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A spatial autoregressive approach for wake field prediction across a wind farm

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Abstract. To reduce the operation and maintenance cost for wind farms, turbine operators are actively developing strategies to, among others, reduce service cost, maximise power production, and prolong lifetime of components and super-structure. All of these tasks require a wind farm model that can accurately predict turbine behaviours in response to the changing environment. Recent studies focus on developing data-based methods for predictive maintenance purposes. This paper proposes a data-based model that aims to capture the spatial and temporal wind variations across a wind farm, as a means to predict the interactions between operating turbines and the environment, which can be useful for wind farm performance monitoring. The proposed method is a Gaussian process-based spatial autoregressive model, which reflects our physical understanding of the wake effect while taking the advantage of a stochastic data-driven learner. In the case study of a simulated wind farm, the proposed model (named here a GP-SPARX model) provides the best predictive accuracy in comparison to two other spatial autoregressive regression models, showing its capability of capturing nonlinear correlations and its potential as a low-cost wake field predictor given inputs from weather station measurements.

Keywords: spatial autoregressive model · GP-SPARX · wind turbine wake field modelling.

1 Introduction

Once a wind turbine farm is built and starts operating, the next step is to monitor how true turbine performance differs from the expectations in the design phase, and to consider how a cost reduction strategy can be developed accordingly. Wind power cost reduction is achieved by minimising the average cost per unit of electricity generation for a turbine over its lifetime, which can be divided into three objectives: (a) reducing service cost, (b) maximising power production, and (c) prolonging turbine lifetime. Each task is associated with an optimisation study, in which a model that describes the normal wind farm performance under various conditions is crucial.

To describe turbine performance across a wind farm, a model should be able to capture the wake effect that governs the spatial and temporal wind variations. In the case of a horizontal axis turbine, the wake effect refers to decreased wind speed and increased turbulence intensity as the wind passes through the turbine rotor. The existence of wakes can severely affect the power output of downstream turbines, e.g. up to 80% of the power from the free stream wind may be lost in compactly spaced wind farms such as Lillgrund [5]. Wake loss results from a series of nonlinear processes – how the wind drives the motion in turbine rotors, which, in turn, disrupts the downstream wind flows through the formation of wakes; these processes repeat themselves as the wind passes through more turbines downstream.

The development of physics-based models which represent the governing laws of wake behaviours can be dated back to almost four decades ago. There are two main types of physics-based wake models: analytical and CFD. Analytical wake models tend to describe long-term mean variations, thus, are commonly used to design wind farm layout; whereas CFD-based techniques focus on local complex phenomena such as wake turbulence, which are usually used in the design of, e.g. wind turbine blades [1]. Both types of models contribute significantly to our physical understanding of the complex wake phenomena.

As opposed to physics-based models, data-based models aim to determine the connections between a set of input and output data, without requiring deep physical understanding of the underlying system. Recent studies have looked into data-based wake models for various tasks in wind farm control and monitoring, such as wake steering, power down-regulation, and wind farm performance detection. Among them, Neural Networks (NN) are the most popular data-based algorithm used, owing to their flexibility to fit to a range of differing predictive tasks [7, 16, 18, 20]. In [19] a hierarchical model based on spline regression is used as an alternative, as its simplicity allows it to be integrated with a higher level spatial model. The authors of the this paper have also used Gaussian process (GP) regression for data-based wake modelling in [13, 14], for its advantage as a stochastic algorithm.

The motivation of the current study is to propose a data-based method that is able to accurately capture the spatial and temporal correlations across a wind farm, as a means of modelling the interactions between the environment and turbine response. A successful model of such interaction is beneficial to wind farm performance monitoring, and also applicable to wind farm control tasks. The method proposed in previous studies [13, 14] is successful in capturing the spatio-temporal correlations in a wind farm, but requires input data from multiple reference locations. Hence, this paper proposes a new method, again based on a GP, that aims to capture the spatial and temporal variations across the wind farm with input from only one reference location (the free stream wind speed at the front of the farm). To do so, the new model has a spatial autoregressive structure that reflects our physical understanding of the wake effect.

Many other problems across disciplines are also spatially autoregressive, it is therefore useful to briefly review the commonly used methods in other fields. Most of the well-known spatial autoregressive (SAR) models in the areas of spatial econometrics and geostatistics are developed based on the study given by Cliff and Ord [2], in which they highlighted two main types of analysis associated with a SAR model: (a) estimating model parameters and (b) detecting spatial autocorrelation within data. Following Cliff and Ord, Griffith developed methods of scientific visualisation to explore the spatial autocorrelation in georeferenced data [8], whereas, Lee [11] and LeSage [12] investigated the methods to estimate and interpret model parameters, respectively. The general form of a standard SAR model remains the same throughout these studies, which is characterised by predefined functional forms and weight matrices. Similar to the case of all other deterministic models, substantial work is required to decide which functional form of the SAR models is best for a specific problem. The predefined weight matrices in SAR models specify prior knowledge in spatial correlations [12], which provides a parsimonious solution but might introduce subjective bias. The standard SAR models do not account for temporal correlations without a spatio-temporal extension, however, the SAR-based spatio-temporal models inherit the same traits [12] and, thus, constraints. Here we propose a stochastic version of spatial autoregressive model based on a GP – a GP-SPARX model – to address the spatio-temporal modelling tasks in the context of wind farm wake fields.

The layout of the paper is as follows. Section 2 introduces the simulation data used in the analysis. Section 3 provides descriptions for all the spatial autoregressive models being investigated in this paper. Sections 4 presents the prediction results obtained from the models, and conclusions are drawn in Section 5.

2 A small simulated wind farm

In this study, the effectiveness of the proposed spatial autoregressive approach in predicting wake fields is evaluated using data from a small simulated wind farm. Simulated data are used for this initial study to validate that the model form suggested can account for known physics and so that we can say, at this stage, that any unsatisfactory prediction is not associated with actual anomalies in the farm that may be present in operational data.

The small wind farm illustrated in Fig. 1 is simulated based on our physical understanding of the wake effect, which, to some extent, is in line with the most commonly known analytical wake models [1, 9, 10]. When the wind flows across a turbine, the wind velocity first drops due to the energy loss at the rotor and then recovers as the turbine wake progresses downstream. The same process repeats itself as the wind continues to encounter more turbines downstream. The wake of an upstream turbine, therefore, affects that of the downstream neighbour, and similarly, the wind speed measured at an upstream turbine affects that at the downstream neighbour. As a result, it is considered that the wind speeds measured at all turbine locations are connected in a spatially autoregressive way.

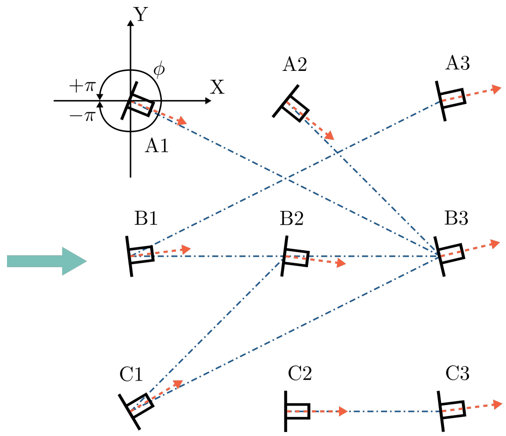

Fig. 1: A small simulated wind farm.

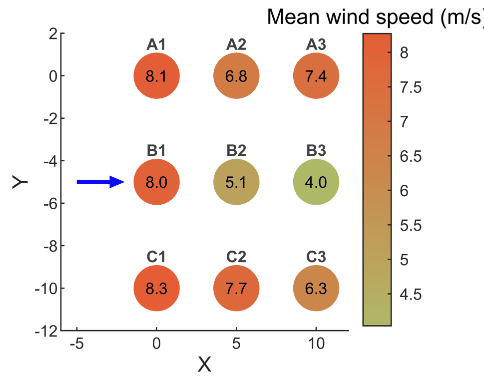

Fig. 2: Mean wind speed map of the simulated farm.

Turbine	Effective upstream neighbour(s)
B2	B1, C1
A3	B1
B3	A1, A2, B2, C1
C3	C2

Table 1: Effective upstream neighbours of turbines in wakes.

However, the spatial (auto-)correlation across a wind farm differs from those commonly described in spatial econometrics and geostatistics in that it is directional. In this context, this refers to the assumption that only upstream values have an effect on the downstream ones, not the other way round. The direction of spatial effect is determined by the free stream wind direction. To better understand the definitions of upstream and downstream members, examples are illustrated in Fig. 1. Given the free stream wind direction shown as the blue arrow on the left, the lag-1 (direct line of sight) upstream neighbours of turbine A2 include tubines A1, B1 and C1, and there is no lag-2 upstream neighbour for A2. For turbine A3, the lag-1 upstream neighbours include A2, B1, B2, and C2, and the lag-2 upstream neighbours include A1 and C1. Thus, a lag-1 upstream neighbour refers to any turbine positioned in an upstream location that is able to affect the downstream turbine of interest without interruption; whereas, a lag-2 upstream neighbour is separated from the downstream turbine by a lag-1 neighbour. For the small simulated farm used in this analysis, the largest spatial lag available is 2.

Individual turbine angles are also important, as they determine whether and to what extent an upstream wake affects a downstream turbine. An *effective upstream neighbour* of a turbine is therefore defined as an upstream neighbour with a wake in which the downstream turbine is shadowed. All effective neighbours are connected by blue dash-dotted lines in Fig. 1. It is seen that five out of nine turbines are not shadowed by any upstream wake – A1, B1, C1, A2 and C2. For the remaining four turbines under wake shadowing, their effective lag-1 upstream neighbours are listed in Table 1. It is noticed that only turbine B3 has an effective lag-2 upstream neighbour, B1. Therefore, it is assumed that the lag-1 neighbours provide the predominant wake effects in this initial study.

The identification of effective upstream neighbours requires information on turbine locations (fixed in time) and angles (possibly time-variant). In this preliminary analysis, simulated data are created based on the assumptions of fixed global wind direction (blue arrow in Fig. 1) and time-invariant individual turbine angles (red dashed arrows in Fig. 1). The spatio-temporal wind variations are modelled as follows. The time variations in the free stream wind are modelled as a linear combination of sinusoidal functions and i.i.d. Gaussian white noise. The spatial correlations between turbine locations are simulated using a reduced polynomial NARX model of order 3, based on the following empirical laws. For a downstream turbine at position s, (a) there are four exogenous variables – the free stream wind speed v_∞, the turbine angles at effective lag-1 upstream positions θ_{s-1}, the angle at the current turbine θ_s, and the distance between the lag-1 neighbours and the current position $d_{s-1,s}$; (b) v_∞ only exists in a first order term, while θ_{s-1}, θ_s and $d_{s-1,s}$ only exist in terms of order 2 or higher; (c) any term of order 2 or higher contains more than one variable, to simulate the (nonlinear) interactions between variables; (d) the maximum order of the autoregressive form of the dependent variable u_{s-1} in any term is 1 [9, 10]. The mean values of the simulated wind speed can be visualised in Fig. 2.

3 Spatial autoregressive models

Spatial autoregressive models of various levels of complexity are tested and compared in this section. Note that the deterministic versions of spatial autoregressive (SPARX) models used here are designed to be directly comparable to the stochastic version based on a GP.

3.1 Linear SPARX model

In a wind farm of S turbines, the turbine at location $s = 1, \ldots, S$ has a total of M effective upstream neighbours, i.e. M turbines at location $s-1$. The linear SPARX model is

$$u_s = \beta_{0_1} v_\infty + \sum_{m=1}^{M} \beta_{m_1} u_{s-1_m} \tag{1}$$

where u_s is a $T \times 1$ vector representing the time series of wind variations at position s, and u_{s-1_m} denotes the observed wind time series at the m^{th} effective upstream turbine. The free stream wind speed v_∞ is the only exogenous input included. The model parameters are given as β with various subscripts.

3.2 Nonlinear SPARX model

A simple nonlinear model is created by adding the squared wind speed terms to Eq. 1,

$$u_s = \beta_{0_1} v_\infty + \sum_{m=1}^{M} \beta_{m_1} u_{s-1_m} + \beta_{0_2} v_\infty^2 + \sum_{m=1}^{M} \beta_{m_2} u_{s-1_m}^2 \tag{2}$$

It is worth noting that the terms of squared wind speeds are not included in the original polynomial NARX model that is used to simulate the wind farm. Thus, Eq. 2 presents one of the simplest functional forms for nonlinear SPARX models, without knowledge of the governing physics.

3.3 GP-SPARX model

A stochastic SPARX model can be obtained by replacing the deterministic functions with a GP, i.e.

$$\begin{aligned} u_s &= f(\mathbf{x}) \\ f(\mathbf{x}) &\sim \mathcal{GP}\left(\mathbf{0}, k_{SE}(\mathbf{x}, \mathbf{x}')\right) \end{aligned} \tag{3}$$

where the inputs $\mathbf{x} = [v_\infty, u_{s-1_1}, \ldots, u_{s-1_M}]$ are the same as the case in Eq. 1. Here, a GP with a zero mean function and a squared exponential covariance function is used to model the wind variations at each turbine location. The reader is referred to [15] for more details on GP regression.

4 Result and comparison

The SPARX models mentioned previously are used to predict the wind variations across a simulated wind farm, and the results are discussed in this section. Here, the focus is to make predictions based on Model Predicted Output (MPO), where the model predicted values are fed into the next iteration of the autoregressive computation [17], i.e.

$$u_s^* = f\left(v_\infty, u_{s-1_1}^*, \ldots, u_{s-1_M}^*\right)$$

The superscript * denotes model predictions. MPO predictions on testing data are a demanding test of the models, for good model performance is only possible if the spatial correlations are correctly captured. In the current context, the MPO results indicate how well the models can potentially predict the entire wake field given inputs from the meteorological masts.

Fig. 3 gives an indication of the training and testing data (in terms of free stream wind speed v_∞) used in this analysis. It can be seen that the training data roughly cover the range of wind speed variations in the testing period, such that the chosen testing set assesses the models' ability to interpolate within a specific range.

The accuracy of MPO predictions is summarised in Fig. 4-6. Among the three models introduced earlier in Section 3, the GP-SPARX model provides the most accurate MPO predictions in terms of normalised mean squared error (NMSE), with a close second being the nonlinear SPARX model. What is common across all three NMSE maps is that the error values for the "front row" turbines (A1, B1, C1, A2 and C2) are relatively higher than the rest. The fact that these turbines have no upstream neighbours means that the predicted wind speeds at these positions are only based on the free stream reference input, without the aid of model predicted upstream values, which seems to have caused the less accurate predictions.

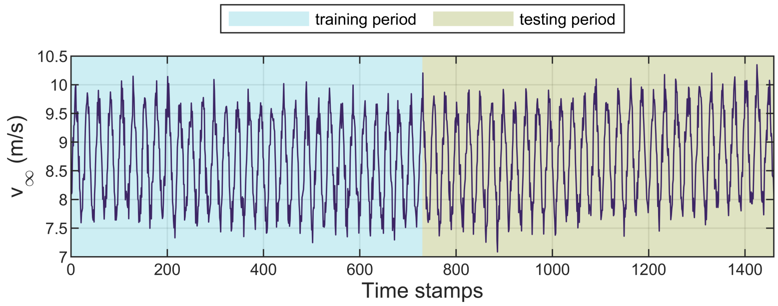

Fig. 3: Time series of free stream wind speed.

Difference in model predictive capability can be seen in the results at the wake shadowed turbines (B2, A3, B3, and C3). The linear SPARX model provides the lowest accuracy at turbine B3, the turbine with the highest number of effective upstream neighbours, while the predictions at the remaining three shadowed locations are similarly accurate (Fig. 4). However, in the case of both nonlinear SPARX and GP-SPARX models (Fig. 5-6), the predictive accuracy seems to be inversely correlated with the number of effective upstream neighbours (indicated in Table 1). The reason of this difference between linear and nonlinear models can be obtained from the known physics in the simulated wind farm. As mentioned earlier in Section 2, the spatial correlations between turbines are estimated by an nonlinear polynomial NARX model, with the nonlinearity associated with the autoregressive terms. The higher the number of effective upstream neighbours, the more autoregressive terms there are in the simulation model, and, thus, the higher degree of nonlinearity involved. The increased level of nonlinearity due to increased effective upstream neighbours is better captured by the nonlinear models.

Between the nonlinear models, the GP-SPARX gives more accurate MPO predictions across all positions compared to the nonlinear SPARX model, but the difference in NMSE values are small (Fig. 5-6). To better understand this difference, the time series of predicted wind speeds (in a chosen time window) at the "most nonlinear" turbine location, B3, are compared in Fig. 7. It is shown that both the nonlinear SPARX and the mean GP-SPARX predictions are markedly closer to the true values than the linear prediction, which is in line with the NMSE results shown earlier. Although the predictions by nonlinear SPARX and GP-SPARX models follow roughly the same trend, the GP-SPARX tends to provide a more accurate prediction during some periods, e.g. at time stamps 861-865 and 885-887 in Fig. 7. In addition to a more accurate mean prediction, GP-SPARX also provides a predicted confidence interval, in which most true data points lie. A satisfactory value of -6.8210 is given as the mean standard log loss (MSLL).

To complete the comparison, the corresponding cost to the three predictive models are looked into, as summarised in Table 2. Note that the computational complexity refers to the running time for the computation of each u_s. In the case of both linear SPARX and nonlinear SPARX models, the parameters are estimated via the least squares method. For a turbine with M effective upstream neighbours, the input to the linear SPARX model is a $T \times (M+1)$ matrix, and a $T \times 2(M+1)$ matrix for the nonlinear SPARX model, where T is the number of training time

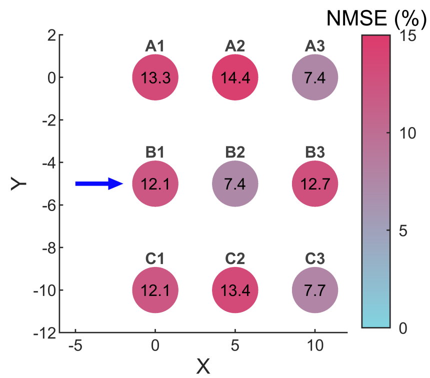

Fig. 4: NMSE of MPO predictions given by the linear SPARX model.

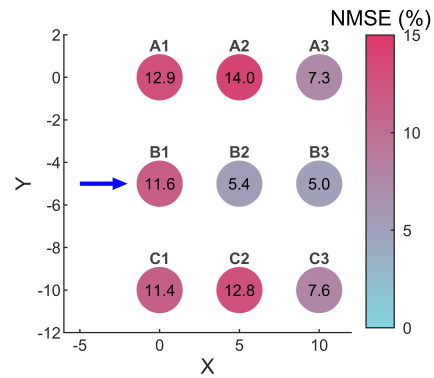

Fig. 5: NMSE of MPO predictions given by the nonlinear SPARX model.

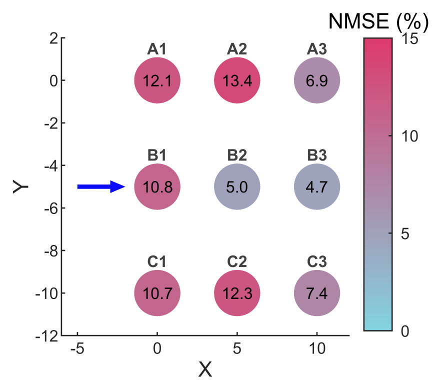

Fig. 6: NMSE of MPO predictions given by the GP-SPARX model.

stamps (i.e. training samples). Since $T >> M$ ($T = 730$ and $0 \leq M \leq 4$), the computation of least squares estimate is dominated by matrix multiplication. In contrast, GP is notorious for its basic complexity of $O\left(T^3\right)$, because of the inversion of a $T \times T$ matrix. However, given the relatively small training data set used in this preliminary study, the cost difference is hardly noticeable. One disadvantage of the nonlinear SPARX approach is that computational complexity will multiply if a higher order functional form is chosen. On the contrary, the cost of GP-SPARX remains unchanged unless more exogenous inputs are added.

Table 2: The computational cost of the predictive models.

	Linear SPARX	Nonlinear SPARX	GP-SPARX
Computational complexity	$O\left((M+1)^2 T\right)$	$O\left(4(M+1)^2 T\right)$	$O\left(T^3\right)$

5 Conclusions

This paper demonstrates how the physical understanding of the wake effect can contribute to the formation of predictive data-driven models – in terms of the

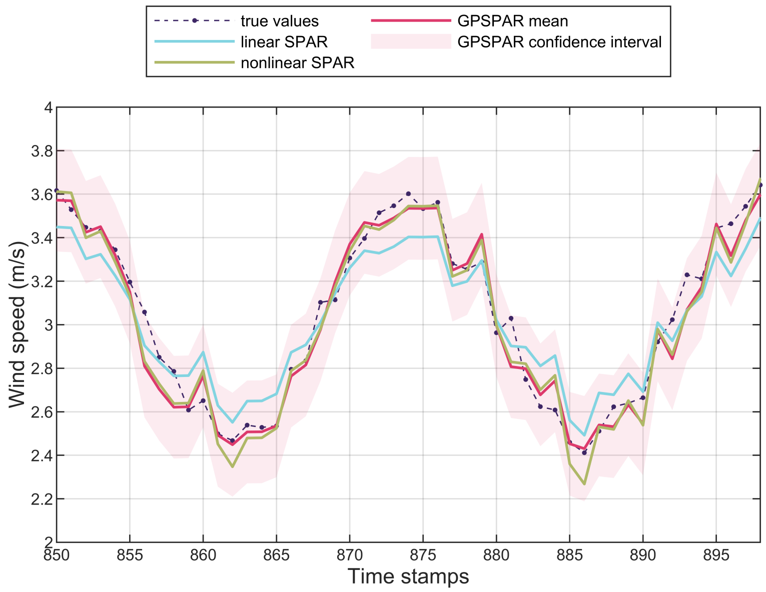

Fig. 7: Time series of wind speed predictions at turbine B3.

functional forms of regression models and the way in which GP models are used. A Gaussian process-based spatial autoregressive (GP-SPARX) approach is proposed in this paper, as a potential method to predict the spatial and temporal variations across a wind farm, given only one spatial reference, e.g. weather station measurements. In addition to the free stream wind speed that is used as an exogenous input, information about turbine positions and angles is given to the model in terms of effective upstream neighbours, in order to determine the spatial autoregressive process. It is demonstrated that the GP-SPARX model is able to capture nonlinear spatial correlations with acceptable accuracy. The predictive confidence intervals given by GP-SPARX can also be used as thresholds for detecting performance anomalies across a farm, in a way similar to [3, 6].

It must be noted that this paper presents a simple initial study with heavy assumptions in the simulation, including (a) a fixed free stream wind direction, (b) time-invariant turbine angles, and (c) simplified spatio-temporal correlations. The success of the proposed method in this simple study encourages us to further investigate the performance of GP-SPARX in a more complex setting, such as a simulated farm with more realistic assumptions and data collected from operating wind farms. In term of model design, the next step can be to incorporate physics into GP, in a similar manner as [4].

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