MIC2022
22nd Machining Innovations Conference for Aerospace Industry

Proceedings are available on SSRN: www.ssrn.com

SSRN Proceedings Hosted on SSRN.com

MIC Procedia (2022) 094-100

22nd Machining Innovations Conference for Aerospace Industry 2022 (MIC 2022), November 30th and December 1st 2022, Hannover, Germany

Active chatter suppression through virtual inerter-based passive absorber control

Muhammet Ozsoy[a]*, Hakan Dogan[a,b], Erdem Ozturk[c], David J. Wagg[a], Neil D. Sims[a]

[a]Department of Mechanical Engineering, The University of Sheffield, Mappin Street, Sheffield S1 3JD, UK
[b]Department of Mechanical Engineering, University of Bath, Bath, BA2 7AY, UK
[c]Advanced Manufacturing Research Center, The University of Sheffield, Wallis Way, Rotherham, Sheffield S60 5TZ, UK

* Corresponding author. *E-mail address: mozsoy1@sheffield.ac.uk*

Abstract

The role of inerter-based devices has generated considerable interest in terms of suppressing the vibrations in machines and structures. The inerter is a mechanical device that generates force proportional to the relative acceleration between its terminals. Recently, it has been shown that inerter-based dynamic vibration absorbers (IDVAs, for the mass ratios between 0 and 0.2) can improve the chatter suppression performance compared to a traditional tuned mass damper (TMD) for the same mass ratios. This study proposes an IDVA applied to machining operations as a novel active control method to increase chatter suppression performance. Considering the TMD application as a virtual passive absorber (VPA) method in active control, IDVAs can be potentially employed in the same framework. A proof-mass actuator, which is mounted on a beam that is designed to support a flexible structure, is proposed. Once the IDVA parameters are optimised, a time-domain model is applied to explore the actuator saturation effects. The effect of an IDVA as a novel active control method on chatter stability is then evaluated. The simulated stability lobe diagram shows that the IDVA increases the absolute chatter stability by just above 20%. To validate the simulation results, an experimental setup is designed including a flexible workpiece to be machined and a proof-mass actuator assembled using a beam. In summary, it is shown that inerter-based dynamic vibration absorbers, as an active control method, can successfully be implemented to improve the chatter suppression performance and critical limiting depth of cut.

Keywords: Active chatter control; inerter-based dynamic vibration absorber; chatter stability; proof-mass actuator

1. Introduction

Regenerative chatter is one of the most detrimental issues that reduce productivity in machining operations. It leads to excessive vibrations, noise, poor surface quality, damage or even breakage of the tool. After understanding the theory underlying the chatter mechanism [1,2], different chatter avoidance or suppression methods have been proposed. Among the different methods, passive control has frequently been applied as an effective method. Tuned mass dampers (TMDs) have been employed to develop the dynamic response of the most flexible modes of the machining systems in order to mitigate chatter vibration during the cutting process. Rivin and Kang [3] analysed the design of cantilever tools to achieve high stiffness with low effective mass, which allows high mass ratios for TMDs. Tarng et al. [4] employed a piezoelectric inertia actuator in turning as a TMD by manually adjusting the natural frequency of the absorber equal to the natural frequency of the cutting tool. However, as the absolute chatter stability is proportional to the real part of the FRF, the best performance can be obtained by considering the real part. Sims [5] presented an analytical tuning strategy to obtain the best performance for TMDs in machining operations. The effectiveness of Sims' method over classical Den Hartog's method has also been shown in chatter suppression in a boring bar [6,7]. In milling operations, passive dampers have been mostly employed on the workpieces to suppress chatter that occurs due to the thin-wall parts [8–11]. Moreover, researchers have investigated multiple TMDs [12,13], two degree-of-freedom TMDs [14,15], and nonlinear TMDs with a friction element [16] and a cubic stiffness [17] to increase chatter suppression performance of a single degree-of-freedom (SDOF) TMD in turning and milling operations.

The introduction of the inerter by Smith [18] has provided passive control devices with better performance. The inerter, a relatively new mechanical device, produces inertial forces proportional to the relative acceleration between its terminals—see also [19,20]. The integration of the inerter into the passive layouts with the traditional stiffness and damping elements has led to improvement in the dynamic performance in a wide range of fields such as civil engineering structures [21–25], and vehicle and train suspension systems [26-29]. The benefits of using inerters in milling operations have also been investigated. Wang et al. [30] used inerters to mitigate vibrations in milling. However, only the forced vibration response of the machining system rather than the chatter stability has been considered. Chatter stability improvement using inerters in milling operations has been considered by Dogan et al. [31]. They numerically analysed four different inerter-based dynamic vibration absorbers and have shown the possibility of chatter suppression improvements.

However, the passive control approaches require manual adaptation considering the change in the system's configuration. The stability of the machining process can be degraded due to the variation of the dynamic properties of the system. This issue can be overcome by active control applications [32]. For instance, researchers [33,34] proposed active control methods to suppress the chatter vibrations. They applied direct velocity feedback successfully, which is relatively easy to apply. Also, model-based control methods [35,36] were presented to improve the chatter stability. However, the structural properties must be identified to employ the model-based control methods. Recently, Ozsoy et al. [37] presented six different control methods using an inertial (proof-mass) actuator to improve the chatter stability in milling. It has been shown that the active control applications can improve the chatter stability and critical limiting depth of cut. Also, they stated that there is no significant difference in performance improvement between the control methods as long as the control parameters have been optimised for the machining process.

The virtual version of TMD, called the virtual passive absorber (VPA) approach, was considered by Huyanan and Sims [38] to mitigate the chatter vibrations in milling. They presented three control methods, the skyhook, virtual passive absorber (VPA), and virtual active tuned mass damper. An inertial actuator was attached to a workpiece. VPA had better performance according to the frequency response function (FRF) results as it was not affected by the measurement noise. It is possible to improve chatter stability performance integrating the inerter concept into the VPA approach. For this, one of the IDVA layouts in [31], which already showed the chatter performance improvement in milling, can be employed. In this paper, an IDVA layout is applied as a virtual passive absorber approach to improve absolute chatter stability in milling operations. The actuation force of the passive layout is realised through a proof-mass actuator and the performance improvement is shown numerically and experimentally.

In Section 2, machining stability theory and the mechanical model for the IDVA are presented. Section 3 considers the numerical optimisation method to obtain the optimal design parameters. Then, results of numerical analyses are given in Section 4 and experimental verification is presented in Section 5. Section 6 presents a discussion and finally, conclusions are drawn in Section 7.

2. Theory

1.1. *Machining dynamics*

Understanding of regenerative chatter was first presented by Tobias and Fishwick [1] and Tlusty and Polacek [2] for continuous cutting operations. It was shown that the stability limit is inversely proportional to the real part of the system's frequency response function (FRF). Then, it was extended to milling operation, where the amplitude and orientation of the cutting force vary due to the rotating tool. Regenerative chatter mechanism is explained using an SDOF cutting tool and rigid workpiece as shown in Figure 1. Due to the flexible cutting tool, the instantaneous chip thickness $h_j(t)$ varies depending on the displacements of the tool in previous and current cuts. Assuming that the cutting force exerted on the tool-workpiece is proportional to the chip area, this change in the instantaneous chip thickness leads to the dynamic cutting force. The chosen cutting conditions (e.g. axial depth of cut and spindle speed) could cause instability. In that case, the cutting forces and the vibrations grow exponentially.

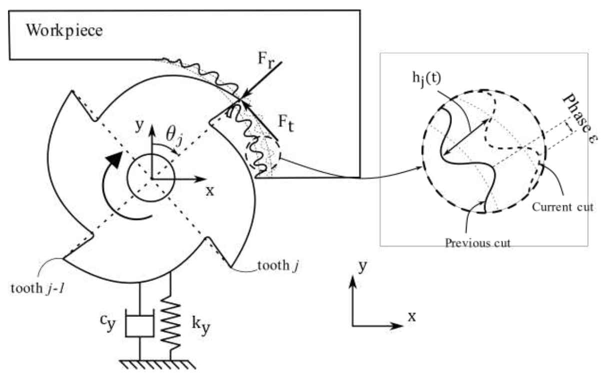

Figure 1- Schematic view showing the regenerative mechanism in an SDOF milling operation

The analytical prediction for the stability limits of a milling operation can accurately be determined by applying the zero-order approach, where the directional dynamic milling force coefficients are averaged using the Fourier series expansion. For an SDOF system (flexible in the y-direction), the stability limit is defined as [39–41]:

$$a_{lim} = \frac{-1}{\left(\frac{N}{2\pi}\right)\alpha_{yy}K_t\mathrm{Re}[G_y(i\omega_c)]}, \quad (1)$$

where a_{lim} is the axial depth of cut at stability limit, N is the number of flutes, K_t is the tangential cutting stiffness, $G_y(i\omega_c)$

is the FRF of the machining system, and ω_c is the chatter frequency. $\propto_{yy}$ is directional coefficient in the y-direction, which comes from the single frequency solution of the dynamic milling coefficient matrix. The relationship between the spindle speed and the chatter frequency is written as

$$2\pi k + \varepsilon = \omega_c \tau, \tag{2}$$

where k is the integer number of oscillations between each pass, ε is the phase of the oscillations, and τ is the spindle period. All parameters in Eq. 1 are related to cutting conditions except the FRF of the machining system $G_y(i\omega_c)$. Depending on the cutting conditions, the sign of $\propto_{yy}$ can be negative or positive and thus, the critical depth of cut a_{lim} is specified by the most negative or positive real part of $G_y(i\omega_c)$. It is worth noting that the method presented here is identically applicable for negative values of $\propto_{yy}$. This paper will focus on the case where $\propto_{yy}$ is positive. The most negative real part, which defines the stability boundary in this case, can be manipulated using active or passive control methods.

1.2. *Inerter-based dynamic vibration absorber*

Chatter stability limit is determined utilising the most flexible modes of the system. If the system has one dominant vibration mode, the machining system can be modelled as SDOF system. A milling system with an IDVA can be represented as illustrated in Fig. 2. The milling system is modelled as a SDOF system excited by the cutting forces and has parameters, mass M kg, spring stiffness K N/m, and viscous damping C Ns/m. The IDVA, which has a mass m kg, an outer spring k_o N/m, an inner spring k_i N/m, an inerter b kg, and a damper c Ns/m, is attached to the machining system to suppress chatter vibration. This layout involves no grounded inerter connection which makes possible the use of this passive layout in a similar manner to a classical TMD. It has been examined for machining chatter suppression compared to other layouts and shown one of the best performances [31]. The acting force on the machining system from the inerter-based passive layout is realised by using an actuator in an active control manner.

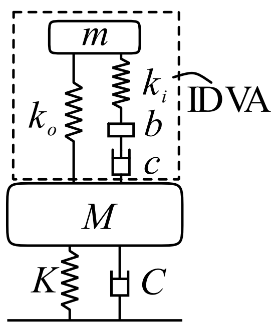

Figure 2 - Mechanical model of the milling system controlled with IDVA consisting of an inerter, an outer spring, an inner spring, and a damper.

The transfer function of the SDOF system controlled with the IDVA, shown in Fig. 1, is written as

$$G(s) = \frac{X(s)}{F_{cut}(s)} = \frac{1/M}{s^2+2\xi\omega_n s+\omega_n^2+A(s)/M}, \tag{3}$$

where $\omega_n = \sqrt{K/M}$ is the natural frequency of the machining system, $\xi = C/(2\sqrt{MK})$ is the damping ratio of the machining system, and $A(s) = F_a(s)/X(s)$ is the transfer function of the auxiliary system, which is given in Appendix A. Additional to the natural frequency ω_n and the damping ratio ξ of the machining system, a series of non-dimensional parameters are defined to derive the non-dimensional transfer function: mass ratio $\mu = m/M$, inertance-to-mass ratio $\delta = b/m$, auxiliary damping ratio $\zeta = c/(2\sqrt{mk})$, corner frequency ratio $\eta = \omega_b/\omega_m$, natural frequency ratio $\gamma = \omega_m/\omega_n$, and $\lambda = \omega/\omega_n$, where $\omega_m = \sqrt{k/m}$ and $\omega_b = \sqrt{k_i/b}$. Using the non-dimensional parameters above and substituting $s = j\omega$, the non-dimensional transfer function can be written in the form of

$$\tilde{G}(j\lambda) = \frac{R_N+jI_N}{R_D+jI_D}, \tag{4}$$

where the full expressions for R_N, R_D, I_N, and I_D are given in Appendix A.

3. **Numerical optimisation for chatter stability**

Tuning parameters is significant to obtain the best results. To obtain the optimum parameters, numerical optimisation is conducted. The stability limit of the milling operation is inversely proportional to the real part FRF as given in Eq.1. The objective is to maximise the most negative real part. Therefore, the optimisation problem for a known mass ratio is described as

$$min_{\gamma,\zeta,\delta,\eta}\left(\mathrm{Re}\{\tilde{G}(j\lambda)\}\right). \tag{5}$$

which is to be solved for $[\gamma, \zeta, \delta, \eta]$ to maximise the most negative real part of the FRF by utilising Self-adaptive Differential Evolution (SaDE) algorithm [42,43] in Matlab.

The optimisation problem in Eq. 5 is to solve for the modal parameters of the milling system in the experimental setup, as will be presented in Section 5. The experimental setup consists of a workpiece clamped to flexure, a relatively rigid cutting tool, and an actuator supported by a beam. The workpiece-beam-actuator system is the most flexible part of the milling system. An impact hammer test was applied to identify the dynamic parameters of the most flexible part using the impact hammer Dytran 5800B2 and the accelerometer PCB 353B18. The natural frequency, modal mass and structural damping ratio were found as 129.29 Hz, 20.31 kg, and 0.0134, respectively. The first mode of the workpiece-beam-actuator system was found as discrete and far from the second mode. Hence, the milling system is assumed to be a SDOF system.

The mass ratio was chosen considering the maximum actuator force, which is 27 N. Considering the dynamic parameters of the milling system, the optimal design parameters for mass ratio $\mu = 0.0055$ are given in Table 1. The maximum force that the proof-mass actuator provides was considered in the choice of the mass ratio.

Table 1. Optimal design parameters of the IDVA for $f_n = 129.29$ Hz, $M = 20.31$kg, $\xi = 0.0134$, and $\mu = 0.0055$ obtained from SaDE.

	γ	ζ	δ	η
TMD	1.0325	0.0473	-	-
IDVA	1.0404	0.0428	0.0145	1.0028

4. Numerical results

The negative real parts of the FRFs for the uncontrolled system (dotted black line), systems controlled with the TMD (solid black line) and the IDVA (solid blue line) for the optimal design parameters in Table 1 is presented in Figure 3. The most negative real part is reduced to -4.60×10^{-7} m/N with use of the TMD and further reduced to -3.79×10^{-7} m/N with the use of IDVA, which refers to 17.6% improvement. Using the FRFs in Fig. 3, the stability lobe diagrams (SLDs) for the cutting parameters in Table 2 are presented in Fig. 4. Similar to FRFs' improvements, the absolute stability limit a_{lim} with the IDVA is improved by 21.4% (from 3.62 mm to 4.39 mm) compared to the absolute stability limit obtained by the TMD.

Table 2. Structural and machining parameters

Structural Parameters		Machining Parameters	
Natural frequency	129.29 Hz	Tool diameter	16 mm
Damping ratio	1.34 %	Number of teeth	4
Stiffness	$1.34\text{x}10^7$ N/m	Tool helix angle	30°
Machining Parameters		Material	Al-7075-T6
Milling type	Down milling	Cutting stiffness K_r	$180\text{x}10^6$ N/mm^2
Radial immersion	Half immersion	Cutting stiffness K_t	$660\text{x}10^6$ N/mm^2
Feed per tooth	0.05 mm		

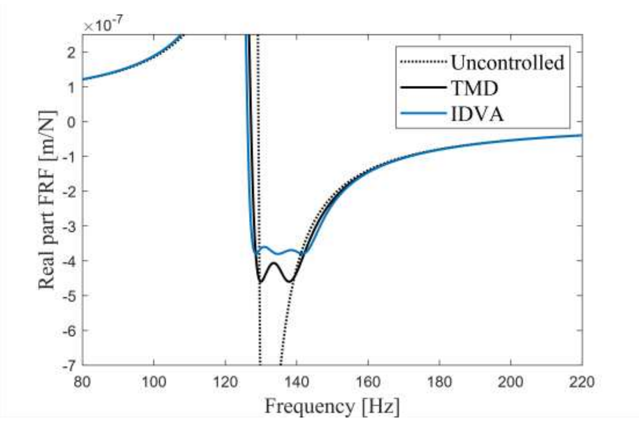

Figure 3 - FRFs of uncontrolled system, and systems with the TMD and IDVA for the design parameters obtained by SaDE.

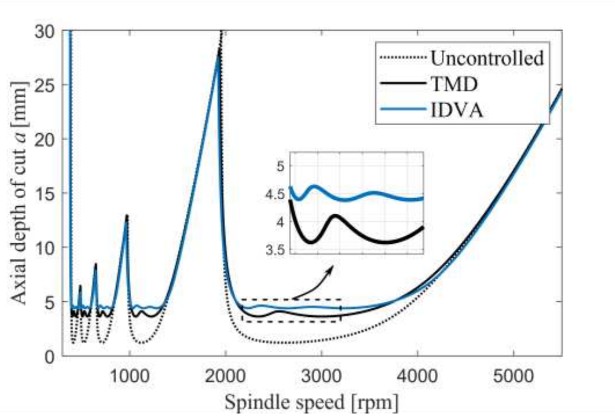

Figure 4 - Stability lobe diagrams for the uncontrolled system, and the systems with TMD and IDVA for the FRFs shown in Fig. 3.

5. Machining trials for experimental verification

The flexible workpiece was fixed to a CNC table as illustrated in Fig. 5. An inertial actuator was attached to the workpiece through a beam to support the flexible direction. An accelerometer (PCB 353B18) and a hall effect sensor were utilised to measure the acceleration of the workpiece and to detect the chatter by once per revolution [44], respectively. In order to collect and process the data, NI DAQ USB-4431 was used. The dimension of the aluminium block was 100x100x300 mm. In order to detect the chatter, once per revolution and the fast Fourier Transform (FFT) spectrum were used.

The milling experiments were performed to verify the chatter stability for uncontrolled and controlled cases. Firstly, the cutting parameters were chosen for the uncontrolled case where the chatter boundary was predicted to be 1.2 mm, as shown in Fig. 6. The parameters at 1100 rpm, 2 mm depth of cut were selected for the uncontrolled case, verified as a chatter cut as seen in Fig. 7 where the tooth-pass (circle), run-out (square) and chatter (diamond) frequencies can be seen in the FFT spectrums.

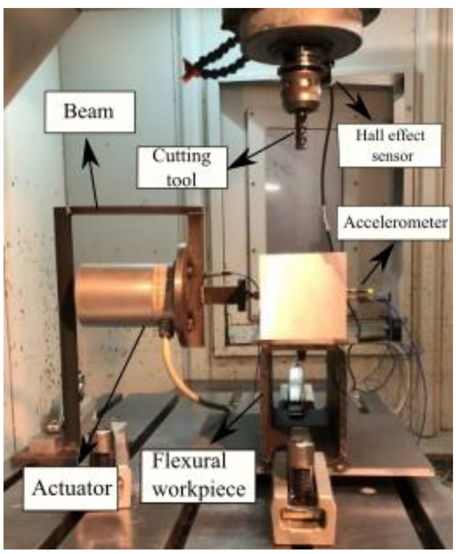

Figure 5 - The experimental setup used in milling trials

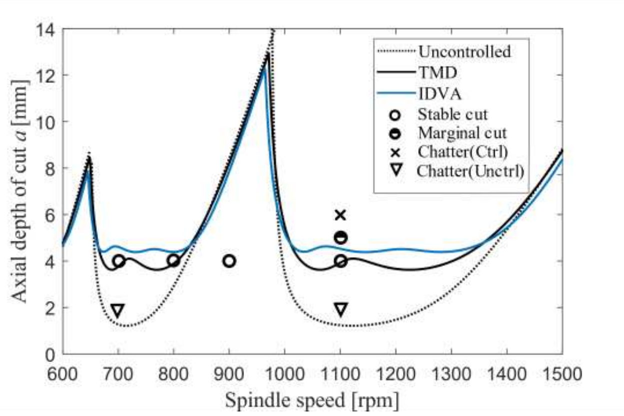

Figure 6 - Numerically obtained SLDs where experimental cuts are marked.

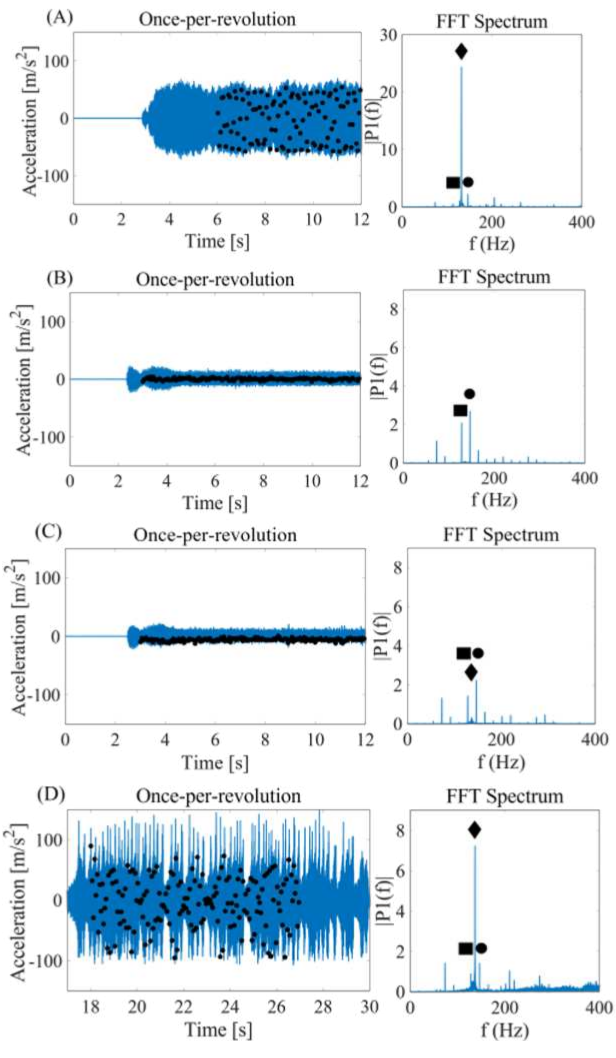

Figure 7- Experimental cuts: (A) uncontrolled, 1100 rpm, 2 mm doc, (B) controlled, 1100 rpm, 4 mm doc, (C) controlled, 1100 rpm, 5 mm doc, (D) controlled, 1100 rpm, 6 mm doc. Tooth-pass, run-out and chatter frequencies are represented by ●,■,◆, respectively.

Then, the experiments were carried out for the controlled case. In order to verify the control effect on the critical limiting depth of cut, the parameters at 1100 rpm, 4 mm depth of cut (stable), 1100 rpm, 5 mm depth of cut (marginal) and 1100 rpm, 6 mm depth of cut (chatter) were chosen. The chatter boundary was verified experimentally as it can be seen from FFT spectrums for the related cuts in Fig. 7. There are only tooth-pass and run-out frequencies for the parameters 1100 rpm and 4 depth of cut mm in the FFT spectrum, which meant to be a stable cut between TMD and IDVA SLDs. A chatter frequency that has a lower magnitude value than the tooth-pass frequencies, was observed for 5 mm depth of cut at the same spindle speed, which meant to be a marginal cut. A chatter cut was obtained with 6 mm depth of cut, which has a chatter frequency with larger magnitude value than the other frequencies' in the FFT spectrum.

6. Discussion

The numerical and experimentally verified results show that the IDVA can improve the absolute chatter stability limit by just above 20% compared to a traditional TMD. The performance improvement has been limited due to the small mass ratio (mu=0.0055) chosen. It can be increased up to 40% for higher mass ratios [31]. The main reason for the choice of a small mass ratio is the maximum actuator force. The proof-mass actuator employed in the experiment has only 27 N maximum actuation force. Hence, the choice of a higher mass ratio would lead to the saturation of the actuation force in the actuator since the actuator force of the layout is proportional to the mass ratio. Actuators that can produce higher maximum actuation forces or the machining systems with smaller modal masses will provide better performance because they allow having higher mass ratios.

The optimal design parameters for the IDVA have been obtained for the absolute stability limit. Therefore, the improvement has been shown for only the absolute stability limit. There has been no improvement in the deep stable region between two lobes, which is generally preferable to operate the cutting operations there due to the high depth of cut values. The improvement in these regions needs a different objective function to be optimised.

The optimisation was conducted for the maximisation of the most negative part of the FRF as only the case where the sign of $\propto_{yy}$ is negative. The analysis conducted in this paper is identically applicable for milling operations with positive sign of $\propto_{yy}$ as well as turning operations. Experimental verification of the improved performance was important to indicate the potential of this approach. Although the machining system is a damped system and the computational time for the optimisation was very short (under a minute), this method would be more convenient to apply with an analytical optimisation method aiming at the real part of the FRF similar to Sims' method [5]. In that way, the optimal design parameters for the inerter-based virtual passive absorber approach can easily be adapted for changing modal parameters of the machining system due to the material removal.

7. Conclusion

This study applied an active control through an inerter-based virtual passive absorber approach in order to increase the chatter stability in milling. SaDE was utilised to find the optimal design parameters. The numerical analysis showed that an IDVA could improve the absolute chatter stability by just above 20% compared to TMD. This improvement was limited due to the maximum actuator force used in the experiment. Finally, the numerical result was experimentally verified by cutting trials.

This paper focused on only one inerter-based layout. It is possible that another inerter-based layout could produce less actuation force for the same mass ratio. Also, the effect of the changing modal parameters of the milling system due to the material removal on the chatter performance has not been investigated. These two points should be considered in the future.

To conclude, the proposed IDVA is a feasible active control method in order to improve the chatter stability in milling.

Acknowledgements

MO and HD would like to acknowledge the support from the Turkish Ministry of National Education by providing the scholarships.

Appendix A. Expressions for the FRF

$A(s) = F_a(s)/X(s)$ in Eq.3 is the transfer function of the auxiliary system in Fig.1:

$$A(s) = \frac{bm(c+k_o)s^4 + mk_o(b+c)s^3 + mk_o(c+k_o+k_i)s^2}{bms^4 + m(b+c)s^3 + [k_o(b+m) + m(c+k_i) + bck_i]s^2 + k_o(c+b)s + k_o(k_o+k_i+c)}$$

The full expressions for the terms in Eq. 4:

$$R_N = -2\delta\eta^2\gamma^2\lambda^2\zeta + 2\eta^2\gamma^4\xi - 2\eta^2\gamma^2\lambda^2\xi - 2\gamma^2\lambda^2\xi + 2\lambda^4\xi$$
$$I_N = \delta\eta^2\gamma^3\lambda - \delta\eta^2\lambda^3\gamma$$
$$\begin{aligned} R_D = {} & 2\delta\eta^2\gamma^2\lambda^4\mu\zeta + 2\delta\eta^2\gamma^2\lambda^4\xi - 2\eta^2\gamma^4\lambda^2\mu\xi - 2\eta^2\gamma^4\lambda^2\xi + 2\eta^2\gamma^2\lambda^4\xi - 2\delta\eta^2\gamma^2\lambda^2\xi + 2\gamma^2\lambda^4\mu\xi \\ & + 2\eta^2\gamma^4\xi - 2\eta^2\gamma^2\lambda^2\xi + 2\gamma^2\lambda^4\xi - 2\lambda^6\xi - 2\gamma^2\lambda^2\xi + 2\lambda^4\xi + 2\zeta\lambda^4\delta\eta^2\gamma \\ & - 2\zeta\lambda^2\delta\gamma^3\eta^2 \end{aligned}$$
$$\begin{aligned} I_D = {} & -\delta\eta^2\gamma^3\lambda^3\mu - \delta\eta^2\gamma^3\lambda^3 + \delta\eta^2\gamma\lambda^5 + \delta\eta^2\gamma^3\lambda - \delta\eta^2\gamma\lambda^3 + 4\xi\zeta\lambda^5 - 4\xi\zeta\lambda^3\gamma^2 - 4\xi\zeta\lambda^3\delta\eta^2\gamma^2 \\ & + 4\xi\zeta\lambda\gamma^4\eta^2 - 4\xi\zeta\lambda^3\eta^2\gamma^2 \end{aligned}$$

References

[1] S. A. Tobias and W. Fishwick, "Theory of Regenerative Machine Tool Chatter," *Eng.*, 1958.

[2] J. Tlusty and M. Polacek, "The Stability of Machine Tools Against Self Excited Vibrations in Machining," *Int. Res. Prod. Eng.*, pp. 465–474, 1963.

[3] E. I. Rivin and H. Kang, "Enhancement of dynamic stability of cantilever tooling structures," *Int. J. Math. Tools Manufact.*, vol. 32, no. 4, pp. 539–561, 1992.

[4] Y. S. Tarng, J. Y. Kao, and E. C. Lee, "Chatter suppression in turning operations with a tuned vibration absorber," *J. Mater. Process. Technol.*, vol. 105, pp. 55–60, 2000.

[5] N. D. Sims, "Vibration absorbers for chatter suppression: A new analytical tuning methodology," *J. Sound Vib.*, vol. 301, pp. 592–607, 2007.

[6] M. H. H. Miguélez, L. Rubio, J. A. A. Loya, and J. Fernández-Sáez, "Improvement of chatter stability in boring operations with passive vibration absorbers," *Int. J. Mech. Sci.*, vol. 52, no. 10, pp. 1376–1384, Oct. 2010.

[7] L. Rubio, J. A. Loya, M. H. Miguélez, and J. Fernández-Sáez, "Optimization of passive vibration absorbers to reduce chatter in boring," *Mech. Syst. Signal Process.*, vol. 41, no. 1–2, pp. 691–704, 2013.

[8] Y. Yang, D. Xu, and Q. Liu, "Milling vibration attenuation by eddy current damping," *Int J Adv Manuf Technol*, vol. 81, pp. 445–454, 2015.

[9] Y. Yang, R. Xie, and Q. Liu, "Design of a passive damper with tunable stiffness and its application in thin-walled part milling," *Int. J. Adv. Manuf. Technol.*, vol. 89, no. 9–12, pp. 2713–2720, 2017.

[10] H. Yuan, M. Wan, Y. Yang, and W. H. Zhang, "A tunable passive damper for suppressing chatters in thin-wall milling by considering the varying modal parameters of the workpiece," *Int. J. Adv. Manuf. Technol.*, vol. 104, no. 9–12, pp. 4605–4616, 2019.

[11] H. Yuan, M. Wan, and Y. Yang, "Design of a tunable mass damper for mitigating vibrations in milling of cylindrical parts," *Chinese J. Aeronaut.*, Jan. 2019.

[12] Y. Yang, J. Muñoa, and Y. Altintas, "Optimization of multiple tuned mass dampers to suppress machine tool chatter," *Int. J. Mach. Tools Manuf.*, vol. 50, pp. 834–842, 2010.

[13] Y. Nakano, H. Takahara, and E. Kondo, "Countermeasure against chatter in end milling operations using multiple dynamic absorbers," *J. Sound Vib.*, vol. 332, pp. 1626–1638, 2013.

[14] Y. Yang, W. Dai, and Q. Liu, "Design and implementation of two-degree-of-freedom tuned mass damper in milling vibration mitigation," *J. Sound Vib.*, vol. 335, pp. 78–88, 2014.

[15] Y. Yang, W. Dai, and Q. Liu, "Design and machining application of a two-DOF magnetic tuned mass damper," *Int. J. Adv. Manuf. Technol.*, vol. 89, no. 5–8, pp. 1635–1643, 2017.

[16] M. Wang, T. Zan, Y. Yang, and R. Fei, "Design and implementation of nonlinear TMD for chatter suppression: An application in turning processes," *Int. J. Mach. Tools Manuf.*, vol. 50, no. 5, pp. 474–479, 2010.

[17] G. Habib, G. Kerschen, and G. Stépán, "Chatter mitigation using the nonlinear tuned vibration absorber," *Int. J. Non. Linear. Mech.*, vol. 91, no. January, pp. 103–112, 2017.

[18] M. C. Smith, "Synthesis of mechanical networks: The inerter," *IEEE Trans. Automat. Contr.*, vol. 47, no. 10, pp. 1648–1662, 2002.

[19] M. C. Smith, "The Inerter: A Retrospective," *Annu. Rev. Control. Robot. Auton. Syst.*, vol. 3, no. 1, pp. 361–391, 2020.

[20] D. J. Wagg, "A review of the mechanical inerter : historical context , physical realisations and nonlinear applications," *Nonlinear Dyn.*, vol. 104, no. 1, pp. 13–34, 2021.

[21] K. Ikago, K. Saito, and N. Inoue, "Seismic control of single-degree-of-freedom structure using tuned viscous mass damper," *Earthq. Eng. Struct. Dyn.*, vol. 41, pp. 453–474, 2012.

[22] I. F. Lazar, S. A. Neild, and D. J. Wagg, "Using an inerter-based device for structural vibration suppression," *Earthq. Eng. Struct. Dyn.*, vol. 43, no. 8, pp. 1129–1147, 2014.

[23] A. Giaralis and L. Marian, "Use of inerter devices for weight reduction of tuned mass-dampers for seismic protection of multi-story building: the Tuned Mass-Damper-Interter (TMDI)," *Act. Passiv. Smart Struct. Integr. Syst. 2016*, vol. 9799, no. April 2016, p. 97991G, 2016.

[24] D. De Domenico and G. Ricciardi, "Optimal design and seismic performance of tuned mass damper inerter (TMDI) for structures with nonlinear base isolation systems," *Earthq. Eng. Struct. Dyn.*, vol. 47, no. 12, pp. 2539–2560, 2018.

[25] P. Deastra, D. Wagg, N. Sims, and M. Akbar, "Tuned inerter dampers with linear hysteretic damping," *Earthq. Eng. Struct. Dyn.*, vol. 49, no. 12, pp. 1216–1235, 2020.

[26] M. C. Smith and F.-C. Wang, "Performance Benefits in Passive Vehicle Suspensions Employing Inerters," *Veh. Syst. Dyn.*, vol. 42, no. 4, pp. 235–257, 2004.

[27] Y. Hu, M. Z. Q Chen, and Z. Shu, "Passive vehicle suspensions employing inerters with multiple performance requirements," *J. Sound Vib.*, vol. 333, no. 8, pp. 2212–2225, 2014.

[28] J. Z. Jiang, A. Z. Maramoros-Sanchez, R. M. Goodall, and M. C. Smith, "Passive suspension incorporating inerters for railway vehicles," *Veh. Syst. Dyn.*, vol. 50, pp. 263–276, 2012.

[29] J. Z. Jiang, A. Z. Matamoros-Sanchez, A. Zolotas, R. M. Goodall, and M. C. Smith, "Passive suspensions for ride quality improvement of two-axle railway vehicles," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 229, no. 3, pp. 315–329, 2015.

[30] F. C. Wang, C. H. Lee, and R. Q. Zheng, "Benefits of the inerter in vibration suppression of a milling machine," *J. Franklin Inst.*, vol. 356, no. 14, pp. 7689–7703, 2019.

[31] H. Dogan, N. D. Sims, and D. J. Wagg, "Investigation of the inerter-based dynamic vibration absorber for machining chatter suppression," *J. Phys. Conf. Ser.*, vol. 1264, 2019.

[32] S. Tobias, Machine Tool Vibration. Blackie, London, 1965.

[33] J. Munoa, I. Mancisidor, N. Loix, L. Uriarte, R. Barcena, and M. Zatarain, "Chatter suppression in ram type travelling column milling machines using a biaxial inertial actuator," *CIRP Annals-Manufacturing Technology*, vol. 62, no. 1, pp. 407–410, 2013.

[34] J. Munoa, X. Beudaert, K. Erkorkmaz, A. Iglesias, A. Barrios, and M. Zatarain, "Active suppression of structural chatter vibrations

using machine drives and accelerometers," *CIRP Annals*, vol. 64, no. 1, pp. 385–388, 2015.

[35] M. Zaeh, R. Kleinwort, P. Fagerer, and Y. Altintas, "Automatic tuning of active vibration control systems using inertial actuators," *CIRP Annals*, vol. 66, no. 1, pp. 365–368, 2017.

[36] N. Van de Wouw, N. van Dijk, A. Schiffler, H. Nijmeijer, and E. Abele, "Experimental validation of robust chatter control for high-speed milling processes," in *Time Delay Systems*. Springer, 2017, pp. 315–331.

[37] Ozsoy, M., Sims, N. D., & Ozturk, E. (2022). "Robotically assisted active vibration control in milling: A feasibility study". *Mechanical Systems and Signal Processing*, 177, 109152.

[38] Huyanan, S., & Sims, N. D. (2007). "Vibration control strategies for proof-mass actuators," *Journal of Vibration and Control*, 13(12), 1785-1806.

[39] Y. Altintas and E. Budak, "Analytical Prediction of Stability Lobes in Milling," *Ann. CIRP*, vol. 44, no. 1, pp. 357–362, 1995, Accessed: Jan. 29, 2018.

[40] E. Budak and Y. Altintas, "Analytical Prediction of Chatter Stability in Milling — Part I : General Formulation," *J. Dyn. Syst. Meas. Control*, vol. 120, pp. 22–30, 1998.

[41] E. Budak and Y. Altintas, "Analytical prediction of chatter stability in milling - Part II: Application of the general formulation to common milling systems," *J. Dyn. Syst. Meas. Control*, vol. 120, pp. 31–36, 1998.

[42] A. K. Qin, V. L. Huang, and P. N. Suganthan, "(SADE)Adaptation for Global Numerical Optimization," *IEEE Commun. Mag.*, vol. 13, no. 2, pp. 398–417, 2009.

[43] K. Worden and G. Manson, "On the identification of hysteretic systems. Part I: Fitness landscapes and evolutionary identification," *Mech. Syst. Signal Process.*, vol. 29, pp. 201–212, 2012.

[44] Schmitz, T. L. (2003). Chatter recognition by a statistical evaluation of the synchronously sampled audio signal. Journal of Sound and Vibration, 262(3), 721-730.