of the remaining budget allocates one third to each of the non-salient boxes. One out of 21 Wilcoxon signed rank tests (7 treatments x 3 nonsalient boxes) appears significant at the 5 percent level (the far right box in treatment IV).

The summary statistics shown in Table 3 reveal sufficient variation across the different treatments that can address our hypotheses.[18] First, H1, the NE-symmetry prediction, requires that the within-subject means equal 0.25 in treatments IS and IF. This hypothesis seems to receive support in the treatment with completely symmetric battlefields (IS). In the IF treatment, H3, the alternative prediction of label salience, appears to be supported. Second, H2, the NE-asymmetry prediction, seems to be rejected in favour of H4 and H5, the value salience and the low value salience predictions, respectively. Furthermore, our data suggest that the effect of value salience is higher than that of label salience. All of this suggests support for the salient target hypothesis. Moreover, this pattern appears to be robust in comparing IS to AS, IF to AF, and IV to AV in line with H6. We test H1-H6 formally in section 4.2.

Before proceeding, we address a confounding factor that could potentially affect the interpretation of these tests. By using the within-subject mean share allocation as the unit of observation, we abstract away from potential time trends arising from the interaction with other subjects. The data appear to support this position. For each treatment, we test whether changes in the within-subject mean and standard deviation between periods 1-10 and 11-20 are different from zero. Table 4 shows that, overall, the repeated nature of our experiment does not affect the mean allocation share though it reduces the dispersion around the mean in most treatments.

[17]The mean share at the session level is an alternative unit of observation. This more conservative unit deals with potential dependencies arising from the interaction between subjects at the cost of reducing the number of observations. The lack of time trends presented below suggests that concerns about potential dependencies are limited in our data and, consequently, the within-subject mean is an informative unit of observation.

[18]We employ one-way Anova to test the joint null hypothesis that all treatments have the same mean share to the target box. Results show that the averages are not the same across treatments (1-10 periods $p-value$: .0001; 11-20 periods $p-value$:< .0001; all periods $p-value$:< .0001). Similar results are obtained using a Kruskal-Wallis test to address concerns about normality.

Table 3: Summary statistics

Statistic	**Periods**	**Within-subject mean share**						
		IS	**IF**	**IV**	**IL**	**AS**	**AF**	**AV**
	All							
N		32	32	32	32	32	32	32
Mean		0.257	0.273	0.317	0.193	0.248	0.261	0.33
std. dev.		0.029	0.053	0.076	0.05	0.021	0.025	0.121
25 percent		0.244	0.244	0.269	0.159	0.238	0.242	0.276
50 percent(median)		0.255	0.262	0.3	0.203	0.249	0.255	0.332
75 percent		0.267	0.296	0.356	0.239	0.261	0.272	0.402
	1-10							
N		32	32	32	32	32	32	32
Mean		0.257	0.282	0.316	0.194	0.244	0.262	0.32
std. dev.		0.056	0.062	0.085	0.054	0.036	0.03	0.142
25 percent		0.219	0.244	0.26	0.15	0.222	0.243	0.269
50 percent(median)		0.261	0.266	0.308	0.197	0.244	0.258	0.322
75 percent		0.275	0.307	0.359	0.237	0.264	0.279	0.39
	11-20							
N		32	32	32	32	32	32	32
Mean		0.256	0.264	0.317	0.191	0.252	0.26	0.339
std. dev.		0.027	0.063	0.086	0.073	0.025	0.036	0.115
25 percent		0.236	0.242	0.266	0.143	0.24	0.245	0.265
50 percent(median)		0.251	0.254	0.299	0.191	0.25	0.255	0.344
75 percent		0.275	0.286	0.372	0.235	0.263	0.27	0.404

Note: The unit of observation is the within-subject mean share $s^i_{t,u}$ allocated by subject i to the target box over periods t to u. Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box.

Table 4: Individual changes between first and second half of the session

Treatments:	**IS**	**IF**	**IV**	**IL**	**AS**	**AF**	**AV**
change between periods 1-10 and 11-20 in means							
mean	-0.002	-0.019	0.001	-0.003	0.008	-0.003	0.019
median	0.001	-0.024	-0.009	-0.003	0.011	0	0.029
t-test of zero difference	0.871	0.117	0.939	0.821	0.356	0.723	0.226
signed rank test of zero difference	0.575	0.058	0.41	0.282	0.135	0.808	0.071
change between periods 1-10 and 11-20 in standard deviations							
mean	-0.006	-0.029	-0.04	-0.018	-0.017	-0.008	-0.016
median	-0.002	-0.012	-0.018	-0.015	-0.017	-0.013	-0.015
t-test of zero difference	0.468	0.04**	0.003***	0.026**	0.022**	0.236	0.275
signed rank test of zero difference	0.531	0.025**	0.008***	0.029**	0.013**	0.137	0.175

Note: The values in the cells corresponding to rows of the statistical tests are p-values. *** and ** denote statistical significance at 1 percent and 5 percent, respectively. Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box.

4.2 Main results

Figure 3 illustrates the within-subject mean share allocated to the target box by players with a budget of 200 tokens with the corresponding 95 percent confidence interval. A formal comparison across treatments can be made by examining whether the theoretical predicted mean under the null hypothesis lies within the 95 percent confidence interval generated by the data in the specific treatment. If the predicted mean under the null hypothesis lies outside the 95 percent confidence interval in the direction predicted by the alternative hypothesis, we reject the null hypothesis in favour of the alternative hypothesis. Otherwise, we cannot reject the null hypothesis.

First, in the IS and IF treatments, H1, the NE prediction, requires $\bar{s}_{iT} = 0.25$. In Figure 3, checking whether 0.25 (blue dashed line) is within the 95 percent confidence interval for the within-subject mean allocation share to the target box in treatments IS and IF, we obtain result 1.

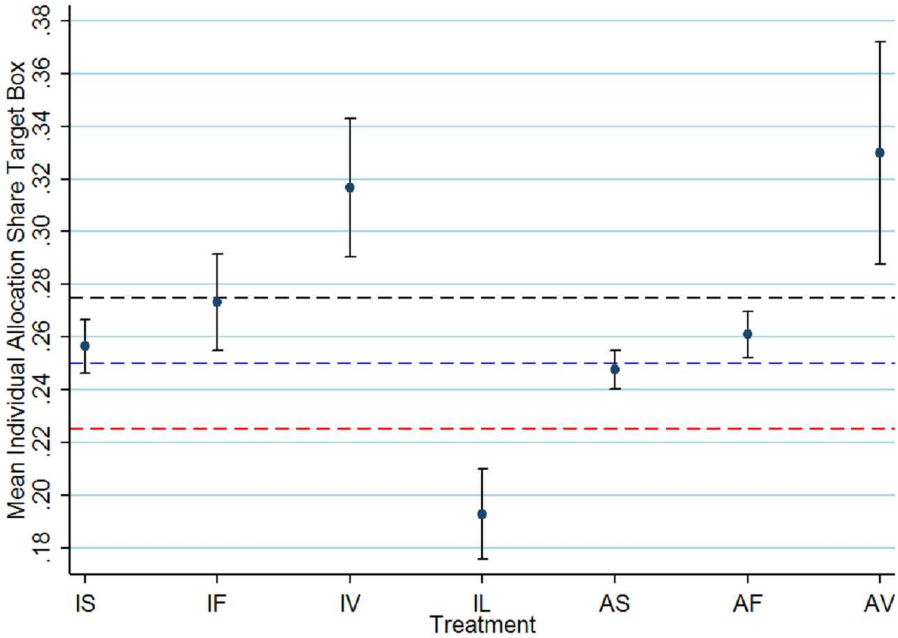
Result 1: In IS, the mean allocation share is consistent with H1, the NE-symmetry prediction. In IF, the mean allocation share is consistent with H3, the label salience prediction. Therefore, our results in the IS and IF treatments support the salient target hypothesis.

Our experimental design controls for some of the sources of minor deviations from equilibrium play in previous studies.[19] The IS treatment finds support for equilibrium play. In the IF treatment, the Nash equilibrium hypothesis would require again a share of 0.25 in the target box, while the salient target hypothesis requires a finding of label salience. The lower bound of the confidence interval of the IF treatment does not reach 0.25, consistent with H3, the label salience prediction. From the perspective of equilibrium play this result is interesting because the games are ex-ante formally the same as in the completely symmetric case. But the salience of the label appears to be sufficient to trigger over-allocation to the target box.

Second, in examining the effect of value asymmetry in the IV treatment, H2, the NE-asymmetry prediction, requires that 0.275 (black dashed line) lie within the 95 percent confidence interval of the within-subject

[19] Arad and Rubinstein (2012) find that people tend to under-allocate in the first and last battlefields in a row while Chowdhury *et al.* (2013) find that people tend to over-allocate in the battlefields to the left and under-allocate in the ones to the right.

Figure 3: **Mean individual allocation share**



Note: The vertical segments represent the 95% confidence intervals around the mean individual allocation share. Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box.

mean allocation share to the target box. In contrast, H4, the value salience prediction, requires that 0.275 lie below the lower bound of the 95 percent confidence interval for that treatment. The latter is clearly observed in Figure 3.

Result 2: In IV, the mean allocation share is inconsistent with H2, the NE-asymmetry prediction, and consistent with H4, the value salience prediction. Therefore, our results in IV support the salient target hypothesis.

Results 1 and 2 suggest that the support for equilibrium found in the literature is not robust even when small asymmetries are introduced in the boxes through labels (a change of colour) or values (a change in 1.5 points: roughly 7 cents on the dollar). Fortunately, deviation from equilibrium behaviour occurs in a systematic direction that is predicted by the salient target hypothesis. In addition, our results also suggest that asymmetry in values is more prominent than asymmetry in labels. With asymmetric values, the mean allocation share deviates from the equilibrium prediction by 0.047, or 17.4 percent of the equilibrium level. With asymmetric labels the mean allocation share deviates from the equilibrium prediction by 0.023, or 9.2 percent of the equilibrium level. Although this figure is particular to the asymmetry induced in our experiment, the size of the effect with asymmetric values is expected to vary depending on the asymmetry of the values. In the presence of asymmetries, it is reasonable to think that, as the values become increasingly symmetric, the size of the effect in asymmetric value treatments should approach a lower bound at the level of label asymmetry treatments.

We now evaluate the robustness of these results. Although our motivating examples and main treatments focus on the positive relationship between salience and allocations, one implication of the salient target hypothesis is that lower values in the target box should lead to under-allocation. We evaluate this statement by comparing the mean share allocated to the target box in IL to the prediction in H2 – requiring that 0.225 (red dashed line) lie within the 95 percent confidence interval of the within-subject mean allocation share to the target box. In contrast, H5 suggests that 0.225 lie above the 95 percent confidence interval. This is clearly observed in Figure 3.

Result 3: In IL, the mean allocation share is inconsistent with H2, the NE-asymmetry prediction, and consistent with H5, the low value salience prediction. Therefore, our results in IL support the salient target hypothesis in both the positive and negative domains of the relationship between salience and allocations.

Finally, we compare the mean share allocated to the target box in treatments with identical budgets and asymmetric budgets. This comparison tests H6, that our results are robust. This comparison is restricted to players with 200 tokens, the only common type between treatments with identical and asymmetric budgets. Before stating the result of this comparison, we report how different the mean share allocated to the target box by weak players, those with 160 tokens in the asymmetric budget treatments, is from that of players with 200 tokens in both the identical and asymmetric budget treatments. We first run t-tests and rank sum tests comparing the within-subject mean allocation share between weak players and players in treatments with identical budgets. The corresponding p-values appear in Table 5. The results show that every $p-value$ is higher than 0.10. Thus, there is no significant difference between the means of weak players and players with a symmetric budget of 200 tokens.

Table 5: P-values tests within-subject mean allocation between symmetric and weak players

Treatments	**Tests**	**Periods**		
		1 - 10	**11 - 20**	**All**
IS & AS	t-test	0.46	0.66	0.38
	rank sum	0.79	0.72	0.25
IF & AF	t-test	0.4	0.83	0.57
	rank sum	0.85	0.82	0.82
IV & AV	t-test	0.7	0.98	0.87
	rank sum	0.44	0.74	0.6

Note: The unit of observation is the within-subject mean share $\bar{s}^i_{t,u}$ allocated by subject i to the target box over periods t to u. These tests are based on thirty two observations. Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or higher value labelled (V). For example, IF means identical budgets and a focal box.

More importantly, as the behaviour of strong players is a strategic response to the behaviour of weak players, we compare weak players to strong players. We perform t-tests and signed rank tests with null hypothesis that the difference of paired observations is not different from zero. Results in Table 6 show that there is no significant difference. Thus, we are confident that any potential difference in the behaviour of subjects with a budget of 200 tokens between treatments with identical budgets and asymmetric budgets is due to the difference in budgets.

In examining the effect of budget asymmetry across treatments, H6 requires that Results 1 and 2 extend to

Table 6: Share difference between strong and weak players

Periods	1-10	11-20	All
Mean	-0.0053	0.0073	0.001
Median	-0.0062	0.0055	-0.0005
standard deviation	0.025	0.04	0.03
t-test of zero difference ($p - value$)	0.48	0.54	0.91
signed rank test of zero difference ($p - value$)	0.53	0.75	0.69

Note: The unit of observation is the within-session difference $\Delta\bar{x}_{t,u} = \bar{x}_{t,u}^{strong} - \bar{x}_{t,u}^{weak}$, where $\bar{x}_{t,u}^{type}$ is the mean of the share allocated by all type players to the target box over periods t to u ("first 10" = 1 to 10, "last 10" = 11 to 20, "all" = 1 to 20).

asymmetric budgets. So, we compare allocations in AS, AF and AV to the Nash predictions in H1 and H2. The same patterns arise in the asymmetric condition, as shown in Figure 3.

Result 4: In AS , AF, and AV the same pattern arises as in Results 1 and 2: 0.25 and 0.275 lie below the confidence interval of the mean allocation share in AF and AV, respectively; whereas, 0.25 lies within the confidence interval in AS. This is consistent with H6.

This indicates that our Results 1 and 2 supporting the salient target hypothesis are robust.

Finally, given support for the salient target hypothesis, a natural question arises regarding the consequences of observed behaviour in terms of the subjects' expected payoffs.[20] Table 7 shows for each treatment the change in expected payoffs due to four different measures of deviations of realized behaviour from equilibrium or payoff optimizing behaviour. The column labelled "$\mid \Pi(\mathbf{s_1}, \mathbf{s_2}) - \Pi(\mathbf{s_1^*}, \mathbf{s_2^*}) \mid$" indicates the average absolute value of the difference between equilibrium expected payoffs and realized expected payoffs. Given that the game is constant-sum, this difference will be nonpositive for one player and nonnegative for the other. The column "$\Pi(\mathbf{s_1}, \mathbf{s_2^*}) - \Pi(\mathbf{s_1^*}, \mathbf{s_2^*})$" shows the average loss in the expected payoffs of a subject from playing his realized share allocation against the equilibrium share allocation of the rival when compared to the case where both players play their equilibrium share allocations. The columns labelled "$\Pi(\mathbf{s_1}, \mathbf{s_2}) - \Pi(\mathbf{s_1^*}, \mathbf{s_2})$" and "$\Pi(\mathbf{s_1}, \mathbf{s_2}) - \Pi(\mathbf{r_1}, \mathbf{s_2})$" both assume that the rival subject plays the realized share allocation. The former indicates the difference in expected payoffs when the actual share allocation is chosen by the subject compared to the case where he chooses the equilibrium share allocation.[21] The latter shows the difference in expected payoffs when the actual share allocation is chosen compared to the case where the subject best responds to the realized allocation of the rival.

The average absolute value of the difference between equilibrium expected payoffs and realized expected payoffs (column "$\mid \Pi(\mathbf{s_1}, \mathbf{s_2}) - \Pi(\mathbf{s_1^*}, \mathbf{s_2^*}) \mid$") necessarily represents the largest magnitude of the four measures (but does not distinguish between gains and losses). The equilibrium expected payoff is 30 and 33.333 for players in treatments with identical budgets and strong players, respectively. Thus, the size of the average absolute value of the difference between equilibrium and realized expected payoffs relative to equilibrium expected payoffs is between 2.5% and 5.2%, depending on the treatment.[22] Assuming that a player's rival

[20]Expected payoffs are less noisy than actual earnings because the latter depend on the random realization of a lottery for each battlefield. To calculate expected payoffs conditional on the realized allocation shares, we retain our previous assumption that the portion of the budget not placed in the target box is distributed equally among the remaining boxes. Further individual analysis appears in Appendix D.

[21]Notice that this measure is necessarily negative. Although the sum of the realized expected payoffs across a pair of rivals is equal to the total sum of the box values, the sum of the two players' expected payoffs from playing the equilibrium allocation against the realized allocations of their rival is greater than the sum of the box values.

[22]We employ a one-way ANOVA to test the null hypothesis that all our main treatments with identical budgets have the same average absolute value. The results cannot reject this null hypothesis.

Table 7: Deviations in Expected Payoffs

Treatment	$\mid \Pi(\mathbf{s_1},\mathbf{s_2}) - \Pi(\mathbf{s_1^*},\mathbf{s_2^*}) \mid$	$\Pi(\mathbf{s_1},\mathbf{s_2^*}) - \Pi(\mathbf{s_1^*},\mathbf{s_2^*})$	$\Pi(\mathbf{s_1},\mathbf{s_2}) - \Pi(\mathbf{s_1^*},\mathbf{s_2})$	$\Pi(\mathbf{s_1},\mathbf{s_2}) - \Pi(\mathbf{r_1},\mathbf{s_2})$
IS	0.914	-0.592	-0.592	-0.698
IF	0.966	-0.613	-0.613	-0.655
IV	0.803	-0.596	-0.596	-0.668
IL	0.973	-0.583	-0.583	-0.725
AS	0.762	-0.505	-0.507	-0.586
AF	0.941	-0.423	-0.425	-0.643
AV	1.790	-1.364	-1.282	-1.573

Note: Expected payoffs are calculated based on the allocation to the target box and holding the remaining allocations constant. Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box.

chooses the equilibrium allocation, the loss from playing the realized strategy rather than equilibrium (column "$\mid \Pi(\mathbf{s_1},\mathbf{s_2^*}) - \Pi(\mathbf{s_1^*},\mathbf{s_2^*}) \mid$") relative to equilibrium expected payoffs is between 1.4% and 4.1%, depending on the treatment. Similar magnitudes arise when assuming that players anticipate their rival's realized allocation and we investigate the deviation in expected payoff from not playing the equilibrium allocation (column "$\mid \Pi(\mathbf{s_1},\mathbf{s_2}) - \Pi(\mathbf{s_1^*},\mathbf{s_2}) \mid$"). In fact, for treatments with symmetric budgets the numbers in both columns must be the same by construction. Finally, assuming that a player anticipates their rival's realized allocation, not best responding and playing their realized allocation yields a relative loss when compared to equilibrium expected payoff of between 1.9% and 4.7%.

These measures indicate that both the actual and potential gains and losses from realized deviations from optimal behaviour are not large and are not significantly different across treatments. These results open up the possibility that observed behaviour might be consistent with some form of imperfect optimization that might be reasonably captured by a solution concept such as ϵ-equilibrium. We discuss this possibility in the next section.

5 Discussion

Our experiment provides the first controlled evidence of the robustness of equilibrium play in the presence of salience. Deviations from equilibrium follow the salient target hypothesis, which asserts that the strategic allocation of a player's budget in a multi-battle contest is influenced by salient battlefields. In our experiment, salience arises from a particular asymmetry of one of the battlefields with respect to the others. We examine asymmetries of values and labels. Our results suggest that saliently labelled and high value battlefields receive higher allocations in comparison to both the Nash equilibrium prediction and the case in which labels and values are symmetric. This over-allocation is larger when the asymmetry involves a higher value.[23]

One implication of our results is that they suggest a reinterpretation of recent experimental evidence on Colonel Blotto games providing partial support for Nash equilibrium behaviour. Previous studies have

[23] Risk aversion might conceivably lead subjects to over-allocate to a subset of battlefields in order to increase the likelihood of securing these battlefields at the expense of reducing the likelihood of winning a number of battlefields. Thus, behaviour in IV, AV and IL could appear consistent with such an explanation. However, there would be limitations in rationalizing data in our other treatments. There is no over-allocation in IS and AS. Furthermore, this explanation cannot account for why subjects over-allocate in the salient battlefield in IF and AF rather than any other battlefield or set of battlefields – when every battlefield is indistinguishable in terms of value.

indicated that positional effects may lead to small deviations from equilibrium play. Our design controls for positional effects and equilibrium behaviour in fact arises in the symmetric case. In stark contrast, subjects deviate from equilibrium behaviour in the presence of asymmetries in battlefields. More concretely, subjects over-allocate to saliently labelled or high value battlefields. Duffy and Matros (2017) also study Colonel Blotto games with lottery CSF and heterogeneous battlefield values, but where *all* battlefields have different values. In their treatment in which subjects maximize the sum of values of battlefields won, they claim qualitative, but not quantitative, support for the equilibrium prediction.[24] Qualitative support is equated with higher valued battlefields receiving higher allocation shares or, more formally, the coincidence of the ranks of battlefield values and allocation shares. Our results also preserve the identity of battlefield value and share rankings. However, qualitative support for equilibrium behaviour is difficult to reconcile in light of our results on label asymmetry. These results suggest an unambiguous quantitative and qualitative rejection of equilibrium behaviour in the presence of a salient battlefield.[25]

To rationalize our result of over-allocation to the salient battlefield as a best-response to an opponent's anticipated or actual allocation, one would have to expect the opponent to allocate an implausibly large share of his budget to the salient battlefield. In particular, assuming symmetric allocations across the non-salient battlefields (as suggested by the experimental evidence that subjects do not distinguish positions), the realized mean share of at least 0.275 in the IF treatment is supported only with the expectation that the opponent places at least slightly above 80% of his budget in the salient battlefield (see Figure 2). Similarly, in the IV treatment, the realized mean share of at least 0.31 is supported only with the expectation that the opponent places at least slightly above 82% of his budget in the salient battlefield. In our data, less than 0.5% of the observations are consistent with this possibility in both treatments. And, feedback between games should provide enough information for subjects to adjust their allocations according to best responses. This does not appear to be the case. This apparent inconsistency between observed behaviour and best responses in treatments with salience but support for equilibrium behaviour in treatments without salience is a novel result. Previously reported evidence, in hide-and-seek games with salience, did not support equilibrium but appeared consistent with best responses (Rubinstein and Tversky, 1993; Crawford and Iriberri, 2007). This contrast suggests that our evidence makes a stronger case for the role of salience outside of the narrow confines of coordination games. Subjects in our experiment over-allocate to saliently labelled and high value battlefields, while both hiders and seekers choose non-salient options. In our treatment with a unique battlefield with a lower value subjects under-allocate to the low value battlefield.

Another possible way to rationalize our results is invoking imperfect optimization and, in particular, the ϵ-equilibrium solution concept (see, for example, Baye and Morgan, 2004, for a rationalization of actual behavior in price setting games). An ϵ-equilibrium is a strategy profile such that no player can unilaterally deviate and gain more than ϵ from his payoff in the prescribed strategy. The set of ϵ-equilibria for our main treatments are presented in Appendix D. For any $\epsilon > 0$, our Colonel Blotto game presents a multiplicity of ϵ-equilibria. Aside from the Nash equilibrium itself, in our view, there is no convincing *ex-ante* equilibrium selection criterion to choose one ϵ-equilibrium over another because, independently of the expectations of a rival's strategy, a player can always ensure the max-min payoff. However, the concentration of allocation shares above the equilibrium predictions in IF and IV − central to Results 1 and 2 − could be interpreted to have emerged because, for small ϵ, the range of ϵ-equilibria above the Nash equilibrium is larger than below,

[24] Duffy and Matros (2017) also include treatments in which players maximize the probability of winning battlefields yielding more than one-half the total value of all battlefields.

[25] In light of our evidence, a reinterpretation of the Duffy and Matros (2017) data suggests some support for the salient target hypothesis. In their treatment with heterogeneous values and a value maximization objective, subjects appear to over-allocate relative to equilibrium in the two highest valued battlefields and under-allocate in the two lowest valued battlefields.

because deviations to larger allocations are less costly. Furthermore, it has been brought to our attention by a referee that the concept of ϵ-equilibrium could also provide a rationalisation of the over-allocation in IF (and AF) but not in IS (or AS). These treatments have exactly the same expected payoff functions and, therefore, the same set of ϵ-equilibria. Thus, the referee suggests that, as a result of labelling in IF (and AF), a different ϵ-equilibria could be selected in the IF treatment compared to the IS treatment. The data in IL complicates this picture.[26] Although under-allocation in IL is also rationalisable by ϵ-equilibrium, it is unclear why this pattern emerges given the similarities with other treatments in terms of expected payoff functions. In particular, the expected payoff function in IL also exhibits less costly deviations to larger allocations than the Nash equilibrium than to smaller allocations, for small ϵ. Stronger claims about the behavioural implications of the shape of expected payoff functions and the set of ϵ-equilibria appear unwarranted, and more research is needed.

The results of our laboratory experiments supporting the salient target hypothesis suggest some lessons for the field. First, for the given objective functions and CSF examined, when salience is induced through labels or a higher value there is a tendency for the opponent to over allocate resources to the salient target relative to the Nash equilibrium level. The extent that this tendency can be exploited will vary across contexts, but in the models examined here the behaviour exhibited would suggest an optimal response that involves reducing the allocation to the salient battlefield below the equilibrium level. Of course, this prescription will generally depend on the nature of the multi-battle contest and may differ when other objective functions and CSF are employed. Second, to the extent that salience or focality can be manipulated ex ante, there may be a strong incentive to do so. Because the salient targets we have studied in our main treatments attract resources, pre-play expenditures by a player to make an existing target salient may prove useful in causing the rival to sub-optimally divert resources. Again, the extent that this may be exploited will vary with the nature of the multi-battle contest. Exploring these issues under alternative assumptions on contest structure remains a topic for future experimental work.

[26]This complication also apply to other models of noisy or imperfect optimization such as Quantal Response Equilibrium.

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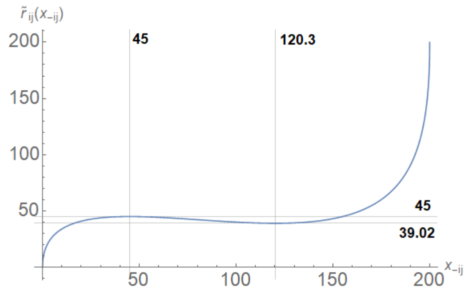
Appendix A: Out of equilibrium behavior with a single low value battlefield

As in the other two cases, the combination of parameters involves $n = 4$ and $X_A = X_B = 200$, but sets the value of the salient battlefield $v_j = 13.5$ and the remaining battlefields $v_k = 15.5$, $k \neq j$. Figures A1 below indicate the corresponding conditional best response function. For this configuration of valuations, $\tilde{r}_{ij}(x_{-ij})$ is initially increasing in x_{-ij} (holding $-i$'s allocations to the remaining battlefields equal) up to the equilibrium levels $x_{-ij} = x_{ij} = 45$. After these values of x_{-ij}, player i's conditional best response to x_{-ij} decreases slowly until x_{-ij} reaches approximately 120.3 (at which the optimal response $\tilde{r}_{ij}(x_{-ij})$ is approximately 39.023). For larger values of x_{-ij}, $\tilde{r}_{ij}(x_{-ij})$ increases in x_{-ij}, approaching 200 as x_{-ij} approaches the complete budget $X_{-i} = 200$. For values of x_{-ij} close to 200, player $-i$ is placing so little on the other battlefields $k \neq j$ that player i is able to win those battlefields with near certainty with a very small allocation. Consequently, his optimal response is to place almost all of this budget in battlefield j.

Figure A1: Best response



The function follows a pattern similar to the symmetric and asymmetric high value case. Thus, a player should under-allocate as a response to other players under-allocating. The average allocation to the target box in our IL treatment is 38.55. This represents the best response to a common opponents' allocation of 16.5 to the target box. It appears unlikely that this accurately reflects the expectations of opponent behaviour, as only 2% of the observations are consistent with this possibility.

Appendix B: Instructions

The instructions below were distributed in the completely symmetric case (IS). Instructions for other treatments are just variations highlighted between brackets in the text.

GENERAL INSTRUCTIONS

Thanks for taking part in this experiment.

You are requested not to communicate during the experiment. If you are not sure about what you are being asked to do, raise your hand and the experimenter will come to your desk to help you. You will be paid $7 for turning up plus whatever you earn in the experiment. You will be presented with 20 decision problems. At the end of the experiment, five of these problems will be randomly picked and you will be paid accordingly. You will be paid at the end of the experiment in private and in cash.

YOUR DECISION

You will be assigned as either participant 1 or participant 2 throughout the experiment. In each decision problem, you will be paired with a randomly selected person whose participant number is different from yours. You will never be told who you have been paired with. Everyone in this room is reading the same set of instructions.

In each decision problem, both participants will receive 200 tokens [**budget asymmetry treatments**: In each decision problem, participant 1 will receive 200 tokens and participant 2 will receive 160 tokens]. Each participant will choose how to allocate their tokens among four boxes. The four boxes are labelled and shown in a row. The position of the boxes is determined randomly and their labels are white circles [**focal treatments**: The position of the boxes is determined randomly and one box is labelled with a black circle and the others with a white circle]. You and the participant you have been paired with in that decision problem see the same display of boxes.

Each participant must allocate every token before proceeding to the next decision problem. You do that by writing the number of tokens you want to allocate to certain box in the slot below. You can use decimal numbers.

YOUR EARNINGS

In every decision problem, each box is worth 15 points [**high value asymmetry treatments**: In every decision problem, one box is worth 16.5 points and the other three 14.5 points each] [**low value asymmetry treatments**: In every decision problem, one box is worth 13.5 points and the other three 15.5 points each]. If that decision problem is selected at the end of the experiment, you will earn the points of the boxes you win, which will be converted into cash at a rate of $1 per 20 points. Your objective is to earn as many points as possible.

How do you win a particular box?

Your chances of winning a particular box are given by the number of tokens that you allocated divided by the total number of tokens allocated by you and the other participant. In case both participants allocate zero to the same box, each participant has 50*percent* chance of winning that box. After allocating the tokens, these proportions are shown in a roulette below each box and the computer will spin an arrow. You win if the arrow finishes in your part of the roulette.

After every decision problem, you will see the allocation of your tokens, the allocation of the other participant's tokens, which boxes you win and your *possible* earnings in that decision problem.

Before starting the experiment, a questionnaire will check that the instructions were clear. Please complete the questionnaire and raise your hand. The experimenter will come to pick it up and clarify any doubt.

Appendix C: Summary Statistics individual average allocation across locations

Table C1: Summary Statistics for Individual Allocations Across Locations

Treatment	Statistics	Position			
		Far Left	Left	Right	Far Right
IS	N	32	32	32	32
	Mean	0.258	0.253	0.245	0.244
	std.dev	0.029	0.03	0.027	0.018
	25 percent	0.241	0.233	0.224	0.229
	50 percent (median)	0.252	0.246	0.25	0.243
	75 percent	0.277	0.27	0.262	0.256
IF	N	32	32	32	32
	Mean	0.253	0.247	0.255	0.246
	std.dev	0.022	0.023	0.024	0.024
	25 percent	0.239	0.232	0.241	0.23
	50 percent (median)	0.248	0.249	0.255	0.249
	75 percent	0.268	0.257	0.272	0.262
IV	N	32	32	32	32
	Mean	0.251	0.246	0.252	0.251
	std.dev	0.022	0.019	0.017	0.019
	25 percent	0.236	0.232	0.237	0.239
	50 percent (median)	0.251	0.245	0.251	0.249
	75 percent	0.262	0.258	0.264	0.258
IL	N	32	32	32	32
	Mean	0.248	0.255	0.248	0.249
	std.dev	0.027	0.019	0.018	0.021
	25 percent	0.237	0.246	0.238	0.235
	50 percent (median)	0.25	0.251	0.25	0.249
	75 percent	0.254	0.264	0.26	0.265
AS	N	32	32	32	32
	Mean	0.247	0.255	0.248	0.25
	std.dev	0.026	0.018	0.017	0.02
	25 percent	0.238	0.24	0.241	0.244
	50 percent (median)	0.249	0.257	0.251	0.25
	75 percent	0.259	0.264	0.256	0.257
AF	N	32	32	32	32
	Mean	0.259	0.251	0.247	0.244
	std.dev	0.025	0.02	0.016	0.019
	25 percent	0.244	0.244	0.24	0.234
	50 percent (median)	0.255	0.252	0.248	0.242
	75 percent	0.275	0.262	0.253	0.254
AV	N	32	32	32	32
	Mean	0.249	0.257	0.247	0.247
	std.dev	0.023	0.027	0.027	0.025
	25 percent	0.232	0.238	0.237	0.231
	50 percent (median)	0.247	0.253	0.252	0.245
	75 percent	0.269	0.274	0.266	0.264

Note: The unit of observation is the within-subject mean share s_i allocated by subject i to the box in the far left, left, right and far right positions. Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box.

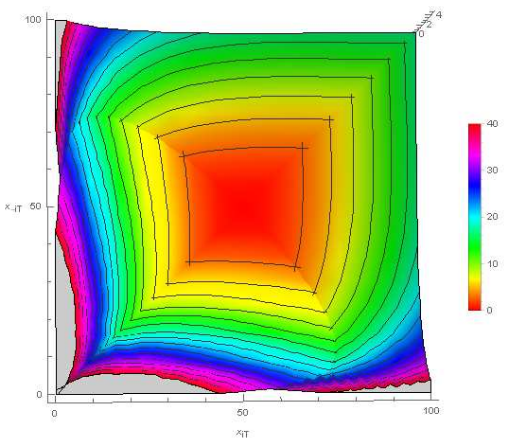
Appendix D: ϵ-equilibrium and effects of deviation on individual expected payoffs

Following the analysis in Appendix A, in our games, a pair of share allocations $(\mathbf{x_i^*}, \mathbf{x_{-i}^*})$ is a ϵ-equilibrium if for every player i the following property holds

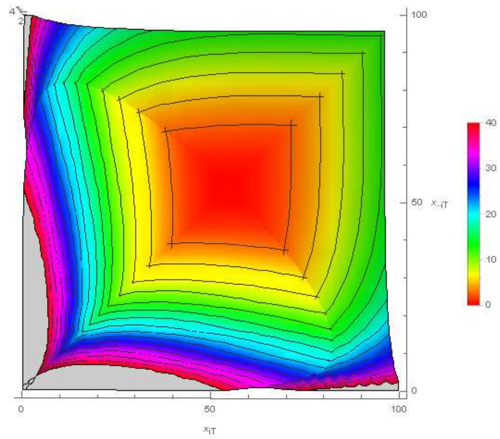
$$\Pi_i(\mathbf{x_i^*}, \mathbf{x_{-i}^*}) \geq \Pi_i(\mathbf{x_i'}, \mathbf{x_{-i}^*}) - \epsilon \quad \forall \mathbf{x_i'}$$

Figure D1 provides the graphical representation of the set of ϵ-equilibria for $0 \leq x_{iT} \leq 100$. More concretely, the horizontal and vertical axes in each graph represent the budget allocation for player i and $-i$ to the target box T, respectively. The level of ϵ is depicted by the coldness of the colour (i.e., higher ϵ are depicted by colder colours). As mentioned in Section 5, the set of ϵ-equilibria, for a given ϵ is larger above the Nash equilibrium than below (as is evident in examining symmetric ϵ-equilibria for a given ϵ, which lie on the diagonal of the graph). This arises because payoffs functions are steeper below than above the Nash equilibrium.

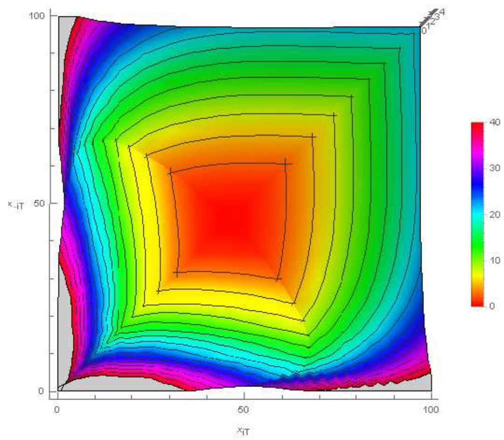
Figure D1: Topology of Epsilon equilibrium



(a) IS treatment



(b) IF treatment



(c) IL treatment

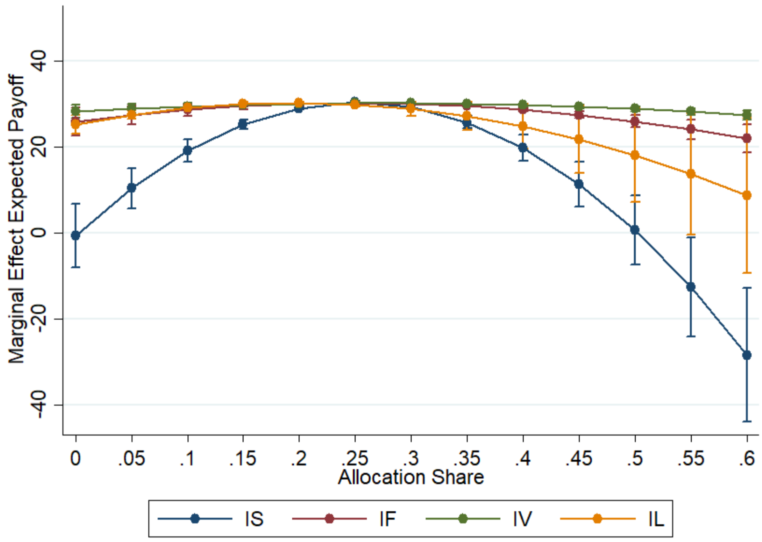
Note: The set of ϵ-equilibria for $0 \leq x_{iT} \leq 100$ for IS (panel a), IF (panel b), and IL (panel c). The horizontal and vertical axes in each graph represent the budget allocation for player i and $-i$, respectively. The level of $\epsilon \in [0, 40]$ is depicted by the coldness of the colour (i.e., higher ϵ are depicted by colder colours).

To study the effect that systematic deviations from Nash Equilibrium have on individual expected payoffs, we look at the relationship between individual expected payoffs and the share allocated to the salient box. The theoretical relationship of allocation share and expected payoff is quadratic around Nash equilibrium. Thus, we include a quadratic term in the following OLS model, which also allows heterogeneous effect across treatments:

$$EP_i = \alpha_1 + \alpha_2 \bar{s}_i + \alpha_3 \bar{s}_i^2 + \sum_j \beta_j M_{ji} + \sum_j \gamma_j M_{ji} \bar{s}_i + \sum_j \delta_j M_{ji} \bar{s}_i^2 + u_i$$

Here EP_i is the within-subject average expected payoff, $\bar{s}_i$ is the within-subject average of the allocation share, and M_{ji} is an indicator variable for treatment $j \in \{IS, IF, IV, IL\}$. The marginal effects of this regression appear in Figure D2. There is a clear difference between the treatment with no salience (IS) and the other treatments. Deviations from equilibrium reduce payoffs less in the treatments with salience. Deviations below equilibrium reduce payoffs less, as one would expect from the fact that such deviations are best responses to deviations above equilibrium.

Figure D2: Marginal Effect of Allocation Share on Expected Payoff



Note: In each subgraph, the vertical axis represents the expected payoff and the horizontal axis is the share allocation. The unit of observation is the within-session individual mean. Each treatment is represented by a two letter acronym. The first letter indicates whether budgets are identical (I) or asymmetric (A) and the second letter indicates whether boxes are symmetric (S), one is focal (F) or asymmetric in values – with the case in which the target has higher value labelled (V) and the case in which it is of low value labelled (L). For example, IF means identical budgets and a focal box.