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1 A sterility–mortality tolerance trade-off leads to
2 within-population variation in host tolerance

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4 **Abstract**

5 While experimental studies have demonstrated within-population variation in host
6 tolerance to parasitism, theoretical studies rarely predict for polymorphism to arise.
7 However, most theoretical models do not consider the crucial distinction between toler-
8 ance to the effects of infection-induced deaths (mortality tolerance) and tolerance to the
9 parasite-induced reduction in the reproduction of infected hosts (sterility tolerance).
10 While some studies have examined trade-offs between host tolerance and resistance
11 mechanisms, none has considered a correlation within different tolerance mechanisms.
12 We assume that sterility tolerance and mortality tolerance are directly traded-off in a
13 host population subjected to a pathogen and use adaptive dynamics to study their evo-
14 lutionary behaviour. We find that such a trade-off between the two tolerance strategies
15 can drive the host population to branch into dimorphic strains, leading to coexistence
16 of strains with sterile hosts that have low mortality and fully fertile with high mortality
17 rates. Further, we find a wider range of trade-off shapes allows branching at interme-
18 diate or high infected population size. Our other significant finding is that sterility
19 tolerance is maximised (and mortality tolerance minimised) at an intermediate disease-
20 induced mortality rate. Additionally, evolution entirely reverses the disease prevalence
21 pattern corresponding to the recovery rate, compared to when no strategies evolve.

22 We provide novel predictions on the evolutionary behaviour of two tolerance strategies
23 concerning such a trade-off.

24 **Keywords**— host, parasitism, sterility tolerance, trade-off, fecundity, branching

25 1 Introduction

26 In response to parasitism, hosts often respond by evolving defence strategies to limit any reduction
27 in fitness. Host defences can broadly be divided into two types: resistance (a host’s ability to
28 reduce infection spread or the parasite burden) and tolerance (a host’s ability to limit damage due
29 to infection) (Roy and Kirchner, 2000; Råberg et al., 2009). Mechanistically, resistance can evolve as
30 avoidance, increased recovery, or decreased parasite replication rate (Boots and Bowers, 1999; Boots
31 et al., 2009; Miller et al., 2007). Similarly, tolerance can also take different forms, notably either
32 reducing the parasite impact on host mortality or on host reproduction (Restif and Koella, 2004;
33 Best et al., 2008, 2010a, 2017; Pagán and García-Arenal, 2018). Both resistance and tolerance
34 strategies act to increase the host’s fitness but can have distinct evolutionary implications (Roy
35 and Kirchner, 2000). In particular, the occurrence of polymorphisms has been widely identified in
36 models of resistance mechanisms (Boots and Haraguchi, 1999; Best et al., 2010b; Boots et al., 2012;
37 Hoyle et al., 2012; Best et al., 2017), but very few have detected polymorphism in tolerance (Best
38 et al., 2008, 2010a; Ferris and Best, 2019). A number of host-parasite evolutionary models have
39 discussed and compared the evolution of different resistance mechanisms (Antonovics and Thrall,
40 1994; Boots and Haraguchi, 1999; Miller et al., 2007; Carval and Ferriere, 2010), but the distinction
41 between the two forms of tolerance - reducing mortality or sterility effects - is comparatively less
42 studied (Best et al., 2008, 2010a).

43 The form of tolerance that reduces parasite impact on host mortality is referred to as “mortality
44 tolerance” and has been well studied both theoretically (Miller et al., 2005, 2007; Best et al.,
45 2014), and experimentally (Mauricio et al., 1997; Tiffin and Rausher, 1999; Roy and Kirchner,
46 2000). Another form of tolerance that reduces parasite implications on host’s reproduction is termed
47 as “sterility tolerance” and is less explored (Abbate et al., 2015), with studies largely limited to

48 exploring the impact of sterilising pathogens on host evolution either theoretically (Best et al.,
49 2008, 2010a, 2017; Kada and Lion, 2015; Janoušková and Berc, 2018; Bartlett and Boots, 2021),
50 or experimentally (Sloan et al., 2008; Lafferty and Kuris, 2009; Vale and Little, 2012; Kutzer et al.,
51 2018; Montes et al., 2020). There are critical differences between these two arms of tolerance -
52 mortality and sterility - as a host defense strategy. In general, only mortality tolerance creates
53 a positive feedback on the fitness of horizontally-transmitted pathogens (but see Vitale and Best
54 (2019)), whereas sterility tolerance is either neutral or costly to such pathogens' fitness, depending
55 on the trade-off considered (Best et al., 2008; Boots et al., 2009). A negative feedback can cause
56 a decline in the parasite prevalence, leading to negative frequency-dependence and the potential
57 for the coexistence of polymorphic host strains (Roy and Kirchner, 2000). This means that strains
58 with different levels of tolerance to pathogen-induced sterility can coexist within host populations
59 (Best et al., 2008). As such, there are possibilities of polymorphism in sterility tolerance, but not
60 in mortality tolerance (unless external conditions like seasonality are imposed, see Ferris and Best
61 (2019) for instance). An experiment on pea aphid genotypes against fungal pathogens supports
62 this theory, as they found no variation among mortality tolerance traits but did so within traits of
63 fecundity tolerance (Parker et al., 2014). Willink and Svensson (2017) also found that two female
64 morph types in *I. elegans* evolve different tolerance levels to fecundity reduction caused by parasitic
65 mites. This gives rise to an unresolved question of whether correlations between investment in
66 sterility and mortality tolerance could lead to within-population variation in mortality tolerance.

67 The theoretical literature is largely based on the assumption that evolving defense is costly, sug-
68 gesting trade-offs between defence strategies and other host fitness attributes (Boots and Haraguchi,
69 1999; Restif and Koella, 2003, 2004; Donnelly et al., 2015). Nonetheless, there is evidence of trade-
70 offs between mechanisms of resistance and tolerance as well (Fineblum and Rausher, 1995; Pilson,
71 2000; Agrawal et al., 2004; Råberg et al., 2007; Baucom and Mauricio, 2008; Mikaberidze and Mc-
72 Donald, 2020), but there has been a less theoretical investigation of such a scenario (Restif and
73 Koella, 2004; Best et al., 2008, 2017; Singh and Best, 2021). Investment in sterility tolerance has
74 previously been assumed to be bought at the cost of host characteristics such as increased natu-
75 ral death rate (Best et al., 2008, 2010a) or reduced intrinsic birth rate (Restif and Koella, 2004).

76 A model by [Best et al. \(2017\)](#) explored the consequences of varying infected hosts fecundity on
77 mortality tolerance and found that high fecundity levels select for greater investment in mortality
78 tolerance. Further, a negative correlation between host fecundity and longevity has been studied in
79 theory from a pest control perspective ([Berec and Maxin, 2012](#); [Janoušková and Berec, 2018](#)), or
80 with a focus on host resource allocation ([Janoušková and Berec, 2020](#)). In parallel to these studies,
81 other theoretical works have indicated that hosts could adjust their resource allocation between
82 reproduction and survival following infection ([Hochberg et al., 1992](#); [Hurd, 2001](#); [Gandon et al.,](#)
83 [2002](#); [Bonds, 2006](#); [Leventhal et al., 2014](#)). [Budischak and Cressler \(2018\)](#) considered models of
84 sterility vs mortality tolerance in a resource-dependent context, and some experiments have inves-
85 tigated the association between these two components of tolerance ([Pagán et al., 2008](#); [Pagán and](#)
86 [García-Arenal, 2018](#); [Montes et al., 2020](#)). Another study by [Pike et al. \(2019\)](#) found population-
87 level mortality to be negatively correlated with an investment in fecundity following staph exposure,
88 thus suggesting a fecundity-mortality trade-off in the wild type *N2* strains of *C. elegans* that were
89 exposed to *S. aureus*. While these correlations between fecundity and mortality of infected hosts
90 have been found in various contexts, the balance of host strategies of tolerance to parasite implica-
91 tions on either of these traits (i.e, correlations between mortality tolerance and sterility tolerance)
92 are lacking. As such, experimental evidence of a direct trade-off between both tolerance forms as
93 two arms of defense is rare. Here we model a host-parasite evolutionary scenario where the host
94 obeys such a trade-off and aim to provide useful insights for future empirical investigations.

95 Theoretical models have examined the evolution of tolerance to parasite-induced mortality and
96 sterility as independent adaptive traits, but with an assumption that evolving these strategies is
97 costly to other host fitness traits ([Restif and Koella, 2003, 2004](#); [Miller et al., 2005](#); [Best et al.,](#)
98 [2008, 2010a, 2017](#); [Vitale and Best, 2019](#)). Therefore, we have no clear predictions of what will
99 happen when these two arms of tolerance are directly traded-off with one another. Forming such a
100 sterility-mortality tolerance trade-off as the basis of this study, we explore the interrelation between
101 epidemiological feedbacks and evolutionary outcomes under this trade-off. Importantly, we demon-
102 strate that the negative feedback created by sterility tolerance on parasite prevalence can lead to
103 polymorphism in mortality tolerance through evolutionary branching for a wide range of trade-offs

104 and parameter values. We also compare disease prevalence patterns with and without evolving host
105 defense strategies.

106 **2 The model**

107 We extend a classic host-parasite SIS model from [Anderson and May \(1981\)](#) by considering a trade-
108 off between tolerance to disease-induced mortality and tolerance to disease-induced reduction in
109 the host's reproduction. We also keep general assumptions such as the density-dependent contact
110 process between susceptible and infected hosts and a well-mixed, homogeneous population of hosts.
111 The population dynamics governing the densities of susceptible hosts X and infected hosts Y is
112 given below:

$$\begin{aligned}\frac{dX}{dt} &= (a - q(X + Y))(X + fY) - \beta XY - bX + \gamma Y, \\ \frac{dY}{dt} &= \beta XY - ((\alpha - \tau) + b + \gamma)Y.\end{aligned}\tag{1}$$

113 Parameters are described in Table (1). All hosts reproduce by rate a and parasite reduce the
114 reproduction of infected hosts by a factor denoted by f , such as high f indicates that infected
115 hosts reproduce more and low f indicates that they reproduce less, with $0 < f < 1$. All hosts
116 die with natural death rate b , and q denotes the impact of crowding on the host birth rate. The
117 disease transmits with a coefficient β , and α is the additional death rate of infected hosts caused
118 by parasitic infection, also known as virulence. Further, γ is the rate at which infected hosts can
119 recover from the infection and move into susceptible state again.

120 In addition to the basic assumptions, we assume that the host evolves tolerance to both: impact
121 of disease on fertility and on additional mortality of infected hosts. Tolerance to disease-induced
122 sterility will be evolved by increasing f , and tolerance to mortality is given by a reduction τ in
123 the infection-induced mortality rate α . More simply put, high f means the host is more tolerant
124 to parasite impact on its reproduction (and that infected hosts reproduce more but cannot evolve
125 compensatory reproduction i.e. $f < 1$), and high τ implies that the host is more tolerant to the

Parameters	Definition	Default value
a	Host birth rate	2.5
b	Host natural death rate	0.05
q	Crowding effect	0.5
α	Disease induced mortality rate (virulence)	2
β	Infection transmission coefficient	1
γ	Recovery rate of infected hosts	0.3
f	Sterility tolerance	varies
τ	Mortality tolerance	varies
$\tau'(f^*)$	trade-off gradient	-1.6122

Table 1: Description of parameters

126 deaths caused by the parasite (i.e., reduced mortality). τ and f are further related by a trade-off
127 function which is defined in a later subsection. We choose our parameters such that the parasite
128 persists in the system ($R_0 = \frac{\beta(a-b)}{q(\alpha+b+\gamma-\tau)} > 1$) at an endemic equilibrium (X^*, Y^*) , where

$$X^* = \frac{\alpha + b + \gamma - \tau}{\beta},$$

$$Y^* = \frac{-(1+f)q\beta(b + \alpha + \gamma - \tau) + \beta^2(-b + af - \alpha + \tau) + \sqrt{A}}{2fq\beta^2},$$

129 and $A = \beta^2(-4fq(-a\beta + b(q + \beta) + q(\alpha + \gamma - \tau))(b + \alpha + \gamma - \tau) + (b(q + fq + \beta) + (1 + f)q(\alpha +$
130 $\gamma - \tau) - \beta(af - \alpha + \tau))^2$.

131 2.1 Adaptive dynamics

132 We use the adaptive dynamics framework (Metz et al., 1995; Geritz et al., 1997, 1998) to model the
133 evolution of two forms of tolerance as defense strategies against parasitism. This method involves
134 assuming small, rare mutations occurring in order to invade the resident host at its set environment
135 (equilibrium). A mutant strain with strategy $(f_m, \tau(f_m)) = (f_m, \tau_m)$ tries to invade the resident
136 equilibrium strategy $(f, \tau(f))$, and can achieve so if its fitness (long-term exponential growth rate of
137 the mutant) is positive in the given environment. Here we use the expression for a fitness proxy that
138 has been proved to be sign-equivalent to that of the mutant's growth rate or fitness by Hoyle et al.
139 (2012). The formula for fitness proxy is calculated using the method described in the appendix of

140 [Best et al. \(2017\)](#) and is given by

$$s(f, f_m) = (\alpha + b + \gamma - \tau_m)(a - q(X^* + Y^*) - b - \beta Y^*) + \beta Y^*(af_m - qf_m(X^* + Y^*) + \gamma), \quad (2)$$

141 where X^* and Y^* are the susceptible and infected equilibrium densities, respectively, and are the
142 functions of f . To achieve stable investment in tolerance strategies, we look for singular strategies;
143 the points where the derivative of the fitness expression with respect to the mutant strategy also
144 known as the fitness gradient ($\left. \frac{\partial s}{\partial f_m} \right|_{f_m=f=f^*}$) is zero. These are the potential points where evo-
145 lution of a trait stops, potentially temporarily ([Metz et al., 1995](#); [Geritz et al., 1998](#)). Then two
146 stability conditions obtained from the second order derivatives of the fitness gradient determine the
147 evolutionary outcome of evolving strategies. First is the evolutionary stability (ES) that requires
148 $\left. \frac{\partial^2 s}{\partial f_m^2} \right|_{f=f_m=f^*} < 0$, and second is convergence stability (CS) that must have $\left. \frac{\partial^2 s}{\partial f_m^2} + \frac{\partial^2 s}{\partial f \partial f_m} \right|_{f=f_m=f^*} < 0$.
149 Evolutionary stability concerns whether further mutations can invade a strategy and convergence
150 stability concerns if the strategy is evolutionary attracting. If a singularity is both evolutionary and
151 convergent stable, it is called a continuously stable strategy (CSS) and is the endpoint of evolution
152 ([Eshel, 1983](#)). On the other hand, a singular strategy that is attracting but can be invaded by a
153 nearby mutant, i.e., has convergence stability but not evolutionary stability, leads to evolutionary
154 branching. This means that the population evolves towards singularity, but when nearby, branches
155 into two distinct strains. A singularity that is neither evolutionary stable nor convergent stable is
156 referred to as a repeller ([Metz et al., 1995](#); [Geritz et al., 1997, 1998](#)).

157 **2.2 Sterility tolerance-mortality tolerance trade-off**

158 Trade-offs have been widely used to predict the evolutionary behaviour of ecological systems ([Bowers](#)
159 [et al., 2005](#)). So whether a singular strategy is a CSS, branching point, or a repeller can be
160 determined by the trade-off shape. Fixing the singularity at a point, we can choose the trade-off
161 curvature to get the relevant evolutionary behaviour.

162 Here we assume a trade-off function that links two forms of tolerance as different defense strate-
163 gies such that the benefit from an increased investment in either of the tolerance strategy comes

164 at the cost of a reduced investment in another one. So, if the host increases its reproduction by
 165 increasing tolerance to parasite-induced sterility (f), tolerance to mortality viz. $\tau(f)$ will decrease
 166 (or mortality will increase), and the converse holds as well. The trade-off function is of the same
 167 form as used in previous literature (Hoyle et al., 2012; Vitale and Best, 2019), and is given by

$$\tau(f) = \tau^* - \frac{\tau'(f^*)^2}{\tau''(f^*)} \left(1 - e^{\frac{\tau''(f^*)(f-f^*)}{\tau'(f^*)}} \right). \quad (3)$$

168 Here, $\tau^* = \tau(f^*)$, $\tau'(f^*) = \frac{\partial \tau}{\partial f}|_{f=f^*}$, $\tau''(f^*) = \frac{\partial^2 \tau}{\partial f^2}|_{f=f^*}$, and (f^*, τ^*) is the singular strategy.
 169 Assuming that (f^*, τ^*) is fixed at $(0.5, 1)$, we can chose the slope $\tau'(f^*)$, and curvature $\tau''(f^*)$ of
 170 the trade-off curve such that the chosen strategy is a continuously stable strategy (CSS), using the
 171 conditions of ES and CS. So $\tau'(f^*)$ is calculated such that f^* is a singular strategy i.e., fitness
 172 gradient is zero at f^* and value of $\tau''(f^*)$ is chosen such that f^* is a CSS. We use this trade-off
 173 function to observe the variation in singular points and to show different evolutionary outcomes,
 174 depending on its curvature.

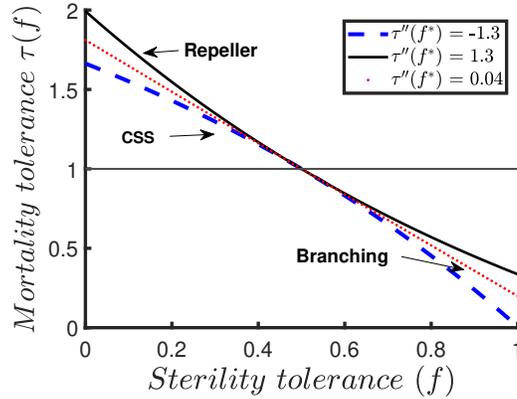


Figure 1: *Examples of trade-off curves that lead to different evolutionary outcomes corresponding to different curvature values, and gradient $\tau'(f^*) = -1.6122$. As such, $\tau''(f^*) = -1.3$ gives a CSS, $\tau''(f^*) = 1.3$ gives a repeller and $\tau''(f^*) = 0.04$ leads to evolutionary branching. The thin black line passing through 1 is the value of constant mortality tolerance at $f^* = 0.5$.*

175 In Fig. 1, we indicate three different trade-off shapes, that lead to distinct evolutionary outcomes
 176 for when the singular point is fixed at $(f^*, \tau^*) = (0.5, 1)$. If the trade-off is a concave-shaped function

177 such as the dashed line in Fig. 1 (i.e., when investment in sterility tolerance becomes increasingly
 178 costly), the singularity will be a CSS: host population evolves towards this point and does not
 179 change with further mutations. On the other hand, evolutionary branching occurs for a range of
 180 slightly convex or weakly decelerating trade-off shapes (close to the dotted line). We also found the
 181 occurrence of branching for nearly linear trade-off shapes. Furthermore, for a strongly decelerating
 182 trade-off such as the dark line in Fig. 1, the singularity will be a repeller: population evolves to
 183 either maximum or minimum investments in both arms of defense.

184 3 Results

185 3.1 Branching

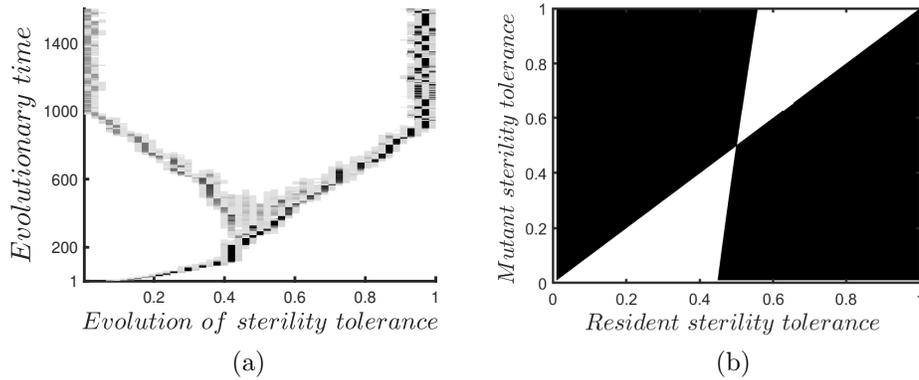


Figure 2: (a) Simulation output for the evolution of sterility tolerance when directly traded-off with mortality tolerance. The relative darkness of shading represents the relative susceptible population densities of the host. (b) Corresponding PIP plot with resident strategy on the x -axis and mutant's strategy on y -axis. Shaded part indicates the probable invasion regions of the invading species (mutant host). $\tau''(f^*) = 0.04$, for strategy $(f^*, \tau^*) = (0.5, 1)$, and remaining parameters are same as in table 1.

186 We used the numerical simulation technique from Hoyle et al. (2012) to demonstrate the oc-
 187 currence of stable dimorphic strains (branching) in our model system (Fig. 2a). We found that for
 188 weekly decelerating trade-offs, evolutionary branching in the two tolerance mechanisms can occur
 189 for a wide range of parameters (Fig. 3). This means that the host strains with maximum and min-

190 imum sterility tolerance can coexist in the population. An initially monomorphic host population
 191 evolves towards the branching point but when close to it, branches into two sub-populations or
 192 strains with distinct tolerance levels. One of the strains has minimal sterility tolerance and high
 193 mortality tolerance, whereas the other has maximum sterility tolerance but low mortality tolerance
 194 (extreme dimorphic strains).

195 Given alongside is the pairwise invadability plot (PIP) (see Metz et al. (1995); Geritz et al. (1997)
 196 for details on the construction of PIP) in which the black region indicates where the mutant can
 197 invade the resident host and white region is where the invasion is impossible (Fig. 2b). The point of
 198 intersection is the branching point and strains from either side of this point can invade the resident,
 199 but disruptive selection leads to evolutionary branching. Furthermore, the dark grey shades in
 200 Fig. 2a indicate higher susceptible population densities, and light grey indicates lower population
 201 densities. We observe that the strain with maximum sterility tolerance is darker, i.e., has higher
 202 susceptible densities. This suggests that the sub-population in which infected hosts reproduce fully
 203 but have higher chances of dying is larger than the one in which infected hosts are sterile.

204 Now we explore the parameter range space that allows evolutionary branching. We follow
 205 the method detailed by Kisdi (2006) that involves checking the sign of a quantity M to predict
 206 the mutual invadability of distinct traits (also see Best et al. (2008, 2010a)). The conditions of
 207 evolutionary stability (ES), and convergent stability (CS) are analytically expressed as

$$\begin{aligned}
 ES &= \left. \frac{\partial^2 s}{\partial f_m^2} \right|_{f^*} < 0, \\
 CS = ES + M &= \left. \frac{\partial^2 s}{\partial f_m^2} + \frac{\partial^2 s}{\partial f \partial f_m} \right|_{f^*} < 0.
 \end{aligned}$$

208 So, to get branching at a singular strategy, we need $CS < 0$ but $ES > 0$. At a fixed singular
 209 strategy (f^*, τ^*) , we can write ES and M as functions of the trade-off. In that case, ES will be a
 210 function of the trade-off curvature, but M is not. For a set of parameters at which $M < 0$, we can
 211 always choose an appropriate trade-off curvature that satisfies the required conditions and leads to
 212 branching. The more negative M is, the greater the range of trade-offs that can allow branching to
 213 occur.

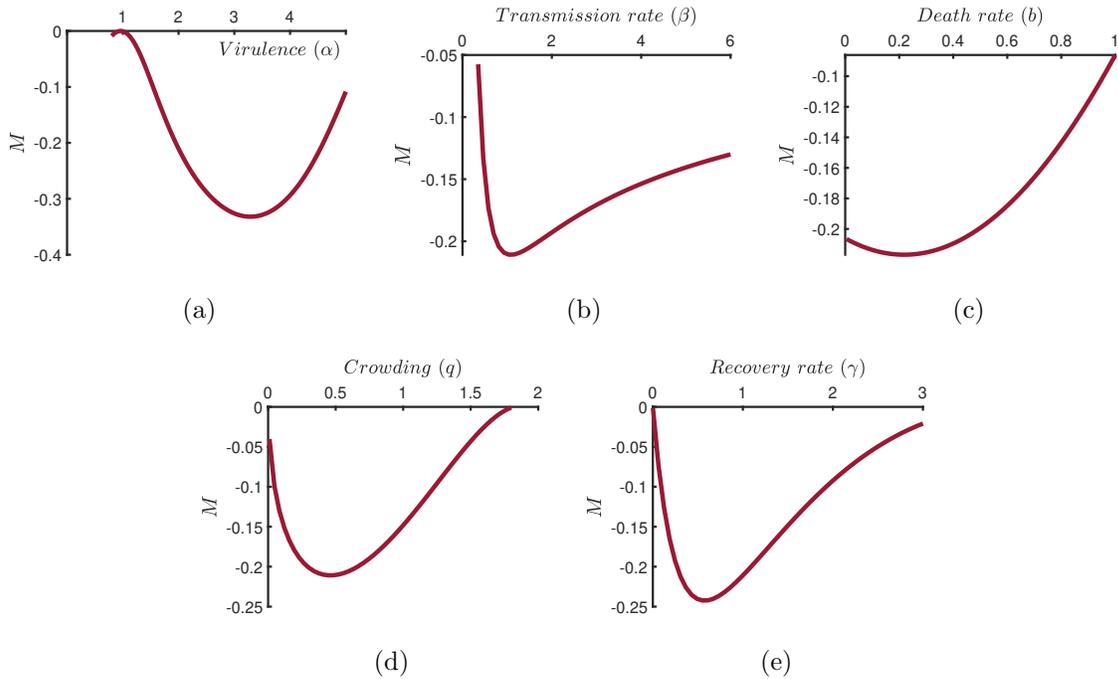


Figure 3: Plots showing the sign of mutual invadability expression (M) corresponding to different parameters. Singular strategy is fixed at $(f^*, \tau^*) = (0.5, 1)$, and remaining parameters are same as in Table 1.

214 To examine the potential of branching under different ecological conditions, we check the sign of
 215 M corresponding to different model parameters (Fig. 3). Here we calculate the trade-off gradient at
 216 each value of the varying parameter such the chosen point $(f^*, \tau^*) = (0.5, 1)$ is singular. We found
 217 that M is negative for a wide range of parameters and attains greater negative values at intermediate
 218 values of the displayed parameters (Fig. 3a-3e). The parameter ranges where a number of trade-off
 219 curvatures exist for which the singular point is CS but not ES (a branching point) coincides with an
 220 intermediate or high average infected population size (low b , low/intermediate q and γ , intermediate
 221 α and intermediate/high β). This suggests that the infected population size (or infection prevalence)
 222 could be a driver of the host variation in sterility and mortality tolerance when linked with such a
 223 trade-off.

224 3.2 Evolution drives CSS and disease prevalence patterns

225 Next, we focus on CSS points to study stable investments in defense mechanisms, i.e., consider the
 226 accelerating trade-off. We then examine the role of evolution in driving the selection of two tolerance
 227 mechanisms by creating feedback on disease prevalence under varying ecological conditions. Initially,
 228 we demonstrate how virulence in the form of additional mortality drives the evolutionary dynamics.
 229 For an accelerating trade-off, we found that the host evolves highest sterility tolerance and lowest
 230 mortality tolerance at intermediate levels of disease-induced mortality rate (Fig. 4a). Initially, as
 231 virulence starts to increase, hosts compensate for the loss due to additional deaths by increasing
 232 reproduction. As long as the virulence is not too high and infected hosts live long enough to
 233 reproduce, the host shall benefit more by increasing fecundity, such that the maximum sterility
 234 tolerance is evolved at intermediate virulence. However, with further increments in virulence, host
 235 lifespan decreases rapidly and chances to reproduce become very low, making sterility tolerance
 236 an inadequate strategy. As such, increasing fecundity is not enough to maintain fitness at high
 237 virulence, and the host has to increase its tolerance to the additional mortality instead.

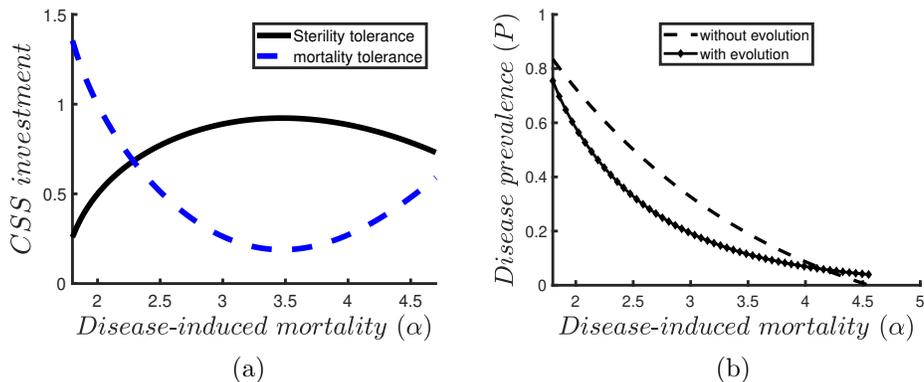


Figure 4: (a) CSS investment variation in sterility and mortality tolerance along with varying virulence α . (b) Disease prevalence plot with evolution (prevalence at corresponding CSS investment) and without evolution (prevalence at equilibrium values of X^* and Y^* for f ranging from 0.01 to 1, and τ taking values as per the trade-off), as α varies. Parameters are same as in Table 1, for $\tau'(f^*) = -1.6122$, $\tau''(f^*) = -1.3$, and strategy $(f^*, \tau(f^*)) = (0.5, 1)$.

238 The host investment in either of the tolerance strategies (CSS) depends significantly upon the
 239 disease prevalence $P = Y^*/(X^* + Y^*)$, where X^* and Y^* denote the susceptible and infected hosts'

240 densities at CSS points. The adjacent plot shows the disease prevalence corresponding to varying
 241 α when there is no evolution (dashed line) and prevalence at corresponding CSS points (solid line)
 242 (Fig. 4b). We found the prevalence to continuously decrease with increasing virulence in both cases,
 243 although the decline is sharper with no evolution. As virulence α starts to increase, the lifespan of
 244 infected hosts $1/(b + \alpha + \gamma - \tau)$ reduces and prevalence drops. Even when mortality turns too high
 245 and mortality tolerance starts to increase again (Fig. 4a), the prevalence continues to fall.

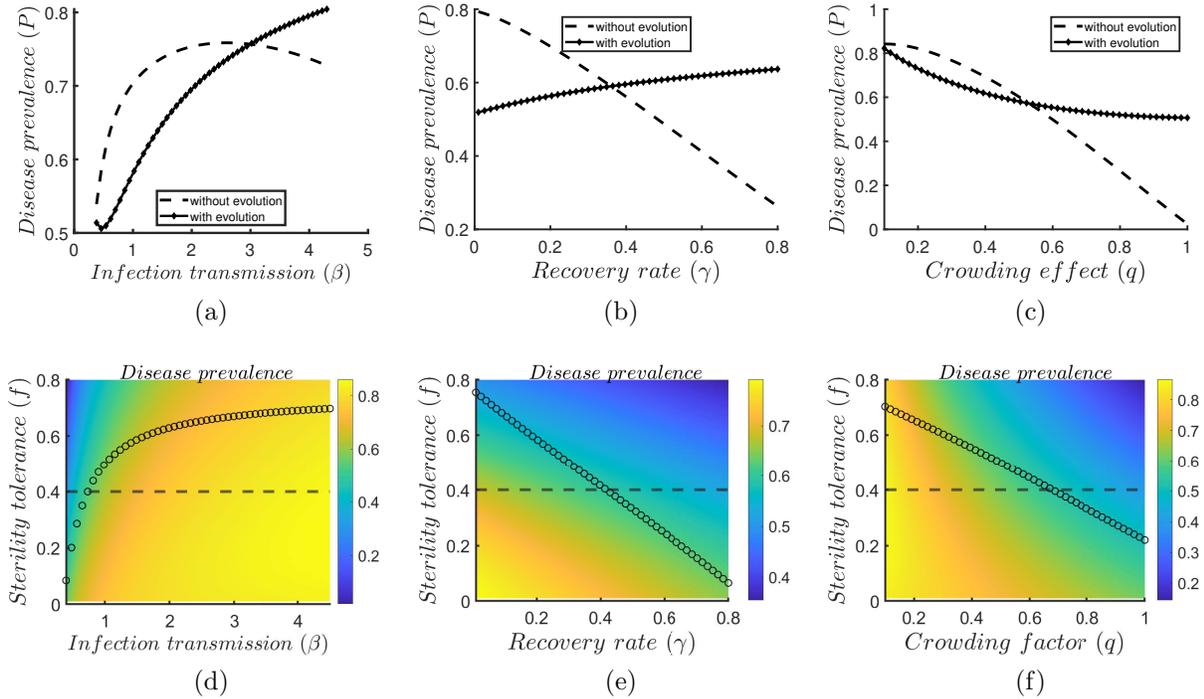


Figure 5: *Patterns displaying how evolution drives the disease prevalence for varying β , γ , and q . The top row shows the difference in prevalence with and without evolution (plots a, b, c). For no evolution case, prevalence P is calculated at the values of f ranging from 0.01 to 1, and for evolution case, P is calculated at the corresponding CSS points. The coloured surfaces show prevalence overlayed with evolving strategies i.e. CSS points (plots d, e, f). Remaining parameters are the same as in table 1, for singular strategy $(f^*, \tau(f^*)) = (0.5, 1)$ and $\tau''(f^*) = -1.3$.*

246 Next, we discuss how evolution affects the patterns of disease prevalence with respect to infection
 247 transmission rate, crowding effect and recovery rate due to the feedback on tolerance investments
 248 (Fig. 5). In the top row, dashed lines represent prevalence when there is no evolution of either

249 form of tolerance and diamond-shaped dots represent prevalence when host evolves sterility and
250 mortality tolerance against parasitic consequences (Fig. 5a-5c). In the second row, we have the
251 coloured surface plots that represent disease prevalence levels through a colour gradient, for when
252 there is no evolution. Plots are overlaid with points (black rings) indicating the CSS investment in
253 sterility tolerance along with respective parameters on the x-axis (Fig. 5d-5f). The dashed horizontal
254 line indicates constant level of investment when there is no evolution. The path followed by dashed
255 line and the black rings can be compared to see how evolution drives the prevalence.

256 For non-evolving strategies, we found that the disease prevalence initially increases with increas-
257 ing transmission rate β , but starts to decline when β goes too high, forming a concave down shape
258 (Fig. 5a). When the host evolves, following a tiny downward bump in the beginning, prevalence
259 continues to increase along with β (Fig. 5a). As transmission increases, more hosts move from
260 susceptible to infected state, thereby increasing the average infected density and hence prevalence.
261 When transmission is too high and no tolerance mechanism evolves, an increased average infected
262 density leads to higher mortality, creating a negative feedback on prevalence. With evolution,
263 however, as transmission increases, the host increases its reproduction which comes at the cost of
264 greater infected mortality. While this additional mortality would push prevalence even lower, the
265 increased reproduction will indirectly lead to a larger infected population, reversing the negative
266 feedback. The corresponding surface plot demonstrates this behaviour of increased reproduction at
267 high transmission, where sterility tolerance is an increasing saturating function of β (Fig. 5d).

268 We further found that evolution completely reverses the disease prevalence pattern correspond-
269 ing to the recovery rate γ . As such, prevalence is a rapid decreasing function of γ without evolution
270 but an increasing function of γ when the host evolves (Fig. 5b). From Fig. 5e, we clearly see that
271 the black dashed line goes from higher to lower prevalence, but evolving strategies denoted by black
272 rings go from lower to higher prevalence. When there is no evolution, increasing recovery rate sim-
273 ply indicates fewer hosts in the infected state, thus lowering the prevalence. When the host evolves,
274 however, we see that increasing recovery leads to a rapid drop in sterility tolerance and hence a rise
275 in mortality tolerance (Fig. 5e). This is driven by high recovery rate leading to less selection for
276 sterility tolerance since hosts can contribute to fitness when they return to susceptible state. The

277 increase in mortality tolerance outweighs the increase in recovery to lead to higher prevalence.

278 Finally, we found that prevalence rapidly decreases with increasing crowding when there is
279 no evolution, but it is a slowly decreasing saturating function of crowding when strategies evolve
280 (Fig. 5c). From the corresponding surface plot, we observe that sterility tolerance is a rapidly
281 decreasing function of q (Fig. 5f). It is understood that increasing crowding acts on net births
282 and reduces overall infected density (varying q only affects the equilibrium density of infected hosts
283 Y^* as X^* is free of q), thus lowering the prevalence. Reduced overall reproduction due to smaller
284 infected population size leads to lower sterility tolerance when competition is high. To maintain
285 the fitness, mortality tolerance increases, thus slowing down the reduction in prevalence (evolution
286 case, Fig. 5c).

287 4 Discussion

288 The fitness costs of parasites on their hosts can generally be classified into reduced fecundity or
289 mortality of hosts. Here we studied a host attacked by a parasite that adversely affects both
290 fecundity and mortality, and we assume that the host can respond by evolving tolerance to both
291 forms of parasitic impact but is subject to a sterility-mortality tolerance trade-off. Using this
292 modelling framework, we identify the following key results: (i) stable dimorphism can arise for a
293 weakly decelerating trade-off at which the most fecund and sterile host strains can coexist in the
294 population; (ii) a wider range of trade-off shapes can lead to branching for parameters corresponding
295 to intermediate/high infected population sizes; (iii) the host evolves maximum sterility tolerance
296 and minimum mortality tolerance at intermediate virulence; and (iv) evolution changes patterns
297 of disease prevalence creating a feedback on the CSS investments, where the prevalence pattern
298 corresponding to recovery rate completely reverses.

299 Existing theory predicts that due to positive frequency dependence and positive impact on par-
300 asite fitness, the mechanisms of tolerance do not lead to polymorphisms or evolutionary branching
301 in standard models (Roy and Kirchner, 2000; Miller et al., 2005) (but see Ferris and Best (2019)
302 when there is seasonality). However, only tolerance to mortality has a positive impact on parasite

303 prevalence, whereas tolerance to sterility is either neutral or costly to the parasite and thus could
304 lead to polymorphic strains (Best et al., 2008, 2010a). In this study, we have shown the existence
305 of dimorphism through evolutionary branching for a direct trade-off between two arms of tolerance,
306 with no additional cost to any other host life-history trait. Note that the branching in mortality
307 tolerance follows from the branching in traits conferring sterility tolerance due to the trade-off. So,
308 in one of the existing strains, infected hosts cannot reproduce ($f = 0$), but they are most protected
309 against infection-induced mortality and are least likely to die. In contrast, the infected hosts in an-
310 other strain reproduce fully ($f = 1$) but are more prone to death due to infection. This supports the
311 theoretical predictions of Antonovics and Thrall (1994) and Bowers et al. (1994) that dimorphism
312 can only occur in two dissimilar strains. Evidence of polymorphisms in both forms of tolerance is
313 widely available in the plant-pathogen literature. Populations of *Arabidopsis thaliana* infected by
314 *Cucumber mosaic virus* (CMV) displayed large genetic variation in sterility tolerance (tolerance to
315 the effects on host progeny production) between and within-host populations (Pagán et al., 2007,
316 2008; Montes et al., 2019), whereas Montes et al. (2020) detected polymorphism in both mortality
317 and fecundity tolerance. Koskela et al. (2002) also found genetic variation in sterility tolerance
318 of *Urtica dioica* to *Cuscuta europea* measured in terms of reproductive biomass. Further, Vijayan
319 et al. (2017) investigated the evolution of mortality tolerance (as expected time until death after
320 infection) in *A. thaliana* and *Brassica juncea* to *Turnip mosaic virus* (TMV) and found genetic
321 variation in this tolerance trait among host species. A recent study also suggested the possibility
322 of variation in different tolerance strategies between unprotected (hosts exposed to food bacterium)
323 or protected (hosts exposed to food bacterium plus *E. faecalis*) treatments of hosts (Rafaluk-Mohr
324 et al., 2022). While numerous experimental studies have demonstrated within-population variation
325 in host tolerance, few theoretical studies have ever demonstrated the evolution of polymorphism in
326 tolerance.

327 We further found that branching can occur for a substantial region of parameter space, but a
328 wider range of trade-off shapes leading to branching exists at parameters corresponding to inter-
329 mediate or high infected population size. In previous work, Best et al. (2010a) discovered that a
330 broader range of trade-off shapes could lead to branching in sterility tolerance at low intrinsic death

331 rates (indicating high infected population density). Furthermore, Ferris and Best (2019) had similar
332 findings for the host mortality tolerance in a seasonal environment with infected fecundity added
333 to their model, suggesting that parasites that temporarily sterilise their hosts are more likely to
334 promote diversity. They concluded that branching in host tolerance is more likely in a fluctuating
335 environment with a high average infected population size. Since evidence of tolerance mechanisms
336 leading to branching are rare (Best et al., 2010a; Ferris and Best, 2019), possibilities of branching
337 have been mostly discussed in models of host resistance evolution (Boots and Haraguchi, 1999; Hoyle
338 et al., 2012; Toor and Best, 2015; Best et al., 2017). For example, for both avoidance and clearance
339 models, Best et al. (2017) predicted that when parasite-induced sterility is not too low, a number
340 of trade-off curvatures allowed branching and that the potential for branching decreased with in-
341 creasing fecundity of infected hosts. The relationship between branching and infected population
342 density has been demonstrated experimentally by Blanchet et al. (2010), where they found higher
343 variation in host tolerance in a wild dace population with a high parasite burden. Our findings are
344 consistent with the trend of higher chances of diversity at high infected population sizes, suggesting
345 that branching in any host tolerance strategy requires high infection prevalence.

346 The allocation to different tolerance mechanisms of the host depends upon the cost and how
347 virulent/deadly the parasite is. For instance, when resistance (via reduced transmission) is traded-
348 off with mortality tolerance, hosts infected with low virulent parasites experience selection for
349 greater mortality tolerance than those infected by highly virulent parasites (Singh and Best, 2021).
350 Similarly, mortality tolerance evolves in an experimental system with a ‘protected’ treatment in
351 which virulence is low, whereas fecundity tolerance evolves in an unprotected treatment in which
352 virulence is high (Rafaluk-Mohr et al., 2022). We observed that the host initially shows similar
353 behaviour in our model, but then the pattern reverses, thus evolving maximum sterility tolerance
354 and minimum mortality tolerance at intermediate virulence. Best et al. (2010a) had a similar finding
355 where they considered increased fecundity comes at the cost of an increased natural death rate and
356 obtained maximum sterility tolerance at intermediate levels of virulence. On the other hand, we
357 found prevalence to continuously decrease with increasing α , suggesting lower infected equilibrium
358 densities at high virulence, in alignment with the theoretical findings of Miller (2006) and Miller

359 [et al. \(2007\)](#). However, a study by [Thrall et al. \(1998\)](#) on the evolution of sexual and non-sexual
360 transmission modes identified that for a fixed level of sterility, population densities are minimized
361 at intermediate levels of virulence. This suggests that further work is needed to understand the
362 different impacts of tolerance on disease prevalence under different biological conditions.

363 In combination with genetic constraints, epidemiological feedbacks can produce a wide range
364 of potential evolutionary outcomes. Here we identify that increased intra-host crowding leads to
365 monotonically decreasing pattern for both disease prevalence and sterility tolerance. So, a host with
366 high carrying capacity will be more tolerant to the parasitic effects on fertility. This is analogous
367 to the results of [Donnelly et al. \(2015\)](#) that infected density and prevalence decrease monotonically
368 with increasing crowding. [Krist \(2001\)](#) found that if parasite castration diminishes the density of
369 snails in highly prevalent populations, reduced competition for resources could increase the energy
370 available for reproduction, indicating the selection of high sterility tolerance at low crowding. Other
371 empirical works made a similar inference ([Goulson and Cory, 1995](#); [Reilly and Hajek, 2008](#); [Lindsey
372 et al., 2009](#)). Likewise, theory has typically explored how tolerance varies along gradients of different
373 epidemiological parameters. Transmission rate is one of the most commonly explored gradients, and
374 the investment in sterility or mortality tolerance is predicted to increase with infectivity ([Boots and
375 Bowers, 1999](#); [Miller et al., 2005, 2007](#); [Best et al., 2010a](#)). On the other hand, hosts evolved highest
376 mortality tolerance at a high recovery rate when infected with a sterilising parasite, but at low or
377 intermediate recovery rates when parasite impacts on sterility were low ([Best et al., 2017](#)). Here
378 we found that sterility tolerance is selected in response to high transmission rate and low recovery
379 rate. So when infected with a highly infectious parasite, the host will benefit more by increasing
380 its reproductive efforts. However, quick recovery from the infection will lead to lower selection for
381 sterility tolerance as the infected hosts can reproduce after becoming susceptible again. Additionally,
382 we discovered that the evolutionary trend with varying recovery rate completely reversed compared
383 to when no defence evolved. Therefore, the importance of recovery rate in influencing tolerance
384 selection highlights the need for empirical data sets that explicitly measure recovery rate.

385 A number of parasites have been found to affect both reproduction and survival in their hosts.
386 For example, the bacterium *Pasteuria ramosa* can castrate its host *Daphnia magna*, and also leads to

387 its premature death (Vale and Little, 2012; Jensen et al., 2006). Other examples include parasitic
388 nematode *Trichostrongylus tenuis* in red grouse, fungal infections caused by *Puccinia* spp. in
389 European weeds (Roy and Kirchner, 2000), and bank voles and wood mice infected by the cowpox
390 virus (Feore et al., 1997). On the other hand, there is enough empirical evidence to support the
391 idea that the reproductive abilities of the host can be costly for its survival under infection. For
392 instance, females of mealworm beetle *Tenebrio molitor* infected by the rat tapeworm *Hymenolepis*
393 *diminuta* had reduced reproduction, but their lifespan significantly increased (Hurd, 2001). Despite
394 the empirical evidence (Pike et al., 2019; Montes et al., 2020), few of the theoretical models to date
395 specifically addressed a reproduction-survival (or equivalently, a sterility-mortality) trade-off and
396 those who did (Best et al., 2008, 2010a), considered survival as a host fitness trait and did not
397 recognise the effects of parasite-induced mortality. Our model assumes that the host's response to
398 pathogen's impact on its mortality evolves along with their impact on the fecundity, and trade-off
399 amounts for the distributed allocation of host defense between these two parasitic repercussions.
400 As such, our sterility-mortality tolerance trade-off considers both forms of parasite impacts and is
401 the first trade-off of its kind in theory.

402 Given the limited empirical studies on the evolution of tolerance to both components of infection-
403 induced fitness loss -sterility and mortality, theory can provide excellent insights for future empirical
404 research and a better understanding of their implications. Here we stressed how epidemiological
405 feedbacks drive the evolution of two linked tolerance mechanisms and discovered that polymorphism
406 could occur in the traits of mortality and sterility tolerance for a wide range of trade-offs and
407 parameter values. We would encourage the development of experimental testing concerning such
408 trade-offs in real systems, particularly where high within-population variation has been found. We
409 also highlighted the need for studies on the impact of recovery rate on tolerance investments, which
410 could play a crucial role in host evolution but are seldom examined in the literature. In real-
411 life systems, the long-term behaviour of host-parasite interactions is directly linked to the interplay
412 between host and parasite evolutionary characteristics, i.e., the coevolutionary dynamics. Therefore,
413 the future work may incorporate the coevolution of host and parasite for a similar trade-off function
414 or when sterility and mortality tolerance evolves together but with costs to other host-life history

415 traits.

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