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A Novel Oblique Impact Model for Unified Particle Breakage Master Curve

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Abstract

Many experimental and numerical studies have been performed on the impact breakage of particulate solids, leading to a variety of impact breakage models developed to predict breakage probability. Ideally, impact breakage models would be mechanistic in nature, mathematically simple and inclusive of critical breakage parameters. In this paper, a critical review of the most widely used impact breakage models is presented, with the conclusion that the majority of existing breakage models inadequately predict breakage probability under oblique impact. In this work, a novel oblique impact model is proposed where the effect of impact angle is considered by the equivalent velocity. A breakage database compiled from the literature is deployed to interrogate the validity of the proposed model across a variety of oblique impact circumstances. In this way, the new oblique impact model is shown to provide excellent predictions of breakage probability, requiring only one set of fitting parameters under various impact angles. The unique feature of this oblique impact model is not necessarily required to be used with any specific normal impact breakage models, but can instead be universally applied with any of the assessed normal impact breakage models to establish unified breakage master curves for any oblique impact.

Keywords: Oblique impact model; Breakage model assessment; Unified master curve; Impact angle; Equivalent velocity

1 Introduction

The prediction of particle impact breakage has been a longstanding topic across many engineering fields. Particle breakage is widely observed in numerous phenomena such as rock falls in geotechnical engineering (Ye et al., 2021), ball mill in mineral engineering (De Carvalho and Tavares, 2013), catalyst attrition in chemical engineering (Boerefijn et al., 2000), impact mill in pharmaceutical engineering (Li et al., 2020). Central to a fundamental understanding of particle impact breakage is the identification of critical breakage parameters, which can then be incorporated and formulated in mathematical or theoretical models. These breakage parameters can be briefly divided into particle (i.e. material) parameters and impact (i.e. process) parameters. The particle parameters include but are not limited to particle size (Shi, 2016), particle structure (Ge et al., 2019; Wang et al., 2015), moisture content (Mueller et al., 2011) and mechanical properties, i.e. hardness and fracture toughness (Wang et al., 2021a). Larger particles are more prone to impact breakage than smaller particle due to higher crack density in larger particles (Shi, 2016).

The process parameters are mainly composed of impact velocity (Evans et al., 1978) either impact energy (Tavares, 2004), impact angle (Portnikov et al., 2018; Wang et al., 2021a) and impact frequency (Bwalya and Chimwani, 2020; Rozenblat et al., 2013; Tavares, 2009; Zhang et al., 2022). Impact velocity is typically recognized as the most influential impact parameter in particle breakage. Increasing the impact velocity transitions the breakage mode from chipping to fragmentation (Mueller et al., 2014; Subero and Ghadiri, 2001; Wang, 2016). Damage accumulation or strength degradation occurs in the majority of comminution systems due to multiple impact loading events, i.e. impact frequency. A distinctive feature of impact frequency is that particles undergoing lower impact velocity and multiple impacts have the same consequence of particles with single impact with higher impact velocity (Wang et al., 2021c). In other words, the breakage probability will be increased with increasing impact number under identical impact velocities.

Impact angle is another critical impact parameter in particle breakage, which is defined by the acute angle between the particle impact direction and the impact target (Wang et al., 2021a). It has been shown that impact angle becomes increasingly important with increasing impact velocity (Cheong et al., 2003). Whilst the breakage probability is defined by an experimental means, the damage ratio, i.e. the extent of broken bonds, is usually adopted by DEM simulation of oblique impact (Chen et al., 2022; Moreno et al., 2003; Wang et al., 2022). DEM simulations of agglomerate breakage under oblique impact indicate the damage ratio is predominantly dependent on the normal velocity, whilst tangential velocity has little effect. However, the size distribution of fragments was affected by the impact angle. Note that the above conclusion is only applicable to spherical agglomerates under relatively low impact velocity (Moreno et al., 2003). By that time, the DEM study of particle shape was only intended as spherical and the spectrum of impact velocity is varied from 1 m/s to 4.8 m/s. The effect of impact angle for non-spherical particles beyond the relatively low impact velocity is not investigated in their study. For non-spherical agglomerates, the damage model and degree depends on not only impact velocity, impact angle but also impact orientation (Liu et al., 2010). This is because of the synergic effect of impact orientation and impact angle resulting in varying contact modes upon impact. The amount of debris produced was shown to be highly sensitive to the impact location for non-spherical agglomerates (Liu et al., 2010). It is important to highlight the notable difference of breakage evaluation by breakage ratio from experiments compared to damage ratio from DEM simulations. In experiments, breakage ratio is traditionally quantified by the ratio of debris mass to the total mass of mother particles; The damage ratio is calculated based on the amount of broken bonds in the particle assembly. Due to this disparity, very few literature work made direct comparison of the breakage results between experimental and DEM simulations.

A series of impact breakage models have been developed to account for breakage parameters such as particle size, impact velocity or energy and impact number. In particular, a breakage model developed by Vogel and Peukert is able to quantify the breakage probability of various particles through the use of a breakage master curve (Vogel and Peukert, 2003). The breakage master curve for five polymers, limestone and glass spheres with different sizes was successfully constructed with two model parameters. The first model parameter f_{Mat} defines the resistance of the particle to external stresses, whilst the second model parameter $W_{m,min}$ denotes the mass-specific threshold energy for particle to break. Whilst the Vogel and Puekert model was initially developed to predict breakage probability, the modification of its form was enabled to describe the breakage index t_{10} (%) concerning the particle size distribution. The modified breakage index t_{10} has the advantage of only requiring one set of model parameters, with particle size and breakage properties incorporated explicitly. Despite its success in establishing a unified master curve for impact velocity and impact number, the Vogel and Peukert model falls short of constructing a master curve for impact angle. However, recent studies have shown increasing amount of research regarding oblique impact breakage (Cavalcanti et al., 2021; Wang et al., 2021a).

The principal objectives of this paper are two-fold: (a) to provide a critical review and assessment of existing impact breakage models relevant to the oblique impact, and (b) to demonstrate the universality of a novel oblique impact model with the chosen set of literature database. The former part serves to scrutinize assumptions, expression and application between existing particle breakage models. The difference between chipping and fragmentation models is clarified and the significance of impact angle is highlighted, quantification of which is lacking in the literature. The summary of existing breakage models identifies the detriment of ignoring impact angle, as its importance has been experimentally and computationally observed but rarely considered in a theoretically based model. A unique feature of the novel oblique impact model is that it is not necessarily appended to any specific normal impact breakage models. Instead, it can be universally used with all the assessed breakage models assessed in this paper, to establish unified breakage master curves subject to various impact angles. As the majority of existing breakage models are inadequate to describe oblique impact conditions, and oblique impact is a substantial contribution to milling processes, this is a substantial contribution to the particle breakage field.

2 Overview of impact breakage models

2.1 Impact breakage models

Prior to the overview of impact breakage models, it is noteworthy to distinguish breakage patterns, i.e. chipping and fragmentation, as a function of impact velocity. Chipping is also termed as surface breakage, whilst fragmentation is termed as body breakage by other researchers (Kotzur et al., 2018; Tavares, 2021). The transition from chipping to fragmentation with increasing impact velocity is illustrated in Figure 1. Under low impact velocity, the particle will undergo loss of debris, but still sustain its entity. Above a certain threshold impact velocity, the particle undergoes fragmentation which was simulated using an elasto-dynamic finite element method (Andrews and Kim, 1999). The particle will be broken to several pieces of fragments as the threshold impact velocity is surpassed. This results from the propagation of median and radial cracks throughout the entire body of particles (Kotzur et al., 2018). There exists several ways to define breakage probability, but the calculation of mass percentage of particles lower than the initial particle size is usually adopted (Antonyuk et al., 2006). As a result, the breakage probability will be defined as 100% when the size of impacted particle falls below the lower limit of the initial size distribution. There are no mechanistic-based methods to distinguish the impact velocity threshold for the breakage pattern transition. However, as a rule of thumb, chipping can be defined with less than 10% mass loss, beyond which fragmentation occurs with increased impact velocity (Cavalcanti et al., 2021). As such, the threshold of breakage probability for chipping is 46% with equivalent breakage size, converted from 10 % mass loss of initial particles.

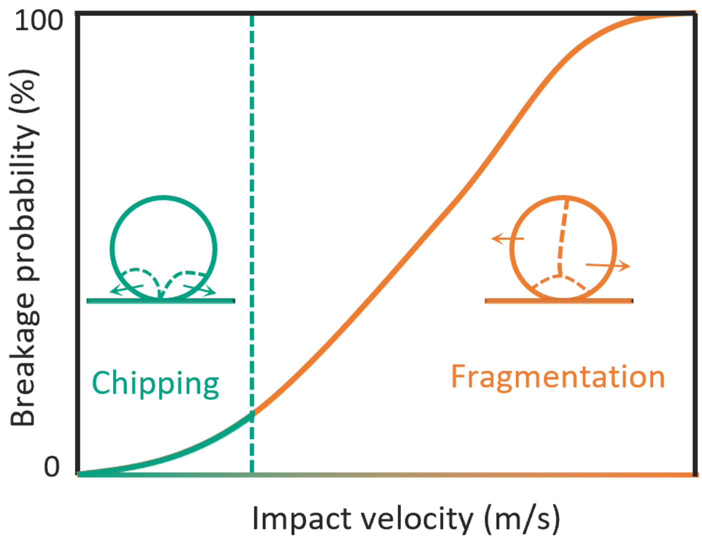


Figure 1 Breakage pattern as a function of impact velocity (modified from Wang et al., 2021b)

Over the last decades, there have been significant efforts in developing particle breakage models to elaborate the influence of critical breakage parameters on breakage probability. Previous studies on the breakage models mainly focused on the normal impact. As a consequence, most of the breakage models are initially applied to normal impact, and only the normal component velocity is considered under oblique impact.

The chipping model can be expressed as a function of impact velocity using power functions. The fragmentation model is usually developed based on Weibull distribution (A. D. Salman et al., 2003) or logistic distribution (Petukhov and Kalman, 2004) or log-normal distribution (Tavares and King, 1998). Comprehensive assessment of impact breakage models has been carried out by several researchers (Rozenblat et al., 2012; Wang et al., 2021a, 2021b). Rozenblat et al. shortlisted 6 breakage models including Petukhov and Kalman model (Petukhov and Kalman, 2004), Salman et al. model (A. Salman et al., 2003), Pocock et al. model (Pocock et al., 1998), Duo et al. model (Duo et al., 1996), Boerefijn et al. model (Boerefijn et al., 2000) and Cleaver and Ghadiri model (Cleaver et al., 1993). Only the first three models are used for model assessment of fragmentation, as the last three models are intended for chipping. Whilst these three fragmentation models exhibit nearly the same fitting quality against the experimental data, model simplicity and statistical meaning of model parameters are the basis in the appropriate model selection (Rozenblat et al., 2012). An assessment of breakage models to predict the particle size distribution was performed in an impact pin mill, in the context of a population balance model (Wang et al., 2021b). These assessed models include the Weichert model, Pocock model, Vogel and Peukert model, Antonyuk et al. model and Portnikov-Kalman model (Wang et al., 2021b). These five models are shown to give close agreement with particle size distribution in the impact pin mill. In particular, the logistic distribution of the Portnikov-Kalman model was identified as the strongest performer from a statistical performance viewpoint. Despite all these advancements, consideration of impact angle in either the particle scale or the process scale is insufficient. In view of the focal point in the present work, only the breakage models relevant to oblique impact will be presented for brevity.

A Weibull-based breakage probability was developed by Weichert as a function of mass-specific energy and it gives:

$$P = 1 - \exp(-cd^2 W_m^Z) \tag{1}$$

where $W_m = 0.5 * v^2$ is the mass-specific energy where v denotes the impact velocity; d is the particle diameter; c and z are fitting parameters.

Another Weibull-based function to describe the breakage probability is given (A. Salman et al., 2003)

$$P = 1 - \exp(-(v/a)^b) \tag{2}$$

where v is the impact velocity whereas a and b are the fitting parameters.

The difference between Weichert model and Salman et al. model lies in the expression of mass-specific energy and impact velocity respectively.

A lognormal distribution function can be used to describe a breakage probability on the basis of specific fracture energy (Pocock et al., 1998):

$$P_E = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\ln W_{m,kin} - E_{50,i}}{\sqrt{2}\sigma}\right)\right] \tag{3}$$

where P_E denotes the breakage probability as a function of specific energy; $W_{m,kin}$, $E_{50,i}$ and σ denote the mass specific energy, median mass specific energy and standard deviation of the specific energy.

A modified form of normal impact breakage models is developed to consider the impact angle and it gives (Portnikov et al., 2018):

$$P = 1 - \frac{1}{1 + (v/v_{50})^b} \tag{4}$$

where P is the breakage probability, selection function; v_{50} denotes the median impact velocity resulting in 50% of particle impact breakage. b is the logistic parameter to describe scattering of breakage data.

The relationship between impact angle θ and the median impact velocity v_{50} gives (Portnikov et al., 2018):

$$v_{50} = A_v exp(-x/d_0)[4.4\, exp(-\theta/18.1) + 1] \quad (5)$$

where A_v and d_0 are fitting parameters; x is the particle size.

Eq. (5) indicates that v_{50} is varied with regard to impact angle θ and increases with the decrease of impact angle.

A chipping model, which is called surface breakage in the original source (Cavalcanti et al., 2021) was proposed based on DEM simulation of oblique collision and it gives:

$$\tilde{\varepsilon} = 100\ k\ d\ e\ E_{loss} \quad (6)$$

where k denotes the Hertzian stiffness of the contact target; d is not clearly specified in the original source; e denotes the fraction of loss energy in a collision event; E_{loss} is the mass-specific collision energy, calculated using DEM. The surface breakage is also called attrition and abrasion in the literature (Tavares, 2009), indicating a small amount of breakage due to debris loss under low impact velocity. Particle undergoing surface breakage keeps its size relatively unchanged along with fine progeny produced as the mass loss.

The mass-based energy loss E_{loss} is further expressed by

$$E_{loss} = \frac{E'_{loss}}{m_p} \quad (7)$$

where E'_{loss} denotes the total energy loss in the collision, calculated from DEM and m_p is the mass of a pellet.

When oblique impact occurs with an impact angle θ, the total energy loss gives

$$E'_{loss} = W_m(3 * 10^{-7}\theta^3 - 1.3 * 10^{-4}\theta^2 + 1.7 * 10^{-2}\theta) \quad (8)$$

where W_m is the specific impact energy, equal to $v^2/2$. Eq. (8) indicates the total energy loss abides by a polynomial distribution with respect to the impact angle θ.

Although Eqs. (5) and (8) take into account the impact angle, a main challenge remains whether a master curve can be established with a single set of fitting parameters regarding impact angle. Vogel and Peukert (Vogel and Peukert, 2003) developed a master curve of breakage probability by unifying several parameters in a single predictive line as:

$$P_x = 1 - \exp\{-f_{Mat} x n (W_{m,kin} - W_{m,min})\} \tag{9}$$

where P_x denotes the breakage probability; f_{Mat} denotes the resistance of the particle against the external load; $W_{m,kin}$ is mass-specific kinetic energy; x and n are particle size and impact number. $W_{m,min}$ represents the mass-specific threshold energy for particle to break.

A generic form of chipping models is summarized (Wang et al., 2021a)

$$P_x = \frac{\rho^a v^b x^c H^d}{{k_c}^e} \tag{10}$$

where ρ and x are particle density and particle size; v is the impact velocity usually spanning in relatively low regime. H and k_c are the particle hardness and fracture toughness.

a, b, c, d, and e are the exponent of the abovementioned parameters. The existing chipping models (Evans et al., 1978; Evans and Wilshaw, 1976; Ghadiri and Zhang, 2002) are rooted from the same mechanical foundation, i.e. indentation fracture process. The notable difference within these chipping models is the varying velocity exponent b. As the impact velocity is the most influential parameter, Eq. (10) can be further simplified when the mechanical properties are not known:

$$P_x = m * v^b \tag{11}$$

where $m = \frac{\rho^a x^c H^d}{{k_c}^e}$ is treated as a single lumping parameter.

It has been nearly two decades for Vogel and Peukert model to construct a unified master curve including particle size, impact energy, impact frequency. This model has been successfully used with a wide application into materials like limestone, glass spheres, and polymethyl methacrylate (PMMA). However, the impact angle as another critical breakage parameter is excluded in this model. To address this challenge, a novel oblique impact model is thus developed with an attempt to establish a unified master curve for oblique impact breakage. A summary of critical parameters considered in the existing normal breakage models is shown in Table 1.

Table 1 Critical parameters considered by existing normal impact breakage models (Modified from Wang et al., 2021b)

Number	Breakage model	Mathematical form	Particle size	Mechanical property	Impact velocity	Impact frequency	Impact angle
1	Pocock et al., 1998	Lognormal	Yes	Yes	Yes	Yes	No
2	Salman et al., 2003	Weibull	No	No	Yes	No	No
3	Vogel and Peukert, 2003	Weibull	Yes	Yes	Yes	Yes	No
4	Portnikov-Kalman, 2018	Logistic	Yes	Yes	Yes	Yes	Yes
5	Wang et al., 2021a	Power	Yes	Yes	Yes	No	Yes

2.2 A novel oblique impact model

Appreciable progress of oblique impact breakage has been made with concluding remarks that normal component velocity is dominant in particle breakage (Salman et al., 1995; Wang, 2016). However, in most cases, the understanding gained is based on experimental breakage tests or computational DEM simulations. A mechanistic-based breakage model subject to oblique impact is not yet available. Recent work for oblique impact model development was performed, where the contribution of tangential velocity component is justified (Wang et al., 2021a). The main equations of the developed oblique impact model are briefly recalled for the sake of completeness. Further details about the analytical formulations can be found in the original publication (Wang et al., 2021a).

The normal component v_n and tangential component v_t of an impact velocity v with impact angle θ can be given by

$$v_n = v sin\theta \tag{12}$$

$$v_t = v cos\theta \tag{13}$$

Regardless of impact angle θ, the resultant impact velocity is hereby given by

$$v = v\sqrt{sin^2\theta + cos^2\theta} \tag{14}$$

Accordingly, the normal impact force F_n and the tangential impact force F_t are expected to arise from the normal component velocity v_n and tangential component velocity v_t.

As a result, the resultant impact force F is given by

$$F = \sqrt{F_n^2 + F_t^2} \tag{15}$$

It is well known that the breakage induced by F_n and F_t are differing despite the same input value and time characteristics. Hence, it is more appropriate to propose an equivalent impact force F_e where the breakage caused by F_n can be comparable to that caused by F_t and it gives

$$F_e = \sqrt{F_n^2 + \alpha^2 F_t^2} \tag{16}$$

where α is the fitting parameter to correlate the breakage caused by tangential force F_t. The schematic of impact velocity and impact force with normal and tangential components is shown in Figure 2.

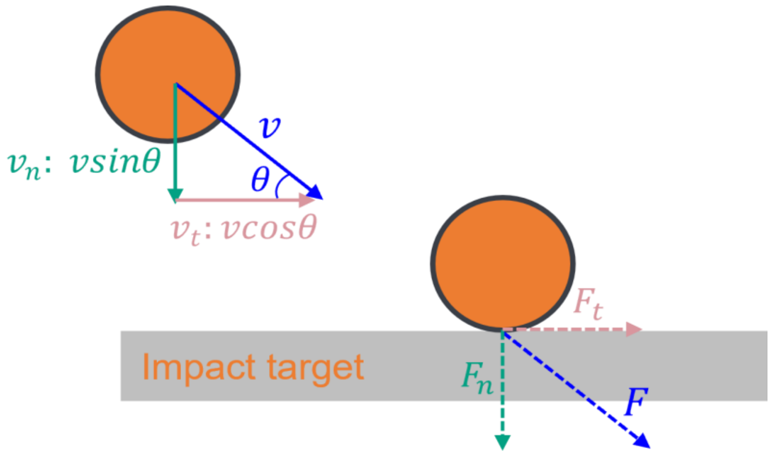


Figure 2 *Schematic of impact velocity and impact force with normal and tangential components under oblique impact*

Furthermore, the activation of tangential impact force F_t is relied on contact friction and more specifically is determined by the dynamic friction coefficient between the particle and the impact target, i.e. $F_t \leq \mu F_n$. Similar to the equivalent impact force F_e, an effective tangential velocity associate with the effective tangential impact force F_t can be formulated by

$$v_{te} = \mu v sin\theta cos\theta \tag{17}$$

Analogue to Eq. (17), the equivalent velocity v_{eq} can hereby be defined as below:

$$v_{eq} = \sqrt{v_n^2 + \alpha^2 v_{te}^2} \tag{18}$$

Substituting Eq. (17) into Eq. (18), it evolves

$$v_{eq} = v\sqrt{sin^2\theta + \psi^2 sin^2\theta cos^2\theta} \tag{19}$$

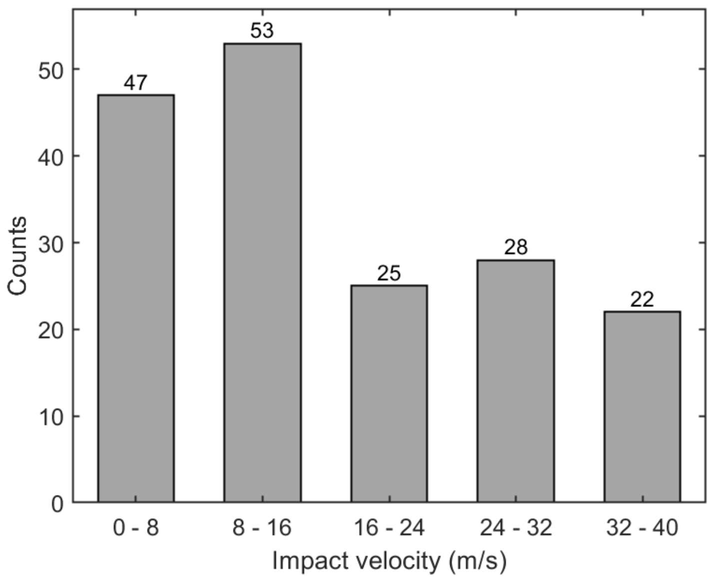
Comparing Eq. (19) with Eqs. (12) and (14), it can be found that equivalent velocity falls in between the normal velocity and the impact velocity.

The lumped parameter $\psi = \alpha\mu$ reflects the combination of frictional behaviour and the correlation with tangential velocity. The equivalent velocity in Eq. (19) can be appended to any existing breakage models. A simple treatment is to replace the normal velocity with equivalent velocity under oblique impact conditions. However, the success of Eq. (19) has only been established with a limited amount of breakage models and breakage database (Wang et al., 2021a). This study is to pursue a comprehensive assessment of the proposed oblique impact model in a wide spectrum of breakage database from the literature.

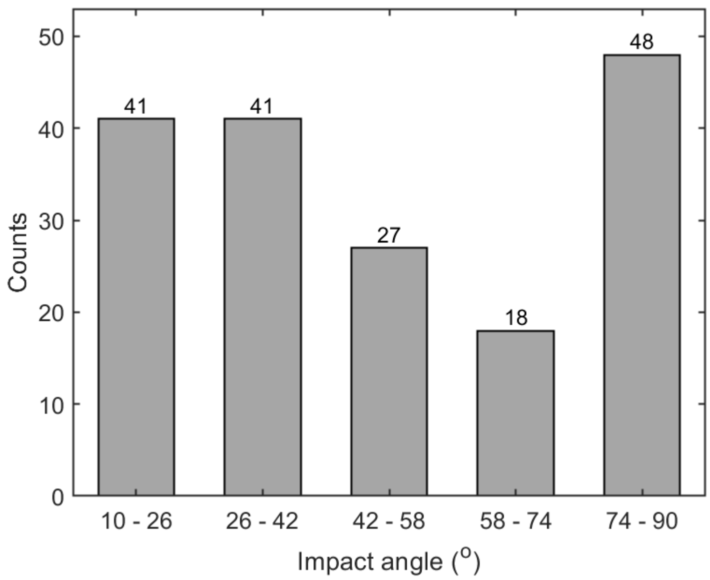
3 Literature database

A wide variety of literature database was collected and used for oblique model assessment from an extensive scope of relevant scholarly work. The database covers 5 types of particles and 175 datapoints with the impact angle spanning from 10^{o} to 90^{o}, which can be found in the appended link of Excel file. The distribution of the collective data of impact angles is plotted in Figure 3. In Figure 3a, the impact velocity is varied from 1.2 m/s to 40 m/s and the impact velocity between 32 m/s and 40 m/s accounts for the least amount amongst the five bin size regimes. In Figure 3b, the impact angles are categorized into 5 bin sizes with the incremental value of 16^{o}. The impact angles between 74^{o} and 90^{o} accounts for the largest fraction whilst the impact angles between 58^{o} and 74^{o} is least represented. The detailed features of test conditions in the breakage database are reported in the Appendix. In Figure 3c, the lower breakage probabilities from zero to 0.2, 0.4-0.6 have the largest proportion of test data. The breakage probabilities between 0.8 and 1.0 have the minimum number of 19 test points, which corresponds to the least amount of impact velocity ranging 32 m/s to 40 m/s in Figure 3a.

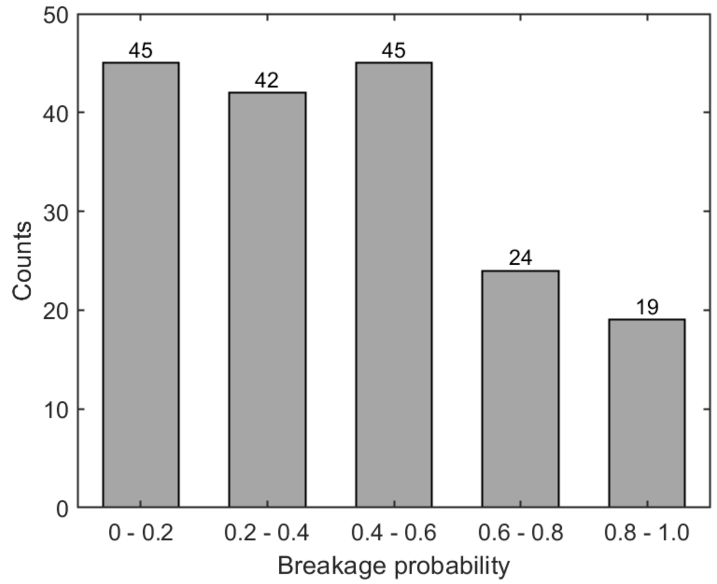
It is clear that the combined database spans a wide range of key features, and therefore provides a robust benchmark for the oblique impact model assessment. It is encouraged that future data be gathered in under-represented data classifications. For example, the impact breakage tests under the impact angles 58^{o}-74^{o} are encouraged for future work.


47
53
25
28
22
Counts
Impact velocity (m/s)
0 - 8
8 - 16
16 - 24
24 - 32
32 - 40

(a)


41
41
27
18
48
Counts
Impact angle (o)
10 - 26
26 - 42
42 - 58
58 - 74
74 - 90

(b)



(c)

Figure 3 Histogram distribution of key factors in the test database (a) Range of impact velocity (b) Range of impact angle (c) Range of breakage probability

3.1 Experimental database

The experimental database is composed of breakage results from three publications (Jägers et al., 2021; Portnikov et al., 2018; Salman et al., 2002). The test materials under oblique impact include 2.36-3.35 mm salt particles (Portnikov et al., 2018), 3.2 mm fertiliser particles (Salman et al., 2002), and 15 mm wood pellet (Jägers et al., 2021). Note that the breakage probability in the database from (Salman et al., 2002) is defined by the number of unbroken particles. To maintain the consistency with the other data source, this is converted by calculation to the breakage probability. In their original source, the breakage ratios are plotted as a function of impact velocity (Jägers et al., 2021; Portnikov et al., 2018; Salman et al., 2002). These experimental results clearly indicate the breakage ratio is varied as function of impact angles irrespective of particle size. The breakage ratios show the maximum value under normal impact angle and then diminish significantly with the decrease of impact angle.

3.2 DEM database

The DEM database includes the breakage data reported by Moreno et al. 2003 and Ye et al. 2021. In Moreno et al. 2003, the extent of breakage is characterised by damage ratio, which is defined as the ratio of broken contact numbers to the initial contact numbers (Moreno et al., 2003). The damage ratio is plotted as a function of six impact angles, i.e. 30°, 45°, 60°, 70°, 80°, 90° and the impact velocity is varied from 1.15 m/s to 3.41 m/s subject to the six impact angles. The general trend of their study indicates increasing damage ratio with increased impact velocity for the same impact angle. Moreover, the increase of damage ratio is observed when the impact angle is increased from 30° to 90°. The breakage data from (Ye et al., 2021) covers the damage ratio spanning from zero until 0.8, resulted from the impact velocity between 4 m/s to 14 m/s subject to five impact angles, i.e. 15°, 35°, 55°, 75°, and 90°. Similar conclusions were drawn; increase of impact angle resulted in an increased damage ratio for the same impact velocity. In particular, lower impact angles demonstrated a widening gap of damage ratio compared to that under normal impact given the same impact velocity. Figure 4 depicts the proportion of experimental and DEM database amongst the literature database.

In view of both experimental and DEM database, it is firmly believed that a wide spectrum of oblique impact is covered in the present study, which is expected to sufficiently satisfy the amount of data for the oblique impact model assessment.

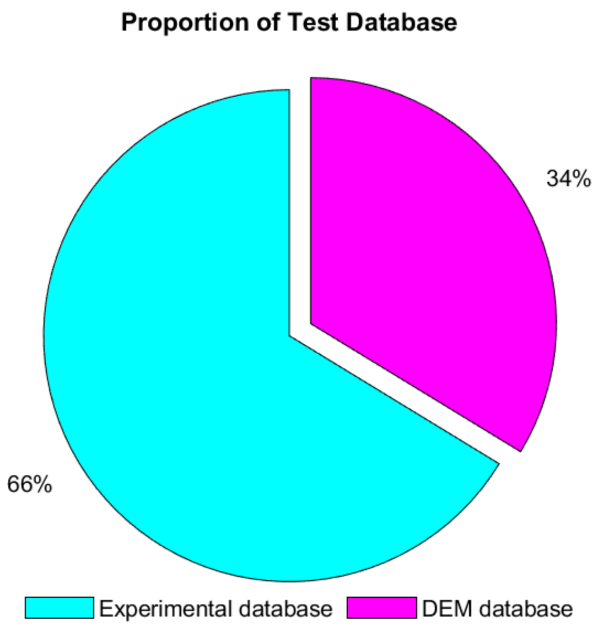


Figure 4 Proportion of experimental and DEM database in the scoped literature

4 Assessment of the oblique impact breakage model

4.1 Model assessment procedure

The breakage model assessment procedure can be divided as the following steps (shown in Figure 5). The first step is to identify the breakage pattern, i.e. chipping or fragmentation. The second step is to choose the appropriate model expression. A variety of breakage models under normal impact was developed for chipping and fragmentation in the literature. In this stage, mathematical simplicity and physical meaning of model parameters are given priority in the model selection. In this work, the breakage models will be directly selected from the original source of the breakage data. For the original source without specification of breakage models, Vogel and Peukert model is assigned for the test data from (Ye et al., 2021), due to the success of this model in the construction of the unified breakage master curve. The third step is to estimate the fitting parameters in the breakage models against the normal impact breakage data. The fourth step is to adopt the equivalent velocity replacing impact velocity in the breakage models as selected in the second step. The optimal value of friction mobilisation parameter ψ will be achieved against the breakage data under various impact angles.

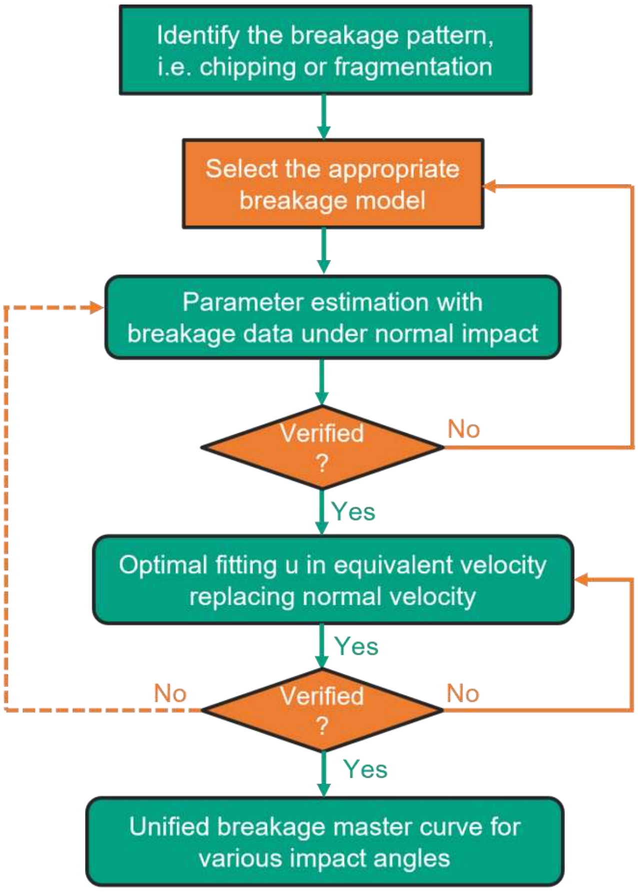


Figure 5 Model assessment flowchart for unified breakage master curve

4.2 Particle breakage master curve

4.2.1 Statistical performance

The statistical performance of each model as assessed by the collected data is given in Table 2. Table 2 first shows the normal impact models corresponding to the individual data source, where the model parameters are fitted with only the normal impact data. Then the fitting parameter ψ in the proposed oblique impact model is achieved with the oblique impact breakage dataset. Despite differing normal impact models used for normal impact data fitting, utilizing the equivalent velocity from the proposed oblique impact model gives rise to ψ^2. As only one set of fitting parameters is required for various impact angles in the sourced dataset, the fitting efficiency is thus significantly improved as compared to individual parameter fitting subject to every impact angle. For example, there are total 7 impact angles considered in the dataset from (Jägers et al., 2021). The model fitting efficiency is considerably improved by 86% using one set of fitting parameters in Table 2, compared to the conventional seven sets of fitting parameters with regards to the seven impact angles accordingly.

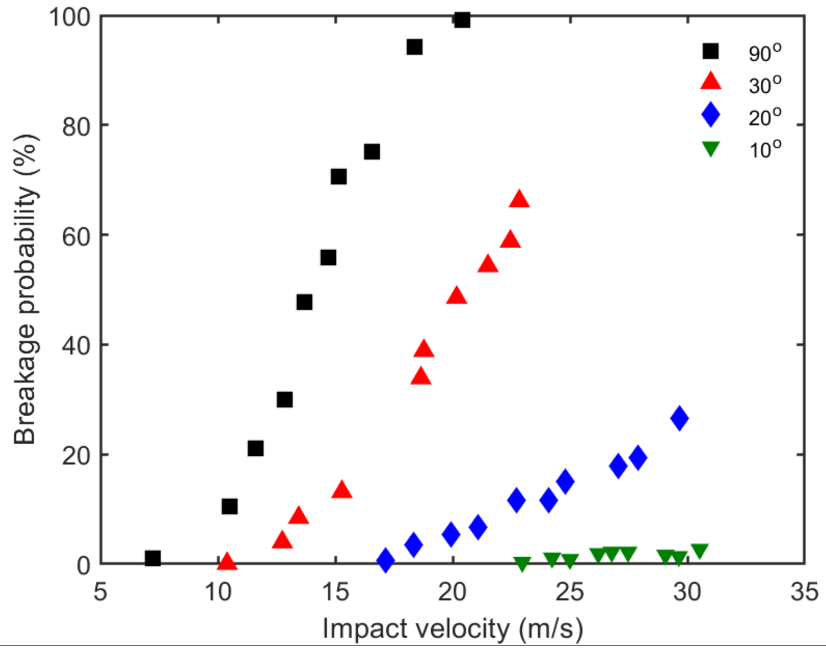
Table 2 Fitting parameter and reduced efforts in parameter estimation

Data source	Normal impact model and fitting parameters		Oblique impact model and fitting parameters	Breakage type	Improved fitting efficiency - (Set of fitting parameters with and without the oblique impact model)
(Salman et al., 2002)	Salman model		(Wang et al., 2021a) $\psi^2 = 0.89$	Fragmentation	75% - (1:4)
	m	15.14			
	n	5.25			
	R^2	0.995			
(Moreno et al., 2003)	Wang model		(Wang et al., 2021a) $\psi^2 = 0.031$	Chipping	83% - (1:6)
	m	0.12			
	n	1.2			
	R^2	0.995			
(Portnikov et al., 2018)	Portnikov model		(Wang et al., 2021a) $\psi^2 = 0.39$	Fragmentation	86% - (1:7)
	v_{50}	11.9			
	b	2.25			
	R^2	0.995			
(Jägers et al., 2021)	Portnikov model		(Wang et al., 2021a) $\psi^2 = 0.71$	Fragmentation	86% - (1:7)
	v_{50}	19.98			
	b	2.55			
	R^2	0.998			
(Ye et al., 2021)	Vogel and Peukert model		(Wang et al., 2021a) $\psi^2 = 1.06$	Fragmentation	83% - (1:6)
	f_{Mat} (kg/Jm)	0.33			
	$W_{m,min}$ (J/kg)	0.52			
	R^2	0.991			

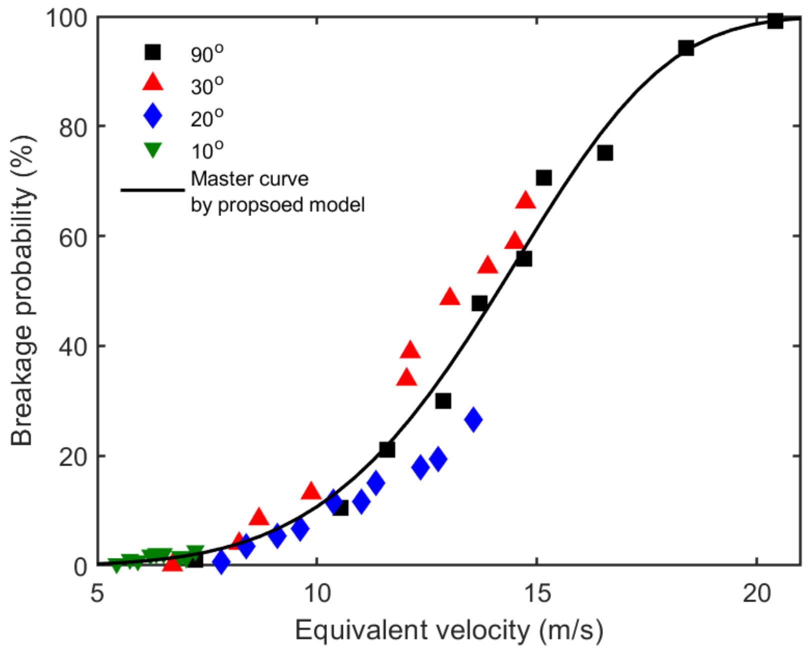
4.2.2 Graphical comparison

The predicted breakage probability using the equivalent velocity is compared with the test database to assess the applicability of the proposed oblique impact model in Figures 6, 7, 8, 9 and 10. An example of chipping model assessment with the test data from (Moreno et al., 2003) is presented in Figure 7. The breakage data is reasonably assumed to follow a power trend under normal impact. The velocity exponent is determined as 1.2 using the nonlinear least squares method. Whilst keeping the velocity exponent fitted from normal impact constant, the impact velocity is replaced with the equivalent normal velocity to fit the oblique model parameters against the test data from the five impact angles 80°, 70 °, 60 °, 45 °, 30 °. In Figure 7a, without the adoption of equivalent velocity proposed in the present work, the breakage probability under various impact angles differs markedly with a large scatter. The principal cause for the scatter is attributed that the contribution of tangential velocity is not considered in majority of existing breakage models. As a stark contract, Figure 7b clearly shows a unified breakage master curve of chipping database under various impact angles. This is attributed to the equivalent velocity where the contribution from tangential velocity can be rationalized. Taking the fragmentation database from (Jägers et al., 2021) as another example, the breakage ratio predicted using the equivalent velocity are compared with the test results in Figure 9. Figure 9a is the plot of reported breakage probability of wood pellet with particle length 15 mm in dataset 2 and Figure 9b is the predicted breakage probability by equivalent normal velocity under seven impact angles, 90°, 70°, 50°, 45°, 40°, 30°, 20°.

In Figure 9a, the breakage probability of wood pellets at seven impact angles are nearly the same at the impact velocity 5 m/s. When the impact velocity is increased as 15 m/s, the breakage rate under oblique impact becomes more scattered as a function of impact angles. In particular, the breakage probability is increased with the increase of impact angle and the scatter of breakage probability under oblique impact is widening with higher impact velocity until 40 m/s. The breakage dataset from (Jägers et al., 2021) clearly indicates a fragmentation mechanism and hence Portnikov et model is appropriately used for the parameter fitting under normal impact. Following the model assessment procedure, Figure 9b displays a unified breakage master curve as a function of equivalent velocity. Likewise, the unified breakage master curves using the equivalent velocity in Eq. (19) are also observed in Figures 5, 7, and 9 for the breakage data from (Portnikov et al., 2018; Salman et al., 2002; Ye et al., 2021). The predicted and surveyed breakage probabilities by means of equivalent velocity and literature database respectively, are depicted in Figure 11. This parity plot displays the surveyed test data on the horizontal axis and predicted values on the vertical axis. This again indicates a strong predictive accuracy from the oblique impact model where the total 175 data points follows the diagonally linear (1:1) line.

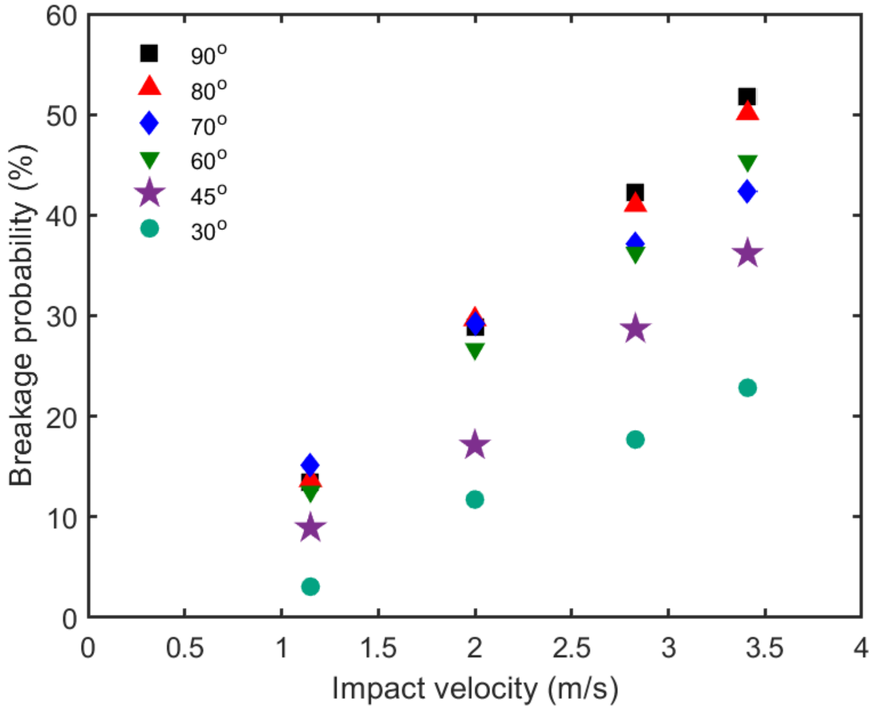


(a)

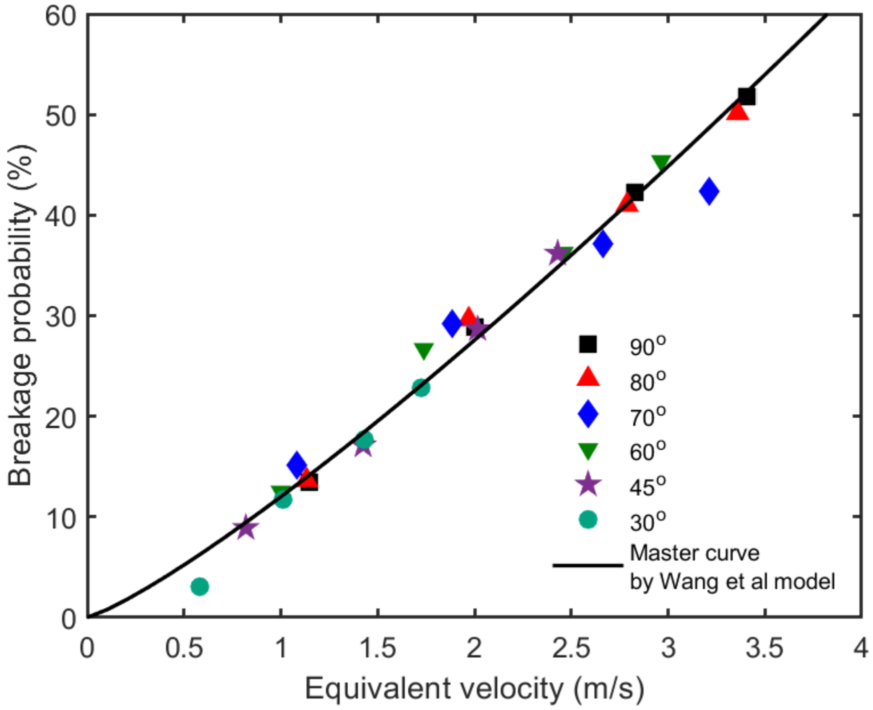


(b)

Figure 6 (a) Breakage database from Salman et al. 2002 versus (b) master curve using proposed oblique impact model

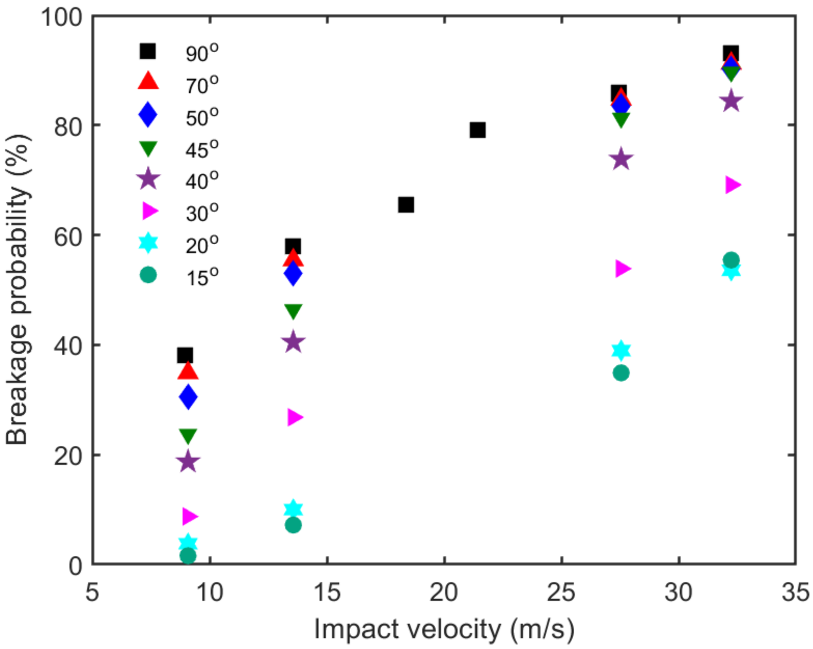


(a)

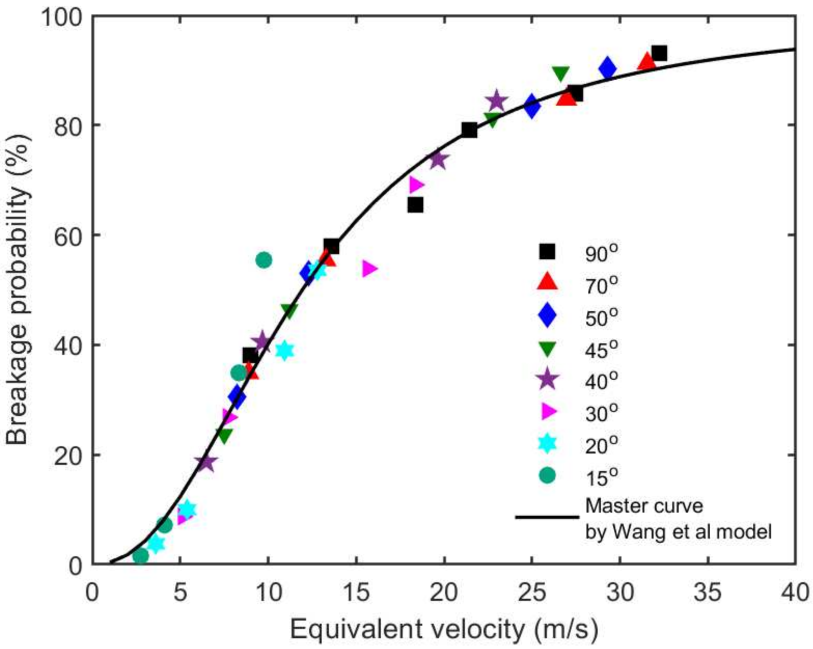


(b)

Figure 7 (a) Breakage database from Moreno et al. 2003 versus (b) master curve using proposed oblique impact model

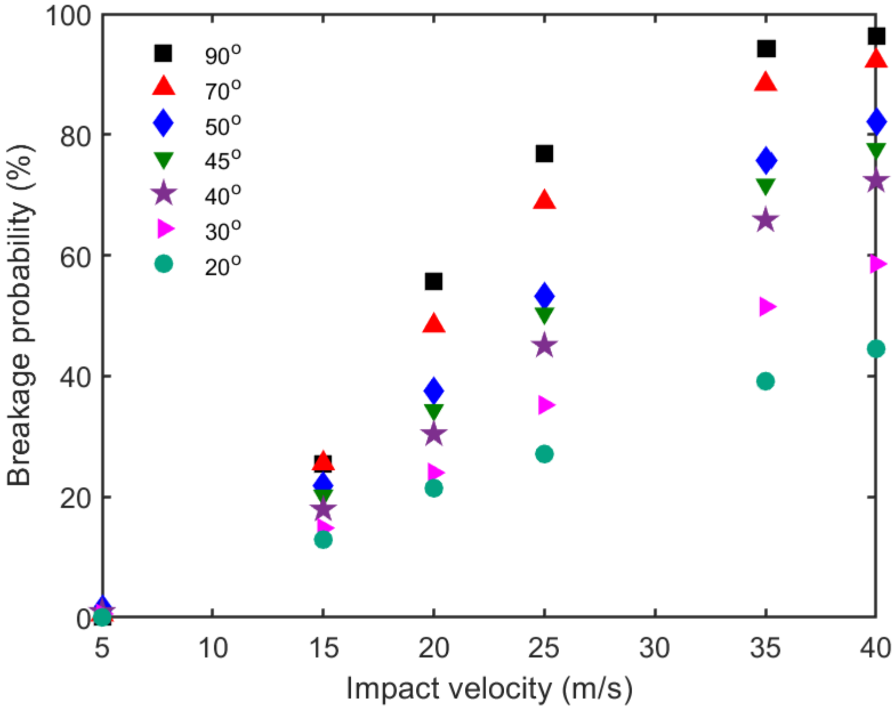


(a)

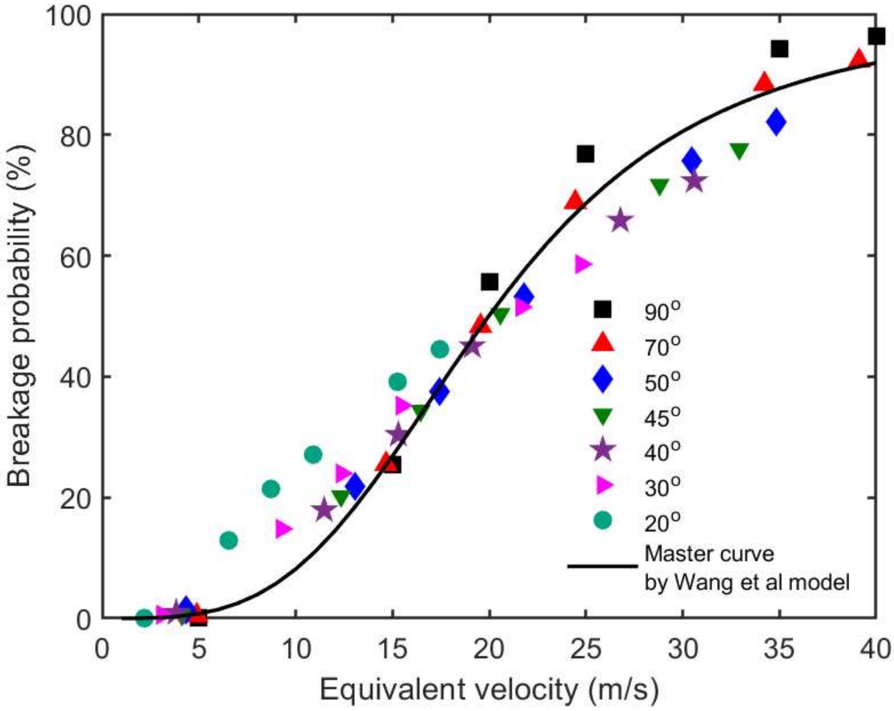


(b)

Figure 8 (a) Breakage database from Portnikov et al. 2018 versus (b) master curve using proposed oblique impact model

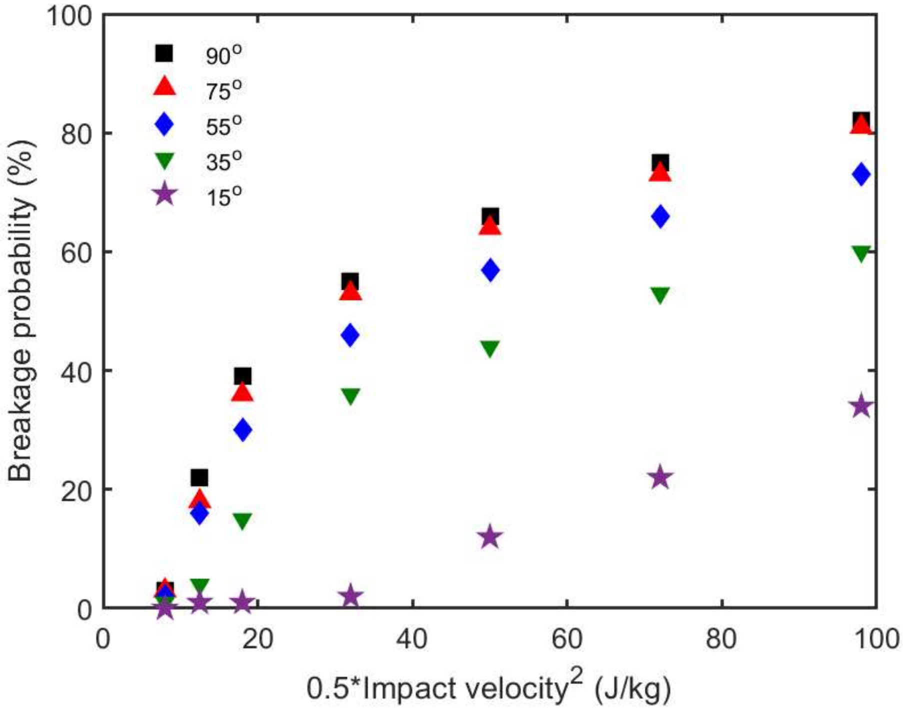


(a)

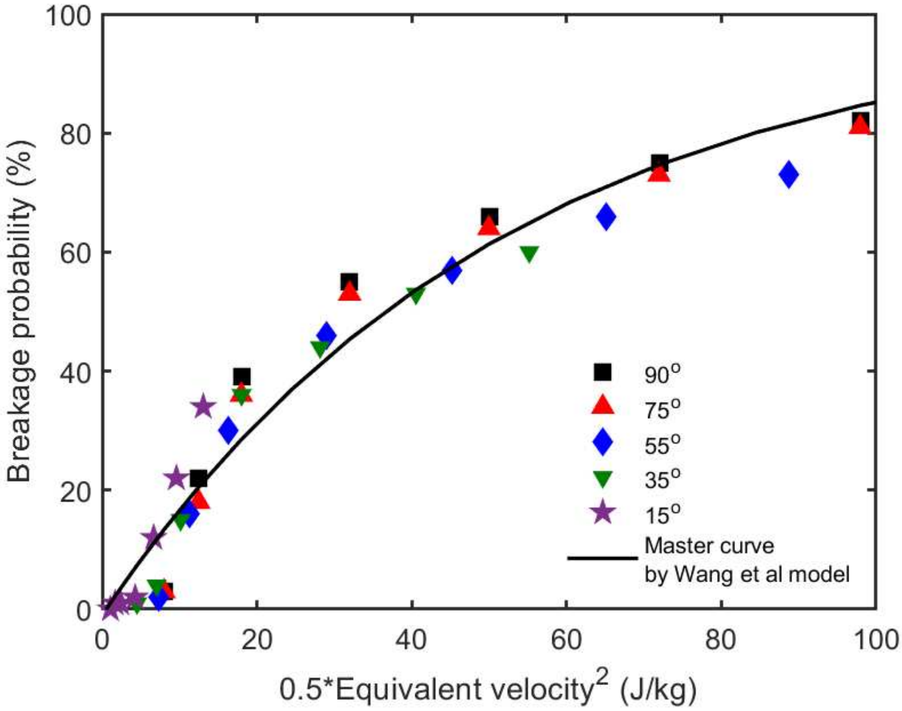


(b)

Figure 9 (a) Breakage database from Jägers et al. 2021 versus (b) master curve using proposed oblique impact model



(a)



(b)

Figure 10 (a) Breakage database from Ye et al. 2021 versus (b) master curve using proposed oblique impact model

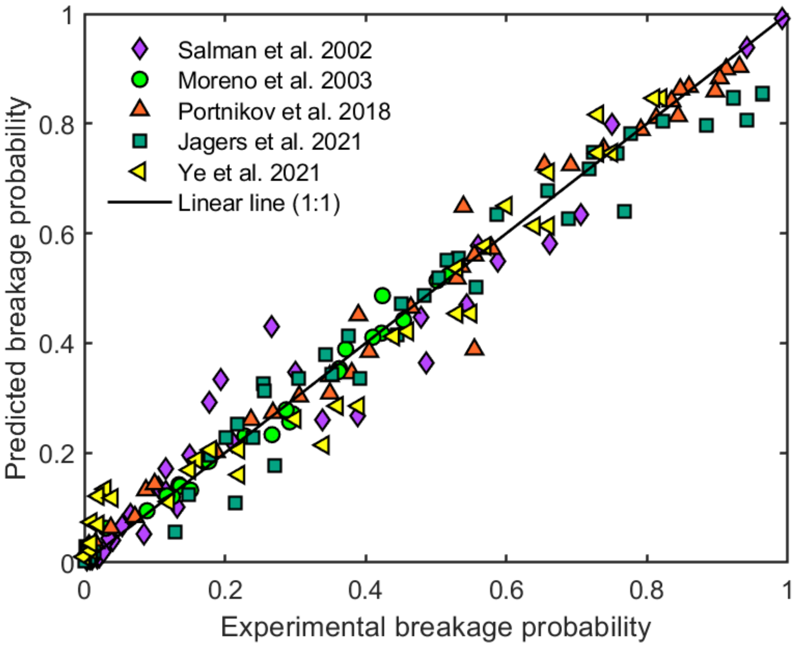


Figure 11 Experimental vs predicted breakage probability for all the breakage database

4.3 Discussion

Results from the oblique impact model assessment clearly show the universality of the proposed equivalent velocity in unifying the master curves with all the deployed test data. Compared to the conventional experimental studies, the proposed oblique impact model provides a theoretical solution for oblique impact conditions. Significant improvement of breakage probability prediction can be made by only one set of fitting parameters for various impact angles. A mapping regime of breakage probability subject to oblique impact can thus be readily established with the calibrated parameters setup. For instance, with the calibrated parameters against the dataset from (Jägers et al., 2021), the breakage probability map based on Portnikov et al. model is plotted in Figure 12 where the contribution of both impact velocity and impact angle can be quantified. In the preceding study any further attempts of oblique impact breakage beyond the test data have to be experimentally performed as hindsight. As shown in Figure 12, the proposed oblique impact model is capable of predicting breakage probability with consideration of both impact velocity and impact angle. The oblique impact model is also promising for coupling DEM with other computational techniques such as computational fluid dynamics (CFD) or population balance model (PBM). Despite the insights of particle collision information from DEM, the critical information such as the distribution of impact angle and its role in the multiphase interaction has not yet been effectively explored. The issue regarding inadequacy or ignorance of impact angle in DEM-CFD or DEM-PBM coupling will be especially significant where the oblique impact in the particulate processes such as fluid bed granulation and dry milling is frequent. Another challenge remains whether the proposed equivalent velocity in the oblique impact model can be used to unify the breakage function, i.e. fragment size distribution with respect to varying impact angles. This forms a potential research topic for further work, to examine whether a unified curve of particle size distribution can be similarly constructed given varying impact angles.

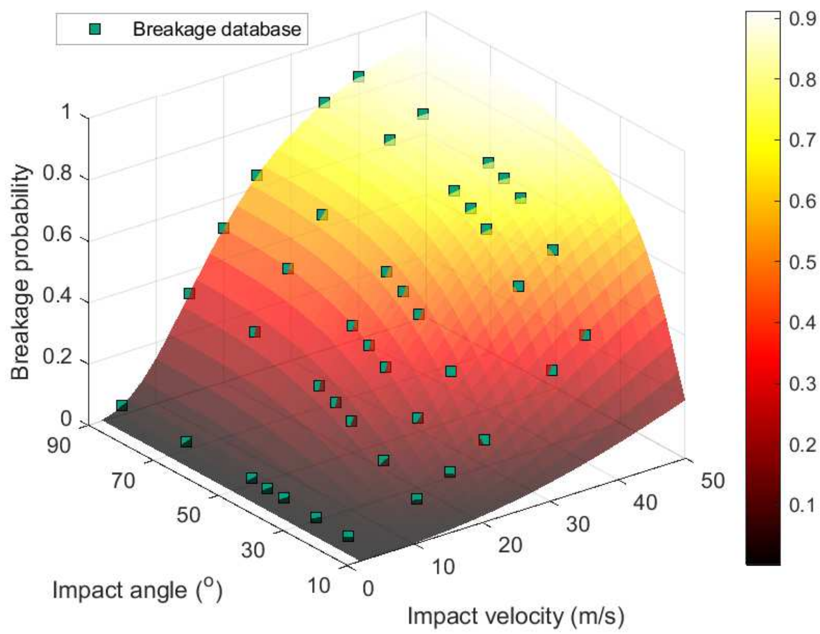


Figure 12 Breakage mapping regime created using the proposed oblique impact model based on the database from (Jägers et al., 2021)

5 Conclusions

This paper has presented a simple and effective oblique impact model where the breakage master curve can be invariably established for various impact angles. The motivation behind this developed model is driven by the omission of tangential velocity component in the conventional breakage models. The breakage probability is likely to be underestimated when calculated considering only from the normal velocity component, ignoring the tangential velocity component. The novelty of the proposed model lies in the consideration of tangential velocity component, the physical consideration of friction coefficient, and most importantly a unified breakage master curve using the paradigm of equivalent velocity.

The assessment of breakage models under oblique impact was conducted using the collected breakage database from the literature. The developed oblique impact model is shown to be generally applicable in all the oblique impact circumstances. This is the first oblique impact model where the breakage probaiblity subject for oblique impacts can be unfied with a master curve, overcoming the experimental limitations and considerably improving the fitting efficiency and predictive accuary. The developed oblique impact model is therefore recommended for future exploration of particle dynamics, where oblique impacts are significant. It is expected this model will be of particular use in future DEM-CFD or DEM-PBM coupling scenarios.

Nomenclature

a	Fitting parameter, -
A_v	Fitting parameter, -
b	Fitting parameter in Eqs. (2) and (4), -
c	Fitting parameter, -
d	Particle diameter in Eq. (1), mm
d_0	Fitting parameters, -
e	Fraction of loss energy in Eq. (6), -
$a, b, c, d,$ e	Exponent in Eq. (10), -
E	Mass specific energy, J/kg
$E_{50,i}$	Median mass specific energy, J/kg
E_{loss}	Mass-specific collision energy, J/kg
E'_{loss}	Total energy loss in the collision, J/kg
f_{Mat}	Resistance of the particle against the external stressing, $kgJ^{-1}m^{-1}$
F	Impact force, N
F_n	Normal impact force, N
F_t	Tangential impact force, N
F_e	Equivalent impact force, N
H	Particle hardness, GPa

k	Hertzian stiffness of the contact target, -
k_c	Fracture toughness, MPa.m$^{1/2}$
m_p	Mass of a pellet, g
m	A single lumping parameter, -
n	Impact number in Eq. (9), -
P_E	Breakage probability as a function of specific energy, -
P	Breakage probability, selection function, -
P_x	Breakage probability, -
t_{10}	Breakage index (%), -
v	Impact velocity, -
v_n	Normal component of impact velocity, -
v_t	Tangential component of impact velocity, -
v_{50}	Median impact velocity resulting in 50% of particle breakage, m/s
v_e	Equivalent velocity, -
$\mathrm{W_m}$	The mass-specific energy, -
$W_{m,min}$	Mass-specific threshold energy for particle to break, -
$W_{m,kin}$	Mass-specific kinetic energy, -
x	Particle size, -
z	Fitting parameter, -

Greek symbols

σ	Standard deviation of the specific energy, J/kg
θ	Impact angle, ^{o}
ρ	Particle density, kg/m^3
α	Fitting parameter, -
μ	Dynamic friction coefficient, -

Appendix

Table A1 Features of test conditions from the literature breakage database

Database No.	Source literature	Source data	Test particle and diameter	Test method	Breakage pattern	Amount of data
1	(Salman et al., 2002)	Figure 4	Fertiliser particle, 3.2 mm	Experimental Horizontal impact	Fragmentation	40
2	(Moreno et al., 2003)	Figure 3	Agglomerate, 1.814 mm	DEM	Chipping	24
3	(Portnikov et al., 2018)	Figure 4	Salt particle, 2.36-3.35 mm	Experimental Horizontal impact	Fragmentation	34
4	(Jägers et al., 2021)	Figure 6b	Wood pellet, 15 mm	Experimental Horizontal impact	Fragmentation	42
5	(Ye et al., 2021)	Figure 12a	Marble sphere, 58 mm	DEM	Fragmentation	35

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