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# 1 Lower Mantle Heat Flow Controls the Longitudinal 2 Structure of Earth's Magnetic Field

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4 **Thermal interactions between Earth's core and mantle provide the power that**  
5 **maintains the geomagnetic field. However, the observational expression of**  
6 **these interactions and their unique potential to link magnetic field behaviour**  
7 **and deep Earth processes has remained uncertain for decades. Here we show**  
8 **that recent global time-dependent magnetic field models spanning tens of thou-**  
9 **sands of years combined with numerical simulations indicate how the mantle**  
10 **controls core dynamics. Simulations of rapidly rotating turbulent dynamo ac-**  
11 **tion with strong imposed lateral variations in CMB heat flow reproduce the**  
12 **morphology and secular variation of Earth's modern field, and the inferred**  
13 **large-scale flow structure at the top of the core. These simulations reveal**  
14 **that the long-term detectable signature of thermal core-mantle interactions**  
15 **are equatorial patches of reverse flux, rather than the high-latitude patches**  
16 **suggested by less Earth-like simulations. Comparison of our simulations with**  
17 **observation models also suggest that the amplitude of the present-day hemi-**  
18 **spheric imbalance in secular variation is anomalously large.**

## 19 **Introduction**

20 Earth's global magnetic field has persisted throughout the majority of its history, generated  
21 by a dynamo process in the liquid core that derives its power from the slow loss of heat to  
22 the mantle (1). Convection within the mantle is characterised by much longer timescales and  
23 much longer wavelengths than core convection and lateral variations in the temperature distri-  
24 bution of the lowermost mantle result in a heterogeneous pattern of heat flow at the core-mantle  
25 boundary (CMB), with enhanced/suppressed heat flux where the lowermost mantle is anoma-  
26 lous cold/hot (2, 3). Yet the extent to which mantle heat flow controls the geodynamo and  
27 geomagnetic field has been debated for decades. Variations in reversal frequency (4), apparent  
28 preferred longitudes of transitional virtual geomagnetic poles (5), and persistently weak field  
29 changes in the Pacific (6), all argued to result from the mantle's influence on core dynamics,  
30 have been disputed by both observational and modelling studies (7). However, recently a major  
31 observational limitation—the lack of continuous global time-dependent representations of the  
32 field covering more than a few core turnover times—has been addressed by new models, such  
33 as GGF100k, covering the last 100 kyrs (8). By combining these models with numerical sim-  
34 ulations of core dynamics we find that it is now possible to identify the magnetic signature of  
35 thermal core-mantle interaction and its links to core dynamics.

36 A prominent feature of the geomagnetic field in high-resolution models covering the last  
37 400 years (9) are the four high-latitude flux patches that appear at longitudes where mantle heat  
38 flow is expected to be anomalous high. Convergent downwelling resulting from the locally el-  
39 evated heat flow could cause intense flux patches to persistently concentrate around preferred  
40 longitudes (7). However, while similar patches must be represented in realistic simulations of  
41 core dynamics, observational models that precede the past four centuries (10) find that they  
42 are not stationary, indicating that they could be transient features. Nevertheless, observational

43 studies agree that long-wavelength structure of Earth's time-averaged magnetic field contains  
44 substantial non-zonal structure (10). Free convection in the core may give rise to magnetic  
45 field structures with lifetimes comparable to the advective time scale (a few centuries); how-  
46 ever, since mantle structure persists over geologically long times its influence should appear in  
47 sufficiently long-term averages of Earth's magnetic field.

48 Another feature of the recent magnetic field is the preference for stronger secular variation  
49 (SV) in the Atlantic hemisphere (11), a signature that has persisted for at least a few centuries  
50 (9). Paleomagnetic reconstructions of Earth's magnetic field do not recover instantaneous SV;  
51 however, they can provide measures of field variability throughout their duration. For example,  
52 the paleosecular variation index ( $P_i$ ) (12) is a non-dimensional measure of the paleo-field's  
53 deviation from the expected dipole strength and structure at a given location (see Methods), and  
54 temporal evolution of  $P_i$  provides insight into patterns of field variation over the last 100 kyr  
55 (10). Although periods of enhanced activity in the Atlantic hemisphere are seen in long-term  
56 records, so too are periods of enhanced Pacific activity (13–16); as with the structure of the field  
57 itself, successful simulations must contain non-zonal SV features that are relatively long-lived  
58 but not strictly stationary.

59 Secular variation arises from the interaction of the magnetic field and fluid flow at the top  
60 of the outer core. Models derived from the inversion of secular variation data can thus provide  
61 insight into core flow, although the details of the recovered flow depend on the treatment of  
62 the data and the assumptions used to break the inherent non-uniqueness of the inversion (17).  
63 Nevertheless, some features are consistently seen in these observationally derived models such  
64 as relatively strong westward flow beneath the equatorial Atlantic, whereas flow beneath the  
65 equatorial Pacific is generally weak or eastward (17–20). The large-scale flow is often charac-  
66 terised as an equatorially symmetric eccentric gyre, with the westward Atlantic flow deviating  
67 towards the poles near American longitudes, bypassing the Pacific at high latitudes, and return-

68 ing towards the equator by Indian longitudes (19, 21, 22).

69 Many previous studies have obtained simulated fields that are morphologically similar to the  
70 present geomagnetic field (23–25); however, only one has also reported a match to the pattern  
71 of modern SV (26). This result was obtained by adding two features to the standard geodynamo  
72 model setup: gravitational coupling between the inner core and mantle, and a hemispheric pat-  
73 tern of inner core growth due to convective translation. Although the strength of gravitational  
74 coupling is relatively well constrained (27), recent determinations of core material properties  
75 suggest that purely thermal convection of the inner core is highly unlikely and that purely com-  
76 positional or doubly-diffusive thermochemical convection were more likely before the inner  
77 core grew to half its present size (28–31). Here we instead seek solutions that match the field  
78 and SV morphology based on a single well-established mechanism: lateral variations in heat  
79 flow at the CMB.

## 80 **Results**

81 Our previous work has systematically investigated the effects of different patterns and ampli-  
82 tudes of outer boundary forcing on bottom-driven non-magnetic rotating convection in spherical  
83 shell geometry (32–34), a configuration that provides a simple analogue for core dynamics. Us-  
84 ing knowledge of the regime diagram for homogeneous convection in the same setup (35) has  
85 allowed us to run a targeted suite of dynamo simulations in the rapidly rotating and turbu-  
86 lent dynamical regime that is thought to characterise Earth’s core. We therefore consider six  
87 simulations (see Methods) with Ekman number  $E = 10^{-5}$ , and two values of the Rayleigh  
88 number  $Ra = \{2000, 6000\}$ . The magnetic Prandtl number  $Pm = 1$  is set to achieve a  
89 Quasi-Geostrophic and Magnetic-Archimedian-Coriolis (QG-MAC) force balance and a mag-  
90 netic Reynolds number  $Rm \sim 1000$ , as is expected in the core (1, 36, 37). Two simula-  
91 tions employ homogeneous boundary conditions while four impose a pattern of CMB heat

92 flux heterogeneity derived from mantle seismic tomography (38) with amplitude described by  
93  $q^* = (q_{\max} - q_{\min})/q_{\text{ave}} = \{2.3, 5.0\}$  (where,  $q_{\max}$ ,  $q_{\min}$ ,  $q_{\text{ave}}$  are the maximum, minimum, and  
94 average values of CMB heat flux, respectively; see Methods). The models have been run for  
95 10's of thousands of simulation years, sufficient time to resolve the long-term time-averaged  
96 behaviour of the non-zonal field (25). Performing such simulations in the rapidly rotating pa-  
97 rameter regime is computationally expensive, the six runs presented here requiring a total of  
98 14.6 million cpu hours.

99 The GGF100k reconstruction (8) resolves only the longest wavelength features of Earth's  
100 field and thus we begin by considering the time-averaged field of our simulations truncated to  
101 spherical harmonic degree and order 4 (Figure 1, supplemental figure 1). The time-averaged  
102 field of our  $Ra = 2000$ ,  $q^* = 0$  run lacks the non-zonal structure evident for Earth (e.g.,  
103 figure 11 of (10), supplemental figure 1a). Our  $Ra = 6000$ ,  $q^* = 0$  run is a multipolar reversing  
104 case and thus does not have a meaningful time-averaged field. Heterogeneous mantle forcing in  
105 our  $q^* \neq 0$  runs organises the flow near the top of the core, introducing non-zonal structure into  
106 the time-averaged magnetic field, although the precise strength and location of this non-zonal  
107 structure varies between our  $q^* \neq 0$  simulations. Nevertheless, the non-zonal structure observed  
108 in GGF100k (such as low radial flux under South America, which is also seen in our  $q^* \neq 0$   
109 simulations) is clearly far greater than that of our homogeneous model once it has been averaged  
110 over many advection times. This suggests that some factor other than the internal dynamics of  
111 the fluid core is responsible for the long-term non-zonal features of the field.

112 The spatial and temporal structure of the modern field (e.g., the last 400 years as described  
113 in *gufm1*) are better resolved than its long-term behaviour, and we now investigate whether the  
114  $q^* \neq 0$  simulations can match the geometry and secular variation of the modern field. We  
115 evaluate the temporal evolution of geometric features of the radial CMB magnetic field in our  
116 simulations with a set of widely-employed compliance criteria (24) (see Methods). While the

117 choice of criteria is subjective and these measures do not assess all observable properties of the  
118 dynamo behaviour (24, 25), they are straightforward to compute given a spherical harmonic rep-  
119 resentation of the magnetic field and are useful for suggesting periods of simulation behaviour  
120 that are suitable for more detailed analysis. The measures of field structure from 400-year win-  
121 dows of the simulations are compared to the values obtained from analysis of *gufm1* (9) to  
122 provide a  $\chi^2$  measure of agreement between simulations and Earth's modern geomagnetic field.  
123 These criteria are complemented by a measure of hemispheric imbalance in SV ( $H_{sv}$ ) and its  
124 variation which provides a  $\chi^2$  measure of compliance with respect to the quiet Pacific secu-  
125 lar variation (16). With the exception of the non-dipole-dominated solution with  $Ra = 6000$   
126 and  $q^* = 0$  all simulations produce periods of good or excellent agreement with the modern  
127 geomagnetic field (supplemental figure 2). The  $Ra = 2000$ ,  $q^* = 5.0$  simulation is usually non-  
128 compliant with the modern field, unlike the other three  $q^* \neq 0$  simulations. All simulations also  
129 have 400-year windows characterised by quiet Pacific SV, although the hemispheric imbalance  
130 is generally less than that derived from *gufm1*.

131 Figure 2 compares the magnetic field and SV of the 400-year window from the  $Ra = 2000$ ,  
132  $q^* = 2.3$  simulation with the lowest  $\chi^2$  totals across all five measures to the structure of field and  
133 secular variation in *gufm1*. This window occurred at approximately 25,700 model years and, as  
134 expected from the low  $\chi^2$  value, reproduces many characteristics of Earth's modern geomag-  
135 netic field, such as patches of intense flux at high latitude and quiet SV in the central Pacific.  
136 The pattern of flow from this 400-year window has more structure than the time-averaged flow  
137 of the full run (figure 2c,f) although certain features arising from the CMB heterogeneity, such  
138 as the promotion of downwelling between approximately  $30^\circ$ – $50^\circ$  west can be seen in both.

139 Including boundary heterogeneity alters the time-averaged structure of the magnetic and  
140 velocity fields near the top of the core introducing persistent longitudinal structure. The hetero-  
141 geneous boundary forcing drives flow that, in the time average, produces a large equatorially

142 symmetric gyre with westward flow at mid-to-low latitudes under Africa and the Atlantic (fig-  
143 ure 2c). The flow from the gyre diverts poleward at American longitudes thereby avoiding the  
144 Pacific, which is characterised by weak time-averaged flows when  $q^* = 2.3$ . The hemispheric  
145 difference in the time-averaged flows near the surface of the core in the cases with  $q^* \neq 0$  might  
146 be expected to result in persistent longitudinal differences in the hemispheric balance of secu-  
147 lar variation. However, there is no evidence for a preferred hemisphere of secular variation in  
148 our simulations (supplemental table 2) and, therefore, on average they provide a fairly poor fit  
149 (supplemental table 1) to the quiet Pacific secular variation associated with the historic geomag-  
150 netic field. Direct measurement of secular variation requires continuous observation of Earth's  
151 magnetic field, possible only in the modern era. However, the cumulative effect of SV can be es-  
152 timated from time-dependent field models constructed from paleomagnetic and archeomagnetic  
153 samples (13–15). These models are necessarily smoothed due to the unavoidable limitations in  
154 the spatial and temporal sampling of the data, but they do not indicate that there is hemispheric  
155 structure in geomagnetic variability on thousand-year time scales (16).

156 The longitudinal structure of the paleosecular variation index,  $P_i$ , provides another view on  
157 the hemispheric balance of temporal activity, one that can be compared directly with observa-  
158 tional models such as GGF100k. The variability of  $P_i$  is lower in our  $Ra = 2000$ ,  $q^* = 2.3$   
159 simulation than in GGF100k (Figure 3); however, the simulation and observational models have  
160 similar median values. Both the simulated and observed fields have times of  $P_i$  being high in  
161 the Atlantic hemisphere and low in the Pacific hemisphere, but they also have times with the  
162 opposite imbalance. Maps of mean  $P_i$  value (supplemental figure 4) show regional differences  
163 in paleosecular activity in both the simulations and observations.  $P_i$  tends to be larger at higher  
164 latitudes in our simulations and somewhat low over the equatorial Pacific in our  $q^* \neq 0$  cases.  
165 However, there are not large differences between  $P_i$  distributions at different longitudes in either  
166 GGF100k or our simulations. For example, the median longitudinal  $P_i$  values in GGF100k and

167 the three  $q^* \neq 0$  simulations that match modern field structure and SV never differ significantly  
168 from 0.06, the mean value obtained from temporally and spatially averaging *gufm1*.

169 Times when the  $Ra = 2000$ ,  $q^* = 2.3$  simulation exhibits poor compliance relative to  
170 *gufm1* primarily arise due to the emergence of very strong flux patches in certain high-latitude  
171 locations, a signature which can be seen in the total time-averaged field of the simulation (Fig-  
172 ure 1b). Although emphasis has previously been placed on the persistence of strong high-  
173 latitude flux patches (39–41), the time-averaged fields of the heterogeneous cases also have  
174 non-zonal structure at equatorial latitudes that is absent from the homogeneous case (figure 4).  
175 Both  $Ra = 2000$ ,  $q^* \neq 0$  cases have a pair of reverse flux patches straddling the equator roughly  
176 beneath South America, with a similar structure on the CMB beneath the Indian Ocean. As is  
177 the case for the high-latitude patches, the detailed strength, structure, and location of these low-  
178 latitude features varies with  $Ra$  and  $q^*$  but they are present in all of our  $q^* \neq 0$  cases (see also  
179 supplemental figure 3).

## 180 Discussion

181 We find that dynamo simulations with strong lateral CMB heat flux variations successfully  
182 reproduce the main features of the large-scale field morphology and paleosecular activity de-  
183 scribed in observation models of the modern field and the GGF100k model spanning the last  
184 100 kyrs. Unlike other studies (42) our simulations were not tuned to produce Earth-like fields;  
185 the control parameters were chosen to sit within the appropriate dynamic regime for Earth’s  
186 core and a CMB heat flux heterogeneity pattern derived from seismic tomography imposed.  
187 The bulk dynamics of our simulations obeys a QG-MAC balance, and previous work (36, 43)  
188 has shown that this balance is maintained in simulations sampling a uni-dimensional path in  
189 parameter space that leads towards parameters more similar to Earth’s core. The large-scale  
190 dynamics at the top of the core in our simulations do not depend on inertial or viscous ef-

191 facts (44), which both become weaker as core conditions are approached. Previous work has  
192 also found that small-scale free convection in the core’s interior does not disrupt large-scale  
193 boundary-forced patterns (34, 36, 43). Therefore, we have reason to believe that the behaviour  
194 we observe is robust. The detailed time-dependent dynamics, and hence the compliance with  
195 Earth’s field, do vary between our  $q^* \neq 0$  cases; however, they have similar large scale features  
196 of mantle-induced flow and structure in the outermost core, and all produce instances where  
197 both field and SV morphology comply with geomagnetic observations. Both the CMB heat flux  
198 heterogeneity imposed by the mantle and the internal core dynamics affect the compliance of  
199 the resultant geodynamo. A more extensive suite of simulations could determine what balance  
200 of these factors is required for Earth-like behaviour; however, the fact that three of the four  
201 heterogeneous cases have long stretches of time with good or excellent compliance suggest that  
202 these results do not depend on a delicate balance of conditions.

203 Earlier studies, at higher Ekman number, tended to find that the dynamo would fail in sim-  
204 ulations with large heterogeneous boundary forcing (23, 45). However, as in (46), we find that  
205 our simulations maintain a dynamo despite peak-to-peak variations in heat flux being larger  
206 than the average. Indeed, for the  $Ra = 6000$  cases the inclusion of boundary heterogeneity acts  
207 to stabilise the dynamo, as the homogeneously forced case was in a multi-polar reversing state.  
208 As in previous work (23, 40, 41, 45, 46), the boundary heterogeneity organises flow near the top  
209 of the core, in our simulations this results in the large-scale time-average flow forming an ec-  
210 centric gyre (22, 47) without recourse to variations in lower mantle electrical conductivity (48)  
211 or inner core translation (16, 26).

212 Early heterogeneously forced simulations often favoured the formation of four quasi-stationary  
213 high latitude flux patches, with equatorial symmetric pairs at American and Siberian longi-  
214 tudes (40). However, the relative strength and stability of these pairs varied with model param-  
215 eters, such that a hemispheric imbalance in the time averaged field structure may arise (40, 46).

216 In our simulations, which similarly use a pattern derived from mantle tomography (38), strong  
217 heterogeneous boundary forcing tends to promote one pair of high latitude flux patches near the  
218 dateline ( $180^\circ$  longitude), with the patch southeast of New Zealand generally the stronger of  
219 the two. Core-mantle boundary heterogeneity may impart longitudinal structure into the long  
220 term average of Earth's magnetic field, but the combination of our results and previous work  
221 indicates that the location of that structure need not be a simple reflection of mantle thermal  
222 structure as it also depends on the balance of forces within the core.

223 We also find that mantle influence on the core results in persistent non-zonal structure at  
224 low latitudes. In studies at higher  $E$  and with  $Ra$  that is only slightly supercritical (23, 40) the  
225 wavelength of convective rolls in the fluid core was much larger than in our simulations. In  
226 those studies, the long-wavelength mantle pattern could couple to the large scale core flow and  
227 produce a locked dynamo state with nearly steady flows spanning the fluid shell. The difference  
228 between the scales of free convection and mantle forcing in our simulations mean that a locked  
229 dynamo does not emerge. However, enhanced short-wavelength convective activity does occur  
230 at low latitudes between the LLVPs, where the seismic model predicts relatively cool mantle  
231 material and hence high CMB heat flux that promotes downwelling at the top of the core. This  
232 results in the time-averaged magnetic fields of our  $q^* \neq 0$  simulations tending to have pairs of  
233 equator-straddling reverse flux patches beneath South America and the Indian Ocean (figure 4).  
234 Because of the temporal variability of the flow and field in our simulations the prevalence of  
235 reverse flux patches in these locations is not necessarily obvious in shorter time averages (e.g.,  
236 the 400-year window of figure 2b) and thus may be more observable in paleomagnetic data than  
237 in the modern field.

238 The Pacific and African LLVPs are taken to be anomalously hot in our model and thus tend  
239 to suppress convection in the outermost core underneath them, particularly when  $q^* = 5$ . The  
240 seismic velocity anomalies, and hence our inferred CMB heat flux anomalies, differ between

241 the two LLVPs, with stronger anomalies in the Pacific. Their geometries also differ, with the  
242 Pacific LLVP elongated longitudinally, whereas the African LLVP is elongated latitudinally. In  
243 our simulations small scale convective velocities, and hence short-wavelength variations of  $B_r$ ,  
244 tend to be weaker beneath the anomalously hot LLVPs, which could result in weaker observed  
245 secular variation. The difference in LLVP geometry and amplitude between the Pacific and  
246 African hemispheres might then provide a mechanism for explaining why secular variation of  
247 the modern geomagnetic field in the Pacific has been anomalously quiet (49).

248 Regional patterns of field structure and (paleo)secular variation differ somewhat between  
249 our simulations and it is not computationally feasible to explore a wide range of patterns and  
250 amplitudes of CMB heat flux heterogeneity. The consequences of uneven spatial and temporal  
251 sampling and smoothing in paleomagnetic field models (10) also limits our ability to resolve  
252 fine details of the structure and dynamics of Earth’s field prior to the modern observational  
253 era. Nevertheless, we find that there is no statistically significant preference for a hemispheric  
254 difference in secular variation in our  $q^* \neq 0$  simulations, Holocene field models (16), or the pa-  
255 leosecular activity index of GGF100k. This suggests that, although there is long-term non-zonal  
256 structure in Earth’s magnetic field, the hemispheric imbalance in secular variation observed for  
257 recent times is anomalously large. However, persistent features of the flow and field that arise  
258 from the mantle control, such as the eccentric gyre and low-latitude reverse flux patches, should  
259 be expected in the geological past for as long as the current distribution of LLVPs has been  
260 present.

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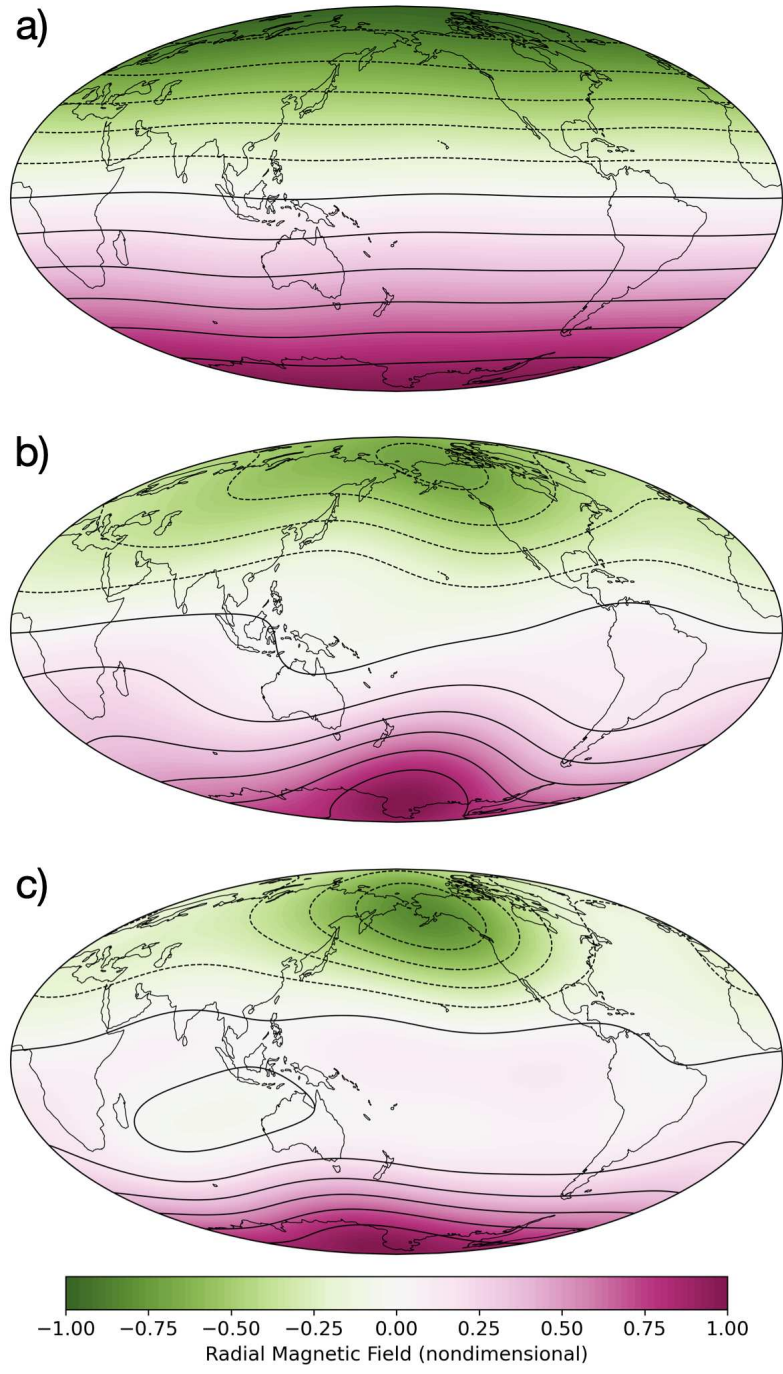


Figure 1: Time-averaged magnetic fields for our simulations with  $Ra = 2000$  and  $q^* = 0.0, 2.3, 5.0$  (a,b,c). The radial component of the magnetic field on the CMB truncated at spherical harmonic degree and order 4. All plots use the same colour scale.

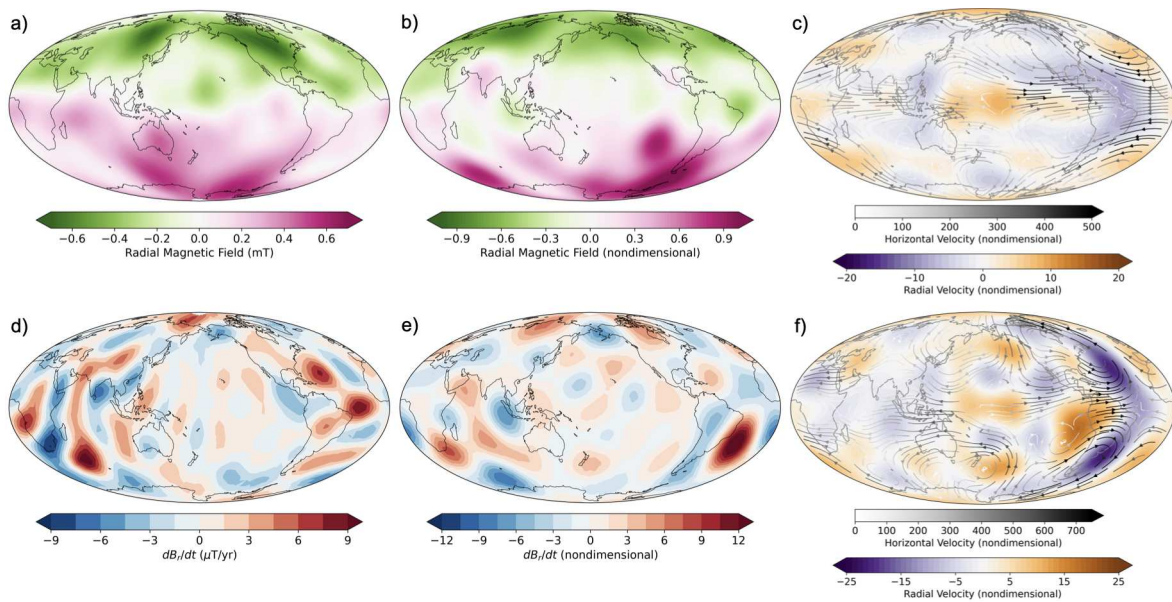


Figure 2: Comparison of our  $Ra = 2000$  and  $q^* = 2.3$  with *gufm1*. Time-averaged radial magnetic field at the core-mantle boundary from *gufm1* (a) and the best window of our simulation (b). Snapshot of secular variation in 1990 from *gufm1* (d) and the best window of our simulation (e). Time-averaged flow near the top of the core from the entire run of our simulation (c) and the best window (f). Magnetic and velocity fields are truncated at spherical harmonic degree and order 8.

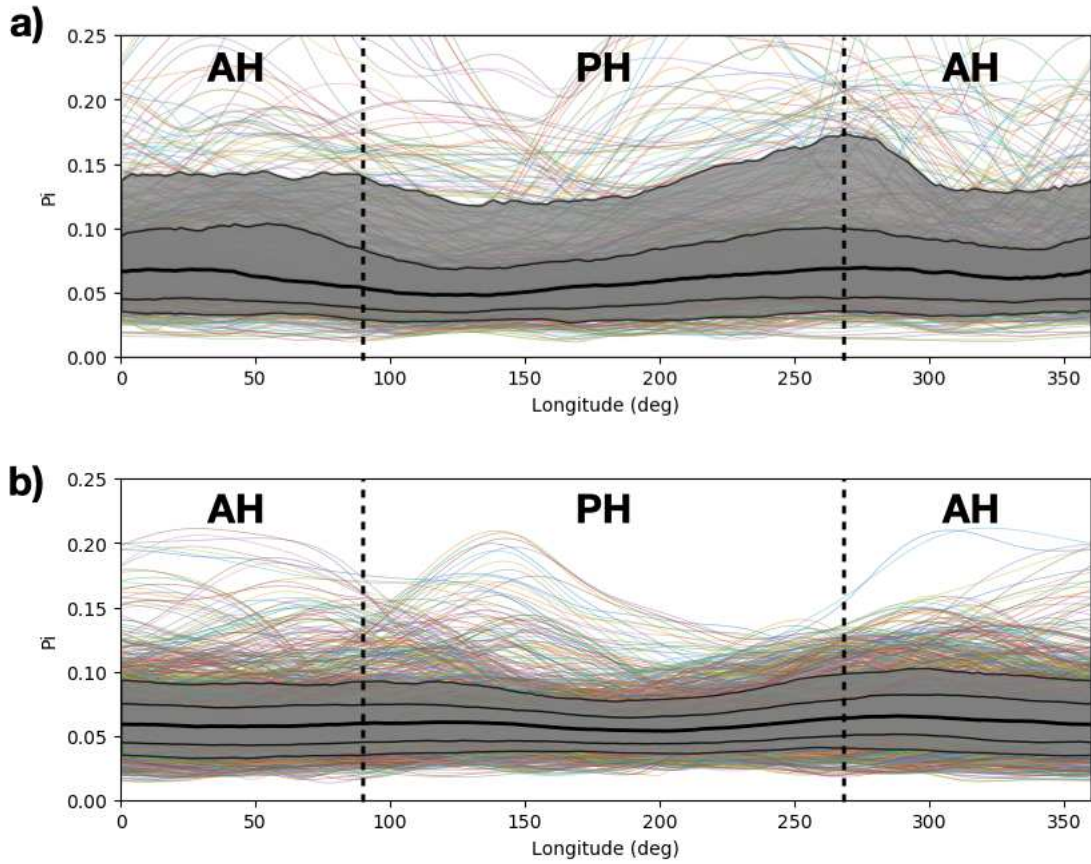


Figure 3: The longitudinal variation in paleosecular variation index ( $P_i$ ) in the GGF100k observational model (a), and our  $Ra = 2000$  and  $q^* = 2.3$  simulation (b). Coloured lines are  $P_i$  calculated at individual time points, thick black line is the median, grey bands indicate the 10–90, and 25–75 percentiles. Vertical dashed lines designate the boundaries between the Pacific and Atlantic hemispheres.

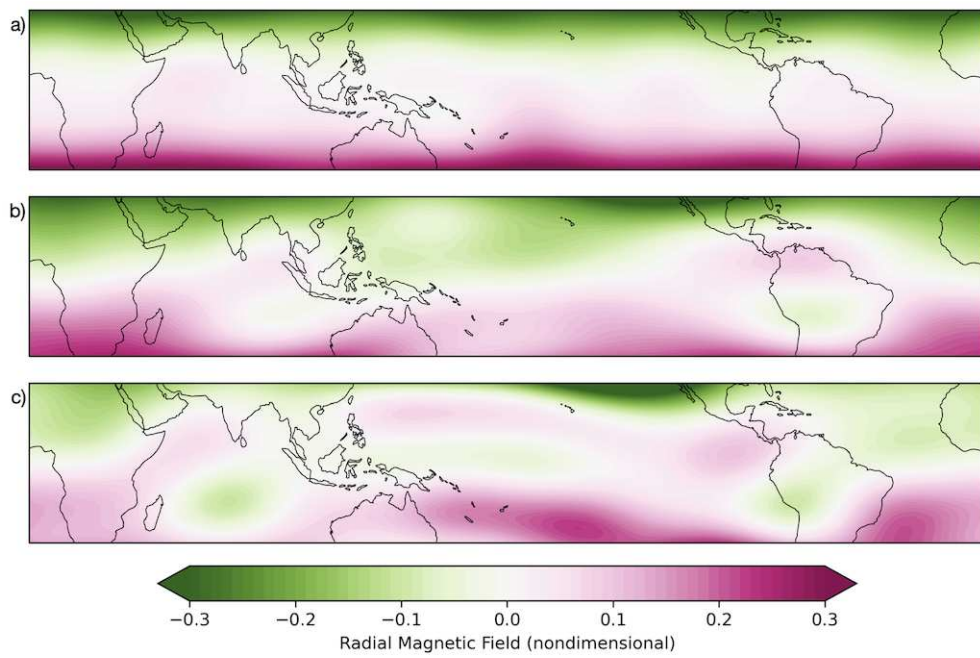


Figure 4: Time-averaged radial magnetic field at the core-mantle boundary in the equatorial regions of the simulations with  $Ra = 2000$  and  $q^* = 0, 2.3, 5$  (a,b,c). All plots use the same colour scale for the (non-dimensional) magnetic field strength and are truncated at spherical harmonic degree and order 8.

# Supplementary Information for: Lower Mantle Heat Flow Controls the Longitudinal Structure of Earth's Magnetic Field

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## Methods

### Numerical Simulations

We numerically solve the magnetohydrodynamic equations for rotating convection in a spherical shell under the Boussinesq approximation. These equations include the conservation of momentum, energy, and mass, the magnetic induction equation, and an equation of state, with the influence of density variations ignored other than as a source of buoyancy. Nondimensionalisation of this system of equations suggests a set of control parameters for the simulations. The Ekman number,  $E = \nu/2\Omega L^2$ , describes the ratio of viscous to Coriolis forces (where,  $\nu$  is kinematic viscosity of the fluid,  $\Omega$  rotation rate, and  $L$  a characteristic length-scale, taken to be the thickness of the spherical shell). A Rayleigh number,  $Ra = \alpha g_o \beta / 2\Omega \kappa$ , describes the strength of buoyant driving relative to dissipation (where,  $\alpha$  is the thermal expansivity of the fluid,  $g_o$  is gravitational acceleration on the outer boundary,  $\kappa$  is the thermal diffusivity of the fluid, and  $\beta$  is a measure of the average heat flux through the outer boundary). The Prandtl number,  $Pr = \nu/\kappa$ , and magnetic Prandtl number,  $Pm = \nu/\eta$ , relate the thermal, momentum, and magnetic diffusivities of the fluid (where,  $\eta$  is magnetic diffusivity). The simulations are

20 driven by thermal fixed-flux boundary conditions that are homogeneous on the ICB and hetero-  
21 geneous on the CMB, with no internal heat source or sink. The amplitude of CMB heat flux  
22 heterogeneity is described by  $q^* = (q_{\max} - q_{\min})/q_{\text{ave}}$  (where,  $q_{\max}$ ,  $q_{\min}$ ,  $q_{\text{ave}}$  are the maximum,  
23 minimum, and average values of CMB heat flux, respectively).

24 The amplitude of Earth's CMB heat flux variations is difficult to estimate because it must be  
25 inferred from seismic tomography while accounting for the possibility of both thermal and com-  
26 positional variations in the lower mantle. The nature of LLVPs is uncertain; however, the gen-  
27 eral view is that these features are anomalously hot, even if they have a substantial compositional  
28 contribution to their origin (1). Lateral variations in core-mantle heat flow due to the thermo-  
29 chemical variations of the lowermost mantle have been estimated in studies combining insight  
30 from seismic observations, mineral physics, and mantle convection simulations, which sug-  
31 gested a minimum heat flux of  $q_{\min} \approx 0 \text{ mW m}^{-2}$  and  $q_{\max} \geq 200 \text{ mW m}^{-2}$  (2–4). The adiabatic  
32 gradient at the top of the core is  $\partial T_a / \partial r = g\gamma T / \phi \approx -0.875 \pm 0.125 \text{ K km}^{-1}$  with the seismic  
33 parameter,  $\phi$ , and gravity taken from PREM (5) and estimates for the Grüneisen parameter  $\gamma$  of  
34 1.3–1.5 (6). Combined with the uncertainty in core thermal conductivity (7, 8), this gives a plau-  
35 sible range of adiabatic heat flux at the top of the core of  $q_a = -k\partial T_a / \partial r \approx 15 - 100 \text{ mW m}^{-2}$ ,  
36 implying that hot LLVPs in the lower mantle will result in a subadiabatic heat flux across the  
37 CMB. Overall,  $q^*$  of at least order 1 is expected and it could be larger than the values we con-  
38 sider.

39 Solidification of the inner core releases light elements (9) that likely remain trapped within  
40 the fluid core, providing an additional source of buoyancy at depth. If convection in Earth's core  
41 is dominated by compositional buoyancy, then a homogeneous zero-flux condition for compo-  
42 sition at the CMB might reduce the impact of thermal heterogeneity. Conversely, light elements  
43 trapped in Earth's core might act to raise the effective value of  $q^*$  as, like heat conducted along  
44 the adiabat, they represent a homogeneous source of buoyancy that is not available to promote

45 convection at the CMB. The overall impact of double-diffusive convection on Earth’s core dy-  
 46 namics will depend on the balance of the thermal and compositional driving and their boundary  
 47 conditions. An exploration of double-diffusive geodynamo conditions is beyond the scope of  
 48 this work; however, previous work has found that a wide variety of such simulations can repro-  
 49 duce Earth-like geomagnetic fields (10).

50 The dynamic conditions of Earth’s outer core are characterised by low Ekman, low Prandtl,  
 51 and high Rayleigh numbers (11). Earth-like values of these parameters are not computation-  
 52 ally accessible; however, we consider six simulations designed to have the appropriate bal-  
 53 ance of forces. Our simulations are characterised by  $E = 10^{-5}$ ,  $Pr = 0.2$ ,  $Pm = 1$ ,  
 54  $Ra = \{2000, 6000\}$ , and  $q^* = \{0, 2.3, 5.0\}$  (Table 1). The pseudo-spectral method used in  
 55 this work is described in more detail in (12). Velocity and magnetic field are decomposed  
 56 into toroidal and poloidal scalars, so that the divergence-free conditions are exactly satisfied.  
 57 All scalars are expanded in Schmidt-normalised spherical harmonics and represented in ra-  
 58 dius by second-order finite differences. The finite difference points are located at the zeros of  
 59 the Chebyshev polynomials, giving finer spacing near the boundaries of the fluid core. Time  
 60 stepping is accomplished in spectral space using a predictor–corrector scheme that treats dif-  
 61 fusion terms implicitly, while the Coriolis, buoyancy and nonlinear terms are treated explic-  
 62 itly. Nonlinear terms are transformed into real space at each time step using the spherical  
 63 transform method (13). At each radius multiplications are performed on a Gauss–Legendre  
 64 grid with  $(3/2)\ell_{\max}$  colatitude points and  $3\ell_{\max}$  longitude points. For all simulations the num-  
 65 ber of radial grid points,  $N_r = 256$ , and the maximum spherical harmonic degree and order,  
 66  $\ell_{\max} = m_{\max} = 192$ .

67 After removal of the initial transient, the resultant turbulent flows achieve an Earth-like  
 68 (14, 15) magnetic Reynolds number ( $Rm$  of order  $10^3$ ), are strongly influenced by rotation  
 69 (Rossby number,  $Ro$ , of order  $10^{-2}$ ), and generate relatively strong magnetic fields (as measured

70 by the Elsasser number,  $\Lambda$ ). For the  $Ra = 2000$  cases the average magnetic energy (ME) is a  
71 few times greater than the average kinetic energy (KE), while for the  $Ra = 6000$  cases they  
72 are roughly equal for the heterogeneous boundary cases. These simulations are, therefore, in  
73 the strong-field dynamo regime appropriate to Earth’s core (16). The  $Ra = 6000$ ,  $q^* = 0$   
74 case is an unstable frequently reversing dynamo with a relatively weak magnetic field. We use  
75 the magnetic diffusion time to re-scale the time and all cases have been run for a few tens of  
76 thousands of years in order to obtain robust estimates of the time-averaged behaviour (17).

## 77 **Measures of Magnetic Field and SV Structure**

78 To compare geometric features of the radial CMB magnetic field between simulations and ob-  
79 servational field models we compute the four compliance criteria of (18): the ratio of the power  
80 of the axial dipole to the non-axial dipole field (AD/NAD); the ratio of power in odd versus  
81 even spherical harmonic degrees (O/E); the ratio of zonal to non-zonal power (Z/NZ); and a  
82 factor quantifying how strongly radial flux is concentrated into localised patches (FCF). Previ-  
83 ous work (18) established a target value for each of these compliance criteria ( $C_i$ ) by averaging  
84 the over the 400-year long *gufm1* model (19) as well as an estimate of reasonable variability  
85 ( $\sigma_i$ ). For each characteristic the agreement of a simulation relative to the Earth is calculated as  
86  $\chi_i^2 = \left[ \left( \ln(C_i^{\text{sim}}) - \ln(C_i^{\text{gufm1}}) \right) / \ln(\sigma_i) \right]^2$ . The total semblance ( $\chi^2 = \Sigma \chi_i^2$ ) of a simulated  
87 field compared to *gufm1* can then be classified as excellent ( $\chi^2 \leq 2$ ), good ( $2 < \chi^2 \leq 4$ ),  
88 marginal ( $4 < \chi^2 \leq 8$ ), or non-compliant ( $8 < \chi^2$ ). The time-averaged value of the measures  
89 and the total semblance for each run are reported in Tables 1 and 2.

90 The modern geomagnetic field has stronger secular variation (SV) in the Atlantic than in  
91 the Pacific hemisphere (20, 21). We have previously constructed a measure of this hemispheric  
92 imbalance ( $H_{\text{sv}}$ ) and its variation which provides a  $\chi^2$  measure of compliance with respect  
93 to the quiet Pacific secular variation (22). We determine the pattern of secular variation in

94 our simulations by calculating the centred differences of Gauss coefficients from successive  
95 snapshots. We truncate the radial magnetic field at the CMB to spherical harmonic degree and  
96 order 8 and average our simulations over consecutive 400-year windows to calculate statistics  
97 of the compliance criteria throughout the runs.

98 The paleosecular variations index ( $P_i$ ) developed in (23) is a non-dimensional measure that  
99 can be constructed from observation of Earth's field at a given location and time.  $P_i$  depends  
100 on the departure of the observed virtual geomagnetic dipole latitude ( $\lambda_p$ ) from true north, and  
101 on the departure of the observed virtual dipole moment (VDM) from the present-day value. For  
102 example, if the VDM at a location is equal to the reference value of  $80 \text{ ZAm}^2$ , then a doubling  
103 of  $P_i$  from 0.05 to 0.10 corresponds to a change in  $\lambda_p$  from  $81^\circ$  to  $72^\circ$ . At each time point in  
104 GGF100k and the simulations  $P_i$  is calculated on a two degree by two degree latitude-longitude  
105 grid and these values are averaged over latitude to produce the longitudinal variation in  $P_i$  at  
106 each point in time. The average character of  $P_i$  at a given longitude is taken to be the median  
107 value from all time points, with the variability described by the 25-75 and 10-90 percentile  
108 values.

109 Spatial variability in field activity is also shown in maps of the mean value of  $P_i$  over the  
110 time span of each model (Supplemental Figure 4). The simulations allow for uniform high  
111 spatio-temporal resolution; whereas, paleomagnetic data such as that used in GGF100k are  
112 unevenly distributed in both space and time (24, 25). We compare to the observationally de-  
113 rived spherical harmonic model rather than the data directly; so, we have not down-sampled  
114 our simulation output to match the GGF100k sampling pattern. The simulations also provide  
115 essentially instantaneous measurements of the magnetic field and its time derivative; whereas,  
116 paleomagnetic records are variably smoothed in time depending on, for example, sedimenta-  
117 tion rates during magnetisation acquisition, or the availability of tightly bound absolute age  
118 constraints. Regional variation in this inherent temporal smoothing of the data (e.g., due to

119 different geological settings) could result in regional variation of  $P_i$  in a paleo-field model that  
120 would be unrelated to the true variability of the geomagnetic field. Differences in the spatial and  
121 temporal sampling of data must also be considered when comparing models such as GGF100k  
122 with field models based on modern (observatory or satellite) observations. Despite extensive  
123 computational effort, we are also only able to simulate a few combinations of mantle heat flux  
124 heterogeneity and bulk core dynamics for the length of time required to obtain useful long-term  
125 statistics. To help mitigate these considerations we mainly focus on the long wavelength and  
126 time-averaged features of the simulated and observationally derived magnetic field models and  
127 on features that are common across all of our  $q^* \neq 0$  simulations.

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## Additional Figures and Tables

Table 1: Dynamo Parameters that Vary Between Runs

$Ra$	$q^*$	$\Lambda$	$Rm$	$Ro$	ME/KE	Mean CC $\chi^2$	Mean $H_{SV}$ $\chi^2$
2000	0.0	34	889	0.018	2.2	2.12	3.79
2000	2.3	39	851	0.017	2.7	5.39	4.24
2000	5.0	50	830	0.017	3.4	14.51	4.11
6000	0.0	26	1848	0.037	0.37	60.14	3.94
6000	2.3	43	1608	0.032	0.84	1.05	2.88
6000	5.0	62	1483	0.030	1.4	3.30	3.88

Table 2: Geomagnetic Field Measures

$Ra$	$q^*$	AD/NAD		O/E		Z/NZ		FCF		$H_{sv}$	
		mean	$\sigma$	mean	$\sigma$	mean	$\sigma$	mean	$\sigma$	mean	$\sigma$
<i>gufm1</i>	<i>gufm1</i>	1.4	2.0	1.0	2.0	0.15	2.5	1.5	1.5	-0.24	0.07
2000	0.0	1.27	0.28	1.88	0.50	0.21	0.13	2.38	0.87	0.03	0.12
2000	2.3	0.78	0.26	1.30	0.33	0.17	0.07	4.58	2.35	0.05	0.12
2000	5.0	0.41	0.10	1.01	0.21	0.23	0.11	9.57	3.70	0.05	0.11
6000	0.0	0.07	0.09	0.82	0.19	0.16	0.11	2.29	0.87	0.04	0.09
6000	2.3	1.23	0.26	1.47	0.34	0.26	0.08	1.54	0.34	-0.04	0.07
6000	5.0	0.92	0.23	1.56	0.42	0.26	0.09	3.10	1.60	0.03	0.04

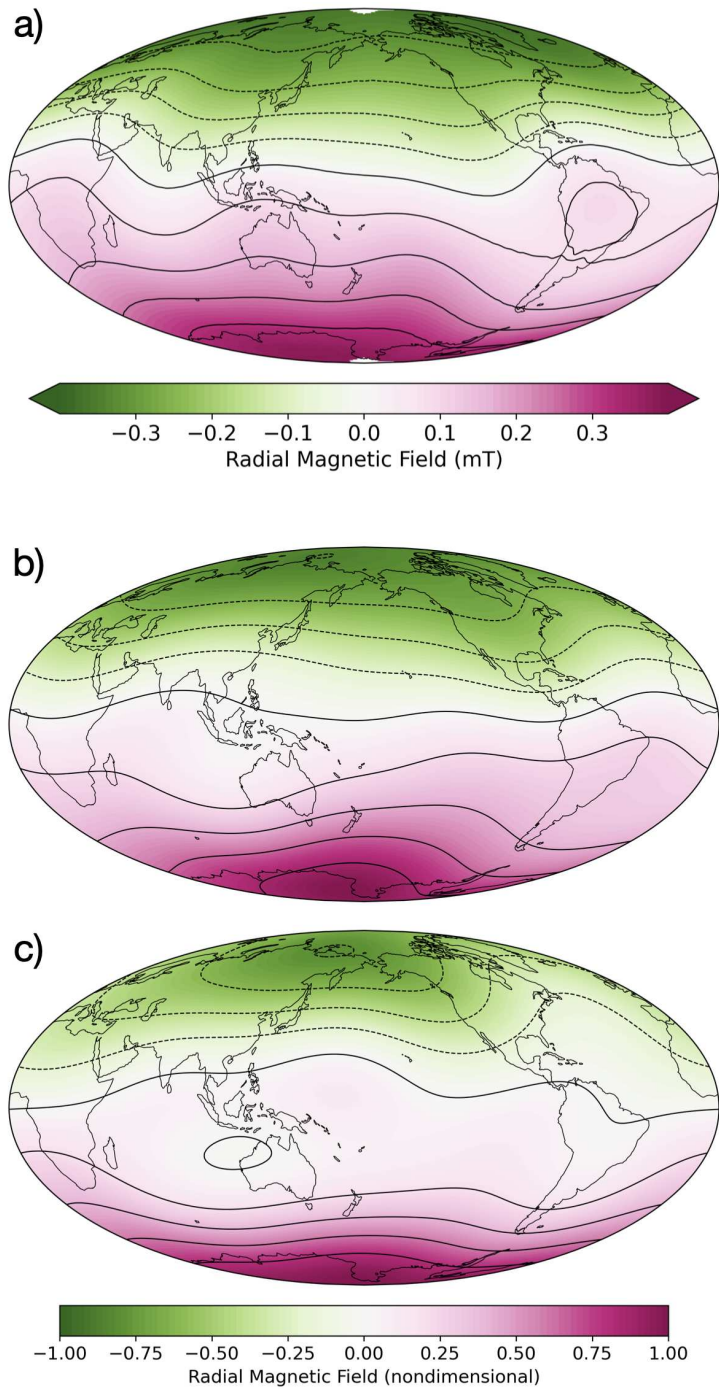


Figure 1: Time-averaged magnetic fields for GGF100k (a) and our simulations with  $Ra = 6000$  and  $q^* = 2.3, 5.0$  (b,c). The radial component of the magnetic field on the CMB truncated at spherical harmonic degree and order 4. Both simulation plots use the same colour scale.

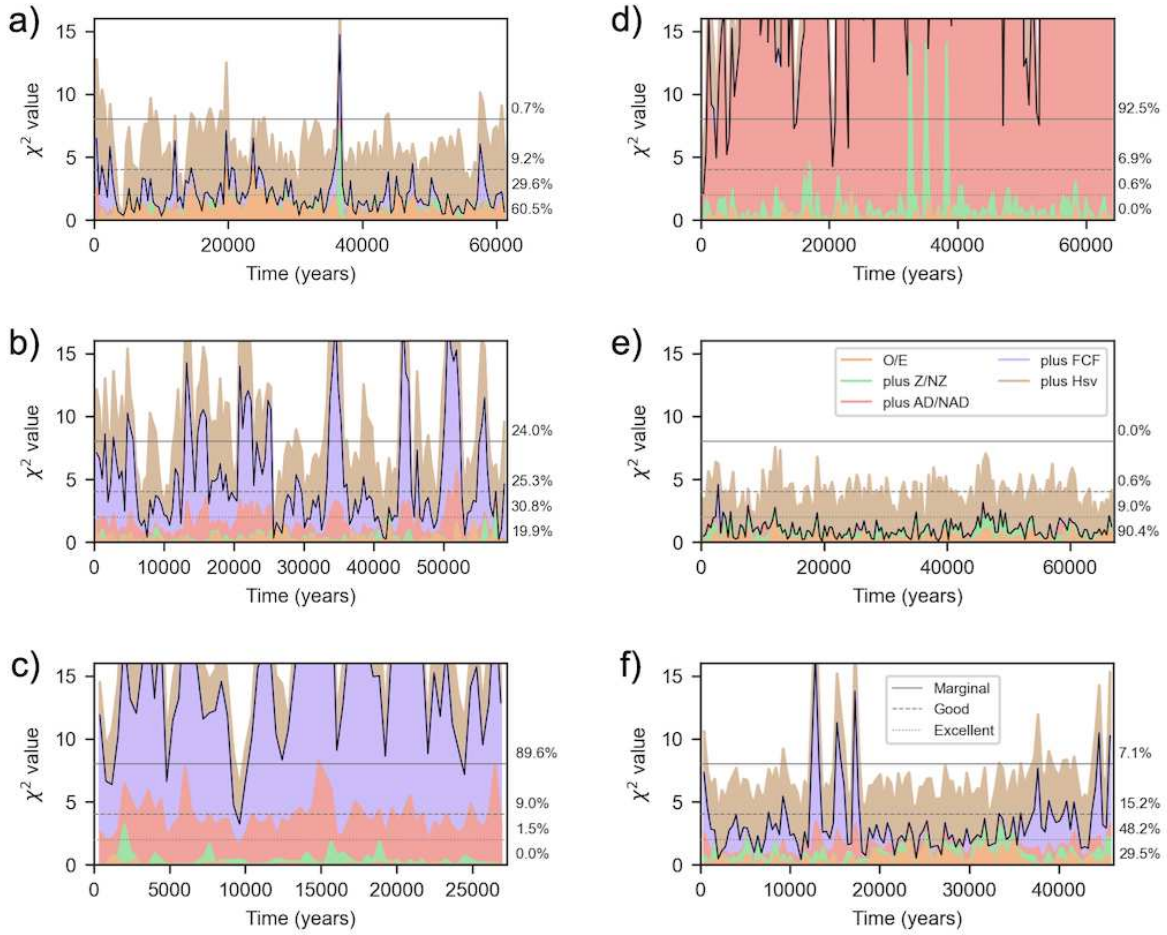


Figure 2: Evolution of the contributions to magnetic field and secular variation semblance over our simulations. The  $\chi^2$  contribution from the O/E, Z/NZ, AD/NAD, FCF, and  $H_{sv}$  measures are given by the orange, green, red, purple, and brown filled areas, respectively. The black solid line highlights the sum of the four compliance criteria for the magnetic field geometry and the grey horizontal lines indicate the values below which this total compliance is considered excellent, good, or marginal in comparison with Earth as derived from *gufm1*. Values to the right of each panel indicate the percentage of 400-year windows that fall in each compliance band. Simulations have  $Ra = 2000$  (panels a,b,c) or  $Ra = 6000$  (panels d,e,f) and  $q^* = 0.0$  (a,d),  $q^* = 2.3$  (b,e), or  $q^* = 5.0$  (c,f).

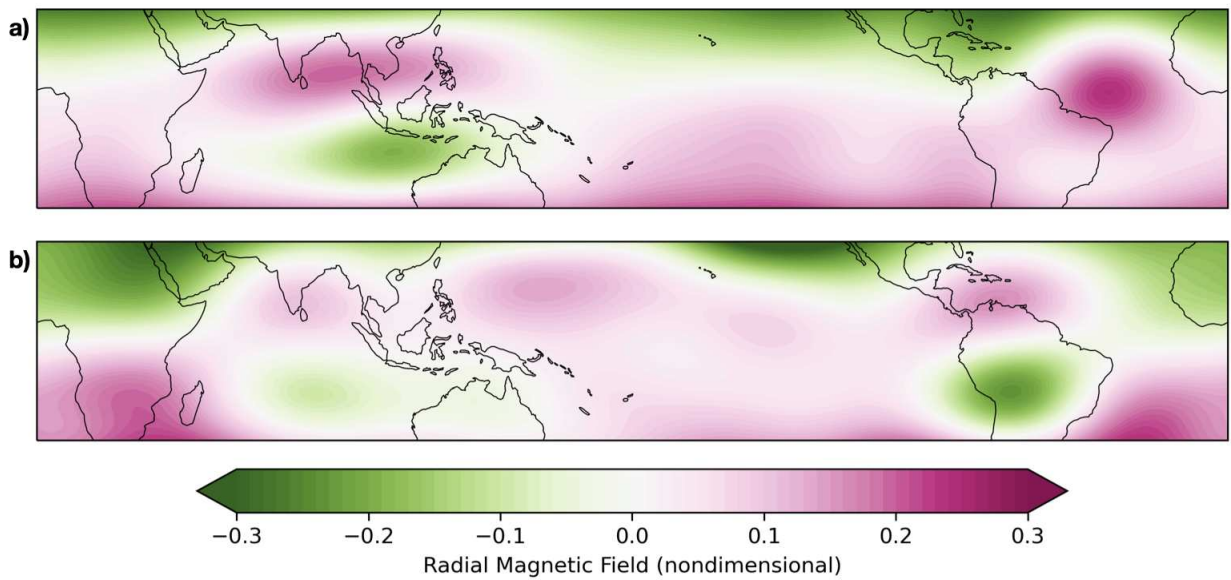


Figure 3: Time-averaged radial magnetic field at the core-mantle boundary in the equatorial regions of the simulations with  $Ra = 6000$  and  $q^* = 2.3, 5$  (a,b). Both plots use the same colour scale for the (non-dimensional) magnetic field strength and are truncated at spherical harmonic degree and order 8.

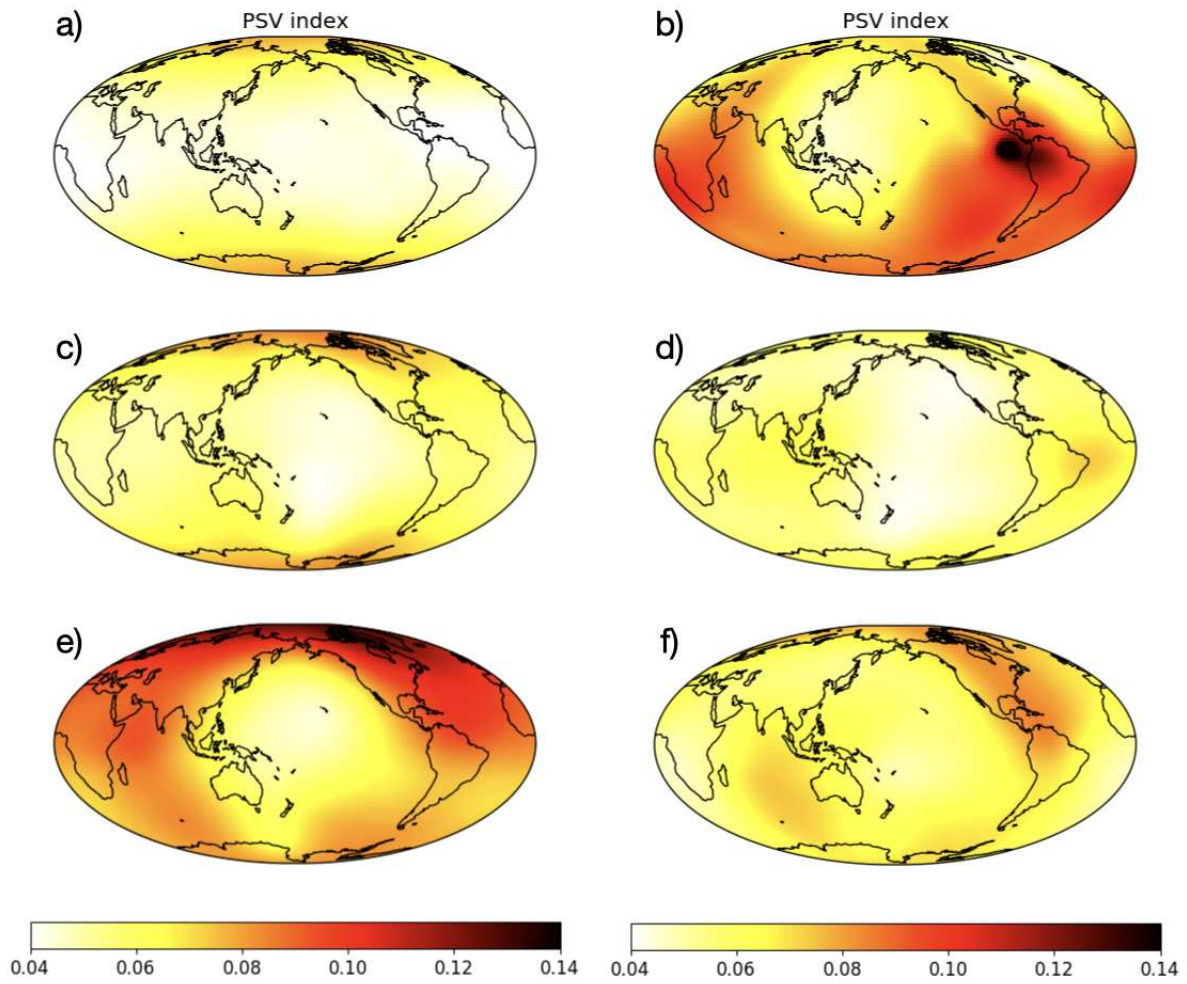


Figure 4: Time-averaged values of the paleosecular variation index from our simulations ( $Ra = 2000$  and  $q^* = 0, 2.3, 5$  (a,c,e);  $Ra = 6000$  and  $q^* = 2.3, 5$  (d,f) and GGF100k (b). All plots use the same colour scale.