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General analytical solutions for one-dimensional nonlinear consolidation of saturated clay under non-isothermal distribution condition

Jiangshan Li[1], Wenhao Jiang[1, 2], Shangqi Ge[3, 4, *], Xiaohui Chen[4], Xin Chen[1, 2], Lei Liu[1]

1. State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan, 430071, China.

2. University of Chinese Academy of Sciences, Beijing, 100049, China.

3. College of Civil Engineering and Architecture, Zhejiang University, Hangzhou, 310058, China.

4. School of Civil Engineering, University of Leeds, Leeds, LS2 9JT, UK.

*. Corresponding author, E-mail: geshangqi@zju.edu.cn

Abstract: The fluctuation of temperature leads to the changes of physical-mechanical properties of clayey soils. In some practical projects such as landfills, the compacted clay liner is usually subjected to a non-isothermal distribution state. For one-dimensional nonlinear consolidation process of saturated clay under non-isothermal distribution condition, the general analytical solutions considering time-dependent loading are derived for the first time, where the methods of algebraic transformation and separation variable are used. Moreover, two forms of boundary conditions are included according to engineering practice. Referring to the proposed general analytical solutions, the expressions for the analytical solutions under instantaneous loading pattern and single-stage linear loading pattern are developed. Besides, the correctness of the presented analytical solutions is validated by comparing with the existing analytical solutions and finite difference solutions. Based on the proposed analytical solutions, the influence of temperature gradient, final loading and loading time on the consolidation behaviors is analyzed. It is found that the increase in temperature gradient accelerates the consolidation rate, and the average volume compressibility coefficient decreases by 65.4% when final loading increases from 50kPa to 500kPa. In conclusion, the analytical solutions proposed in this study are more comprehensive and can be applied in different engineering cases.

Key words: general analytical solutions; saturated clay; nonlinear consolidation; non-isothermal distribution; loading pattern; consolidation behavior

1 INTRODUCTION

Consolidation theory of soils is one of the most basic theories in geotechnical engineering. Since Terzaghi[1] proposed the one-dimensional small-strain consolidation theory, many scholars have carried out experimental and theoretical studies on the consolidation characteristics of soils[2-8]. The consolidation theory evolved gradually from linear constitutive relationship to nonlinear constitutive relationship[5-8]. Besides, multi-layered soils[5,9-10], time-dependent loading[7,11-12], and continuous drainage boundary were considered[13-15]. Nevertheless, the effects of temperature on the one-dimensional consolidation behaviors were ignored in the above consolidation theories.

Numerous experimental studies have found that the change of temperature would lead to the variation in physical-mechanical properties of soils[16-23], which resulted in many geotechnical engineering problems such as soils deformation, strength loss and stability failures. For example, the physical-mechanical properties of saturated and compacted clay used as a liner system in the landfills or nuclear waste disposal sites can be obviously affected by temperature[16,21,24-26]. Meanwhile, the compressibility and permeability of soft soils usually show nonlinear changes during the consolidation process, especially for saturated clay[3-4,27-28]. Therefore, it is necessary to investigate the one-dimensional nonlinear consolidation process of clay by taking into consideration the effects of temperature.

To explore the consolidation characteristics of clay considering the effects of temperature, many related experimental studies were carried out[19,29-37]. Paaswell carried out the heating test of soils under constant loading and found that the increase in temperature d led to the soil settlement, where the concept of "thermal consolidation" was proposed for the first time[29]. Some experimental researches showed that the influence of temperature on the compressibility of clay was related to the stress history[33-34], and the pre-consolidation pressure of soils gradually decreased with increasing the temperature[30,36]. Furthermore, the generation and dissipation of excess pore water pressure (EPWP) could be affected by temperature[31]. Besides, Di Donna and Laloui discovered that the primary consolidation coefficient and permeability coefficient of soils increased under thermal cycling loading, thus accelerating the consolidation rate[19].

Meanwhile, the related analytical theories of soils consolidation process considering temperature field were also investigated[38-43]. Based on the thermal-hydraulic-mechanical coupling consolidation theory, Bai and Abousleiman developed the analytical solution for one-dimensional thermal consolidation of soils by finite Fourier transformation[38]. Shen and Wu obtained the analytical solution for one-dimensional consolidation of soils with thermo-mechanical coupling by a variable separation method[39]. Liu et al. developed an analytical solution for one-dimensional nonlinear consolidation of soft soils under constant heat sources, where the secondary consolidation of soils was considered[40]. Subsequently, a semi-analytical solution under varied loading and constant heat source was proposed, where the top boundary was assumed to be a semi-permeable boundary[42]. Moreover, the creep and thermal consolidation behaviors of saturated clays were introduced into the consolidation process[43]. However, the above theoretical studies mainly considered the consolidation process with thermal loading, and the effects of temperature on the permeability were mostly neglected. At present, the non-isothermal distribution condition has not been introduced into the consolidation theory, in which the effects of temperature on the permeability should also be considered.

For the barrier system of landfills and geological disposal of wastes, the saturated and compacted clay is commonly used[16,21,24,26,44]. The chemical reaction process of the medium produces large amounts of heat and increases the temperature in the internal field. In contrast, the temperature of the soils and groundwater outside the field is generally lower[16,24,26,45-47]. Therefore, clay liner is usually subjected to a non-isothermal distribution condition. Moreover, in the areas with an extremely high temperature or extremely low temperature, the surface clay is greatly affected by the ambient temperature, while the temperature of the deep layer of clay remains stable[41,48], thereby leading to a non-isothermal distribution condition of clay. To sum up, it is necessary to consider the non-isothermal distribution condition in the consolidation theory of clay.

In this study, the governing equation for one-dimensional nonlinear consolidation of clay under non-isothermal distribution conditions is derived with some assumptions. Then, the general analytical solutions are proposed under time-dependent loading for the first time using the methods of algebraic transformation and variable separation, where two forms of boundary conditions are included. In addition, the proposed analytical solutions are demonstrated by comparing with the existing analytical solutions and finite difference solutions. Finally, the influence of temperature gradient, final loading and loading time on the consolidation behaviors is analyzed based on the analytical solutions developed in this study.

2 MATHEMATICAL MODEL

2.1 SCHEMATIC DIAGRAM AND BASIC ASSUMPTIONS

The schematic diagram for one-dimensional nonlinear consolidation process of saturated clay under non-isothermal distribution condition is shown in Figure 1. Two forms of drainage boundary conditions are considered: single-sided drainage case and double-sided drainage case. The case that the top boundary is pervious and the bottom boundary is impervious is denoted as PTIB case, while the case that both boundaries are pervious is denoted as PTPB case. As shown in Figure 1, z represents the downward vertical coordinate, and $q(t)$ represents the time-dependent loading subjected to the clay with a thickness of L. Moreover, to consider the effects of non-isothermal distribution condition on the consolidation process[49], it is assumed that the temperature T in clay is linearly distributed with depth and rapidly reaches a stable state[46-47,50-51], where T_a and T_b represent the temperature at the top and bottom boundary, respectively.

Experimental studies have shown that the permeability coefficient of clay increased with the increase of temperature, and it is general believed that the variation in permeability coefficient is mainly related to the change of dynamic viscosity coefficient[16,20,52-53]. Meanwhile, some experimental results showed that in a certain range of temperature (e.g. 20~80°C), the pre-consolidation pressure of clay decreased slightly with the increase of temperature, and the effect of temperature on the compressibility index was negligible[19,30,33-34]. Furthermore, Zhang et al. discovered that the compressibility curves of clay at different temperatures remained almost unchanged after conducting consolidation experiments on two types of clay samples at different temperatures[35]. In addition, it is widely assumed that the initial effective stress is constant with depth for the nonlinear consolidation theory of clay, and the strain of soil particles and water under surcharge loading can be ignored[2-3,6-8]. In this case, the compressibility index is assumed to be constant to develop the analytical solutions considering the low compressibility of saturated clay. Besides, the following assumptions are made[2-3,6-8,28,47-48]:

(1) The clay is homogeneous and saturated during the consolidation process;

(2) The assumption of small-strain is established;

(3) Soil particles and pore water are incompressible, and only the vertical drainage is considered;

(4) The seepage of water in clay follows Darcy’s law;

(5) The initial effective stress of clay remains constant along the depth;

(6) The temperature distribution in clay will rapidly stabilize, and the effect of heat conduction can be neglected during the consolidation process;

(7) During the consolidation process, the following logarithm relations ($e-\log\sigma'$ and $e-\log k_v$) are defined to reflect the nonlinear variations of compressibility and permeability[7,28,40]:

$$e=e_0-C_{cT}\log(\sigma'/\sigma_0') \tag{1}$$

$$e=e_0+C_{kT}\log(k_v/k_{v0}) \tag{2}$$

where e and e_0 are the void ratio and initial void ratio of clay, respectively; σ' and σ_0' are the effective stress and the initial effective stress of clay, respectively; C_{cT} is the compressibility index of clay; k_v and k_{v0} are the permeability coefficient and the initial permeability coefficient of clay, respectively; C_{kT} is the permeability index of clay. Referring to the existing studies[2,3,40], C_{cT} and C_{kT} are assumed to be constants.

2.2 GOVERNING EQUATION

The permeability coefficient k_v of clay is affected by void ratio e and temperature T. Based on the existing researches[16,20,52-53], the relationship of permeability coefficient k_v with temperature T can be written as:

$$\frac{k_v(T)}{k_v(R)}=\frac{K_v(T)\eta_R}{K_v(R)\eta_T} \tag{3}$$

where R is the reference temperature (e.g. 20°C); K_v is the intrinsic permeability coefficient; η_R and η_T are the dynamic viscosity coefficients at temperature R and T, respectively.

For clay, it is generally considered that the intrinsic permeability coefficient K_v remains constant in a certain temperature range (e.g. 20~60°C), and the change of permeability coefficient is attributed to the change of the dynamic viscosity coefficient[16,20,52-53]. Thus, Eq. (3) can be rewritten as:

$$\frac{k_v(T)}{k_v(R)}=\frac{\eta_R}{\eta_T} \tag{4}$$

Under standard atmospheric pressure, the dynamic viscosity coefficients of water at different temperatures are shown in Table 1 (i.e. the temperature range of 0°C~100°C is considered).

According to Table 1, it is found that the following relationship can be used to approximately describe the change of dynamic viscosity coefficient with temperature:

$$\frac{\eta_R}{\eta_T} = 0.029T + 0.428 \qquad (R^2 = 0.993) \tag{5}$$

According to Eq. (2) and Eq. (3), the relationship between the permeability coefficient k_v and temperature T can be expressed as:

$$k_v(T) = k_v(R)(0.029T + 0.428) \tag{6}$$

Since the rapid conduction process of temperature, based on the related researches[46-47,50-51], the distribution of temperature T in clay can be expressed as:

$$T(z) = T_a + Mz \tag{7}$$

where $M = (T_b - T_a)/L$, M represents the temperature gradient, the unit of M is $°\mathrm{C/m}$. It should be noted that to develop the analytical solutions, the case of $M = 0\,°\mathrm{C/m}$ is not considered in this study (i.e. the isothermal distribution condition is not considered).

Combining Eq. (6) with Eq. (7), the temperature-dependent permeability coefficient $k_v(T)$ can be transformed into the depth-dependent permeability coefficient $k_v(z)$:

$$k_v(z) = k_{v,R} \times (\theta z + Q_a) \tag{8}$$

where $\theta = 0.029M$, θ represents the permeability coefficient gradient, and the unit of θ is $1/\mathrm{m}$; $k_{v,R}$ is the permeability coefficient at the reference temperature R; $Q_a = 0.029T_a + 0.428$, $k_v(0) = Q_a k_{v,R}$, $k_v(0)$ is the permeability coefficient at the top boundary of clay.

According to Eq. (1), Eq. (2) and Eq. (8), the permeability coefficient k_v can be further expressed as:

$$k_v = k_{v0,R}(\theta z + Q_a)\left(\frac{\sigma_0'}{\sigma'}\right)^{C_{cT}/C_{kT}} \tag{9}$$

where $k_{v0,R}$ is the initial permeability coefficient at the reference temperature R.

Based on the above assumptions, the governing equation for one-dimensional nonlinear consolidation can be written as:

$$\frac{\partial}{\partial z}\left(\frac{k_v}{\gamma_w}\frac{\partial u}{\partial z}\right) = -\frac{1}{1+e_0}\frac{\partial e}{\partial t} \tag{10}$$

where u represents the excess pore water pressure (EPWP); γ_w is the unit weight of water.

According to the principle of effective stress and applied surcharge loading $q(t)$, the effective stress can be written as:

$$\sigma' = \sigma_0' + q(t) - u \tag{11}$$

Combined with Eq. (1), Eq. (9) and Eq. (10), the governing Eq. (11) can be rewritten as:

$$\frac{\partial}{\partial z}\left[\frac{k_{v0,R}}{\gamma_w}(\theta z + Q_a)\left(\frac{\sigma_0'}{\sigma'}\right)^{C_{cT}/C_{kT}}\frac{\partial u}{\partial z}\right] = m_{v0}\frac{\sigma_0'}{\sigma'}\left(\frac{\partial u}{\partial t} - \frac{dq}{dt}\right) \tag{12}$$

where $m_{v0} = \frac{C_{cT}}{\ln 10 \times (1+e_0)\sigma_0'}$, m_{v0} is the initial volume compressibility coefficient.

The initial condition can be expressed as:

$$u(z,0) = q_0 \tag{13}$$

where $q_0 = q(0)$, q_0 is the surcharge loading at the initial moment.

The boundary conditions can be presented as:

$$u(0,t) = 0,\ \frac{\partial u(L,t)}{\partial z} = 0 \quad \text{(for PTIB case)} \tag{14a}$$

$$u(0,t) = 0,\ u(L,t) = 0 \quad \text{(for PTPB case)} \tag{14b}$$

3 ANALYTICAL SOLUTIONS FOR MATHEMATICAL MODEL

3.1 GENERAL ANALYTICAL SOLUTIONS

When the logarithm relations ($e - \log\sigma'$ and $e - \log k_v$) are adopted to describe the nonlinear variations in compressibility and permeability, the compressibility index and permeability index are usually close[2-3,40]. To develop the analytical solutions for the governing Eq. (12), it is assumed that $C_{cT}/C_{kT} = 1$. Therefore, the governing Eq. (12) can be further developed as:

$$C_{v0,R}\left\{(\theta z+Q_a)\left[\frac{\partial^2 u}{\partial z^2}+\frac{1}{\sigma'}\left(\frac{\partial u}{\partial z}\right)^2\right]+\theta\frac{\partial u}{\partial z}\right\}=\frac{\partial u}{\partial t}-\frac{dq}{dt} \tag{15}$$

where $C_{v0,R}=\frac{k_{v0,R}}{m_{v0}\gamma_w}$, $C_{v0,R}$ is defined as the initial consolidation coefficient corresponding to the reference temperature R.

By introducing a variable $Z=\sqrt{\theta z+Q_a}$ and considering the chain rule of derivative, the following transformation can be obtained:

$$\frac{\partial u}{\partial z}=\frac{\partial u}{\partial Z}\frac{\theta}{2Z} \tag{16}$$

The governing equation can be further derived by introducing Eq. (16) into Eq. (15), which leads to Eq. (17):

$$\lambda^2\left[\frac{\partial^2 u}{\partial Z^2}+\frac{\partial u}{\partial Z}\frac{1}{Z}+\frac{1}{\sigma'}\left(\frac{\partial u}{\partial Z}\right)^2\right]=\frac{\partial u}{\partial t}-\frac{dq}{dt} \tag{17}$$

where $\lambda=\frac{1}{2}\sqrt{C_{v0,R}\theta^2}$, λ represents a constant coefficient.

Thus, the corresponding initial conditions and boundary conditions can be rewritten as:

$$u(Z,0)=q_0 \tag{18}$$

$$u(Z_0,t)=0,\ \frac{\partial u(Z_1,t)}{\partial Z}=0\ \text{(for PTIB case)} \tag{19a}$$

$$u(Z_0,t)=0,\ u(Z_1,t)=0\ \text{(for PTPB case)} \tag{19b}$$

where $Z_0=\sqrt{Q_a}$, $Z_1=\sqrt{\theta L+Q_a}$.

To obtain the analytical solution, a new variable $w(Z,t)$ is introduced, leading to Eq. (20):

$$w(Z,t)=\ln\left[\frac{\sigma'}{\sigma_0'+q(t)}\right]=\ln\left[\frac{\sigma_0'+q(t)-u}{\sigma_0'+q(t)}\right] \tag{20}$$

According to Eq. (20), the governing Eq. (17) can be further expressed as:

$$\lambda^2\left[\frac{\partial^2 w}{\partial Z^2}+\frac{\partial w}{\partial Z}\frac{1}{Z}\right]=\frac{\partial w}{\partial t}+F(t) \tag{21}$$

where $F(t)=\frac{1}{\sigma_0'+q(t)}\frac{dq}{dt}$.

Again, the corresponding initial conditions and boundary conditions can be rewritten as:

$$w(Z,0)=w_0 \tag{22}$$

$$w(Z_0,t)=0,\ \frac{\partial w(Z_1,t)}{\partial Z}=0 \quad \text{(for PTIB case)} \tag{23a}$$

$$w(Z_0,t)=0,\ w(Z_1,t)=0 \quad \text{(for PTPB case)} \tag{23b}$$

where $w_0=\ln\left(\frac{\sigma_0'}{\sigma_0'+q_0}\right)$.

According to the forms of governing Eq. (21) and corresponding conditions, and referring to variable separation method used by Zhu and Yin[54], it is assumed that:

$$w(Z,t)=\sum_{m=1}^{\infty}H_m(Z)T_m(t) \tag{24}$$

where $H_m(Z)$ and $T_m(t)$ are functions related to Z and t, respectively.

By introducing Eq. (24) into Eq. (21), Eq. (25) can be obtained:

$$\lambda^2\left[\frac{\partial^2 H_m(Z)T_m(t)}{\partial Z^2}+\frac{\partial H_m(Z)T_m(t)}{\partial Z}\frac{1}{Z}\right]=\frac{\partial H_m(Z)T_m(t)}{\partial t}+F_mH_m(Z)F(t) \tag{25}$$

where $1=\sum_{m=1}^{\infty}F_mH_m(Z)$, F_m is an undetermined constant related to m.

Based on Eq. (25), the relationships related to $H_m(Z)$ and $T_m(t)$ can be written as:

$$\frac{\partial^2 H_m(Z)}{\partial Z^2}+\frac{\partial H_m(Z)}{\partial Z}\frac{1}{Z}=-\frac{\beta_m^2}{\lambda^2}H_m(Z) \tag{26}$$

$$\frac{\partial T_m(t)}{\partial t}+F_mF(t)=-\beta_m^2T_m(t) \tag{27}$$

where β_m is an undetermined constant related to m.

According to the form of governing Eq. (26), it can be determined that the general solution of $H_m(Z)$ is:

$$H_m(Z)=A_mJ_0\left(\frac{\beta_m}{\lambda}Z\right)+B_mY_0\left(\frac{\beta_m}{\lambda}Z\right) \tag{28}$$

where J_0 and Y_0 are the Bessel functions of the first and second kind of zero order, respectively; A_m and B_m are undetermined constants related to m, respectively.

According to the form of governing Eq. (27), the general solution of $T_m(t)$ is expressed as:

$$T_m(t)=\exp(-\beta_m^2t)\left[C_m-\int_0^t F_mF(\tau)\exp(\beta_m^2\tau)\,d\tau\right] \tag{29}$$

where C_m is an undetermined constant related to m.

Combined with Eq. (28) and Eq. (29), the expression of $w(Z,t)$ is shown as:

$$w(Z,t) = \sum_{m=1}^{\infty} W_{0m}(Z)\exp(-\beta_m^2 t)\left[C_m - \int_0^t F_m F(\tau)\exp(\beta_m^2 \tau)\, d\tau\right] \quad (30)$$

where $W_{0m}(Z) = A_m J_0\left(\frac{\beta_m}{\lambda} Z\right) + B_m Y_0\left(\frac{\beta_m}{\lambda} Z\right)$.

When PTIB case is considered, the transcendental equation of β_m can be expressed as:

$$J_0\left(\frac{\beta_m}{\lambda} Z_0\right) Y_1\left(\frac{\beta_m}{\lambda} Z_1\right) - Y_0\left(\frac{\beta_m}{\lambda} Z_0\right) J_1\left(\frac{\beta_m}{\lambda} Z_1\right) = 0 \quad (31)$$

where J_1 and Y_1 are the Bessel functions of the first and second kind of one order, respectively.

After determining β_m in Eq. (31) and introducing $A_m = 1$, the expression of B_m can be developed as:

$$B_m = -J_0\left(\frac{\beta_m}{\lambda} Z_0\right) / Y_0\left(\frac{\beta_m}{\lambda} Z_0\right) \quad (32)$$

When PTPB case is considered, the transcendental equation of β_m is determined as:

$$J_0\left(\frac{\beta_m}{\lambda} Z_0\right) Y_0\left(\frac{\beta_m}{\lambda} Z_1\right) - Y_0\left(\frac{\beta_m}{\lambda} Z_0\right) J_0\left(\frac{\beta_m}{\lambda} Z_1\right) = 0 \quad (33)$$

Similarly, introducing $A_m = 1$, the expression of B_m is shown as:

$$B_m = -J_0\left(\frac{\beta_m}{\lambda} Z_0\right) / Y_0\left(\frac{\beta_m}{\lambda} Z_0\right) \quad (34)$$

Although the expressions of B_m are the same in PTIB and PTPB cases, it should be pointed out that the transcendental equation for determining β_m is completely different (i.e. Eq. (31) and Eq. (33) are different), which means the values of B_m are different for the two cases.

Referring to the related researches[54-56], the expression for F_m can be written as:

$$F_m = \frac{\int_{Z_0}^{Z_1} Z W_{0m}(Z) dZ}{\int_{Z_0}^{Z_1} Z W_{0m}(Z) W_{0m}(Z) dZ} \quad (35)$$

Similarly, the expression for C_m is shown as:

$$C_m = \frac{\int_{Z_0}^{Z_1} w_0 Z W_{0m}(Z) dZ}{\int_{Z_0}^{Z_1} Z W_{0m}(Z) W_{0m}(Z) dZ} = w_0 F_m \quad (36)$$

Therefore, combined with $Z = \sqrt{\theta z + Q_a}$, the expression for $w(z,t)$ can be written as:

$$w(z,t) = \sum_{m=1}^{\infty} W_{0m}\left(\sqrt{\theta z + Q_a}\right)\exp(-\beta_m^2 t) F_m\left[w_0 - \int_0^t \frac{1}{\sigma_0' + q(\tau)}\frac{dq}{d\tau}\exp(\beta_m^2 \tau)\, d\tau\right] \quad (37)$$

Combined with Eq. (20), the expression of $u(z,t)$ can be written as:

$$u(z,t) = [\sigma_0' + q(t)] \times \left[1 - \mathrm{e}^{w(z,t)}\right] \tag{38}$$

Hence, the expression for the settlement of clay at any time t can be written as:

$$S(t) = \frac{C_{cT}}{(1+e_0)} \int_0^L \log\left(\frac{\sigma'}{\sigma_0'}\right) dz = \frac{C_{cT}}{(1+e_0)} \int_0^L \log\left(\frac{\sigma' + q(t) - u}{\sigma_0'}\right) dz = \frac{C_{cT} L}{(1+e_0) I} \sum_{i=1}^{I} \log\left(\frac{2\sigma_0' + 2q(t) - u(z_i,t) - u(z_{i-1},t)}{2\sigma_0'}\right) \tag{39}$$

where $z_i = H \times i/I$, $i = 1, 2, 3, \cdots, I$, and I is the number of intervals of clay.

The magnitude of the final loading is denoted as q_u. Thus, the expression for the final settlement is shown as:

$$S_\infty = \frac{C_{cT} L}{(1+e_0)} \log\left(\frac{\sigma_0' + q_u}{\sigma_0'}\right) \tag{40}$$

Therefore, the consolidation degree U_s defined by settlement is:

$$U_s = S(t)/S_\infty \tag{41}$$

The consolidation degree U_p defined by EPWP is:

$$U_p = \frac{\int_0^H [q(t) - u]\, \mathrm{d}z}{\int_0^H q_u\, \mathrm{d}z} = \frac{1}{q_u I} \sum_{i=1}^{I} \left\{ q(t) - \frac{[u(z_i,t) + u(z_{i-1},t)]}{2} \right\} \tag{42}$$

Eq. (38), Eq. (41) and Eq. (42) are the general analytical solutions for the one-dimensional nonlinear consolidation of clay under non-isothermal distribution condition. When the drainage boundary condition and the variation patterns of surcharge loading are determined, the expression for analytical solutions can be obtained based on the above corresponding equations.

3.2 EXPRESSIONS FOR TWO LOADING PATTERNS

For the surcharge preloading method, instantaneous loading and linear loading are the most common patterns in the geotechnical engineering[3,7,40]. Figure 2 shows the variation of surcharge loading $q(t)$ with time t under two loading patterns, where Figure 2(A) represents the instantaneous loading pattern and Figure 2(B) represents the single-stage linear loading pattern, respectively. Based on the general analytical solutions proposed in the above section, the expressions for the analytical solutions of clay under two loading patterns in Figure 2 are given below.

3.2.1 INSTANTANEOUS LOADING PATTERN

For the instantaneous loading pattern, $q(t) = q_u$, we have:

$$w_0 = -\ln N_u \tag{43}$$

where $N_u = (\sigma_0' + q_u)/\sigma_0'$, N_u represents the ratio of final effective stress to initial effective stress. N_u is a parameter reflecting the nonlinear consolidation characteristics of saturated clay.

The expression of $w(z,t)$ can be written as:

$$w(z,t) = \sum_{m=1}^{\infty} W_{0m}\left(\sqrt{\theta z + Q_a}\right)\exp(-\beta_m^2 t) w_0 F_m \tag{44}$$

Therefore, combined with Eq. (44), the expression of $u(z,t)$ is:

$$u(z,t) = N_u \sigma_0' \left\{1 - \exp\left[\sum_{m=1}^{\infty} W_{0m}\left(\sqrt{\theta z + Q_a}\right)\exp(-\beta_m^2 t) w_0 F_m\right]\right\} \tag{45}$$

3.2.2 SINGLE-STAGE LINEAR LOADING PATTERN

For single-stage linear loading pattern, the expression of $q(t)$ can be expressed as:

$$q(t) = \begin{cases} q_u\, t/t_c, & 0 \le t \le t_c \\ q_u, & t > t_c \end{cases} \tag{46}$$

where t_c represents the linear loading time.

Combined with Eq. (37) and Eq. (46), it is known that $w_0 = 0$, and the expression of $w(z,t)$ is:

$$w(z,t) = \begin{cases} \sum_{m=1}^{\infty} W_{0m}\left(\sqrt{\theta z + Q_a}\right)\exp(-\beta_m^2 t) F_m C_1, 0 \le t \le t_c \\ \sum_{m=1}^{\infty} W_{0m}\left(\sqrt{\theta z + Q_a}\right)\exp(-\beta_m^2 t) F_m C_2, t > t_c \end{cases} \tag{47}$$

In Eq. (47), the expression of C_1 and C_2 is:

$$C_1 = -\exp\left(-\frac{\beta_m^2 t_c}{N_u - 1}\right)\left[\ln T + \sum_{k=1}^{\infty} \frac{\beta_m^{2k} t_c^k (T^k - 1)}{k!k(N_u - 1)^k}\right] \tag{48}$$

$$C_2 = -\exp\left(-\frac{\beta_m^2 t_c}{N_u - 1}\right)\left[\ln N_u + \sum_{k=1}^{\infty} \frac{\beta_m^{2k} t_c^k (N_u^k - 1)}{k!k(N_u - 1)^k}\right] \tag{49}$$

where $T = [(N_u - 1)t + t_c]/t_c$.

Therefore, the expression of $u(z,t)$ is:

$$u(z,t) = \begin{cases} (\sigma_0' + q_u\, t/t_c)\{1 - \exp[\sum_{m=1}^{\infty} W_{0m}(\sqrt{\theta z + Q_a})\exp(-\beta_m^2 t)F_m C_1]\}, 0 \le t \le t_c \\ N_u \sigma_0'\{1 - \exp[\sum_{m=1}^{\infty} W_{0m}(\sqrt{\theta z + Q_a})\exp(-\beta_m^2 t)F_m C_2]\}, \quad t > t_c \end{cases} \tag{50}$$

Once the expression of EPWP u is determined, the consolidation degree U_s defined by settlement and U_p defined by EPWP can be obtained by Eq. (41) and Eq. (42), respectively. It should be noted that the determination of parameters in the above expressions should be combined with the corresponding boundary conditions.

4 VERIFICATION

4.1 COMPARISON WITH THE EXISTING ANALYTICAL SOLUTIONS

Xie et al. studied the one-dimensional nonlinear consolidation process of clayey soil under time-dependent loading, where the analytical solutions for single-layer clay under instantaneous loading pattern and single-stage linear loading pattern were presented[3]. When the instantaneous loading pattern is considered, the expressions of EPWP u and the consolidation degree U_s defined by settlement for PTIB case are given as[3]:

$$u(z,t) = \frac{q_u N_u}{(N_u - 1)}(1 - N_u^{-P}) \tag{50}$$

$$U_s = 1 - \sum_{m=1}^{\infty} \frac{2}{M^2}\exp(-M^2 T_v) \tag{51}$$

where $P = \sum_{m=1}^{\infty} \frac{2}{M}\sin\left(\frac{Mz}{H}\right)\exp(-M^2 T_v)$, $T_v = C_{v0,R} t / H^2$, $M = (2m-1)\pi/2$.

When single-stage linear loading pattern is considered, the expressions of u and U_s are given as[3]:

$$u(z,t) = \begin{cases} \frac{q_u T}{(N_u - 1)}[1 - \exp(-P_1)], 0 \le t \le t_c \\ \frac{q_u N_u}{(N_u - 1)}[1 - \exp(-P_2)], t > t_c \end{cases} \tag{52}$$

$$U_s = \begin{cases} \frac{1}{\ln N_u}\left[\ln T - \sum_{m=1}^{\infty} \frac{2Q_1}{M^2} \exp(-M^2 T_v)\right], 0 \le t \le t_c \\ \frac{1}{\ln N_u}\left[\ln N_u - \sum_{m=1}^{\infty} \frac{2Q_2}{M^2} \exp(-M^2 T_v)\right], t > t_c \end{cases} \tag{53}$$

In Eqs. (52)～(53), the expressions of related variables are:

$$P_1 = \sum_{m=1}^{\infty} \frac{2Q_1}{M} \sin\left(\frac{Mz}{H}\right) \exp(-M^2 T_v) \tag{54a}$$

$$P_2 = \sum_{m=1}^{\infty} \frac{2Q_2}{M} \sin\left(\frac{Mz}{H}\right) \exp(-M^2 T_v) \tag{54b}$$

$$Q_1 = \exp\left(-\frac{M^2 T_{vc}}{N_u - 1}\right)\left[\ln T + \sum_{k=1}^{\infty} \frac{(M^2 T_{vc})^k (T^k - 1)}{k! k (N_u - 1)^k}\right] \tag{54c}$$

$$Q_2 = \exp\left(-\frac{M^2 T_{vc}}{N_u - 1}\right)\left[\ln N_u + \sum_{k=1}^{\infty} \frac{(M^2 T_{vc})^k (N_u^k - 1)}{k! k (N_u - 1)^k}\right] \tag{54d}$$

$$T = \frac{T_{vc} + (N_u - 1) T_v}{T_{vc}} \tag{54e}$$

where $T_{vc} = C_{v0,R} t_c / H^2$.

Similarly, only H in the above expressions needs to be changed to $H/2$ for PTPB case[3,7].

To verify the correctness of the proposed analytical solutions in this study, two forms of drainage boundary conditions and two loading patterns are adopted for comparison. The calculation results of the analytical solutions under different temperature gradients are compared with the analytical solutions proposed by Xie et al.[3] As shown in Table 2, the calculation parameters of clay are selected for comparative analysis.

Figure 3 shows the comparison between the analytical solutions proposed in this study and the existing analytical solutions under two loading patterns[3]. When $M = 10$ °C/m or $M = -10$°C/m, the curves of consolidation degree U_s defined by settlement of the proposed analytical solutions in this study are significantly different from the analytical solutions proposed by Xie et al.[3] However, when the temperature gradient M approaches to 0°C/m (e.g. $M = 0.01$°C/m or $M = -0.01$°C/m), the curves of U_s under the two analytical solutions are basically the same regardless of the loading patterns. Although the expressions for the two analytical solutions are completely different due to the consideration of non-isothermal distribution conditions in this study, the comparison shows that the proposed analytical solutions can be degenerated into the analytical solutions proposed by Xie et al. when the non-isothermal distribution condition is not considered[3], which verifies the correctness of the analytical solutions presented in this study.

4.2 COMPARISON WITH THE FINITE DIFFERENCE SOLUTIONS

Due to the strong stability and high calculation accuracy of the finite difference method for solving partial differential equations, this method is widely used for the consolidation analysis of soft soils[4,40,57-58]. To further verify the correctness of the analytical solutions presented in this study, the governing Eq. (13) can be solved by the finite difference method with the corresponding initial conditions and boundary conditions[57].

Figure 4 shows the comparison of the consolidation degree U_p defined by EPWP with time using two calculation methods, where the parameters shown in Table 2 are adopted. Under different temperature gradients M, the curves of U_p calculated by the analytical solutions proposed in this study are in good agreement with the curves of U_p calculated by the finite difference solutions, which also verifies the correctness of the analytical solutions proposed in this study.

5 CONSOLIDATION BEHAVIORS ANALYSIS

To analyze the consolidation characteristics of saturated clay under non-isothermal distribution conditions, the effects of temperature gradient M, final loading q_u and loading time t_c on the consolidation behaviors are investigated under PTPB case. In the analysis, except for the specified parameters, the parameters shown in Table 2 are used.

5.1 THE EFFECTS OF TEMPERATURE GRADIENT

Figure 5 shows the distribution of EPWP with depth under different values of temperature gradients M considering the single-stage linear loading pattern. It is observed that EPWP decreases with the increase of M at the same depth, which indicates that the increase of temperature accelerates the dissipation of EPWP. The difference of EPWP curves under different values of M is more obvious at the bottom boundary than at the top boundary, which is mainly because the permeability coefficient of clay near the bottom boundary is larger than that of clay near the top boundary.

Figure 6 presents the variation in consolidation degree U_s defined by settlement with time under different values of temperature gradient M considering the single-stage linear loading pattern. It is observed that U_s increases with the increase of M, which is consistent with variation in EPWP with time in Figure 5. If t_{90} is defined as the time when the U_s reaches 90%, the t_{90} under $M = 10℃/\mathrm{m}$ is about 21.2% shorter than the t_{90} under $M = 0.01℃/\mathrm{m}$. This is mainly because the existence of temperature gradients M improves the permeability of clay, thus accelerating the consolidation rate. In short, the temperature gradient M has a great impact on the consolidation behaviors of clay, and the influence of temperature should be considered in consolidation theory when the non-isothermal distribution condition exists.

5.2 THE EFFECTS OF FINAL LOADING

For the one-dimensional nonlinear consolidation process, the consolidation characteristics will be affected by final loading q_u. To investigate the effects of q_u on the consolidation behaviors, Figure 7 shows the variation in consolidation degree U_s defined by settlement and consolidation degree U_p defined by EPWP with time under different values of q_u. It can be seen that U_s is always larger than U_p under two loading patterns due to the gradual decrease in compressibility during the nonlinear consolidation process. Meanwhile, U_p decreases with the increase of q_u, which is mainly because the impact of the decrease in permeability is greater than that of the decrease in compressibility.

In Figure 7(A), when the instantaneous loading pattern is adopted, the curves of U_s under different q_u are almost the same, which is consistent with the variation laws under the instantaneous loading pattern without considering the effects of temperature[2-3]. In Figure 7(B), when the single-stage linear loading pattern is adopted, U_s increases slightly with the increase of q_u at the initial stage of consolidation, but the curves of U_s gradually tend to be consistent at the end of the linear loading stage. The main reason is that the increase in q_u accelerates the settlement rate of clay during the stage of linear loading.

To further analyze the influence of q_u on the one-dimensional nonlinear consolidation behaviors, $S_q(t) = S(t)/q_u$ is defined, which represents the settlement under unit loading at any time t (e.g. $S_{q,f} = S_\infty/q_u$, $S_{q,f}$ represents the final settlement under unit loading). Figure 8 presents the variation in settlement S and S_q (the ratio of settlement to final loading) with time under different values of q_u, where the instantaneous loading pattern is adopted. It is found that the settlement increases with the increase in q_u, while the values of S_q decrease with the increase in q_u. In fact, the decrease in S_q reflects the decrease in compressibility, which indicates that the increase of surcharge loading significantly decreases the compressibility of clay during the nonlinear consolidation process[2-3,6-7].

To more accurately investigate the one-dimensional nonlinear consolidation characteristics, Table 3 shows the value of $S_{q,f}$ under different q_u. It can be seen that the value of $S_{q,f}$ under $q_u = 500\text{kPa}$ is 0.346 times smaller than that of $q_u = 50\text{kPa}$, which means that the average volume compressibility coefficient decreases by 65.4%, when the final loading increases from 50kPa to 500kPa. To sum up, the increase of q_u can significantly decrease the compressibility and permeability of clay, which significantly affects the nonlinear consolidation process and final settlement.

5.3 THE EFFECTS OF LOADING TIME

To further explore the nonlinear consolidation characteristics of clay under the single-stage linear loading pattern, the variation of EPWP with time under different values of loading time t_c is given in Figure 9. It is observed that the time of reaching the peak value prolongs with the increase in t_c, where the peak value of EPWP also decreases. When the linear loading stage is finished, EPWP gradually decreases with time, and the influence of t_c on the dissipation of EPWP gradually decreases in the consolidation process.

Figure 10 shows the variation in consolidation degree U_s defined by settlement with different t_c. It is seen that U_s decreases with the increase in t_c, which is consistent with the variation features of EPWP in Figure 9. This is mainly because the surcharge loading applied in unit time decreases with the increase in t_c. The linear loading pattern is widely adopted in geotechnical engineering[7-8,14]. Therefore, the analytical solutions proposed in this study are more practical.

6 CONCLUSIONS

In this study, the general analytical solutions for one-dimensional nonlinear consolidation process of saturated clay under non-isothermal distribution conditions are derived with some assumptions for the first time, where the time-dependent loading and two forms of boundary conditions are considered. The proposed analytical solutions are verified by comparing with the existing analytical solutions and finite difference solutions. Based on the parameter analysis of consolidation behaviors for PTPB case under instantaneous and single-stage linear loading patterns, the following conclusions are obtained:

(1) EPWP decreases with the increase of temperature gradient M, thus increasing the consolidation degree U_s defined by settlement. The time that U_s reaches 90% under $M = 10°\mathrm{C/m}$ is about 21.2% shorter than that of $M = 0.01°\mathrm{C/m}$, which indicates that the influence of temperature should be considered in consolidation theory when the non-isothermal distribution

condition exists.

(2) During the nonlinear consolidation process, the consolidation degree U_s defined by settlement is always larger than the consolidation degree U_P defined by EPWP, and U_P decreases with the increase of final loading q_u. When the instantaneous loading pattern is adopted, the curves of U_s under different q_u are almost the same. However, when considering the single-stage linear loading pattern, the increase of q_u will slightly increase U_s at the initial stage.

(3) The settlement S increases with the increase of final loading q_u, while the values of S_q (the ratio of S to q_u) decrease with the increase of q_u, which is mainly due to the reduced compressibility. When q_u increases from 50kPa to 500kPa, the average volume compressibility coefficient decreases by 65.4%.

(4) For the consolidation process under the single-stage linear loading pattern, the consolidation degree U_s decreases with the increase of t_c, which is mainly because the surcharge loading applied in unit time decreases with the increase of t_c. Nevertheless, the increase of t_c reduces the maximum EPWP generated in the clay.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author, upon reasonable request.

STATEMENT OF CONTRIBUTION

Jiangshan Li: Conceptualization, Writing original draft, Funding acquisition.

Wenhao Jiang: Writing - review and editing, Software, Methodology.

Shangqi Ge: Writing - review and editing, Software, Supervision.

Xiaohui Chen: Conceptualization, Review and editing, Data curation.

Xin Chen: Review and editing, Validation.

Lei Liu: Review and editing, Investigation.

ORCID

Jiangshan Li: https://orcid.org/0000-0003-0055-7397

Shangqi Ge: https://orcid.org/0000-0002-6863-5075

Xiaohui Chen: https://orcid.org/0000-0002-2053-2448

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TABLE 1 Dynamic viscosity coefficients of water under different temperatures[59-60]

Temperature, T/°C	Dynamic viscosity coefficient, η/(10^{-3}Pa/s)
0	1.781
5	1.519
10	1.307
20	1.002
30	0.798
40	0.653
50	0.547
60	0.466
70	0.404
80	0.354
90	0.315
100	0.282

TABLE 2 Calculation parameters of clay

Parameter	Value
Thickness, L/m	4.0
Initial void ratio, e_0	0.8
Reference temperature, R/°C	20
Permeability coefficient, $k_{v0,R}$/(m/s)	4.0×10^{-10}
Temperature at top boundary, T_a/°C	45
Initial effective stress, σ_0'/kPa	50
Temperature gradient, M/(°C/m)	10
Compressibility index, C_{cT}	0.26
Permeability index, C_{kT}	0.26
Final loading, q_u/kPa	250
Loading time, t_c/day	200

TABLE 3 The values of $S_{q,f}$ for different q_u

Parameter	$S_{q,f}$/($\times10^{-3}$m/kPa)
$q_u = 50$kPa	3.479
$q_u = 200$kPa	2.019
$q_u = 350$kPa	1.491
$q_u = 500$kPa	1.203