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On the relation between corruption and market competition

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Abstract

We construct a model where bureaucrats are corruptible, in the sense that they may accept bribes in order to mislead the authorities on the actual circumstances of firms that do not comply with a government regulation. We show that corruption increases the number of competing firms. We also show that increased competition (i) increases the likelihood that corruptible bureaucrats will actually transgress, and (ii) increases the bribe that corrupt bureaucrats will demand from each firm. These results are consistent with existing evidence showing a positive relation between competition and corruption, and that this causal relation operates in both directions.

KEYWORDS
competition, corruption, market entry

JEL CLASSIFICATION
D43, D73, H23, L20

1 | INTRODUCTION

According to Transparency International, corruption can be defined as *"the abuse of entrusted power for private gain"*.[1] It involves individuals who engage in fraudulent rent-seeking activities by exploiting their positions in public administration. By its very nature, corruption can infringe on a country's social, economic as well as political domains, while its repercussions can be potentially far reaching (e.g., Aidt, 2003; Dimant & Tosato, 2018; Shleifer & Vishny, 1993). In this study, our focus is on the interdependence between corruption incentives, bribery, and firms' decisions to compete in a market and comply with its regulations.

The relation between market competition and corruption has already received some attention. In a framework where corrupt officials receive bribes in order to issue licence fees that permit firms to compete in a market, Bliss and Di Tella (1997) find that different measures of competition intensity have ambiguous effects on both the number of competing firms and on the magnitude of corruption. The idea that corrupt bureaucrats demand bribes in exchange for entry licenses is also a feature of subsequent research on the issue. Emerson (2006) obtains multiple equilibria where high (low) corruption is associated with low (high) entry. This is because of reinforcing effects where bribes shrink

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profits, while reduced entry allows bureaucrats to extract a greater surplus from existing competitors. In the model of Amir and Burr (2015), the assumption of pre-existing firms in the market generates additional effects through which the number of new entrants under corruption may be higher.

These studies yield conflicting results on the relation between corruption and market entry. However, the existing evidence on this issue is not equally ambiguous. For instance, while the empirical investigation of Ades and Di Tella (1999) shows that different measures of competition are inversely related to corruption, there is also a plethora of more recent studies that present evidence in support of a positive relation between competition and corruption. The studies by Alexeev and Song (2013) and Diaby and Sylwester (2015) show evidence that measures of competition—including the number of competing firms—are positively and significantly associated with bribe payments to corrupt officials. Dreher and Gassebner (2013) present evidence that corruption facilitates entry in economies where market activity is significantly regulated, while, in a similar vein, Bologna (2017) uses empirical evidence to show a positive relation between corruption and market competition in environments where institutional quality is poor. The study of Bennett et al. (2013) offers further empirically-supported arguments that accord with the idea of a positive relation between competition and the incidence of corruption.[2] It is worth emphasizing that this evidence does not solely capture a "grease the wheels" scenario—the idea that corruption can reduce firms' operating costs by facilitating the reduction of red tape, regulatory burden etc. (e.g., Leff, 1964; Shleifer & Vishny, 1993). In fact, some of these studies show that the causal effect of this relation operates on the opposite direction, that is, market competition causes an increase of bribery and corruption (Alexeev & Song, 2013; Diaby & Sylwester, 2015).

To the best of our knowledge, the existing literature lacks an analytical framework that illuminates circumstances and mechanisms under which the degree of market competition is not only positive but, more importantly, bidirectionally related to bribery and corruption. So far, the focus has been mainly restricted to the causal effect of corruption on market entry. Whatever the reason for this neglect, the previously mentioned empirical studies reveal that the evidence in support of a two-way causal positive relation has gained some momentum and is certainly too compelling to be ignored. Clearly, there is a need for further theoretical investigations that can shed more light on the conditions that underpin the corruption-competition nexus, hence filling an important void in the literature. This study is one such attempt.

We analyze the joint determination of the incidence of corruption—manifested in the bribe-seeking behavior of corrupt bureaucrats—and the number of competing firms in a market. Our departure from previous research on this issue is twofold. Firstly, rather than assuming that bureaucrats directly control market entry, we consider a case where corruption allows firms to circumvent regulations in a manner that reduces their operating costs. To be more precise, we envisage a scenario where, in exchange for bribes, corrupt bureaucrats facilitate firms in evading the payments associated with the implementation of a specific regulation. If anything, this is an empirically relevant case, as pointed out by Gino et al. (2013). It warrants attention given that evasion of all sorts (e.g., tax payments) is an important facet of corruption in the corporate world. Crocker and Slemrod (2005) cite evidence that corporate tax underreporting in the United States amounts to almost $37.5 billion, while Joulfaian (2009) and Alm et al. (2016) present evidence showing that corporate tax evasion is more pronounced in economies where corruption among tax officials is a common occurrence. Secondly, in contrast to the models of Bliss and Di Tella (1997), Emerson (2006), and Amir and Burr (2015), in our framework bureaucrats are potentially corruptible. In other words, in addition to deciding the amount of bribes they will request in exchange for their fraudulent deeds, bureaucrats will also decide endogenously on whether to take advantage of their position and seek these bribes, or to abstain from such wrongdoing and behave honestly.

In our model, firms can produce goods using either a "bad" technology or a "good" one. This distinction may capture regulatory aspects—compliance with health and safety regulations or environmental standards, for instance—in light of the negative externalities associated with the use of the "bad" technology. Firms that employ this technology are liable to a penalty, but its adoption is costless; firms that employ the "good" technology are exempt from this penalty, but have to incur the cost of its adoption. Bureaucrats are entrusted with the tasks of inspecting and reporting the technology employed by firms, and taking the appropriate action, that is, collecting the penalty or not. Nevertheless, there is a moral hazard problem inherent to this setting: In exchange for a bribe, bureaucrats may offer to firms that employ the "bad" technology, the opportunity of fabricating their true circumstances. As expected from an environment where corrupt bureaucrats offer cost-reducing opportunities, corruption increases the number of competing firms, because the opportunity to bribe bureaucrats and avoid the penalty associated with the use of the "bad" technology increases a firm's potential profitability.

In broad terms, the causal direction of the previously mentioned outcome is yet another manifestation of a 'grease the wheels' scenario. The existing literature has already pointed out the role of bribery as some sort of an informal tax that allows governments to keep both public sector salaries and firms' costs, through 'effective' taxation and 'red tape', low (Leff, 1964; Shleifer & Vishny, 1993). To some extent, our model captures the government's (un)willingness to confront corruption through the probability that cases of 'hidden' non-compliance, facilitated by bribery, are detected and penalized. Given this, however, our model delves deeper into the underlying characteristics and the implications of collusions between public officials and firms, whose purpose is to mislead the authorities. The reason why this is important is twofold: First, there is empirical evidence to suggest that the lack of regulatory compliance is largely due to such illegal agreements, rather than being a mere side effect of governmental reluctance or oversight. In fact, the recent evidence by Xu et al. (2021) shows that anti-corruption campaigns can be effective in inducing greater regulatory compliance, exactly by curbing the instances of fraudulent collusion between managers and public officials. Second, existing models have not explicitly accounted for the causal impact of market competition on corruption and bribery, despite evidence to the contrary (e.g., Alexeev & Song, 2013; Diaby & Sylwester, 2015). By explicitly considering the bureaucrats' decision-making process, our model fills this gap between theory and evidence.

Indeed, our model's outcomes are not restricted to a 'grease the wheels' mechanism; they go way beyond this in a novel and significant manner. We show that a larger number of competing firms (i) make it more likely that a bureaucrat will decide to abuse his position of power by seeking bribes from the firms he inspects, and (ii) increase the bribe that the bureaucrat will request from each non-compliant firm in exchange for his misdeeds. The latter effect is driven by the bureaucrat's optimal response to the risk of being apprehended: He collects bribes from a lower fraction of the firms he inspects, in order to reduce his exposure, but at the same time he compensates by demanding a larger payment from each bribe-paying firm. Furthermore, we also show that a measure of the frequency of bribery in our model—that is, the number of firms colluding with bureaucrats to hide their non-compliance—is also higher in a more competitive environment.

In light of these outcomes, our model offers analytical and explicit mechanisms that direct our attention to the two-way causal relation between competition, bribery and the incidence of corruption. Put differently, our message is not restricted to the suggestion that the positive correlation between competition and corruption is attributed to merely the one causing the other. Our underlying message is in fact stronger. The two-way causal relation is a source of a feedback mechanism that generates a more powerful argument in explaining the positive relation between competition and corruption: Increased corruption may actually be entrenched to more competitive markets, and vice versa.

The main focus of our analysis is positive, rather than normative in nature. The objective is to examine the endogenous interrelation between managerial decisions on whether to compete in a market and comply with its regulations, and the public officials' incentives toward bribe-seeking behavior. Nevertheless, we also provide some policy implications. Specifically, we examine how a policy maker would choose the efficiency of the government's monitoring and detection technology, to maximize consumer surplus. This is a process that entails a trade-off: On the one hand, corruption intensifies the negative externality associated with the more widespread adoption of the "bad" technology; on the other hand, it intensifies competition which, in turn, suppresses prices. Our analysis indicates that, even when policy makers have perfect control on how many cases of unlawful collusion can be detected—effectively, controlling corruption incentives—they will not opt for the most efficient detection technology. In other words, the interplay between bureaucratic corruption and managerial decisions on compliance and firm entry, emerges as an additional explanation on why, despite statements to the contrary, many governments are not forceful enough in trying to curtail the incidence of bribery and corruption.

The remainder of the study is organized as follows: Section 2 presents an overview of the model. In Section 3, we analyze the circumstances surrounding bribery and corruption. Section 4 shows how the incidence of corruption and market entry are jointly determined, while Section 5 provides the welfare analysis. In Section 6, we conclude.

2 | THE MODEL

Consider a market of n competing firms that incur a fixed cost of entry $f > 0$. Upon entry, each firm supplies q_j units of a homogeneous good. Firms face the inverse demand function

$$p = \kappa - Q, \tag{1}$$

where $\kappa > 0$, p is the good's price and Q is the good's aggregate demand. Market clearing requires that $Q = \int_0^n q_j \mathrm{d}j$, meaning that we can use Equation (1) to express a firm's variable profit, denoted v_j, according to

$$v_j = \left(\varkappa - \int_0^n q_j \mathrm{d}j\right) q_j - m q_j, \tag{2}$$

where $m > 0$ is the per unit cost of production.

Since the good supplied from the industry is homogeneous, it is useful to think of the firms as Cournot competitors that choose the quantity they produce in order to maximize their variable profit. Therefore

$$\frac{\partial v_j}{\partial q_j} = 0 \Leftrightarrow \varkappa - \int_0^n q_j \, \mathrm{d}j - q_j^* - m = 0. \tag{3}$$

Combining the market clearing condition $Q = \int_0^n q_j \mathrm{d}j$ with Equations (1) and (3), it follows that the equilibrium is symmetric; that is, $q_j^* = q^* \forall j$. Using Equation (3), we get

$$q^* = \frac{\varkappa - m}{1 + n}, \tag{4}$$

where $\varkappa > m$ is assumed to ensure that each firm produces a strictly positive quantity of output. Given $Q = nq^*$, we can substitute Equation (4) in (1) to get

$$p = \frac{\varkappa + nm}{1 + n} \equiv p(n), \tag{5}$$

such that $p'(n) < 0$. The variable profit of a firm equals $v = (p - m)q^*$. Substituting Equation (4) in (5), we get

$$v = \left(\frac{\varkappa - m}{1 + n}\right)^2. \tag{6}$$

As expected, the firm's variable profit is lower when the number of competitors in the market is higher. With a higher number of competitors, total supply (i.e., nq^*) increases. For the market to clear, the price of the good has to fall in order for consumer demand to absorb this additional supply. The reduction in price has a detrimental effect on each firm's revenue and, therefore, variable profit.

2.1 | Regulation and technology choice

There are two technologies available for each firm to choose how to produce: a "good" one and a "bad" one. This distinction can capture a broad range of scenarios. For instance, the "good" and "bad" technologies can be interpreted as a firm's decision to comply or not with a health and safety regulation. Alternatively, they may capture a firm's choice to adopt either an environmentally-friendly technology or a polluting one. Henceforth, we shall be referring to the "good" technology as Type-G and to the "bad" technology as Type-B. To keep the analysis tightly focused, we will abstract from any differences in productivity with regard to these technologies. What differentiates them is that, unlike the Type-G technology, the Type-B one is a source of negative externalities. These differences will come into play more explicitly in Section 5 where we undertake a welfare analysis.

The Type-B technology can be adopted at zero cost. However, firms that employ it are liable to a penalty (e.g., a tax or a fine), equal to $t > 0$. The Type-G technology relieves firms from the obligation to pay the penalty. Nevertheless, its adoption is costly in the sense that it requires a firm-specific fixed cost c_j. This cost is random and realized only after a firm makes its decision to compete in the industry. It is also independently and identically distributed across firms. We will assume that the continuous random variable c_j has a uniform distribution with support on the $[0, 1]$ interval.

At this point, we should emphasize that the assumption of uncertain compliance costs, the revelation of which occurs after entry, is not a mere technical device in the model's construction. On the contrary, it finds support from several empirical studies. Some of these studies identify government regulation per se as a potential source of uncertainty, in light of the time it takes from the entry decision to a firm's actual operation. Pindyck (1993) highlights the unpredictability of changes in government regulations among the factors that generate cost uncertainty for firms, while

Becker and Henderson (2000) pinpoint the variance of air quality regulations and how this materializes in cost uncertainty through varying equipment requirements. In a similar vein, Purvis et al. (1995) argue that *"compliance is often a moving target as environmental regulations are likely to evolve"* (Purvis et al., 1995; p.541); they also argue that the uncertainty about compliance requirements generates uncertainty regarding the cost and the performance of technologies that are adopted as a means of compliance. The study of Oliva et al. (2020) elaborates on the technology-based arguments of government regulation as a source of uncertainty: They claim that *"benefits of both the technology and costs associated with the follow-through decision may be unknown to the potential adopter. New information arrives between the take-up and follow-through decisions in the form of learning about the technology [...] or in the form of shocks to the opportunity cost of follow-through"*, (Oliva et al., 2020; p.617). In this respect, they echo Pindyck's (1993) claim that *"uncertainty can only be resolved by undertaking the project; actual costs and construction time unfold as the project proceeds"* (Pindyck, 1993; p.54). These arguments offer credence to our model's assumption that uncertainty is resolved following a firm's decision to enter and compete in a market. Given the above, it is perhaps little surprise that several empirical studies (e.g., Becker & Henderson, 2000; Fuss et al., 2008; Oliva et al., 2020; Purvis et al., 1995) show evidence that the cost uncertainty associated with regulation compliance is a significant factor in firms' decision-making.[3]

There are also several empirically motivated arguments on why government regulations can have differentiated impacts across different firms' costs of compliance. One reason can be geographical location, and the fact that regulatory requirements and implementation can differ across regions. Becker (2011) offers supporting evidence for the case of the United States; this fact is also extensively analyzed by the European Commission (2015), whose report also mentions *"divergences between countries due to varying interpretations of EU legal requirements during transposition of European Directives into national legislation"* (European Commission, 2015; p.102). Another important source of compliance cost heterogeneity emanates from differences in entrepreneurial ability (e.g., Wu & Knott, 2006). For example, Scholz (1984) hints at the differentiated ability to interpret accurately all the requirements associated with regulatory codes, which are complex and, in many instances, ambiguous. Furthermore, the aforementioned arguments—that is, differences in location and entrepreneurial aptitude—also have repercussions about how costly is a firm's effort to successfully mobilize and combine the resources, such as labor, capital, and material, necessary for its operations to comply with regulatory requirements (e.g., Newell & Stavins, 2003; Pindyck, 1993).

For simplicity and clarity, and given the distribution of the random adoption cost of the Type-G technology, we introduce the following restriction on the penalty imposed for the adoption of the Type-B technology:

Assumption 1. $t < 1$

One reason why this may apply is because the government's decision to induce all firms into the adoption of the Type-G technology—by setting $t > 1$ – may deter entry for everyone, since each firm's entry decision is made prior to the realization of c_j, that is, it is based on expectations.[4] Another reason may be because the government is not aware of the upper bound of the distribution of c_j. Whatever the reason, note that in Section 4 we argue that our results remain qualitatively intact, even after relaxing Assumption 1.

The government cannot directly observe the technology of each firm. For this reason, it delegates this task to bureaucrats who monitor firms and verify the technology they employ. We assume that the government hires δ bureaucrats, where $\delta < n$, and offers a salary $w > 0$ to each of them, in exchange for their services. Bureaucrats are instructed to inspect the technology adopted by each firm and, therefore, decide on whether a penalty should be imposed or not. All firms will have their technology choice verified, meaning that each bureaucrat will monitor $\frac{n}{\delta}$ firms. As we shall see later, however, firms and bureaucrats may enter into an illegal agreement to conceal the true circumstances (regarding the type of technology adopted) from the authorities. This opportunity arises because bureaucrats are corruptible, thus they may be willing to fabricate a firm's circumstances in exchange for a bribe. If the authorities detect this illegal activity, both parties—the firm and the bureaucrat involved in the collusion—will face sanctions.

The timing and information structures are as follows: Stage 1 is when firms pay the fixed cost and enter the market. In Stage 2, each firm observes its c_j and makes its technology choice, that is, whether to comply (Type-G technology) or not (Type-B technology). Subsequently, firms produce goods and sell them in the market. In Stage 3, each firm is inspected by a bureaucrat. In terms of information, the bureaucrat who inspects each firm can observe and verify whether the firm complies with the government regulation or not; the bureaucrat cannot observe the circumstances that induced the firm to make this choice. Put differently, the bureaucrat can observe if a firm adopted either the Type-G or the Type-B technology, but is not aware of what was each firm's c_j at the time it made its technology choice.[5] The information on which the bureaucrat will act is only his observation of whether a firm complies or not. Given these, for a firm that adopted the Type-G technology, this is when the events that determine its overall profitability end. The firm that adopted the Type-B technology, and as long it is inspected by a bureaucrat who is not willing to accept a bribe, will pay the penalty t and the events that determine its overall profitability will end. Nevertheless, for firms who are inspected by corrupted bureaucrats, there are also events that apply to Stage 4; This is the stage when firms who have illegally masqueraded as adopting the Type-G technology, while they have actually adopted Type-B, may be detected and revealed. For firms that are not detected, nothing changes in relation to Stage 3; those who do, however, will face additional penalties that will determine their overall profitability. Their detection will also reveal the identity of the corrupt bureaucrat, who will also face sanctions.[6] This sequence of events is also illustrated in Figure 1.

2.2 | Market entry in the absence of corruption

For illustrative purposes, we will present the model's outcomes under a benchmark scenario where none of the bureaucrats are corruptible. This may be because they have strong moral considerations that prevent them from taking advantage of their position to seek illegal rents. In this case, all bureaucrats will truthfully report the actual circumstances of the firms they monitor.

In this setting, let us consider a firm that has decided to compete in the market. Given that the firm will be inspected in Stage 3, during the second stage it will adopt the Type-G technology if $v - c_j \geq v - t$ holds. Consequently, a firm j will be willing to use the Type-G technology as long as

$$c_j \leq t. \tag{7}$$

Intuitively, the condition is that the cost of its adoption must be lower than, or at least equal to, the penalty from the adoption of the Type-B technology, that is, the amount t. Given Equation (7), firms that face $c_j \in [0, t]$ will opt for the adoption of the Type-G technology whereas firms that face $c_j \in (t, 1]$ will adopt the Type-B technology.

Now let us analyze the choice of a firm that considers entry during the first stage. The firm's expected profit is given by

$$\pi_j = v - \int_0^t c_j \mathrm{d}c_j - t\int_t^1 \mathrm{d}c_j = v - \frac{t^2}{2} - t(1-t) = v - \mu^{NC} \equiv \pi^{NC}, \tag{8}$$

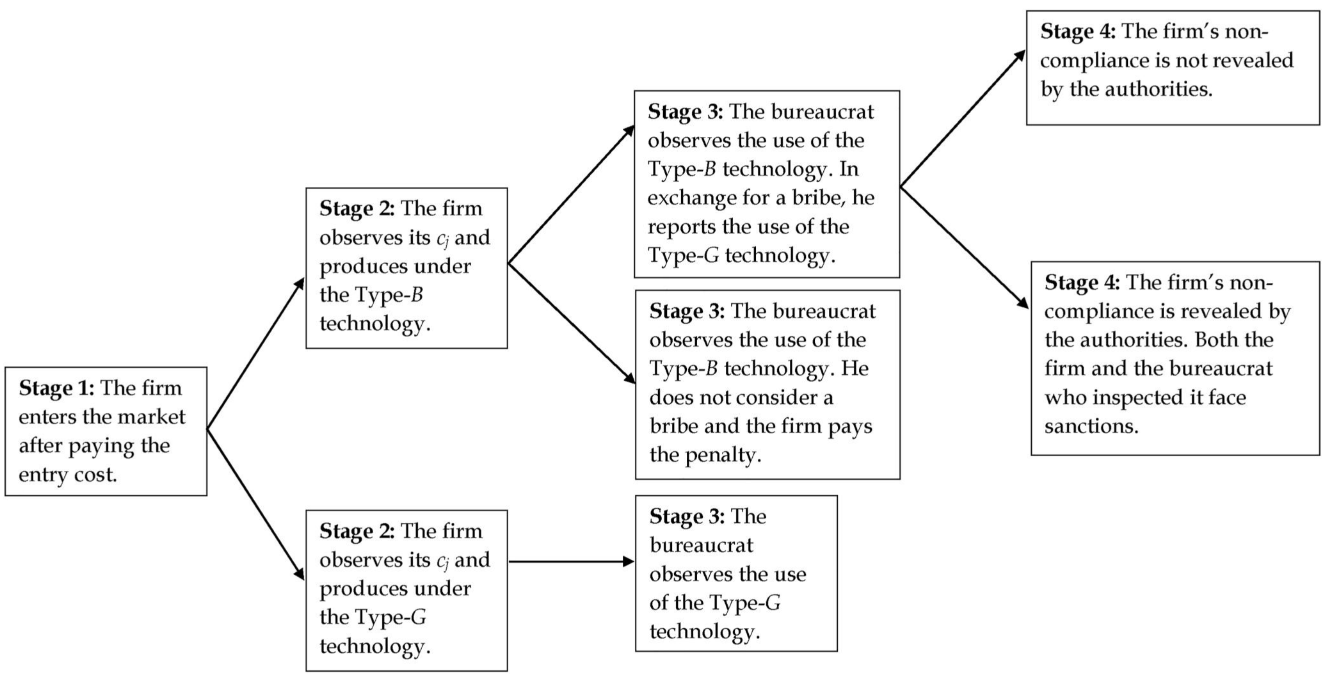

FIGURE 1 The sequence of events

where

$$\mu^{NC} = \frac{t(2-t)}{2}. \tag{9}$$

Potential entrants will wish to pay the fixed cost of entry and compete in the market as long as $\pi^{NC} \geq f$. Therefore, given $\frac{\partial \pi^{NC}}{\partial n} = \frac{\partial v}{\partial n} < 0$, the equilibrium number of firms will be determined by the zero-profit condition $\pi^{NC} = f$. Using Equations (6) and (8), it follows that the number of entrants can be calculated as[7]

$$n^{NC} = \frac{\kappa - m}{\sqrt{f + \mu^{NC}}} - 1. \tag{10}$$

The result in Equation (10) gives the equilibrium number of competing firms in the scenario where all bureaucrats are not corruptible, thus the parties that are involved in the choice and verification of the technology employed, that is, firms and bureaucrats, behave honestly. Nevertheless, this is just a baseline case. In an alternative scenario where bureaucrats are corruptible, the delegation of monitoring to them generates a moral hazard issue with potentially important implications for the model's outcomes. This is the issue to which we now turn.

3 | CORRUPTIBLE BUREAUCRATS AND BRIBERY

Suppose that a bureaucrat would be willing to accept a bribe in order conceal the actual circumstances relevant to the technology choice of the firm he inspects. Specifically, by paying a bribe $b > 0$ to the official, the firm can avoid paying the penalty, despite the fact that it can choose not to incur the adoption cost of a Type-G production method. Instead, the official who accepts the bribe will report that the firm employs the Type-G production technology, while in reality this is not the case.

Of course, the risk underlying this illegal practice is that it may be eventually detected by the authorities. Formally, let $g \in [0, 1]$ be the probability that a firm employing the Type-B technology is revealed as having unlawfully misrepresented its circumstances. When a firm is proven guilty of such misdemeanor, it will ultimately have to pay the penalty associated with the use of a Type-B technology, having already paid a bribe to avoid this penalty in the first place. Naturally, this revelation will automatically lead to the detection and punishment of the bureaucrat who transgressed by colluding with the firm in exchange for a bribe. We will delve into a more detailed description of the issues surrounding a corruptible bureaucrat's behavior and actions at a later point. For now, we focus on a firm contemplating which technology to adopt and whether to bribe the bureaucrat who inspects it.

Consider a firm that is inspected by a corruptible bureaucrat in Stage 3. In this case, the firm can pay a bribe to conceal the adoption of the Type-B technology. However, the firm also understands that there is a chance of this misdemeanor being revealed in Stage 4. Therefore, the Stage 2 expected payoff from adopting the Type-B technology, while anticipating a collusion with a corrupt bureaucrat, is $v - (b + gt)$, that is, the amount that remains from variable profits, after subtracting the bribe and the expected penalty in the event that the firm's malefaction is revealed. Of course, if the bribe is excessively high, paying the penalty associated with the adoption of the Type-B technology—a decision that returns a payoff $v - t$ – is a preferrable choice. To rule out this uninteresting case, henceforth we restrict our attention to circumstances where $b < (1-g)t$. With this in mind, the Stage 2 choice of a firm is between adopting the Type-G technology—a decision that returns a payoff $v - c_j$ – and colluding with the bureaucrat to conceal its adoption of the Type-B technology. It follows that a firm j will be willing to adopt the Type-G technology as long as

$$c_j \leq b + gt \equiv \widehat{c}. \tag{11}$$

In this scenario, firms with $c_j \in [0, \widehat{c}]$ will choose the Type-G technology, whereas firms with $c_j \in (\widehat{c}, 1]$ will choose the Type-B technology and, in the third stage, bribe bureaucrats to facilitate them in deceiving the authorities. Taking account of Equation (11), the number of firms willing to engage in a fraudulent collusion with bureaucrats can be found from

$$n \int_{\widehat{c}}^{1} \mathrm{d}c_j = n\gamma, \tag{12}$$

where

$$\gamma = 1 - (b + gt), \tag{13}$$

is the probability that the firm inspected by a corruptible bureaucrat will be willing to pay a bribe. This is an outcome that leads to

Proposition 1. *A higher bribe reduces a firm's willingness to collude with a bureaucrat in concealing the adoption of a "bad" technology. Therefore, it also reduces the number of firms out of which bureaucrats can potentially extract bribes.*

Proof. It follows from Equation (12), which is increasing in γ, and Equation (13), from which it is evident that $\frac{\partial \gamma}{\partial b} < 0$. $\square$

This result is straightforward to explain. A higher bribe will reduce the expected profit when the firm opts for the adoption of the Type-B technology and agrees to the corrupt official's demands in order to mislead the government. As a result, more firms will find the adoption of the Type-G technology to be a more desirable option in terms of profitability.

The aforementioned outcome is one of the important factors in determining a bureaucrat's behavior, to which we now turn our attention. A corruptible bureaucrat will contemplate engaging in the type of fraudulent collusion that we described previously. In addition to his salary w, there is also the opportunity to earn illegal rents from the firms that are willing to bribe him in order to mislead the authorities. Recall that each bureaucrat will monitor $\frac{n}{\delta}$ firms. Taking into account the previous analysis and discussion, the probability that a firm will be willing to offer him a bribe is γ. Furthermore, we use $\beta \in (0, 1)$ to denote the probability that a corrupted bureaucrat will be detected and punished for his nefarious activities. With regard to apprehended bureaucrats, the penalty for their misconduct is that they are dismissed without pay and they also lose all their ill-gotten gains. Irrespective of whether he is apprehended or not, the corrupted bureaucrat faces a fixed utility cost $l > 0$. This cost may capture various elements such as moral concerns, the anxiety and fear associated with the possible detection and punishment etc. Given this discussion, the expected utility u^C of a corrupted official is

$$u^C = (1 - \beta)\left(w + \gamma \frac{n}{\delta} b\right) - l. \tag{14}$$

It should be noted that, although potentially corruptible, the bureaucrat has the option to abstain from seeking illegal rents, hence behaving honestly. In this case he will enjoy utility equal to

$$u^{NC} = w. \tag{15}$$

Obviously, bribe-taking increases a bureaucrat's exposure and risk of detection. This is because the moment a firm is revealed as having misled the authorities, this will immediately expose the identity of the bureaucrat who inspected it originally. Recall that each corrupt bureaucrat expects to collect bribes from $\gamma \frac{n}{\delta}$ firms. Since the probability of detection for a guilty firm is g, the probability of revelation of a corrupted bureaucrat's malfeasance is[8]

$$\beta = 1 - (1 - g)^{\gamma \frac{n}{\delta}}. \tag{16}$$

Substituting Equations (13) and (16) in Equation (14), we can write the corrupted bureaucrat's expected utility as follows:

$$u^C = (1 - g)^{(1 - b - gt)\frac{n}{\delta}} \left[w + (1 - b - gt)\frac{n}{\delta} b\right] - l. \tag{17}$$

The bureaucrat will demand a bribe that maximizes his expected utility in Equation (17).[9] When deciding the bribe that he will ask in order to conceal the true characteristics of the firm he monitors, he will take account of some conflicting effects on his expected utility. On the one hand, a higher bribe will directly increase the amount of ill-gotten gains. On the other hand, it will reduce the potential pool of firms out of which he can extract illegal rents. This is because some of these firms will find it more advantageous to actually adopt the Type-G technology (thus having no need to bribe officials at all). The latter consideration has an additional effect on the bureaucrat's expected utility: By reducing the number of firms that will be willing to collude, it also reduces the probability that a bureaucrat will be eventually caught and punished for his wrongdoing. This is because it will reduce the number of links with firms that may be eventually proven to have evaded the penalty of not complying with regulations.

To obtain the bureaucrat's optimal bribe, we use Equation (17) calculate $\frac{\partial u^C}{\partial b}$. This calculation yields $\frac{n}{\delta}(1-g)^{(1-b-gt)\frac{n}{\delta}}H(b)$, where

$$H(b) = \alpha(g)w + 1 - gt - \alpha(g)\frac{n}{\delta}b^2 - \left[2 - \alpha(g)\frac{n}{\delta}(1-gt)\right]b, \tag{18}$$

and

$$\alpha(g) = -\ln(1-g) \geq 0. \tag{19}$$

Before we proceed, we will consider some technical restrictions (Assumptions 2–4) and results (Lemmas 1–2), the objective of which is to eventually derive a unique interior solution to the bureaucrat's maximization problem. On the outset, it should be emphasized that the Assumptions 2–4 that will follow are not necessary, but they are sufficient in ensuring a unique interior solution to the bureaucrat's maximization problem. Recall that in the process of choosing the bribe, the bureaucrat faces a complex trade-off that involves three distinct effects on his utility. The set of assumptions that follow are sufficient to ensure that, even under this complexity, the bureaucrats utility maximization problem satisfies the conditions for a maximum. With these in mind, we begin by defining the composite term

$$\theta = \frac{\alpha(g)(1-gt)}{2}, \tag{20}$$

and impose the following:

Assumption 2. $t \in \left(\frac{1}{\sqrt{2}(1-g)+g}, 1\right)$

This assumption merely imposes a lower bound on the penalty associated with the use of the Type-B technology. It also allows us to derive the result in

Lemma 1. $\theta < \frac{1}{4}$.

Proof. See Appendix A.1. □

Based on Lemma 1, in what follows we impose the following restriction:

Assumption 3. $\delta \in (\theta n, n)$.

Again, this assumption is merely a lower bound on the number of bureaucrats. While not a necessary one for the subsequent analysis, it is sufficient to guarantee the result in

Lemma 2. $H'(b) < 0$.

Proof. From Equation (18), check that $H'(b) = -\alpha(g)\frac{n}{\delta}2b - \left[2 - \alpha(g)\frac{n}{\delta}(1-gt)\right]$. By virtue of Equation (20) and Lemma 1, Assumption 3 is sufficient to ensure that $H'(b) < 0$. □

Recall that the maximum bribe that a bureaucrat can demand is equal to $(1-g)t$. Any bribe above this level would imply that those firms for which the adoption cost is too high to consider the implementation of the Type-G technology, would prefer to have their circumstances truthfully reported and subsequently pay the penalty, rather than paying the bureaucrat in order to conceal this information. To rule out this possibility, we define

$$\widehat{w} = \frac{2(1-g)^2t^2 - (1-gt)^2}{\alpha(g)(1-gt)}, \tag{21}$$

where $\widehat{w} > 0$ by virtue of Assumption 2. Given Equation (21), we impose.

Assumption 4. $w < \widehat{w}$

This restriction is not an alien one, in light of existing empirical evidence that identifies relatively low public sector salaries as a major factor behind public officials' incentives to be corrupt (e.g., Rijckeghem & Weder, 2001). Once more, we emphasize that this is just one of the sufficient conditions that ensure a unique interior solution to the bureaucrat's maximization problem, as we show in.

Lemma 3. *There exists* $b^* \in (0, (1-g)t)$ *such that*

$$\frac{\partial u^C}{\partial b}\begin{cases} >0 & if \quad b < b^* \\ <0 & if \quad b > b^* \end{cases}.$$

Proof. See Appendix A.2. □

The solution b^* is the bribe that the corrupt bureaucrat will demand from each firm, in exchange for his contribution in facilitating the firm to evade the penalty of non-compliance. It is found by setting $H(b) = 0$ in Equation (18). That is,

$$b^* = \frac{\sqrt{\left[\frac{2\delta}{a(g)n} - (1-gt)\right]^2 + \frac{4\delta[a(g)w + 1 - gt]}{a(g)n}} - \left[\frac{2\delta}{a(g)n} - (1-gt)\right]}{2} \equiv b(n). \tag{22}$$

Based on Equation (22), we can infer the result in.

Proposition 2. *More competition increases bribery. Specifically, the bribe that a corrupt bureaucrat demands from each firm is increasing in the number of competing firms.*

Proof. See Appendix A.3. □

This is a novel result of our model—one that finds support from the existing evidence however (e.g., Alexeev & Song, 2013; Diaby & Sylwester, 2015). A higher number of firms increases the pool of potential bribe payers. In this case, this effect also increases the probability that a bureaucrat will be eventually apprehended for his fraudulent behavior. By increasing the bribe, he mitigates this latter effect while, at the same time, increasing the potential gains from his misconduct. Put differently, the corrupt bureaucrat's optimal response to the risk of being apprehended is to collect bribes from a lower fraction of the firms he inspects while, at the same time, demanding a larger bribe from each of these firms.

Of course, at this point we should mention an important distinction when we consider the impact of market competition on bribery. According to Méndez and Sepúlveda (2010), the total amount of bribes collected by corrupted bureaucrats does not only reflect the amount of each bribe but also the frequency of bribe-taking. On the outset, it is possible that the bribe paid per illegal collusion is high whereas, at the same time, the number of bribe-exchanging agreements may be low. Later, however, we show that (in our model) the number of firms willing to bribe bureaucrats, to hide their non-compliance, is also higher in a more competitive market.

4 | CORRUPTION INCENTIVES AND MARKET ENTRY

Let us consider the choice of a firm that considers entry during Stage 1, and focus on the case where bureaucrats are willing to accept bribes. In this case, we can use Equation (11) and $b^* = b(n) < (1-g)t$ to write a firm's expected profit as

$$\pi_j = v - \left(\int_0^{\widehat{c}} c_j dc_j + [b(n) + gt] \int_{\widehat{c}}^1 dc_j \right) = v - \mu^C \equiv \pi^C, \tag{23}$$

where

$$\mu^C = \mu^{NC} - [(1-g)t - b(n)] \left[\frac{(1-g)t - b(n)}{2} + 1 - t \right]. \tag{24}$$

The expressions in Equations (23) and (24) direct us to the result in.

Lemma 4. *It is* $\pi^C > \pi^{NC}$ $\forall n$ *and* $\frac{\partial \pi^C}{\partial n} < 0$.

Proof. $\pi^C > \pi^{NC}$ follows from a straightforward comparison between Equations (8) and (9) and Equations (23) and (24). Using Equation (6) and Equations (23) and (24), we can calculate $\frac{\partial \pi^C}{\partial n} = -\frac{2(\kappa - m)^2}{(1+n)^3} - b'(n)[(1-g)t - b(n) + 1 - t]$. This is unambiguously negative given that $b'(n) > 0$ (see Proposition 2). □

It should be noted that there are two distinct mechanisms that cause a negative relation between the number of competing firms and expected total profits. In addition to the standard, negative impact of increased competition on variable profits, there is also the increased bribe that a firm contemplating entry anticipates to pay in the event that it enters into a fraudulent agreement with a corrupt bureaucrat. More importantly, the preceding analysis allows us to infer the number of firms that will pay the fixed entry cost, hence competing in the market. Indeed, having established that $\frac{\partial \pi^C}{\partial n} < 0$, the equilibrium number of firms, denoted n^C, will be determined by the zero-profit condition $\pi^C = f$. Contrary to the case where there is no corruption (see Equation (10)), here the functional relation between π^C and n is too complicated to yield an analytical solution. This is not problematic though, because the results presented in Lemma 4 point unambiguously to the fact that

$$n^C > n^{NC}. \tag{25}$$

The intuition behind the outcome in Equation (25) is easy to understand. The bribe that corrupt bureaucrats seek in return for their misconduct ensures that, for firms adopting the Type-B technology, the expected cost of non-compliance is lower. In other words, the presence of corruption offers opportunities that reduce a firm's expected operating costs. As a result, the expected total profit increases, thus enticing more firms in the market. We present this outcome more formally in.

Proposition 3. *The number of firms entering and competing in the market is higher in the presence of corrupted, bribe-seeking bureaucrats.*

Proof. It follows from the result in Equation (25). □

A corruptible bureaucrat will take advantage of his position of power, thus seeking illegal rents, only if the expected utility when doing so is higher compared to the utility that he will enjoy if he remains honest. To see the characteristics that drive this decision, define the composite term $z(n) = [1 - b(n) - gt]\frac{n}{\delta}$ and use Equation (22) to write it as

$$z(n) = \frac{\frac{2}{\alpha(g)} + \frac{n}{\delta}(1-gt) - \sqrt{\left[\frac{2}{\alpha(g)} - \frac{n}{\delta}(1-gt)\right]^2 + \frac{4n[\alpha(g)w + 1 - gt]}{\delta \alpha(g)}}}{2}. \tag{26}$$

Of course, given Equations (12) and (13), the expression in Equation (26) is the number of firms from which a corrupted bureaucrat collects bribes. Using Equations (15) and (17), we can now define $\widehat{U}(n) = u^C - u^{NC}$, that is,

$$\widehat{U}(n) = (1-g)^{z(n)}[w + z(n)b(n)] - l - w, \tag{27}$$

an expression through which we can establish.

Lemma 5. *$z'(n) > 0$ and $U'(n) > 0$.*

Proof. See Appendix A.4. □

The result of Lemma 5 has an important implication about how market competition affects a bureaucrat's incentive to be corrupt. We can formalize this argument through.

Proposition 4. *An increase in the number of competing firms increases the likelihood that a corruptible bureaucrat will actually transgress. When this happens, the frequency of bribe-taking, that is, the number of firms from which a corrupted bureaucrat receives bribes, is higher.*

Proof. It follows from Lemma 5. □

In terms of intuition, a more competitive market has a number of distinct effects on a corrupted bureaucrat's utility. It increases the pool of potential bribe payers, but it also increases the bureaucrat's exposure to the risk of being seized for his transgression. However, the bureaucrat mitigates the latter possibility by demanding a higher bribe per firm, thus accepting more bribes from a lower fraction of the firms he inspects. Note, however, that although the fraction of bribe-paying firms falls, this is a lower fraction of a higher total number of firms. Ultimately, the positive impact of a higher n dominates, hence the outcome in Proposition 4.

Now, notice that, by virtue of Equations (26) and (27), it is $U(0) = -l < 0$. Obviously, for our analysis to be meaningful, the moral costs, anxiety etc. emanating from a bureaucrat's misconduct should not be so strong as to deter him from ever considering abusing his position of power. Otherwise, any consideration of "corruptible" bureaucrats would be irrelevant; no one would optimally decide to be corrupt. Obviously, this is a scenario that, in addition to being of limited interest, is also very unrealistic. As it transpires, a sufficient condition to avoid such circumstances is by imposing.

Assumption 5. *$l < u^C(n^C) - w$.*

Together with Lemma 5, this assumption ensures the existence of $\widehat{n} \in (0, n^C)$, defined from $U(\widehat{n}) = 0$. Once more, the complexity of the model's expressions does not allow this to be shown analytically. We know, however, that the following applies:

Lemma 6. *Corruptible bureaucrats behave honestly if $n < \widehat{n}$, but become corrupted if $n > \widehat{n}$.*

Proof. It follows from $U(0) < 0$, Lemma 5 and Assumption 5. □

We now have all the results and mechanisms that are necessary to present and discuss the main upshot of our analysis. We do this in.

Proposition 5. *The incidence of corruption is both a cause and an effect of an increased number of competing firms. Consequently, an equilibrium with corruption is associated with more intense competition among firms, and vice versa, compared to an equilibrium without corruption.*

Proof. Recall that n^{NC}, n^C and $\widehat{n}$ are all composite parameter terms. With this in mind, consider the case where these parameter configurations satisfy $n^{NC} > \widehat{n}$. In this case, we can allude to Equation (25) and Lemma 5 to establish that an equilibrium without corruption does not exist. This is because under all circumstances, equilibrium entry falls in the region where bureaucrats find it optimal to be corrupt when such opportunity is given to them. It follows that the only possible solution for market entry is n^C. Indeed, this conjecture is verified by the fact that $n^C > \widehat{n}$ holds. In the alternative scenario where parameter configurations satisfy $n^{NC} < \widehat{n}$ we have multiple equilibria in the sense that two scenarios represent possible equilibrium outcomes. By virtue of Lemma 5, an equilibrium without corruption and n^{NC} competing firms is consistent with $n^{NC} < \widehat{n}$, whereas an equilibrium with corruption and a higher number of competing firms is n^C is also consistent with $n^{NC} < \widehat{n}$, given Equation (25) and $\widehat{n} < n^C$. □

The anticipation of paying bribes as a means of circumventing government regulation, hence increasing expected profitability, is a major factor in determining how many firms will enter and compete in the market. If this number is sufficiently high, it can entice bureaucrats into abusing the power entrusted to them, hence seeking to increase their personal wealth through the collection of bribes. What emerges under such circumstances is a powerful propagation mechanism that can establish bureaucratic corruption as a permanent feature of a market with more competing firms. Even when multiple equilibria exist, they are consistent with the positive, two-way causal relation between corruption and market entry. This multiplicity arises under parameter configurations that generate a case of self-fulfilling prophecies. If potential entrants expect that bureaucrats will (will not) accept bribes, the resulting entry in the industry will be sufficient to motivate bureaucrats to seek (not to seek) illegal rents through bribery, thus verifying the initial expectation.

Once more, we emphasize that these outcomes are consistent with a plethora of empirical studies whose results concur with a positive relation between the incidence of corruption, bribery, and market competition. Our model's novelty is that it demonstrates analytically the two-way causal nature of this relation. In this respect, it indicates the possibility that increased corruption incentives are entrenched to more competitive markets, and vice versa.

Before we complete the analysis of this section, we will devote some discussion to argue that our results remain qualitatively identical when the restriction $t<1$ is relaxed. Although we view this as a less realistic case, we present its corresponding implications for reasons of concreteness. Firstly, let us begin with the case where bureaucrats and firms behave honestly. Naturally, the condition $t\geq 1$ implies that all potential entrants will choose to adopt the Type-G technology. This is because the payment of the fine is always the more costly option, whatever the realized cost of implementing the Type-G technology. In this case, $\mu^{NC}=1/2$ and market entry in the absence of corruption is $n^{NC}=(\kappa-m/\sqrt{f+0.5})-1$. Now let us consider the case where corruption is an equilibrium phenomenon in the sense that bureaucrats are seeking bribes in order to conceal the actual circumstances of firms which are willing to offer them. Although all firms would be willing to adopt the Type-G technology in the absence of corruption, the opportunity offered by corrupt bureaucrats allow some of them to use the costless Type-B technology while claiming to do otherwise. It is straightforward to verify that the analysis and results are the same, as long as the different restriction $g<1/t$ holds. This will be required in this setting to make the story non-trivial. Indeed, if this condition does not hold then all firms will choose not to pay bribes as adopting the Type-G production technology is always a less costly option. Under $g<1/t$, however, it is $\mu^{C}=\mu^{NC}-\left([1-(b+gt)]^2/2\right)$ meaning that the equilibrium number of firms under corruption will still satisfy $n^{C}>n^{NC}$. The bureaucrat's problem is also unaffected in relation to what was presented in the original version of the model, hence implying that all our results can survive qualitatively intact.

5 | WELFARE ANALYSIS

So far, our analysis and discussion has been positive rather than normative; our objective was to uncover the mechanisms that underpin the empirically-observed, bidirectional relation between corruption, bribery and market competition. Obviously, one can think of various policy implications from an analysis such as ours—implications that emanate from the impact of the corruption-competition nexus on welfare. To see this, recall that the distinction between the two technologies was meant to capture the idea that the "bad" technology is responsible for a negative externality. With this in mind, one can in principle identify two opposing effects of corruption on consumer's welfare: On the one hand, it magnifies the negative externality by leading to increased market entry and to a higher fraction of firms that employ the "bad" technology; on the other hand, the same increase in market entry will intensify competition, thus leading to a reduction of prices and a corresponding increase of consumer surplus.

How will a government who cares about consumer welfare respond to this trade-off? The objective of this section is to address this question. To do this, however, we first need to become explicit about consumers' preferences, and to specify these preferences in a manner that is consistent with the inverse demand function that we adopted in our model (see Equation (1)). For this reason, we follow Martin (2009) and assume that there is a unit mass of consumers whose preferences are characterized by the utility function

$$V=\left(\varkappa Q-\frac{Q^2}{2}\right)+x-E, \tag{28}$$

where $\varkappa>0$, Q is the quantity of the market's good that consumers purchase, x is the quantity of a different good purchased by consumers (acting as the numéraire), while E captures the negative externality that stems from the use of the Type-B technology by (some of) the market's competing firms. Consumers who have disposable income y maximize Equation (28) subject to the budget constraint

$$pQ+x=y, \tag{29}$$

taking p and E as given. It is straightforward to establish that the solution to the consumer's maximization problem will lead to the same linear inverse demand function that we adopted in Equation (1).

With regard to the negative externality, we will assume that each firm's use of the Type-B technology results in a negative externality of $e>0$. Under an equilibrium with corruption, and given Equation (13), the number of firms who will adopt this technology is $\gamma n^C=[1-b(n^C)-gt]n^C$. It follows that the overall negative externality is

$$E=\left[1-b\left(n^C\right)-gt\right]n^C e. \tag{30}$$

Next, we can substitute Equations (1), (29) and (30) in Equation (28). It follows that we can write consumers' welfare as

$$V=\frac{\left[\varkappa-p\left(n^C\right)\right]^2}{2}+y-\left[1-b\left(n^C\right)-gt\right]n^C e, \tag{31}$$

where $p(n^C)$ is defined in Equation (5). Note that, by virtue of Equation (22), the optimal bribe is also a function of the detection probability, that is, $b^*=b(n^C,g)$, meaning that from Equations (23) and (24) and $\pi^C=f$, the number of competing firms will also be a function of g, that is, $n^C=N(g)$. Using these together with Equation (5), we can write consumers' welfare in Equation (31) as a function of the government's detection probability. That is

$$V(g)=\frac{(\varkappa-m)^2}{2}\left[\frac{N(g)}{1+N(g)}\right]^2+y-[1-b(N(g),g)-gt]N(g)e. \tag{32}$$

We will employ Equation (32) to consider a government that controls g as a policy instrument. Specifically, we will consider an initial stage (Stage 0) during which the government chooses g to maximize consumers' welfare. The question is whether the welfare-maximizing policy will result in either $g=1$ or $g<1$ – the latter case being indicative of a government willing to 'accept' some level of corruption. Unfortunately, the complexity of the model's solution creates an insurmountable difficulty in undertaking this process analytically. Indeed, the system of equations in Equation (22) and $\pi^C=f$ is so complex that, while we know that it generates a unique equilibrium pair n^C and b^*, it is impossible to obtain closed-form solutions. The only way to deal with this obstacle, thus being able to meaningfully undertake the welfare analysis, is to do so by means of a numerical example. Specifically, we will set permissible values for all the model's parameters, apart from g which we will allow to take all permissible values within $[0,1]$ in order to check how $V(g)$ varies with it.

Our baseline numerical values are $\varkappa=30$, $m=20$, $y=80$, $t=0.9$, $\delta=1$, $f=23$, $w=1$ and $e=0.5$. Given these, the relation between consumers' welfare and g is illustrated in Figure 2a. As we can see, the value of g that maximizes welfare is below 1, at roughly 0.8. In other words, even under a case where policy makers have a significant control of corruption incentives (after all, $g=1$ implies that no bureaucrat will have the incentive to be corrupt—see Equation (27)), they will choose not to adopt a policy that will eliminate the incidence of corruption. The underlying relation between corruption and firms' decisions to compete in the market is the key to this outcome: By intensifying competition, corruption may actually have some positive impact on consumer welfare. Of course, the policy makers' response depends on the relative benefits and costs of corruption for consumer welfare. One expects that if the relative strength of the negative externality becomes higher, then policy makers will be ever more forceful in trying to restrain

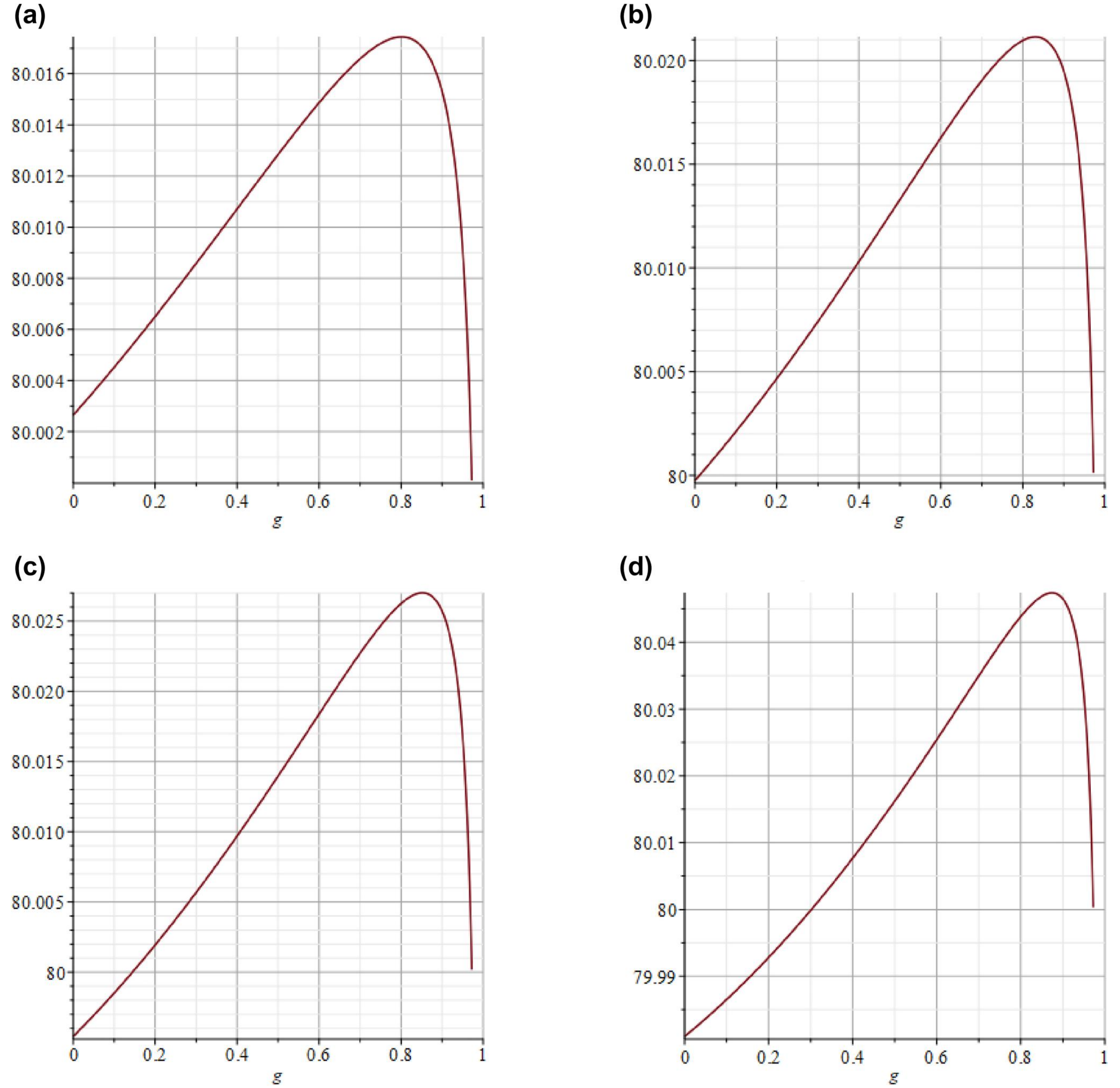

FIGURE 2 (a) $e = 0.5$; (b) $e = 0.7$; (c) $e = 1$; (d) $e = 2$

the incidence of corruption. Indeed, as Figure 2b–d show, increasing the value of e leads to correspondingly higher optimal values for g.

In the previous examples, the optimal value of g was indeed relatively high, indicating a probability of corruption detection of at least 80%. It should be noted, however, that our focus was on consumer welfare. Naturally, one would expect that a consideration of total welfare, that is, including producers' expected profits and bureaucrats' utility, will add elements that could further reduce a government's willingness to combat corruption. After all, producers and bureaucrats should be better off—in expected terms—by the ability to illegally collude and face a lower likelihood of being sanctioned for such offenses. Indeed, if we use the baseline scenario but undertake a numerical analysis of total welfare, which includes expected profits (from Equation (23)) and bureaucrats' utility (from Equation (17)); including a numerical value for $l = 0.1$), we can see that the optimal detection probability falls significantly to roughly 0.5 (see Figure 3).

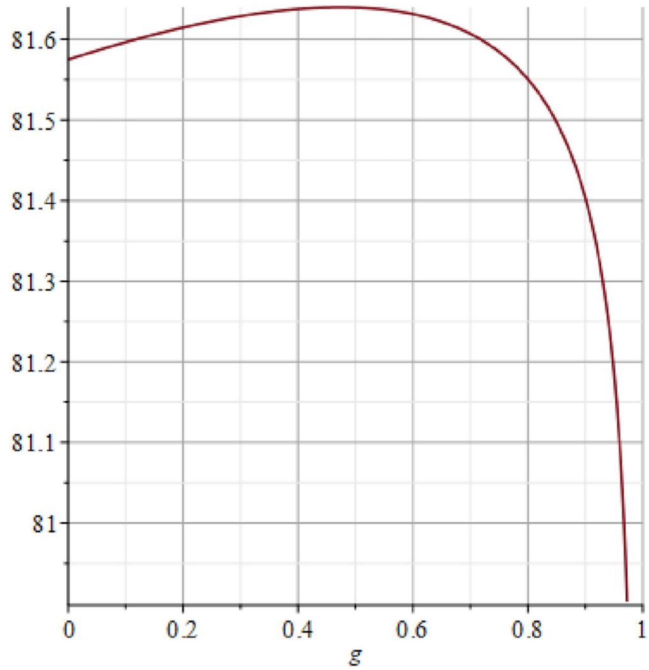

FIGURE 3 Total welfare in the baseline numerical example

Finally, we should note that the underlying assumption in our model, and in the previous numerical examples, is that the cost of compliance is what varies across firms—not the negative externality associated with non-compliance. That is, the Type-*B* technology entails the same externality from each non-compliant firm. An alternative scenario is one whereby firms who face higher costs of compliance are also those that generate greater negative externalities in the event that they do not comply. To examine the implications of this approach in our original numerical example about consumer surplus, we define $e(c_i) = ec_i$ $(e > 0)$ and replace Equation (30) by the following:

$$E = \int_{\hat{c}}^{1} e(c_i) n^C di = \frac{[1 - b(n^C) - gt]^2 n^C e}{2}. \tag{33}$$

Using the parameter values of the baseline case, the relation between consumers' welfare and g is illustrated in Figure 4. We can see a sharp decline in the optimal detection probability compared to the original case of a fixed external cost per firm. This is attributed to the fact that some of the non-compliant firms generate reduced externalities—in overall, the aggregate negative externality is lower compared to the baseline scenario.

6 | CONCLUSION

There is now mounting empirical evidence suggesting a positive relation between market competition and corruption. Motivated by this evidence, our contribution in this study is the following: By deviating from the existing literature and assuming that corrupt bureaucrats facilitate the evasion of penalty payments, rather than regulating market entry directly, we show the possibility of a self-reinforcing cycle, whereby corruption leads to an increase of the number of firms that compete in the market, whereas the same increase in market competition raises the incentives of bureaucrats to engage in corrupt activities, as well as the bribes they seek in exchange for their misconduct.

We should note that, had we treated bribes and the incidence of corruption as exogenous, the model's main mechanism would have been quite simple: Corruption would cause higher market entry by allowing firms to avoid regulations, thus lowering their costs. By endogenizing these aspects of public officials' decisions, we have demonstrated that increased market entry can lead, in turn, to a higher incidence of corruption and increased bribes. Unavoidably, this approach raises the model's complication. However, we view this as a necessary cost to highlight mechanisms that are consistent with the existing evidence, which shows that the relation between market competition and corruption is two-way causal.

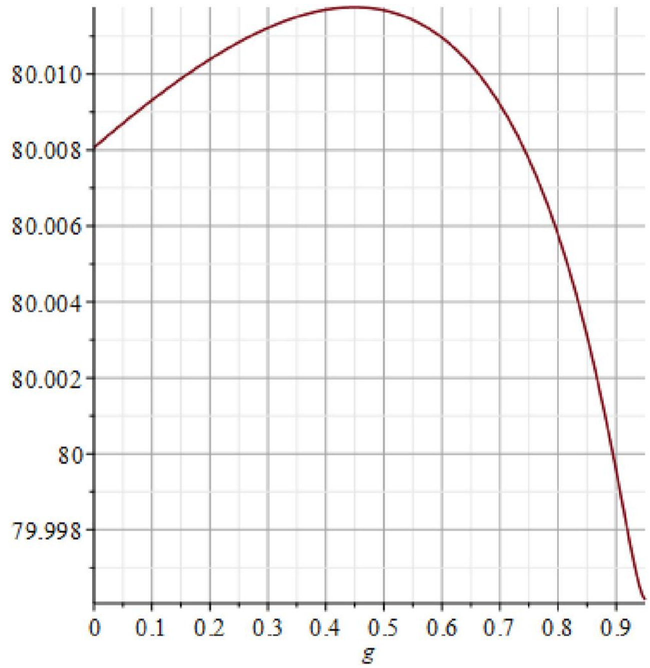

FIGURE 4 Consumer welfare under varying externality per firm

In addition to offering analytical mechanisms and intuition behind empirically-supported outcomes, our study also raises awareness to some interesting policy implications. Particularly, the interrelation between firms' decisions on whether to compete in a market and comply with its regulations, and the public officials' incentives toward bribe-seeking behavior, offers an alternative explanation on why policy makers may be reluctant to devote more effort and resources in restraining the incidence of corruption. This may happen when policy makers assess that the negative repercussions of corruption are not sufficiently strong to counter the contribution of increased competition on reduced prices.

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ENDNOTES

1 http://www.transparency.org/whoweare/organisation/faqs_on_corruption#defineCorruption.

2 Similar ideas are echoed in Méon and Weill (2010) and Sethi and Sama (1998)—the latter claiming that more competitive environments make it more likely that firms will engage in ethically-questionable practices. Xu and Yano (2017) show that anti-corruption efforts by governments do not appear to have a desirable effect when firms operate in heavily regulated industries, while Gokalp et al. (2017) hint at a positive relation between the degree of competition and corporate tax evasion.

3 It should be noted that the introduction of cost uncertainty in models of firm entry and behavior is a well-established paradigm. See Waldman (1987); Maskin (1999); Wambach (1999); Février and Linnemer (2004); Wu and Knott (2006); and Dharmapala et al. (2011) among others.

4 This will happen if $(\kappa - m)/\sqrt{f+0.5} < 1$ holds.

5 One can think of alternative information structures. For example, if the bureaucrat has knowledge on the firm's c_j that applied when it made its technology choice, then this can affect his actions: He can condition the bribe he requests on this information, demanding higher bribes from firm owners who benefited the most from non-compliance, that is, those with higher adoption costs. While it is reasonable to assume that inspection reveals if a firm complies or not, the applicability of an assumption whereby the bureaucrat does not only understand that the firm benefited from non-compliance, but he also has knowledge on exactly by how much each firm benefited, is not as clear cut; hence, our approach in this study. Naturally, an investigation of this alternative information structure, and its implications, represents an avenue for future research.

[6] The underlying assumption in our framework is that regulation does not involve the issuing of permits. In other words, firms do not need to provide proof of compliance before being given permission to produce. If this was the case, then this choice would be embedded, thus indistinguishable, from entry decisions. It could also affect our results, as a more competitive, thus less profitable, environment affects the bureaucrats' scope for bribe requests. Nevertheless, our objective is not to analyze what is, in effect, an environment of entry permits. It should be noted that our approach is consistent with actual cases where regulation compliance is inspected post-entry, while firms already undertake production activities. See the following: https://www.gov.uk/guidance/environmental-protection-inspection, https://www.hse.gov.uk/involvement/inspections.htm.

[7] Of course, we assume $(\kappa - m)/\sqrt{f + \mu^{NC}} > 1$.

[8] In fact, the specification in Equation (10) draws from Chakraborty et al. (2010) who examine the economic implications of infectious disease transmission. To see the analogy, consider the probability of a bureaucrat being revealed and sanctioned for their misconduct, that is, β (by analogy, the probability of being 'infected'). The factors that jointly determine this are as follows. First, the number of firms from which the bureaucrat collects bribes, that is, $\gamma\delta/n$ (by analogy, the number of 'potentially infectious' interactions). Second, the probability that one of these firms will be caught having tried to mislead the authorities, that is, g (by analogy, the probability that one of these interactions will 'infect' the bureaucrat), since a firm's revelation will immediately reveal the identity of the bureaucrat who inspected it.

[9] In this context, the bureaucrat has all the bargaining power in choosing the bribe he will demand in order to conceal the true circumstances of firms that wish to evade the penalty associated with the adoption of the Type-B technology. Given the nature of the collusion, this is not an unusual assumption and it has been employed by other theoretical analyses in which tax payers bribe officials in order to avoid (part of) their tax burden (e.g., Blackburn, Bose, & Haque, 2006).

[10] Assumption 2 guarantees that $2(1-g)t - (1-gt) \geq 0$. Therefore, the RHS of this expression is positive, meaning that we can square both sides without changing the sign of the inequality.

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SUPPORTING INFORMATION

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APPENDIX

A.1 | Proof of Lemma 1

Given that θ is decreasing in t, it is sufficient to show that this holds for the lowest possible value of t. By virtue of Assumption 2, this is $t=[\sqrt{2}(1-g)+g]^{-1}$. Substituting this together with Equations (19) in (20), we get the expression

$$\theta=\frac{-\ln(1-g)}{\sqrt{2}}\frac{1-g}{\sqrt{2}(1-g)+g}=\theta(g)\geq 0. \tag{A1}$$

Given the expression in Equation (A1), note that $\theta(0)=\theta(1)=0$. Differentiating with respect to g yields

$$\theta'(g)=\frac{\ln(1-g)+\sqrt{2}(1-g)+g}{\sqrt{2}\left[\sqrt{2}(1-g)+g\right]^2}. \tag{A2}$$

Taking account that $\theta(0)=\theta(1)=0$, $\theta(g)>0\ \ \forall g\in(0,1)$ and Equation (A2), we infer that the value of $\widehat{g}$ that maximizes $\theta(g)$ must satisfy

$$\theta'(\widehat{g})=0\Rightarrow -\ln(1-\widehat{g})=\sqrt{2}(1-\widehat{g})+\widehat{g}. \tag{A3}$$

The term $-\ln(1-g)$ is monotonically increasing, taking values from $-\ln(1)=0$ to $\lim\limits_{g\to 1}(-\ln(1-g))\to+\infty$, whereas the term $\sqrt{2}(1-g)+g$ is monotonically decreasing taking values from $\sqrt{2}$ to 1. By the single crossing property, there is indeed a unique $\widehat{g}\in(0,1)$ that satisfies Equation (A3)—in fact, using a calculator we can find that $\widehat{g}\simeq 0.678$. Substituting this back to Equation (A1), we can show that the maximum possible value of θ satisfies

$$\theta(\widehat{g})\simeq\frac{1-0.678}{\sqrt{2}}<\frac{1}{4},$$

thus completing the proof. $\square$

A.2 | Proof of Lemma 3

The sign of the expression in Equation (18) determines the sign of $\frac{\partial u^C}{\partial b}$. Using Equation (18), we can check that $H(0)=\alpha(g)\ +1-gt>0$. Together with Lemma 2, we infer that there is a unique b^*, determined from $H(b^*)=0$, such that $H(b)>0\Rightarrow\frac{\partial u^C}{\partial b}>0$ for $b<b^*$ and $H(b)<0\Rightarrow\frac{\partial u^C}{\partial b}<0$ for $b>b^*$. Setting $H(b)=0$ in Equation (18) yields the solution for b^* that is presented in Equation (22). Of course, we want this solution to satisfy $b^*<(1-g)t$. For this to happen, it must be the case that[10]

$$\begin{gathered}
\sqrt{\left[\frac{2\delta}{a(g)n}-(1-gt)\right]^2+\frac{4\delta[a(g)w+1-gt]}{a(g)n}}<2(1-g)t+\left[\frac{2\delta}{a(g)n}-(1-gt)\right]\Rightarrow\\
\frac{\delta[a(g)w+1-gt]}{a(g)n}<(1-g)^2t^2+(1-g)t\left[\frac{2\delta}{a(g)n}-(1-gt)\right]\Rightarrow\\
\frac{\delta[a(g)w+1-gt]}{a(g)n}<(1-g)t\frac{2\delta}{a(g)n}-(1-g)t(1-t)\Rightarrow\\
a(g)w+1-gt<2(1-g)t-\frac{\alpha(g)n}{\delta}(1-g)t(1-t)\Rightarrow\\
a(g)w+1-gt<2(1-g)t-\frac{\alpha(g)n}{\delta}(1-g)t(1-t)\Rightarrow\\
w<\frac{2(1-g)t-(1-gt)}{\alpha(g)}-\frac{n}{\delta}(1-g)t(1-t).
\end{gathered} \tag{A4}$$

Obviously, the RHS of the expression in Equation (A4) is increasing in δ. Therefore, it is sufficient to show that Equation (A4) holds for the minimum possible value of δ. From Equation (20) and Assumption 3, it follows that this value is $\delta = \frac{\alpha(g)(1-gt)}{2}n$. Substituting in Equation (A4), we can rewrite the condition as

$$\begin{gathered}
w < \frac{2(1-g)t-(1-gt)}{\alpha(g)} - \frac{2(1-g)t(1-t)}{\alpha(g)(1-gt)} \Rightarrow \\
w < \frac{2(1-g)t(1-gt)-(1-gt)^2-2(1-g)t(1-t)}{\alpha(g)(1-gt)} \Rightarrow \\
w < \frac{2(1-g)^2t^2-(1-gt)^2}{\alpha(g)(1-gt)}.
\end{gathered} \tag{A5}$$

The expression in Equation (21) together with Assumption 4 imply that the condition in Equation (A5) holds, thus completing the proof. □

A.3 | Proof of Proposition 2

We can use Equation (21) to calculate the derivative

$$\begin{gathered}
b'(n) = \frac{1}{2}\left\{ -\frac{2\left[\frac{2\delta}{\alpha(g)n}-(1-gt)\right]\frac{2\delta}{\alpha(g)n^2}+\frac{4\delta[\alpha(g)w+1-gt]}{\alpha(g)n^2}}{2\sqrt{\left[\frac{2\delta}{\alpha(g)n}-(1-gt)\right]^2+\frac{4\delta[\alpha(g)w+1-gt]}{\alpha(g)n}}} + \frac{2\delta}{\alpha(g)n^2} \right\} \Rightarrow \\
b'(n) = \frac{1}{2}\frac{2\delta}{\alpha(g)n^2}\left\{ -\frac{2\left[\frac{2\delta}{\alpha(g)n}-(1-gt)\right]+2[\alpha(g)w+1-gt]}{2\sqrt{\left[\frac{2\delta}{\alpha(g)n}-(1-gt)\right]^2+\frac{4\delta[\alpha(g)w+1-gt]}{\alpha(g)n}}} + 1 \right\} \Rightarrow \\
b'(n) = \frac{\delta}{\alpha(g)n^2}\left\{ 1 - \frac{\frac{2\delta}{\alpha(g)n}+\alpha(g)w}{\sqrt{\left[\frac{2\delta}{\alpha(g)n}-(1-gt)\right]^2+\frac{4\delta[\alpha(g)w+1-gt]}{\alpha(g)n}}} \right\}.
\end{gathered} \tag{A6}$$

For this expression to be positive, we have to show that

$$\begin{gathered}
\sqrt{\left[\frac{2\delta}{\alpha(g)n}-(1-gt)\right]^2+\frac{4\delta[\alpha(g)w+1-gt]}{\alpha(g)n}} > \frac{2\delta}{\alpha(g)n}+\alpha(g)w \Rightarrow \\
\frac{4\delta[\alpha(g)w+1-gt]}{\alpha(g)n} > \left[\frac{2\delta}{\alpha(g)n}+\alpha(g)w\right]^2 - \left[\frac{2\delta}{\alpha(g)n}-(1-gt)\right]^2 \Rightarrow \\
\frac{4\delta[\alpha(g)w+1-gt]}{\alpha(g)n} > [\alpha(g)w+1-gt]\left[\frac{4\delta}{\alpha(g)n}+\alpha(g)w-(1-gt)\right] \Rightarrow \\
\frac{4\delta}{\alpha(g)n} > \frac{4\delta}{\alpha(g)n}+\alpha(g)w-(1-gt) \Rightarrow \\
w < \frac{1-gt}{\alpha(g)}.
\end{gathered} \tag{A7}$$

Given Assumption 4, it is sufficient to show that Equation (A7) holds for $w = \widehat{w}$. Substituting Equation (21) in Equation (A7), we get

$$\frac{2(1-g)^2t^2-(1-gt)^2}{\alpha(g)(1-gt)}<\frac{1-gt}{\alpha(g)}\Rightarrow$$
$$(1-g)t<1-gt\Rightarrow$$
$$t<1.$$

This is indeed true by virtue of Assumption 1, thus completing the proof. □

A.4 | Proof of Lemma 5

From Equation (27) we can derive

$$\widehat{U}'(n)=(1-g)^{z(n)}\{z(n)b'(n)+z'(n)[b(n)(1-\alpha(g)z(n))-\alpha(g)w]\}. \tag{A8}$$

We have already proven that $b'(n)>0$. Now, let us consider $z'(n)$ for which we can use Equation (26) to derive

$$z'(n)=\frac{1}{2}\left\{\frac{1-gt}{\delta}-\frac{-2\left[\frac{2}{\alpha(g)}-\frac{n}{\delta}(1-gt)\right]\frac{1-gt}{\delta}+\frac{4[\alpha(g)w+1-gt]}{\alpha(g)\delta}}{2\sqrt{\left[\frac{2}{\alpha(g)}-\frac{n}{\delta}(1-gt)\right]^2+\frac{4n[\alpha(g)w+1-gt]}{\alpha(g)\delta}}}\right\}\Rightarrow$$

$$z'(n)=\frac{1}{2\delta}\left\{1-gt-\frac{-\left[\frac{2}{\alpha(g)}-\frac{n}{\delta}(1-gt)\right](1-gt)+\frac{2[\alpha(g)w+1-gt]}{\alpha(g)}}{\sqrt{\left[\frac{2}{\alpha(g)}-\frac{n}{\delta}(1-gt)\right]^2+\frac{4n[\alpha(g)w+1-gt]}{\alpha(g)\delta}}}\right\}\Rightarrow \tag{A9}$$

$$z'(n)=\frac{1}{2\delta}\left\{1-gt-\frac{2w+\frac{n}{\delta}(1-gt)^2}{\sqrt{\left[\frac{2}{\alpha(g)}-\frac{n}{\delta}(1-gt)\right]^2+\frac{4n[\alpha(g)w+1-gt]}{\alpha(g)\delta}}}\right\}.$$

The expression for $z'(n)$ in Equation (A9) will be a positive one as long as

$$\sqrt{\left[\frac{2}{\alpha(g)}-\frac{n}{\delta}(1-gt)\right]^2+\frac{4n[\alpha(g)\omega+1-gt]}{\alpha(g)\delta}}>\frac{2w}{1-gt}+\frac{n}{\delta}(1-gt)\Rightarrow$$
$$\frac{4n[\alpha(g)\omega+1-gt]}{\alpha(g)\delta}>\left[\frac{2w}{1-gt}+\frac{n}{\delta}(1-gt)\right]^2-\left[\frac{2}{\alpha(g)}-\frac{n}{\delta}(1-gt)\right]^2\Rightarrow$$
$$\frac{4n[\alpha(g)\omega+1-gt]}{\alpha(g)\delta}>4\left[\frac{w}{1-gt}-\frac{1}{\alpha(g)}+\frac{n}{\delta}(1-gt)\right]\frac{\alpha(g)\omega+1-gt}{\alpha(g)(1-gt)}\Rightarrow$$
$$n>\frac{\alpha(g)w\delta-(1-gt)\delta+n\alpha(g)(1-gt)^2}{\alpha(g)(1-gt)^2}\Rightarrow$$
$$w<\frac{1-gt}{\alpha(g)}.$$

This is something that we have already proven though (see Equation (A7) and the analysis that followed it); therefore, it is indeed $z'(n)>0$. Together with $b'(n)>0$, we can use Equation (A8) to infer that, for $\widehat{U}'(n)>0$, it is sufficient to show that the following condition holds:

$$b(n)[1-\alpha(g)z(n)]>\alpha(g)w. \tag{A10}$$

To show this, we begin by defining the composite term

$$\xi = \sqrt{\left[\frac{2\delta}{a(g)n} - (1-gt)\right]^2 + \frac{4\delta[a(g)w + 1 - gt]}{a(g)n}}. \tag{A11}$$

and using it together with Equations (21) and (26) to write

$$b(n) = \frac{\xi - \left[\frac{2\delta}{a(g)n} - (1-gt)\right]}{2}, \tag{A12}$$

and

$$\begin{aligned}
1-\alpha(g)z(n) &= 1 - \frac{2 + \frac{n}{\delta}a(g)(1-gt) - a(g)\sqrt{\left[\frac{2}{\alpha(g)} - \frac{n}{\delta}(1-gt)\right]^2 + \frac{4n[\alpha(g)w + 1 - gt]}{\delta\alpha(g)}}}{2} \\
&= \frac{-\frac{n}{\delta}a(g)(1-gt) - a(g)\frac{n}{\delta}\sqrt{\left[\frac{2\delta}{a(g)n} - (1-gt)\right]^2 + \frac{4\delta[a(g)w + 1 - gt)}{a(g)n}}}{2} \Rightarrow \\
1-\alpha(g)z(n) &= \frac{a(g)n}{2\delta}[\xi - (1-gt)].
\end{aligned} \tag{A13}$$

Substituting Equations (A12) and (A13), we can rewrite the condition in Equation (A10) as

$$\left\{\xi - \left[\frac{2\delta}{a(g)n} - (1-gt)\right]\right\}[\xi - (1-gt)] > \frac{4w\delta}{n} \Rightarrow$$

$$[\xi + (1-gt)][\xi - (1-gt)] - \frac{2\delta}{a(g)n}[\xi - (1-gt)] > \frac{4w\delta}{n} \Rightarrow$$

$$\xi^2 - (1-gt)^2 - \frac{2\delta}{a(g)n}[\xi - (1-gt)] > \frac{4w\delta}{n} \Rightarrow$$

$$\left[\frac{2\delta}{a(g)n} - (1-gt)\right]^2 + \frac{4\delta[a(g)w + 1 - gt)}{a(g)n} - (1-gt)^2 - \frac{2\delta}{a(g)n}\left[\xi - (1-gt)\right] > \frac{4w\delta}{n} \Rightarrow$$

$$\left[\frac{2\delta}{a(g)n}\right]^2 - \frac{4\delta}{a(g)n}(1-gt) + (1-gt)^2 + \frac{4\delta}{a(g)n}(1-gt) - (1-gt)^2 - \frac{2\delta}{a(g)n}[\xi - (1-gt)] > 0 \Rightarrow$$

$$\frac{2\delta}{a(g)n}\left[\frac{2\delta}{a(g)n} - \xi + 1 - gt\right] > 0 \Rightarrow$$

$$1 - gt > \xi - \frac{2\delta}{a(g)n} \Rightarrow$$

$$2(1-gt) > \xi - \left[\frac{2\delta}{a(g)n} - (1-gt)\right] \Rightarrow$$

$$2(1-gt) > \sqrt{\left[\frac{2\delta}{a(g)n} - \left(1-gt\right)\right]^2 + \frac{4\delta[a(g)w + 1 - gt)}{a(g)n}} - \left[\frac{2\delta}{a(g)n} - (1-gt)\right] \Rightarrow$$

$$2(1-gt) > 2b(n) \Rightarrow$$

$$b(n) < 1 - gt.$$

This is a condition that holds because, by virtue of Lemma 3, we know that $b(n) < (1-g)t$ which is lower than $1-gt$. Therefore, it is indeed true that $\widehat{U}'(n) > 0$. □