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RESEARCH ARTICLE

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Numerical modelling of subglacial ribs, drumlins, herringbones, and mega-scale glacial lineations reveals their developmental trajectories and transitions

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Abstract

Initially a matter of intellectual curiosity, but now important for understanding ice-sheet dynamics, the formation of subglacial bedforms has been a subject of scientific enquiry for over a century. Here, we use a numerical model of the coupled flow of ice, water, and subglacial sediment to explore the formation of subglacial ribs (i.e., ribbed moraine), drumlins and mega-scale glacial lineations (MSGLs). The model produces instabilities at the ice–bed interface, which result in landforms resembling subglacial ribs and drumlins. We find that a behavioural trajectory is present. Initially subglacial ribs form, which can either develop into fields of organized drumlins, or herringbone-type structures misaligned with ice flow. We present potential examples of these misaligned bedforms in deglaciated landscapes, the presence of which means caution should be taken when interpreting cross-cutting bedforms to reconstruct ice flow directions. Under unvarying ice flow parameters, MSGLs failed to appear in our experiments. However, drumlin fields can elongate into MSGLs in our model if low ice–bed coupling conditions are imposed. The conditions under which drumlins elongate into MSGLs are analogous to those found beneath contemporary ice streams, providing the first mechanism, rather than just an association, for linking MSGLs with ice stream flow. We conclude that the instability theory, as realized in this numerical model, is sufficient to explain the fundamental mechanics and process-interactions that lead to the initiation of subglacial bedforms, the development of the distinctive types of bedform patterns, and their evolutionary trajectories. We therefore suggest that the first part of the longstanding 'drumlin problem' – how and why they come into existence – is now solved. However, much remains to be discovered regarding the exact sedimentary and hydrological processes involved.

KEYWORDS
bedform, drumlin, ice sheet, MSGL, ribbed moraine, subglacial

1 | INTRODUCTION

Subglacial bedforms, repetitive landforms formed at the base of an ice sheet or glacier as a result of the movement of subglacial sediments, are abundant in areas of former glaciation (e.g., Clark, Ely, Spagnolo, et al., 2018; Dyke & Morris, 1988; Kalm, 2012). They can also be found in the forefields of some retreating glaciers (e.g., Hart, 1995; Johnson et al., 2010), and have been imaged beneath contemporary ice sheets (Holschuh et al., 2020; King et al., 2009). The formation of subglacial bedforms, drumlins in particular, has been a subject of scientific enquiry since the late 19th century (e.g., Davis, 1884; Tarr, 1894; Upham, 1895), with numerous hypotheses for their formation subsequently proposed (see Boulton, 1987; Clark, 2010; Hindmarsh, 1998b; Iverson et al., 2017; Shaw, 1983; Smalley & Unwin, 1968).

Understanding their genesis began as a matter of intellectual curiosity. Whilst this geomorphological puzzle remains, glaciological studies have highlighted the importance of the subglacial environment in regulating ice flow (e.g., Lliboutry, 1968; Weertman, 1957). Ice–sediment–topography feedbacks during subglacial bedform genesis may have important implications for basal roughness (Baranowski, 1979; Schoof, 2002), subglacial drainage (Schoof, 2010; Stevens et al., 2022), and subglacial sediment deformation (Alley et al., 1986; Boulton et al., 1974; Smalley & Unwin, 1968). Indeed, the contribution of basal motion (deformation and sliding) to ice flow is still a large uncertainty in the numerical ice-sheet models used to forecast the behaviour of ice sheets in our warming world (Ritz et al., 2015; Zoet & Iverson, 2020). Furthermore, subglacial bedforms are a key tool for reconstructing the behaviour of former ice sheets, with inferences about palaeo-ice-sheet conditions often made using assumptions about bedform genesis (Clark, 1997; Kleman & Borgström, 1996). The data gathered from such reconstructions can be used to train and improve ice-sheet models (e.g., Ely et al., 2021; Gandy et al., 2019). Therefore, understanding the genesis of subglacial bedforms is no longer a purely academic problem, but one that may provide useful insight for predicting the contribution of current ice masses to sea level rise.

Traditionally, subglacial bedforms have been grouped into individual classes, the most common categories (among a plethora of nomenclature; Figure 1) of which are: drumlins (e.g., Clark et al., 2009), subglacial ribs or Rogen moraine (Dunlop & Clark, 2006), mega-scale glacial lineations (MSGLs; Clark, 1993; Spagnolo et al., 2014), and flutes (Ely et al., 2017; Ives & Iverson, 2019; van der Meer, 1997). Despite well-known labels, the morphometric properties of subglacial bedforms suggest they belong to a continuum of size and shape (Aario, 1977; Rose, 1987), whereby each category (with the exception of flutes) has overlapping size and morphological properties with those adjacent (Ely et al., 2016). This morphometric commonality,

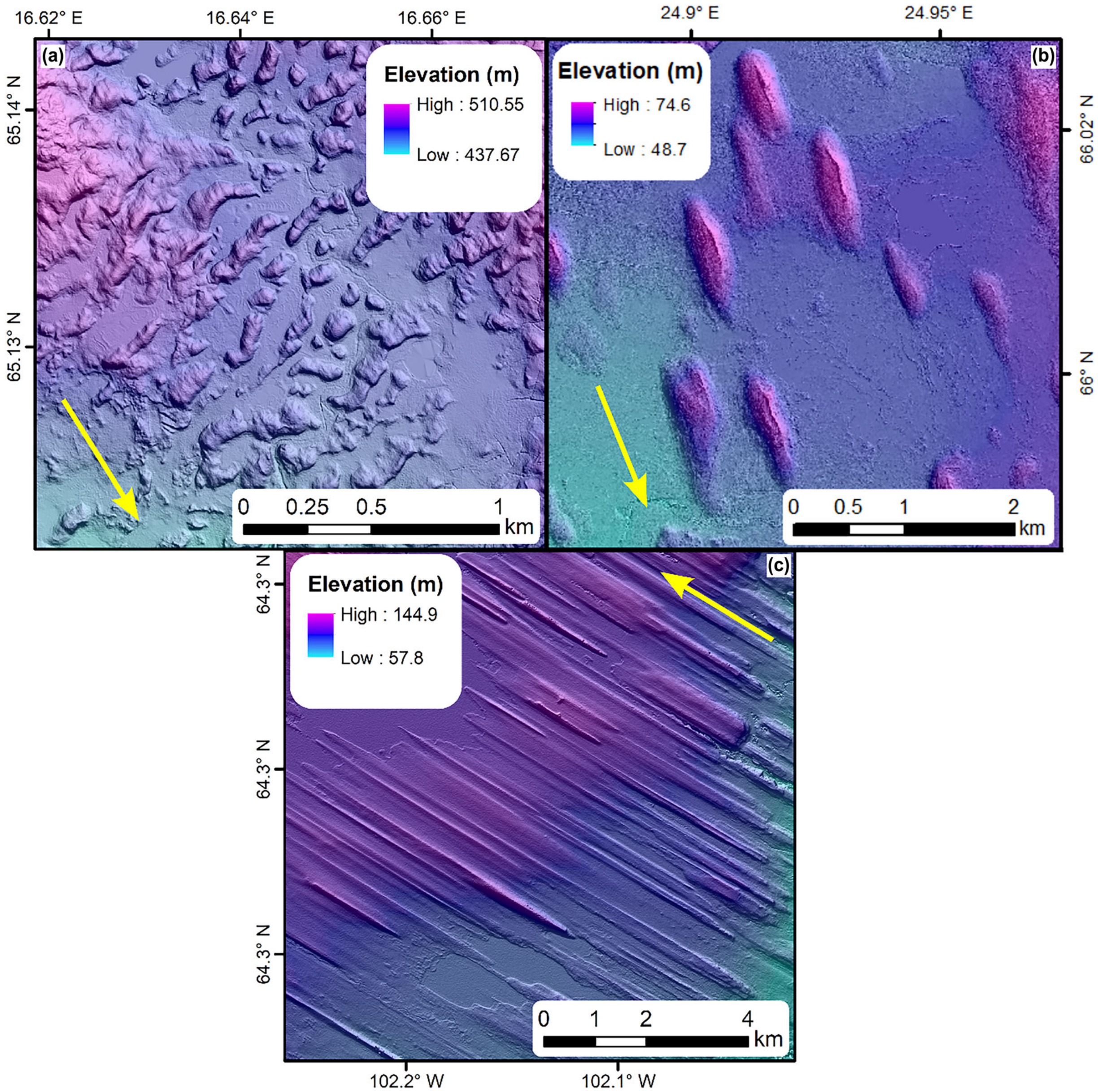

FIGURE 1 Shaded relief maps of subglacial bedforms, demonstrating the three main types studied in this article. Arrows denote approximate former ice-flow direction. (a) Subglacial ribs (often called ribbed or Rogen moraine) in Sweden (GSD-Höjddata, grid 2 + © Lantmäteriet). (b) Drumlins in Finland (National Land Survey of Finland elevation model 2 m 11/19). (c) Mega-scale glacial lineations in Canada (Porter et al., 2018).

together with their regular spacing (Clark, Ely, Spagnolo, et al., 2018; Dunlop & Clark, 2006; Spagnolo et al., 2014), paints a picture of simplicity, which has been used to support notions of a common formation process. Fowler (2018) describes this view as equicausality, whereby a single mechanism, or set of mechanisms, is invoked to explain the array of observed subglacial bedforms (e.g., Menzies, 1987; Shaw, 2002). However, the equicausal view is often at odds with findings from sedimentological-based studies (e.g., Möller & Dowling, 2018), which highlight a large diversity of internal composition within subglacial bedforms (Stokes et al., 2013). These findings often lead to the conclusion that multiple different processes lead to the formation of subglacial bedforms (e.g., Möller & Dowling, 2018). Indeed, a smorgasbord of separate mechanisms has been proposed for different bedform types (Boulton, 1987; Clark et al., 2003; Hättestrand & Kleman, 1999) and subtypes (e.g., Lindén et al., 2008; Möller, 2006). Such interpretations are termed equifinality in Fowler (2018).

Usefully, Fowler (2018) identifies a third viewpoint, which can be used to reconcile the equfinialists and the equicausalists: equimorphology, whereby a single model can represent all subglacial bedforming processes, and also produce an array of subglacial bedforms. This is the philosophy of the instability theory of subglacial bedform generation (the subject of this article), which has been developed in a series of articles that can be traced back to Hindmarsh (1998a) and Fowler (2000). It is perhaps useful to clarify when explaining equimorphology that any model includes many necessary simplifications; that is it is far from a complete description of reality. This approach means that equimorphological models cannot explain all the specifics of the phenomena they aim to explore, as many detailed processes are abstracted through the equimorphological lens. As such, the initial aim of the equimorphologist is to build a model with valid underlying concepts, rather than one that can explain every observation (Fowler, 2018). For subglacial bedforms, this means that complex processes, such as individual modes of subglacial sediment transport (e.g., cavity infill, sediment stacking, melt-out, etc.), may be generalized into a few equations in a model, reliant upon a common description of subglacial transport relating movement to basal shear stress and water pressure. Furthermore, practical simplifications must be made for models to be built; especially where poorly understood processes are involved (such as the closure rate of a subglacial water film; Fowler & Chapwanya, 2014), or highly debated ingredients are present (such as the rheology of subglacial till; Fowler, 2003).

To visualize the results of the instability theory and test scenarios, numerical solutions are required. This adds a further complication to the modelling approach; appropriate numerical tools must be chosen to solve the governing equations. The choice of numerical scheme is especially pertinent to the instability theory of subglacial bedforms for two reasons: (i) the presence of instabilities in the system means that small differences in numerical computation can alter the final result dramatically; and (ii) the thickness of the material layers of sediment (metres), water (millimetres) and ice (kilometres) differ by orders of magnitude. This means that when evaluating a model simulation of subglacial bedforms against observations, one must question whether discrepancies are the result of an incorrect formulation of the model, an oversimplification of the model, or inadequacies in the numerical techniques used. To complicate matters even further, numerical models may produce answers that resemble observations, but contain incorrect ingredients. For example, the recent model of Barchyn et al. (2016) seems to replicate the spectrum of observed subglacial bedforms, but the limited physics and questionable constituent parts (namely the relation between effective pressure and bed slope) limit the utility and correctness of this model (Fowler, 2018).

Subglacial bedform genesis is a difficult process to study, both mathematically (Fannon et al., 2017; Fowler, 2018) and conceptually (Clark, 2010; Dowling, 2016). Clearly, numerous complex interactions are involved, and the 'drumlin problem' (as the formation of subglacial bedforms is often referred to) is compounded by our limited ability to observe the processes involved in real-time. Because of this, developers of the instability theory of subglacial bedform genesis have sought the minimum number of ingredients required to form an adequate explanation. This approach will inevitably result in a minimal explanation of subglacial bedform genesis. Once this goal is achieved, more complexity can be added if seeking an explanation for specific observations. The latest iteration of the physics-based instability model of subglacial bedforms is described in Fannon et al. (2017), along with a few numerical solutions of the model to provide example simulations of relief development of some bedforms.

Here, we use the model described in Fannon et al. (2017), performing more experiments and which expand upon the range of parameters demonstrated thus far. This creates a compendium of model outputs, which we loosely compare to observations to gain insight. Furthermore, we aim to provide a qualitative, non-mathematical description of the fundamental instability mechanisms that give rise to the complex instabilities present under a landform-generating ice sheet. Our aims therefore are three-fold, to: (i) provide insight into the relevant instability mechanisms at play under ice sheets, (ii) evaluate the progress that the instability theory model has made in achieving a minimum explanation for subglacial bedforms (e.g., are all the basic ingredients present?); and (iii) assess the lessons that the instability model has for interpreting subglacial bedforms.

2 | METHODOLOGY AND METHODS

The instability theory for subglacial bedform generation has been developed in a series of articles tracing back to Hindmarsh (1998a, 1998b, 1999) and Fowler (2000), both of whom examined the coupled flow of sediment and ice in two dimensions (vertical and downstream). Subsequent articles have added further ingredients to the model, including the infilling of subglacial cavities (Fowler, 2009), the third dimension (Chapwanya et al., 2011; Fowler, 2010a), and the inclusion of subglacial water (Fowler, 2010b). The latest iteration of the model is presented in Fannon et al. (2017), and we add no further ingredients or adaptations to the physics of the model here. As the model is mathematically complex, has been described thoroughly in the aforementioned articles, and our aim is not to build or improve upon the model, we provide only a qualitative description here which focuses on the key ingredients. Readers interested in a complete derivation of the equations involved are directed to Fannon et al. (2017) and Fannon (2020).

2.1 | Instability mechanisms

Instability theory, defined broadly, is a process by which arbitrarily small perturbations at the interface between deformable substances grow. Perturbations tend to grow initially in a wave-like pattern, with maximal growth typically occurring over preferential wavelengths (i.e., bump spacings) which are dependent upon material properties, and on flow speeds or force magnitudes. As the instability progresses, this initial wave-like deformation may then evolve into more complex forms. Such unstable interactions lead to the dynamic shaping of the ice, water and till layers into constantly evolving patterns. The final form of the till layer, after retreat of the ice sheet, is left behind as an observable, undulating landform pattern; subglacial bedforms.

Numerous examples of instabilities exist in geomorphology, such as in the formation of sand dunes (Elbelrhiti et al., 2005), rills on hillslopes (Smith & Bretherton, 1972), and cusps on beaches (Werner & Fink, 1993). Elsewhere in nature, instabilities are responsible for numerous patterns including in clouds (Baumgarten & Fritts, 2014), animal stripes (Turing, 1952), snowflake formation (Libbrecht, 2003), and filaments within the structure of nebulae (Hester, 2008). A common denominator between these phenomena is the large-scale regularity of the patterns they form, with a preferred spacing between component elements; a characteristic that also exists for subglacial bedforms (Clark, Ely, Spagnolo, et al., 2018b).

Within the subglacial environment the three main materials (ice, water, and till) are all mobile, each possessing a different density, viscosity, and yield stress. All three components are subject to shear due to the motion of the overlying ice, which drives their evolution. Their interactions, therefore, may be subject to at least three classical instability mechanisms: Kelvin–Helmholtz instability (Helmholtz, 1868; Kelvin, 1871), which arises due to velocity-driven shear stress; Saffman–Taylor instability due to differing viscosity (Saffman & Taylor, 1958); and Rayleigh–Taylor instability due to differing density (Lord Rayleigh, 1882; Taylor, 1950). In most cases, the dominant instability mechanism within the subglacial environment can be expected to be Kelvin–Helmholtz instability (Helmholtz, 1868; Kelvin, 1871). This instability mechanism occurs when one fluid, or other deformable substance, passes over another with a velocity differential, imparting shear stress at the interface. This instability occurs during the formation of wind-induced sea-surface waves (Kuznetsov & Lushnikov, 1995). A classical example for Kelvin–Helmholtz instability, illustrated in Figure 2, involves two fluids moving in opposite directions. Kelvin–Helmholtz instability also arises where: (i) the two materials are moving in the same direction with differing velocities; (ii) the fluid is unconstrained, since flow over bumps still induces acceleration (e.g., air flowing over an aerofoil or mountain); or (iii) one or both of the substances are a particulate (e.g., sand) or other deformable substance. For the subglacial environment, the flow of ice over till will create a natural shear between the two components, inducing Kelvin–Helmholtz instability.

Given the variety of materials, interfaces, and interactions at the ice–bed interface, it is unsurprising that instabilities have been proposed to occur (Fowler, 2000; Hindmarsh, 1998a, 1998b, 1999). The potential for three different types of instabilities, across three materials, makes instabilities in the subglacial environment a complex and likely mathematically intractable problem. Owing to this complexity, the link between cause and effect is difficult to derive, as small changes lead to different outcomes.

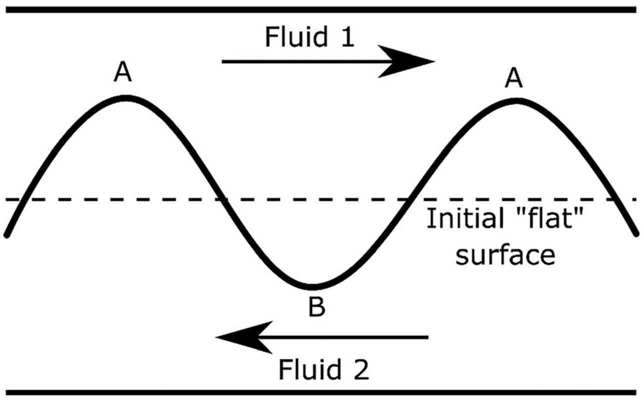

FIGURE 2 Classical example of Kelvin–Helmholtz instability at a fluid–fluid interface, demonstrating the transition from an initial 'flat' surface to an undulating one. Two fluids are moving in opposite directions, constrained by walls above and below, and separated by a slightly perturbed interface. At position A fluid 1 has a reduced cross-sectional area available for flow, so conservation of mass dictates that it flows more rapidly, reducing pressure in the fluid, and working to draw the interface upwards. Fluid 2, meanwhile, reduces speed at A, increasing its fluid pressure, and further pushing the interface upwards. At position B the situation is reversed, with both fluids working to push the interface downwards. By this mechanism small perturbations at the interface grow. The same instability arises if two fluids are flowing in the same direction at different velocities.

2.2 | Model overview

An overview of the basic model configuration is shown in Figure 3. A water film of thickness on the order millimetres lies between a layer of till and the overlying ice. The mathematical model consists of the combined governing physics for each component in a set of coupled partial differential equations. As with any numerical model, some assumptions and simplifications are made. For instance, ice is considered as a fluid flowing with a constant viscosity, to avoid the computational complexity of solving for non-linear ice flow. The flow of ice over the water-saturated till exerts a basal shear stress, which then generates a sliding velocity via a sliding law which also depends on the effective normal stress. It is this basal shear stress, induced through ice flow, which drives bedform initiation and evolution.

Subglacial water is considered to exist as a thin film, of order millimetres in thickness. Water flows through the film slowly, subject to a local Poiseuille flow law, driven by changes in the hydraulic head. Water flows into the film due to basal ice melt (an input parameter in the range of millimetres per year) and is exchanged between the water and till layers due to changes in till porosity; squeezed till will reduce its porosity, yielding water into the film. Conversely, till expansion will increase porosity and suck water from the film via capillary action. Till porosity, and therefore the flux of water between the till and film layers, evolves as a function of effective pressure. The thickness of the water layer will evolve dynamically as the model progresses, naturally forming regions of preferential flow; however, the model does not allow for channelized drainage systems to form. This is a key practical simplification within the model, and is made primarily due to considerations of scale.

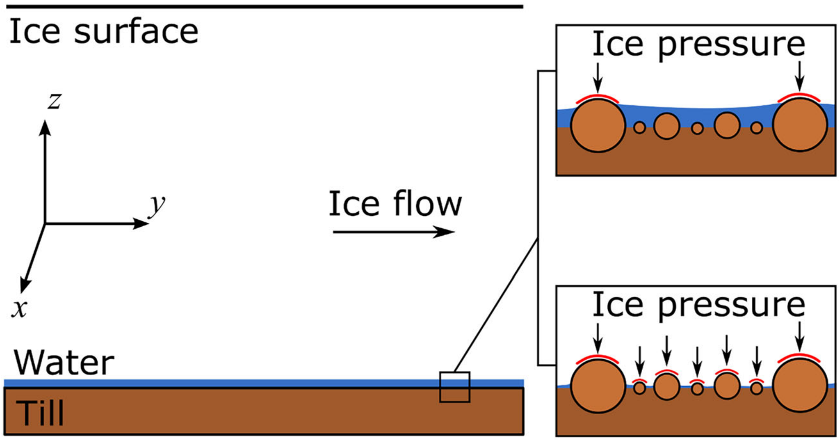

FIGURE 3 Schematic of the model configuration. The model describes the coupled flow of till, sediment, and water at the ice–bed interface. The amount of traction transferred from flowing ice into the till is dependent upon the effective pressure: the ratio between ice-overburden pressure acting downwards and subglacial water pressure opposing this.

The transfer of effective pressure into the till, via the basal shear stress imposed by the overlying ice motion, takes place primarily through the uppermost clasts of the till; a concept described in Creyts & Schoof (2009). A thinner film of water leads to increased direct contact between the till and ice (Figure 3), and therefore increased transfer of effective pressure: the difference between ice-overburden pressure and subglacial water pressure. The model assumes a coupling between effective pressure and water film thickness, defined as

$$\lambda hN = 1, \tag{1}$$

where h is the film thickness, N is the effective pressure, and λ is the parameter coupling the two (all of which are non-dimensionalized). Small values of λ represent a strong transfer of effective pressure into the till; that is, a stronger coupling between basal shear stress and till deformation. Conversely, larger values of λ represent a weaker coupling, with less transfer of effective pressure. The precise value of λ remains an unknown, yet it can have a significant effect on bedform generation.

In the model, till can flow once a yield stress criterion is exceeded. Sediment flow is described by the Exner equation and considers: (i) the advection of till due to shear from the overlying ice; (ii) till squeezing due to the effective pressure gradient; and (iii) bedload transport of sediment by stream flow. The transport of sediment via the water film is driven by shear between the water and till layers, with the bedload transport function based on a power law. Since λ has a role in determining the degree of coupling between the ice and till, its value is critical in all three modelled modes of till transport. Although this model does not fully represent the complexity of till rheology, which itself is subject to significant debate (Fowler, 2003; Iverson et al., 1998; Tulaczyk, 2006), it allows for a mathematical description of till, and its interaction with the other layers, to be formulated.

The numerical solution to the system of equations generated by the mathematical model takes place on a two-dimensional (2D) rectangular horizontal grid with uniform node separation, utilizing a mixture of finite differencing and spectral methods; see Fannon et al. (2017) for more detail. Periodic boundary conditions are implemented. This approach allows a domain of finite size to emulate the behaviour of an equivalent-size portion within an infinite domain. In the context of our simulations, the validity of periodic boundary conditions relies on choosing a domain size that is larger than that of the largest generated bedforms. If, for example, a single drumlin, rib, or other bedform is generated that spans the entire solution domain in the direction of ice flow, the bedform may join up with itself, creating an artificial and non-physical 'looped' feature. To mitigate this risk, we aim to choose the largest domain size that is viable with the available computational resources, whilst maintaining a good spatial resolution. The model is initialized by assigning a constant ice-surface velocity, and physical properties for each material (density, viscosity, porosity, etc.), along with an initial thickness for the ice, water and till layers. These layers, which are ostensibly of uniform initial thickness, are then perturbed by imposing quasi-random perturbations of very small amplitude onto the till surface.

We aim to use the earlier-described numerical model to investigate the generation of subglacial bedforms. We do not provide an exhaustive parameterization of the model with resulting outputs (parameters used in each experiment shown are listed in Supporting Information Table S1); neither do we attempt to hypothesize suitable ranges of parameters in which one particular bedform type will appear (in fact our experiments suggest that most bedform types are able to form under a variety of conditions). Instead, we aim to demonstrate the range of subglacial landforms which can be predicted by the model when using reasonable assumptions for initial conditions, and to highlight the evolutionary path of these bedforms from one type towards another; that is, their 'behavioural trajectory'. As well as helping guide us regarding the skill of the model at producing realistic bedforms, we can also glean information about how different bedform types evolve into others, something that has yet to be observed in nature, but that has been widely supposed to happen.

3 | RESULTS

We present here a selection of results from our numerical model in the form of maps of till-surface elevation, taken at various times during a simulation. The presented results highlight the typical behaviour we have observed during our extensive investigations of over 200 model runs. A common behaviour observed from our numerical experiments is shown in Figure 4. The initial configuration of small-amplitude perturbations (Figure 4a) grows rapidly, over the course of around 75 years, before rearranging itself into transverse ribs (Figure 4b) over the course of the following 20 to 100 years. These

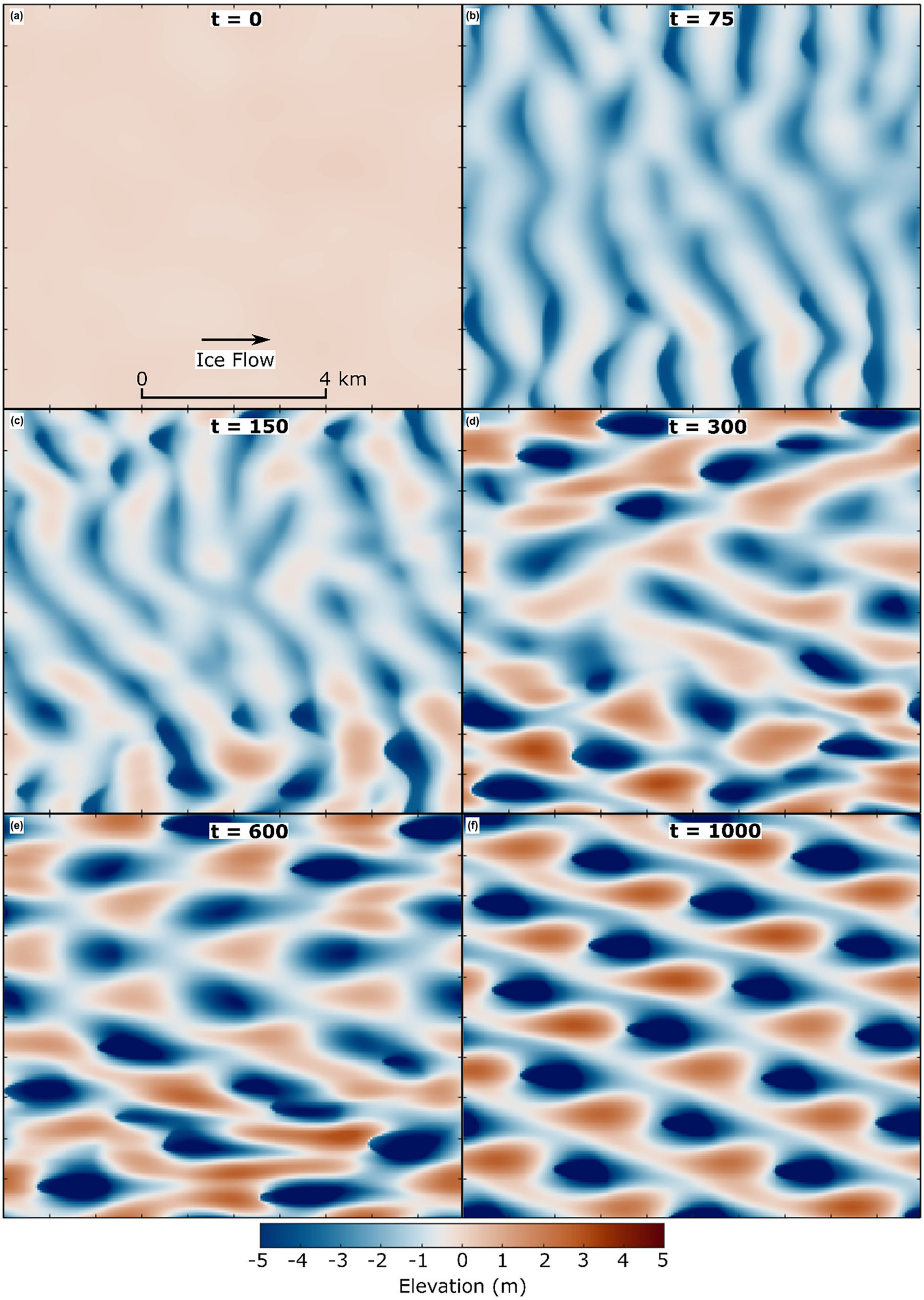

FIGURE 4 Typical behavioural trajectory of bedform generation from a nearly flat surface through ribs, breached ribs, to drumlins (the red bumps). The initial configuration (a) contains small quasi-random perturbations to the till surface of magnitude ∼0.2 m. transverse ribs then develop (b), at an amplitude significantly higher than the initial perturbation (∼3 m). The ribs then become slightly more diagonal and begin to lose integrity (c). As the ribs break apart, isolated occurrences of drumlins and quasi-circular bedforms occur (d). The ribs organize into a drumlin field (e). This drumlin field continues to vary in shape, organization and spacing through time. Eventually the drumlins organize into a regular spacing and shape that no longer alters through time, reaching a maximum amplitude (∼12 m) (f). Note that all observed landforms migrate in the direction of ice flow, at a rate of roughly 30 m a^{-1} in this case. For a movie of this simulation, see Supporting Information Video S1.

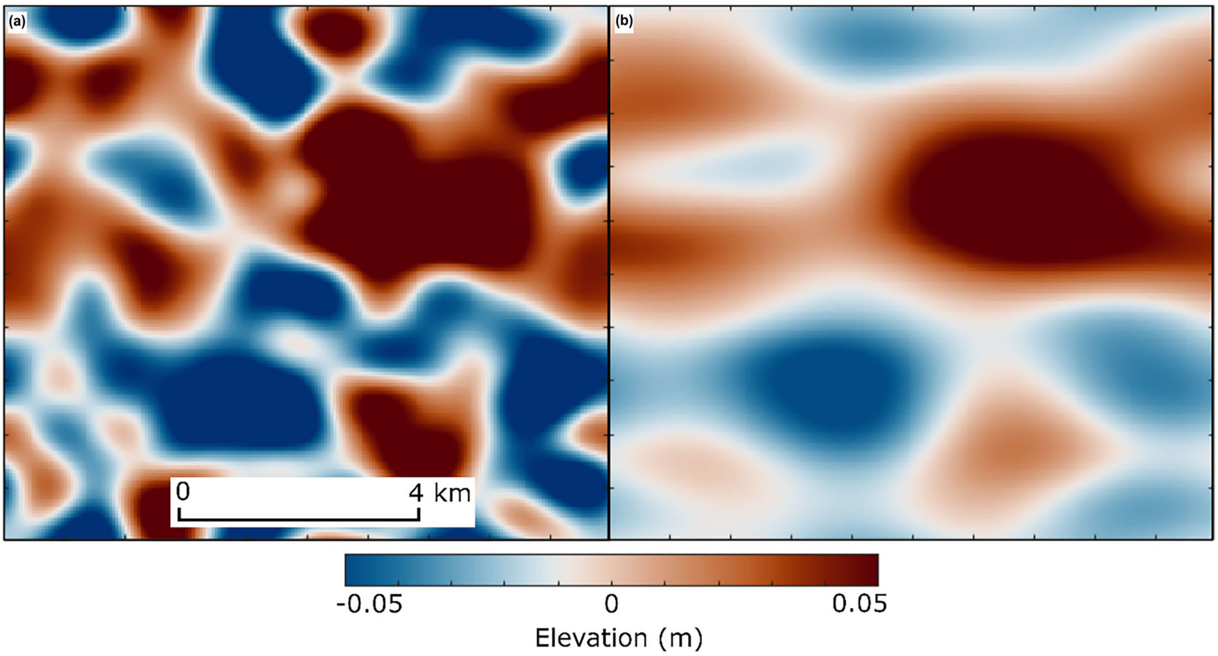

FIGURE 5 A stable configuration of the till surface, whereby no bedforms emerge. In this case a weaker coupling of ice to till (low effective pressure) from a thicker water film leads to a lack of bedform development. The low-amplitude initial configuration (a) is smoothed out and consolidates into a large smooth mound of sediment with very low amplitude (0.1 m) (b). This mound subsequently holds form, translating very slowly downstream, and reduces further in amplitude. Such conditions of weak coupling can smooth or erase any sedimentary bumps or bedforms. For a movie of this simulation, see Supporting Information Video S2.

ribs migrate downstream, as, in fact, do all landforms predicted by this model. Downstream migration speeds are in the order of 10 to 100 m a^{-1}, at a fraction of the ice velocity that is dependent upon the degree of subglacial coupling (typically around 30 to 60%). After around 100 years or so the ribs start to become less perpendicular to the ice-flow direction, and begin to lose their integrity (Figure 4c). After around 200 to 300 years, the ribs break apart, forming isolated quasi-circular bedforms and drumlins (Figure 4d). These 'breached ribs' are observed in virtually all our simulations, acting as a transitory state between more regular bedforms; a bridge between ribs and the final bedform configuration. After around 600 years, these breached ribs begin to re-organize into drumlins, with a more regular shape and spacing (Figure 4e). As the simulation continues, the drumlin field continues to evolve, with drumlins changing shape and colliding with each other through time due to the different travel speeds of drumlins of different thickness (Supporting Information Video S1). Eventually, a configuration of highly-regular drumlins is created, which continue to migrate downstream without changing size or shape (Figure 4f).

We observe the formation of ribs in every single experiment in which the instability mechanism is active. Depending on parameter values, these ribs and amorphous bedforms may differ in spacing, amplitude, or other properties. Ribs are invariably observed to then break down into fields of breached ribs, analogous to those in Figure 4(d). The bedforms that they then evolve into can differ greatly; however, ribs are always the starting point. In our simulations, ribs are the therefore the initial configuration for subglacial bedforms, in that it is these forms that first develop a relief above the quasi-flat initial conditions. Ribs exist with a relatively short lifespan, soon being superseded by other forms.

The most commonly observed final bedform configuration in our models is the rearranging-drumlin field, an example of which is shown in Figure 4(e). Here, drumlin-like bedforms are present, having a reasonably consistent spacing and amplitude. However, they are continuously varying in shape as they move downstream, with individual drumlins continuing to interact with each other, exchanging sediment and changing shape as the pathways of subglacial water flow evolve with the changing topography. Perfectly regular drumlin fields, which we define as drumlins of consistent shape and spacing that simply translate downstream (such as shown in Figure 4f), do not always emerge from our simulations. Once this steady shape is produced, no further behavioural modification is observed. The drumlins continue to migrate in the direction of ice flow but, under these idealized conditions, they do not deform, elongate, or change their amplitude or spacing. The flow of subglacial water reaches a stable equilibrium, taking preferred pathways through the drumlin field.

Under some parameter combinations, no instability is observed, as is demonstrated in Figure 5. Here the coupling between basal shear stress and effective pressure across the water film is weakened, by imposing a larger value of λ (see Equation 1). The resulting solution sees the water film lubricate the ice sheet to such a degree that instability mechanisms do not cause the small initial perturbations to grow. Instead they are smoothed, and are then slowly damped out over time. In our experiments, such solutions are commonplace for values of $\lambda > 2.0$. Bedforms arise with λ values between 0.8 and 1.5, suggesting there is a window under which coupling conditions are conducive to bedforming; just enough grip between ice and till but not too much.

The effect of changing the small initial perturbation applied to the till layer is demonstrated in Figure 6, highlighting a fundamental property of instability mechanisms. Here, two different initial configurations are shown (Figure 6a,b), along with their evolution into ribs, at $t = 75$ years (Figure 6c,d), and their configuration at $t = 1500$ years as

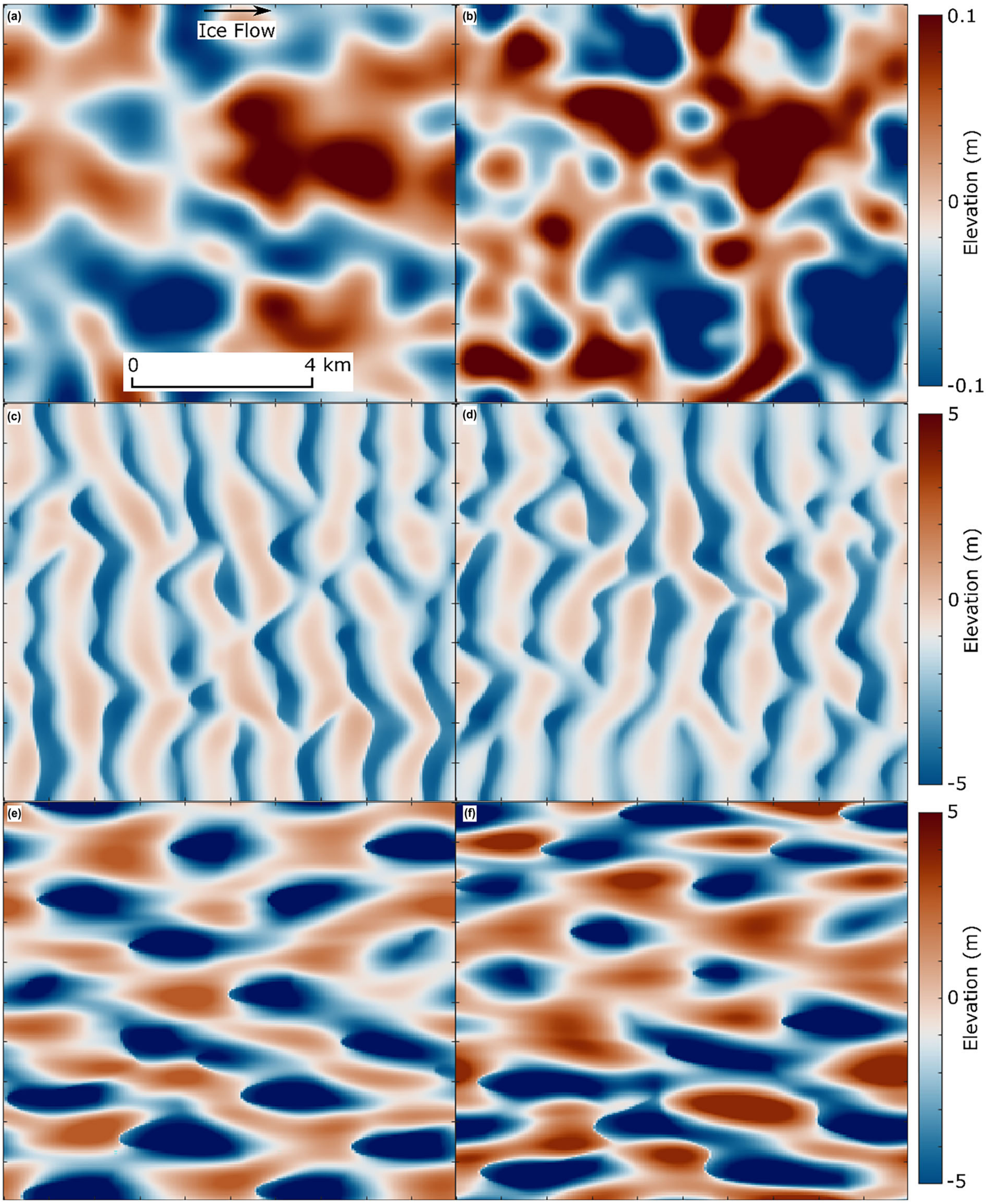

FIGURE 6 Different initial bed topographic conditions with identical physical parameters of ice flow and bed coupling, for example, can lead to broadly similar outcomes but ones that differ in specific bedform placement. The two initial conditions (a) and (b) give rise to different configurations of ribs (c) and (d), but exhibit broadly the same macroscopic properties; size, spacing, and amplitude. After 1500 years, both configurations take the form of a rearranging drumlin field (e) and (f), again with similar macroscopic properties. This demonstrates that the final structures are driven by the interacting physics of ice, water, and till rather than the initial bed configuration of the system. For a movie of these simulations, see Supporting Information Video S3a and Supporting Information Video S3b.

a field of rearranging drumlins (Figure 6e,f). Given the inherently unstable nature of the physics at work, the two pairs of solution fields shown, that is, the two rib and drumlin fields, are noticeably different in the placement and configuration of the individual features. However, they represent broadly the same type of bedforms, with broadly the same macroscopic properties (amplitude, spacing, shape, etc.). Therefore, the development of identifiable bedform structures is driven by the physics of the ice, water and till, rather than the initial configuration of the system.

Occasionally, we observed large, diagonally orientated structures arising from our results (e.g., Figure 7), which resembled those reported as MSGLs in Fannon et al. (2017). However, the appearance

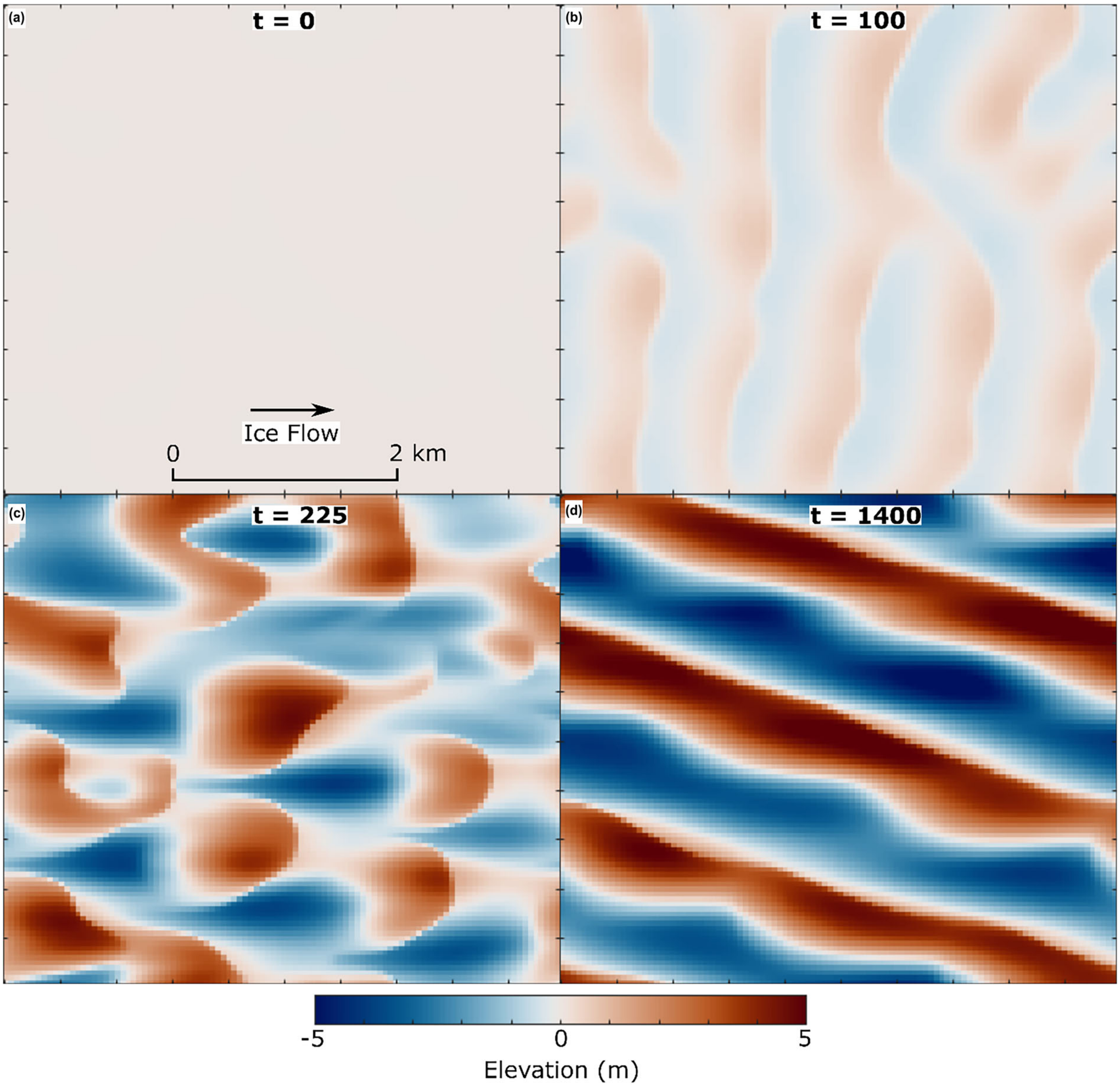

FIGURE 7 Example of a diagonally orientated bedforms. Domain size is 5 km by 5 km, and solution parameters are identical to those presented later on a larger domain (Figure 8). The solution progresses from the initial nearly flat condition ($t = 0$), through ribs ($t = 100$), and amorphous bedforms (c). The amorphous bedforms then coalesce into large structures (d); however, the domain size in this case is potentially insufficient, leading to an output whereby a single large bedform is predicted, which exits the domain at the right and re-enters at the left several times. This single bedform is therefore interacting with itself in a non-physical way as a result of the periodic boundary conditions, and its dominance precludes the development of any other features. For a movie of this simulation, see Supporting Information Video S4.

of these structures requires careful attention due to the periodic boundary conditions imposed here (Section 2.2). Periodic boundary conditions feed the solution field at the outflow boundary back into the solution field at the inflow boundary. When the size of bedform features surpasses the size of the computational domain, numerical artefacts can lead to non-physical results. Consider the case that a particular bedform has grown beyond the length of the computational domain. In this case the leading (downstream) point of the bedform will exit the domain via the outflow boundary, and be fed back into the domain at the inflow boundary, before the trailing (upstream) end of the bedform has fully entered the domain. In such cases bedforms may interact with themselves in a manner not reflective of reality. Figure 7 presents an example of such behaviour.

To examine the formation of diagonal structures further, and in an attempt to counteract the periodic boundary conditions, we conducted an experiment using the same parameters as those in the experiment presented in Figure 7, but using a larger domain (20 km by 20 km in Figure 8 compared to 5 km by 5 km in Figure 7). This experiment initially produced large, diagonal structures which did not extend beyond the domain (Figure 8). The initial evolution of this model roughly follows what we would consider the 'standard' development trajectory from our simulations (e.g., Figure 4); the perturbations of the initial surface (Figure 8a) develop quickly into a field of transverse ribs (Figure 8b), which grow in amplitude before breaking apart and forming a field of amorphous bedforms with a roughly even spacing (Figure 8c). However, these amorphous bedforms do not then go on to self-organize into any form of drumlin field; instead they begin to merge with their nearest neighbours, forming herringbone-type patterns (Figure 8d,e) with two orientations of diagonal ribs. We note that the diagonally orientated bedforms form toward the centre of

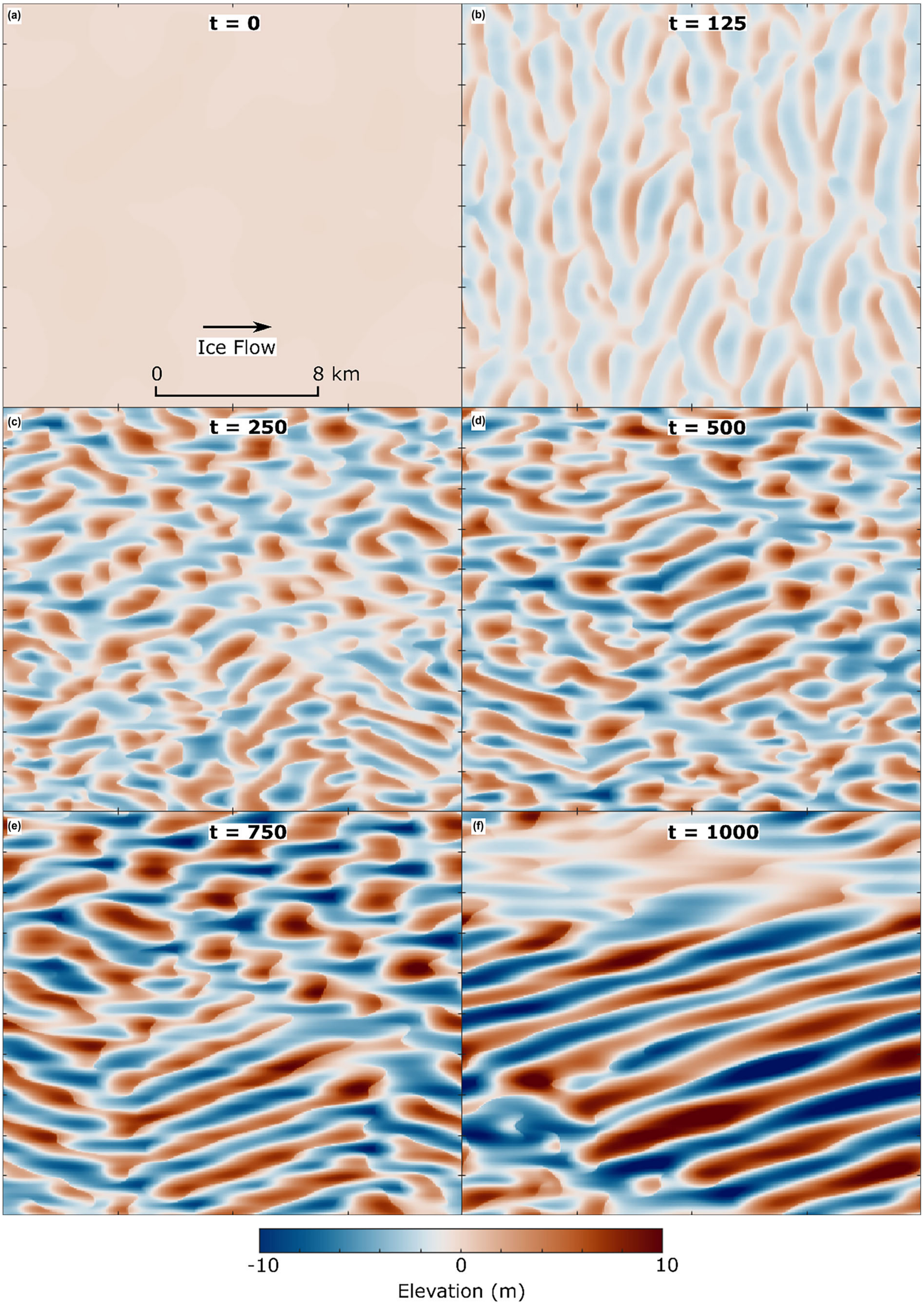

FIGURE 8 Development of herringbone bedforms. The nearly flat initial configuration (a) grows rapidly in amplitude, first forming a field of ribs (b), which continue to grow in amplitude and break into amorphous bedforms (c). The amorphous bedforms then begin to merge with their nearest neighbours, forming longer structures oriented at an angle to the ice-flow direction (d). The merging continues, forming more elongated structures in a herringbone-type pattern (e). Over time these structures merge into highly elongated formations, reaching the edges of the simulation domain where the problem of periodic boundary conditions reoccurs (f). For a movie of this simulation, see Supporting Information Video S5.

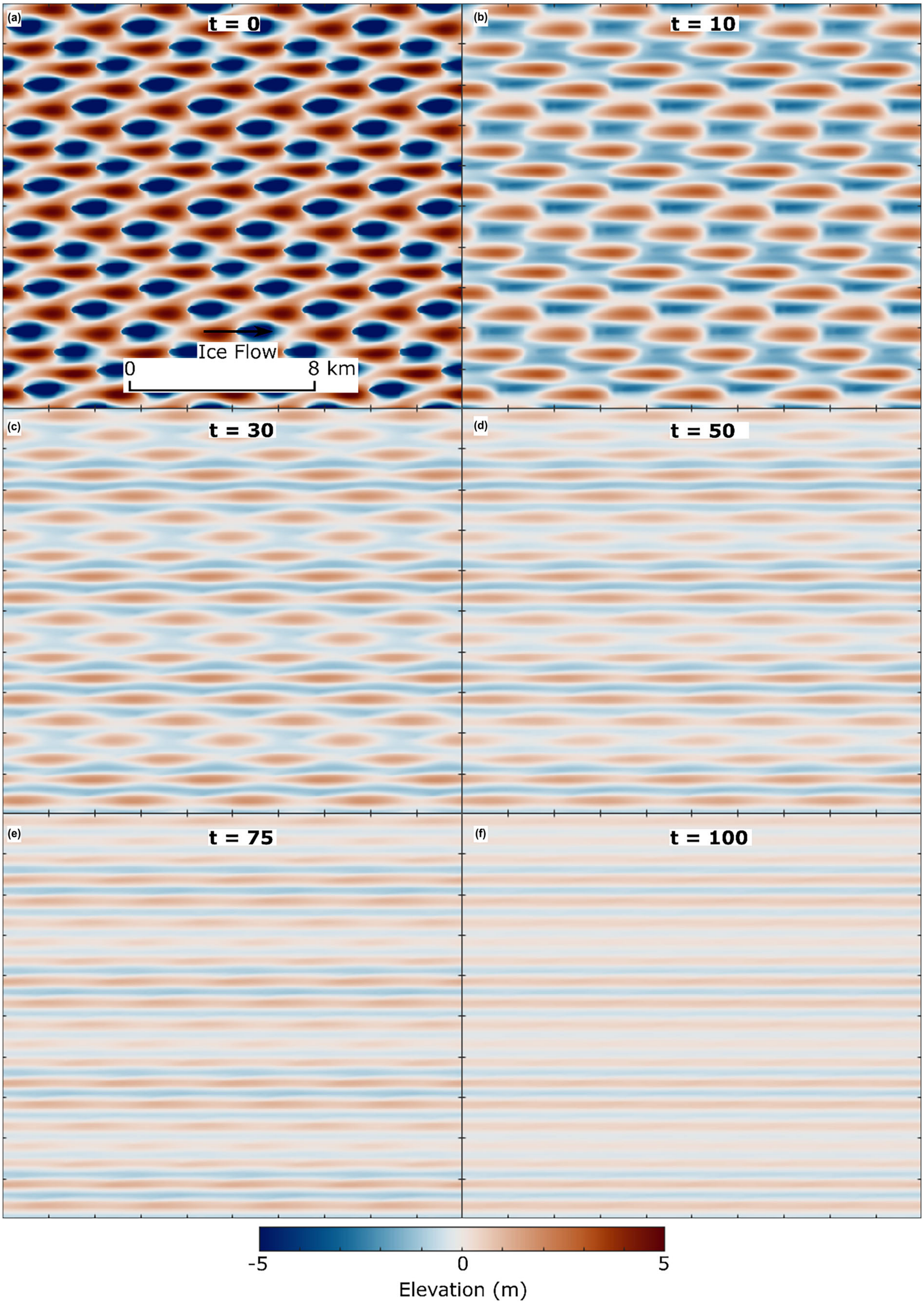

FIGURE 9 A transition from a drumlin field into features resembling mega-scale glacial lineations (MSGLs) with an increase in bedform length as their amplitude decreases. The initial drumlin field (a), which has an amplitude of ∼10 m, is restarted under parameters corresponding to well-lubricated till and/or a thick film of subglacial water. The response is that the drumlins first elongate (b, c, d) and then rapidly form into MSGL-like forms with a much-diminished amplitude (∼2 m; e, f). Note that the elongation and merging of drumlins occurs within the domain so this is not an artefact of periodic boundary conditions. For a movie of this simulation, see Supporting Information Video S6.

the domain (Figure 8d), the furthest point away from the domain edge and the effects of periodic boundary conditions. As the merging continues and the structures grow in size, a preferential orientation eventually emerges, and the large and diagonally orientated structures shown in Figure 8(f) are formed. We henceforth refer to these features as 'herringbones', given their diagonal orientation and structure. These herringbone bedforms are observed to be constantly shifting and evolving. We have observed herringbones in several simulations, and in each case, they have formed at an angle to the direction of ice flow. However, their occurrence is notably less frequent than that of a regular or rearranging drumlin field, occurring under conditions when the sediment is particularly well-lubricated and/or mobile.

In the experiments reported so far, highly elongated structures that are aligned with the ice-flow direction, that would be recognizable to a geomorphologist as MSGLs, were frustratingly absent. This led us to consider three possibilities: (i) the herringbone patterns are MSGLs, but periodic boundary conditions cause them to become misaligned; (ii) we had not explored a conditions conducive to MSGL formation; and (iii) the model is unable to produce MSGLs in its current state. Observations of downstream transitions from drumlins to MSGLs (e.g., ÓCofaigh et al., 2002; Sookhan et al., 2021), the morphological continuum between drumlins and MSGLs (Ely et al., 2016; Stokes et al., 2013), and the association between MSGLs and ice streaming (Clark, 1993; King et al., 2009), led us to hypothesize that MGSLs may arise from a change in conditions over a field of drumlins. To test this hypothesis (possibility (ii) earlier), we restarted drumlin-producing simulations with a large range of newly imposed parameters. The resulting outcomes varied; no alteration to the drumlin field, complete eradication of the drumlin field, and only slight modification of the initial drumlin field, were all observed. However, in some experiments, notably those with higher till water content and/or a thicker film of subglacial water, MSGL-like forms developed from the initial drumlin field. An example of this experiment is shown in Figure 9. Here, drumlins in the initial field (Figure 9a), elongate under the new parameters (Figure 9b,c), and are rearranged to form features resembling MSGLs (Figure 9e). The parameters under which the change from drumlins to MSGL-like forms occurs corresponds to high and low degrees of coupling between the ice and the bed. Under these conditions, the bed is damped towards a flatter state, with bedforms becoming lower in relief, and the overall bed becoming smoother (Figure 9). Thus, between drumlins and these MSGL-like forms, there is a corresponding decrease in bedform amplitude through time (Figure 9). This smoothing is rapid compared to the timescales involved in producing other bedform types, happening over the course of a few decades..

4 | DISCUSSION

4.1 | Does the instability model of subglacial bedforming provide a minimum explanation for ribs, drumlins and mega-scale glacial lineations?

A flowchart summarizing the general behaviour and our interpretation of our bedforming simulations is presented in Figure 10. When an instability is operating, the initial perturbations rapidly grow from their random pattern into dominant wavelengths resembling subglacial bedforms. As such, the initial perturbations act only to catalyse the instability. In such cases, ribs always occur first (Figure 10), and they always then break down into a field of breached ribs, populated with sporadic drumlins. This slightly disorganized collection of amorphous bumps, ribs and drumlins often then increase in spatial organization into a more regularly spaced array of organized bedforms. They usually then evolve into a field of constantly rearranging drumlins. The rearranging drumlins usually continue as they are, constantly interacting with each other and changing shape through time. More rarely however, they transition into a field of drumlins with steady shapes which simply translate downstream, maintaining their sizes and shapes through time. This is as if a 'steady-state' shape has finally evolved.

Forms resembling ribs and drumlins were readily produced by our model. However, our results raise two possibilities for the origin of MSGLs. The first is that the diagonally orientated bedforms produced by Fannon et al. (2017), and replicated here (Figure 7), can be interpreted as MSGLs forming under consistent flow conditions. If we consider this possibility to be true, then the following question arises: why are these landforms (Figure 7) misaligned with ice flow unlike observations of MSGLs? One candidate is the periodic boundary conditions used in the model, which unrealistically permits the transverse flux of water at domain edges. Perhaps all the MSGL mechanisms are correctly working in these simulations, but water flow in and out of

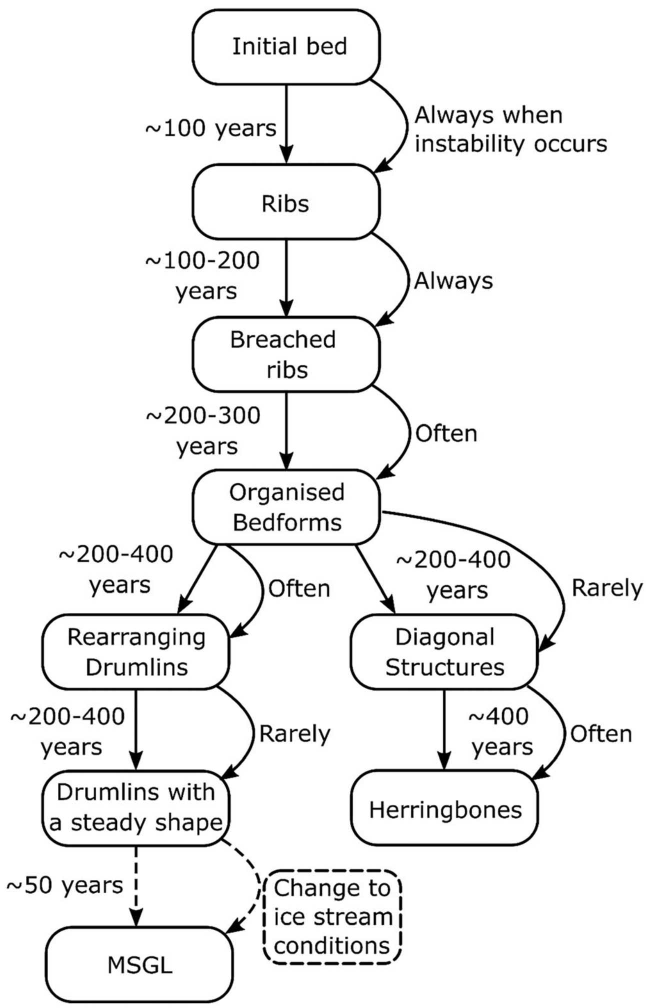

FIGURE 10 Flowchart showing the common behaviours observed in our modelling and the trajectory of typical bedform development paths evident in the simulations.

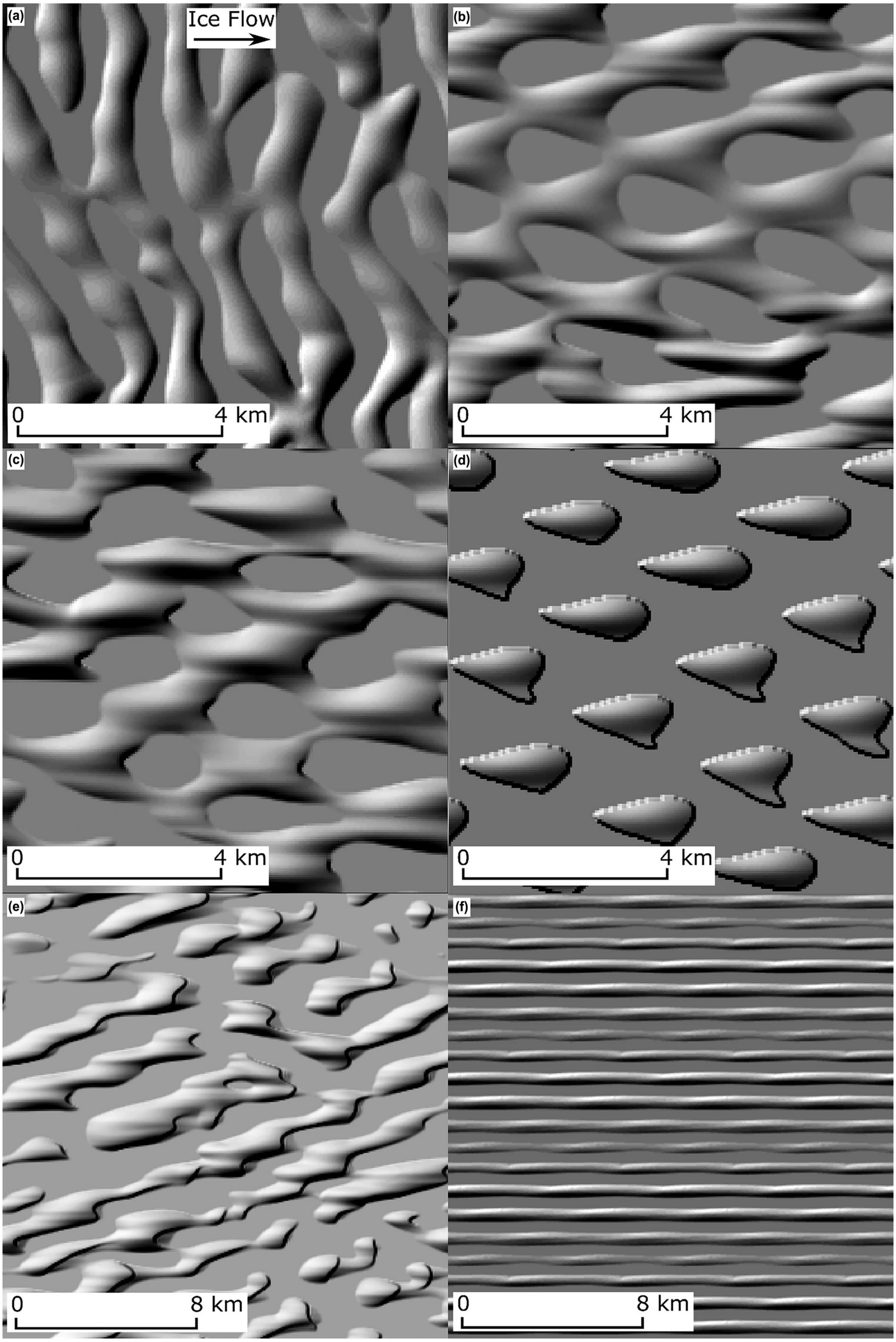

FIGURE 11 Legend on next page.

FIGURE 11 Shaded relief maps of the most commonly observed modelled subglacial bedforms, with the troughs cosmetically filled in to represent postglacial infill (see Section 4.1). (a) Subglacial ribs, which appear in every simulation where an instability operates. (b) Breached ribs, streamlined in places with occasional drumlin-like features emerging. (c) Rearranging drumlin fields, which alter their size, shape, and structure through time. (d) A regular drumlin field, whose constituent drumlins do not alter in size and shape through time, even though they are actively migrating. (e) Herringbone bedforms misaligned with flow direction. (f) MSGL, found when the model was restarted with a drumlin field, and a better-lubricated till and/or a thicker subglacial water film.

the domain edges skews the orientation of the bedforms and this is just an annoying artefact of the model set up. To investigate this, we took the simple approach of running simulations on a much larger domain (Figure 8) where such edge effects would be less pronounced. We note that in these experiments, bedforms diagonally aligned with ice flow originate toward the centre of our domain (Figure 8d), in a region where the effects of periodic boundary conditions are minimized. These features do become elongate like MSGLs, but only once there are significant regions of the bedforms interacting with the domain sides (Figure 8f). Furthermore, the across-stream spacing of these diagonal features (∼2 km, Figure 8) is an order of magnitude higher than that measured for MSGLs (∼200 m, Spagnolo et al., 2014). These factors, and our observations of diagonally-aligned bedforms in nature presented later (e.g., Figure 13), leads us to the following interpretation: the diagonally aligned bedforms are not the consequence of periodic boundary conditions, but nor are they MSGLs. Instead, we interpret the diagonally aligned bedforms produced by the model as their own category of bedforms, which we henceforth refer to as subglacial herringbones. Though this herringbone interpretation is our working hypothesis, a potential avenue for future work is to develop a numerical solution where the lateral flow of water is impeded, as would be the case in nature.

The second possibility implies that MSGLs are a transitory state, which form in response to a change in subglacial conditions. In our modelling, ribs, breached ribs, drumlins, and herringbones all occur under unvarying flow conditions and we never observed MSGLs aligned with ice flow arising under such constant conditions. To generate MSGL-like features in our model, a reduction in the degree of coupling between the ice and the bed is required (Figure 10). After changing such conditions, the model usually produces MSGL-like features by attenuating the drumlins into longer and more subdued forms of lower relief. With the elapse of time the MSGL-like features dampen in relief towards a flat bed. Our interpretation is that these MSGL-like forms are more representative of observed MSGLs. This is based on their alignment to ice-flow direction, and their morphological and scale similarity to observed MSGLs. Furthermore, the conditions we impose to get a field of drumlins to rearrange into MSGLs (Figure 10), are analogous to those associated with ice streaming. This association between MSGLs and ice stream conditions is elaborated on later (Section 4.2). In a similar manner to herringbones, this hypothesis of MSGLs as essentially elongated drumlins (e.g., Figure 9) remains our working hypothesis that awaits further testing.

In all cases, we find that modelled bedforms exist as a series of peaks and troughs, resembling continuous waveforms. While a strong similarity in size, shape and diversity exists between these modelled and subglacial bedforms observed in deglaciated landscapes (e.g., Figure 1), a specific detail is different. Rather than waveforms, the classical appearance of drumlins, for example, is that of blisters on a flat landscape (Clark, 2010) and without many obvious troughs. We attribute this difference as largely cosmetic, being due to troughs usually becoming filled by post-glacial sediment, water or bogs. This is elaborated on in Spagnolo et al. (2012). Figure 11 therefore plots hillshaded relief models of the common variants of the modelled bedforms with all troughs infilled. Many of the features in Figure 11 look remarkably similar to those observed in nature (e.g., Figure 1).

The earlier-mentioned suggests that the model contains at least the minimum amount of physical ingredients required to transform a nearly flat initial bed into a field of subglacial bedforms that resemble observations. Does this mean that the formation of subglacial bedforms has been solved by the instability model? The answer to this is likely to vary depending upon which of the three philosophical views identified by Fowler (2018) is held by the reader. To an equimorphologist, the instability theory is likely a plausible first-order explanation for subglacial bedforms. The model is built by combining well-studied physical processes with proven mathematical modelling techniques, and although it contains many simplifications of the real world, it can reproduce the fundamental repetitive regular nature of various bedform fields (as observed in Clark, Ely, Spagnolo, et al. [2018] and shown in Figures 4, 6, 8, 9 and 11) over realistic length and timescales (though the equimorphologist is likely to be intrigued by our use of Popperian inference when interpreting model output; Fowler, 2018). Despite this, those with the equicausal and equifinal philosophies may be left wondering how (or even if) the instability theory can explain their observations of subglacial ribs and drumlins. We address these two viewpoints, and the implications that the instability theory has for subglacial bedform interpretation, in the following sections (Sections 4.2 and 4.3). This discussion also highlights the difficulty of integrating observations with instability models.

4.2 | Implications for interpreting bedform morphology and palaeo-landscapes

Broadly, studies of subglacial bedform morphology have led to the following observations: (i) bedform size and position is regular (Clark, Ely, Spagnolo, et al., 2018; Dunlop & Clark, 2006; Hill, 1973; Spagnolo et al., 2014, 2017); (ii) subglacial bedforms tend towards named categories, each of which has its preferred scale and spacing (e.g., ribs in Dunlop and Clark [2006], drumlins in Clark et al. [2009] and MSGLs in Spagnolo et al. [2014]); but (iii) these categories grade into each other in a morphological continuum (Aario, 1977; Ely et al., 2016; Rose, 1987). As we elaborate on later, these observations are consistent with the instability model of bedform formation, and our results here.

As outlined earlier (Section 2.2), regularity is a common trait of phenomena which arise through an instability (Clark, Ely, Spagnolo, et al., 2018). Furthermore, that ribs always precede drumlins may provide an explanation for the observation that drumlins are often arranged in transverse bands of low and high density

(Clark, Ely, Spagnolo, et al., 2018; Hill, 1973). This banding may be a remnant of rib ridges, or due to a mixture of rib and drumlin forming instabilities occurring concurrently (e.g., Figure 6e). Similarly, MSGLs also have a regular spacing of ~270 m (Spagnolo et al., 2014) measured orthogonal to their long axes. A measure of drumlin spacing (nearest neighbour distance) is remarkably similar ~300–350 m (Clark, Ely, Spagnolo, et al., 2018). This regularity of positioning and spacing is consistent with our model results (Figure 9); whereby MSGLs owe their regularity and spacing to the drumlin field from which they evolve.

We find that ribs and drumlins are the most common results from the model; all models where instability is present initially evolve into ribs and through time these generally tend to evolve toward drumlin-like features (Figure 4). That the modelled bed tends to relax to or get stuck in a number of common configurations may explain observation (ii) earlier, that in nature, subglacial bedforms tend towards specific named categories, each of which has its preferred scale and spacing. Perhaps our more frequently realized model states (e.g., rib, drumlin or MSGL) are actually the preferred states of the subglacial bed, hence we observe them more often in deglaciated landscapes than the more complex transitory states (e.g., Figure 8d) that we have yet to attach names to. Our model results also provide an explanation for the following qualitative observation based on our experience of mapping palaeo-ice-sheet beds (e.g., Clark et al., 2000; Clark, Ely, Greenwood, et al., 2018; Ely et al., 2016; Stokes et al., 2009): drumlins are more common than either ribs or MSGLs. In our model, subglacial ribs are short-lived, breaking down into drumlin-like features. Once formed, drumlins usually remain persistent, forming an arrangement with a consistent size and shape (Figure 4f). In our modelling, MSGLs require drumlins to have first occurred, and then to experience a change in basal conditions. Hence, drumlins are the most prevalent landform both in the landscape and in our model results. This rationale also helps explain why transitions between bedform types (e.g., ribs to drumlins, drumlins to MSGLs), though observed (Ely et al., 2016), are rarer; the transitional forms evolve rapidly into the more commonly named categories. Alternatively, this may simply be a consequence of bias during data collection, with observers preferring to categorize landforms into pre-existing nomenclature, and more attention should be paid to finding transitory features.

That subglacial bedforms are part of a morphological continuum (i.e., observation (iii) earlier), is supported and explained by our model results, which show that subglacial ribs can transition into drumlins over time, and during a period of bed smoothing, that drumlins can relax into MSGLs. This morphological transition has been observed to occur along palaeo-ice stream tracks (ÓCofaigh et al., 2002; Hess & Briner, 2009; Sookhan et al., 2021) and has been modelled in Barchyn et al. (2016). Notably, our modelling results never produce the opposite trajectory; drumlins never transition into subglacial ribs. In fact, our results suggest that the formation of a drumlin field, while not inevitable, represents the end-point to the behavioural trajectory under constant conditions; that is, when formed, a drumlin field does not further evolve unless conditions change. To our knowledge, there are no observations from the palaeo-record of gradual downstream transitions from drumlins back to subglacial ribs, which would suggest that such a time-evolution occurs. Indeed, in rare cases where ribs have been found downstream of streamlined bedforms, the transition is abrupt and appears overprinted, reflecting different phases of bedform development (Stokes et al., 2008). From our results, and observations of bedforms, we therefore infer that a trajectory of bedforms exists, with ribs evolving first, before breaking down and eventually forming drumlins, usually via a stage of more complex and diagonally arranged forms. Drumlins are then a prerequisite for MSGL formation, elongating into MSGLs if basal conditions that cause bed smoothing occur. The rapid evolution of drumlins into MSGLs when these conditions arise is consistent with inferences of rapid MSGL development from studies of the Marguerite Bay palaeo-ice stream (Jamieson et al., 2016).

Based on the consistency between model simulations and landscape observations regarding regularity, spacing, scale, transitions between named bedform types and behavioural trajectories, the instability theory likely satisfies the equicausalists; a single set of mechanisms can explain subglacial bedforms. However, for those that study the morphology of subglacial bedforms a common quest has also been to find specific properties of the ice, water and/or till at the time of formation that can be used to explain different morphological variants (e.g., Barchyn et al., 2016; Rattas & Piotrowski, 2003; Stokes & Clark, 2002). The most invoked ice-to-landform relationship is between ice velocity and either bedform type or length (Briner, 2007; Hart, 1999; Stokes & Clark, 2002). This inference is based on observations that subglacial ribs are found in the interior of ice sheets (Dunlop & Clark, 2006; Hättestrand & Kleman, 1999), where ice velocities are suspected to be slow, and that the length of subglacial lineations (drumlins and MSGLs) often corresponds to inferred variations in ice velocity, with long lineations being associated with ice streams (Stokes & Clark, 2002). The ultimate end point of this logic would be that one could invert from the morphology of subglacial bedforms found in a palaeo-landscape to estimate the properties of the ice sheet that formed them. Unfortunately, the instability hypothesis of bedform formation limits the extent to which subglacial bedform morphology can be used to invert for ice flow conditions. In our model, small perturbations to the system configuration can alter bedform morphology (e.g., Figure 9), and different combinations of parameters can lead to similar end results. We have resisted listing parameter combinations used in each experiment throughout for this very reason; a change in one parameter likely affects many different parts of the system. In this case, the link between cause and effect is blurred. Furthermore, the trajectory of subglacial bedforms means that the duration for which bedforming conditions operate is crucial, consistent with observations that bedform length relates to cumulative basal slip distance (Zoet et al., 2021). Fast ice flow of only a few years may create subglacial ribs, with persistent slow flow creating a similar outcome.

For bedforms formed by other geomorphic agents (e.g., fluvial, aeolian and marine bedforms), phase diagrams have been used to connect properties of flow to the bedform variant produced (e.g., Ashley, 1990; Duran Vinent et al., 2019; Koller et al., 2017; Perillo et al., 2014; Southard & Boguchwal, 1990). In many cases, this is derived from controlled flume or numerical modelling experiments (Duran Vinent et al., 2019; Nishimori & Tanaka, 2003). Complexity arises due to changes in sediment composition (Carling, 1999; Schindler et al., 2015) and availability (Gao et al., 2015), and vegetation cover (Baas & Nield, 2010). Furthermore, phase diagrams may be impossible to derive when bedforms are formed under unsteady conditions (Allen & Collinson, 1974). In comparison to their aeolian, fluvial, and marine counterparts, subglacial bedforms have three unique characteristics which make the construction of a phase diagram difficult. First and foremost, the formation of subglacial bedforms involves two geomorphic agents (water and ice), whereas bedforms in other environments have just a single forming agent (water for marine and fluvial bedforms, air for aeolian bedforms). Second, we have few observations of subglacial bedforms actively forming, from which flow conditions could be derived. Finally, grain size is likely to play an important role in both the coupling between the ice and the bed (Creyts & Schoof, 2009), and the deformational response of the sediment (Hooke & Iverson, 1995). Therefore, the inhomogeneity and mixed grain size of subglacial sediment, especially when compared to sand-based aeolian dunes, further complicates any link between flow conditions and bedform morphology. Thus, our current understanding prohibits the development of a phase-diagram approach for understanding the conditions for subglacial bedforming.

A link between MSGLs and fast-flowing ice has often been postulated in the literature. This inference is based upon the logic that their long length and highly parallel arrangement requires fast flow (Clark, 1993), and that they are found within palaeo-ice stream trunks (Stokes & Clark, 2002), at the front of active ice streams (Canals et al., 2000), or beneath contemporary Antarctic ice streams (Holschuh et al., 2020; King et al., 2009). However, a physical mechanism that explains the association between MSGLs and fast-flowing ice has been lacking. Our experiments show that drumlins are smeared out to become MSGLs only when the till is sufficiently water-laden and/or there is a thick subglacial water layer. High water content within subglacial sediment or at the ice–bed interface has been found beneath fast-flowing ice streams (Engelhardt et al., 1990; Schlegel et al., 2022). Indeed, the mechanisms by which ice flows fast within an ice stream – sliding (Engelhardt & Kamb, 1997), wet-bedded till deformation (Alley et al., 1986), or a combination of the two (Tulaczyk et al., 2001) – are a consequence of a wet subglacial environment. This creates an association, but not a causal process link, between MSGLs and fast-ice flow; both require a comparatively wet subglacial environment. In this interpretation, the formation of MSGLs is not a direct consequence of fast ice. Instead, both are a by-product of the amount of water in the subglacial environment.

Our interpretation of our model results suggests that drumlins are a prerequisite for MSGL formation. In Figure 12, we envisage two process histories under which drumlins are elongated into MSGLs. The first involves the switching on of an ice stream (Figure 12a). Here, a drumlin field develops under ice-sheet flow. The switching on of an ice stream causes the bed to switch to smoothing conditions, leading to the pre-existing drumlins elongating into MSGLs (Figure 12a). Given that some drumlins will be more resistant to remoulding than others, this ice-stream-switch mechanism potentially explains mixed assemblages where drumlins and MSGLs can be found neighbouring each other, such as at the Dubawnt Lake Ice Stream (e.g., Stokes et al., 2013). In the second circumstance, ribs and drumlins are seeded (i.e., initiate and grow) at the onset of an ice stream, or at positions within the ice stream where basal conditions are conducive to their formation (Figure 12b). From these seeding points the bedforms migrate downstream into the main ice stream trunk where they are smoothed into MSGLs. We anticipate that this process history is more likely to produce consistent MSGL fields, such as those observed beneath the Rutford Ice Stream (e.g., King et al., 2009). These scenarios are, of course, simplifications of the real pathways under which MSGLs may develop. Indeed, the history of an individual field of bedforms is likely to reflect a spatiotemporally complex history relating to changing subglacial conditions, perhaps punctuated by events such as subglacial floods (Fowler et al., 2013). Thus, the bulk morphology of a bedform field may reflect the cumulative stochastic variations in subglacial conditions (Hillier et al., 2016), as has been suggested from the characteristics of their size and shape distributions (Ely et al., 2018; Fowler et al., 2013; Hillier et al., 2013).

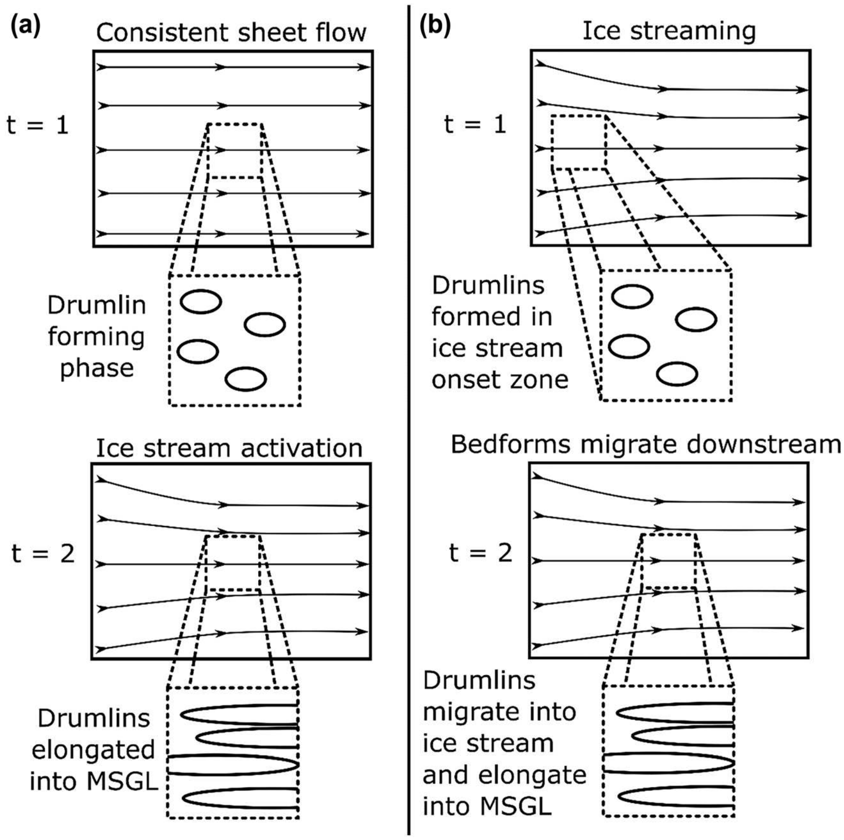

FIGURE 12 Proposed process histories which lead to MSGLs. (a) Ice-sheet flow leads to the formation of drumlins. Later, ice stream activation causes these drumlins to elongate into MSGLs. (b) Drumlins are seeded at the onset of an ice stream which then migrate into the main faster flowing ice stream trunk, where they are elongated into MSGLs.

A further result of note is that, under certain conditions, no bedforming instabilities are predicted to occur (see Figure 5). Where the coupling between basal shear stress and effective pressure across the water film is weakened within the numerical model, bedforming instabilities do not operate. The switching on and off of conditions conducive to bedform formation may help explain the preservation of landforms (e.g., Clark, 1999). Furthermore, the preservation of landforms beneath ice may explain why the morphology of bedforms beneath some Antarctic ice streams appears to be unrelated to current glaciological conditions (Holschuh et al., 2020), and why little to no change has been detected between radar surveys of some contemporary ice stream beds (Davies et al., 2018).

Our model also produces herringbone-type subglacial bedforms (e.g., Figures 8 and 11). These features generate diagonal-to-flow elements that occur under a uniform and unvarying flow direction. That subglacial bedforms misaligned with ice-flow direction can form raises potential complications for reconstructing palaeo-ice-flow direction. These model simulations prompted us to seek observations of herringbone-type arrangements in the landscape, to check whether the model result is realistic. A similar herringbone arrangement has been noted at a larger scale, for the 'traction-ribs' of Stokes et al. (2016). Potential herringbone candidates in deglaciated landscapes are shown in Figure 13. Further work is required to ascertain the origin of these features. Previously, these examples are likely to have been interpreted as arising from remoulding and cross-cutting under different ice-flow directions as commonly observed and interpreted on ice sheet beds (Clark, 1993). Here our modelling suggests that, in addition to cross-cutting bedforms from multiple ice-flow directions, true unidirectional subglacial herringbones are likely to exist, complicating the task of reconstructing past ice-flow directions of such regions. We do not yet know how to distinguish true cross-cutting drumlins from subglacial herringbones.

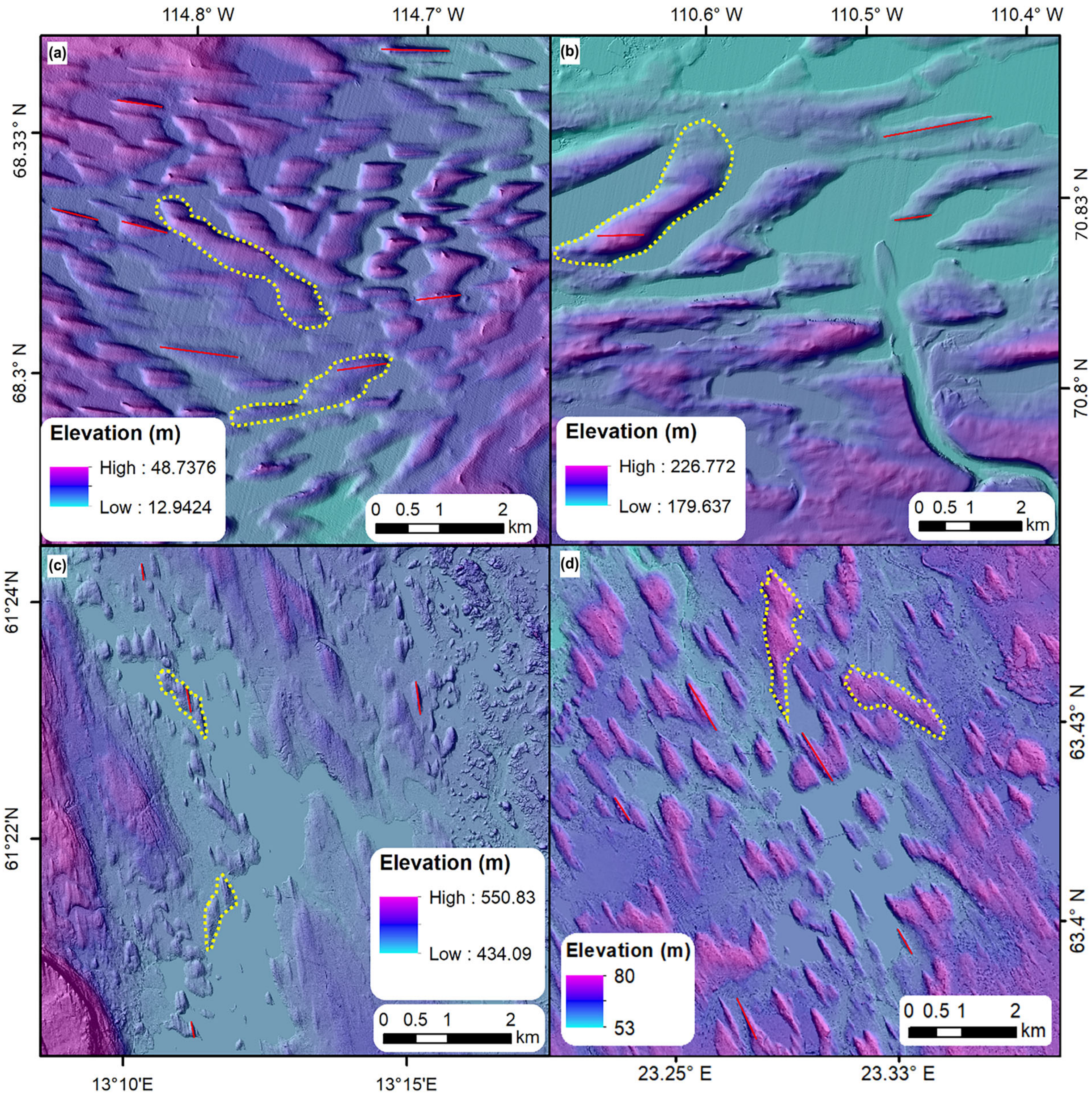

FIGURE 13 Potential herringbone-type bedforms in Canada, Sweden and Finland which resemble those found in model simulations. To aid interpretation, the long axes of some drumlins are mapped as lines in red and the outlines of potential herringbone features are highlighted by yellow dotted lines. (a) Northwest territories, Canada (Porter et al., 2018). (b) Victoria Island, Canada (Porter et al., 2018). (c) Sweden (GSD-Höjddata, grid 2 + © Lantmäteriet). (d) Finland (National Land Survey of Finland elevation model 2 m 11/19).

4.3 | Does the diversity of internal structure, composition and setting of subglacial bedforms contradict the instability model of subglacial bedforming; are alternatives required?

Studies of the internal sedimentary composition and structure of subglacial bedforms often highlight large variations (Stokes et al., 2011), often leading to an equifinal interpretation (e.g., Lindén et al., 2008; Möller, 2010; Möller & Dowling, 2018) whereby bedforms in different places, whilst looking morphologically similar, are reconstructed as being created by different processes. This differs from the equimorphological view of the instability model in the level of explanation sought for subglacial bedform formation. Proponents of the instability model are satisfied that bedform formation is explained once the reductive criteria of a near-flat bed being transformed into a field of bedforms has been achieved (e.g., Figure 4). The aim of a sedimentologist is more holistic, to derive physical explanations that explain the sequence of sediments observed within bedforms. This often includes sediments unrelated to the bed shaping processes that the instability model concerns itself with, such as sorted fluvial sediments that predate bedform shaping (Stokes et al., 2013) or veneers of sediment draped over the bedform surface that postdate bedforming (Stokes et al., 2011). Thus, when addressing the question of bedform formation, these two viewpoints have slightly different questions in mind. Preoccupying the mind of a modeller is: how was bedform relief generated and shaped? Whilst the sedimentologist asks: what is the history of processes that led to the sequence of sediments observed within bedforms? A result of this divergence in approach is that the instability model frustratingly leaves many questions unanswered because explanations of field-scale sedimentological observations are left unexplained. The instability model in its current form cannot be used to predict the composite histories of erosional, depositional, and deformational processes recorded in sections through subglacial bedforms – this is a more grandiose aim than the model was designed to achieve. It is not unforeseeable that eventually a model may be developed which can address the detail found within bedforms or recreate specific process histories. Until a model is capable of directly addressing these questions, we here peruse the following helpful line of enquiry: could the instability model reasonably be adapted to accommodate the diversity of internal structures found within subglacial bedforms, and the range of settings in which they are found? Indeed, for these reasons, an array of alternative explanations for drumlin formation exists. Finite space precludes us from discussing them all, so we restrict ourselves to a discussion of recent and persistent alternative explanations.

A common observation of subglacial bedforms, especially drumlins, is that some are nucleated around a core composed of bedrock (e.g., Boyce & Eyles, 1991; Hart, 1997; Meehan et al., 1997). Alternatively, inhomogeneities in sediment have been envisaged (e.g., Boulton, 1987; Smalley & Unwin, 1968). Conceptually, the nucleation of bedforms around bedrock and sediment cores has been considered within a deforming bed framework (Boulton, 1987). It is therefore possible to envisage performing an interesting set of experiments within the instability model in the future, which includes variable sediment strength and thickness. However, if one is of the opinion that all drumlins are triggered by a bedrock obstacle, then the need for an instability-based model may seem superfluous, as such forms do not require an instability to determine their location (e.g., Möller, 1987; Möller & Dowling, 2018). At the heart of this problem is whether drumlins are individual elements, each of which can arise and be explained on their own, or whether they are part of a spatially organized array with internal regularity. In the latter case, explanation is arguably required at the field rather than individual landform scale. To tackle this, Clark (2010) made the perhaps useful distinction between true instability-triggered 'emergent' drumlins, 'obstacle' drumlins formed around bedrock obstacles, and 'clones' formed around inhomogeneities in sediment.

Though Clark (2010) argues that the widespread patterning of drumlins implies that emergent drumlins are likely the most common variety, the balance between these categories is likely to vary between locations. Indeed, the erodent layer hypothesis provides a tribological explanation for drumlins and MSGLs formed in regions of thin till blanketing bedrock (Eyles & Boyce, 1998; Eyles et al., 2016), which mainly fall under the obstacle variant of Clark (2010). The erodent layer hypothesis views streamlined bedforms as the erosional remnants of a thin till layer moving across a bedrock surface, like the small-scale wearing features found when two materials move in relative motion to each other. Currently, the instability model does not include a bedrock layer beneath the till, or account for net sediment erosion. Indeed, many subglacial bedforms form in areas of thin till layers, so such an adaptation would be interesting to include in the future. Though previous work has suggested that instabilities do not operate in such thin-sediment settings (Evans et al., 2015; Eyles et al., 2016), this remains to be tested. Here, we note that the instability and erodent layer hypothesis are perhaps compatible; undoubtedly the movement of subglacial sediment over bedrock has a geomorphic effect, though the organization of such a process into a field of drumlins likely requires an instability. Perhaps obstacles themselves can be organized by erodent till deformation. Furthermore, a challenge for the erodent layer hypothesis is to account for the formation of depositional drumlins and MSGLs (e.g., Menzies et al., 2016; Spagnolo et al., 2016).

In all our model experiments where the instability operates, the resultant bedforms migrate downstream. This appears to be at odds with observations of stationary bedforms, both inferred from bedform-internal composition (Stokes et al., 2011), and repeat observations of MSGL beneath Antarctica (Schlegel et al., 2022). Though Stokes et al. (2013) provide explanations for how the instability model may account for the varied internal structure of bedforms (stationary and non-stationary), these ideas are yet to be implemented numerically. Thus, a mechanism for stopping drumlin migration is missing from the model. Perhaps the first target for future generations of the instability model would be to implement more complex feedbacks between till rheology, effective pressure in the till, and the degree to which sliding is spatially inhomogeneous (as is apparent from the Rutford Ice Stream observations [Schlegel et al., 2022]).

The drumlins found in the foreland of the surging Múlajökull glacier, Iceland, provide a further interesting case study of bedform formation to compare to the instability model (Benediktsson et al., 2016; Johnson et al., 2010; McCracken et al., 2016). Included in the impressive array of observations are over 2000 measurements of till density, which indicate that effective pressure was higher in inter-drumlin regions than on top of drumlins, during the quiescent periods between surges. This pattern of effective pressure derived from these measurements is the opposite prediction to the instability model. Unfortunately, in its current form, the instability model has not been adapted to explain how stresses might alter between surge and non-surge states (ice flow is currently constant through time). Fowler (2018) suggests that to do so would be a tricky (but not impossible) task, which would require a model that accounts for the formation of R-channels between drumlins, causing till squeezing during a non-surge event. Perhaps a stronger observational test of the instability model would be to measure basal conditions during periods of relatively steady flow and drumlin formation, a setting more akin to the situations modelled throughout this article. Remote probes that access the bed seem a promising manner by which such measurements could be obtained (e.g., Hart et al., 2019). Meanwhile, Iverson et al. (2017) developed a model of drumlin growth which successfully explains the observations at Múlajökull. In this model, drumlins grow and migrate downstream through a combination of melt-out derived deposition during the surge phase, and with the erosion of drumlin flanks during quiescence (Iverson et al., 2017). However, the wider applicability of this model to the formation of subglacial bedforms is questionable. Based on the observation that portals of water often surround the edges of drumlins at the glacier margin (McCracken et al., 2016), pre-existing undulations, bounded by subglacial channels, are imposed at the start of the model (Iverson et al., 2017). The length and width of these undulations does not change during the course of a simulation. That pre-existing undulations bounded by channels are required to initialize the model creates a chicken and egg conundrum: did channels organize the spacing of undulations, or were channels guided by undulations? We suggest that an instability is required to set the initial spacing of drumlins, the development of which is later modulated by surge dynamics.

One criticism of the instability model concerns the rheology of till. The instability model assumes a viscous rheology. However, laboratory experiments show that till rheology is likely plastic or Coulomb-plastic (Iverson, 2010; Iverson et al., 1998; Tulaczyk et al., 2000). The choice of a viscous rheology can be considered a pragmatic choice (Fowler, 2018), as instability theory assumes bedform formation only occurs once sediment is moving (e.g., for a plastic rheology, once the yield-stress has been exceeded). Furthermore, the instabilities (e.g., Kelvin–Helmholtz) involved should operate regardless of the till rheology (Schoof, 2007a). However, future work should be undertaken to understand whether differing till rheology fundamentally changes characteristics of the bedforms that arise from instabilities. Other criticisms of the instability model have led to model improvements. As covered by Fannon et al. (2017), the criticisms from Schoof (Schoof, 2007a, 2007b) led to the development of three-dimensional instabilities (Fowler & Chapwanya, 2014), a consideration of cavitation (Fowler, 2009), and an explanation for internal stratification (Stokes et al., 2013).

Finally, an alternative explanation for subglacial bedforms suggests they are unrelated to till deformation or glacial processes. Instead, the mega-flood hypothesis attributes the formation of subglacial bedforms to large subglacial floods (Shaw, 2002; Shaw et al., 1989). The volumes of water required make this hypothesis unfeasible (ÓCofaigh et al., 2010). However, it is perhaps interesting to note that the envisaged mechanisms of eroding sediment and creating cavities during a flood are an instability in themselves. It therefore seems this hypothesis also invoked an instability, though possibly in a practically unobtainable manner.

5 | CONCLUSIONS

Using a numerical model which is well-grounded in physics, and which tracks the movement and coupling between sediment, water and ice, we demonstrate the occurrence of instabilities in the topography of the sediment–ice interface. Our model simulations of this interface produce bed undulations resembling observations of real subglacial bedforms including ribs, drumlins and MSGLs. The experiments reveal a behavioural trajectory of subglacial bedforming. Subglacial ribs occur first in the sequence. These often break down into amorphous and more complex bedforms, which interact with each other to organize into a more regular spacing and shape. The bed then evolves either into a field of migrating drumlins, the spacing and shape of which remains constant throughout time, or into elongate structures with diagonal ridges. We call these subglacial herringbones. Since they are misaligned with ice-flow direction, caution should be taken when reconstructing palaeo-ice-flow directions. Subglacial ribs, drumlins, amorphous bedforms and herringbones all occurred in model simulations under non-varying ice flow conditions, but MSGLs were not found to arise. To form MSGLs, we perturbed conditions over a modelled field of drumlins, finding that under conditions of high and low ice-bed coupling drumlins became smoothed and attenuated into MSGLs. We liken these modelled conditions to those found beneath ice streams, providing the first physical mechanism behind the association of MSGLs and fast ice flow conditions.

We have shown that the instability model of subglacial bedforming can produce ribs, drumlins and MSGLs of appropriate size and shape, and can simulate transitions between these named types along with less well-known transitional types that we suggest will be found or recognized on ice-sheet beds. We add subglacial herringbones as a new named type. We therefore propose that the instability model of subglacial bedforming has achieved its goal of providing a minimum explanation for the formation of subglacial bedforms. To explain process histories and the sedimentary properties and architecture at a specific place remains challenging and is currently beyond the scope of this model. Though we have demonstrated that the instability model provides a solution to the first part of the longstanding 'drumlin problem', more complexity needs to be added to the model to explain specific observations.

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AUTHOR CONTRIBUTIONS

Ely and Clark conceptualized the project and acquired funding. Stevens conducted the numerical modelling experiments, under the supervision of Ely and Clark. Butcher provided data for the manuscript. Ely led the writing of the initial draft, and all authors contributed to the writing through reviewing and editing.

DATA AVAILABILITY STATEMENT

All output from the numerical modelling is provided as figures and as supplementary movies to the manuscript.

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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