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Extending the Hoyle-State Paradigm to $^{12}C + ^{12}C$ Fusion

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Carbon burning is a key step in the evolution of massive stars, Type 1a supernovae and superbursts in x-ray binary systems. Determining the $^{12}C + ^{12}C$ fusion cross section at relevant energies by extrapolation of direct measurements is challenging due to resonances at and below the Coulomb barrier. A study of the $^{24}Mg(\alpha, \alpha')^{24}Mg$ reaction has identified several 0^+ states in ^{24}Mg, close to the $^{12}C + ^{12}C$ threshold, which predominantly decay to $^{20}Ne(\text{ground state}) + \alpha$. These states were not observed in $^{20}Ne(\alpha, \alpha_0)^{20}Ne$ resonance scattering suggesting that they may have a dominant $^{12}C + ^{12}C$ cluster structure. Given the very low angular momentum associated with sub-barrier fusion, these states may play a decisive role in $^{12}C + ^{12}C$ fusion in analogy to the Hoyle state in helium burning. We present estimates of updated $^{12}C + ^{12}C$ fusion reaction rates.

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In the 1950s, Sir Fred Hoyle made one of the most sensational predictions in physics: that the abundance of carbon in our universe mandates the existence of a 0^+ resonance state in triple-α fusion just above the 3α threshold of ^{12}C [1]. Experiments found the predicted state [2] leading to the Hoyle-state paradigm to the center of our understanding of massive star evolution for nearly 70 years. The Hoyle state is a diffuse and extended state [3] appearing in no simple nuclear structure model (e.g., the shell model). Descriptions focus on cluster models based on three α particles [4] and more exotic scenarios such as Bose-Einstein condensate of α particles [5]. Despite the extensive theoretical and experimental studies of its properties, e.g., Ref. [6], there is no consensus on its structure, save that clustering is mandatory to its understanding. As the flagship of cluster structures in nuclei, the Hoyle-state paradigm was extended by Ikeda [7] to encompass light α-conjugate nuclei with predictions that cluster structures built from α particles (e.g., ^{12}C) appear at the decay thresholds of the components. Here, we address whether such threshold cluster states in ^{24}Mg, formed, e.g., from two ^{12}C nuclei, play the critical role in carbon burning in massive stars, in analogy to how the Hoyle state controls helium burning. An outline of the analogy we intend to draw is presented in Fig. 1.

As with the Hoyle state, the fusion of heavy ions such as $^{12}C + ^{12}C$ poses a strong challenge, not only in its implications for nuclear structure but also the key role played in the evolution of massive stars. Unlike nearly all other heavy-ion fusion reactions, the $^{12}C + ^{12}C$ reaction exhibits strongly resonant behavior [9] at and below the Coulomb barrier down to the lowest energies explored. Such resonances have been connected with similar resonant behavior seen in $^{12}C + ^{12}C$ elastic and inelastic scattering, and breakup [10–13]. One view is that such resonances are narrow molecular "doorway" states connected to $^{12}C + ^{12}C$ structures. Others conclude that the phenomenon is an artefact of the low level density in this compound system [14]. Even if the molecular hypothesis is accepted, the experimental data confound a simple picture as there are simply too many resonances. Elaborate models have been constructed around vibrational excitations of an underlying rotational excitation of a dumbbell-shaped $^{12}C + ^{12}C$ molecule or in terms of the additional degree of freedom of

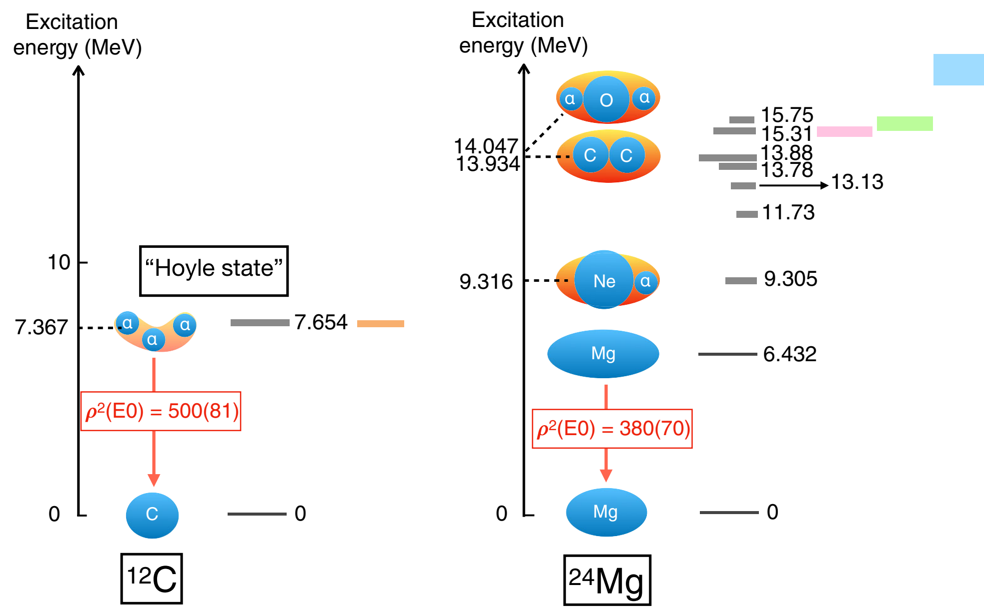

FIG. 1. Analogy between the Hoyle state in ^{12}C as the driver of 3α fusion and candidate cluster states in ^{24}Mg as drivers of $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion. Left: The Hoyle state in ^{12}C with its associated E0 decay to the ground state, suggesting a large radial difference between the states. The illustration to the left of the experimental state shows the Hoyle state as a bent arm of three α particles close to the 3α threshold. To the right, the Gamow window is shown (in orange) for helium burning around $T_9 = 0.1$. Right: A subset of the excited ^{24}Mg 0^+ states that are strongly populated by $^{24}\mathrm{Mg}(\alpha,\alpha')^{24}\mathrm{Mg}$. The size of the bars is proportional to the fraction of the energy-weighted sum rule (EWSR). On the left are schematic diagrams of the predicted ^{24}Mg cluster structures and their breakup thresholds. (For simplicity of presentation, the $^{16}\mathrm{O}+{}^{8}\mathrm{Be}$ configuration is omitted.) The 6.432-MeV 0^+ state suggested to be a highly deformed counterpart to the ^{24}Mg ground state is indicated along with its associated E0 decay [8]. On the right are the Gamow windows corresponding to superbursts (pink; $T_9 = 0.4$), massive stars (green; $T_9 = 0.6$), and Type 1a supernovae (blue; $T_9 = 1.8$).

internal excitation of one or both of the ^{12}C components [15]. Faced with such complexity and resonances persisting to the lowest energies, recent research has focused on brute-force measurements of $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion [16–19] with the hope that such measurements can be extrapolated down to the astrophysically important energies. However, the confounding aspect of the resonances means that it is still unclear if $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion exhibits sub-barrier hindrance in common with most other heavy-ion fusion reactions [14]. The existence of such hindrance would play an important role in defining the fusion probability for three astrophysical scenarios of interest, which, in ascending order of typical burning temperature are (1) superbursts in low-mass x-ray binary systems [20], (2) nucleosynthesis in massive stars [21,22], and (3) explosive burning in Type Ia supernovae [23] (see typical Gamow windows for such processes in Fig. 1). As as example of the wider astrophysical impact, the preexplosive $^{26}\mathrm{Al}/^{60}\mathrm{Fe}$ ratio sensitively depends on the neutron seed inventory for carbon burning [24]; decreased carbon fusion tends to support higher production of these important long-lived radioisotopes in massive stars.

The largest hurdle in our understanding of $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion is in accessing the extremely low cross section deep within the Gamow window for massive stars and superbursts. No feasible technique permits direct measurements with the present beam intensities and target performance. Indirect studies of the cross section have been attempted [25], but matching the results with existing direct measurements is challenging and the applicability of the reaction model has been challenged [26,27].

In the present Letter, we again use an indirect approach motivated by the Hoyle-state paradigm, which plays a critical role at the intersection of nuclear structure and astrophysics. $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion is mediated through even-spin, natural-parity isoscalar states since it involves the fusion of two identical isoscalar bosons. The $^{24}\mathrm{Mg}(\alpha,\alpha')^{24}\mathrm{Mg}$ reaction is an attractive probe for studying the mediating states since it favors the population of isoscalar, natural-parity states. Antisymmetrized molecular dynamics calculations predict that 0^+ states associated with cluster structures in ^{24}Mg should be strongly populated by the $^{24}\mathrm{Mg}(\alpha,\alpha')^{24}\mathrm{Mg}$ reaction [28,29]. Finally, the relative population of low-spin states is enhanced by using $^{24}\mathrm{Mg}(\alpha,\alpha')^{24}\mathrm{Mg}$ at small angles; in the Gamow window for massive stars, CCFULL [30,31] calculations suggest that the $\ell = 0$ and $\ell = 2$ angular momenta dominate $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion.

A 200-MeV beam of α particles was extracted from the Separated-Sector Cyclotron at iThemba LABS, Cape Town, and transported down a dispersion-matched beamline to the target position of the K600 Q2D magnetic

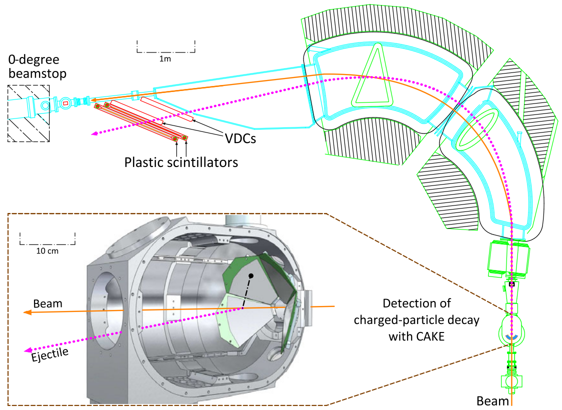

FIG. 2. Diagram of the K600 and the CAKE in 0-degree mode. In this experiment only four of the possible five CAKE detectors were used. The configuration shown is that used in the present experiment.

spectrometer [32] at which an enriched ^{24}Mg foil was located. Data were taken at two angles, with the $\pm 4°$ aperture into the K600 centered at 0° and 4°. For the 4° measurement, the beam was stopped in a Faraday cup located next to the aperture into the spectrometer. For the 0° measurement (see Fig. 2) the unreacted beam and scattered α particles both passed into the K600. The unreacted beam passed the focal plane and was stopped in a Faraday cup. Inelastically scattered α particles were detected at the focal plane of the K600. In the 0° measurement, light charged particles emitted from ^{24}Mg excited states were detected in the Coincidence Array for K600 Experiments (CAKE) of double-sided silicon strip detectors.

The focal plane was calibrated using well-known states in ^{24}Mg and the CAKE using a ^{228}Th source. Excitation-energy spectra gated on the various decay channels were constructed using the CAKE particle identification and excitation energy-decay energy loci. The experimental apparatus was simulated using GEANT4 [33] to extract the CAKE efficiency. Excitation-energy spectra for inclusive and exclusive data were generated, including a background subtraction for the inclusive spectrum [32]. Coincidence spectra for the α-particle and proton decays to the ground and first excited states of ^{20}Ne (^{23}Na) are shown in Fig. 3. Branching ratios for charged-particle decays were extracted by comparing the number of counts in the coincidence spectra relative to the equivalent peak in the inclusive spectrum. Since $J = 0$ states decay isotropically, the angular correlation functions for this case are simple and do not require correction for factors such as the maximum ejectile scattering angle. A number of overlapping states must be included in the determination of the branching ratios because of the high level density. More details will be provided in a future publication.

$J^\pi = 0^+$ levels in (α, α') reactions may be identified by comparing the 0° and small-angle spectra [34,35], and we have recently reported a number of $J^\pi = 0^+$ states in ^{24}Mg using this method [34]. Our focus here is on a subset of states at or above the threshold for breakup into $^{12}\mathrm{C} + {}^{12}\mathrm{C}$: $E_x = 13.78(3)$, 13.88(3), 15.31(4), and 15.75(4) MeV [36]. These states are strongly populated in the $^{24}\mathrm{Mg}(\alpha, \alpha')^{24}\mathrm{Mg}$ reaction. Their population as % EWSR and the branching ratio for breakup into the available channels are presented in Table I.

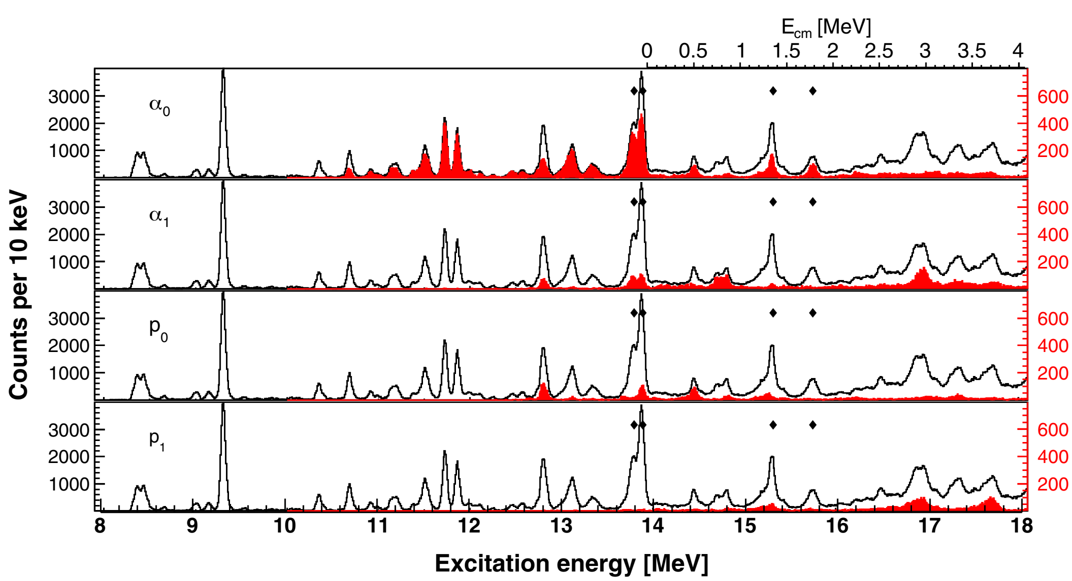

FIG. 3. Excitation-energy spectra for states excited in the $^{24}\mathrm{Mg}(\alpha, \alpha')^{24}\mathrm{Mg}$ reaction at 0°. The inclusive data, shown in black, are the same in each panel. Superimposed are the coincidence data for different breakup channels. From top to bottom: α_0, α_1, p_0, and p_1. The center-of-mass energy for the colliding $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ system is given at the top of top panel. The locations of the four $J^\pi = 0^+$ states of particular importance in the current Letter are marked with black diamonds.

TABLE I. Properties of the four $J^\pi = 0^+$ states in ^{24}Mg discussed in the present Letter including the % EWSR from the $^{24}\mathrm{Mg}(\alpha,\alpha')^{24}\mathrm{Mg}$ reaction and their branching ratio into α_0, α_1, p_0, and p_1 decays. The excitation energies include systematic uncertainties not present in the analysis in Ref. [34]. The uncertainties in breakup fractions only include the statistical errors. There is systematic uncertainty of around 10% due to the choice of the background in the inclusive spectrum. This results in a scaling of the overall branching ratios but does not change in the relative branching.

E_x (MeV)	% EWSR	B_{α_0}	B_{α_1}	B_{p_0}	B_{p_1}	$B_{p>1}$
13.78(3)	1.7(3)	88(2)	22(1)			
13.88(3)	2.6(5)	61(1)	13(1)	12(1)		
15.31(4)	1.9(4)	45(2)	10(1)		7(1)	7(1)
15.75(4)	1.1(2)	87(4)		5(1)	4(1)	

Strikingly, the $J^\pi = 0^+$ levels at $E_x = 13.78(3)$ and 13.88(3) MeV lie close to the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$, $^{16}\mathrm{O} + 2\alpha$, and $^{16}\mathrm{O} + {}^{16}\mathrm{Be}$ thresholds, while two other strongly populated $J^\pi = 0^+$ states at 15.31(4) and 15.75(4) MeV have energies within the Gamow window for superbursts and massive stars (see Fig. 1). This is in accord with the antisymmetrized molecular dynamics calculations of Chiba and Kimura [28], which predict that cluster states related to the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ and $^{16}\mathrm{O} + 2\alpha$ configurations should be strongly populated through components of these configurations in the ^{24}Mg ground state.

While we cannot directly prove that the reported 0^+ states represent the clustered configurations, there is strong evidence that these states have a peculiar structure. Despite comprehensive data on resonance scattering measurements of α particles from ^{20}Ne [38–40], there is no evidence for population of these four $J^\pi = 0^+$ states. Our coincidence data show these states predominantly decay via the α_0 channel (see Table I). The nonobservation of these states in resonance scattering must be due to small α-particle widths to the ^{20}Ne ground state and, by extension, the other partial widths must also be small since the α_0 channel dominates. The usual reasons for a narrow width—low penetrability through the Coulomb and angular-momentum barrier, and isospin selectivity—cannot explain the small widths since the energies are relatively high, there is no angular-momentum barrier, and the states must be isoscalar since they are strongly populated in $^{24}\mathrm{Mg}(\alpha,\alpha')^{24}\mathrm{Mg}$. Instead, the relatively narrow width of these states must be due to peculiarities of their structure. The binuclear fusion in the Harvey prescription [41–44] models the two colliding nuclei in their lowest states colliding along the z axis. The oscillator quanta perpendicular to the z axis are frozen and the z oscillator quanta are varied to satisfy the Pauli exclusion principle. There are different relative configurations for the fusion of two oblate ^{12}C nuclei. Some configurations populate ^{24}Mg states which have a small overlap with the ^{20}Ne ground state, resulting in reduced α-particle decay widths.

The four 0^+ states in ^{24}Mg are expected to have strong $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ cluster components along with other components such as $^{16}\mathrm{O} + 2\alpha/{}^{8}\mathrm{Be}$ since they lie close to these cluster thresholds [7]. The parallel with the Hoyle-state paradigm where the states appear at the breakup threshold is clear and, indeed, a 0^+ state also appears at the threshold for $^{20}\mathrm{Ne} + \alpha$ breakup. However, the marked difference from the Hoyle state is that the 13.78(3)- and 13.88(3)-MeV states can only contribution to $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ fusion through their high-energy tails, which are expected to contribute weakly since these states are rather narrow. In addition, the Coulomb barrier is rather high compared to the resonance energy so the possibility of a "ghost" contribution is small. In practice, the 15.31(4)- and 15.75(4)-MeV states are in the Gamow window for $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ fusion in superbursts and massive stars and could play a critical role, based on their suspected structural properties.

We have evaluated the impact of these potential resonances on carbon fusion rates following Mori *et al.* [45], comparing to the rates of Caughlan and Fowler [46] (Fig. 4). We assumed a reduced width of $\theta^2 = 0.1$, a typical value for cluster states (see, e.g., Ref. [47]). By scaling the interaction radius of $^{12}\mathrm{C} + {}^{13}\mathrm{C}$ scattering by $(12/13)^{1/3}$ [48], we find $R = 7.2$ fm. The cross sections were numerically integrated in energy including energy dependence of the partial widths. There is an enhancement

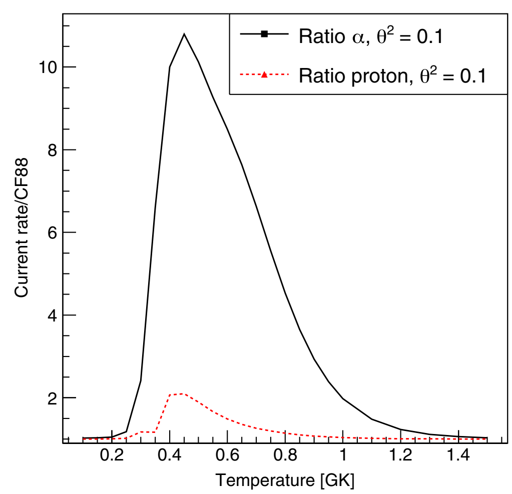

FIG. 4. The ratio of the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ fusion rate calculated in the present Letter (see text for details) to the reference rates of Caughlan and Fowler [46]: the solid black curve shows the impact on the $^{12}\mathrm{C}(^{12}\mathrm{C},\alpha)^{20}\mathrm{Ne}$ reaction rate and the broken red curve that for $(^{12}\mathrm{C}(^{12}\mathrm{C},p)^{23}\mathrm{Na})$. The parameters used in the calculation were $\Gamma = 10$ keV for both states, with $R = 7.2$ fm (see the text). The reduced width was taken to be $\theta^2 = 0.1$ and the carbon width for the states was computed using this value along with the penetrability corresponding to the resonance energies in Table I.

of up to an order of magnitude in the $^{12}C(^{12}C, \alpha)^{20}Ne$ reaction rate with a more modest increase in the $^{12}C(^{12}C, p)^{23}Na$ rate.

The impact of these new resonances on nucleosynthesis in massive stars and Type Ia supernovae is likely to be strongly influenced by the markedly different alpha-particle and proton branching. At higher energies, the average $\alpha : p$ ratio is 13∶7. In the present study, we find that α-particle production is enhanced compared to protons. The effect of the branching on the *s* process is somewhat complex (see Refs. [21,22]) and dedicated astrophysical simulations are required. Of particular interest is the production of the long-lived radioisotopes ^{26}Al and ^{60}Fe; synthesis of these elements in massive stars depends sensitively on the neutron seed generation that is strongly modified by carbon fusion.

In conclusion, we have sought to determine whether the Hoyle-state paradigm can be extended to fusion in other α-conjugate systems such as $^{12}C + ^{12}C$. From a study of the $^{24}Mg(\alpha, \alpha')^{24}Mg$ reaction at very forward angles, we identified four 0^+ states in ^{24}Mg near the $^{12}C + ^{12}C$ breakup threshold. We argue that some or all of these states have a $^{12}C + ^{12}C$ cluster structure. There are clear parallels with the historical prediction of the Hoyle state since Cooper, Steiner, and Brown [20] posited the existence of a low-energy resonance at around $E_{cm} = 1.5$ MeV, the region where two 0^+ states are observed in the present Letter, in $^{12}C + ^{12}C$ fusion to solve discrepancies between observational and theoretical superburst studies. The observed states predominantly decay by α-particle emission, with implications for the *s* process in the carbon-burning shell of massive stars. While the two 0^+ states are only a subset of potential resonances in the relevant Gamow windows, their unusual structure may mean that they play the critical role. Further studies are warranted to characterize their properties in more detail, but the value of interpreting these resonances in the Hoyle paradigm is clear from the present Letter.

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