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# Sustainable Housing Provision: A Case for the Vertical Extension of Steel Framed Buildings

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## Abstract

The UK is experiencing unprecedented housing demand, with traditional provision in suburbanised clusters now known to have negative environmental, economic and social impacts. An alternative to this is residential densification through vertical extension; generating a more sustainable urban form whilst also serving to increase circularity of material flows within the construction industry and reduce whole-life carbon and energy requirements. As a result of their relative abundance, inherent durability, and ease of adaptation, multi storey steel framed buildings are particularly pertinent in this context. It is also likely that sufficient reserve structural capacity exists within this typology; resulting from overdesign, the employment of simplified analytic design methods, and the limited number of section sizes available for use. The relative portion of reserve buckling capacity originating from these sources is assessed using a modified version of the effective length method to account for column continuity in multi-storey steel frames. This reveals the consideration of column continuity to contribute an average of 1.23 kN/m<sup>2</sup> of reserve buckling capacity across typical office buildings, with the amount resulting from section size limitations varying with design scenario.

## Keywords

Housing, Residential Accommodation, Densification, Urban Sprawl, Adaptability, Vertical Extension, Multi-storey, Steel-frame, Office Building, Buckling Capacity.

## 1 Introduction

Rising house prices mean that, in September 2019, the average cost of a 'first-time' home in the UK reached almost £200,000 [1]. This is the result of a historic under-supply of affordable residential accommodation, something which has also contributed directly to the near-record levels of homelessness currently observed in the UK [2]. An ever-growing demand and amassed deficit of over 4.8-million homes now means that, in order to meet the 15-year target set by the current government, a total of 380,000 new homes must be built each year [3]. Considering that just over half of this number were completed in the year proceeding June 2019 [4], this represents a formidable task for the UK construction industry.

As well as practical issues surrounding the construction of such large quantities of residential accommodation, additional challenges arise from the UK's commitment to the Paris accord [5], and the resulting legal requirement for the construction industry to reach net zero emissions by 2050 [6]. This pertains both operational and embodied carbon, dictating that potential solutions to the housing crisis must provide sustainable, energy efficient and cost effective residential accommodation through the use of reduced and/or low-carbon materials.

### 1.1 Residential densification vs urban sprawl

When attempting to generate a more sustainable urban form, the unsuitability of traditional housing provision in decentralised clusters is widely acknowledged, with this approach known to have numerous negative environmental, social and economic impacts [7]. Perhaps the most salient of these in the context of low-carbon residential accommodation provision is the heavy reliance of outwards urban growth on personal vehicular travel [8] [9]. As well as associated segregation and public-health concerns, logic dictates that this has negative impacts upon energy consumption and greenhouse gas emissions; a postulation which is investigated for low (19 dwellings/ha) and high (150 dwellings/ha) density settlements by Norman et al. (2006) [10]. This study reveals per-capita greenhouse gas emissions and energy usage attributable to public and personal transportation to be 3.7 times higher in low density settlements. Building operations (heating, cooling and electricity use) in low density residential developments were also found to consume almost twice the energy per capita of comparable high density developments [10]. This result is generally thought to be attributable to the fact low-density settlements have a greater proportion of external wall area [11], and because high density living typically sees a smaller floor area allocated per capita.

Additional environmental concerns surrounding low-density housing arise from the fact that, as these suburban clusters continue to

expand, they spread into rural and semi-rural areas to consume previously undeveloped land [12]. This is commonly referred to as urban sprawl, resulting in the destruction of natural habitats [13] and widespread surface sealing [7]; the process of reducing the natural permeability of land through development. In contrast, in the case of residential accommodation provision through urban densification, these negative impacts are largely avoided.

A high density urban form also poses significant economic benefits in comparison with low-density suburban clusters, impacting both individual members of society and the economies of entire areas. As suburban settlements grow in size, their ability to provide retail and leisure opportunities increases; reducing the reliance of their inhabitants upon similar provisions made in the city centre. This has the effect of reducing expenditure within these areas; resulting in further de-investment and their consequential decline [12]. In the long-term this also has negative environmental and social impacts, with duplicate provisions of the same amenity being made within each cluster rather than in a single centralised location. A similar pattern may be observed for public services, whereby the form of suburban settlements results in increased material consumption and operational energy requirements [14]. This dictates that increasing residential density has a positive impact upon the economics of public service provision, with the per capita costs of most public services increasing with the extent of land across which a settlement is distributed [15].

As well as the associated social, economic and environmental benefits of providing housing through urban densification, it is clear that this approach is consistent with the trend of rural-urban migration observed in the UK and globally. This exemplified by the fact that almost 70% of the world's population is expected to reside in cities in the next 30 years, compared to just 55% at present [16].

## 1.2 Densification and vertical extension

Although it is evident that urban consolidation through the residential densification of cities poses significant benefits, there are numerous approaches through which this may be achieved. The first of these, termed 'infilling', relates to construction upon previously undeveloped land between existing buildings; most commonly open, public spaces such as parks, squares and pedestrianised areas. When considering the health benefits and recreational opportunity associated with access to such areas [17][18][19] infill development is rarely a favourable approach. The replacement of open parkland and shared public space also shares many of the negative environmental implications associated with urban sprawl, as discussed in section 1.1. Within the highly urbanised setting of the inner-city, these are also often exacerbated, with concerns relating to the provision of ecological opportunity and the treatment of rainfall run-off being particularly prevalent [20].

Land recycling is one alternative to this, referring to the redevelopment of existing developed land for economic purpose [21]. This includes brownfield sites (land that was once developed but has since fallen into disuse) as well as areas which, whilst still in use, are generally underutilised. The generation of residential accommodation through the redevelopment of brownfield sites has been a widely used and somewhat successful strategy in the recent past, driven by a desire to prevent urban sprawl and protect greenfield land. Despite this, and partially as a result of the ongoing consumption of brownfield sites, only a portion of the required residential accommodation may be yielded through this approach; around 60% as reported by Campaign to Protect Rural England (2019) [22]. Developments of this kind are also typically more traditional in nature, characterised by single-unit masonry properties clustered within suburban housing estates, a building typology and urban form

known to have large whole-life carbon consumption and operational energy outputs as discussed in section 1.1. The continual cycle of demolition and reconstruction also has negative environmental and economic impacts resulting from its associated waste generation, carbon emissions and energy/material consumption. When considering that over 60% of the UK's waste results from construction, demolition and excavation [23] this means that, as well as the requirement to limit global temperature rise to 1.5°C and ensure net-zero emissions by 2050 [5][6], land recycling is detrimental to ongoing efforts to increase the circularity of material flows within the construction industry.

It is predicted that, before 2050, half of all emissions associated with new constructions (around 11% of global carbon emissions) will be attributable to material production and construction phases [24]. This dictates that reducing upfront carbon emissions is key in ensuring the sustainability of future housing provision, something which may be achieved through the adaptation of existing infrastructure. This serves to curtail the cycle of demolition and reconstruction through the re-use of whole buildings; improving energy and material efficiency via a significantly less energy intensive construction process and the utilisation of existing superstructures. As well as increasing the circularity of material flows within the construction industry, this prolongs the lifespan of the existing building and resultantly the period for which the embodied carbon from its original construction is utilised [25]. This result is listed by Allwood et al. (2012) as a key strategy in increasing material efficiency in the construction industry, further exemplifying the environmental benefits of adaptive re-use [26].

Within building adaptation, the vertical extension of existing structures has notable additional benefits. These typically result from the fact that it serves to yield new usable floor space rather than simply renovating and/or repurposing existing space; something which is of particular importance when considering its use in the provision of large quantities of residential accommodation. This also enables housing provision through vertical extension to be carried out as either a within- or across-use adaptation, whereby a building's existing use is retained or withdrawn respectively [27]. As well as offering greater scope for environmentally and economically beneficial mixed-use space [28], this allows vertical extensions to be completed as part of a wider renovation scheme (where an existing building is vacant), or with negligible impact upon the existing building (in instances where the existing building must remain operational throughout construction). The latter of these is of increased significance in city centres where vacancy rates are low and typical building uses (e.g. retail, office and residential) dictate the unfeasibility of prolonged closures.

The relative benefits of a combined vertical extension and renovation scheme in comparison with alternative adaptations are assessed in recent work [29] which conducts both profit and life cycle analyses (LCA's) for each of four different renovation strategies (minimalist, code-compliant, low-energy renovation and low energy renovation plus vertical extension). Although more technically challenging, this reveals that a combined low-energy renovation and vertical extension scheme offers the highest return on investment per m<sup>2</sup> of gross floor area. This adaptation strategy is also found to have the lowest environmental impact in terms of global warming potential (kg eq CO<sub>2</sub>/m<sup>2</sup>) and total energy demand (MJ/m<sup>2</sup>), both of which consider building operation and material manufacture but neglect the refurbishment process itself [29].

The concept of integrating a vertical extension scheme within a more widespread renovation project as a means improving its envi-

ronmental and economic impacts has also been considered by Nilsson (2017) [30], who added that, should the technical challenges associated with vertical extension be addressed, it is likely to become a vital tool in the sustainable regeneration of cities. Of these challenges, many are mitigated in the context of steel-framed buildings, with this particular structural form also offering significant opportunity for the residential densification of city centres in a materially and energy efficient manner.

## 2 The role of steel framed buildings

The relative abundance of multi-storey steel framed buildings in UK cities dictates that they have a significant role to play in residential accommodation provision through vertical extension. This is exemplified by the fact that they have enjoyed an annual share of over 40% of the non-residential multi-storey frame market for the past 35 years, with the present portion being more than 65% [31]. When considering office buildings of two or more storeys this value rises to over 70% which is significantly greater than the next largest share of 22% for in-situ concrete frames [32]. In addition to the significant rate at which multi-storey steel framed buildings have historically been constructed in the UK, their inherent durability means that they now make up a large portion of the present building stock. This durability also means that, despite most buildings originally being designed with an intended working life of 50 years (category 4 in BS EN 1990) [33], there is significant scope for this to be extended well beyond this duration. In instances where 'warm frame' construction is used, the potential for this is increased further [34].

There is also evidence to suggest that, despite having a typical intended working life of 50 years, many steel-framed structures are demolished before this point [35]. The most commonly cited reason for this is that the building no longer meets present needs [35], suggesting that vertical extension may offer a means of saving buildings otherwise destined for demolition, ultimately prolonging the lifespan of multi-storey steel framed structures. Contrasting with alternate structural solutions such as timber and reinforced concrete, steel framed structures may also be reinforced with relative ease. In the context of vertical extension (where the primary concern is the ability of the building's columns to withstand increased axial loads), this is exemplified by the process of flange-tip reinforcement [36]. This inherent suitability for structural adaptation also poses noted benefits when considering complications arising at the interface between the existing building and extended portion, with load transfer structures typically being required.

More salient than the ease of structurally retrofitting multi storey steel frames is the likelihood that they already possess sufficient reserve structural capacity to allow for future vertical extension. This is discussed extensively in existing literature and originates from the notion that steel framed structures are typically over-designed and, as such, more remote from failure than is required by current design codes. This is substantiated in a study by Moynihan and Allwood (2014), which found the average utilisation ratio (U/R) of all beams and columns in 23 steel framed buildings to be 0.40 and 0.49 respectively [37]. In this work, critical U/R's for each element were taken as the largest of those calculated for 6 key design criteria (axial force; shear force; moment resistance; buckling resistance; combined axial and moment buckling resistance; deflection) and calculated using a relationship in the general form of equation (1).

$$\text{Utilisation Ratio} = \frac{\text{Actual Performance Value}}{\text{Maximum Permissible Performance Value}} \quad (1)$$

Rationalisation is identified within this work as the primary cause of underutilisation. This results from the fact that most clients wish to

be provided with the most economical solution; something which is rarely conducive with the most structurally efficient form [38]. To achieve this, rationalisation sees a reduced number of different structural members used in a simple and repetitive configuration [39] to reduce procurement, fabrication and assembly costs. As the required section size for the most onerous design situation must be adopted in each instance, this results that many structural members are over-sized; inducing reserve capacity within the structure. The effects of this process are worsened by the fact that the universal column (UC) and beam (UB) members typically used within multi-storey steel framed buildings are only available in a limited number of different section sizes (46 and 106 for UC and UB sections respectively) [40]. Even prior to the process of rationalisation this commonly sees the use of section sizes with significantly more capacity than is required.

Codes of practice serve as an additional source of underutilisation and reserve structural capacity within buildings. This can be demonstrated by the transition from superseded British Standards to Eurocodes [33] which dictate that the combined effect of imposed and permanent loads is to be calculated as per equation (2).

$$\sum_{j \geq 1} \gamma_{G,j} G_{k,j} + \gamma_Q Q_{k,1} + \sum_{i > 1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \quad (2)$$

Where  $G_k$  and  $Q_k$  are permanent and variable loads,  $\gamma_G$  and  $\gamma_Q$  are their respective partial factors, and  $\psi$  is a combination factor.

In cases with just one variable action, this sees partial factors of 1.35 and 1.50 being applied to the permanent and variable load respectively, contrasting with the factors of 1.40 and 1.60 previously applied when designing to British Standards [41]. When considering that the assumed design resistance of steel members remains unchanged across these two publications, this highlights a clear presence of reserve structural capacity. Within the Eurocodes there is also scope to demonstrate further reserve capacity of multi-storey steel framed structures. This relates to the use of load combination factors ( $\psi$ , as in Equation 2), as well as the application of reduction factors to unfavourable permanent loads and situations where imposed loads of the same category are used over several storeys [33]. As recommended in the UK National Annex to Eurocode 1 this reduction factor is taken to be 0.925, having the effect of reducing the imposed load partial factor from 1.35 to 1.25. In particular, this offers significant scope for the realisation of reserve capacity in across-use vertical extension projects within which loading becomes consistent across floor levels it was previously not.

A similar - but more general - philosophy relating to the amelioration of structural analysis techniques over time may be applied to buildings constructed prior to the adoption of British Standards. This suggests that, should structures originally designed in accordance with more primitive codes of practice be re-appraised using modern analysis techniques, large reserve capacities may be identified. This is the case even when permissible stresses are taken as those assumed at the time of construction.

The temporal increase of the permissible stress assumed when designing multi-storey steel framed structures is another potential source of reserve capacity. This is due to the fact that, although accounted for with decreasing material safety factors [42], the continual advancement of steel production is typically underrepresented in design guidance [43]. The reactive nature of codes of practice and their infrequent publication also mean that there is an inherent delay in the representation of each advancement; further contributing to underutilisation in historic structures.

When considering the typically over-conservative nature of loads assumed in the design process, there is an additional likelihood of reserve capacity in multi-storey steel framed buildings. This disparity is exemplified in London office buildings, where the average assumed imposed load of 17 structures was found to be 4.38kN/m<sup>2</sup> despite the actual load experienced by typical office styles averaging just 1.50kN/m<sup>2</sup> [44]. Combined with the changing demands of office buildings associated with digitisation, the results of this study also suggest that the current recommendation of 2.5kN/m<sup>2</sup> made in Eurocodes [33] may soon be superseded. This would result in further reserve capacity in all structures designed using this assumption. A similar effect also results from the common practice of providing sufficient resistance for the placement of plant at any point across a building's roof structure despite this rarely being the case.

Structural engineers' continued preference for analytical methods over numerical alternatives also results in underutilisation in steel frame constructions, an effect which is worsened by these being encouraged in current codes of practice. The main drawback of these is that, in order to be sufficiently simple to allow for completion using hand calculations, they are typically based upon numerous underlying assumptions. This ultimately has a detrimental effect on accuracy, as exemplified by the common practice of modelling steel frame structural connections as either fully rigid or nominally pinned. In reality neither of these idealised cases are pragmatic, with semi rigid analysis whereby converging members are modelled as rotational springs offering a more accurate alternative [45]. Despite this, due to their requirement for knowledge of beam-column connection depths prior to their determination, such techniques are rarely used. This introduces the potential for the identification of reserve capacity in multi-storey steel framed structures should they be re-appraised post-construction using semi-rigid analysis. The neglect of the effects of adjoining members in analytic methods also means that continuous columns are generally misrepresented; typically being modelled as a series of pin-ended columns of length equal to the frames inter-storey height.

### 3 Column continuity and the modified effective length method

As economical column lengths of 8-12m (2-3 storeys) are common within multi-storey steel frames [46], the aforementioned misrepresentation of column continuity in analytic design methods has the potential to result in reserve structural capacity in a significant number of cases. This is true for instances where connections are taken either as fully rigid or nominally pinned.

For the former of these, Eurocodes [47] and accompanying NCCI 'buckling lengths of columns: rigorous approach' (SN008a) [48] recommend that the combined effect of members converging at each end of a column are accounted for through the summation of their individual rotational stiffnesses. In this process column stiffness coefficients are taken simply as their nominal rotational stiffness ( $I/L$ ), whereas effective stiffnesses are used for beams in order account for the presence of axial loads and varying far end restraint conditions. From these, distribution factors in the form of equation (3) are calculated for both the upper and lower column nodes.

$$\eta_i = \frac{K_c + K_i}{K_c + K_i + K_{i,1} + K_{i,2}} \quad (3)$$

Where  $K_c$  is the stiffness coefficient of the column under analysis and  $K_i$ ,  $K_{i,1}$  and  $K_{i,2}$  are the stiffness coefficients of the column and beams converging at node  $i$  respectively. The use of nominal rotational stiffnesses for adjoining columns within this relationship represents the situation where columns above and below the critical

section buckle simultaneously. This is evidently unlikely to be the case in reality, with adjoining columns serving to either stabilise or destabilise the section in consideration [49].

To account for this, Gantes and Mageirou (2005) propose a modification to SN008a whereby, as with beams, effective stiffnesses are used for adjoining columns [50]. Similar to Wood [51] this utilises slope deflection equations to derive a set of enhanced effective stiffness expressions; increasing accuracy as well as the range of far end conditions covered, as shown in table 1. For consistency the notation used by Gantes and Mageirou for stiffness ( $I/L$ ), compressive force ( $N$ ), and Euler buckling load ( $N_E$ ) have been replaced here by those found in SN008a.

**Table 1** Converging member effective stiffness coefficients using SN008a and Gantes and Mageirou (43)

Far end condition	SN008a [48]	Gantes and Mageirou [50]
<b>Fixed support</b>	$\frac{I}{L} \left(1 - 0.4 \frac{N}{N_E}\right)$	$\frac{I}{L} \left(1 - 0.33 \frac{N}{N_E}\right)$
<b>Pinned support</b>	$0.75 \frac{I}{L} \left(1 - \frac{N}{N_E}\right)$	$0.75 \frac{I}{L} \left(1 - 0.66 \frac{N}{N_E}\right)$
<b>Single curvature</b>	$0.5 \frac{I}{L} \left(1 - \frac{N}{N_E}\right)$	$0.5 \frac{I}{L} \left(1 - 0.82 \frac{N}{N_E}\right)$
<b>Double curvature</b>	$1.5 \frac{I}{L} \left(1 - 0.2 \frac{N}{N_E}\right)$	$1.5 \frac{I}{L} \left(1 - 0.16 \frac{N}{N_E}\right)$
<b>Pinned roller support</b>	-	$0.25 \frac{I}{L} \left(0 - 9.87 \frac{N}{N_E}\right)$
<b>Fixed roller support</b>	-	$0.25 \frac{I}{L} \left(1 - 0.82 \frac{N}{N_E}\right)$

The primary drawback of the use of this suite of formulae is that, in order to calculate the effective rotational stiffness of an adjoining column, the compressive force ( $N$ ) present within it at failure of the critical column must be known. Webber et al. (2015) overcome this by initially assuming the critical column to fail at its Euler buckling load, as given in equation (4).

$$N_E = \frac{\pi^2 EI}{L_{cr}^2} \quad (4)$$

Where  $L/L_{cr}$  is conservatively approximated to 1.0 or 0.7 for sway and non-sway cases respectively. This result is then used to determine the compressive force present within adjoining columns at failure of the critical column using its design compressive force and that of the column which has reached critical load. This is given for adjoining and critical column lengths  $BC$  and  $CD$  in equation (5).

$$N_{BC} = \frac{N_{d,BC}}{N_{d,CD}} N_{c,CD} \quad (5)$$

Where  $N_{BC}$  is the compressive force in the adjoining column at failure,  $N_{d,BC}$  and  $N_{d,CD}$  are the design compressive forces in the adjoining and critical columns respectively, and  $N_{c,CD}$  is the approximated critical failure load given by equation (4). Substituting this relationship into the equations in table 1 yields a second set of effective column stiffness coefficients for which knowledge of the compressive force in the adjoining column at failure is not required. These are shown in table 2 for non-sway cases where adjoining and critical columns have the same  $EI$  value.

**Table 2** Modified effective stiffness coefficients for adjoining columns with various far end conditions [52]

Far end condition	Effective stiffness coefficient of column <i>BC</i> adjoining critical column <i>CD</i>
<b>Fixed support</b>	$\frac{I_{BC}}{L_{BC}} \left( 1 - 0.33 \frac{N_{d,BC}}{N_{d,CD}} \left( \frac{L_{BC}}{0.7L_{CD}} \right)^2 \right)$
<b>Pinned support</b>	$0.75 \frac{I_{BC}}{L_{BC}} \left( 1 - 0.66 \frac{N_{d,BC}}{N_{d,CD}} \left( \frac{L_{BC}}{0.7L_{CD}} \right)^2 \right)$
<b>Single curvature</b>	$0.5 \frac{I_{BC}}{L_{BC}} \left( 1 - 0.82 \frac{N_{d,BC}}{N_{d,CD}} \left( \frac{L_{BC}}{0.7L_{CD}} \right)^2 \right)$

Using these revised effective column stiffness formulae, it is exemplified by Webber et al. (2015) that the proposed alterations to SN008a serve to significantly increase its accuracy. Predicted column buckling capacities are measured against outputs from eigenvalue buckling analyses in Autodesk Robot Structural Analysis to show the proposed approach to be within 1.3% and 0.6% of these for the 2 non-sway cases considered [52]. This is significantly more accurate than when using the unmodified effective length procedure, which is found to be erroneous by 83% and 21% for the same frame configurations. It should also be noted here that the proposed method consistently predicts column criticality correctly, unlike the unmodified procedure in SN008a.

Despite being derived using the assumption of fully rigid connections, as the expressions for modified effective stiffness coefficients provided in table 2 represent non-sway cases where adjoining and critical columns have the same  $EI$  value, they may also be used to account for column continuity in instances where connections are modelled as pinned. This requires the stiffness coefficient of all beam-column connections to be taken as zero, reducing the distribution factor formula provided in SN008a to equation (6).

$$\eta_c = \frac{K_{CD}}{K_{CD} + K''_{BC}} \quad (6)$$

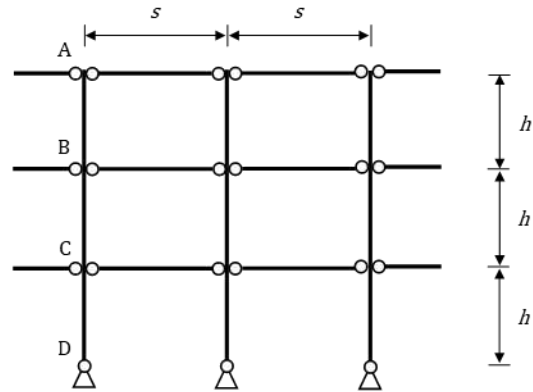
Where  $K_{CD}$  is the nominal stiffness of the column section under analysis ( $I_{CD}/L_{CD}$ ), and  $K''_{BC}$  is the effective rotational stiffness of the adjoining column section; calculated using the revised formulae in table 2.

This means that, although valuable in exemplifying the likelihood of reserve structural capacity in multi-storey steel-framed structures designed with fully rigid connections, the work of Webber et al. (2015) does not explore the effects the proposed modifications have on the design buckling capacity of nominally pinned columns used in typical real-world applications. This results from the testing of the proposed modifications on conceptual frame configurations unlikely to be observed in reality, and the reporting of column buckling capacity in terms of Euler buckling load [52].

#### 4 The effect of column continuity on design buckling capacities in multi-Storey steel framed buildings

In order to analyse the effect that the consideration of column continuity has on the presence of reserve structural capacity in multi-storey steel frames, effective column stiffness coefficients have been used to calculate the enhanced design buckling capacity of nominally pinned columns used in a number of different design scenarios. The focus of these is placed upon office buildings; selected due to their aforementioned relative abundance [32] and likely suitability for future vertical extension [44]. To allow for systematic analysis, a generalised non-sway frame as shown in figure 1 is

adopted, consisting of a continuous 3-storey column loaded symmetrically at each floor level about both its major and minor axes by nominally pinned beams. This removes the requirement to consider the effect of actual and nominal moments on the buckling capacity of the column, resulting that it is modelled in pure compression. Inter-storey height ( $h$ ) and assumed loading criteria are consistent across each storey, meaning that the ground floor column section is critical in all instances as a result of it experiencing the largest load whilst being equally stiff. The bi-axial symmetry of the frame also dictates that only buckling in the columns minor axis is considered, whilst its assumption as a braced non-sway frame negates the requirement to consider second order effects.



**Figure 1** Generalised 3-storey continuous column structural frame configuration

Across all design scenarios, an imposed load of 2.5 kN/m<sup>2</sup> is assumed for 'general office' usage, with an additional 1.0 kN/m<sup>2</sup> added to account for moveable partitions [53]. The total permanent load is taken as 4.2 kN/m<sup>2</sup>, comprising 0.5 kN/m<sup>2</sup> for structural steelwork; 3.0 kN/m<sup>2</sup> for a typical trapezoidal profile composite floor; and 0.7 kN/m<sup>2</sup> for ceiling, services and flooring [54]. From the relationship in equation (2) and partial factors taken from BS EN 1993-1-1, this results in a combined action effect of 11.03 kN/m<sup>2</sup>. Using this value, storey height and column spacing ( $h$  and  $s$  as in figure 1) are varied alternately to simulate a range of different design scenarios. Based upon typical structural grids and floor-floor heights commonly found within multi-storey steel framed office buildings [55] [56] this sees storey height varied between 3.0 and 7.5 m inclusively whilst column spacing remains at a constant value of 7.5 m. Following this, storey height is held at a fixed value of 3.75 m whilst column spacing is varied between 5.0 and 10.0 m inclusively. In each of these instances increments of 0.1 m are used.

For each design scenario permutation the maximum design compressive force ( $N_{Ed}$ ) experienced by the column is calculated as the product of the design action effect and floor area attributable to each column ( $s \times s$ ), summed across all 3 floors. An initial column sizing process indicative of the analytic methods commonly used by structural engineers is then followed, with a suitable column size being selected for each design situation using the process outlined in Appendix A.3 of SCI P362 [57]. Implemented in MATLAB for computational efficiency this sees sequentially stronger universal column sections selected for analysis until clause 6.46 of BS EN 1993-1-1 ( $N_{b,Rd} > N_{Ed}$ ) is satisfied. It is also the case that the section in consideration must be classified as 1, 2 or 3 and have sufficient cross sectional axial capacity ( $N_{Ed}/A < f_y$ ).

In this process, consistent with typical design practice, the ground floor column section is initially assumed to be nominally pinned at both ends, resulting that its effective and actual lengths are equal. This is used in the calculation of the columns critical buckling load ( $N_{cr}$ ), which follows equation 4 due to its equality to the Euler buck-

ling capacity for section classes 1-3. Following this, section dimensions are used to determine the columns corresponding imperfection factor ( $\alpha$ ) as per table 6.2 in BS EN 1993-1-1, and its non-dimensional slenderness ( $\bar{\lambda}$ ) calculated using equation (7).

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} \quad (7)$$

Where  $A$  is the column's cross sectional area and  $f_y$  is the yield strength of the selected steel grade (taken as 355 N/mm<sup>2</sup> in all cases). From this, the minor axis flexural buckling reduction factor ( $\chi$ ) can be determined, allowing the design buckling resistance ( $N_{b,Rd}$ ) of the selected section to be calculated using equation (8).

$$N_{b,Rd} = \frac{\chi Af_y}{\gamma_{M1}} \quad (8)$$

Where  $\gamma_{M1}$  is the partial factor for a member's resistance to instability, taken as 1.00 [58]. This resistance is then compared to the previously calculated design compressive force ( $N_{Ed}$ ) to assess the suitability of the selected section size using clause 6.46 of BS EN 1993-1-1 as above.

Following the determination of the lightest UC section offering sufficient buckling resistance for a given design scenario, the proposed solution is re-analysed using the principles of column continuity introduced by Webber et al. (2015) [52] and adapted for nominally pinned beam-column joints in equation (6). In all instances the adjoining column length's far end (node  $B$ ) is conservatively modelled as pinned, dictating the use of the relationship found in the second row of table 2 in this process. The same is true for the columns base node ( $D$ ), meaning its distribution factor ( $\eta_D$ ) is taken to be 1.

Due to the overly-conservative nature of the formulae provided in SN008a and impracticalities in using design charts to determine large numbers of effective length ratios, relationships developed by Smyrell (1993) [59] have been used for this purpose. These provide an accurate approximation of a column's effective length ratio ( $L/L_{cr}$ ) from its corresponding nodal distribution factor combination ( $\eta_C, \eta_D$ ). This allows for the subsequent recalculation of its Euler buckling capacity and design buckling resistance following section 6.3.1 of BS EN 1993-1-1 as outlined above. Reserve capacity is then taken to be the difference between this and the buckling resistance required by the present design scenario.

## 5 Results and discussion

Reserve buckling capacities are reported per m<sup>2</sup> of floor area attributable to the column at each floor level,  $s \times s$  (not be confused with the total area acting upon the column across all 3-floors as used in the determination of maximum design compressive forces). This serves to give an indication of the magnitude of uniformly distributed load which may be safely exerted by the construction of any additional storeys. As seen in figures 2 and 3, the relative portions of reserve buckling capacity resulting from the consideration of column continuity and limitations in the number of UC sections available for selection have also been disaggregated for clarity.

It can be seen from figures 2 and 3 that the reserve buckling capacity resulting from limitations in the number of available section sizes follows a cyclic pattern. This sees it decrease incrementally by a similar amount (averaging 0.98 kN/m<sup>2</sup> and 0.86 kN/m<sup>2</sup> for varying column spacing and storey height respectively) as the design scenario analysed becomes more onerous (i.e. as column spacing or storey height increases). Following this, there is a sharp and sudden rise in

reserve capacity before the sequential decrease proceeds. This results from the aperiodic adoption of the next largest section size when that currently under consideration can no longer provide sufficient resistance. The cause of this differs in instances of varying column spacing and storey height; resulting from the increase in design action whilst resistance remains constant in the former, and the inverse in the latter. This effect means that each cluster in figures 2 and 3 can be taken to represent a single section size

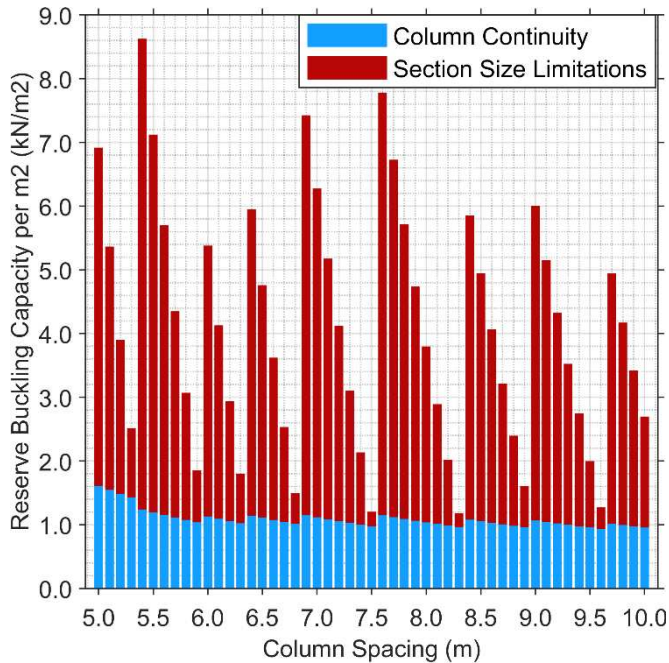
Figures 2 and 3 also reveal the consideration of column continuity to yield a relatively constant addition of reserve structural capacity across all design scenarios considered, with the mean values being 1.09 kN/m<sup>2</sup> and 1.40 kN/m<sup>2</sup> for varying column spacing and storey height respectively. Although not sufficient to allow for vertical extension in itself, this represents a significant contribution which can be shown to be present in all instances where multi-storey continuous columns are used. This suggests that, in combination with additional sources of reserve structural capacity as discussed in section 2, it is probable that sufficient capacity will be present in most multi storey steel framed office buildings to allow for future vertical extension.

In the case of increasing column spacing, reserve capacity resulting from the consideration of column continuity follows a similar cyclic pattern to that caused by section size limitations. This is despite the fact that an increase in column spacing results in a reduction in effective column length and increase in design buckling capacity and is instead a feature of the reporting of reserve buckling capacities in a per m<sup>2</sup> functional unit. In contrast, as storey height is varied within each section size cluster, the reserve buckling capacity resulting from the consideration of column continuity either increases or decreases incrementally depending upon the section size in consideration. Notwithstanding this, and as a result of the consistent and comparatively minor nature of reserve capacities resulting from consideration of column continuity, a similar cyclic pattern to that seen for section size limitations is observed for variations in total reserve buckling capacities with both column spacing and storey height.

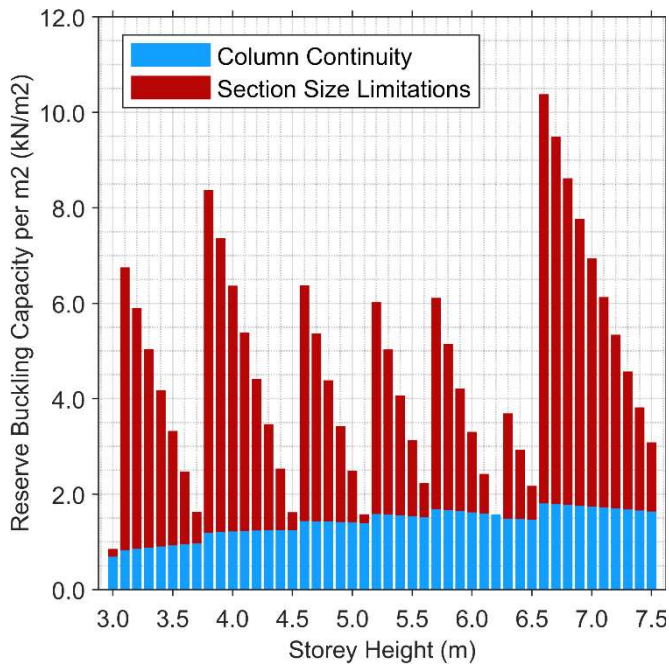
This dictates that there are large variations in total reserve buckling capacities, with values ranging from 1.18 kN/m<sup>2</sup> to 8.63 kN/m<sup>2</sup> for varying column spacing and 0.85 kN/m<sup>2</sup> to 10.38 kN/m<sup>2</sup> for varying storey height. This shows that there is significant scope for future vertical extension in instances where continuous column sections are used in design scenarios requiring only a small portion of their offered resistance. Considering the typical combined action effects associated with multi-storey steel framed residential accommodation buildings, the identified reserve buckling capacities indicate that a single storey may be added in some cases even if the extended portion utilises the same structural solution as the existing building. If a more lightweight alternative is used however, there is the potential for a greater number of storeys to be added across a larger number of more onerous design scenarios.

This is the case even without the consideration of additional sources of reserve capacity and the potential for structural remediation as discussed in section 2; with these serving to increase the potential for vertical extension further. Conceptually, as rationalisation sees the adoption of section sizes with greater resistance than is required whilst design actions remain constant, it may be thought of as the process of increasing a columns reserve buckling capacity to a value just greater than that obtained for the least onerous design scenario in the next largest section size cluster. This represents the situation in figure 3 whereby any of the reserve buckling capacities between 6.75 kN/m<sup>2</sup> and 1.62 kN/m<sup>2</sup> for storey heights between 3.1 m and

3.7 m are enhanced to a value greater than that of 8.37 kN/m<sup>2</sup>, obtained for a storey height of 3.8 m. A similar result would be observed in the instance of structural remediation of columns, with this process also serving to increase design resistances whilst actions remain constant.



**Figure 2** Reserve design buckling capacities resulting from section size limitations and consideration of column continuity in columns of varying spacing.



**Figure 3** Reserve design buckling capacity resulting from section size limitations and consideration of column continuity in columns of varying length.

In addition to the sources of reserve structural capacity discussed in section 2 there is scope to increase the portion identified through the consideration of column continuity further. This results from simplifying assumptions used in using the modified effective length method which ultimately have a detrimental effect on accuracy. The first of these relates to the assumption of the column base as a pinned support, something which is highly unlikely to be the case in reality. This means that only the rotational stiffness of the column's upper node is considered, leading to the over-estimation of the effective length of the critical column section. In order to overcome

this, the partial rigidity of the columns base may be accounted for using a distribution factor between 0 and 1, as obtained when column nodes are modelled as rotational springs [45].

An additional simplifying limitation of the adopted methodology is the assumption of adjoining column's far ends as nominally pinned, a process which means that the columns continuity beyond this point is not considered. Exemplified using notation found in figure 1, this dictates that the resistance offered to the adjoining column (*BC*) by the section above this (*AB*) is not represented. This ultimately results that the buckling load of the adjoining column (*BC*) at failure and the resistance this provides to the critical column (*CD*) are underestimated. To account for this, the modified effective length approach may be applied in series to first calculate the Euler buckling load of the uppermost column section and each below this consecutively. A similar limitation to this results from the assumption used by Webber et al. (2015) [52] that the critical column fails at a load equal to its Euler buckling capacity. The effects of this may be reduced by taking the buckling capacity identified by the initial modified effective length analysis and using this as the assumed failure load ( $N_{c,CD}$  as in equation (5)) in a secondary iterative analysis.

## 6 Conclusion

Existing literature suggests that the future residential densification of city centres offers a more sustainable and environmentally benign alternative to traditional housing provision in suburbanised clusters. The adaptation of existing buildings through vertical extension provides a suitable means of achieving this, whilst also serving to increasing the circularity of material flows within the construction industry and the reduce whole-life carbon emissions and energy consumption of buildings.

Multi-storey steel framed buildings are identified as particularly pertinent in the context of vertical extension as a result of their relative abundance, inherent durability, and suitability for adaptation and structural retrofit. An increased likelihood of the presence of reserve structural capacity in such structures has also been highlighted. This results from the process of rationalisation; limitations in the number of section sizes available for use; the amelioration of structural analysis techniques with time; the over-conservative nature of assumed design actions; and simplifications made in the analytic design processes typically used by designers.

The widespread presence of small ( $\approx 1.0$  kN/m<sup>2</sup>) reserve structural capacities can be demonstrated for typical multi-storey steel framed office buildings constructed using continuous columns through the consideration of the restraining effect that this has upon the buckling capacity of the critical portion. This is exemplified by modelling inter-storey column continuity as a fully rigid connection and employing a modified version of the effective length method to account for the presence of axial loads in adjoining column sections. Although this is not sufficient to allow for vertical extension in itself, when combined with reserve capacity resulting from the limited number of section sizes available, adequate capacity is available to allow for extension in certain design cases. This suggests that, should additional sources of reserve structural capacity be considered, it is probable that sufficient reserve structural capacity will be present in most multi storey steel framed office buildings to allow for future vertical extension. The likelihood of this is increased further should a less onerous structural form be used for the extended portion of the building.

This introduces a requirement for further work to affirm the typical reserve structural capacities in multi-storey steel framed buildings arising from the sources listed above and discussed in section 2. Due



to the anticipated variability in these in comparison with the consideration of column continuity, this should be conducted using a suite of case study structures rather than idealised frame solutions as adopted in this work.

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