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# Toward scalable benchmark problems for multi-objective multidisciplinary optimization

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***Abstract*—Scalability in disciplines is an important consideration for multidisciplinary design optimization (MDO). Very few benchmark problems for multi-objective MDO exist in the literature, none of which are readily scalable. In this study, we introduce a new scalable benchmark problem that extends an existing well-known multi-objective benchmark problem. We show that scaling the number of disciplines in the problem, implying an increasing number of decision variables, does produce substantive changes in convergence ability. We also show that the accuracy of the multidisciplinary analysis (MDA) solver has an impact on the convergence ability of the multi-objective optimization algorithm, which is particularly noticeable when moving from 7 to 14 disciplines. Modification of standard (non-MDO) multi-objective benchmark problems is a promising approach to developing scalable multi-objective MDO benchmarks.**

***Index Terms*—multidisciplinary design optimization, multi-objective optimization, benchmark problems, scalability**

## I. Introduction

Multidisciplinary design optimization (MDO) is a field of research that deals with the optimization of systems involving multiple disciplines, subsystems or components. Environmental, socio-economic, institutional and engineered systems are commonly partitioned into smaller interacting subsystems that may be connected in a variety of ways. The performance of the overall system may differ from the performance of the individual components due to their interactions. One of the key challenges in MDO is to model these interactions and, in addition, there might be a need to account for non-comparable and conflicting criteria [1]. A typical example is the design of an aircraft wing involving disciplines that deal with the aerodynamics and structural integrity of the aircraft [2]. Another example is a robotic fish where the overall system has been decomposed into four disciplines, namely hydrodynamics, propulsion, weight and equilibrium, and energy [3].

A standard constrained single-objective optimization problem contains design variables, an objective function to be either minimised or maximised, and for feasibility a candidate design might need to satisfy one or more constraints. An MDO problem contains these characteristics and in addition the behaviour of each component or discipline is modelled by the use of discipline analysis. Each discipline analysis consists of running a simulation model that represents some phenomena associated with the given discipline, and the procedure often consists of solving a set of equations (e.g. Navier-Stokes equations in fluid mechanics). The outputs generated by one discipline analysis might be required as an input to run the simulation model of another discipline analysis. These interactions between disciplines are modelled by the use of linking variables (also known in the literature as coupling or response variables).

For dealing with MDO problems there are several approaches in the literature, and these have been classified into architectures by Martins and Lambe's 2013 seminal survey paper [4]. Depending on the strategy adopted, there might be a need to create copies of the linking variables in order to allow the discipline analysis to run in parallel—one example is the Individual Discipline Feasible (IDF) architecture [5]. In such cases it might be required to enforce consistency constraints to ensure that the values provided by the copies are consistent with the outputs emanating from the discipline analysis. In other approaches, the discipline analysis could be run in a sequence to avoid the use of linking variable copies—as is the case with the Multidisciplinary Feasible (MDF) architecture [5].

Multi-objective problems (MOPs) are important in the field of optimization because there is rarely only one objective to optimize in real-world or synthetic problems. MOP benchmarks are well-studied and there are already several test suites, which include problems that are scalable in complexity, objectives, design variables, and other factors. However, multi-objectivity in MDO has not received the same amount of attention. There are no scalable multi-objective, multidisciplinary benchmark problems in the existing literature and, as a result, research in real-world MDO problems is limited.

This paper aims to discuss some of the issues arising from scaling multi-objective MDO problems in the number of disciplines by modifying the well-known ZDT test suite based on the approach used by Tedford and Martins [6] for single-objective problems. Although we only modify ZDT1 for brevity, the same principle could be applied to the other problems in the ZDT test suite, and perhaps to other test suites

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as well. The remainder of the paper is organised as follows. An introduction to MDO architectures, with special focus on the MDF approach, is provided in Section II. Section III discusses existing multi-objective and multidisciplinary benchmark problems and test suites available in the literature. A proposed multi-objective MDO problem with scalable number of disciplines based on the ZDT1 test problem is described in Section IV. The experimental setup used in this paper is in Section V. The experimental results in Section VI includes the effects of introducing varying number of disciplines to the problem (Section VI-A), and the influence of solver accuracy on convergence (Section VI-B). The paper concludes with a discussion and future work in Section VII.

## II. Overview of MDO architectures

MDO architectures are well documented in the literature. In [4] the authors have outlined various different architectures and these were categorised as either monolithic or distributed. In the monolithic approach a single optimization problem is solved, whereas in the distributed approach the overall optimization problem is partitioned into smaller subproblems each containing its own subset of decision variables, constraints and objectives. The focus of this paper is on the MDF monolithic architecture where the optimization problem contains multiple conflicting objectives.

We now describe a single-objective optimization problem resulting from the MDF architecture. The problem contains global, local and linking variables. If a variable is global it means that it is accessible to all disciplines, while a local variable is only accessible to one. The constraints (if any) can also be either global or local, and the objective function is assumed to be global. Linking variables are used to model the interactions of the whole system and need to be exchanged between the disciplines. In this sense, each linking variable is supplied by a discipline as a response of the analysis conducted in the disciplinary model for a given design decision. Mathematically, the optimization problem is expressed as:

$$\begin{aligned} \min \quad & f_0(\mathbf{x}, \mathbf{y}(\mathbf{x}, \mathbf{y}), \mathbf{z}) \\ \text{w.r.t} \quad & \mathbf{x}, \mathbf{z} \\ \text{s.t.} \quad & \mathbf{c}_0(\mathbf{x}, \mathbf{y}(\mathbf{x}, \mathbf{y}), \mathbf{z}) \geq 0 \\ & \mathbf{c}_i(\mathbf{x}, \mathbf{y}(\mathbf{x}, \mathbf{y}_{j \neq i}), \mathbf{z}) \geq 0 \quad \text{for } i = 1, ..., N. \end{aligned} \tag{1}$$

In Equation 1 the subscripts 0 and $i$ are used to indicate if a function or variable is either a global or local one, respectively. For a total of $N$ disciplines $\mathbf{x}$ is a vector that contains all local variables, that is, $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)^T$, and at the $i$th discipline ($i \in \{1, \ldots, N\}$) there are $n_{x_i}$ local variables given by the vector $\mathbf{x}_i = (x_{i,1}, \ldots, x_{i,n_{x_i}})^T$. A total of $n_z$ global variables are in the vector $\mathbf{z} = (z_1, \ldots, z_{n_z})^T$. Similarly to $\mathbf{x}$, the vector $\mathbf{y}$ contains all linking variables per discipline, that is, $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_N)^T$, and at the $i$th discipline there are a total of $n_{y_i}$ linking variables given by the vector $\mathbf{y}_i = (y_{i,1}, \ldots, y_{i,n_{y_i}})^T$. $\mathbf{c}_0$ and $\mathbf{c}_i$ are both vectors representing the global and local constraints, respectively, and $f_0$ is the objective function.

One of the characteristics of the MDF architecture is that each discipline is solved in turn, often by the use of the Gauss–Seidel multidisciplinary analysis (MDA) procedure [7] or by the use of Newton-based methods. This implies that the different discipline analyses cannot be conducted in parallel but it ensures consistency between the linking variables inputs and outputs (this is particularly important if there are circular dependencies between the disciplines as is the case in this paper's problem formulation). One of the drawbacks of this architecture is that a full MDA needs to be conducted for each candidate design, and the MDA needs to be run until a consistent set of linking variables are found. Not having a consistent set of linking variables after each analysis can have an impact on the convergence of the optimization algorithm, as demonstrated in this paper.

## III. Overview of multi-objective and multidisciplinary test problems

### A. MDO benchmarks

*1) NASA test suite:* The NASA test suite [8] was developed by the Langley Research Centre in the 1990s and contains 14 test problems, some of which are still in use today. These include the heart dipole problem, the combustion of propane problem, the scalable problem, and others. Even though the NASA test suite is considered an important base for the development of MDO architectures and strategies, many of the test problems are no longer available first-hand due to lack of website maintenance, and the test suite is outdated and does not reflect the state of MDO research today. This means that some of the test problems can only be obtained through secondary sources, some of which are expressed using non-standard notation. Additionally, the problems in the NASA test suite are single-objective and are scalable in the number of decision variables and disciplines. Therefore, the test suite is currently not suited for dealing with problems with multiple conflicting objectives. The Golinski speed reducer, one of the single-objective NASA test suite problems, has been reformulated by transforming the objective function, as well as the constraints, into multiple objectives [9].

*2) Other benchmark problems:* MDO problems that are not associated with any test suite include the Sellar problem [10], which was developed as a 2-discipline, single-objective problem, often used for testing different architectures and optimization algorithms. Each discipline contains one equation within its MDA, as well as constraints on the linking variables. The Sellar problem has been extensively discussed within the MDO literature, although finds little use beyond basic architecture testing due to its small size and simplicity. Other problems such as those presented in [11] and solved in [12] are more applicable to distributed architectures.

*3) Use of MDO in real-world problems:* There are many instances of MDO architectures and strategies being used in real-world or industrial problems. Aerospace design problems have especially been subject to an MDO approach, although MDO has also been used for other applications such as backhoes [13], robotic fish [3], automotive vehicles [14], building envelope design [15], and others. While these problems could be adapted into MDO benchmarks, it is common to find that the MDA equations are unavailable (e.g. due to use of proprietary software).

### B. MOP benchmarks

MOP benchmarks contain two or more objectives to be optimized at once and are a point of interest for optimization researchers because of the complexities involved in algorithm design, the role of the decision maker, and other aspects. As a result, many MOP benchmark problems and test suites have been developed for use in research and algorithm testing.

*1) BBOB:* The Bi-objective Black Box optimization Benchmarking (BBOB-biobj) test suite [16], implemented in the Comparing Continuous Optimizers (COCO) platform [17], contains 55 test problems with differing properties, such as problem conditioning, separability, modality, problem structure control, and scalability in the number of design variables. In addition there are parameters that transform the decision variables (e.g. by the use of rotation transformations).

*2) ZDT problems:* The ZDT problems [18] are named after their creators, Zitzler, Deb and Thiele. These consist of 6 benchmark problems with two objective functions and a scalable number of design variables. The problems are composed of some combination of building block functions which differ slightly depending on the problem at hand. The relationships between the design variables are easy to handle due to their summative nature (i.e. see the '$g(\bullet)$' functions for each problem).

*3) DTLZ and WFG problems:* The Deb-Thiele-Laumanns-Zitzler (DTLZ) [19] test suite presents a more difficult challenge to optimization engines. Similarly to ZDT, an auxiliary equation $g(\bullet) \geq 0$ is used to integrate a larger number of design variables into the problem by multiplying it with the first $M-1$ design variables. The main difference between the ZDT and DTLZ problems is that the DTLZ problems demonstrate scalability in objectives.

The Walking Fish Group (WFG) [20] problems are designed to introduce other properties into multi-objective benchmarks. These properties are multi-modality, bias and separability in parameters, and features of the Pareto front such as deception, degeneracy, and shape. These problems consist of 'building blocks', like the ZDT and DTLZ test problems, that centre around transformations of design variables and the Pareto front. The design variables undergo several transformation, each one focused on a particular property, making the problems easy to customise where necessary.

*4) Other MOPs:* In addition to the named test suites above, there are many alternative multi-objective benchmark problems that may be useful for more limited purposes. These include problems such as Fleming and Fonseca's 2-objective problem [21], the Schaffer functions [22], Van Velduizen's test suite [23], and so on.

## IV. Proposed multi-objective MDO problem with scalable number of disciplines

For clarity, we will first establish a separate notation for the design variables in the problem. The decision vector is $\mathbf{w} = (w_1, \ldots, w_{n_v})^T$, and the original ZDT1 problem is expressed as:

$$\begin{aligned} \min \quad & f_1(\mathbf{w}), f_2(\mathbf{w}) \\ f_1(\mathbf{w}) &= w_1 \\ f_2(\mathbf{w}) &= g(\mathbf{w}) h(f_1(\mathbf{w}), g(\mathbf{w})) \\ g(\mathbf{w}) &= 1 + \frac{9}{n_v - 1} \left( \sum_{i=2}^{n_v} w_i \right) \\ h(f_1(\mathbf{w}), g(\mathbf{w})) &= 1 - \sqrt{\frac{f_1(\mathbf{w})}{g(\mathbf{w})}} \end{aligned} \tag{2}$$

where $n_v$ is the number of decision variables, and $0 \leq w_i \leq 1$ for all $i = 1, \ldots, n_v$. The Pareto optimal solution set for the ZDT1 problem is given by

$$f_2(\mathbf{w}) = 1 - \sqrt{f_1(\mathbf{w})}. \tag{3}$$

The ZDT1 problem is used here to explore scalability in disciplines within certain bounds. These bounds are dictated by the user of the benchmark via the chosen number of variables $n_v$, number of design and linking variables for each discipline, and the total number of disciplines. In this study, each discipline must contain at least one local variable and one linking variable, and there must be at least one global variable. However, some considerations first must be established to illustrate the constraints of the problem. The original ZDT1 problem contains $n_v$ design variables, and all design variables except the first, $w_1$, are separable in the $g(\mathbf{w})$ function (Equation 2). This means that we can exchange values within this function freely, without worrying about the effect of variable sequencing on either the disciplinary analyses or the objective functions, as long as $w_1$ remains the same. Extending this problem via the $n_v$ variable allows the number of design variables to have no upper bound. As a result, it is simple to exchange both linking, global and local design variables for the design variables in the original ZDT1 problem by simply summing all global (excluding $z_1$), local and linking variables in the $g(\mathbf{x}, \mathbf{y}, \mathbf{z})$ function. Additionally, there is no possibility for interactions between same-discipline local and linking variables that would prevent the optimizer from converging. The optimization problem is given in (4) and the MDA equations are given in (5).

$$\begin{aligned} \min \quad & f_1(\mathbf{z}), f_2(\mathbf{x}, \mathbf{y}, \mathbf{z}) \\ f_1(\mathbf{z}) &= z_1 \\ f_2(\mathbf{x}, \mathbf{y}, \mathbf{z}) &= g(\mathbf{x}, \mathbf{y}, \mathbf{z}) h(f_1(\mathbf{z}), g(\mathbf{x}, \mathbf{y}, \mathbf{z})) \\ g(\mathbf{x}, \mathbf{y}, \mathbf{z}) &= 1 + \frac{9}{n_v} \left( \sum_{i=2}^{n_z} z_i + \sum_{i=1}^{N} \sum_{j=1}^{n_{x_i}} x_{i,j} + \sum_{i=1}^{N} \sum_{j=1}^{n_{y_i}} y_{i,j} \right) \\ h(f_1(\mathbf{z}), g(\mathbf{x}, \mathbf{y}, \mathbf{z})) &= 1 - \sqrt{\frac{f_1(\mathbf{z})}{g(\mathbf{x}, \mathbf{y}, \mathbf{z})}} \end{aligned} \tag{4}$$

$$
\text{s.t.} \quad 0 \le z_i \le 1, \quad i = 1, \ldots, n_z \\
0 \le x_{i,j} \le 1, \quad i = 1, \ldots, N \text{ and } j = 1, \ldots, n_{x_i} \\
0 \le y_{i,j} \le 1, \quad i = 1, \ldots, N \text{ and } j = 1, \ldots, n_{y_i} \\
\sum_{i=1}^{N} n_{x_i} + \sum_{i=1}^{N} n_{y_i} + n_z = n_v + 1
$$

where

$$
\mathbf{y}_i(\mathbf{x}_i, \mathbf{y}_j, \mathbf{z}) = -D_i^{-1}(A_i \mathbf{x}_i + C_i \mathbf{z} - B_i \mathbf{y}_j), \text{ and } j = i - 1. \quad (5)
$$

Following the approach in [6], the MDA consists of a system of linear equations, and solving them determines the value of the linking variables for each discipline. The inputs to these equations are the local design variables, global design variables, and the linking variables obtained from the other disciplines. Weights equations $A$, $B$, $C$ and $D$ are used to customise the equations for each discipline and have dimensions $n_{x_i} \times n_{y_i}$, $n_{y_j} \times n_{y_i}$, $n_z \times n_{y_i}$ and $n_{y_i} \times n_{y_i}$ respectively. Matrix $D$ must be invertible, and in this paper is an identity matrix. Matrices $A$, $B$ and $C$ are randomly generated, with values varying between 1 and 11. The rows of these weights matrices are then normalised to sum to 1 by using Equation 6 for some generalised matrix $M$, where $\bar{M}$ is the non-normalised version of the weights matrices. $A$ and $C$ are concatenated, then the rows of this matrix are multiplied by 2 to enforce the bound constraints on the linking variables within the MDA equations.

$$
M = \frac{\bar{M}_{i,j}}{sum(row(\bar{M}))} \quad (6)
$$

While it is possible to include the linking variables from more than one discipline, for simplicity, a circular relationship between the preceding disciplines has been adopted.

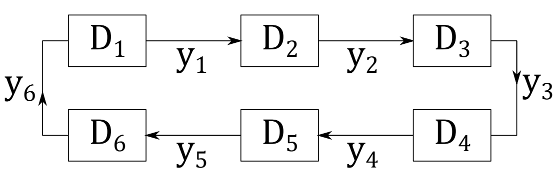



Fig. 1: An example of the linking relationships in a 6-discipline system, where $D_i$ represents the subsystem number.

Figure 1 is given as diagrammatic example of the disciplinary relationships used in Equation 5, for a six-discipline problem. This was chosen because it is easily scalable for all design and linking variables, as well as number of disciplines, and guarantees convergence.

## V. EXPERIMENTAL SETUP

The experiments were undertaken using the OpenMDAO [24] and PyOptSparse [25] packages in Python. The MDF architecture was used to frame the problem. The non-dominated sorting genetic algorithm 2 (NSGA-II) algorithm [26] has been used to solve the optimization problem in (4). We used the default settings suggested by the pyOptSparse software[1], that is, a population size of 100, crossover and mutation probabilities set to 60% and 20%, respectively. Based on initial experiments we have observed good convergence after 100 generations, and we have set this as the termination criterion. Two solvers were used in the MDA routines—a Newton nonlinear solver with an iteration limit of 1000 and no relaxation factor, and a direct solver. Both were provided by the OpenMDAO python package. For the purposes of statistical significance, each experiment was run 21 times, and the run with the median inverted generational distance (IGD) was chosen for analysis.

In this paper, the total number of design and linking variables are modified to maintain similarity in subsystem size across all disciplines. In this case study, each discipline contains 5 linking variables. These values are specified in Table I. For the sake of simplicity, the number of linking and local design variables in each discipline is the same, i.e. $n_{x_i} = n_{y_i}$. The initial values for the design variables were randomly generated values between 0 and 1. These values are used to initialise the solver-optimizer cycle in OpenMDAO. The processed $A$, $B$ and $C$ matrices as used in these investigations are available in the project's GitHub repository. [2].

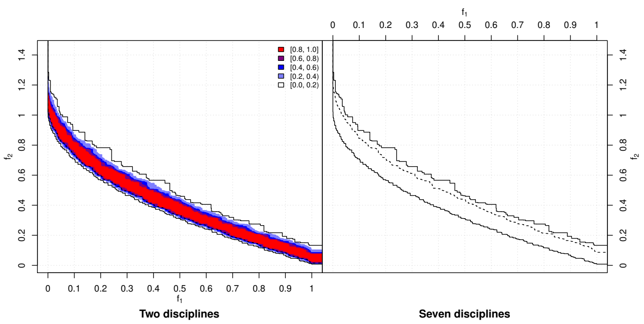
TABLE I: Experimental setup for the modified ZDT1 problem.

| **Number of disciplines** N | **Number of global variables** $\mathbf{n_z}$ | **Number of local & linking variables per sub-problem** $\mathbf{n_{x_i}, n_{y_i}}$ | **Total variables per problem** $\mathbf{n_v} + 1$ |
|---|---|---|---|
| 2 | 10 | 5, 5 | 30 |
| 7 | 10 | 5, 5 | 80 |
| 14 | 10 | 5, 5 | 150 |

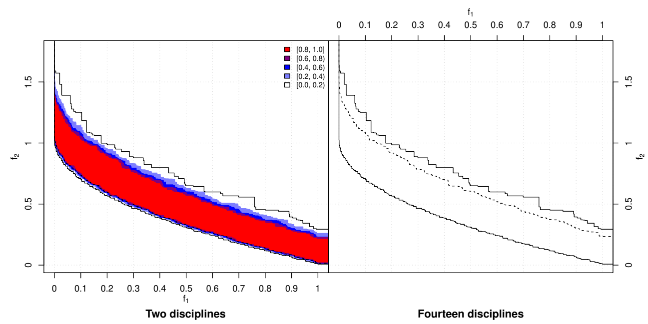
To evaluate the quality of solutions sets, we use the following indicators:

1) Hypervolume metric: used to measure both the convergence towards and diversity across the Pareto front. The metric quantifies the volume defined by the boundary between the obtained solution set at each generation, and a reference point. The reference point is set to $(1.0, 7.0)$ for all runs. To determine the hypervolume we use a dimension-sweep algorithm [27].
2) Convergence of the variables (global, linking and decision variables): the Tchebychev scalarisation metric, defined by $\text{argmin}_i \ \max_j \ \alpha_j f_j^i$, is used to select the solution from amongst the available alternatives across the Pareto front. $\alpha$ is set to 0.5 in order to target a point in the middle of the Pareto-optimal front.
3) Differences between the empirical attainment functions (EAF) [28] when comparing two algorithms across multiple runs. This metric provides visual information of where an algorithm has done better than the other across the Pareto front.

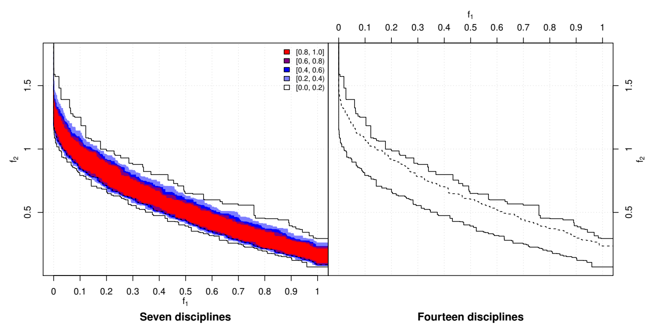
[1]pyOptSparse provides an implementation of NSGA-II and the default settings are shown in https://mdolab-pyoptsparse.readthedocs-hosted.com/en/latest/optimizers/NSGA2.html.



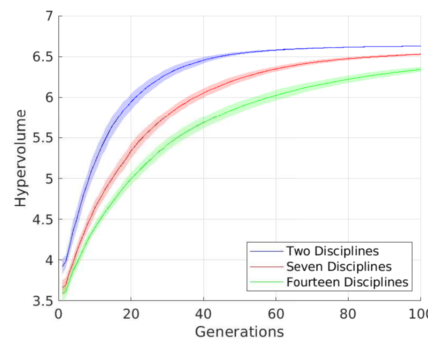

(a) EAF: Two versus seven disciplines




(b) EAF: Two versus fourteen disciplines




(c) EAF: Seven versus fourteen disciplines




(d) Hypervolume

Fig. 2: Influence of the number of disciplines on the convergence of the optimizer. The non-dominated solutions obtained at the end of the optimization run, for a total of 21 runs, are compared by using the empirical attainment function. For each subfigure, the plot in the left highlights the differences in favour of case 1, and the plot in the right highlights the differences in favour of case 2. The colour level encodes the magnitude of the observed differences. The lines in the left, centre and right correspond to the best, median and worst attainment surfaces, respectively. In (d) the approximation to the true Pareto-optimal front is measured by the hypervolume metric (the higher the better). The shaded region is the standard deviation across runs.

## VI. EXPERIMENTAL RESULTS

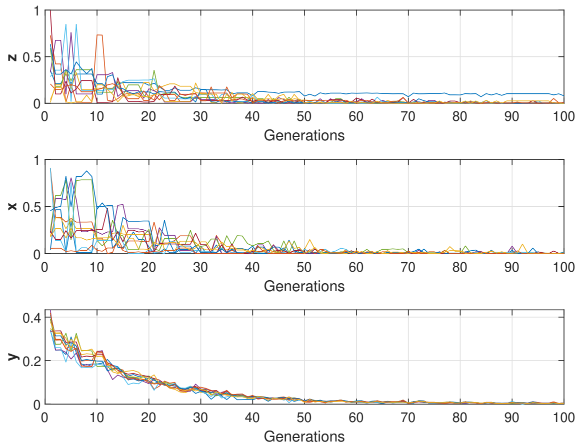
### *A. An increase in the number of disciplines has a deteriorating impact on the convergence of the optimization algorithm*

In this section, we study the effect of scaling the number of disciplines, and therefore also the number of variables, in the convergence of the optimization algorithm when applied to problem (4). The obtained results in Figure 2 show that the convergence deteriorates with an increase in the number of disciplines from 2 to 14, in that:

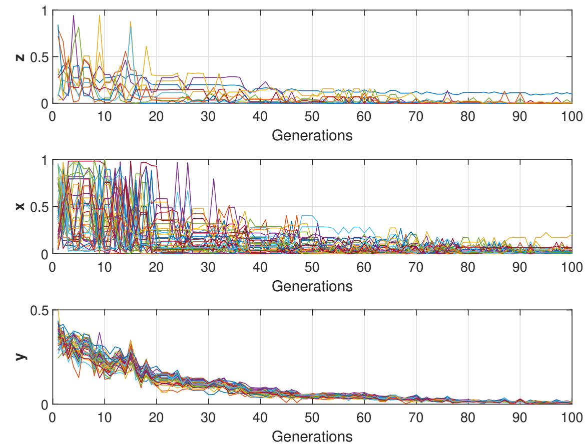
1) The differences between the EAFs is shown in Figures 2a, 2b, 2c, and the magnitude of the EAFs differences clearly favours the problems with fewer disciplines.
2) The hypervolume metric captured along the generations in Figure 2d shows that the lower the number of disciplines, the faster the convergence, and also that the variance across runs remain stable with an increase in the number of disciplines.

Figure 3 shows the convergence of the global, local and linking variables along the optimization run. This is shown separately for the two-discipline, seven-discipline and 14-discipline problem in Figures 3a, 3b and 3c. In the two-discipline problem, it can be seen that all variables except $z_1$ converge towards 0 by the 100th generation, while the global variable $z_1$ converges towards 0.08. In the seven discipline problem, the linking variables convergence towards 0, but have a much shallower curve than in the two discipline problem. The local variables also deviate further from 0 than they did in the two discipline problem. $z_1$ converges towards 0.08, and the global variables approach 0 as they did in the two discipline problem. In the fourteen discipline problem, many of the variables have failed to settle by the 100th generation, with the linking variable values varying between 0 and 0.1 by the end of the run. The local variables values vary between 0 and 0.6 by the end of the run, and the global variables between 0 and 0.3. $z_1$ converges towards 0.3. It is clear that as the number of disciplines increases, it takes longer for the optimization algorithm to converge towards the optima.

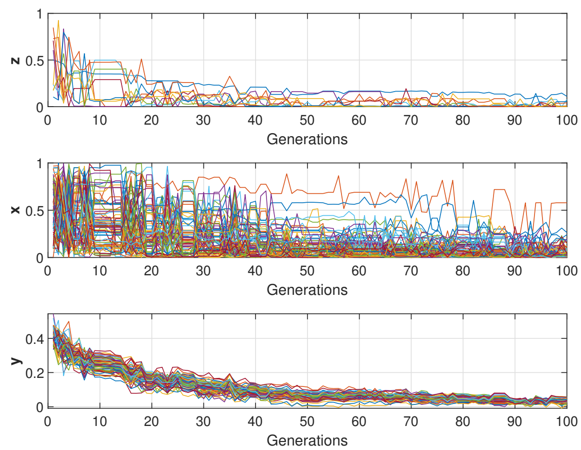
[2]github.com/vj2Sheffield/SSCI_MDO_test_problems




(a) Convergence: Two disciplines




(b) Convergence: Seven disciplines




(c) Convergence: Fourteen disciplines

Fig. 3: Monitoring the convergence of the decision variables along generations. In each subplot, convergence is shown for (a) global design variables, (b) local design variables, and (c) linking variables.

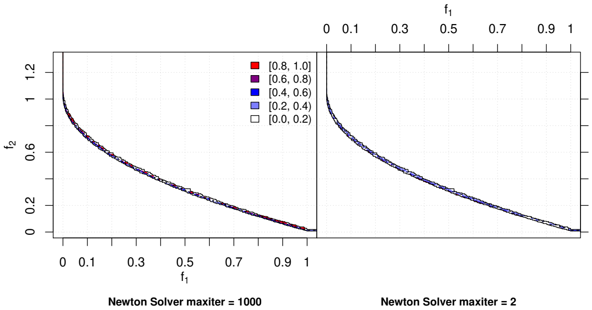
### *B. The MDA solver accuracy has a higher impact on the convergence with an increase in the number of disciplines*

In this section we study the impact that the accuracy of the MDA solver can have on the convergence of the optimization algorithm. In the previous section the maximum number of iterations for the Newton method, used to solve the MDA, was set to 1000. In here it is set to only 2. The obtained results are shown in Figure 4 and indicate that the convergence deteriorates with an increase in the number of disciplines, as captured by the differences of the empirical attainment functions (EAFs). The deterioration in terms of performance is particularly visible for the 14-discipline case as shown in Figure 4c, but can be negligible for cases with a lower number of disciplines, as is the case with 2 and 7 disciplines (Figures 4a and 4b, respectively).

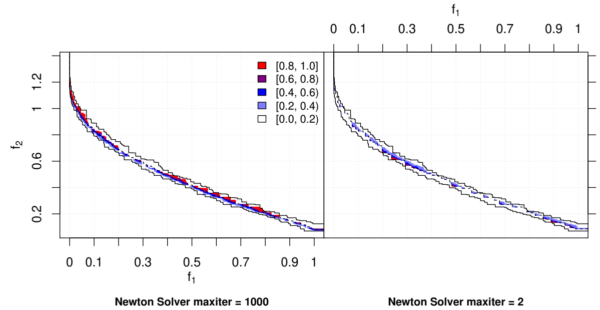
This result is expected given that as more disciplines are involved in the MDA, the bigger is the system of equations that needs to be solved. As the number of equations increases, the MDA solver is likely to require a higher number of iterations to ensure good convergence. We have shown that if the MDA solver lacks convergence, it can have an impact on the ability of the multi-objective optimization algorithm to find a good approximation to the Pareto-optimal front.

## VII. Summary and Future Work

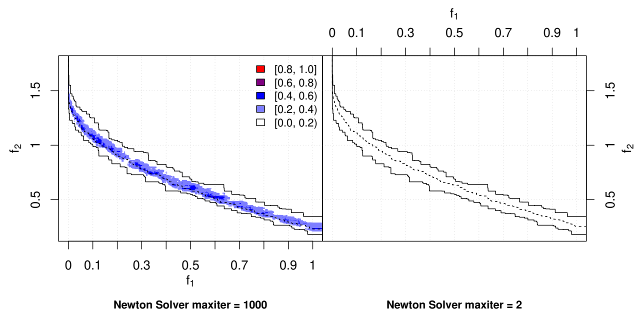
In this paper, we have proposed an MDO formulation based on the ZDT1 test problem, scalable in the number of disciplines, design variables and linking variables. This approach can be applied to other ZDT problems with ease by modifying the input variables of the objectives and $g(\bullet)$ functions, and introducing linking variables. However, the ZDT test problems




(a) Two disciplines




(b) Seven disciplines




(c) Fourteen disciplines

Fig. 4: Influence of the MDA solver accuracy on the optimizer's convergence. The maximum number of iterations used by the Newton's method to solve the MDA has been set to 1000 and 2, corresponding respectively to high accuracy and low accuracy. The non-dominated solutions obtained at the end of the optimization run, for a total of 21 runs, are compared by using the empirical attainment function. For each subfigure, the plot in the left highlights the differences in favour of case 1, and the plot in the right highlights the differences in favour of case 2. The colour level encodes the magnitude of the observed differences. The lines in the left, centre and right correspond to the best, median and worst attainment surfaces, respectively.

are not scalable in the number of objectives, and may lack the complexity found in other test problems that better resemble real-world problems (e.g. the WFG test suite). Nevertheless, this study provides a good starting point by showing how to construct a simple bi-objective scalable MDO problem, which has allowed us to investigate the impact of two fundamental issues inherent to the MDO architectures from a previously unexplored multi-objective context.

In future work we will consider alternative MDO architectures to MDF, such as the distributed approaches (e.g. collaborative optimization and analytical target cascading) [4]. While these formulations tend to result in higher computational time and worse convergence properties, they offer important real-world features such as design variable privacy and increased disciplinary autonomy. Other test suites that could be converted into MDO formulations are the WFG and BBOB problems. Future work will also include investigating the performance of multi-objective optimization algorithms other than NSGA-II, and how suitable these are when applied to different MDO architectures.

## REFERENCES

[1] K. Klamroth, S. Mostaghim, B. Naujoks, S. Poles, R. Purshouse, G. Rudolph, S. Ruzika, S. Sayın, M. M. Wiecek, and X. Yao, "Multiobjective optimization for interwoven systems," *Journal of Multi-Criteria Decision Analysis*, vol. 24, no. 1–2, pp. 71–81, 2017.

[2] S. A. Ning and I. Kroo, "Multidisciplinary considerations in the design of wings and wing tip devices," *Journal of Aircraft*, vol. 47, pp. 534–543, 2010.

[3] H. Chen, W. Li, W. Cui, P. Yang, and L. Chen, "Multi-objective multidisciplinary design optimization of a robotic fish system," *Journal of Marine Science and Engineering*, no. 5, 2021.

[4] J. R. R. A. Martins and A. B. Lambe, "Multidisciplinary design optimization: A survey of architectures," *AIAA Journal*, vol. 51, no. 9, pp. 2049–2075, 2013.

[5] E. J. Cramer, J. E. Dennis, Jr., P. D. Frank, R. M. Lewis, and G. R. Shubin, "Problem formulation for multidisciplinary optimization," *SIAM Journal on Optimization*, vol. 4, no. 4, pp. 754–776, 1994.

[6] N. P. Tedford and J. R. R. A. Martins, "Benchmarking multidisciplinary design optimization algorithms," *Optimization and Engineering*, vol. 11, pp. 159–183, 2010.

[7] C. L. Bloebaum, "Coupling strength-based system reduction for complex engineering design," *Structural Optimization*, vol. 10, pp. 113—-121, 1995.

[8] S. Padula, N. Alexandrov, and L. Green, "MDO test suite at NASA Langley Research Center," in *6th Symposium on Multidisciplinary Analysis and Optimization*, pp. 410–420, 1996.

[9] M. Farnsworth, A. Tiwari, M. Zhu, and E. Benkhelifa, *A Multi-objective and Multidisciplinary Optimisation Algorithm for Microelectromechanical Systems*, pp. 205–238. Cham: Springer International Publishing, 2018.

[10] R. Sellar, S. Batill, and J. Renaud, "Response surface based, concurrent subspace optimization for multidisciplinary system design," in *34th Aerospace Sciences Meeting and Exhibit*, 1996.

[11] K. Klamroth, S. Mostaghim, B. Naujoks, S. Poles, R. Purshouse, G. Rudolph, S. Ruzika, S. Sayın, M. M. Wiecek, and X. Yao, "Multiobjective optimization for interwoven systems," *Journal of Multi-Criteria Decision Analysis*, vol. 24, no. 1-2, pp. 71–81, 2017.

[12] J. A. Duro, Y. Yan, R. Purshouse, and P. Fleming, "Collaborative multi-objective optimization for distributed design of complex products," 04 2018.

[13] Y. Guang-qiu, L. Shu-wen, and D. B. Perlurst, "Multidisciplinary design and collaborative optimization for excavator backhoe device," *Journal of Engineering Science and Technology Review*, vol. 7, no. 2, pp. 99–105, 2014.

[14] R. D. Bäckryd, A.-B. Ryberg, and L. Nilsson, "Multidisciplinary design optimisation methods for automotive structures," *International Journal of Automotive and Mechanical Engineering*, vol. 14, no. 1, pp. 4050–4067, 2017.

[15] D. Yang, M. Turrin, S. Sariyildiz, and Y. Sun, "Sports building envelope optimization using multi-objective multidisciplinary design optimization (m-mdo) techniques: Case of indoor sports building project in china," in *2015 IEEE Congress on Evolutionary Computation (CEC)*, pp. 2269–2278, 2015.

[16] T. Tušar, D. Brockhoff, N. Hansen, and A. Auger, "COCO: The Bi-objective Black Box Optimization Benchmarking (bbob-biobj) test suite." arXiv, 2019.

[17] N. Hansen, A. Auger, R. Ros, O. Mersmann, T. Tušar, and D. Brockhoff, "COCO: a platform for comparing continuous optimizers in a black-box setting," *Optimization Methods and Software*, vol. 36, no. 1, pp. 114–144, 2021.

[18] E. Zitzler, K. Deb, and L. Thiele, "Comparison of multiobjective evolutionary algorithms: Empirical results," *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, 2000.

[19] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, *Scalable Test Problems for Evolutionary Multiobjective Optimization*, ch. Evolutionary Multiobjective Optimization: Theoretical Advances and Applications, pp. 105–145. Springer London, 2005.

[20] S. Huband, P. Hingston, L. Barone, and L. While, "A review of multiobjective test problems and a scalable test problem toolkit," *IEEE Transactions on Evolutionary Computation*, vol. 10, no. 5, pp. 477–506, 2006.

[21] C. M. Fonseca and P. J. Fleming, "An overview of evolutionary algorithms in multiobjective optimization," *Evolutionary Computation*, vol. 3, no. 1, pp. 1–16, 1995.

[22] J. D. Schaffer, "Multiple objective optimization with vector evaluated genetic algorithms," in *Proceedings of the 1st International Conference on Genetic Algorithms*, (USA), pp. 93–100, L. Erlbaum Associates Inc., 1985.

[23] D. A. van Veldhuizen and G. B. Lamont, "Multiobjective evolutionary algorithm test suites," in *Proceedings of the 1999 ACM symposium on Applied computing*, pp. 351–357, 1999.

[24] J. S. Gray, J. T. Hwang, J. R. R. A. Martins, K. T. Moore, and B. A. Naylor, "Openmdao: an open-source framework for multidisciplinary design, analysis, and optimization," *Structural and Multidisciplinary Optimization*, vol. 59, pp. 1075–1104, 2019.

[25] N. Wu, G. Kenway, C. A. Mader, J. Jasa, and J. R. R. A. Martins, "pyoptsparse: A python framework for large-scale constrained nonlinear optimization of sparse systems," *The Journal of Open Source Software*, vol. 5, no. 54, p. 2564, 2020.

[26] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.

[27] C. M. Fonseca, L. Paquete, and M. López-Ibáñez, "An improved dimension–sweep algorithm for the hypervolume indicator," in *Proceedings of the 2006 Congress on Evolutionary Computation (CEC'06)*, pp. 1157–1163, IEEE Press, 2006.

[28] M. López-Ibáñez, L. Paquete, and T. Stützle, *Exploratory Analysis of Stochastic Local Search Algorithms in Biobjective Optimization*, pp. 209–222. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010.