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A methodology to optimise a rail network specification for maximum passenger satisfaction and reduced initial investment

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ABSTRACT

This paper considers the design and construction of new passenger rail networks – a common and expensive task worldwide. Optimising the allocation of initial investment between components of a proposed network is a challenge for planners who wish to reduce the initial investment required whilst maintaining key objectives, such as providing high levels of passenger satisfaction. Previous decision support tools to assist with this challenge typically do not consider the influence of the high-level network specification on passenger satisfaction. Here, a novel optimisation methodology is presented that includes the effects of various factors, e.g. permissible line-speed, train performance, train comfort, and station comfort. The methodology combines Agent-Based Modelling, Bayesian Optimisation, and a model which quantifies passenger experience. To demonstrate its effectiveness, the methodology is illustrated in a hypothetical case-study where it identifies a network specification which substantially improves the satisfaction of the virtual passengers. Furthermore, the network specification reduces the modelled cost of network construction by £5 billion from £64 billion. The case-study demonstrates that the methodology is computationally tractable for realistically sized tasks, and captures the trade-offs between investment and component performance, making it potentially useful to network planners concerned with the satisfaction of passengers.

1. Introduction

Many passenger rail networks are being built or extended worldwide i.e. *network developments*. For example, Sinclair (2019) lists 19 current developments distributed amongst all of the seven populated continents and costing billions of US dollars each. As well as being expensive, developments can also be prone to exceeding their budget for initial investment. Flyvbjerg, Skamris holm, and Buhl (2003) find that the mean increase in investment from the original prediction is 45% with a standard deviation of 38%. Consequently there is often a pressure for designers to reduce the initial investment required for a development, whilst still ensuring key design objectives are met (Kwok et al. 2009). A Value Engineering Strategy (VES) describes how a reduced initial investment budget can be reallocated between components of the network, and defines the network specification. Because rail networks comprise many components that interact in complex ways, selecting a 'good' VES is a challenge for designers. This is a sub-problem of what is often referred to as the Transport Network Design Problem (TNDP). Here for the first time, to the best of the authors' knowledge, a VES is selected with a focus on the design objective of providing maximum passenger satisfaction whilst considering the trade-off with the initial investment

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Nomenclature

i, j, k	optimisation variable indexes
$f(\boldsymbol{x})$	objective function
$g(\boldsymbol{x})$	reduction in initial investment function
$g_T(\boldsymbol{x})$	reduction in initial investment function relating to train performance
$g_C(\boldsymbol{x})$	reduction in initial investment function relating to train comfort
$g_L(\boldsymbol{x})$	reduction in initial investment function relating to line-speed
$g_V(\boldsymbol{x})$	reduction in initial investment function relating to station comfort
t_T	train-agent time step
t_P	passenger-agent time step
$\boldsymbol{x}$	vector of optimisation variables
$\boldsymbol{x}_\eta$	vector of optimisation variables, enumerated in a sequence
$\boldsymbol{x}^*$	optimum vector
X	set containing all vectors, $\boldsymbol{x}$, in search space
x_i	i'th element of $\boldsymbol{x}$
$\alpha(\boldsymbol{x})$	acquisition function
β	target reduction in initial investment
ε_A	Group A Value of Time weighting multiplier
ε_B	Group B Value of Time weighting multiplier
η	objective function evaluations
η_{max}	objective function evaluation budget
$\boldsymbol{\theta}$	network parameters
κ_0	neutral parameter set
λ	passenger load
$\mu(\boldsymbol{x})$	surrogate function
$\sigma(\boldsymbol{x})$	uncertainty function
ω	crowding penalty

Abbreviations

BO	Bayesian Optimisation
DM	Disutility Metric
GA	Genetic Algorithm
HS2	High Speed Two
JTM	Journey Time Metric
MP	Mathematical Programming
TNDP	Transport Network Design Problem
VES	Value Engineering Strategy
VoT	Value of Time

required. This is of concern to many network managers and designers worldwide (Kunimatsu et al. 2012; TNS Political and Social, 2013; Technical Leadership Group 2017). Previous relevant publications in the area of the TNDP have often used Mathematical Programming (MP) to investigate their considered objective. However, such formulations make it difficult to model some features of a passenger journey which affect their satisfaction (Russo 1998; Kanai et al., 2011), for example, the pedestrian movement of passengers within stations, or journey stage conditions such as comfort and crowding. Consequently, there is scope for improving upon the capability of pre-existing TNDP methods and this is the target considered here. Specifically, a novel methodology is presented for optimising the VES with the objective of maximising passenger satisfaction, whilst considering the initial investment available. The methodology is a combination of previously validated methods and tools that allows real-world problems to be solved. Crucially, it allows the inclusion of non-linear relationships, such as the speed-dependent acceleration of trains, and thus enables modelling passenger satisfaction in greater fidelity. Application of this methodology has the potential to reduce the initial investment required for developments, and to improve the resulting service to passengers. The methodology is adaptable to many specific developments of different types, e.g. metro or mainline, and is initially presented in general terms in Section 3. Its effectiveness is the investigated during a case study in Section 3.1.1.

2. Literature review

2.1. Important components and effects to consider

The permissible line-speed of the network and the performance of the train, e.g. the speed-dependent tractive force available to a train and its maximum speed, have an effect upon passenger journey times and hence their satisfaction. However, passenger satisfaction is also affected by the relative time spent in different journey stages (Kunimatsu et al., 2009; Kunimatsu et al. 2012, ARUP, Institute for Transport Studies Leeds, and Accent 2015; Sels et al., 2016), e.g. on a train or in the station, or the crowding level when on a train (Qin 2014; Wardman and Murphy 2015). The Disutility Metric (DM), presented by Kunimatsu et al. (2009, 2012), and the Journey Time Metric (JTM), developed by London Underground Limited and discussed by Chan (2007) and Hickey (2011), quantify these effects upon the satisfaction of an individual passenger. By applying a normalisation for passenger numbers and distance travelled (Hickish, Fletcher, and Harrison 2019a), a whole-network performance metric based on the DM and JTM is well suited to the objective considered in the current paper. The effect of train performance and line-speed upon the determinants of passenger satisfaction can be modelled with an agent-based simulation of individual train and passenger movements (Yao et al. 2013). As well as affecting passenger satisfaction, the line-speed and train performance also affect the initial investment required, e.g. Smith (2019) suggested that one method to reduce initial investment in a high-speed development is to reduce the operational speed. The trade-off between these effects should be considered in a methodology to optimise the VES.

The comfort of trains and stations can also affect passenger satisfaction (Akabal et al., 2017; Huang and Shuai 2018), and Litman (2017) discussed the benefits of considering comfort during the network design process. These points suggest; within a VES optimisation methodology that maximises passenger satisfaction, it is beneficial to include factors relating to journey times and comfort. There is also a dependence between the comfort of train and stations and the initial investment, which is desirable to capture. Oliveira et al. (2019) stated that increasing train comfort might be a cost-effective way to improve passenger satisfaction, an opinion supported by Durrant (2015). Similarly, Preston et al. (2008) have shown that station investment increases demand for travel from passengers. The effect of comfort upon investment should therefore also be considered in the presented methodology.

2.2. Existing methods for optimising the network specification or the Value Engineering Strategy

There is a precedent for tools to assist network managers and designers optimise their network at the system level. For example, Alikhani-Kooshkak et al. (2017) presented a method to optimise train characteristics, however they do not model passenger journeys. Christogiannis and Pyrgidis (2013) optimised the operation of a development for maximum economic profitability, but do not optimise the network specification or minimise the initial investment required. Kwok, Anderson and Ng (2009) presented a methodology for optimising the VES of developments, involving iteratively conducting a Cost-Benefit Analysis (Van Wee 2007) of candidate VESs until an acceptable one is found. However, Kwok et al. did not identify a method to select a VES to evaluate in the next iteration. For a realistically sized task, where there may be a large number of VESs to consider, it may be difficult to select a new VES that improves upon the best-one found already.

The TNDP is concerned with methods to search efficiently through many investment scenarios. There are many sub-problems of the TNDP which are well studied (Farahani et al., 2013), with the current review focussing on problems relating to optimising the characteristics of rail network components (Lai and Shih 2013; Bärmann, Martin, and Schülldorf 2017; Lin et al., 2017; Seyedvakili et al., 2020). The published work most comparable to the current paper is that of Canca et al. (2016), who considered both train fleet size and the characteristics of the lines in their optimisation. However, Canca et al. did not consider a choice between different train characteristics – performance or comfort, for example. The TNDP approaches reviewed above use MP which, as already stated, is not considered suitable for capturing all the effects described in the previous section.

This review has identified the need for a VES optimisation tool that considers the effect upon passenger satisfaction of line-speeds, train performance, and train and station comfort, as well as the trade-off with required investment. Table 1 summarises the functionality of the tools encountered in this literature review, and demonstrates that, to the best of the authors' knowledge, no pre-existing tools are available for this task.

3. A new optimisation methodology

Optimisation of a VES to maximise passenger satisfaction, subject to a constraint on the budget, can be written formally as:

Table 1
The functionality of the decision-support tools included in this literature review.

Reference	Passenger satisfaction	Line-speeds	Train performance	Train comfort	Station comfort	Investment required	Guided VES selection
Alikhani-Kooshkak et al. (2017)	✖	✓	✓	✓	✖	✓	✓
Christogiannis and Pyrgidis (2013)	✓	✖	✖	✖	✖	✖	✓
Kwok et al. (2009)	✓	✓	✓	✓	✓	✓	✖
Lai and Shih (2013)	✖	✓	✓	✖	✖	✓	✓
Canca et al. (2016)	✓	✓	✖	✖	✓	✓	✓
Bärmann et al. (2017)	✖	✓	✓	✖	✖	✓	✓
Lin et al. (2017)	✖	✓	✓	✖	✖	✓	✓
Seyedvakili et al. (2020)	✓	✓	✖	✖	✖	✓	✓
The current publication	✓	✓	✓	✓	✓	✓	✓

$$x^{*} = \underset{x \in X}{\operatorname{argmin}} \ f(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{\lambda}) \tag{1}$$

subject to:

$$g(\boldsymbol{x}) \geq \beta \tag{2}$$

where $\boldsymbol{x}$ denotes the optimisation vector (set of decision variables) representing the VES, X, the search space containing all the feasible VESs, $\boldsymbol{x}^{*}$, the optimum VES. The vector, $\boldsymbol{\theta}$, denotes the parameters describing features of the network not affected by the VES, $\boldsymbol{\lambda}$, the passenger load vector, and β the minimum required reduction in initial investment, i.e. saving. The function $g(\boldsymbol{x})$ quantifies the saving brought about by the VES, compared to a baseline network specification. The function $f(\boldsymbol{x};\ \boldsymbol{\theta},\boldsymbol{\lambda})$ is an objective function which quantifies the performance of the network defined by $\boldsymbol{x}$ and $\boldsymbol{\theta}$, and carrying passenger load described by $\boldsymbol{\lambda}$. Because the objective function quantifies passenger satisfaction with a measure of disutility (Pienaar 1997), described further in the following sections, maximising passenger satisfaction relates to minimising the objective function.

Since this objective function might be non-convex and non-smooth for many practical tasks, it is not possible to guarantee that $\boldsymbol{x}^{*}$ has been located unless candidates belonging to X are exhaustively evaluated. The combinatorial nature of this is computationally expensive for realistic problems and usually infeasible. However, in many practical applications the formally guaranteed optimum solution is not necessary and a 'good' one is sufficient, e.g. the 'best' acceptable solution found during a heuristic search for $\boldsymbol{x}^{*}$. The methodology presented in this paper can be used for tasks such as this, and has a flow of processes illustrated in Fig. 1. An initial candidate, i.e. $\boldsymbol{x}$, is chosen either arbitrarily or based on some prior knowledge. The candidate is then evaluated using the objective function. Unless conditional termination requirements are met, such as a target network score reached or a computational budget expended, the Bayesian Optimisation (BO) method is used to select the next candidate which is then evaluated with the objective function. Fig. 1 illustrates that the cycle of candidate evaluation and selection iterates until termination requirements are met. The candidate with the minimum objective function score found so-far is output as the solution.

3.1. *Calculating the objective function*

Fig. 2 illustrates the flow of processes within the objective function to calculate the effect of a VES upon passenger satisfaction, which is quantified to give the *network score*, i.e. objective function value. Items in the dashed boxes relate to an input to the objective function, items in the dash-dot boxes relate to a process, and items in the solid line boxes relate to an interim or output datum. The processes within the objective function are discussed in more detail in the next sections.

3.1.1. *Network simulation*

A deterministic Agent-Based Model is used to simulate individual passenger and train movements within the network. While the full detail of the model is presented by Hickish et al. (2017), a brief outline is given here. It is important to note that the model is generic and can be tailored to represent a wide variety of specific networks and passenger loads. The simulation environment is defined by the topography of stations and rail lines, as well as the properties of these, e.g. number of platforms, directionality, gradients, and the maximum line speed. There are two types of agent; trains and passengers. For brevity, from here on in we omit the word 'agent' and rely on the context to differentiate between the real-world entity and its virtual representation. Fig. 3 illustrates the flow of information within the model at every time step when simulating the trains. A timetable specifies the route of each train as well as timings. Trains follow their route strictly and attempt to adhere to the timings, but can become delayed because they also obey safety rules restricting

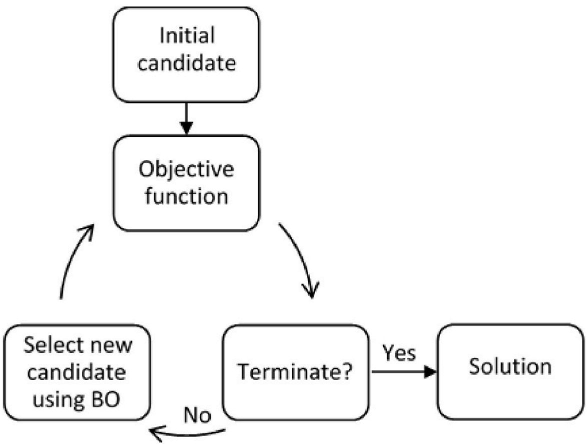

Fig. 1. Illustration of the flow of processes within the presented optimisation methodology. The process starts with an initial candidate and which is then assessed with the objective function. The Bayesian Optimisation (BO) method is used to select new candidates. The algorithm iterates until termination requirements are met, when a solution is output.

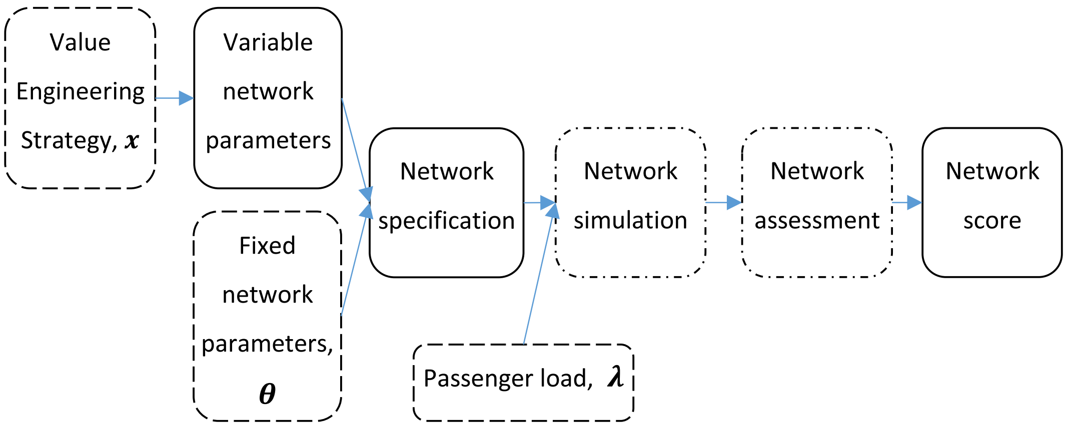

Fig. 2. The processes within the objective function to calculate the effect of a Value Engineering Strategy upon passenger satisfaction. Items in a dashed box are inputs to the objective function, items in a dash-dot box are processes, and items in a solid box are interim or output datum.

their movements. For example, the minimum proximity allowed between trains is determined by their stopping distance plus a safety margin — this represents a 'moving block' system as is being pursued by many countries (Furness et al., 2017; European Commission 2019). Similarly, the train dwell time is dependent on passenger transfer with the platform. Based on the timetable, safety rules, and performance characteristics, each train determines a control action ranging from maximum braking to maximum acceleration. For a given control action, the movement of the trains is determined according to Newton's laws applied to a point-like mass, using a time step defined by the parameter, t_T, and takes into account resistance forces modelled by the Davis formula (Howlett and Pudney 1995) as well as the speed-dependent tractive force available to a train. The simulation environment and the characteristics of the trains are determined by $\boldsymbol{x}$ and $\boldsymbol{\theta}$.

According to λ, passengers are initialised with a pre-defined route and journey start time at which they are assigned to their origin station. Passenger journeys are modelled as a sequence of journey stages, where transfer between the stages is dependent on the current stage of the passenger, its route, and the state of other agents within the environment. Fig. 4 illustrates the stages and how they are connected. Whilst assigned to their origin station, passengers pass through a series of stages modelling the time required for activities such as buying a ticket, moving through the station, and waiting on the platform. Once in the 'On Platform' stage, a passenger waits for the first train to arrive at their current station which will take them on the next leg of their route. However if the train is already 'full' the passenger enters the 'On Platform (Left Behind)' stage and must wait until the first train with space arrives. When a suitable train arrives, the passenger is assigned to the train and travels through the network upon it in the 'On Train' stage. At each station stop the passenger chooses to remain on the train if its next leg coincides with theirs, if it does not they disembark and are assigned to the station. Once assigned to a station that is not their origin, passengers enter the 'Moving Through Station' stage again and then either; return to the 'On Platform' stage to wait for the next suitable train, or, exit the station if it is their destination. Upon exiting the station a passenger is removed from the simulation environment, but the information about their journey is stored for network assessment. Passengers are updated with a time step defined by the value of the parameter, t_P.

The behaviours of trains and passengers have been validated by Hickish et al. (2017), and are not repeated here since the focus is on application within a wider optimisation methodology. The model has four key properties to highlight. First, there are parameters to represent a wide range of network specification characteristics which affect passenger journeys and hence their satisfaction, e.g. train maximum speed and maximum line-speed. Consequently, these factors can be optimised. Second, this model places no restriction on the form of the relationship between variables, i.e. non-linear effects can be included. Third, the model is readily adaptable to include further relationships, e.g. the pedestrian movement of passengers within the station has been simulated in previous work (University of Sheffield 2017) but is not included here. Fourth, representing train and passenger movement at this level of detail comes at an increased computational cost in comparison to a typical MP formulation, but passenger satisfaction is modelled in greater fidelity.

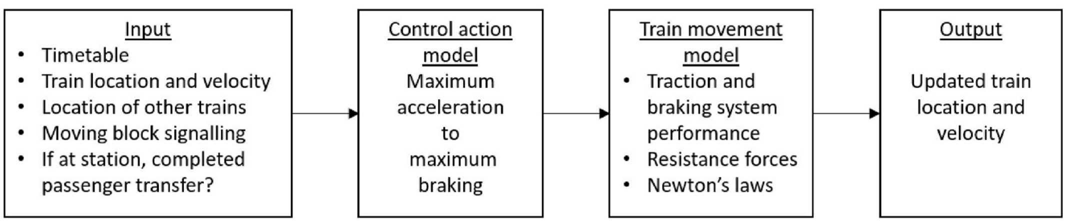

Fig. 3. The processes at every time step when simulating the movement of a train.

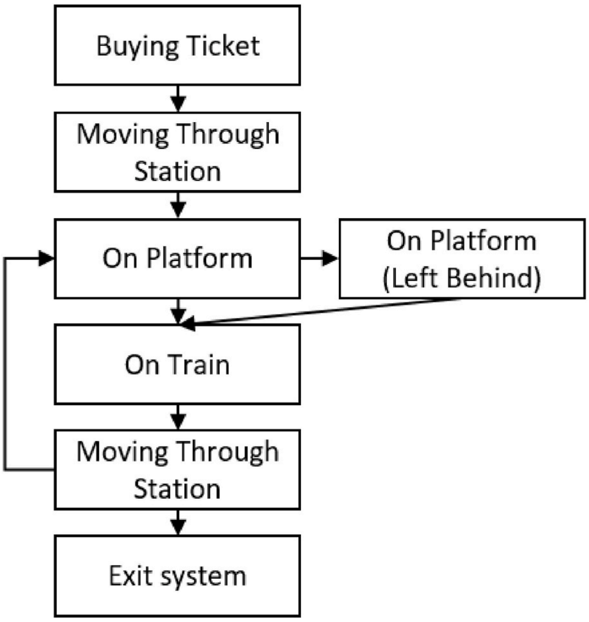

Fig. 4. The flow of stages to model a passenger journey from buying a ticket to exiting the system. Changes between stages occur depending on the route of the passenger and events within the simulation, e.g. the arrival of a train or the passing of a set number of time steps.

3.1.2. Network assessment

To assess the network based on simulation output data, and quantify the satisfaction of the virtual passengers, the JTM (Chan 2007; Hickey 2011) is applied with the normalisation method described by Hickish, Fletcher, and Harrison (2019a). The JTM resolves individual passenger journeys into a series of *passenger states* of varying duration – a passenger state is a combination of the journey stage of the passenger and the crowding conditions. For an individual passenger overall satisfaction is quantified by weighting the time spent in different states and then aggregating the weighted times. The different weightings are determined using the Value of Time (VoT) concept from Transport Economics (ARUP, Institute for Transport Studies Leeds, and Accent 2015), with the values used here shown in Table 2. The weighting for the 'On Train (Crowded)' state contains a crowding penalty, ω, which is calculated with the formula:

$$\omega = \begin{cases} 0\,, & \delta \leq \xi \\ c_1 + c_2\dfrac{\delta - \xi}{\varrho} - c_3\dfrac{\delta\xi - \xi^2}{\varrho^2}, & \xi < \delta \leq \delta_{\max} \end{cases}$$

where δ denotes the number of passengers on the train, ξ, denotes the number of seats on the train, $\delta_{\max}$, denotes the maximum passenger capacity and, ϱ, denotes the crush capacity. The constants c_1 to c_3 are respectively given values of 0.85, 1.915, and 1.03 (Hickish, Fletcher, and Harrison 2019a).

Authors such as Börjesson and Eliasson (2019) and the UK's Department for Transport (2015) state that travel comfort can affect the VoT. Consequently changes in passenger travel comfort, e.g. owing to investment in more comfortable trains, can be modelled by altering the weighting in Table 2. The states in 'Group A' are affected by train comfort and the states in 'Group B' are affected by station comfort. The network score is calculated by aggregating all the individual passenger journey scores and normalising for the number of passengers and their distance travelled. Details of the VoT concept, crowding penalty, normalisation, and validation of this metric can be found in discussion by Wardman (2004), ARUP, Institute for Transport Studies Leeds, and Accent (2015), and Hickish, Fletcher, and Harrison (2019a).

Since passenger and train movements are modelled individually, the objective function described in this section captures the effect upon passenger satisfaction of component characteristics, such as changes to the line-speed or the performance of trains, which affect train and passenger journey times. Other consequential effects to passenger satisfaction from changes in train journey times, such as changes to crowding or waiting times at connection stations, can also be included. Likewise, the effect upon passenger satisfaction

Table 2
The time weightings used for different passenger journey states. The weighting for passengers in the On Train (Crowded) state is dependent on the variable crowding penalty, ω.

Group	A		B			
Journey state	On Train	On Train (Crowded)	On Platform	On Platform (Left Behind)	Moving Through Station	Buying Ticket
Weighting	1	$1 + \omega$	2.5	3	2.7	2.5

owing to changes in the comfort of different components, e.g. trains or stations. Therefore, by including data of the investment associated with different component characteristics, the presented methodology addresses the shortcomings identified in Section 2.2. The methodology is not limited to these components or characteristics and could readily accommodate other relevant effects upon passenger satisfaction and initial investment.

3.2. Bayesian Optimisation

Here a brief overview of the general BO method is given, but for a fuller treatment the reader is referred to Shahriari et al. (2016). BO starts by acknowledging that the objective function is not known *a priori*, and therefore can be viewed as a random process to which a prior probability distribution is assigned. Then follows an iterative sequence of determining the most useful location at which to sample the search space, evaluating the objective function at this location, and then updating the probability distribution by application of Bayes' theorem. There are three key components to the BO method:

1) A model of the objective function, called the *surrogate function* (also referred to as the *proxy function* or *response surface*), $\mu(\boldsymbol{x})$. This is probabilistic over the space of possible surrogates given the observed evaluations of the objective function so-far. To calculate $\mu(\boldsymbol{x})$ it is common to use Gaussian Process regression, for a detailed explanation of which the reader is referred to Seeger (2004).
2) An *uncertainty function*, $\sigma(\boldsymbol{x})$, quantifies the uncertainty in $\mu(\boldsymbol{x})$ at different locations in the search space. An advantage of using Gaussian Process regression to calculate $\mu(\boldsymbol{x})$ is that it inherently supplies this information (Seeger 2004).
3) An *acquisition function*, $\alpha(\boldsymbol{x})$, which calculates the utility of the evaluating the objective in different locations of the search space. This is induced from $\mu(\boldsymbol{x})$ and $\sigma(\boldsymbol{x})$, and its maximum is sought in order to determine the location of the next objective evaluation, i.e. it guides the search. Typically $\alpha(\boldsymbol{x})$ is formulated as a smooth continuous function that can be evaluated, and hence maximised, with low computational cost in comparison to evaluating the objective. One popular method to induce $\alpha(\boldsymbol{x})$ is for it to quantify the 'expected improvement' at $\boldsymbol{x}$. This is the difference between the 'best' objective function value observed so-far and $\mu(\boldsymbol{x})$, and is described further by Shahriari et al. (2016).

The pseudocode of Algorithm 1 demonstrates the sequence by which $\mu(\boldsymbol{x})$, $\sigma(\boldsymbol{x})$, and $\alpha(\boldsymbol{x})$ are updated following new objective function evaluations. The algorithm is iterative and the integer variable, η, is used to enumerate these. The data set, D, denotes all the optimisation vectors evaluated so-far and corresponding objective function values, $\boldsymbol{y}$. Subscripts are applied to distinguish between variable or function values at specific iterations of the algorithm. For example, $\mu_\eta(\boldsymbol{x})$ and $\mu_{\eta+1}(\boldsymbol{x})$, respectively denote the surrogate function at the η'th ('current'), and $\eta+1$ 'th ('next') algorithm iteration. At steps 5 and 11, the surrogate and uncertainty functions are calculated with Gaussian Process regression. Note that when they are updated at step 11, Bayesian inference is additionally used. At step 7, the 'expected improvement' method discussed above is used to induce the acquisition function. It can be seen that the BO method only evaluates the objective function once per iteration, therefore the number of evaluations used during the optimisation is equal to the value of η at the end of Algorithm 1.

Algorithm 1: Bayesian Optimisation		
1	$\eta = 1$	
2	Initialise candidate, $\boldsymbol{x}_1$	
3	$y_1 = f(\boldsymbol{x}_\eta)$	//sample objective function
4	$D_\eta = [\boldsymbol{x}_\eta, y_\eta]$	//data set of corresponding x and y values
5	Calculate $\mu_\eta(\boldsymbol{x})$ and $\sigma_\eta(\boldsymbol{x})$ using D_η	
6	**while** Is-Not-Terminated (η, D_η)	//Boolean flag dependent on η and D_η
7	Induce $\alpha_\eta(\boldsymbol{x})$ using $\mu_\eta(\boldsymbol{x})$ and $\sigma_\eta(\boldsymbol{x})$	
8	$\boldsymbol{x}_{\eta+1} = \underset{x \in X}{\operatorname{argmax}} \alpha_\eta(\boldsymbol{x})$	
9	$y_{\eta+1} = f(\boldsymbol{x}_{\eta+1})$	//sample objective function
10	$D_{\eta+1} = \{D_\eta, [\boldsymbol{x}_{\eta+1}, y_{\eta+1}]\}$	//augment new data to data set
11	Calculate $\mu_{\eta+1}(\boldsymbol{x})$ and $\sigma_{\eta+1}(\boldsymbol{x})$ using $D_{\eta+1}$ and μ_η	
12	$\eta = \eta + 1$	
13	**end**	
14	Return best candidate evaluated so far	

By using information from the aquistion function to guide the search, the BO method can substantially reduce the number of objective function evaluations in comparison to other commonly applied methods, e.g. the Genetic Algorithm (GA) method. For example, Hickish, Fletcher, and Harrison (2019b) have shown that for a family of rail network optimisation test-tasks, a BO implementation required an order of magnitude fewer objective function evaluations than a GA implementation. Since this reduction in objective function evaluations comes at the cost of repeatedly computing $\mu(\boldsymbol{x})$, $\sigma(\boldsymbol{x})$, and $\alpha(\boldsymbol{x})$, the BO method is often best suited to tasks where the objective function is 'expensive-to-compute'. The threshold for this is machine and situation specific, although a value of 1-s computation time is illustrative.

Within the optimisation methodology presented here, the chosen fidelity in passenger satisfaction modelling leads to an 'expensive-to-compute' objective function which we expect the BO method to be well-suited to. Of course, there are other heuristic optimisation algorithm methods which might be well-suited to this methodology (Waibel et al., 2019). However, since no single one will outperform the others for all the specific tasks our generalised methodology might be applied to, the choice is not critical (Wolpert and Macready 1997). The contribution here is to demonstrate that application of a BO-like method allows an Agent-Based Model to be used to compute the objective function — which enables more accurate representation of the real-world problem in comparison to MP.

4. Example application

To highlight the value of a tool to assist with reducing the cost of developments, we demonstrate our methodology on a hypothetical case-study inspired by a planned development of high-speed rail in the UK. We highlight that although the case-study network and costs are based on data relating to the real-world network when available, these have been simplified so we can concentrate on illustrating the application of the methodology in general rather than determining results specific to a single real-world development. Nonetheless the resultant hypothetical network allows an investigation of the suitability of the methodology for assisting network planners. Note that focus lies on initial investment and passenger journey satisfaction only. Effects such as the running costs of energy and maintenance, stimulation of wider regional economic growth, ticket price, variable passenger demand, and relieving crowding on other rail lines, are excluded for clarity, but could be modelled within the same framework.

The topology of the case-study network is shown in Fig. 5, where each station is represented by a circle. We retain the real-world station name so that the reader might more easily cross reference against data sources. The central station is connected to each outer station by two single direction lines – an 'up line' and a 'down line'. Intermediate stations are not considered. The number of platforms at each station are shown inside the corresponding circle, and the line lengths are shown also (HS2 Limited, 2016a, c2019b, Railway Technology c2019b; ARUP n.d). Compared to the baseline train performance (HS2 Limited, 2019) and line-speeds (HS2 Limited, 2016b), the use of trains with lower performance, such as the British Rail Class 374 (Siemens Mobility 2016), and constructing track with lower line-speeds, might reduce the initial investment required. However, effects such as longer train journey times might also affect passenger satisfaction. Here we consider four types of trains and 14 possible line-speeds – thus capturing the trade-off between train performance, line speed, and initial investment. For overall passenger satisfaction, increasing the comfort of stations and trains might offset any increase in journey times and require less initial investment than increasing line-speed and train performance. To capture this, the VoT weighting for passengers in the states relating to Group A or B in Table 2, are optimised separately from a choice of 11 values whilst considering the expense of creating trains and stations with that VoT.

4.1. Formal definition

The optimisation task described in the previous section is represented with nine, integer-valued, optimisation variables, x_i, with index, $i = 1, 2 \dots 9$. The optimisation vector, $\boldsymbol{x}$, expresses the VES with the form:

$$\boldsymbol{x} = [x_1, x_2 \dots x_9] \quad (3)$$

Table 3 lists descriptions of the network characteristics affected by each optimisation variable as well as the upper and lower bounds on each variable. The values of x_1 and x_2 respectively define the train performance and comfort for all trains in the model. The line-speeds of lines connecting each city pair in Fig. 5 are optimised as a pair, and the line-speed applies for the whole length of the line. These simplifications are owing to the lack of data, available to the authors, to model lines at the section level or with a difference between the 'up' and 'down' direction. In practice this data could be included during the application of the methodology and either the speeds of critical sections optimised individually, or multiple sections controlled by one optimisation variable. Consequently there are three optimisation variables associated with line-speed, x_3, x_4 and x_5. Table 3 also lists the effect upon the network specification of the

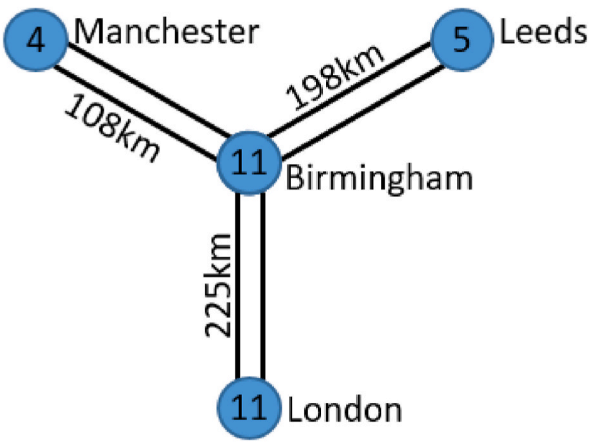

Fig. 5. The topology of the case study network. Each connected station pair is connected by two lines, with line lengths shown. The number of platforms at a station is shown inside each circle.

Table 3
Descriptions of the network characteristics affected by each optimisation variable, the upper and lower bounds on the optimisation variable values, and the effect the variable minimum and maximum upon characteristics of the network.

i	Description	x_i range	Effect of minimum value of x_i	Effect of maximum value of x_i
1	Train performance	$0 \leq x_1 \leq 3$	Baseline design specification	BR Class 395
2	Train comfort	$-5 \leq x_2 \leq 5$	$\alpha_A = 1.05$	$\varepsilon_A = 0.95$
3	London – Birmingham line-speed	$0 \leq x_3 \leq 13$	Line-speed = 360 km/h	Line-speed = 230 km/h
4	Birmingham – Manchester line-speed	$0 \leq x_4 \leq 13$	Line-speed = 360 km/h	Line-speed = 230 km/h
5	Birmingham – Leeds line-speed	$0 \leq x_5 \leq 13$	Line-speed = 360 km/h	Line-speed = 230 km/h
6	London station comfort	$-5 \leq x_6 \leq 5$	$\alpha_B = 1.05$	$\varepsilon_B = 0.95$
7	Birmingham station comfort	$-5 \leq x_7 \leq 5$	$\alpha_B = 1.05$	$\alpha_B = 0.95$
8	Manchester station comfort	$-5 \leq x_8 \leq 5$	$\alpha_B = 1.05$	$\alpha_B = 0.95$
9	Leeds station comfort	$-5 \leq x_9 \leq 5$	$\alpha_B = 1.05$	$\alpha_B = 0.95$

optimisation variable minimum and maximum values. The values of x_1 relate to the performance associated with four different types of train, which are further detailed in Section 4.3 and Table 3. The variables ε_A and ε_B are introduced as multipliers which respectively affect the time weightings in Group A and Group B in Table 2, and are controlled by the value of x_2 , x_6, x_7, x_8 and x_9 respectively. ε_A and ε_B change value in discrete steps of 0.01. Negative values of the optimisation variables result in more comfortable trains and stations. In the model the line-speed of each line pair is set in the range of 360 km/h to 230 km/h, with discrete steps of 10 km/h. By discretising the optimisation variables in this way, the combinatorial nature of the problem leads to a search space which contains 1.5×10^9 candidates. Since evaluating each of these requires the computation of an expensive model, this task is well-suited to the BO method.

The reduction in the investment required for the network specified by the VES, compared to the baseline network specification, is given by:

$$g(\boldsymbol{x}) = g_T(x_1) + g_C(x_2) + \sum_{j=3}^{j=5} g_L(x_j) + \sum_{k=6}^{k=9} g_V(x_k) \tag{4}$$

where $g_T(x)$ denotes the savings function relating to the type of train, $g_C(x)$, the savings function relating to train comfort, $g_L(x)$, the savings function relating to line-speed, and $g_V(x)$ the savings function relating to station comfort. The form of these functions is expanded in Section 4.3.

4.2. Model constants

Interstation run times and trains-per-hour predictions for a network of this scale were provided by the Department for Transport (2016, 2017,p.43) and HS2 Limited (c2019b). In the investigation presented here, all services were modelled to stop at Birmingham. The interstation run times were combined with a 2 min dwell time to create a timetable with equal intervals between trains and constant service pattern between 6am and 11pm. To fulfil this timetable for a passenger load of 117,000 per day, this being scaled from forecast demand for the core of the baseline network (HS2 Limited c2019a), 21 identical trains were modelled. The passenger load has been distributed between the origin-destination pairs according to the relative number of trains between each pair in the case-study timetable, and have had journey start times allocated to reflect hourly variation in travel demand data (Department for Transport, 2012).

4.3. Optimisation variables and investment function values

Table 4 displays the type of train and the maximum speed associated with each value of x_1. The speed dependent tractive force available to a train, and the Davis formula coefficients, were also dependent on the value of x_1. All trains were modelled to have seating and standing capacity for 700 and 250 passengers respectively (HS2 Limited, 2019). Table 4 also displays the value of the train 402 performance savings function for each value of $x1$, these values having been determined by 403 considering the cost of each train type in 2019 prices (Mochida et al., 2010; Siemens Mobility 2016, 404 HS2 Limited, 2019, Railway Museum c2019), scaled on a cost per-carriage basis. Since train prices 405 often represent a commercial package dependent on finance and long-term service

Table 4
The train type, maximum train speed, and savings function value associated with each value of the optimisation variable, x_1.

$x_1 =$	0	1	2	3
Train type	Baseline design specification	BR Class 374	BR Class 373	BR Class 395
Maximum speed (km/hour)	360	320	300	225
$g_T(x_1)$ (£ millions)	0	320	630	1580

agreements, 406 these values are speculative but are adequate for demonstrating the methodology presented in this 407 paper.

The savings functions for train comfort, line-speed and station comfort are respectively given by:

$$g_C(x_2) = \mathfrak{m}_C x_2 \quad (5)$$

$$g_L(x_j) = \mathfrak{m}_L x_j \; for \; j = 3, 4, 5 \quad (6)$$

$$g_V(x_k) = \mathfrak{m}_V x_k \; for \; k = 6, 7, 8, 9 \quad (7)$$

where $\mathfrak{m}_C$, $\mathfrak{m}_L$ and $\mathfrak{m}_V$ are parameters with values shown in Table 5. These values have been calculated by fitting a linear relationship to data relating to initial investment and train performance, train comfort, line-speed, and station comfort. At the time of writing complete data sets, for the component characteristics of HS2 at different initial investments, were unavailable to the authors. So, to calculate the value of $\mathfrak{m}_L$, data for the initial investment and line-speed of the High Speed One and HS2 networks were used (Bodman 2012). To calculate the value of $\mathfrak{m}_C$, a value of £12 million to refurbish one Class 373 train (Railway Technology c2019a) was assumed to be adequate to alter the comfort from the level denoted by $x_2 = 5$ to that denoted by $x_2 = 0$. The same approach was taken with calculating the value of $\mathfrak{m}_V$, using the mean of data for refurbishing six UK stations (Rail Delivery Group 2017). Although these relationships are simplified, they give a basis for capturing the trade-off between initial investment, line-speed, train comfort, and station comfort in the HS2 development, and can readily be replaced by more accurate data, should it become available.

4.4. Experimental method

The methodology described in Section 3 was applied to the optimisation task defined by Equations (1), (2) and (4), and the constraints in Table 3. The parameter β was set to represent a total savings requirement of £5 billion, which is distributed between 54 trains, four stations, and the lines shown in Fig. 5. This is 8% of the cost modelled for the baseline network (Oakervee 2019). To illustrate the 'usefulness' of this saving, it is also 70% of the initial investment that has been estimated as required for a politically sensitive proposed development in the UK – High Speed Three (Tingle 2016). Increasing the value of β would reduce the size of the feasible region of the search space, creating a less challenging optimisation task for demonstrating the presented methodology. Therefore the value of β was chosen to demonstrate the methodology making a substantial saving, whilst balancing the need to demonstrate a meaningful optimisation task.

An initial candidate that satisfied the constraint was arbitrarily chosen, and the objective function was computed using model parameters described in Sections 4.2 and 4.3. The optimisation terminated when the objective function evaluation budget, η_{max}, was exhausted. Each optimisation was repeated independently 24 times with different, uniformly distributed, initial candidates. Computational cost is described using wall clock time. This is machine specific, but is indicative given that an unremarkable machine was used — an Intel Xeon Dual Processor @ 2.4 GHz.

The BO implementation used was the MATLAB R2017b function *bayesopt*, which was applied with an 'Expected-Improvement-Plus' acquisition function and other settings at their default value (The Mathworks Incorporated c2018). A Gaussian Process, calculated with an ARD Matern 5/2 kernel function, is used to calculate $\mu(\boldsymbol{x})$ and $\sigma(\boldsymbol{x})$. The simulation model time-step parameters, t_P and t_T, were assigned values of one and 15 s respectively.

5. Results

Fig. 6 plots markers to display features of the distribution from results of 24 repeats of the optimisation, showing the 'minimum network score found so-far' at every objective function evaluation, enumerated by η up to η_{max}. The y-axis displays the value of the marker of the distribution, in the units of total weighted passenger time per passenger per metre travelled. So that the effect of the VESs can be compared against the 'baseline' case, Fig. 6 also shows the network score for the currently proposed, 'baseline', network specification, i.e. $\boldsymbol{x} = [0, 0, 0, 0, 0, 0, 0, 0, 0]$. Section 3.1.2 describes that the network score is a disutility measure, therefore a reduced network score indicates a better network because disutility is a negative quantity. From $\eta = 50$ all the optimisations found a solution with a 'better' network score than the baseline network. It can be seen that, in general, features of the distribution reduce in value at greater values of η and the range of the distribution is reduced, i.e. the separate optimisations are 'converging' around a minimum. Experiments where η_{max} was increased to 500 found no improvement in the minimum objective function score found by all the optimisations compared to at $\eta = 30$, although greater 'convergence' was observed. The network score of the best and worst-case optimisation at $\eta = 100$, is respectively 8% and 4% less, i.e. better, than the network score when no VES is applied. Whilst this might sound a marginal improvement it is being achieved with a reduced initial investment. These results indicate that - in this case - a budget of 100 evaluations is a good trade-off between computational expense and returning 'good' solutions, this is consistent with previous investigations of the BO method (Li et al., 2017; Hickish, Fletcher, and Harrison 2019b). Depending on the planning window

Table 5
Investment function parameter values.

Parameter	$\mathfrak{m}_C$	$\mathfrak{m}_L$	$\mathfrak{m}_V$
Value (£ millions)	108	0.58 per km	28

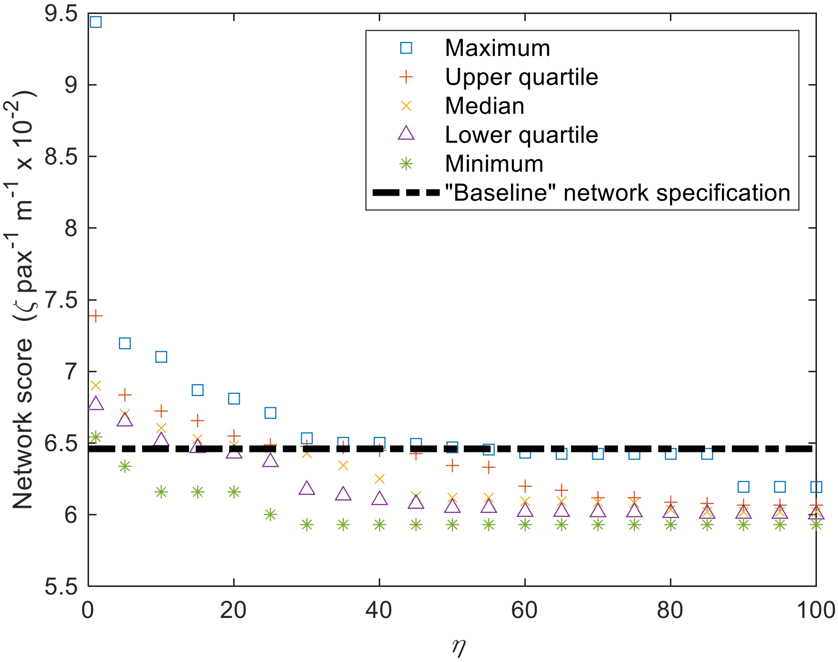

Fig. 6. – *Markers to show the value of features of the distribution, from 24 optimisations, of 'minimum network score found so-far' values at different numbers of objective function evaluations (η).*

and computational resources available to a developer, the computational budget could be modified to reflect the target performance of the solution. To simulate a full day's operation, i.e. one objective function evaluation, required approximately 13 min of computation, i.e. is 'expensive-to compute'. The computational time of the BO implementation — excluding objective evaluations — has previously been found to be $\sim O(\eta^3)$ however at $\eta = 100$ it is less than 2 min (Hickish et al., 2019b), i.e. small in comparison to evaluating the objective function.

For the 'minimum network scores found so-far' distribution from 24 repeat optimisations, Fig. 7 plots the number of times that the vector $\boldsymbol{x}$ contained an optimisation variable (x_i) at a certain value. There is a plot for every optimisation variable, with the value of the optimisation variable shown on the abscissa and the number of times it occurred in the distribution on the ordinate. The results are plotted for the distribution at $\eta = 1$ and $\eta = 100$ and bars for these are plotted next to each other for comparison. There are 24 data points in each series owing to the repeat experiments. Observing the plot for train performance, it can be seen that most initial candidates have $x_1 = 3$, this is because the proportion of candidates that are feasible, i.e. satisfy the constraint given by Equation (2), increases with the value of x_1 and the initial candidates are selected with uniform probability from the pool of feasible candidates. However, at $\eta = 100$ all the optimisations select $x_1 = 2$, as well as indicating the optimum train performance, this suggests the BO implementation is successfully exploring all regions of the search space.

For the train comfort plot in Fig. 7, the variation in frequency of the initial values of x_2 is within the bounds of what would be expected from a small sample size of a uniform probability distribution. This is because the investment associated with x_2 is small compared to the other optimisation variables so there is a weak relationship between its value and the feasibility of the candidate. However the values at $\eta = 100$ show that all the solutions cluster towards increased train comfort despite the increase in investment, with most solutions relating to increasing train comfort by close to the maximum amount. This is because the investment quantity modelled to improve the satisfaction of virtual passengers through increased train comfort is small in comparison the investment modelled to increase model network line-speed. In contrast to train comfort, the plots for station comfort show that the solutions at $\eta =$ 100 cluster around less comfort. This is because a substantial saving can be made in the model with little detriment to virtual passenger satisfaction. The plots relating to line-speed show that most optimisations found a solution to have line-speeds of 300 to 280 km/h between London and Birmingham, whereas most optimisations had speeds less than this for the lines between Birmingham and Manchester, and Birmingham and Leeds. This is because the lines between London and Birmingham line have the greatest passenger load, meaning that shortened journey times here have a large positive effect. As intuitively expected, the line-speed of the optimised network has been approximately 'matched' to the train maximum speed of the optimised network. The plots relating to station comfort show that all the optimisations found solutions that decrease the comfort of stations in order to reduce the initial investment.

5.1. Sensitivity study

To reflect that there is often uncertainty around the monetary costs of a network development (Flyvbjerg, Skamris holm, and Buhl 2003), the optimisation sensitivity to the cost of components was investigated by repeating the optimisation using different values for

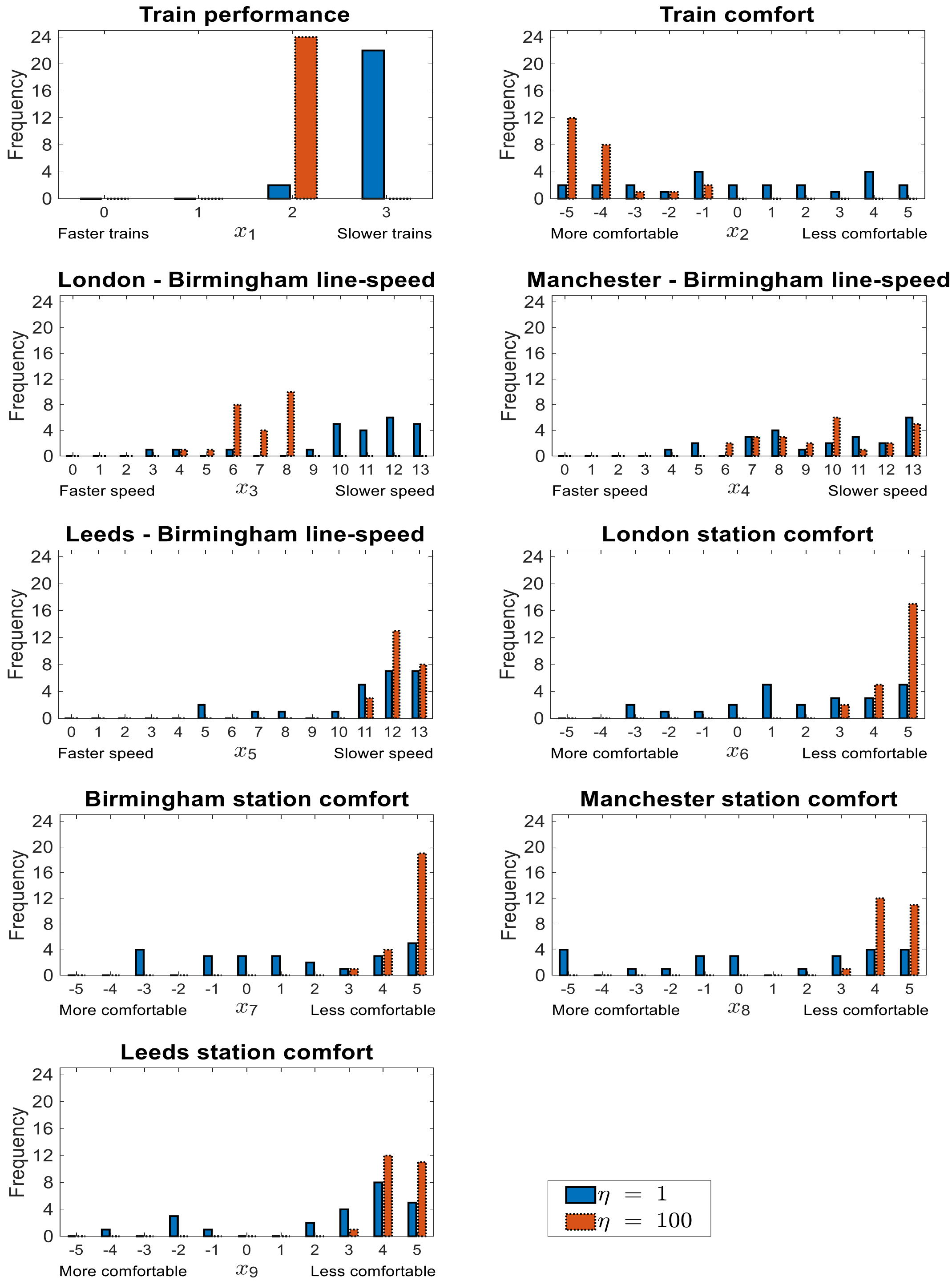

Fig. 7. – *The frequency that each optimisation variable took different values. The values the optimisation variable took, x_i , are shown on the abscissa. The number of times there was a vector (x) with the variable at the value shown by the abscissa, is shown on the ordinate. For every value of x_i , the values shown at the first ($\eta = 1$) and last ($\eta = 100$) algorithm iteration are plotted next to each other for comparison. There are 24 data points in each series owing to repeat experiments.*

the parameters; $\mathfrak{m}_C$, $\mathfrak{m}_L$, and $\mathfrak{m}_V$, and those defining the value of the function $g_T(x_1)$ at $x_1 = 1, 2, 3$. These respectively capture the savings associated with varying train comfort, line-speed, station comfort, and train performance. A two-level, six variable, experiment design was used, the upper and lower levels respectively relating to a 20% increase and decrease on the values shown in Tables 4 and 5, with the original values used as the neutral level. For each set of parameter values the same initialisation conditions were used, but the optimisation was repeated independently 24 times as described in Section 4.4.

Fig. 8 shows plots to compare the best solution found in 24 repeats of the optimisation, i.e. the $\mathbf{x}$ with the lowest objective function value, for all parameter sets where only one parameter value is changed and the others remain at their neutral value. Results from the set in which all parameters are at their neutral value, referred to as set κ_0, are also included therefore there are 13 solutions represented here. Each plot relates to one optimisation variable, x_i, and shows the distribution of values this variable took in the data set of best solutions. Plots for 'Train performance' and 'London station comfort' are not shown since these results are single valued at $x_1 = 2$ and $x_6 = 5$ respectively, indicating the solution is not affected by variations up to a magnitude of 20% in a single savings parameter. For each plot the value of x_i relating to κ_0 is indicated by a darker coloured bar. For the majority of plots the x_i values are distributed with a

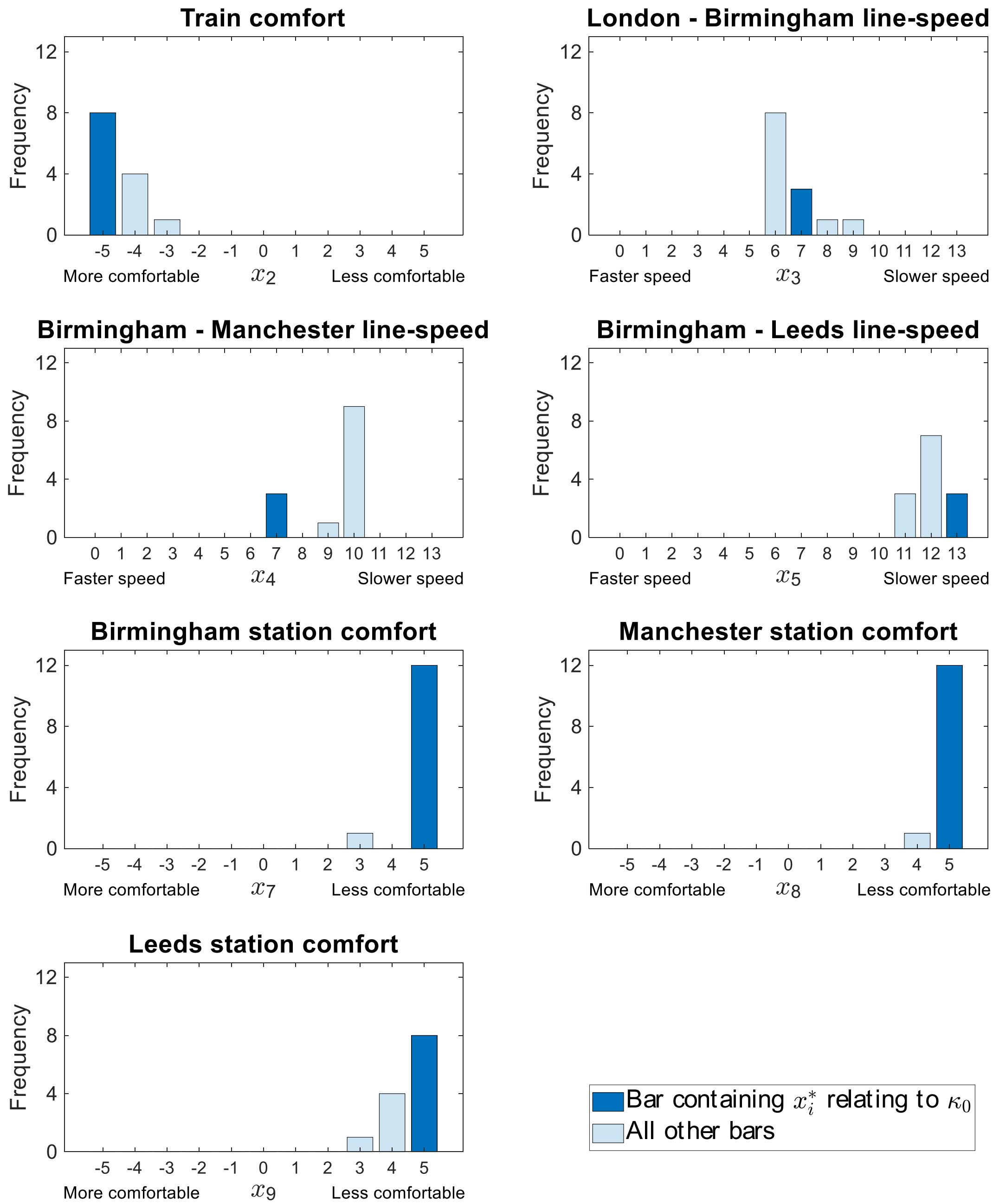

Fig. 8. – *The frequency that each optimisation variable took different values in the distribution of results from a 6 variable, two level, sensitivity study. These results are the best solution found from 24 repeats of the optimisation, for 13 different parameter sets. The values the optimisation variable, x_i, took are shown on the abscissa. The frequency of the results containing a vector ($\mathbf{x}$) with the variable at the value shown by the abscissa, is shown on the ordinate. The result of the optimisation using the 'neutral' parameter set, κ_0, is indicated by a darker coloured bar.*

range of up to 4 – this magnitude is within the bounds of what might reasonably be expected owing to stochastic optimisation algorithm performance, and indicates low sensitivity to parameter sets with only one altered value. The results are consistent with the findings of the previous section, for example the line-speeds of the London – Birmingham link are prioritised over other links. For all the parameter sets, the value of the objective function relating to the best solution is within 2% of the best solution relating to set κ_0.

6. Discussion

It is difficult to make a like-for-like comparison of the presented methodology against other recently published work since, to the best of the authors' knowledge, there are no others in existence which are designed for optimising a VES for maximum passenger satisfaction. The case-study results show that in comparison to methodologies for related TNDP problems, the one presented here has an 'expensive-to-compute' objective function owing to the individualised simulation of passenger journeys. For example, the objective function of Wang et al. (2019) can be inferred to require four orders of magnitude less computation time. Application of the BO method within the presented methodology at least partially mitigates this since fewer evaluations of the 'expensive-to-compute' objective function are needed at the expense of a computational overhead – a worthwhile trade-off in this case. In comparison to 'traditional' MP methodologies for related TDNP problems, an advantage of the presented methodology is its flexibility to readily incorporate additional models of phenomena observed in the real-world, with no limitations on the form of relationships between variables. For example, although passenger demand is treated as fixed in the case-study, a demand model, such as those discussed by Jin et al. (2019), could be incorporated to the network simulation. This would allow the effect of the VES upon passenger demand to be included in the optimisation when the methodology is applied in practice. The case-study highlights the importance, when applying the methodology, of data sets relating to cost of different specifications, and the relationship between VES and passenger comfort. Some of this information can reasonably be assumed to be available to network planners for a specific development since it is used for cost-benefit analyses (Van Wee 2007). In the event of 'missing' information there might often be a business case for obtaining it given the potential for substantial savings in the project cost. Civity Management Consultants (2013) and You et al. (2006) demonstrate two examples where similar datasets and models have been determined.

7. Conclusions

The results support that the presented methodology can assist the identification of a Value Engineering Strategy (VES) which provides a high level of passenger satisfaction under a reduced network development budget. In the example of the hypothetical case-study investigated here it is estimated that the VES identified by the methodology saves £5 billion from the modelled £64 billion construction costs of the network, and improves the satisfaction of virtual passengers by 8%. Whilst an improved network score at lower investment might be counterintuitive, it is plausible if the baseline network relates to one defined by a larger amount of investment that is poorly allocated. Since the combinatorial nature of investment allocation problems means there might often be an extremely large number of candidate allocations, e.g. over a billion in the case-study, there is a danger of a poor allocation or poor VES occurring if tools such as the presented methodology are not used to systematically and efficiently 'explore' the search space. Direct search for a small number of cases is unlikely to reveal similarly good solutions. The VES identified in the case-study involves reducing the model network train speeds whilst increasing their comfort – leading to a saving in the modelled construction costs. This insight demonstrates the benefit of considering multiple network design parameters simultaneously. Here the effect of multiple design parameters upon passenger satisfaction is modelled by applying Agent Based Modelling (ABM) to simulate individual passenger journeys. These are resolved into a series of duration-variable stages with comfort weightings dependent on the behaviour and attributes of other agents. Although other methods could be used for this, the ABM approach is more easily extended to include other relevant contributors to passenger experience e.g. pedestrian movement within stations, in further development of the methodology. The reasonable computational cost of the methodology demonstrated in the case-study means that any optimisation could be repeated quickly in response to changes in the fast-evolving political and fiscal considerations surrounding network developments. Furthermore, it is tractable to conduct a sensitivity study to assist with determining solutions which are robust, thus mitigating the effects of input parameter uncertainty. The methodology could also be adapted to optimise the network specification for minimum investment under a constraint of maintaining target passenger satisfaction – this is an area for future investigation.

Declaration of competing interest

None.

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References

Akabal, F.M., Mohd Masirin, M.I.H., Akasah, Z.A., Rohani, M.M., 2017. Review on selection and suitability of rail transit station design pertaining to public safety. IOP Conf. Ser. Mater. Sci. Eng. 226, 012033.

Alikhani-Kooshkak, R., Tavakkoli-Moghaddam, R., Jamili, A., Ebrahimnejad, S., 2017. Solving a multi-objective train makeup model with locomotive limitation by using a firefly algorithm: a case study. Proc. Inst. Mech. Eng. - Part F J. Rail Rapid Transit 232 (5), 1483–1499.
ARUP. (n.d) Post Consultation Route Birmingham Interchange. Department for Transport, London, UK, Accessed 07/06/2019, http://assets.dft.gov.uk/publications/hs2-maps-20120110/hs2arp00drrw05303issue2.pdf.
ARUP, Institute for Transport Studies Leeds, Accent, 2015. Provision of Market Reserch for Value of Travel Time Savings and Reliability: Non-technical Summary Report. Department for Transport, London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/470229/vtts-phase-2-report-non-technical-summary-issue-august-2015.pdf. (Accessed 20 March 2019).
Bärmann, A., Martin, A., Schülldorf, H., 2017. A Decomposition Method for Multiperiod Railway Network Expansion—With a Case Study for Germany. Transport. Sci. 51 (4), 1102–1121. https://doi.org/10.1287/trsc.2017.0747.
Bodman, A., 2012. Committee of Public Accounts - the Completion and Sale of High Speed 1. Parliament, London, UK. https://publications.parliament.uk/pa/cm201213/cmselect/cmpubacc/464/464we05.htm. (Accessed 5 June 2019).
Börjesson, M., Eliasson, J., 2019. Should values of time be differentiated? Transport Rev. 39 (3), 357–375.
Canca, D., De-Los-Santos, A., Laporte, G., Mesa, J.A., 2016. A general rapid network design, line planning and fleet investment integrated model. Ann. Oper. Res. 246 (1), 127–144.
Chan, J., 2007. Rail Transit OD Matrix Estimation and Journey Time Reliability Metrics Using Automated Fare Data. Massachusetts Institute of Technology.
Christogiannis, E., Pyrgidis, C., 2013. Investigation of the impact of traffic composition on the economic profitability of a new railway corridor. Proc. Inst. Mech. Eng. - Part F J. Rail Rapid Transit 228 (4), 389–401.
Civity Management Consultants, 2013. Further Development of European High Speed Rail Network. https://civity.de/asset/en/sites/2/2018/05/civity_dev_eu_hsr_network_012014.pdf. (Accessed 20 June 2019).
Department for Transport, 2012. Rail Passenger Numbers and Crowding on Weekdays in Major Cities in England and Wales: 2012. gov.uk, London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/252516/rail-passengers-crowding-2012-revised.pdf. (Accessed 13 May 2019).
Department for Transport, 2015. Understanding and Valuing Impacts of Transport Investment. gov.uk, London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/470998/Understanding_and_Valuing_Impacts_of_Transport_Investment.pdf. (Accessed 13 February 2020).
Department for Transport, 2016. High Speed Two: from Crewe to Manchester, the West Midlands to Leeds and beyond. gov.uk, London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/568208/high-speed-two-crewe-manchester-west-midlands-leeds-web-version.pdf. (Accessed 7 June 2019).
Department for Transport, 2017. High Speed Two Phase Two Economic Case. London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/634196/high-speed-two-phase-two-economic-case.pdf. (Accessed 7 June 2019).
Durrant, D.W., 2015. The controversial discourse on speed in the case of HS2. Proc. Inst. Civ. Eng.: Urban Des. Plan. 168 (5), 241–250.
European Commission, 2019. The European deployment plan and national implementation plans. Brussels, Belgium. https://ec.europa.eu/transport/modes/rail/ertms/ertms_deployment_en.
Farahani, R.Z., Miandoabchi, E., Szeto, W.Y., Rashidi, H., 2013. A review of urban transportation network design problems. Eur. J. Oper. Res. 229 (2), 281–302.
Flyvbjerg, B., Skamris holm, M.K., Buhl, S.L., 2003. How common and how large are cost overruns in transport infrastructure projects? Transport Rev. 23 (1), 71–88.
Furness, N., Van Houten, H., Arenas, L., Bartholomeus, M., 2017. ERTMS Level 3: the Game Changer. *IRSE News.* Institution of Railway Signal Engineers, London, UK.
Hickey, S., 2011. Improving the Estimation of Platform Wait Time at the London Underground. Massachusetts Institute of Technology.
Hickish, B., Fletcher, D.I., Harrison, R.F., 2017. Maximising passenger satisfaction through optimised train movements. In: The Stephenson Conference. The Institution of Mechanical Engineers.
Hickish, B., Fletcher, D.I., Harrison, R.F., 2019a. A rail network performance metric to capture passenger experience. J. Rail Transp. Plan. Manag. 11, 100138.
Hickish, B., Fletcher, D.I., Harrison, R.F., 2019b. Investigating Bayesian Optimization for rail network optimization. Int. J. Real. Ther. 1–17.
Howlett, P.G., Pudney, P.J., 1995. Energy-Efficient Train Control. Springer, London, UK).
HS2 Limited, 2016a. Route Engineering Report. High Speed Two Phase 2b West Midlands to Leed. Department for Transport, London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/567616/West_Midlands_to_Leeds_Route_engineering_report.pdf. (Accessed 1 June 2019).
HS2 Limited, 2016b. Route Engineering Report 2016. High Speed Two Phase 2b West Midlands to Leed. Departement for Transport, London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/567616/West_Midlands_to_Leeds_Route_engineering_report.pdf. (Accessed 1 June 2019).
HS2 Limited, 2019. Train Technical Specification. gov.uk, London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/794108/HS2-HS2-RR-SPE-000-000007_P11_TTS_Main_Body__External_.pdf. (Accessed 1 June 2019).
HS2 Limited, 2019a. About Us. Birmingham, UKca. https://www.hs2.org.uk/why/about-us/. (Accessed 8 June 2019).
HS2 Limited, 2019b. Birmingham Curzon Street. Birmingham, UKcb. https://www.hs2.org.uk/stations/birmingham-curzon-street/. (Accessed 7 June 2019).
Huang, W., Shuai, B., 2018. A methodology for calculating the passenger comfort benefits of railway travel. J. Mod. Transp. 26 (2), 107–118.
Jin, G., He, S., Li, J., Li, Y., Guo, X., Xu, H., 2019. An intergrated model for demand forecasting and train stop planning for high-speed rail. Symmetry (11), 720–740, 2019.
Kanai, S., Shiina, K., Harada, S., Tomii, N., 2011. An optimal delay management algorithm from passengers' viewpoints considering the whole railway network. J. Rail Transp. Plan. Manag. 1 (1), 25–37.
Kunimatsu, T., Hirai, C., Tomii, N., Takaba, M., 2009. Evaluation of timetables by estimating passengers' personal disutility using micro-simulation. In: RailZuirch2009 - Third International Seminar on Railway Operations Modelling and Analysis. ETH Zürich.
Kunimatsu, T., Hirai, C., Tomii, N., 2012. Train timetable evaluation from the viewpoint of passengers by microsimulation of train operation and passenger flow. Electr. Eng. Jpn. 181 (4), 51–62.
Kwok, E.C.S., Anderson, P.M., Ng, S.H.S., 2009. Value engineering for railway construction projects: cost driver analysis. Proc. Inst. Mech. Eng. - Part F J. Rail Rapid Transit 224 (1), 45–52.
Lai, Y.-C., Shih, M.-C., 2013. A stochastic multi-period investment selection model to optimize strategic railway capacity planning. J. Adv. Transport. 47 (3), 281–296.
Li, C., Rubín de Celis Leal, D., Rana, S., Gupta, S., Sutti, A., Greenhill, S., Slezak, T., Height, M., Venkatesh, S., 2017. Rapid Bayesian optimisation for synthesis of short polymer fiber materials. Sci. Rep. 7 (1), 5683.
Lin, B., Liu, C., Wang, H., Lin, R., 2017. Modeling the railway network design problem: a novel approach to considering carbon emissions reduction. Transport. Res. Transport Environ. 56, 95–109.
Litman, T., 2017. Build for Comfort, Not Just Speed. Victoria Transport Policy Institute, Victoria, Canada. https://www.vtpi.org/quality.pdf. (Accessed 19 February 2020).
Mochida, T., Yamamoto, N., Goda, K., Matsushita, T., Kamei, T., 2010. Development and maintenance of Class 395 high-speed train for UK high speed 1. Hitachi Rev. 59 (1), 8.
Oakervee, D., 2019. Oakervee Review. gov.uk, London, UK. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/869252/oakervee-review.pdf. (Accessed 10 March 2020).
Oliveira, L., Bruen, C., Birrell, S., Cain, R., 2019. What passengers really want: assessing the value of rail innovation to improve experiences. Transp. Res. Interdiscip. Perspect. 1, 100014.
Pienaar, W.J., 1997. Modelling generalised transit commuter costs for use in urban transport planning. Trans. Built Environ. 30, 8.
Political, T.N.S., Social, 2013. Europeans' Satisfaction with Rail Services. The European Commission, Brussels, Belgium. http://ec.europa.eu/commfrontoffice/publicopinion/flash/fl_382a_en.pdf. (Accessed 8 February 2019).

Preston, J.M., Blainey, S.P., Wardman, M., Chintakayala, P.K., Heywood, C., Sheldon, R., Wall, G.T., 2008. The effect of station enhancements on rail demand. In: European Transport Conference. Association for European Transport.

Qin, F., 2014. Investigating the in-vehicle crowding cost functions for public transit modes. Math. Probl Eng. 1–13, 2014.

Rail Delivery Group, 2017. £5bn Regeneration of Britain's Rail Stations Making Local Economies Stronger. London, UK. https://www.raildeliverygroup.com/media-centre/press-releases/2017/469772907-2017-06-22.html. (Accessed 10 June 2019).

Railway Museum, 2019. Eurostar: the UK's Fastest Train. National Railway Museum, York, UK. https://www.railwaymuseum.org.uk/whats-on/eurostar-uks-fastest-trainc. (Accessed 7 June 2019).

Railway Technology, 2019. Eurostar e320 High-Speed Train. Verdict Media, London, UK. https://www.railway-technology.com/projects/eurostar-e320-high-speed-train/ca. (Accessed 10 June 2019).

Railway Technology, 2019. High Speed 2 (HS2) Railway. Verdict Media, London, UK. https://www.railway-technology.com/projects/high-speed-2-hs2/cb. (Accessed 1 June 2019).

Russo, F., 1998. A model for schedule optimisation in an intercity transportation system. Trans. Built Environ. 37, 12.

Seeger, M., 2004. Gaussian Processes for machine learning. Int. J. Neural Syst. 14 (2), 69–106.

Sels, P., Dewilde, T., Cattrysse, D., Vansteenwegen, P., 2016. Reducing the passenger travel time in practice by the automated construction of a robust railway timetable. Transp. Res. Part B Methodol. 84, 124–156.

Seyedvakili, S., Zakeri, J.-A., Nasr Azadani, S.M., Shafahi, Y., 2020. Long-term railway network planning using a multiperiod network design model. J. Transport. Eng., Part A: Systems 146 (1), 04019054.

Shahriari, B., Swersky, K., Wang, Z., Adams, R.P., de Freitas, N., 2016. Taking the human out of the loop: a review of bayesian optimization. Proc. IEEE 104 (1), 148–175.

Siemens Mobility, 2016. Eurostar e320 high-speed trains for Eurostar International Limited. Siemens AG, Munich, Germany. https://www.mobility.siemens.com/mobility/global/SiteCollectionDocuments/en/rail-solutions/high-speed-and-intercity-trains/velaro/velaro-e320-en.pdf. (Accessed 7 June 2019).

Sinclair, O., 2019–2020. 19 Rail Projects to Watch in 2019. International Rail Journal, Falmouth, UK. https://www.railjournal.com/in_depth/19-rail-projects-to-watch-in-2019, 24-1.

Smith, R., 2019. How to Rescue Britain's HS2 Project. International Railway Journal, Falmouth, UK. (Accessed 27 January 2020).

Technical Leadership Group, 2017. Rail Technical Strategy Capability Delivery Plan. Rail Safety and Standards Board, London, UK. https://www.rssb.co.uk/rts/Documents/2017-01-27-rail-technical-strategy-capability-delivery-plan-brochure.pdf. (Accessed 7 November 2018).

The Mathworks Incorporated, 2018. bayesopt.Natick (MA). USA. https://uk.mathworks.com/help/stats/bayesopt.html. (Accessed 8 March 2018). c.

Tingle, L., 2016. How Much Will HS3 Cost? British Broadcasting Corporation, London, UK. https://www.bbc.co.uk/news/uk-england-35809421. (Accessed 10 March 2020).

University of Sheffield, 2017. Sheffield and Siemens Next Generation Transport Modelling. Sheffield, UK. https://www.sheffield.ac.uk/dcs/latest-news/transport_modelling-1.685272. (Accessed 19 November 2018).

Waibel, C., Worthmann, T., Evins, R., Carmeliet, J., 2019. Building enrage optimization: an extensive benchmark of global search algorithms. Energy Build. 187, 218–240.

Wang, Y., Zhou, Y., Yan, X.D., 2019. Optimizing train-set circulation plan in high-speed railway networks using genetic algorithm. J. Adv. Transport.

Wardman, M., 2004. Public transport values of time. Transport Pol. 11 (4), 363–377.

Wardman, M., Murphy, P., 2015. Passengers' valuations of train seating layout, position and occupancy. Transport. Res. Pol. Pract. 74, 222–238 (Article).

Van Wee, B., 2007. Rail Infrastructure: challenges for Cost–Benefit Analysis and Other ex ante Evaluations. Transport. Plann. Technol. 30 (1), 31–48.

Wolpert, D.F., Macready, W.G., 1997. No free lunch theorems for optimization. IEEE Trans. Evol. Comput. 1, 67–82.

Yao, X., Zhao, P., Qiao, K., 2013. Simulation and evaluation of urban rail transit network based on multi-agent approach. J. Ind. Eng. Manag. 6 (1), 367–379.

You, H., Ryu, T., Oh, K., Yun, M.H., Kim, K.J., 2006. Development of customer satisfaction models for automotive interior materials. Int. J. Ind. Ergon. 36 (4), 323–330.