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Shape Embedding: A Means of Superimposing Alternative Design Descriptions on Shape Models

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ABSTRACT

In 1989, Shapiro and Voelcker introduced three principles for relating shape and function with the goal of establishing an appropriate formal role for geometry in a theory of mechanical design. This technical note contributes to the establishment of such a theory by using an example from their technical note to illustrate the potential value of incorporating shape embedding, used in the shape computation and shape grammar community to support shape synthesis, into CAD packages that are used primarily to support shape definition and analysis.

1. Introduction

Voelcker invited de Pennington to Rochester to attend a Short Course on an Introduction to Solid Modelling in the mid 1970s. The PAP (Production Automation Project at University of Rochester) and Philips CFT (Centre for Technological Research, de Pennington's previous employer, in Eindhoven) had similar aspirations towards Programmable Automation, although the PAP was academic based. The Geometric Modelling Project (GMP) at Leeds was designed in 1978/79 and launched at the end of 1979 with support from the UK government and five companies. The initial planning was influenced by the PAP's PADL-2 project and the research at Leeds focussed more on applications of solid modelling. As a result of the collaboration, Leeds was awarded a UK Research Council Senior Visiting Fellowship in 1981 to support a visit by Voelcker to exchange research ideas and related educational activities. Research discussed with different members of the GMP team included finite element meshing, kinematic analysis and synthesis, and aspects of automatic machining and process planning. McKay joined the GMP project in 1984 where her role was to explore applications of solid modelling in design for manufacturing. As part of this activity she contributed to the development of the STEP standard through its integration committees and as editor of the 1994 edition of ISO10303-41. Her research progressed to product descriptions supporting both design synthesis and manufacturing in the context of new product development systems and so engineering supply chains. She completed her Ph.D. thesis on relationships in product data in 1996 and since then McKay and de Pennington have led a series of research projects exploring interplays between engineering design processes, the networks of organisations that deliver these processes and associated design descriptions. In parallel, Voelcker's research was making major contributions in the area of dimensions and tolerances.

After Voelcker moved to Cornell in 1987, we visited each other every two or three years. While at Leeds, we took the opportunity to share our latest research and seek Voelcker's insights and advice. Circa 2000 these discussions covered relationships between designed product structures (e.g., as represented by the assembly models in CAD systems) and the networks of organisations that produce engineering products. In 2012 Voelcker visited Leeds as part of a trip to the London Olympics. At that stage we were developing research that brought together ideas from the shape computation community (used in shape synthesis) with design descriptions (used in shape analysis and design decision-making) [1]. Our goal was to enable the definition of multiple product structures and so lifecycle processes for a given shape. In advance of his 2016 visit, while preparing a response to a call from the UK EPSRC (Design the Future), McKay made a test case (a brick with two pockets (or two ribs)) to illustrate the need to associate multiple design descriptions (e.g., in the form of manufacturing and design features) with a given shape. Fig. 1 shows the brick and its two coats highlighting the regions of interest for design (the ribs) and manufacture (the pockets). Voelcker was one of the first people to whom we introduced the brick and its coats. Our discussions with him, and his insightful comments, informed the ideas that underpin this note.

This note is inspired by our interactions with Voelcker. In response to Shapiro and Voelcker [3], we propose a mechanism for

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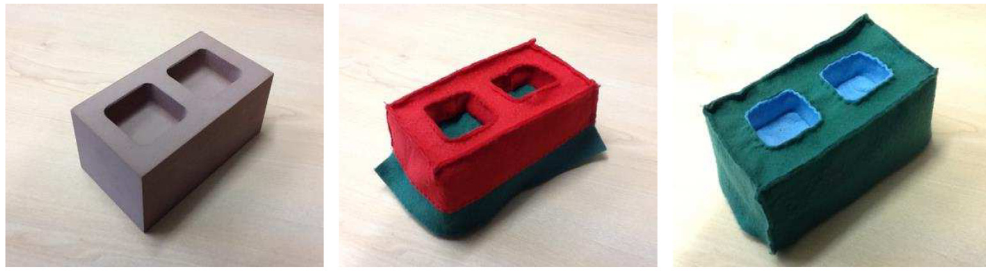

Fig. 1. (a) Brick test case; (b) & (c) Coats for the brick highlighting the designed rib (red) and manufactured pocket (blue) features. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
Source: Reproduced from McKay et al. [2].

superimposing multiple design descriptions, enabling different perspectives to be preserved, on a given geometric model. This paves a way for relating shape and function and so Shapiro & Voelcker's aspiration for a formal role for geometry in a theory of mechanical design. We begin, in Section 2, by introducing the bracket design example used in the 1989 paper and the different design descriptions it embodies. This is followed, in Section 3, with a brief introduction to the mathematical relation of embedding whose application to the case study is illustrated in Section 4. We conclude this note with a discussion of the added value shape embedding provides, challenges for its implementation in shape-based systems and future opportunities in Sections 5 and 6.

2. Multiple design descriptions in Shapiro & Voelcker's bracket design example

Shapiro and Voelcker [3] identify a need for more systematic ways to address the relationship between geometry and function in design descriptions, and introduce three principles for relating function to the geometry of component designs:

> *Principle 1: A system's 'function' is determined by its energy ports, which are generally subsets of its physical boundary, and the energy regimes operating on those ports; both should be fully defined. Principle 1 says nothing about the internal character of the system.*
> *Principle 2: Energy exchanges within a system always may be represented independently of geometry, e.g. via bond graphs.*
> *Principle 3: Principles 1 and 2 must hold for all subsystems defined on combinatorial decompositions of a system.*

To this end they demonstrate the use of Poynting's energy exchange theorem for modelling the mechanical function of discrete parts as energy exchanges through energy ports. For mechanical parts, the shapes of energy ports are subsets of the physical boundary over which energy exchanges occur and it is assumed that the ports are spatially localised and separated. Bond graphs are then used to describe energy exchanges within the system that is formed by the designed part and its environment. Using bond graphs to relate the function of a design with its geometry requires the ability to refer to the shapes of energy ports within the geometry.

Shapiro & Voelcker use the example of a bracket design with three holes: two that mate with two screws and a third that mates with a pivot pin. The holes (shown in Fig. 2(a)) are of known diameter and position and are "carried" by unknown solid geometry. Bosses, shown in Fig. 2(b), are supplementary geometry created to contain the holes and control interfaces with mating parts. For example, the bosses avoid interference between mating parts (the pivot pin and screws) and provide a flat surface for the washers that would typically be used in screw assemblies. Finally, discretionary geometry, e.g., as shown in Fig. 2(c) and (d), is used to bind together the holes and bosses to form the final design. The corresponding bond graph for this design is shown in Fig. 2(e). As can be seen from Fig. 2, in the final design there are two descriptions: one of the two alternative geometric descriptions of the design (either (c) or (d)) and a functional description of the design ((e)). The functional description in (e) refers to the energy ports but not the discretionary geometry. The bond graph only needs to change, or at least be reviewed, if the ports change. Similarly, if the bond graph changes then this indicates a need to review the geometric description of the part. Currently, the descriptions in (c) and (d) are likely to be independent of each other and the bond graph is a third free standing functional description. Thus, there are multiple descriptions of the same design (e.g., (c) and (e), or (d) and (e)). Further, for a given design, since the descriptions are not related to each other, changes in one description are not reflected in others.

For manufacturing, further design descriptions may be needed. For example, if the bracket was to be made by drilling the three holes in a blank then a description for the blank manufacturer would need a description including only the discretionary geometry and bosses. On the other hand, especially for smaller batch sizes, the bracket may be machined from solid in which case a full design description may be needed. In both cases, the discretionary geometry may be varied to accommodate constraints of the manufacturing processes used. This illustrates a wider problem leading to a need for CAD systems to support multiple perspectives on the design of a given part.

3. An introduction to shape embedding and maximal shape representation

A strength of CSG is its unambiguous shape representation which is essential for manufacturing and other downstream processes. For design synthesis, however, ambiguity is needed. For example, three ways in which a pair of overlapping squares can be transformed to create new designs are shown in Fig. 3. For the shape in (a), one of the two 'L' shapes that can be seen (but are not defined explicitly) can be translated to form the shape in (b) whereas one of the squares is rotated to create (c) and the inner square (again, not defined explicitly) is rotated to create (d). The two 'L' shapes and the inner square are said to be emergent shapes because they are not explicitly defined. In the implementation of shape grammars for shape synthesis,

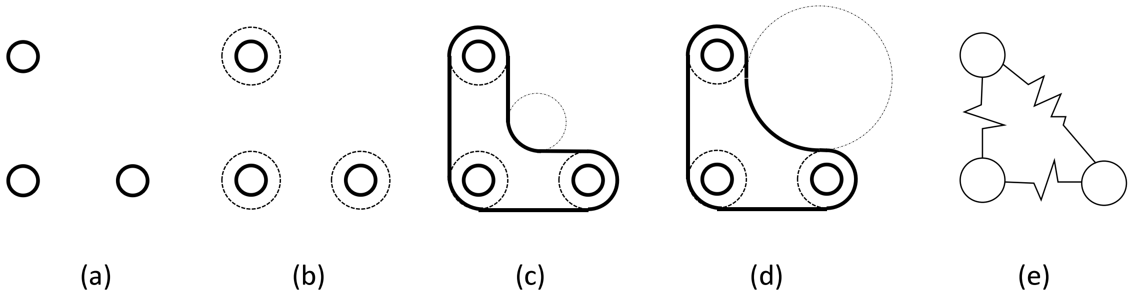

Fig. 2. Bracket design example (Adapted from Shapiro and Voelcker [3]): (a) holes that act as energy ports; (b) holes and bosses; (c) & (d) alternative discretionary geometries; (e) bond graph referring to the holes (reproduced from Shapiro and Voelcker [3]).

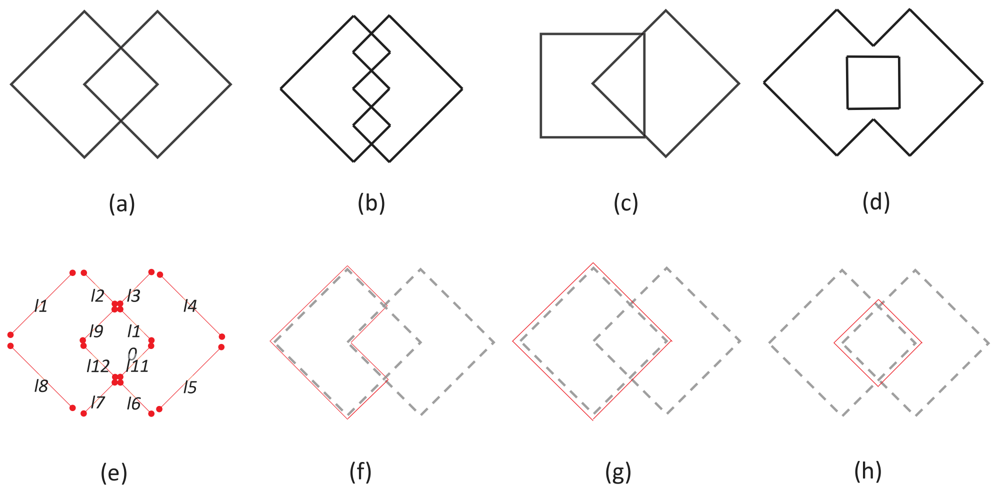

Fig. 3. Examples of shape synthesis and the role of emergent shapes; (a) an initial shape defined by two overlapping squares; (b)–(d) alternative transformations of the initial shape to form new designs; (e) a relatively maximal shape representation of the initial shape; (f)–(h) the sub-shapes that were transformed to form the shapes in (b)–(d) respectively.

a relatively maximal shape representation[1] (as shown in (e)) is used to enable the manipulation of emergent shapes. Given a maximal shape representation, it becomes possible to select any parts of the original shape ("sub-shapes") to define a new shape. Stiny first introduced the idea of shape embedding for design [6] and provides a formal definition that builds on the mathematics of hypercube lattices [7]. With such a lattice it becomes possible to embed all of the sub-shapes in Fig. 3 into a lattice representing the relatively maximal shape and so maintain traceability between the different shape descriptions.

In this note we show how shape embedding could also be useful in design description and analysis applications where, for example, some analyses may relate to the inner square in Fig. 3(a) whereas others may refer to the 'L' shapes, none of which are defined explicitly in Fig. 3(a). More generally, shape embedding allows sub-shapes (whether explicitly defined or not but that are of interest for some reason) to be superimposed on an existing shape description. Relating this to Shapiro & Voelcker's bracket example, if the shapes of the holes shown in Fig. 2(a) could be superimposed on the shape descriptions shown in Fig. 2(c) and (d) then the potential for changes in the holes to be reflected automatically in the bracket shape descriptions becomes possible. More importantly perhaps, it allows references to sub-shapes to be maintained independently of other factors, such as the details of the discretionary geometry. For example, the bond graph in Fig. 2(e) could refer to the shapes of the holes and these references would be independent of the discretionary geometry. In the remainder of this note we illustrate how shape embedding can move us forward towards Shapiro & Voelcker's aspiration to relate function and geometry in a theory of mechanical design.

4. Application of shape embedding to the bracket example

We use the bracket example to illustrate how shape embedding can be used to represent Shapiro & Voelcker's different perspectives on its design. The maximal representation (in 2D) for the bracket design is shown in Fig. 4. However a reduced version, shown in Fig. 5(a), is used here to control the size of the lattice[2] which, for the bracket, is given in Fig. 5(b). This also illustrates that, when used in design analysis, a full maximal representation of the shape is unnecessary. For example, the profile, *P*, in Fig. 5 is not a full maximal representation but it is maximal enough for the purposes of this note. Fig. 6(a) shows a sub-lattice, embedded into the lattice, referring to the holes and 6(b) shows an embedding of the discretionary geometry which can be changed without necessarily affecting the holes. If, for example, there was a need to refer to the sub-shapes forming another hole, say *c3* and *c4* for

1 A shape's maximal representation is the unique and minimal representation of the set of maximal shapes that define the shape [4]. Its relatively maximal shape is a segmentation of non-overlapping parts of its maximal parts based on the intersections between maximal parts of the original shape [5].

2 In general, given a structure with n components, the number of nodes, N, in the full lattice is 2^n and the number of edges is $2N(N-1)$. As a result, the lattice for a shape model with the five parts in Fig. 5 contains 32 nodes and 80 edges whereas the lattice for the 14 part shape in Fig. 4 would contain 16,384 nodes and 114,688 edges. Full explanations of our use of hypercube lattices, including further details of what hypercube lattices are and why they are especially important for this application, are provided in [2], Section 3 and [5].

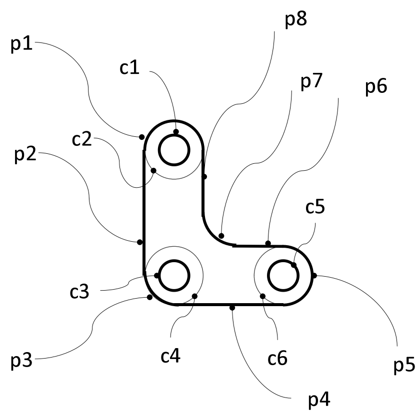

Fig. 4. Shapiro & Voelcker's bracket shape and the parts of its maximal shape representation.

f1, then the lattice could be extended to accommodate these. This would involve replacing *f1* with two elements, *c3* and *c4*, resulting in an $n = 6$ lattice (with 64 nodes, 192 edges and an additional tier to the lattices shown here). Returning to the bond graph in Fig. 2(e), it can now refer to the shapes of the energy ports, e.g., *c1*, through the lattice and in this way a relationship between shape and function is established.

5. Discussion

We have illustrated a technical solution that enables the establishment of relationships between shape and function. This is achieved by defining new groupings of shape elements (from here on referred to as "sub-shapes"[3]) in a given shape model, superimposing these sub-shapes on the original shape model, and then establishing relationships between relevant sub-shapes and elements of the functional description. The solution itself is an application of a general purpose mathematical relation, embedding, which allows new symbols (e.g., references to the sub-shape that forms the bracket's ports in Fig. 2) to be superimposed on parts of an existing shape model (e.g., *c1*, *c3* and *c5* in Fig. 4). The embedding relation (e.g., the sub-shape comprising *c1*, *c3* and *c5*) is defined through a hypercube lattice (e.g., see Figs. 5 and 6) and so provides a formal mechanism for relating shape-based design descriptions. The lattice itself is able to capture all possible combinations of shape elements, and so sub-shapes, in the original shape model. It is important to note, however, that the lattice captures part-whole relationships between shape elements in the shape model but not the geometric relationships between shape elements which remain in the underlying shape model. For this reason, sub-shapes do not, in themselves, need to be valid shapes; rather, they are collections of parts of shapes that are of interest for some reason that may or may not be known at the time of model creation. Two important benefits of this solution for design description and analysis are: (i) the underlying shape model is not changed by the superimposition of new sub-shapes because the lattice acts as a wrapper (or coat, see Fig. 1) for the shape model; and (ii) although lattices representing a fully maximal representation of shape models are likely to be very large, there is unlikely to be a need to create such lattices because only particular sub-lattices are likely to be needed in any given situation.

The feasibility of implementing general purpose hypercube lattices is well established. For example, see [2,5] for details of their implementation for bills of materials and shape grammars respectively. Further, while there are inevitably technical issues to overcome in the implementation of shape embedding as outlined in this note, either interfaced with or as an integral part of today's CAD systems, the addition of lattices would not contradict design requirements for CAD tools, such as those for PADL-2 [8]. In addition, established geometric operations to create the necessary shape representations are available, e.g., see Requicha and Voelcker [9] and as evidenced by the multi-dimensional capabilities of contemporary CAD systems where users can select sub-shapes. Thus, although a general purpose implementation of shape embedding in shape synthesis is not yet available, primarily because there is not yet a general solution for the derivation of the maximal representation of a given shape (see McKay et al. [10]), this is unlikely to prevent the implementation of the ideas introduced in this note because the user already knows which shape elements are of interest and can select them using current CAD system functionalities. Thus, the main implementation challenge lies in making these references to sub-shapes persistent rather than generating a full maximal representation of a given shape. Such a mechanism would need to be able to accommodate changes to the underlying geometry that impact the lattice, e.g., where geometric changes result in topological changes that impact the sub-shapes in the lattice. Although challenging, if implemented, an important benefit would be that the impact of proposed changes to a shape model on information associated with it through the lattice, e.g., function, could be predicted. A further implementation issue lies in the size of the lattice. In shape synthesis and part embedding in BoMs [2], the lattices grow exponentially as the source model is expanded, e.g., new shape elements or parts added. However as noted earlier, unlike shape synthesis, because a complete maximal shape representation is not needed, only the shape elements selected for views of interest need to be captured in the lattice. As a result, the size issue is not as significant as in other lattice applications.

We have used the bracket example provided by Shapiro & Voelcker to demonstrate the potential value of shape embedding to relate function and shape. Even in this apparently simple example, there are other possible applications of shape embedding. For example, Shapiro & Voelcker distinguish between discretionary and non-discretionary geometry. This along with any other classification scheme for sub-shapes in a given shape model could be defined using shape embedding. For instance, engineering features (such as the pockets and ribs in Fig. 1) could be implemented as labelled sub-shapes and so defined using shape embedding although there remain open issues in feature definition such as deciding how the ribs in Fig. 1(b) might be fully bounded and wider philosophical concerns such as those discussed by Casati and Varzi [11]. In this way, shape embedding has the potential to reduce data duplication in design descriptions: in essence, by adding new coats (see Fig. 1) to a given shape model rather than duplicating the shape definition in a new format. We do not suggest, however, that shape embedding could be used to remove all duplication of data. For example, in current practice, relationships between CAD models and other design descriptions are typically handled outside the CAD system itself, e.g., in Product Data and Lifecycle Management systems. For this reason, the impact of change on the validity of other design descriptions would be no different to currently; but it could be reduced where the use of shape embedding is feasible. There may also be wider requirements that need the addressed. For example, in the design descriptions that form technical data packages submitted for certification to regulatory authorities in sectors such as aerospace, if the lattice were included then this, like all other elements of the package would need to be validated and verified.

[3] To maintain consistency in terminology, in this note we refer to these groupings of selected shape elements in a given shape model as "sub-shapes".

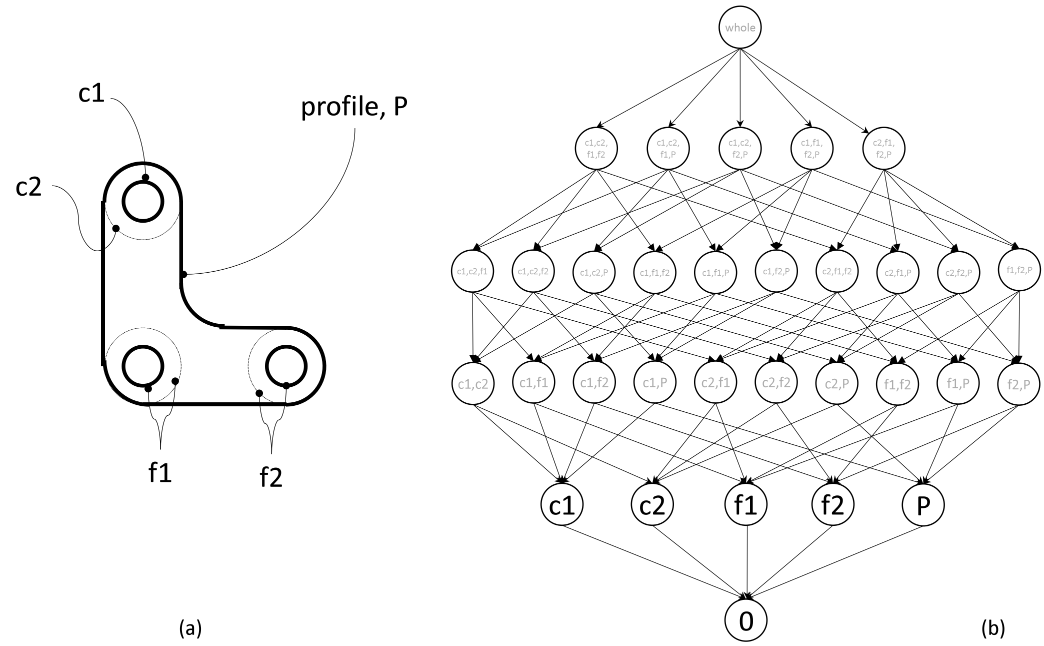

Fig. 5. (a) Shapiro & Voelcker's bracket shape labelled with a reduced set of five shape elements; (b) a corresponding lattice representation of the shape (for clarity, only edges across adjacent tiers are shown in the lattice).

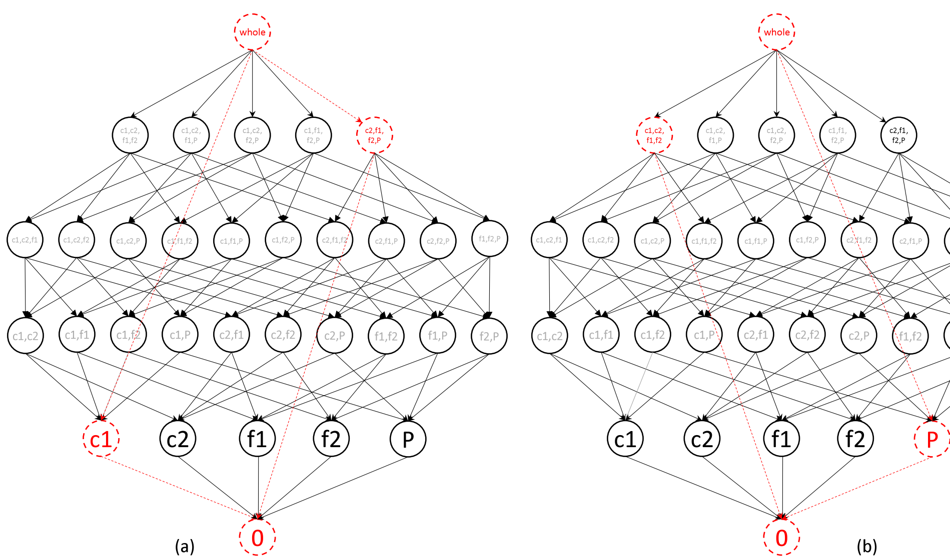

Fig. 6. Lattice representation of the bracket with embeddings (shown in dashed lines) of (a) the hole, c1, which forms one of the energy ports; (b) the profile of the discretionary geometry.

6. Conclusion

We have illustrated an application of a general purpose mathematical relation, embedding, that enables the establishment of relationships between shape and function. The embedding relation is realised through a hypercube lattice which provides a formal implementation mechanism. The feasibility of implementing general purpose hypercube lattices is well established although we anticipate their implementation in this context, which would require integration with current CAD technologies, will include technical and practical challenges. Through this note we have touched on non-technical issues that also need further consideration. Primarily they relate to the management of engineering complexity and change. A clear benefit of being able to create interconnected design descriptions is that visualisations of the impacts of change become more feasible because the information needed to underpin such visualisations is available digitally. However, in real-life examples, e.g., imagine the design of an aero engine, means of visualising change needs substantive further work. In addition, if CAD models come from different sources, marrying these models to create a shared lattice can be a non-trivial problem. Further, in the regulatory environments in which many product development processes operate, uncontrolled change is not permitted meaning that safeguards and effective change management processes would need to be in place. For this reason, the lattice may become a design tool rather than a part of the certified design.

This note also opens technical issues that require further attention. The shapes used here are defined using 1D elements in a 2D space. However, for engineering design, the ultimate application is likely to be of 3D elements in a 3D space where issues related to 3D shape definitions are likely to come to the fore. For example, Silva [12] considers the use of techniques from boundary representation that we anticipate will form important underpinnings to implementations of the ideas introduced in this note. Despite these practical limitations, shape embedding provides the potential to add new information to design descriptions without changing or losing existing ones [13] and so provides a new solution principle for the integration of design descriptions. As opposed to seeking a common underlying meta-model to support so-called "model-based" solutions, shape embedding enables the creation of heterogeneous collections of design descriptions that are connected to each other indirectly, through the lattice. The use of a lattice removes the need for the integrated meta-model and so its underlying ontology [14] on which model-based solutions depend. Shape embedding contributes in a different way; by providing a means to create design languages as and when needed, existing symbolic description mechanisms can be used (e.g., the bond graph in Fig. 2(e)) and new ones established as the needs arise.

The ability to relate function and shape is essential if the engineering design community is to benefit from today's most promising emerging technologies. We conclude with two examples: machine learning in engineering design and feature-based manufacturing. For machine learning, training data is needed to train machine learning applications. Given that the essence of any engineering design activity is the transformation of design requirements into design solutions, training data that includes relationships between shape and function will be a prerequisite for the introduction of general purpose machine learning applications to engineering design. For feature-based manufacturing, Shapiro and Voelcker conclude that the definition of features, including manufacturing features, needed a "purely syntactic system". As outlined in Section 5, a lattice-based mechanism could provide such a system and enable the definition of features as and when needed, and as a complement to traditional feature-based design approaches.

Voelcker's final visit to Leeds was in December 2018. We discussed with him research exploring embedding as a means of associating multiple Bills of Materials with a given design. We conclude this note with an extract from an email Herb sent to us in January 2019 with his version of Yeats' poem "The second coming",

> *I'll close with a quote I stumbled across when poking into mereology.*
> *"Everything is falling apart … the centre cannot hold … and mereology is laid upon the world".*

And typical advice from Herb: *stay vertical.*

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Herb's long marriage to Jean gave him a most supportive partner in pursuing his intellectual curiosity and travels, and she was always a most welcoming host and instrumental in enabling us to maintain our relationship with him. The ideas reported in this note were developed in collaboration with Dr Hau Hing Chau, and our articulation of them has been improved through discussions with him and by responding to comments from the anonymous referees. Our research in this area has been funded by the UK Engineering & Physical Sciences Research Council (EPSRC), currently through Grant: EP/S016406/1, "Assuring the quality of design descriptions through the use of design configuration spaces".

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