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Work-Path Approach Seismic Modelling of Hinge-Controlled Masonry Arches

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Abstract (150 – 200 words)

This work develops a dynamic analysis procedure for hinge-controlled masonry arches subjected to horizontal acceleration profiles. Constructed from the principles of energy conservation, the establishment of equivalent systems, and the path independence of conservative work, a time incremental analysis structure is established for kinematic propagation. Equivalent systems are defined through combining kinematic equilibrium with static deformations of the single degree of freedom mechanism through parametric plotting. This generates the minimum work required to propagate the arch towards collapse. For a constant acceleration above the static limit, energy conservation requires excess work's transformation into kinetic energy. The path independence of work creates a spatial kinetic energy equation which is used to establish the time-domain of the system. Knowing the initial position and kinetic energy thus allows the final position and kinetic energy to be determined for the time increment. A new constant acceleration and time step then propagates the behaviour through the acceleration profile.

Keywords chosen from ICE Publishing list

Dynamics; Brickwork & masonry; Seismic engineering

List of notations

v_i	is the vertical reaction of the i[th] hinge
h_i	is the horizontal reaction of the i[th] hinge
α_i	is the rotation angle of the i[th] hinge
γ_{23}	is the polar rotation deformation of Element 2
θ_{ij}	is the undeformed polar angle between hinges I and j
θ_{ij}'	is the deformed polar angle between hinges I and j
l_{ij}	is the rigid length between hinges I and j
$\boldsymbol{BC}$	is the balance matrix for the equilibrium equation set
$\boldsymbol{r}$	is the reaction vector for the equilibrium equation set
$\boldsymbol{q}$	is the constants vector for the equilibrium equation set
λ_a	is the acceleration collapse multiplier for kinematic equilibrium
λ_{app}	is the applied acceleration multiplier
H_i	is the i[th] hinge
W_{app}	is the applied work
W_{min}	is the minimum work
W_{req}	is the required work
τ_{Ei}	is the centroid point torque from the i[th] element
f_{gi}	is the gravitational force of the i[th] element

F_{app} is the single point applied acceleration force equivalent

F_{min} is the single point minimum acceleration force equivalent

A_j is the polynomial constants for centre mass translation equation

B_j is the polynomial constants for centre mass lever arm rotation equations

C_j is the polynomial constants for minimum work path equation

D_j is the polynomial constants for time domain equation

m_T is the total mass of the arch

m_{Ei} is the total mass of the i^{th} element

ΔPE is the change in gravitational potential energy

KE_1 is the initial kinetic energy

KE_f is the final kinetic energy

l_{Ej} is the lever arm length between the full arch centroid and the j^{th} element's centroid

$\boldsymbol{v}$ is the translational velocity vector

ω is angular velocity

I is the moment of inertia

t is time

Introduction

Efficiency of structural design and analysis is paramount for the successful implementation of any structural system and is further exacerbated for the introduction of novel systems. The masonry arch has the potential to be an advantageous structural system for modern constructions through the technique termed Reinforced Stability Based Design (RSBD) (Stockdale, 2016). This technique defines failure as the loss of stability and introduces safety through the application of reinforcement designed to resist the kinematic motion of the failed arch. This allows the material strengths to become a secondary consideration, establishes the ability to create generalized structural health monitoring systems with minimal calibration time, and provides the potential to significantly extend the serviceable lifespan through the proven longevity of structural masonry (Stockdale, 2012; Angelillo, 2014; Tralli, Alessandri and Milani, 2014). The challenge is that masonry arch analysis does not fit into the linear elastic model and its successful inclusion as a viable design strategy depends upon the development of an efficient and accessible analysis model for both static and dynamic conditions.

Kinematic equilibrium is the evaluation of an equilibrium condition for a defined mechanical state. It is derived from the upper bound theorem of limit analysis (LA), but it differs from the standard application of virtual conditions by directly examining the static condition requirements for a defined mechanical state. Kinematic equilibrium has been introduced through the development of a first-order assessment strategy (Stockdale and Milani, 2019) and the Kinematic Collapse Load Calculator (KCLC) (Stockdale *et al.*, 2018). This approach has proven to be versatile and adaptable: incorporating generic arch geometries (Stockdale and Milani, 2018); addressing non-traditional mechanisms, adapting the analysis model to match experimentation, and obtaining reinforcement capacity requirements for the defined state (Stockdale, Sarhosis and Milani, 2019c); and evaluating static deformations of kinematic conditions (Stockdale, Sarhosis and Milani, 2019b). What is missing in this analysis structure is dynamic modelling.

Existing dynamic analysis methods include both analytical and numerical approaches. The numerical approaches include non-linear finite element modelling (FEM), and the distinct (or discrete) element method (DEM). These approaches have been successful at modelling the dynamic conditions of masonry (Fanning *et al.*, 2005; De Lorenzis, DeJong and Ochsendorf, 2007; DeJong, 2009; Pelà, Aprile and Benedetti, 2009, 2013; Dimitri, De Lorenzis and Zavarise, 2011; DeJong and Dimitrakopoulos, 2014; Dimitri and Tornabene, 2015; Sarhosis, Santis and de Felice, 2016; Gaetani *et al.*, 2017), but they both require a high level of expertise and computational costs. The analytical methods are derived from the upper and lower bound theorems of LA. The lower bound theorem, derived from Hooke's hanging chain analogy and solidified by Heyman's safe theorem (Heyman, 1969), has been utilized for static horizontal testing (Huerta, 2005; DeJong, 2009) but its structure binds it to the stable state. The upper bound theorem of LA applies equivalent horizontal accelerations with an iterative approach to the principles of virtual work and virtual powers for static analyses and dynamic modelling respectively (Oppenheim, 1992; Gilbert and Melbourne, 1994; Clemente, 1998). Additionally, the upper bound has been validated numerically and experimentally for lateral loading (Ochsendorf, 2002; De Luca, Giordano and Mele, 2004; Alexakis and Makris, 2014; Dimitri and Tornabene, 2015; Stockdale, Sarhosis and Milani, 2019c).

The kinematic theorem is structured around the kinematic condition. In fact, the four-hinged arch is by definition a single degree of freedom (SDOF) system. Beginning with the four-hinged mechanism, Oppenheim (Oppenheim, 1992) was able to formulate the exact equations of motion for this condition and use them to study the overturning of an arch during the first half cycle of motion due to a step impulse. This model was expanded (De Lorenzis, DeJong and Ochsendorf, 2007) through the introduction of the assumptions of impact for single rocking blocks (Housner, 1963) to the four-hinged arch model. Applying the same step impulse (Oppenheim, 1992), the second boundary associated with the collapse of the second half cycle was identified as the governing condition (De Lorenzis, DeJong and Ochsendorf, 2007). This model was further expanded (Kollár and Ther, 2019) by removing the four-hinge limitation and evaluating the multi-degree of freedom motions that can exist in systems without hinge control. While the exact solution to motion exists for the SDOF arch structure, its application requires a high level of expertise and has focused on the assessment of the minimum condition.

The objective of this work is to develop a simplified time incremental analysis procedure for the dynamic propagation of a hinge-controlled masonry arche subjected to an overloading horizontal acceleration. First the kinematic equilibrium evaluation of mechanically deformed conditions is utilized to establish the time domain for an overloading acceleration through the development of required work-paths from parametric plotting. The time-incremental analysis procedure is then described in detail. Lastly, the half-cycle failure domain benchmark evaluation established by Oppenheim (Oppenheim, 1992) and a conservation of energy test are employed to validate the approach before concluding this work.

2. Establishing the Time Domain

Beginning with the kinematic equilibrium evaluation and SDOF deformations, this section develops the time-displacement relationship through the path independence of conservative work.

2.1 Kinematic Equilibrium and the KCLC

As stated in the Introduction, the KCLC is an analysis tool designed from and for hinge-controlled masonry arches (Stockdale *et al.*, 2018). It takes the user defined boundary and loading conditions, solves the equilibrium equation set, and checks the results for admissibility. In matrix format, the equilibrium equation set is

$$[BC]\{r\} = \{q\} \tag{1}$$

where ***BC*** is the balance matrix, ***r*** is the reaction vector, and ***q*** is the constants vector. The solution to the reactions and collapse multiplier are

$$\{r\} = [BC]^{-1}\{q\} \tag{2}$$

Figure 1 shows the kinematic equilibrium condition for a constant horizontal acceleration with collapse multiplier λ_a. The definition of rigid elements between hinges allows the force equivalence representation of the accelerations at the centroid of the elements. Taking the sum of the moments about hinges H_1, H_2 and H_3 for elements one through three respectively generates

$$[BC] = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & f_{g1} \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Delta y_{2,1} & \Delta x_{1,2} & 0 & 0 & 0 & 0 & -f_{g1}\Delta y_{CM1,1} \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & f_{g2} \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta y_{3,2} & \Delta x_{2,3} & 0 & 0 & f_{g2}\Delta y_{2,CM2} \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & f_{g3} \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Delta y_{3,4} & -\Delta x_{3,4} & f_{g3}\Delta y_{3,CM3} \end{bmatrix} \quad (3)$$

and

$$\{q\} = [0 \quad f_{g1} \quad -f_{g1}\Delta x_{1,CM1} \quad 0 \quad f_{g2} \quad f_{g2}\Delta x_{2,CM2} \quad 0 \quad f_{g3} \quad -f_{g3}\Delta x_{3,CM3}]^T \quad (4)$$

for the reaction vector

$$\{r\} = [h_1 \quad v_1 \quad h_2 \quad v_2 \quad h_3 \quad v_3 \quad h_4 \quad v_4 \quad \lambda_a]^T \quad (5)$$

For Eqns. 3 and 4, f_{gj} is the gravitational force of the j^{th} element, and the subscripts of the horizontal lever arms, Δx, and the vertical lever arms, Δy, denote the hinges or center of mass locations used (i.e. $\Delta y_{2,1}$ is $(y_2 - y_1)$ and $\Delta x_{1,CM1}$ is $(x_1 - x_{CM1})$, ect.). In Eqn. 5, v_i and h_i are the vertical and horizontal and reactions at the i^{th} hinge respectively. For details on the conditions of admissibility please refer to existing literature (Stockdale *et al.*, 2018; Stockdale, Sarhosis and Milani, 2019c).

2.2 SDOF Deformation

The structure of the equation sets reduces the required information necessary to solve the system to the location of the hinges and the centroid of the elements. This allows the remaining boundary conditions and the rules of motion to be evaluated independent of the equilibrium condition.

Starting from the definition of rigid kinematic motion of pin-connected elements allows the arch-hinge configuration to be represented by three fixed lengths connected by four pins as seen in Figure 2. The motion of the system is bound horizontally and thus allows the rotation at H_4 to be expressed as

$$\alpha_4 = \cos^{-1}\left(\frac{l_{12}}{l_{34}}[\cos(\theta_{12} + \alpha_1) - \cos(\theta_{12})] + \cos(\theta_{43})\right) - \theta_{43} \quad (6)$$

for a given α_1 at H_1 (see Figure 2 for identifying lengths and angles). From this rotation pair the polar change, γ_{23}, of length l_{23} is

$$\gamma_{23} = \theta'_{23} - \theta_{23} \tag{7}$$

and the rotations of the intermittent hinges H_2 and H_3 become

$$\alpha_2 = \alpha_1 + \gamma_{23} \tag{8}$$

and

$$\alpha_3 = \alpha_4 + \gamma_{23} \tag{9}$$

respectively.

2.3 Kinematic Equilibrium of Static Deformations

Figure 3 shows a custom and simplified KCLC constructed for the evaluation of static deformations. Equations 6 through 9 were incorporated into the KCLC and the hinge motion panel was established for the SDOF deformations. Also note that the centroid position information of the whole arch is displayed with both the center of mass (CM) and the center of area (CA) provided. This is to account for non-uniform block masses. Each block is assumed to have a uniform density, but that assumption is not held for the full arch.

User defined deformations are imposed through the hinge motion slider which defines the α_1 rotation and imposes the deformation. When the slider is adjusted, the rigid body rotations are applied about the points H_1 and H_4 for the block elements associated with the respective lengths l_{12} and l_{34}. Then the hinge point H_2 translation is applied to the blocks associated with l_{23} and followed by the calculated rotation α_2. After the deformations are applied, the boundary points and block centroids are updated, and the equilibrium calculation and admissibility check are performed. Figure 4 shows the KCLC and arch-hinge condition of Figure 3 with imposed α_1 rotations of 4°, 8° and 12°. From the deformation sequence an admissible kinematic equilibrium condition through 11° of rotation at H_1 is observed. Also note the capacity reduction of the collapse multiplier and the deformation path of the centroid of the full arch.

2.4 Equivalent Systems

By recording the centroid position of the full arch for imposed α_1 rotations, the position (*x,y*) and increment (*dx,dy*) link is established between the element deformations and a single point system. Imposing the total mass to the centroid point and defining energy conservation generates equivalent systems bound by α_1. Figure 5 shows the centroid deformation path for the same arch-hinge configuration as Figures 2 and 3 for α_1 between 0° and 12° with constant $\Delta\alpha_1$ of 0.1°. A polynomial fit of the deformation path reveals that the path is reasonably represented by

$$y = A_1x^2 + A_2x + A_3 \quad (10)$$

and the slope equation by

$$\frac{dy}{dx} = \frac{1}{2}A_1x + A_2 \quad (11)$$

where the constants A_i are shown in Figure 5.

The deformation of the arch involves both translations and rotations of the elements. In order to account for the element rotations, zero mass lever arms are defined between the centroid of the full arch and the centroid of each element. Since conservation of mass holds for the system, these lever arm lengths are fixed and result in rotational changes about the centroid of the arch. Figure 5 also shows the lever arm rotation angles versus horizontal CM displacement for each element of the arch-hinge configuration under the same deformation sequence as the CM translation. A polynomial fit of the lever arm rotation paths reveals that they are reasonably represented by

$$\theta = B_1x^2 + B_2x + B_3 \quad (12)$$

and the slope equation by

$$\frac{d\theta}{dx} = \frac{1}{2}B_1x + B_2 \quad (13)$$

2.5 Work Path and Potential Energy

Work is path independent for conservative systems. Therefore, the required work to deform the arch can be represented by the work required for the equivalent centroid deformations and lever arm rotations bound by α_1 and $\Delta\alpha_1$. Converting the collapse multiplier into an equivalent force applied at the centroid generates the force-displacement plot shown in Figure 6. Converting the collapse multiplier into equivalent forces at each element's centroid allows the torque-rotation plots to be established as well (see Figure 6).

Work can be expressed as the sum of translational and rotational components

$$W = \int F dx + \int \tau_{E1} d\theta_{E1} + \int \tau_{E2} d\theta_{E2} + \int \tau_{E3} d\theta_{E3} \quad (14)$$

and thus integrating the force-displacement plot and torque rotation plots generates the work path (see Figure 7). Also shown in Figure 7 is the potential energy, *PE*, curve established from

$$\Delta PE = m_T g \Delta y \quad (15)$$

Note that the work required to carry the arch to collapse is greater than the change in potential energy. The reason for this difference is that the formation of the mechanism requires a deformation of the internal thrust prior to and during the progression towards collapse, and it also requires the element rotations. Applying a polynomial fit evaluation to the plotted work path reveals that it is reasonably represented by

$$W_{min}(x) = C_1 x^4 + C_2 x^3 + C_3 x^2 + C_4 x + C_5 \quad (16)$$

where the constants C_i are shown in Figure 7.

2.6 Kinetic Energy

The work path shown in Figure 7 represents the work required to maintain kinematic equilibrium along the path to collapse. If an applied acceleration force, F_{app}, exceeds the limit, F_{min}, established from the collapse multiplier, then the system transitions from stable to mechanical. Assigning rigid elements and ideal hinges therefore requires that the excess energy added to the system be in the form of work, W_{app}, and any of this applied work in excess of the required minimum must be transformed into kinetic energy

$$\Delta KE = W_{app} - W_{req} \quad (17)$$

The required work, W_{req}, to travel from the initial position x_1 to a final position x is

$$W_{req} = W_{min}(x) - W_{min}(x_1) \quad (18)$$

Combining Eqns. 14, 17 and 18 with the constant horizontal acceleration condition produces a final kinetic energy

$$KE_f(x) = KE_1 + F_{app}(x - x_1) - W_{min}(x) + W_{min}(x_1) \quad (19)$$

where KE_1 if the initial kinetic energy.

2.7 Time Domain

Equation 19 establishes a displacement-domain equation of kinetic energy. Kinetic energy can also be expressed as

$$KE = \frac{1}{2} m_T v^2 + \frac{1}{2} I_{E1}\omega_{E1}^2 + \frac{1}{2} I_{E2}\omega_{E2}^2 + \frac{1}{2} I_{E3}\omega_{E3}^2 \quad (20)$$

where $\boldsymbol{v}$ is the velocity vector, and I_{Ej} and ω_{Ej} are the moment of inertia and lever arm angular velocity for the j^{th} element respectively. The velocity vector can be expressed as

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \frac{d\boldsymbol{x}}{dt} + \frac{d\boldsymbol{y}}{dt} \quad (21)$$

and the angular velocities as

$$\omega = \frac{d\theta}{dt} \quad (22)$$

Utilizing Eqns. 11, 13, and 19 through 22 generates

$$KE_f(x) = \frac{1}{2}\left[m_T\left(1 + \frac{1}{2}A_1 x + A_2\right)^2 + \sum_{i=1}^{3} m_{Ei} l_{Ei}^2 \left(\frac{1}{2}B_{1,Ei}x + B_{2,Ei}\right)^2\right]\left(\frac{dx}{dt}\right)^2 \quad (23)$$

where m_{Ei} and l_{Ei} are the i^{th} elements mass and lever arm and the constants $B_{1,Ei}$ and $B_{2,Ei}$ are obtained from Eqn. 13 and Figure 5. Since the developed kinetic energy equation (Eqn. 19) is only dependent on position, the relationship between time and displacement is established by the integral

$$t - t_0 = \int H(x)\,dx \quad (24)$$

where

$$H(x) = \sqrt{\frac{m_T\left(1+\frac{1}{2}A_1x+A_2\right)^2 + \sum_{i=1}^{3} m_{Ei} l_{Ei}^2\left(\frac{1}{2}B_{1,Ei}x+B_{2,Ei}\right)^2}{2KE_f(x)}} \quad (25)$$

Figure 8 shows a plot of H(x) and the area representation of the numeric evaluation of Eqn. 24 with an applied acceleration of $1.14\lambda_a$. Note that the initial time and kinetic energy are both set a zero. Figure 8 also shows the solution to Eqn. 24 which directly defines the relationship between time and horizontal position. Applying a polynomial fit evaluation to the curve reveals

$$x(t) = D_1t^4 + D_2t^3 + D_3t^2 + D_4t + D_5 \quad (26)$$

where the values of constants D_i are shown in the Figure 8. Therefore, given the initial position, kinetic energy and the acceleration's magnitude the displacement can be described as a function of time. Once the displacement is known, the final energies can be obtained.

3. Dynamic Analysis Procedure

Consider an undeformed arch at rest and subjected to a horizontal acceleration that exceeds the stable limit at time t_1. At time t_2 the acceleration magnitude changes. Applying Δt to Eqn. 26 establishes the displacement x_2 at t_2. Then the kinetic energy at time t_2 is obtained from x_2 and Eqn. 23. The displaced position and kinetic energy at time t_2 becomes the initial conditions for the next acceleration value, and thus the arch can be dynamically propagated forward in time.

If at time t_2 the displacement of the arch does not exceed the admissible limit defined by kinematic equilibrium, then the arch has not collapsed, but it is in a kinematic state. In this state the effects of the second acceleration value depend on the equilibrium limit of the deformed condition. Either the acceleration vector exceeds the limit and additional kinetic energy accumulates, or in the accumulated energy will be spent to propagate the arch.

For continued acceleration changes and time steps, the arch will propagate along the deformation path until collapse or zero kinetic energy is reached. If zero kinetic energy is reached the motion will switch directions. This motion reversal results in a negative final kinetic energy for the next time step if the acceleration remains below the equilibrium limit. Equation 25 requires positive kinetic energy in order to establish the time domain and thus motion must be forward facing.

The reversed motion from insufficient acceleration to cause collapse drives the arch back to the undeformed condition. Upon reaching that condition, the elements will experience an impact at the mechanical joints and the hinges will switch joint limits. The impact will result in a dissipation of energy over a finite period of time. The standard parameter to define energy loss during impact is the Coefficient of Restitution (COR), and it is typically determined through one of three models: kinematic, kinetic and energetic (Ahmad, Ismail and Mat, 2016). The kinematic model developed by Newton

$$COR = \frac{KE_f}{KE_i} \tag{26}$$

is the model considered. The time-incremental analysis structure means the impact can be isolated by timesteps and the COR can be applied to the calculation of kinetic energy during transition between the two hinge sets that define motion.

The hinge position switch reverses the mechanism. The equilibrium limit and deformation path are switched to the new mechanism. The reduced kinetic energy and new constant acceleration are set, and the evaluation continues.

Figure 9 shows the flowchart representation of the dynamic analysis procedure developed through combining the described dynamic conditions. For each time step in the defined acceleration sequence the equilibrium limit is established and used to evaluate the work condition. That condition with the previous kinetic energy and position generate a final position and kinetic energy. If kinetic energy reverses, the motion switches. If the arch returns to the original configuration then the COR is applied, and the hinge set is switched. This process is repeated until the end of the acceleration sequence or collapse.

4. Validations of the Work-Path Approach

The developed dynamic analysis procedure was constructed from the principles of energy conservation, the establishment of equivalent systems, and the path independence of conservative work. The equivalent systems were directly defined through the fixed rotations of hinge H_1 and utilized to establish deformation and work paths. The final step is the validations for the analysis structure through Oppenheim's half-cycle collapse line benchmark and the conservation of energy (Oppenheim, 1992).

The Oppenheim arch geometry and the reverse hinge set shown in Figure 10 (Oppenheim, 1992). Note that the hinges switch joint limits, but the mechanical joints are fixed. Examining the deformation sequence of the two configurations establishes the dynamic model for the arch.

4.1 Half Cycle Collapse

In order to evaluate the half-cycle collapse, the system is defined as perfectly plastic upon impact (i.e. COR = 0), and the Oppenheim two-step pulse is applied (Oppenheim, 1992). For each acceleration amplitude, the pulse time was continually increased by 0.02 seconds until a collapse. Figure 11 shows the resulting half-cycle failure domain developed from the described work-path approach as well as Oppenheim's original results. From Figure 11 it can be seen that the increase in static capacity from the upper bound limit is coupled with a small decrease in the recoverable impulse duration. Nonetheless, the behaviour of the arche's half-cycle failure is captured by the work-path approach.

4.2 Conservation of Energy

For the conservation of energy check, the system is defined as perfectly elastic (i.e. COR = 1) and a horizontal acceleration pulse with magnitude of 0.55g is applied for a duration of 0.5 seconds. After the application of the pulse, the analysis continues for 20 seconds at 0.02 second intervals. The horizontal CM displacement and kinetic energy are recorded at each time increment (see Figure 12). A total of 14 full cycles were observed over the 20 seconds with a net energy loss of approximately 0.4 % per cycle. It is postulated that this energy loss occurs from the calculation of the pivot points associated with zero kinetic energy. The establishment of the time domain requires positive kinetic energy and thus reduces the number of available data points for curve fitting in the vicinity of the motion switch. Regardless, the energy has been reasonably conserved under the ideal conditions of elastic impact and perfect hinges. Also note that the hinge configurations are not symmetric. This establishes different work paths and thus different amplitudes and frequencies for the two half-cycles of motion (see Figure 12).

5. Conclusions

Efficiency is of the upmost importance for the success of any structural system and is paramount for the introduction of new structural systems that deviate from modern standards. The argument has been made that the masonry arch has the potential to be an advantageous structural system for modern structural design and construction, but an accessible and efficient analysis platform must be established (Stockdale, 2016). The kinematic equilibrium approach to LA has shown great potential in establishing this efficient and accessible analysis structure for hinge-controlled arches. It has been used to develop a simple and adaptable static analysis software (Stockdale *et al.*, 2018; Stockdale, Sarhosis and Milani, 2019c), allows the incorporation of generic arch geometries (Stockdale and Milani, 2018), analyse the capacity of deformed conditions (Stockdale, Sarhosis and Milani, 2019a), and has been used to formulate first-order assessment strategies (Stockdale and Milani, 2019). Therefore, the final component in the development of this comprehensive analysis structure is the inclusion of the dynamic behaviour.

The focus of this work was to develop and validate this dynamic analysis component for hinge-controlled masonry arches. The developed structure is constructed from the same kinematic equilibrium approach to LA as used for the static conditions. This was achieved through the direct evaluation of work. Utilizing ideal conditions, the work path and ultimately the time domain were established for applied horizontal accelerations and used to formulate the dynamic time incremental analysis structure based upon the assumption of constant acceleration for each time step.

With the inclusion of the dynamic condition, the foundation for a complete and comprehensive analysis structure is established for hinge-controlled masonry arches and brings the utilization of structural masonry one step closer to reality. Now the focus must turn to experimental testing, the incorporation of non-ideal conditions.

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Figure captions

Figure 1. Kinematic equilibrium condition for horizontal acceleration condition.

Figure 2. Rigid pin-connected length equivalent of the four-hinged arch mechanism in the (a) original state and (b) after a deformation.

Figure 3. Custom KCLC with added hinge motion panel and centroid data display.

Figure 4. (a) 4°, (b) 8° and (c) 12° α_1 rotations applied to the arch-hinge condition.

Figure 5. Parametric plots and polynomial fitting of (a) the CM deformation path and (b) the lever arm rotation angles.

Figure 6. Parametric plots and polynomial fits for (a) the required force versus horizontal CM displacement and (b) the required torques versus lever arm rotation angle.

Figure 7. Parametric plot of minimum work and potential energy versus horizontal CM displacement with a polynomial fit.

Figure 8. Plots of (a) H(x) versus horizontal CM displacement with highlighted integration area, and (b) horizontal position versus time with a polynomial fit.

Figure 9. Flowchart of the dynamic analysis procedure.

Figure 10. Oppenheim arch geometry with the (a) original hinge configuration and (b) the hinge reversal from defined joints.

Figure 11. Half-cycle failure domain comparison for the two-step pulse analysis of the Oppenheim arch (Oppenheim, 1992).

Figure 12. Horizontal CM displacement and kinetic energy versus time for applied acceleration pulse.