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Snowmass2021 - Letter of Interest

Cosmology Intertwined IV: The Age of the Universe and its Curvature

Thematic Areas: (check all that apply ☐/■)

■ (CF1) Dark Matter: Particle Like
☐ (CF2) Dark Matter: Wavelike
☐ (CF3) Dark Matter: Cosmic Probes
■ (CF4) Dark Energy and Cosmic Acceleration: The Modern Universe
☐ (CF5) Dark Energy and Cosmic Acceleration: Cosmic Dawn and Before
☐ (CF6) Dark Energy and Cosmic Acceleration: Complementarity of Probes and New Facilities
■ (CF7) Cosmic Probes of Fundamental Physics
☐ (Other) *[Please specify frontier/topical group]*

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Abstract: A precise measurement of the curvature of the Universe is of primeval importance for cosmology since it could not only confirm the paradigm of primordial inflation but also help in discriminating between different early Universe scenarios. The recent observations, while broadly consistent with a spatially flat standard Λ Cold Dark Matter (ΛCDM) model, are showing tensions that still allow (and, in some cases, even suggest) a few percent deviations from a flat universe. In particular, the Planck Cosmic Microwave Background power spectra, assuming the nominal likelihood, prefer a closed universe at more than 99% confidence level. While new physics could be in action, this anomaly may be the result of an unresolved systematic error or just a statistical fluctuation. However, since a positive curvature allows a larger age of the Universe, an accurate determination of the age of the oldest objects provides a smoking gun in confirming or falsifying the current flat ΛCDM model.

The curvature of the Universe – The flat Λ Cold Dark Matter (ΛCDM) cosmological model describes incredibly well the current cosmological observations. However, together with the long standing *Hubble constant* H_0 disagreement[1], and $\sigma_8 - S_8$ tension [2], there are some anomalies in the Planck 2018 cosmological results that deserve further investigations. Between them the most significant from the statistical point of view, is the preference at 3.4σ for a closed Universe[3–5]. Moreover, the Planck dataset also suggest an indication at more than 2σ for Modified Gravity[4;6;7]. This disagreement with the predictions for a flat universe of the standard model is connected with the higher, anomalous, lensing contribution in the Cosmic Microwave Background (CMB) power spectra, characterized by the A_L parameter[4;8], that is strongly degenerate with Ω_k (see Fig. 1). A closed universe also solves a well-know tension above 2σ between the low and high multipoles regions of the angular power spectra [3;9;10]. This indication for curvature can be due to unresolved systematics in the Planck 2018 data, or can be simply due to a statistical fluctuation.

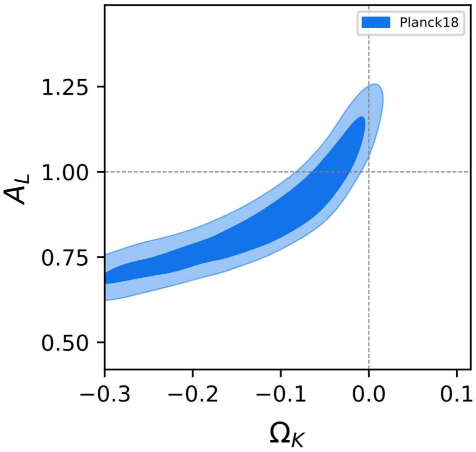

Figure 1: 68% CL and 95% CL contour plots for Ω_k and A_L (from Ref.[3]).

Indeed, while Planck 2018[4] finds $\Omega_k = -0.044^{+0.018}_{-0.015}$[1], i.e. $\Omega_k < 0$ at about 3.4σ ($\Delta\chi^2 \sim -11$), using the official baseline Plik likelihood[11], the evidence is reduced when considering the alternative CamSpec [12] likelihood (see discussion in [13]), albeit with the marginalized constraint still above the 99% CL ($\Omega_k = -0.035^{+0.018}_{-0.013}$). Moreover, the recent results from the ground-based experiment ACT, in combination with data from the WMAP experiment, is fully compatible with a flat universe with $\Omega_k = -0.001^{+0.014}_{-0.010}$ while slightly preferring a closed universe when combined with a portion of the Planck dataset with $\Omega_k = -0.018^{+0.013}_{-0.010}$ [14](see Fig. 2). A closed universe is also preferred by a combination of non-CMB data made by Baryon Acoustic Oscillation (BAO) measurements[15–17], supernovae (SNe) distances from the recent Pantheon catalog[18], and a prior on the baryon density derived from measurements of primordial deuterium[19] assuming Big Bang Nucleosynthesis (BBN), but with a much larger H_0 [3] completely in agreement with the SH0ES collaboration value R19[20]. However, letting the curvature free to vary means to increase both the H_0 and the S_8 tensions[3]. Therefore, at the moment there are not theoretical models that can explain at the same time all the tensions and anomalies we see in the data. On the other hand, a flat universe is preferred also by Planck + BAO, or + CMB lensing[21] or + Pantheon data. However these dataset combinations are in disagreement at more than 3σ when the curvature is free to vary[3;5]. In addition, though the error bars are so large that cannot discriminate between the models, a flat Universe is also in agreement with the analysis made by[22] using the $H(z)$ sample from the cosmic chronometers (CC) and the luminosity distance $D_L(z)$ from the 1598 quasars ($\Omega_k = 0.08 \pm 0.31$) or the Pantheon sample ($\Omega_k = -0.02 \pm 0.14$), in agreement with the previous[23]. Finally, in[24] a combination of BAO+BBN+H0LiCOW provides $\Omega_k = -0.07^{+0.14}_{-0.26}$ with H_0 in agreement with R19, while BAO+BBN+CC gives a positive $\Omega_k = 0.28^{+0.17}_{-0.28}$. In[13] it has been pointed out that is difficult to believe to a possible cosmological data conspiracy towards $\Omega_k = 0$. However, a full agreement of the luminosity distance measurements, like Pantheon or R19, with Planck can be reached also ruling out both, a flat universe and a cosmological constant[25].

The Age of the Universe – The age of the universe is an important piece of the puzzle because it connects H_0 and Ω_m, both of which can be measured in the early and the late universe. The age is not just a

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[1] All the bounds are reported at 68% confidence level in the text.

prediction of the ΛCDM model, that for Planck 2018 is $t_U = 13.800 \pm 0.024$ Gyr, but can also be measured using very old objects. For example, in[26] it is obtained $t_U = 13.35 \pm 0.16(\text{stat.}) \pm 0.5(\text{sys.})$ Gyr using populations of stars in globular clusters. Nevertheless, while robustness and accuracy tests have been done very extensively for CMB and quite extensively for BAO and SNe, for the age of the oldest objects we have to be more careful. For example, one finds the ages of the oldest stars 2MASS J18082002–5104378 B equal to $t_* = 13.535 \pm 0.002$ Gyr[27], but if the scatter among different models to fit for the age is taken into account the age becomes $t_* = 13.0 \pm 0.6$ Gyr[28], and the age of HD 140283 equal to $t_* = 14.46 \pm 0.8$ Gyr[29], but becomes $t_* = 13.5 \pm 0.7$ Gyr[28] using the new Gaia parallaxes instead of original HST parallaxes. Therefore, even if at present there is not real tension between the different t_u determinations, most of the error-bars in the age determination comes from the fact that different stellar models do not really agree with each other at the required level of precision to be really able to help with the tensions in cosmology. Nevertheless, stellar models can/are expected to improve reducing this error significantly, and this could potentially unveil a tension on the age of the Universe. Trying to alleviate it by changing the Planck determination, would interestingly have an effect on the cosmological tensions. A possibility to increase the age of the Universe, to be larger than the age of oldest stars, is by lowering the Hubble constant value, because of the anti-correlation between these parameters[30]. For example, a positive curvature for the Universe, as suggested by Planck 2018, preferring a lower H_0 and worsening significantly the H_0 tension, predicts an older Universe $t_U = 15.31 \pm 0.47$ Gyr. Therefore, it seems that the only way to address the H_0 crisis if R19 is correct is to introduce an extremely recent (after $z \sim 0.1$) departure from ΛCDM, requiring a great deal of fine-tuning.

Future – Detecting a curvature Ω_k different from zero could be due to a local inhomogeneity biasing our bounds[31], and in this case CMB spectral distortions such as the KSZ effect and Compton-y distortions, present a viable method to constrain the curvature at a level potentially detectable by a next-generation experiment. If a curvature Ω_k different from zero is the evidence for a truly superhorizon departure from flatness, this will have profound implication for a broad class of inflationary scenarios. While open universe are easier to obtain in inflationary models[32–36], with a fine-tuning at the level of about one percent one can obtain also a semi-realistic model of a closed inflationary universe[37;38]. In[39] it has been shown that forthcoming surveys, even combined together, are likely to place constraints on the spatial curvature of $\sim 10^{-3}$ at 95% CL at best, but enough for solving the current anomaly in the Planck data. Experiments like Euclid and SKA, instead, may further produce tighter measurements of Ω_k by helping to break parameter degeneracies[40;41].

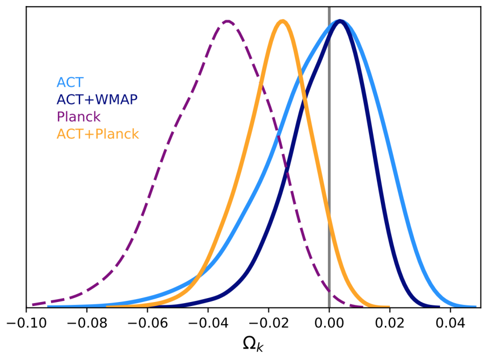

Figure 2: 1D posterior distributions on Ω_k (from Ref. [14]).

Summary – In these four LoIs [1;2;42] we presented a snapshot, at the beginning of the SNOWMASS process, of the concordance ΛCDM model and its connections with the experiment. This is a cutting-edge field in the area of cosmology, with unrestrained growth over the last decade. On the experimental side, we have learned that it is really important to have multiple precise and robust measurements of the same observable, with experiments conducted blind in regard to the expected outcome. This provides a unique opportunity to study similar physics from various points of view. While on the theory side, it is really important having robust and testable predictions for the proposed physical models that can be probed with the data. With the synergy between these two sides, significant progress can be made to answer fundamental physics questions. During the SNOWMASS process we plan to monitor the new advances on the field to come out with a clear roadmap for the coming decades.

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