A Unified Moment-Based Approach for the Evaluation of the Outage Probability with Noise and Interference

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Abstract—**In this paper, we develop a novel moment-based approach for the evaluation of the outage probability (OP) in a generalized fading environment with interference and noise. Our method is based on the derivation of a power series expansion of the OP of the signal-to-interference-plus-noise ratio (SINR). It does not necessitate stringent requirements, the only major ones being the existence of a power series expansion of the cumulative distribution function of the desired user power and the knowledge of the cross-moments of the interferers' powers. The newly derived formula is shown to be applicable for most of the well-practical fading models of the desired user under some assumptions on the parameters of the powers' distributions. A further advantage of our method is that it is applicable irrespective of the nature of the fading models of the interfering powers, the only requirement being the perfect knowledge of their cross-moments. In order to illustrate the wide scope of applicability of our technique, we present a convergence study of the provided formula for the Generalized Gamma and the Rice fading models. Moreover, we show that our analysis has direct bearing on recent multi-channel applications using selection diversity techniques. Finally, we assess by simulations the accuracy of the proposed formula for various fading environments.**

Index Terms—**Moment-based approach, outage probability, generalized fading environment, interference, power series expansion, cross-moments, convergence study, selection diversity.**

I. Introduction

Co-channel interference is among the most limiting factors that negatively affect wireless communication systems' performances. Understanding its impact on the outage probability (OP) is a crucial question that has received a considerable interest over the two last decades. In this respect, several works have addressed the evaluation of the OP in the presence of co-channel interference [1]–[12] from strikingly different view angles. A first group of these works have targeted to develop closed-form expressions for the OP under specific fading models. This approach has in particular been pursued by [1], wherein a probability density function (PDF)-based approach was employed to obtain closed-form expressions of the OP for systems experiencing Nakagami Fading. The same approach has been pursued in [2] which considered the derivation of OP expressions in $\eta-\mu/\eta-\mu$, $\eta-\mu/\kappa-\mu$, and $\kappa-\mu/\eta-\mu$ scenarios with the assumptions that the parameter μ of the $\eta-\mu$ distribution is integer and the co-channel interference are independent. Towards the same goal, methods based on the derivation of the characteristic function have been applied, leading to OP expressions for Nakagami and Rician environments [3], [4]. In the same spirit, composite fading models have recently been considered and various statistics of the signal-to-interference-plus-noise ratio (SINR) have been derived for systems in which the desired signal and co-channel interfering signals follow Rayleigh and K distributions, respectively [5]. Despite considering different fading models, all the aforementioned works consider specific scenarios in order to make the derivations of the OP tractable. In order to handle more complex scenarios, other techniques based on approximation methods [13]–[15] and efficient simulation approaches [16] have been proposed. However, these works are still only applicable for a set of specific scenarios. In order to develop a more unified approach, the authors in [6] have derived a general approach yielding an integral expression of the OP whose integrand is function of the moment generating functions (MGFs) of both desired and interfering powers. This, however, can be quite demanding in practice, requiring one to employ a numerical method for the efficient computation of the integral and to know in closed-form the MGF of both desired and interfering powers. The last requirement does not always hold, being for instance unsatisfied for the Log-normal and the Weibull variates.

Driven by the same motivation of [6], that is to evaluate the OP for generalized fading environments, we propose in the present work to approach this problem using a moment-based method. The advantage of our method is that it avoids the need for an MGF while applying to a wide scope of scenarios. The only two ingredients which are required are $i)$ an infinite power series expansion of the cumulative distribution function (CDF) of the desired user power and $ii)$ expressions for the moments (eventually the cross-moments in the case of correlated interferers) of the co-channel interferers' powers. With these two requirements at hand, the main result is the development of a power series expansion of the OP of the SINR. On the conceptual side, our work is in the same spirit of the work in [17], which has proposed a moment approach based on the Laguerre polynomials to derive a closed-form expression of the PDF, CDF and MGF of a sum of independent random variables (RVs). Our work, however,

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differs from [17] in that it takes into consideration the effect of the interference and additive Gaussian noise. Such a setting cannot be handled by the same techniques of [17]. To the best of the authors' knowledge, our technique can be considered as the first moment based-approach that allows to compute the OP in the presence of co-channel interference and noise while applying to generalized fading environments. Its main features can be summarized as:

- The requirement of having a power series expansion of the CDF of the desired user power does not introduce a serious limitation since it is satisfied by most of the well-practical fading variates under some conditions on the fading parameters. A non exhaustive list of these fading variates includes for instance the Nakagami-m with integer fading severity, the Weibull with integer shape parameters, the Generalized Gamma with integer shape parameters, and the Rice with arbitrary fading parameters.
- The proposed method is applicable irrespective of the fading model of the co-channel interferers, being only based on the knowledge of the moments (or the cross-moments in the correlated case) of the co-channel interferers' powers. It is worth noting that such a requirement is generally satisfied, which supports the unified aspect of the proposed approach. This has to be compared with previous unified approaches, which necessitate restrictive requirements, such as the MGF in the work of [6]. Note that the previous requirement is out of reach for many fading models, especially when the interferences are correlated or a composite fading model is considered. Another benefit of our approach is that it does not include an integral and hence avoids the need for numerical integration methods such as in [6], [7].
- Lastly, it is worth highlighting that our approach yields new OP formulas, under very interesting fading models that have not been studied before in the literature. This is for instance the case when the desired user signal as well as the interfering signals are affected by the Rician fading model with arbitrary parameters and independently of whether the interfering signals are correlated or not. The closest setting to this concerns are the ones studied in [4] and [11] which assume independent co-channel interference with equal average powers.

Having the newly derived formula at hand, it is worth mentioning that the provided OP expression is valid for all threshold values satisfying the convergence of the derived power series expansion. In order to analyze the range of validity of our method, we perform a convergence analysis of the newly derived infinite power series expression when the desired user signal and the interfering signals experience two well-practical fading models, namely the Rician and the Generalized Gamma fading models. More precisely, we formulate sufficient conditions on the parameters of the desired and the interfering powers distributions that guarantee the convergence of the newly derived power series expansion, and hence validate the proposed moment-based approach. As will be shown by simulations, these conditions do not introduce stringent restrictions, being almost valid in the operating range of OPs. Moreover, we show that the scope of applicability of our technique goes beyond single-input-single-output (SISO) systems. As a matter of fact, the same considered model arise in many emerging applications of wireless communications. For the sake of illustration, we show how our results can be applied to approximate the outage probability in single-input-multiple-output (SIMO) using selection diversity (SD) techniques.

The rest of the paper is organized as follows. In section II, the main result establishing the novel moment-based closed-form expression of the OP is given. In Section III, a convergence study of the proposed power series expansion is provided under Rician and Generalized Gamma fading environments. Then, we illustrate in Section IV how our analysis can be used to study the OP in multi-channel receivers using SD techniques. Finally, some selected simulation results are provided in Section V in order to validate the newly derived analytical expression.

II. Moment-Based Approach

In many wireless communication systems, such as cellular systems with co-channel interference, the instantaneous SINR at the desired receiver is expressed as follows:

$$\text{SINR} = \frac{X_0}{\sum_{i=1}^{N} X_i + \eta}, \tag{1}$$

where η is the variance of the additive white Gaussian noise, X_0 is the received power of the useful signal, and $X_1, X_2, ..., X_N$ are the powers of the interfering signals which are allowed to be correlated but assumed independent of X_0. Several metrics have been introduced to assess the performance of wireless communication systems, among which the OP is among the most frequently used in practice. It is defined as the probability that the SINR falls bellow a certain threshold γ_{th}, that is:

$$P_{out}(\gamma_{th}) = P\left(\text{SINR} < \gamma_{th}\right). \tag{2}$$

The aim of the present work is to derive a closed-form expression of the quantity $P_{out}(\gamma_{th})$ under many well-known fading models that are experienced by the desired user signal. Basically, the provided OP expression is built using two ingredients, which are the cross-moments of the interfering powers $\mathbb{E}\left[X_1^{i_1}...X_N^{i_N}\right]$, where $i_1, i_2, ..., i_N$ are arbitrary positive integers, and a power series expansion of the CDF of the desired user power. We will see later that requiring these two ingredients does not introduce in general a serious limitation, since they hold for most of the well-used fading variates.

It is important to stress the fact that while the system model described by (1) has extensively been investigated in the past within the framework of cellular communication systems, it is still triggering the interest, appearing in many emerging applications. Visible light communications [18]–[20] and SIMO systems using SD techniques at the receiver [5] are among the most illustrative representatives. More details regarding the SIMO application will be provided in section IV.

We are now in position to state our main result.

Theorem 1. *Suppose that the CDF of X_0 can be expanded as an infinite sum as follows:*

$$F_{X_0}(x) = \sum_{k=0}^{\infty} a_k x^{b_k}, \quad x \geq 0, \tag{3}$$

where $\{a_k\}_{k\in\mathbb{N}}$ is a sequence of real numbers and $\{b_k\}_{k\in\mathbb{N}}$ is a positive, integer, and strictly increasing sequence. Then, provided that:

$$\sum_{k=0}^{\infty} |a_k| \gamma_{th}^{b_k} \mathbb{E}\left[\left(\sum_{i=1}^{N} X_i + \eta\right)^{b_k}\right] < \infty, \tag{4}$$

the probability $P_{out}(\gamma_{th})$ is given by:

$$P_{out}(\gamma_{th}) = \sum_{k=0}^{\infty} a_k \gamma_{th}^{b_k} \sum_{i_1+\ldots+i_{N+1}=b_k} \binom{b_k}{i_1, \ldots, i_{N+1}} \eta^{i_{N+1}} m_{i_1,i_2,\ldots,i_N}, \tag{5}$$

with $m_{i_1,i_2,\ldots,i_N} = \mathbb{E}\left[\prod_{j=1}^{N} X_j^{i_j}\right]$ is the cross-moment of the random vector $(X_1, X_2, \ldots, X_N)^t$ and $\binom{n}{k_1,k_2,\ldots,k_m}$ is the multinomial coefficient defined as:

$$\binom{n}{k_1, k_2, \ldots, k_m} = \frac{n!}{k_1! k_2! \ldots k_m!}. \tag{6}$$

Proof. Let $f(\cdot)$ and $g(\cdot)$ denote respectively the PDF of X_0 and the joint PDF of the random vector $(X_1, X_2, \ldots X_N)^t$. From the independence of the random vector $(X_1, X_2, \ldots X_N)^t$ and X_0, the quantity $P_{out}(\gamma_{th})$ in (2) can be written as:

$$P_{out}(\gamma_{th}) = \int_{\left\{\frac{x_0}{\sum_{i=1}^{N} x_i + \eta} < \gamma_{th}\right\}} f(x_0) g(x_1, \ldots, x_N) dx_0 dx_1 \ldots dx_N. \tag{7}$$

Now, upon conditioning on $X_1, X_2, \ldots, X_N$, we obtain:

$$\begin{aligned} P_{out}(\gamma_{th}) &= \int_{\mathbb{R}_+^N} \left(\int_{\{x_0 < \gamma_{th}(\sum_{i=1}^{N} x_i + \eta)\}} f(x_0) dx_0 \right) \\ &\quad \times g(x_1, \ldots, x_N) dx_1 \ldots dx_N \\ &= \mathbb{E}\left[F_{X_0}\left(\gamma_{th} \left(\sum_{i=1}^{N} X_i + \eta \right) \right) \right], \end{aligned} \tag{8}$$

where the previous expectation operator $\mathbb{E}[\cdot]$ is taken with respect to the random vector $(X_1, X_2, \ldots X_N)^t$ with joint PDF $g(\cdot)$. Now, combining (8) and (3), we get

$$P_{out}(\gamma_{th}) = \mathbb{E}\left[\sum_{k=0}^{\infty} a_k \gamma_{th}^{b_k} \left(\sum_{i=1}^{N} X_i + \eta \right)^{b_k} \right]. \tag{9}$$

We use now the assumption that $b_k \in \mathbb{N}$, for all $k \in \mathbb{N}$. This enable us to leverage the multinomial formula in [21, Prop. 4.10], thus yielding:

$$P_{out}(\gamma_{th}) = \mathbb{E}\left[\sum_{k=0}^{\infty} a_k \gamma_{th}^{b_k} \sum_{i_1+\ldots+i_{N+1}=b_k} \binom{b_k}{i_1, \ldots, i_{N+1}} \eta^{i_{N+1}} \prod_{1\leq j\leq N} X_j^{i_j} \right]. \tag{10}$$

Finally, using the assumption in (4), we can permute the expectation operator with the infinite sum to get:

$$P_{out}(\gamma_{th}) = \sum_{k=0}^{\infty} a_k \gamma_{th}^{b_k} \sum_{i_1+\ldots+i_{N+1}=b_k} \binom{b_k}{i_1, \ldots, i_{N+1}} \eta^{i_{N+1}} m_{i_1,i_2,\ldots,i_N}, \tag{11}$$

which concludes the proof. □

It is worth mentioning that the assumption that the CDF of the desired user power X_0 possesses a power series expansion as in (3) does not entail a serious limitation to the proposed moment-based approach. In fact, this assumption can be shown to be satisfied for most of the well-known fading models up to a mild assumption on the fading parameters. We can cite for instance the Nakagami-m with integer values of m, the Rice with arbitrary parameters, the Generalized Gamma with integer values of p and d, and the $\kappa-\mu$ with integer values of μ (see Table I). Note that the power CDF expansions shown in this table follow easily from the expansions of the lower incomplete Gamma function $\gamma(\cdot,\cdot)$ in [22] and the Marcum Q function $Q_\mu(\cdot,\cdot)$ in [23].

In the case of independent co-channel interferers, the formula in (5) can be further simplified to become only function of the moments of each co-channel interferer's power instead of the cross-moments in the correlated case.

Corollary 1. Consider the setting of Theorem 1 and assume that the RVs $X_1, X_2, \ldots, X_N$ are independent. Then, the expression in (5) becomes:

$$P_{out}(\gamma_{th}) = \sum_{k=0}^{\infty} a_k \gamma_{th}^{b_k} \sum_{i_1+\ldots+i_{N+1}=b_k} \binom{b_k}{i_1, \ldots, i_{N+1}} \eta^{i_{N+1}} \prod_{j=1}^{N} m_{i_j}, \tag{12}$$

with $m_{i_j} = \mathbb{E}\left[X_j^{i_j}\right]$ is the i_j^{th} moment of the RV X_j, $j = 1, 2, \ldots, N$.

Proof. The proof follows immediately from Theorem 1. □

Theorem 1 reveals an interesting feature of the proposed expression in (5) and (12). Contrary to several existing OP expressions, our formulas do not assume any particular fading model of the co-channel interfering signals. The only requirement is the knowledge of the cross-moments for the case of correlated interference (or the moments in the independent case). Such a result supports the unified aspect of the proposed moment-based approach, in that it can be applied to any interferers' fading model provided the availability of a closed-form expression for their cross-moments. To emphasize the

Table I: Power's CDF Expansion for Different Fading Channels [a]

Fading Type	Power PDF	Power CDF Expansion
Nakagami-m	Gamma distribution $\frac{m^m}{\Omega^m \Gamma(m)} x^{m-1} \exp\left(-\frac{m}{\Omega}x\right)$	$\frac{\gamma(m, \frac{mx}{\Omega})}{\Gamma(m)} =$ $\sum_{n=0}^{\infty} \frac{(-1)^n (\frac{m}{\Omega})^{m+n} x^{m+n}}{\Gamma(m) n! (m+n)}$
Rice	Non-centered Chi-squared distribution $\frac{K+1}{\Omega} \exp\left(-K - \frac{K+1}{\Omega}x\right) I_0 \left(2\sqrt{\frac{K(K+1)x}{\Omega}}\right)$	$1 - Q_1(\sqrt{2K}, \sqrt{\frac{2(K+1)x}{\Omega}}) =$ $\sum_{n=0}^{\infty} \frac{(-1)^n \exp(-K) L_n^{(0)}(K)(1+K)^{n+1} x^{n+1}}{(1+n)! \Omega^{n+1}}$
Weibull	Weibull distribution $k \left(\frac{\beta}{\Omega}\right)^k x^{k-1} \exp\left(-\left(\frac{x\beta}{\Omega}\right)^k\right), \quad \beta = \Gamma\left(1 + \frac{1}{k}\right)$	$1 - \exp\left(-(\frac{\beta}{\Omega}x)^k\right) =$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \beta^{kn} x^{kn}}{n! \Omega^{kn}}$
Generalized Gamma	Generalized Gamma distribution $\frac{p(\frac{\beta}{\Omega})^d}{\Gamma(\frac{d}{p})} x^{d-1} \exp\left(-(\frac{\beta}{\Omega}x)^p\right), \ \beta = \frac{\Gamma\left(\frac{d+1}{p}\right)}{\Gamma\left(\frac{d}{p}\right)}$	$\frac{\gamma\left(\frac{d}{p}, \left(\frac{x\beta}{\Omega}\right)^p\right)}{\Gamma\left(\frac{d}{p}\right)} =$ $\sum_{n=0}^{\infty} \frac{(-1)^n (\frac{\beta}{\Omega})^{d+np} x^{d+np}}{n! (\frac{d}{p}+n) \Gamma(\frac{d}{p})}$
$\kappa - \mu$	Squared $\kappa - \mu$ distribution $\frac{\mu(1+\kappa)^{\frac{\mu+1}{2}} x^{\frac{\mu-1}{2}}}{\Omega^{\frac{\mu+1}{2}} \kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa)} \exp(-\frac{(1+\kappa)\mu x}{\Omega}) I_{\mu-1}(2\mu\sqrt{\frac{\kappa(\kappa+1)x}{\Omega}})$	$1 - Q_\mu(\sqrt{2\kappa\mu}, \sqrt{2(\kappa+1)\frac{\mu x}{\Omega}}) =$ $\sum_{n=0}^{\infty} \frac{(-1)^n \exp(-\kappa\mu) L_n^{(\mu-1)}(\kappa\mu)((1+K)\mu)^{n+\mu} x^{n+\mu}}{\Gamma(\mu+m+1)\Omega^{n+\mu}}$

[a]Functions $\Gamma(\cdot)$ and $I_\xi(\cdot)$ are respectively the Gamma function and the modified Bessel function of the first kind and order ξ [22]. $L_n^\alpha(\cdot)$ are the generalized Laguerre polynomials of degree n and order α [23].

Table II: Moments for Different interfering Powers [a]

Fading Type	n^{th} Power's Moment
Nakagami-m	$\frac{\Omega^n}{m^n \Gamma(m)} \Gamma(m+n)$
Rice	$\frac{\Gamma(1+n)}{(1+K)^n} {}_1F_1(-n, 1; -K) \Omega^n$
Weibull	$\frac{\Omega^n}{\beta^n} \Gamma\left(1 + \frac{n}{k}\right)$
Generalized Gamma	$\frac{\Omega^n}{\beta^n} \frac{\Gamma\left(\frac{d+n}{p}\right)}{\Gamma\left(\frac{d}{p}\right)}$
Log-normal	$\exp\left(n\mu + \frac{1}{2}n^2\sigma^2\right)$
Gamma-Gamma	$\frac{\Gamma(K+n)\Gamma(M+n)}{\Gamma(K)\Gamma(M)} \left(\frac{\Omega}{KM}\right)^n$
Nakagami-q (Hoyt)	$\Gamma(1+n) {}_2F_1 \left(-\frac{n-1}{2}, -\frac{n}{2}; 1, \left(\frac{1-q^2}{1+q^2}\right)^2\right) \Omega^n$
Squared $\kappa - \mu$	$\frac{\Omega^n \Gamma(\mu+n) \exp(-\kappa\mu)}{\Gamma(\mu)[\mu(1+\kappa)]} {}_1F_1(\mu+n, \mu; \kappa\mu)$

[a]The moment expressions are in [24] for the Nakagami-m, the Weibull and the Generalized Gamma fading models, in [25] for the Rice, the Log-normal and the Nakagami-q fading variates, in [26] for the Gamma-Gamma model, and in [27] for the $\kappa - \mu$ fading variate. The functions ${}_1F_1(\cdot, \cdot; \cdot)$ and ${}_2F_1(\cdot, \cdot; \cdot, \cdot)$ are respectively the confluent Hypergeometric and the Hypergeometric functions [22].

large scope of applicability of the proposed moment-based approach, we present in Table II the expression of the moments for some of the well-practical variates such as the Rice, the Generalized Gamma, the Log-normal, the Gamma-Gamma, the Nakagami-q fading, and the $\kappa - \mu$ variates.

III. Convergence region

We have seen in the proof of Theorem 1 that the requirement in (4) is a primordial assumption in order for the newly derived expression (5) to be valid. Thus, it is of paramount importance to investigate the convergence of the power series expansion in (4) by providing sufficient conditions that ensure its convergence, and hence guarantee the validity of the OP expression (5). This constitutes the aim of this part. In particular, we characterize the convergence radius R of the power series (4), which, in passing, corresponds to the convergence radius of the proposed power series formula (5), for two important fading models, namely; the Generalized Gamma and the Rician fading environments.

A. Generalized Gamma Fading Model

We consider the case in which X_0 follows a Generalized Gamma distribution $GG(d_0, p_0, \Omega_0)$ whose PDF is given in

Table I. Similarly, the interference powers $X_1, X_2, ..., X_N$ are eventually correlated with marginals $GG(d_i, p_i, \Omega_i)$, $i = 1, 2, ..., N$. The convergence region of the series expansion in (5) is stated in the following theorem:

Theorem 2. *The convergence radius R of (5) when $X_0 \sim GG(d_0, p_0, \Omega_0)$ and $X_1, X_2, ..., X_N$ are correlated with marginals $GG(d_i, p_i, \Omega_i)$, $i = 1, 2, ..., N$, satisfies:* $R = \begin{cases} +\infty & \text{if } \frac{p_0}{p} < 1, \\ 0 & \text{if } \frac{p_0}{p} > 1. \end{cases}$
In the case where $\frac{p_0}{p} = 1$, the convergence radius satisfies:

$$R \geq \frac{a_0}{a(N+1)} \tag{13}$$

where $p = \min_{1 \leq i \leq N} p_i$, and $a = \max_{1 \leq i \leq N; p_i = p} a_i$ with $a_i = \frac{\Omega_i}{\beta_i}$ and $\beta_i = \frac{\Gamma\left(\frac{d_i+1}{p_i}\right)}{\Gamma\left(\frac{d_i}{p_i}\right)}$, $i = 0, 1, ..., N$.

Proof. The proof is in Appendix A. □

From the result of this theorem, we deduce that if $p_0 < p$, the newly derived moment-based formula (5) is valid for all threshold values $\gamma_{th} > 0$. Such a case may be encountered in practice when the desired user undergoes a higher amount of fading than the interfering users. This, for instance, might occur when the desired user lies in the cell edge and as such is affected by many obstacles, while the interfering users are in the inner cell with sufficiently good channel conditions. However, in the case where $p_0 > p$, this expression fails to converge for all values of γ_{th}. It is also important to note that the result in (13) provides a sufficient condition that ensures the convergence of (5) in the case where $p_0 = p$. In fact, our newly derived formula is valid for all values of γ_{th} that are less than $\frac{a_0}{a(N+1)}$ which corresponds to the lower bound in (13). This lower bound can be thought of, up to a constant factor depending on the distribution parameters, as a ratio between the power of the desired user and that of one of the interferers with parameters p and a. As an outcome of this observation, it seems that the validity region of the provided OP expression would be expanded when the power of the desired user increases or the powers of the interferers with parameters p and a decrease.

B. Rice Fading Model

In this section, we consider the case in which both the desired and the interfering signals experience the Rician fading environment. Hence, the power X_0 of the desired user is then a non-centered Chi-squared distribution with a Rice factor K_0 and an average power Ω_0 (the corresponding PDF is in Table I). The interfering power signals $\{X_i\}_{i=1}^N$ are allowed to be correlated, and follow non-centered Chi-squared distributions with Rice factors $\{K_i\}_{i=1}^N$ and average powers $\{\Omega_i\}_{i=1}^N$. Our aim is again to study the convergence radius of the power series (5) (or equivalently that of (4)) under this Rician fading model. The main result characterizing the convergence radius R of the power series formula (5) under the Rice fading environment is stated in the following theorem.

Theorem 3. *Let X_0 be a non centered Chi-squared distribution with parameters K_0 and Ω_0, and $X_1, X_2, ..., X_N$ be correlated RVs following non centered Chi-squared distributions with parameters $\{K_i\}_{i=1}^N$ and $\{\Omega_i\}_{i=1}^N$. Then, the convergence radius R of (5) satisfies:*

$$R \geq \frac{\alpha_0}{\alpha(N+1)} \tag{14}$$

where $\alpha = \max_{1 \leq i \leq N} \alpha_i$ with $\alpha_i = \frac{\Omega_i}{1+K_i}$, $i = 0, 1, ..., N$.

Proof. The proof is in Appendix B. □

In order for the proposed moment-based expression (5) to be valid, Theorem 2 suggests to consider threshold values that are upper bounded by the $\frac{\alpha_0}{\alpha(N+1)}$. We will see in the numerical results, that, for this range of threshold values, the value of $P_{out}(\gamma_{th})$ will cover the range of operating values of OPs. This goes in favor toward the applicability of the proposed moment-based approach.

Note also that the lower bound in the Rician case is more insightful than the one obtained with the Generalized Gamma fading model. In fact, it can be easily seen that this bound is exactly the ratio between the non line of sight (NLOS) power of the desired user divided by $N+1$ times the the maximum NLOS power over all interferers. It represents, as such, a kind of a "worst case" signal-to-interference ratio over the NLOS path. In light of this observation, this quantity seems to be critical for the validity of the provided OP expression. This can be seen by noting that the provided OP expression would be almost valid if the interferers' power over the LOS path is very high.

IV. Extension to SIMO Receivers

In this section, we show how our proposed moment-based approach can be used to derive OP expressions at SIMO receivers when using SD. Particularly, we consider wireless transmissions between a single-antenna transmitter and a M-antennas receiver that are affected by interference signals coming from N mono-antenna sources. Under this setting, it is shown in [5] that the SINR at the n-th receiver antenna writes as:

$$\text{SINR}_n = \frac{X_0^n}{\sum\limits_{i=1}^{N} X_i^n + \eta}, \tag{15}$$

where X_0^n and X_i^n denote respectively the desired and the i^{th} interferer's power received at the n-th branch, $i = 1, 2, ..., N$ and $n = 1, 2, ..., M$. We assume that X_0^n and $(X_1^n, X_2^n, ..., X_N^n)^t$ are independent for each n. We consider the case in which the receiver uses SD. Under this setting, two types of SD can be distinguished, namely SNR and SINR-based SD techniques [5]. In SNR-based SD, the receiver selects the diversity branch with the highest SNR, or equivalently with the highest $\{X_0^n\}, n = 1, \cdots, M$. Under these circumstances, the SINR at the selected branch is given by:

$$\text{SINR}_{SNR-SD} = \frac{X_{0,SD}}{\sum\limits_{i=1}^{N} X_i^{n_0} + \eta}, \tag{16}$$

where $X_{0,SD} = \max_{1\le n\le M} X_0^n$ and n_0 denotes the index of the selected branch. A second kind of SD is known as SINR-based SD. In this case, the receiver selects the branch with the highest SINR. The SINR at the selected branch is thus given by:

$$\mathrm{SINR}_{SINR-SD} = \max_{1\le n\le M} \mathrm{SINR}_n, \quad (17)$$

In the sequel, we will make use of our results in order to derive expressions for the OP of the SINR at the output of each SD technique.

A. SNR-Based SD

For the sake of simplicity, we assume here that the vectors $\mathbf{x}_n = (X_1^n, \cdots, X_N^n)^T$, $n = 1, 2, ..., M$, are identically distributed but not necessarily independent. Moreover, we assume that the desired signals are independent from the interfering signals. This particularly allows us to treat $\sum_{i=1}^N X_i^{n_0}$ as any sum among $\left\{\sum_{i=1}^N X_i^n\right\}_{n=1}^N$ since they have the same distribution. From the expression of SINR_{SNR-SD} and in order to be able to apply the proposed moment-based approach, it suffices to have a power series expansion of the CDF of the RV $X_{0,SD}$. In fact, under the independence of the desired user signals over each diversity branches, the CDF of $X_{0,SD}$ is given by

$$F_{X_{0,SD}}(x) = \prod_{n=1}^M F_{X_0^n}(x). \quad (18)$$

Knowing the power series expansion for the CDF of each $X_0^n, n = 1, \cdots, M$, i.e., the desired user signal's envelop at each diversity branch is distributed according to one of the distributions listed in Table I, one can easily obtain that of $F_{X_{0,SD}}(\cdot)$ by applying recursively the Cauchy product for power series. For the sake of illustration, take $M = 2$ and assume that the CDFs of X_0^1 and X_0^2 have respectively the expansions $\sum_{k=0}^\infty a_k x^k$ and $\sum_{k=0}^\infty a_k^{'} x^k$ which are valid for $x \ge 0$, then the CDF of $X_{0,SD}$ can be expanded as $\sum_{k=0}^\infty c_k x^k$ where $c_k = \sum_{\ell=0}^k a_\ell a_{k-\ell}^{'}$ and $x \ge 0$. With this expansion at hand, we are able to apply Theorem 1. Thus, we get the following OP expression:

$$P_{out,SD}(\gamma_{th}) = \sum_{k=0}^\infty c_k \gamma_{th}^k \sum_{i_1+\ldots+i_{N+1}=k} \binom{k}{i_1, \ldots, i_{N+1}} \eta^{i_{N+1}} m_{i_1,i_2,\ldots,i_N}, \quad (19)$$

The remaining work is to perform, similarly to Section III, a convergence study of the previous power series.

B. SINR-Based SD

We assume here that the instantaneous SINR at each diversity branches are independent, i.e. the RVs $\{\mathrm{SINR}_n\}_{n=1}^M$ are independent. In this case, the OP of the SINR at the selected branch is given by:

$$P_{out,SD}(\gamma_{th}) = \prod_{n=1}^M P_{out,n}(\gamma_{th}), \quad (20)$$

where $P_{out,n}(\gamma_{th})$ is the OP of $SINR_n$, $n = 1, 2, ..., M$. Assume that for each branch the assumptions of Theorem 1 hold. One can thus obtain a power series expansion for each $P_{out,n}(\gamma_{th})$, $n = 1, \cdots, M$. The power series for $P_{out,SD}(\gamma_{th})$ can be thus obtained by using recursively the Cauchy product method. Furthermore, sufficient conditions guaranteeing the convergence of the power series of $P_{out,SD}(\gamma_{th})$ can be directly obtained from the convergence study carried out for SISO systems. To illustrate this point, let us consider for instance the Rician fading model and denote by Ω_0^n and K_0^n the parameters of the desired user power at the n^{th} diversity branch and Ω_i^n and K_i^n the parameters of the i^{th} interference's power arriving at the n^{th} diversity branch, where $n = 1, 2, ..., M$ and $i = 1, 2, ..., N$. From the analysis made in the previous section, it suffices then to choose a threshold value that is upper bounded by $\min_{1\le n\le M} \frac{\alpha_0^n}{\alpha^n(N+1)}$ where α_0^n and α^n are given, similarly to the ones defined in Theorem 3, by $\alpha^n = \max_{1\le i\le N} \alpha_i^n$ with $\alpha_i^n = \frac{\Omega_i^n}{1+K_i^n}$, $i = 0, 1, ..., N$.

In summary, all this shows that our technique is not restricted to the simple case of SISO systems. Indeed, it can be used to efficiently handle more involved scenarios which are not covered by the existing classical methods.

V. Numerical Results

Some selected simulation results are performed in this part in order to validate the analytical expression derived in (12) corresponding to the particular case where the co-channel interferers experience independent fading channels. In our simulation results, the newly derived power series (12) is approximated using the corresponding truncated series as follows:

$$P_{out}(\gamma_{th}) \approx \sum_{k=0}^{K_{tr}} a_k \gamma_{th}^{b_k} \sum_{i_1+\ldots+i_{N+1}=b_k} \binom{b_k}{i_1, \ldots, i_{N+1}} \eta^{i_{N+1}} \prod_{j=1}^N m_{i_j}, \quad (21)$$

where K_{tr} denotes the order of truncation. Two interesting scenarios will be considered in our simulation results corresponding to two different choices of the fading models: the Rice and the Generalized Gamma fading models. The reason behind such choices is that the Generalized Gamma model includes such many interesting fading models as Nakagami-m, Rayleigh, and Weibull fading variates. Furthermore, the Rician fading environment has not been studied for arbitrary values of Rice factors and average powers, being only studied, to the best of the authors' knowledge, when the interferences' average powers are all equal [4], [11].

In each scenario, we perform two experiments. In the first experiment and for a fixed order of truncation, we validate the newly derived expression (12) by studying the accuracy of (21) as a function of the threshold γ_{th} for different values of fading parameters and different numbers of co-channel interferers. The accuracy of this expression is then studied in the second experiment for different values of K_{tr} and again as a function of γ_{th}.

It is important to point out that the maximum value of the threshold γ_{th}, used in the following simulation results, corresponds the lower bound of the convergence radius R, i.e. $\frac{\alpha_0}{\alpha(N+1)}$ in (14) and $\frac{a_0}{a(N+1)}$ in (13) for respectively the Rice and the Generalized Gamma fading environments. Thus, following the convergence study of the previous section, the convergence of (12) is guaranteed in the considered range of the threshold γ_{th} and hence the proposed moment-based approach is valid.

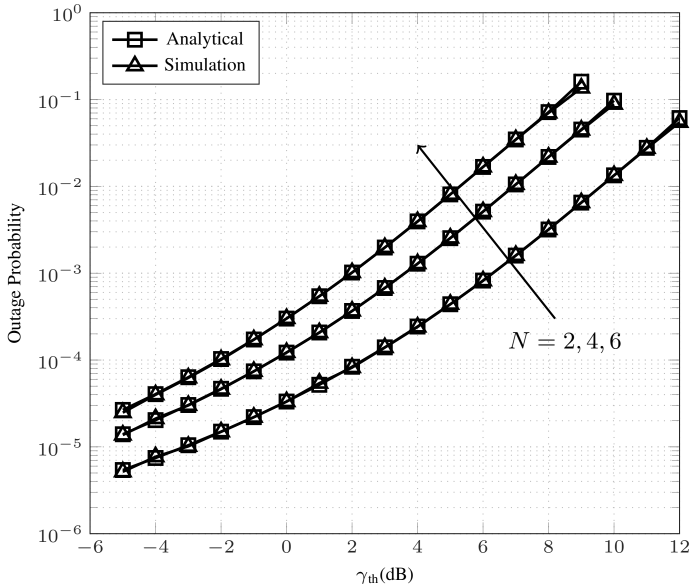

Fig. 1. Outage Probability in a Rician fading environment with $K_{tr} = 10$. Desired user power: $\Omega_0 = 20$ dB, and $K_0 = 10$. Interferers' power: $\Omega_i = 0.5$ dB $i = 1, 2, ..., 6$. $N = 2$: $K = \{6, 6\}$. $N = 4$: $K = \{6, 6, 7, 7\}$. $N = 6$: $K = \{6, 6, 7, 7, 8, 8\}$. Noise variance: $\eta = -10$ dB.

A. Rice Fading Model

The OP of the SINR in (12) is evaluated under the Rice fading model. In this case, we consider that the desired user power follows a non-centered Chi-squared distribution whose PDF given in Table II with Rice factor K_0 and average power Ω_0. The co-channel interferers's powers follow as well independent non-centered Chi-squared distributions with Rice factors $\{K_i\}_{i=1}^N$ and average powers $\{\Omega_i\}_{i=1}^N$. .

It is worth recalling that, in the derivation of the newly derived power series expressions (5) and (12), the sequence $\{b_k\}_{k\in\mathbb{N}}$ is assumed to be a positive, integer, and strictly increasing sequence. Such an assumption holds in the considered Rice fading model since $b_k = k+1$ for all $k \in \mathbb{N}$ (see Table I). Hence, we do not assume any particular values of the desired user power parameters which can be arbitrarily chosen.

In our first experiment, we aim to validate the newly derived analytical expression (12) by studying the accuracy of the corresponding truncated series (21) for different values of the Rice factors and different number of co-channel interferers. To this end, we plot in Fig. 1 the analytical and the simulated value of the OP $P_{out}(\gamma_{th})$ as a function of the threshold γ_{th} and for three different values of the co-channel interferers. Note that the order of truncation is fixed in this experiment and is equal to $K_{tr} = 10$. From this figure, we clearly observe that, for each scenario and for the considered range of threshold values, the analytical and the simulation results coincide perfectly. This goes in favor toward the accuracy of the truncated series expression (21). Moreover, it is worth pointing out that the considered range of threshold values, while properly chosen in order to guarantee the convergence of (12), corresponds as shown by Fig. 1 to the interesting range of OP that are important in practice, i.e. values of $P_{out}(\gamma_{th})$ that are approximately less than 10^{-1}. Such an observation shows that our method can be applied in practice without bearing severe restrictions from the condition in (14).

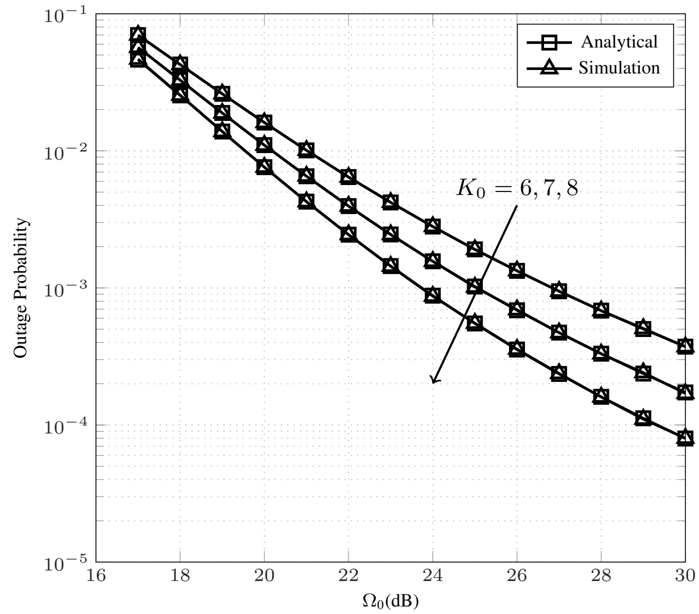

Fig. 2. Outage Probability function of Ω_0 in a Rician fading environment with $K_{tr} = 8$. $N = 4$ independent interferers with $\Omega_i = 0.5$ dB, and $K_i = 5$, $i = 1, 2, 3, 4$. Noise variance: $\eta = -10$ dB and $\gamma_{th} = 5$ dB.

In Fig. 2, we propose to validate the proposed power series formula (12) for different values of the desired user average power Ω_0. To this end, we represent in this figure the OP as a function of Ω_0 for three different values of the Rice factor K_0, when the order of truncation is set to $K_{tr} = 8$ and the threshold γ_{th} is chosen to be equal to 5 dB. It is important to mention that, for the considered simulation parameters, this value of γ_{th} ensures the convergence of the newly derived formula (12), being below the lower bound $\frac{\alpha_0}{\alpha(N+1)}$ of the convergence radius R. This figure shows again a perfect match between the analytical formula and the simulation results. It also allows to shed light on the enhancement in terms of OP when increasing the average power Ω_0 or the Rice factor K_0.

In a second experiment, we aim to analyze the accuracy of the truncated power series (21) for different values of the truncation order. To this end, we plot in Fig. 3 the value of $P_{out}(\gamma_{th})$ as a function of γ_{th} when $K_{tr} = 1, 2, 5, 8$ and under the Rician fading environment. This figure shows that the high accuracy of (21) in retrieving operating ranges of OP, starting from only few terms in the underlying expansion. In particular, it is important to note that when $K_{tr} = 5$, our analytical formula coincides perfectly with the simulation for all values of $P_{out}(\gamma_{th})$ less than 0.02 approximately. Moreover, the

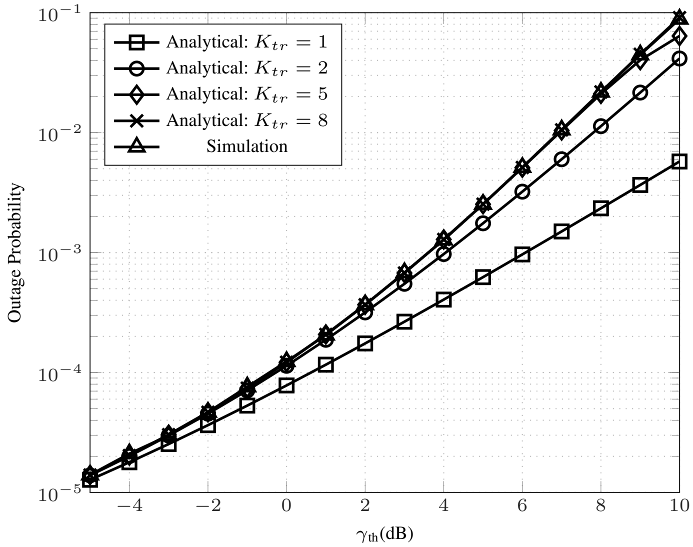

Fig. 3. Outage Probability function of K_{tr} in a Rician fading environment. Desired user power: $\Omega_0 = 20$ dB, and $K_0 = 10$. $N = 4$ independent interferers with $\Omega_i = 0.5$ dB $i = 1, 2, 3, 4$, and $K = \{6, 6, 7, 7\}$. Noise variance: $\eta = -10$ dB.

smaller is the value of γ_{th} the less is the number of truncation order K_{tr} needed to guarantee a fixed accuracy requirement. This fact is expected since, as we decrease γ_{th}, all the curves with different values of K_{tr} converge to an asymptotic curve corresponding to the leading term of the proposed power series expansion (12).

B. Generalized Gamma Fading Model

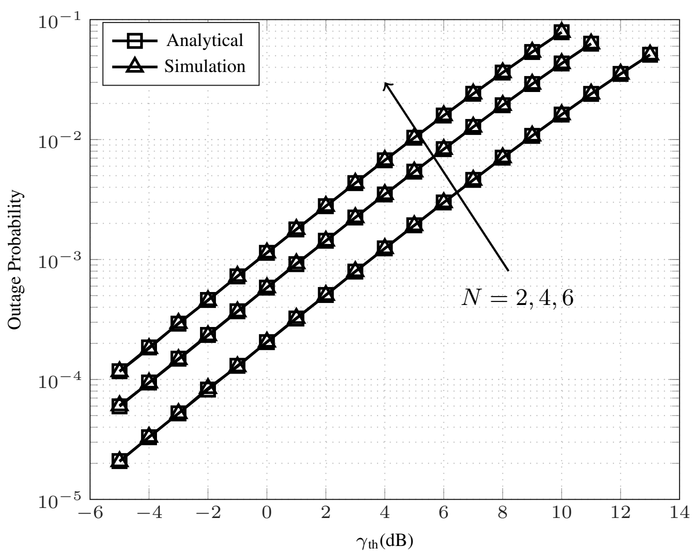

Fig. 4. Outage Probability in a Generalized Gamma fading environment with $K_{tr} = 10$. Desired user power: $\Omega_0 = 25$ dB, $p_0 = 1$, and $d_0 = 2$. Interferers' power: $\Omega_i = 0.5$ dB, and $d_i = 0.5$ $i = 1, 2, ..., 6$. $N = 2$: $p = \{1, 1\}$. $N = 4$: $p = \{1, 1, 1.25, 1.25\}$. $N = 6$: $p = \{1, 1, 1.25, 1.25, 1.5, 1.5\}$. Noise variance: $\eta = -10$ dB.

In this section, we consider the case where the desired user as well as the co-channel interferers experience the Generalized Gamma fading environment. The desired user power follows then a $GG(d_0, p_0, \Omega_0)$ and the co-channel interferers' powers are distributed independently according to $GG(d_i, p_i, \Omega_i)$ $i = 1, 2, ..., N$. Our aim is again to validate the newly derived expression (12) for this particular fading environment.

Note that the requirement of having positive integer values of b_k, $k \in \mathbb{N}$, requires us to introduce some assumptions on the parameters of the desired user power PDF. In fact, from Table 1, the values of b_k is $d_0 + kp_0$, $k \in \mathbb{N}$, and hence the parameters d_0 and p_0 need to be integers in order for the newly derived expression (12) to be valid.

In Fig. 4, we set $K_{tr} = 10$ and compare the analytical formula (12) with the simulation results for different values of the number of co-channel interferers and different values of the parameters of the co-channel interferers' power distributions. In this figure, we easily observe, for each scenario, a good match between the OP provided by the analytical formula and that obtained by simulations. Moreover, as for the previous subsection, Fig. 4 shows that the convergence of the newly derived power series expression (12) is guaranteed for values of $P_{out}(\gamma_{th})$ less than around 0.05 for the three considered scenarios. Thus, the convergence region of (12) covers the operating range of the OP.

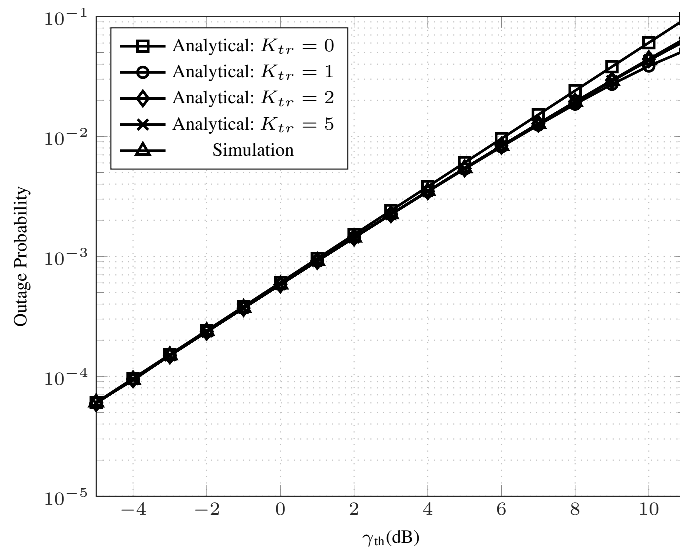

Fig. 5. Outage Probability function of K_{tr} in a Generalized Gamma fading environment. Desired user power: $\Omega_0 = 25$ dB, $p_0 = 1$, and $d_0 = 2$. $N = 4$ independent interferers with $\Omega_i = 0.5$ dB , $d_i = 0.5$ $i = 1, 2, 3, 4$, and $p = \{1, 1, 1.25, 1.25\}$. Noise variance $\eta = -10$ dB.

In a second experiment, we aim to investigate the accuracy of (21) for various values of the order of truncation K_{tr}. Fig. 5 representing $P_{out}(\gamma_{th})$ as a function of γ_{th} with $K_{tr} = 0, 1, 2, 5$, illustrates the obtained results. Interestingly, we point out from this figure that few values of K_{tr} are strongly sufficient to guarantee a good level of accuracy. For instance, the truncated series with $K_{tr} = 2$ coincides perfectly with the simulation in the considered range of OP, and with $K_{tr} = 1$ the accuracy is validated for values of $P_{out}(\gamma_{th})$ that are less than 0.02. Moreover, compared to the results shown in Fig. 3 under the Rician fading environment, we clearly observe that fewer terms (i.e. small values of K_{tr}) are sufficient to ensure a highly accurate result. This might be an outcome of the rapid increase of the powers' moments under the Rician fading environment compared to those of the Generalized Gamma one. Hence, the general term of (12) decrease faster to zero in the Generalized Gamma environment than in the Rician fading case.

C. SINR-Based SD for SIMO Receiver

In this subsection, we aim to validate the proposed moment-based formula in the case of SINR-based SD receivers with $N = 4$ co-channel interferers affected by the Rician fading model. To this end, we plot in Fig. 6 the OP, given by simulations and the proposed moment-based formula, as a function of γ_{th} when the number of branches are $M = 1, 2, 3$. This figure shows for each scenario a perfect match between

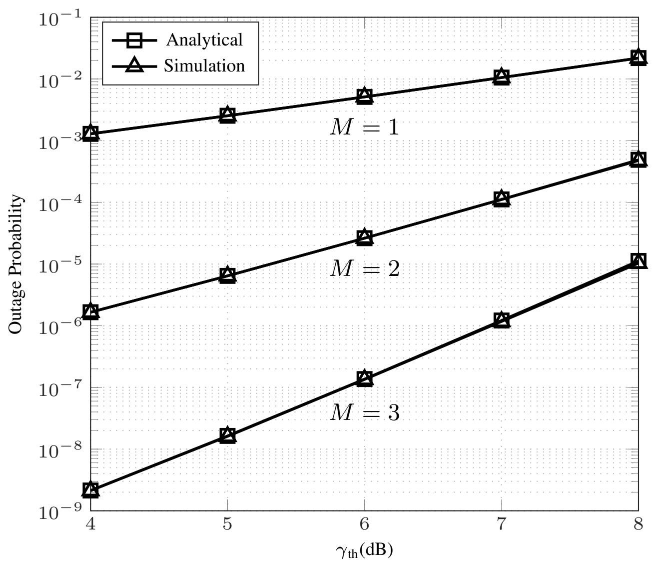

Fig. 6. Outage Probability with M-branch SINR-based SD in a Rician fading environment with $N = 4$ co-channel interferers and $K_{tr} = 10$. Desired user powers are i.i.d: $\Omega_0^n = 20$ dB, and $K_0^n = 10$, $n = 1, 2, ..., M$. Interferers' powers are i.i.d: $\Omega_i^n = 0.5$ dB, and $K^n = \{6, 6, 7, 7\}$, $i = 1, 2, 3, 4$ and $n = 1, 2, ..., M$. Noise variance: $\eta = -10$ dB.

simulated and analytical expressions, thus validating the proposed moment-based formula. As expected, we clearly observe from this figure that the larger is the number of SD branches the smaller is the OP.

VI. Conclusion

We developed in this work a novel moment-based approach of the OP of the SINR when operating over generalized fading channel environments. Our method is based on the derivation of a power series expansion of the OP using only two ingredients namely the power series expansion of the CDF of the desired user power and the cross-moments of the interfering powers. Compared to other existing techniques, it presents two main advantages. First, it is applicable for a wide range of fading models for the desired user signal, under possibly some mild assumptions on the fading parameters, and second, it does not require the knowledge of the fading model of the interfering signals, the cross-moments of the interfering powers being the sole requirement.

We performed also a convergence study of the newly derived power series expansion when operating over two interesting fading models, the Generalized Gamma and the Rice fading environments. To illustrate the wide scope of applicability of our technique, we show how it applies to analyze the OP at receivers using SD techniques. Finally, some selected simulation results were also carried out in order to validate the proposed formula.

Appendix A
Proof of Theorem 2

We start by proving that if $p_0 < p$, the convergence radius R is equal to $+\infty$. The proposed moment-based formula would be thus valid for every threshold values $\gamma_{th} > 0$. To this end, via the use of Jensen's inequality to the convex function x^{b_k}, $k \in \mathbb{N}$, on the positive axis, we have:

$$\begin{aligned}
&\sum_{k=0}^{\infty} |a_k| \gamma_{th}^{b_k} \mathbb{E}\left[\left(\sum_{i=1}^{N} X_i + \eta\right)^{b_k}\right] \\
&= \sum_{k=0}^{\infty} |a_k| \gamma_{th}^{b_k} (N+1)^{b_k} \mathbb{E}\left[\left(\sum_{i=1}^{N} \frac{X_i}{N+1} + \frac{\eta}{N+1}\right)^{b_k}\right] \\
&\leq \sum_{k=0}^{\infty} |a_k| \gamma_{th}^{b_k} (N+1)^{b_k - 1} \left(\sum_{i=1}^{N} \mathbb{E}\left[X_i^{b_k}\right] + \eta^{b_k}\right) \\
&= \sum_{k=0}^{\infty} h_k
\end{aligned} \quad (22)$$

Let us now study the convergence of the power series $\sum_k h_k$. Note that this series could be rewritten as a finite sum of power series:

$$\begin{aligned}
\sum_{k=0}^{\infty} h_k = &\sum_{i=1}^{N} \sum_{k=0}^{\infty} \underbrace{|a_k| \gamma_{th}^{b_k} (N+1)^{b_k - 1} \mathbb{E}\left[X_i^{b_k}\right]}_{h_{k,i}} \\
&+ \sum_{k=0}^{\infty} \underbrace{|a_k| \gamma_{th}^{b_k} (N+1)^{b_k - 1} \eta^{b_k}}_{r_k}
\end{aligned} \quad (23)$$

Hence, the convergence radius of $\sum_k h_k$ is then the minimum over all the convergence radius of the $N + 1$ power series of the previous equation. Let $i \in \{1, 2, ..., N\}$, we apply the D'Alembert test of convergence [28] to the infinite power series of general terms $h_{k,i}$, $k \in \mathbb{N}$. Upon plugging the expressions of a_k, b_k in Table I and the expression of the moments $\mathbb{E}\left[X_1^{b_k}\right]$ in Table II into the expression of $h_{k,i}$, $k \in \mathbb{N}$, we obtain

$$\frac{h_{k+1,i}}{h_{k,i}} \sim \left(\frac{a_i \gamma_{th}}{a_0}\right)^{p_0} \frac{(N+1)^{p_0}}{k+1} \frac{\Gamma\left(\frac{d_i + d_0 + k p_0}{p_i} + \frac{p_0}{p_i}\right)}{\Gamma\left(\frac{d_i + d_0 + k p_0}{p_i}\right)} \quad (24)$$

We employ now the following asymptotic expression of the Gamma function given in [22]:

$$\Gamma(t + a) \sim_{t \to +\infty} t^a \Gamma(t). \quad (25)$$

From this expression, we deduce that:

$$\frac{h_{k+1,i}}{h_{k,i}} \sim \left(\frac{a_i\gamma_{th}}{a_0}\right)^{p_0} \frac{(N+1)^{p_0}}{k+1} \left(\frac{d_i+d_0+kp_0}{p_i}\right)^{\frac{p_0}{p_i}}. \quad (26)$$

Hence, we clearly observe that provided that $p_0 < p$ with $p = \min_{1\leq i\leq N} p_i$, we deduce that for each $i \in \{1,2,...,N\}$, $\frac{h_{k+1,i}}{h_{k,i}}$ goes to zero as k goes to infinity. Consequently, the convergence radius of the power series $\sum_k h_{k,i}$ is ∞ for each $i \in \{1,2,...,N\}$. Similarly, the convergence radius of $\sum_k r_k$ is ∞. In fact, using again the D'Alembert convergence test, we have:

$$\frac{r_{k+1}}{r_k} \sim (\frac{\gamma_{th}}{a_0})^{p_0} \frac{(N+1)^{p_0}}{k+1} \eta^{p_0}, \quad (27)$$

which goes to zero as k goes to infinity. From these two results, we deduce that the power series $\sum_k h_k$ has a convergence radius equal to $+\infty$. Thus, from (22), we conclude that if $p_0 < p$ the convergence radius R is equal to $+\infty$.

In the second part of the proof, we aim to show that if $p_0 > p$, the convergence radius $R = 0$, and thus the proposed moment-based formula is not valid. In fact, let us denote by i_0 the index in the set $\{1,2,...,N\}$ corresponding to the interferer with $p_{i_0} = p$. Using the fact that $b_k \geq 0$, $k \in \mathbb{N}$, it follows that:

$$\sum_{k=0}^{\infty} |a_k|\gamma_{th}^{b_k} \mathbb{E}\left[\left(\sum_{i=1}^{N} X_i + \eta\right)^{b_k}\right] \geq \sum_{k=0}^{\infty} |a_k|\gamma_{th}^{b_k} \mathbb{E}\left[X_{i_0}^{b_k}\right] = \sum_{k=0}^{\infty} g_k \quad (28)$$

Following the same steps as in (26), the D'Alembert convergence test applied to $\sum_k g_k$ yields:

$$\frac{g_{k+1}}{g_k} \sim \left(\frac{a_{i_0}\gamma_{th}}{a_0}\right)^{p_0} \frac{1}{k+1} \left(\frac{d_{i_0}+d_0+kp_0}{p_{i_0}}\right)^{\frac{p_0}{p_{i_0}}}. \quad (29)$$

Thus, given that $p_0 > p_{i_0}$, the previous quantity goes to $+\infty$ as k goes $+\infty$, and therefore the convergence radius R is equal to zero.

Finally, it remains to prove that $R \geq \frac{a_0}{a(N+1)}$ in the case when $p_0 = p$. For that, we study the convergence of each $\sum_k h_{k,i}$ in this particular setting. It is worth mentioning that, for all i such that $p_i > p_0$, the convergence radius of $\sum_k h_{k,i}$ is ∞. This can be proved by following the same calculations in (26). It suffices thus to focus on the case when $p_i = p_0$. In this case, the application of the D'alembert test of convergence along with the approximation in (25) yields:

$$\frac{h_{k+1,i}}{h_{k,i}} \sim \left(\frac{a_i\gamma_{th}}{a_0}\right)^{p_0} \frac{(N+1)^{p_0}}{k+1} \left(\frac{d_i+d_0}{p_0}+k\right). \quad (30)$$

Thus, for $p_i = p$. the convergence radius of $\sum_k h_{k,i}$ is then $\frac{a_0}{a_i(N+1)}$. Recalling that the convergence radius of $\sum_k r_k$ is ∞, we conclude that the convergence of the power series $\sum_k h_k$ is thus given by the minimum over all the convergence radius of those $\sum_k h_{k,i}$ satisfying $p_i = p_0$. Consequently, we conclude that when $p_0 = p$, $R \geq \frac{a_0}{a(N+1)}$.

Appendix B
Proof of Theorem 3

Using the same methodology as in (22), it follows that:

$$\begin{aligned}
&\sum_{k=0}^{\infty} |a_k|\gamma_{th}^{b_k} \mathbb{E}\left[\left(\sum_{i=1}^{N} X_i + \eta\right)^{b_k}\right] \\
&\leq \sum_{k=0}^{\infty} |a_k|\gamma_{th}^{b_k} (N+1)^{b_k-1} \left(\sum_{i=1}^{N} \mathbb{E}\left[X_i^{b_k}\right] + \eta^{b_k}\right) \\
&= \sum_{k=0}^{\infty} \frac{\exp(-K_0)\left|L_n^0(K_0)\right|}{(k+1)!} \left(\frac{\gamma_{th}(1+K_0)}{\Omega_0}\right)^{k+1} (N+1)^k \\
&\times \left(\sum_{i=1}^{N} \mathbb{E}\left[X_i^{k+1}\right] + \eta^{k+1}\right) \\
&\leq \sum_{k=0}^{\infty} \frac{\exp(-K_0/2)}{(k+1)!} \left(\frac{\gamma_{th}(1+K_0)}{\Omega_0}\right)^{k+1} (N+1)^k \\
&\times \left(\sum_{i=1}^{N} \mathbb{E}\left[X_i^{k+1}\right] + \eta^{k+1}\right) \\
&= \sum_{k=0}^{\infty} d_k \quad (31)
\end{aligned}$$

In the previous derivation, we have replaced a_k and b_k, $k \in \mathbb{N}$, by their expressions presented in Table I. Moreover, we have used the inequality $|L_n^0(K_0)| \leq \exp(K_0/2)$ given in [23]. Let us now study the convergence of the power series $\sum_k d_k$. Similarly to the previous proof, we rewrite this power series as a finite sum of power series:

$$\begin{aligned}
\sum_{k=0}^{\infty} d_k &= \sum_{i=1}^{N} \sum_{k=0}^{\infty} d_{k,i} \\
&+ \sum_{k=0}^{\infty} f_k, \quad (32)
\end{aligned}$$

where

$$d_{k,i} = \frac{\exp(-K_0/2)}{(k+1)!} \left(\frac{\gamma_{th}(1+K_0)}{\Omega_0}\right)^{k+1} (N+1)^k \mathbb{E}\left[X_i^{k+1}\right] \quad (33)$$

and

$$f_k = \frac{\exp(-K_0/2)}{(k+1)!} \left(\frac{\gamma_{th}(1+K_0)}{\Omega_0}\right)^{k+1} (N+1)^k \eta^{k+1} \quad (34)$$

We perform now a convergence study of each of the $N+1$ power series. Let $i \in \{1,2,...,N\}$. Plugging the moment corresponding to the Rician model into $d_{k,i}$, we have:

$$\begin{aligned}
d_{k,i} &= \frac{\exp(-K_0/2)}{(k+1)!} \left(\frac{\gamma_{th}(1+K_0)}{\Omega_0}\right)^{k+1} (N+1)^k \mathbb{E}\left[X_i^{k+1}\right] \\
&= \frac{exp(-K_0/2)}{(k+1)!} \left(\frac{\gamma_{th}(1+K_0)}{\Omega_0}\right)^{k+1} (N+1)^k \\
&\times \frac{\Gamma(k+2)}{(1+K_i)^{k+1}} {}_1F_1\left(-(k+1),1;-K_i\right) \Omega_i^{k+1} \quad (35)
\end{aligned}$$

where ${}_1F_1(\cdot,\cdot;\cdot)$ is the Kummer confluent hypergeometric function [22]. This function is related to the Laguerre polynomials of degree n and order zero $L_n^0(\cdot)$ as follows [22]:

$$ {}_1F_1(-n,1;-x) = L_n^0(-x). \tag{36} $$

In [29], an asymptotic expression of $L_n(-x)$ is given for $x > 0$ as follows:

$$ L_n^0(-x) \sim_{n\to+\infty} \frac{1}{2\sqrt{\pi}} \exp(-x/2)(nx)^{-1/4} \exp(2\sqrt{nx}) \tag{37} $$

Using this asymptotic expansion in (35), it follows that

$$ \begin{aligned} d_{k,i} \sim & \frac{\exp(-K_0/2)}{2\sqrt{\pi}} \left(\frac{\gamma_{th}(1+K_0)}{\Omega_0}\right)^{k+1} (N+1)^k \\ & \times \left(\frac{\Omega_i}{1+K_i}\right)^{k+1} \exp(-K_i/2)((k+1)K_i)^{-1/4} \\ & \times \exp(2\sqrt{(k+1)K_i}) \end{aligned} \tag{38} $$

Now, we employ the D'Alembert test of convergence to get:

$$ \begin{aligned} \frac{d_{k+1,i}}{d_{k,i}} \sim & (N+1) \left(\frac{\gamma_{th}(1+K_0)\Omega_i}{\Omega_0(1+K_i)}\right) \\ & \times \exp(2\sqrt{(k+2)K_i} - 2\sqrt{(k+1)K_i}) \\ \sim & (N+1) \left(\frac{\gamma_{th}(1+K_0)\Omega_i}{\Omega_0(1+K_i)}\right) \end{aligned} \tag{39} $$

Consequently, the convergence radius of power series $\sum_k d_{k,i}$ is then $\frac{\Omega_0(1+K_i)}{\Omega_i(1+K_0)(N+1)}$. On the other hand, we can easily prove that the convergence radius of $\sum_k f_k$ is ∞. In fact, we have that:

$$ \frac{f_{k+1}}{f_k} = \frac{1}{k+1} \frac{\gamma_{th}(1+K_0)(N+1)\eta}{\Omega_0}, \tag{40} $$

which tends to zero as k goes to infinity for all values of γ_{th}. The D'Alembert test of convergence shows thus that the convergence radius of $\sum_k f_k$ is ∞. From these two results, we can conclude that the convergence radius of $\sum_k d_k$ is then the minimum over $\left\{\frac{\Omega_0(1+K_i)}{\Omega_i(1+K_0)(N+1)}, i=1,\cdots,N\right\}$ or equivalently the minimum over $\left\{\frac{\alpha_0}{\alpha_i(N+1)}, i=1,\cdots,N\right\}$. Consequently, the convergence radius of $\sum_k d_k$ is $\frac{\alpha_0}{\alpha(N+1)}$ with $\alpha = \max_{1\le i\le N}\alpha_i$. Hence, from (31), the convergence radius R of the proposed moment-based formula (5) satisfies $R \ge \frac{\alpha_0}{\alpha(N+1)}$ which concludes the proof.

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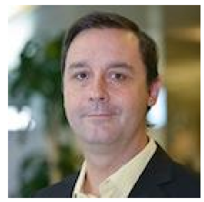

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