



Deposited via The University of Sheffield.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/id/eprint/188228/>

Version: Accepted Version

Article:

Muzikar, P. and Heaton, T.J. (2022) Wiggle matching with correlations. *Radiocarbon*, 64 (1). pp. 187-194. ISSN: 0033-8222

<https://doi.org/10.1017/rdc.2021.106>

This article has been published in a revised form in *Radiocarbon* <https://doi.org/10.1017/RDC.2021.106>. This version is free to view and download for private research and study only. Not for re-distribution, re-sale or use in derivative works. © The Author(s), 2022. Published by Cambridge University Press for the Arizona Board of Regents on behalf of the University of Arizona

Reuse

This article is distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs (CC BY-NC-ND) licence. This licence only allows you to download this work and share it with others as long as you credit the authors, but you can't change the article in any way or use it commercially. More information and the full terms of the licence here: <https://creativecommons.org/licenses/>

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.



Wiggle matching with correlations

Paul Muzikar

Department of Physics and Astronomy, Purdue University
West Lafayette, IN 47907

and

Timothy J. Heaton

School of Mathematics and Statistics, University of Sheffield
Sheffield S3 7RH, UK

August 10, 2021

Abstract

Wiggle matching is an important and powerful technique in radiocarbon dating that can be used to improve the precision of calendar age estimates. All radiocarbon determinations require calibration to provide calendar age estimates. This calibration is achieved by comparing the determinations against a calibration curve $\mu(\cdot)$ to calculate the probability the sample arises from any particular calendar age t . Wiggle matching involves the calibration of a set of radiocarbon determinations taken from samples with known separations between their calendar ages. Since the calendar age separations between samples are known, all the calendar ages are known functions of one particular age, T_1 — commonly the most recent calendar age. Dating the sequence then reduces to considering $p(T_1 = t_1 | data)$, the probability of the calendar age t_1 given the set of radiocarbon determinations. In previous work, a Bayesian approach has been used to derive a nice formula for this quantity under the assumption we have independent pointwise estimates of the calibration curve $\mu(t)$. In this paper, we derive a generalization of this formula showing how to incorporate covariance information from the calibration curve under an assumption of multivariate normality.

20 Introduction

21 Wiggle matching is a powerful technique used in radiocarbon dating to improve the precision
22 with which one can estimate calendar ages of samples (Pearson, 1986). The classic usage of
23 the technique is when seeking to estimate the calendar ages of a series of tree-ring samples
24 for which, due to ring counting, the number of calendar years separating the samples is
25 known precisely. By calibrating multiple determinations jointly within a wiggle match we
26 can improve the precision in our absolute calendar age estimates compared with the estimate
27 we would obtain with just a single determination.

28 Suppose we are dating such a tree-ring sequence consisting of N determinations from
29 consecutive annual rings, such that we know that the true, calendar ages must be $T_1, T_2 =$
30 $T_1 + 1, T_3 = T_1 + 2, \dots, T_N = T_1 + N - 1$. Given T_1 , all the other ages are known and so this
31 is the only unknown we need to estimate. We obtain radiocarbon data for each tree ring,
32 and then try to match the ups and downs in the data to ups and downs in the calibration
33 curve, knowing that we have N consecutive years.

34 Bayesian theory has been applied to make this method quantitative, (see for example,
35 Christen and Litton, 1995; Bronk Ramsey et al., 2001). The net result is an expression, in
36 terms of the data, for $p(T_1 = t_1|data)$, the probability for the unknown age T_1 given the
37 set of N radiocarbon determinations. This expression is also dependent upon the value of
38 the calibration curve $\mu(t_1), \dots, \mu(t_1 + N - 1)$. The formulae given in these papers assume
39 pointwise estimates of the calibration curve which are independent from one another and
40 ignore any potential covariance information between the estimates at adjacent times.

41 The importance of such covariance structure in the calibration curve has long been
42 understood, see for example Blackwell and Buck (2008) and Millard (2008). Millard (2008)
43 discusses wiggle matching and covariance, and provides results of several wiggle matching
44 calculations which incorporate the covariance information from the calibration curve.

45 Our goal in this brief note is to derive a generalization of the usual Bayesian wiggle-
46 match formulation for $p(T_1 = t_1|data)$, to show how this result is modified by the covariance
47 effects under the assumption that the calibration curve is modelled as a multivariate normal
48 distribution. Our final answer, a new expression for $p(T_1 = t_1|data)$, is given by equations
49 (8) - (10). It has the same general structure as the usual formula, and it explicitly shows
50 how the covariance information from the calibration curve enters into the result. The next
51 section contains our derivation. As much as possible, we use the approach and notation
52 already developed for this type of analysis — for examples, see Heaton et al. (2009, 2020),
53 Bronk Ramsey (2015), Niu et al. (2013), and Blackwell and Buck (2008).

54 Bayesian theory

55 We denote the true, unknown calendar ages of our N samples by $\{T_i\}_{i=1}^N$, and the observed
56 radiocarbon determinations (either radiocarbon ages or $^{14}\text{C}/^{12}\text{C}$ ratios) by $\mathbf{y} = (y_1, \dots, y_n)^T$.
57 We denote the unknown true value of the calibration curve (either the true radiocarbon age
58 or $^{14}\text{C}/^{12}\text{C}$ ratio for calendar age t cal BP) in any calendar year by $\mu(t)$. Thus, we have

$$p(y_i|T_i = t_i, \mu(t_i)) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(y_i - \mu(t_i))^2}{2\sigma_i^2}\right), \quad (1)$$

59 where σ_i is the uncertainty in the radiocarbon measurement. We do not know the true value
60 of the calibration curve (i.e., the true value of radiocarbon age or $^{14}\text{C}/^{12}\text{C}$ ratio corresponding
61 to calendar age t cal BP) but we do have pointwise estimates $\hat{\mu}(t)$ in any given year, so that
62 we have

$$\mu(t) = \hat{\mu}(t) + \epsilon(t). \quad (2)$$

63 The $\epsilon(t)$ denotes our uncertainty in the calibration curve at time t . Typically there will be
64 covariance in these values — $\epsilon(t)$ will not be independent of $\epsilon(t+1)$. If we assume that these
65 uncertainties have a multivariate Gaussian form and consider any sequence of N consecutive
66 calendar years $\mathbf{t} = (t, t+1, \dots, t+N-1)^T$:

$$p(\boldsymbol{\epsilon}_t) = (2\pi)^{-N/2} \det(\boldsymbol{\Sigma}_t)^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \boldsymbol{\epsilon}_t^T \boldsymbol{\Sigma}_t^{-1} \boldsymbol{\epsilon}_t\right), \quad (3)$$

67 where $\boldsymbol{\epsilon}_t = (\epsilon(t), \dots, \epsilon(t+N-1))^T$ and $\boldsymbol{\Sigma}_t$ is the covariance matrix which encodes the
68 correlations in the $\boldsymbol{\epsilon}_t$ uncertainties, and in general will depend on t . According to equation
69 (2), for any vector of calendar ages \mathbf{t} , we have $\boldsymbol{\mu}_t = \hat{\boldsymbol{\mu}}_t + \boldsymbol{\epsilon}_t$. Here $\boldsymbol{\mu}_t$ denotes the vector of
70 true radiocarbon ages or $^{14}\text{C}/^{12}\text{C}$ ratios (to correspond to choice of the y_i 's) for the sequence
71 of calendar age \mathbf{t} , and $\hat{\boldsymbol{\mu}}_t$ the corresponding pointwise estimates.

72 We are now ready to begin our derivation. We first note that the quantity we want is
73 $f(t_1) = p(T_1 = t_1 | \text{data}) = p(T_1 = t_1 | \mathbf{y})$. To calculate this we use Bayes' Theorem:

$$f(t_1) = p(T_1 = t_1 | \text{data}) = p(T_1 = t_1 | \mathbf{y}) = a p(\mathbf{y} | t_1) \pi_0(t_1) \quad (4)$$

Here, a is a normalization constant, and $\pi_0(t_1)$ is the prior probability for T_1 . We expand
 $p(\mathbf{y} | t_1)$ to obtain (to save writing, we denote $t_j = t_1 + j - 1$, the calendar age of the j^{th}
determination in the sequence to be wiggle-matched)

$$\begin{aligned} f(t_1) = a \int \dots \int & \left[\left\{ \prod_{j=1}^N \int p(y_j | t_1, \mu(t_j)) p(\mu(t_j) | t_1, \epsilon(t_j)) d\mu(t_j) \right\} \right. \\ & \left. \times p(\epsilon(t_1), \dots, \epsilon(t_N) | t_1) \pi_0(t_1) \right] d\epsilon(t_1) \dots d\epsilon(t_N), \end{aligned} \quad (5)$$

We note that $p(\mu(t_j) | t_1, \epsilon(t_j)) = \delta(\mu(t_j) - \hat{\mu}(t_j) - \epsilon(t_j))$, where $\delta()$ is the Dirac delta function.
Thus we have

$$\begin{aligned} f(t_1) = a \int \dots \int & \left[\left\{ \prod_{j=1}^N \int \exp\left(-\frac{(y_j - \mu(t_j))^2}{2\sigma_j^2}\right) \delta(\mu(t_j) - \hat{\mu}(t_j) - \epsilon(t_j)) d\mu(t_j) \right\} \right. \\ & \left. \times p(\epsilon(t_1), \dots, \epsilon(t_N) | t_1) \pi_0(t_1) \right] d\epsilon(t_1) \dots d\epsilon(t_N). \end{aligned} \quad (6)$$

Note that we keep absorbing into the normalization constant a those factors that do not
depend on t_1 . Doing the integral over $\mu(t_1), \dots, \mu(t_N)$ gives

$$\begin{aligned} f(t_1) = a \int \dots \int & \left[\exp\left(-\sum_{j=1}^N \frac{(y_j - \hat{\mu}(t_j) - \epsilon(t_j))^2}{2\sigma_j^2}\right) \right. \\ & \left. \times \det(\boldsymbol{\Sigma}_{t_1})^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \boldsymbol{\epsilon}_t^T \boldsymbol{\Sigma}_t^{-1} \boldsymbol{\epsilon}_t\right) \pi_0(t_1) \right] d\epsilon(t_1) \dots d\epsilon(t_N). \end{aligned} \quad (7)$$

74 Finally, we do the integral over the $\epsilon(t_1), \dots, \epsilon(t_N)$ to get

$$f(t_1) = a \sqrt{\frac{\det(\boldsymbol{\Sigma}_{t_1}^{-1})}{\det(\mathbf{T}_{t_1})}} \exp\left(-\frac{1}{2}(\hat{\boldsymbol{\mu}}_{t_1} - \mathbf{y})^T \mathbf{W}_{t_1} (\hat{\boldsymbol{\mu}}_{t_1} - \mathbf{y})\right) \pi_0(t_1). \quad (8)$$

75 This is our final answer; it is similar to usual Bayesian result in being a multivariate Gaussian
 76 in $\hat{\boldsymbol{\mu}}_{t_1} - \mathbf{y}$. However, it now involves the $N \times N$ matrix \mathbf{W}_{t_1} , which in general has off-diagonal
 77 terms. This matrix is given by

$$[W_{t_1}]_{ij} = \frac{1}{\sigma_i^2} \delta_{ij} - \frac{1}{\sigma_i^2 \sigma_j^2} [T_{t_1}^{-1}]_{ij} \quad [T_{t_1}]_{ij} = [\Sigma_{t_1}^{-1}]_{ij} + \frac{1}{\sigma_i^2} \delta_{ij}. \quad (9)$$

78 In general these matrices, as well as the determinants of $\boldsymbol{\Sigma}_{t_1}^{-1}$ and \mathbf{T}_{t_1} , will depend on t_1 .

79 The matrix \mathbf{W}_{t_1} can be written in a more compact, elegant form. If we define a diagonal
 80 matrix $D_{ij} = \sigma_i^2 \delta_{ij}$, containing the variances of the radiocarbon determinations \mathbf{y} on the
 81 diagonal, then we may write

$$\mathbf{W}_{t_1} = (\mathbf{D} + \boldsymbol{\Sigma}_{t_1})^{-1}. \quad (10)$$

82 To derive this, we can go back to equation (4), and evaluate $p(\mathbf{y}|t_1)$ in the following way.
 83 We note that the N -dimensional random variable $\mathbf{y} - \hat{\boldsymbol{\mu}}_{t_1}$ can be viewed as the sum of two,
 84 independent, multivariate Gaussians, each with a mean of zero, and with covariance matrices
 85 given by $\boldsymbol{\Sigma}_{t_1}$ and by \mathbf{D} . It is well known that the probability density of this sum is itself a
 86 Gaussian, with a covariance matrix given by $\mathbf{D} + \boldsymbol{\Sigma}_{t_1}$. The equivalence of these two forms
 87 for \mathbf{W}_{t_1} , given in equations (9) and (10), is shown in the Appendix.

88 Special Cases

89 Certain special cases are instructive. First, suppose that there are no correlations among
 90 the $\epsilon(t_j)$. This implies that $\boldsymbol{\Sigma}$ is a diagonal matrix:

$$\Sigma_{ij} = s_j^2 \delta_{ij} \quad (11)$$

91 Thus, s_j^2 is the variance in the calibration curve at time t_j . It is then straightforward to
 92 show that the \mathbf{W} matrix becomes

$$W_{ij} = \frac{1}{\sigma_j^2 + s_j^2} \delta_{ij} \quad (12)$$

93 This is the usual Bayesian result (Christen and Litton, 1995; Bronk Ramsey et al., 2001),
 94 which we recover for this case; any differences in \mathbf{W} from this usual result thus are totally
 95 due to correlations among the $\epsilon(t_j)$.

96 Another special case to consider is if the radiocarbon determinations are very precise,
 97 so that all of our uncertainty comes from the calibration curve, Taking $\sigma_j \rightarrow 0$ then gives
 98 $\mathbf{W} = \boldsymbol{\Sigma}_t^{-1}$. The Gaussian in the wiggle matching formula (8) then directly reflects the
 99 covariance matrix from the calibration curve.

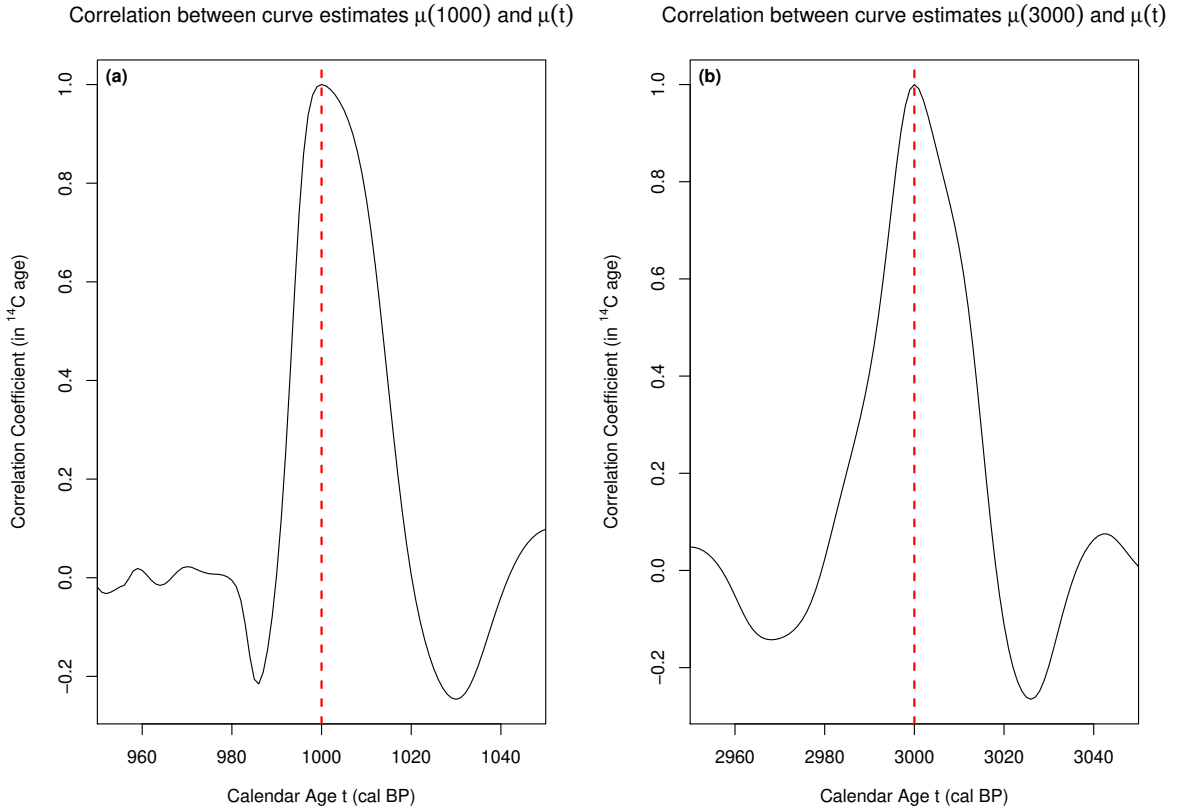


Figure 1: An illustration of the correlation in the IntCal20 calibration curve. Panel a) shows the correlation between the value of the calibration curve at 1000 cal BP, i.e. $\mu(1000)$, and $\mu(t)$ for $950 \leq t \leq 1050$ (50 calendar years either side). Panel b) the correlation between $\mu(3000)$ and $\mu(t)$, for $2950 \leq t \leq 3050$. The quantity plotted on the y-axis is given by equation (15).

100 Role of the covariance matrix Σ

101 It is perhaps not easy to understand the role played in the wiggle matching equation by
 102 the off-diagonal elements of the covariance matrix Σ_{ij} ; these would be $\text{Cov}(\epsilon(t_i), \epsilon(t_j))$ for
 103 $i \neq j$. To gain some insight, we can consider the following special case: suppose that the
 104 off-diagonal elements of Σ are much smaller than the diagonal elements. Thus we write

$$\Sigma_{ij} = s_i^2 \delta_{ij} + G_{ij} \quad (13)$$

105 where $G_{ij} = 0$ for $i \neq j$. If we now assume that the elements of G are small compared to
 106 the diagonal elements, we can derive the following result: to first order in G we have

$$W_{ij} \approx \frac{1}{\sigma_j^2 + s_j^2} \delta_{ij} - \frac{G_{ij}}{(\sigma_i^2 + s_i^2)(\sigma_j^2 + s_j^2)} \quad (14)$$

107 The effect of the diagonal first term is clear: when inserted into equation (8) it will assign
 108 higher probabilities to values of t_1 which make the magnitudes of the various $y_i - \mu(t_i)$ as
 109 small as possible. The second, off-diagonal term will favor values of t_1 which cause the
 110 various products $(y_i - \hat{\mu}(t_i))(y_j - \hat{\mu}(t_j))$ to have the same sign as G_{ij} . Thus, if, for example,
 111 G_{23} is positive, the off-diagonal term will favor values of t_1 which make $(y_2 - \hat{\mu}(t_2))(y_3 - \hat{\mu}(t_3))$
 112 positive.

113 To give some sense of how large the off-diagonal terms can be, we present examples of
 114 the correlations in the IntCal20 curve, and how they decay as the distance between calendar
 115 ages increases. In Figure 1 we plot the correlation between $\mu(t)$ and $\mu(t^*)$ for $t^* = 1000$,
 116 and for $t^* = 3000$ cal BP. In terms of our present notation, the correlation function $C(t, t^*)$
 117 plotted is given by

$$C(t, t^*) = \frac{\langle \epsilon(t)\epsilon(t^*) \rangle}{\sqrt{\langle \epsilon^2(t) \rangle \langle \epsilon^2(t^*) \rangle}} \quad (15)$$

118 where the brackets indicate the expectation over the random variables $\epsilon(t)$.

119 We see that the off-diagonal terms are comparable to the diagonal ones over about 10 -
 120 20 years in both cases; thus, over the range of a short wiggle-match (ca. 20 calendar years),
 121 correlations can be significant.

122 Discussion

123 We have discussed a generalization of the usual Bayesian approach to wiggle matching. The
 124 key ingredients in this formulation are the pointwise mean estimates of the calibration curve
 125 $\hat{\mu}(t)$, and the corresponding curve covariance which for any set of N consecutive years we
 126 assume can be encoded in an $N \times N$ covariance matrix Σ . As we have shown, information
 127 about these quantities can be provided by the constructors of the calibration curve; for
 128 relevant discussion of the newly presented calibration curves, see, for example Heaton et al.
 129 (2020), Reimer et al. (2020) and van der Plicht et al. (2020).

130 **The ultimate origins of the correlated uncertainties in the calibration curve (the off-**
 131 **diagonal elements of Σ_{ij}) should perhaps be discussed. When constructing the radiocarbon**
 132 **calibration curve, we assume that the underlying curve $\mu(t)$ one is trying to estimate is**

133 somewhat smooth, i.e., that the level of ^{14}C in the atmosphere in calendar year t is similar
134 to that in year $t + 1$. This is the basis of most statistical regression techniques and means
135 that information on the value of the curve in one calendar year also informs you about what
136 its value is likely to be in neighbouring years. Indeed, it is this assumption of smoothness
137 which allows you to borrow and combine information from multiple ^{14}C observations over a
138 neighbourhood to strengthen the curve estimate in any individual year and prevent overfitting
139 to the data. Without it, you could make no prediction about the value in a calendar year in
140 which you did not have a direct observation.

141 This smoothness creates a level of dependence in the resultant curve estimate that
142 provides more information than, and goes beyond, just the pointwise intervals for $\hat{\mu}(t)$. It is
143 unlikely that the true atmospheric ^{14}C levels will flip from the top of the curve’s probability
144 interval to the bottom in the space of a single year. Rather if the true value of the curve lies
145 towards the top (bottom) of the probability interval in year t , it is likely to also lie towards
146 the top (bottom) of the interval in nearby times t' . In our notation, $\epsilon(t)$ and $\epsilon(t')$ are likely
147 to have the same sign when t' is in the neighbourhood of t .

148 The level of smoothness, and hence dependence, in the calibration curve will be affected
149 by the nature of the reference ^{14}C data used to construct the calibration curve (to which we
150 aim to adapt) and, in the case of IntCal20, the number and placement of the knots in the
151 underlying spline. Over time spans with a lower density of knots, the resulting estimate for
152 the calibration curve will however be smoother, and this will tend to create longer lasting
153 curve covariance. There are also specific times, during solar proton (SPE or Miyake) type
154 events, where the smoothness of the IntCal curve is permitted to be reduced.

155 The diverse nature of the ^{14}C data used to construct the IntCal curve will also influence
156 the covariance in the final calibration curve estimate. In particular, in the older portion of
157 the calibration curve ($> 14,000$ cal yr BP) where the calendar ages and ^{14}C measurements
158 of the reference data themselves exhibit significant covariance. Here the calibration curve is
159 informed by ^{14}C measurements from floating tree-ring sequences (Adolphi et al., 2017; Turney
160 et al., 2010, 2016) for which the internal chronologies are known, but the absolute ages are
161 not — these are adaptively wiggle-matched, during curve construction, to fit with the rest
162 of the reference ^{14}C data. The calendar ages of the macrofossils from Lake Suigetsu (Bronk
163 Ramsey et al., 2020) also possess covariance, as do the foraminifera from ocean sediments
164 (Bard et al., 2013; Hughen and Heaton, 2020) which are linked to the Hulu Cave timescale
165 by the tying of global abrupt palaeoclimatic events (Heaton et al., 2013). The speleothem
166 ^{14}C measurements (e.g., Cheng et al., 2018; Southon et al., 2012) share a common dead
167 carbon fraction; and the marine based ^{14}C measurements (e.g., Bard et al., 2013; Hughen
168 and Heaton, 2020) also share a ^{14}C offset due to the modelling of the marine reservoir age
169 (Butzin et al., 2020). All these covariances within the constituent IntCal reference data will
170 affect, and tend to increase, the covariance on the final curve estimate $\hat{\mu}(t)$ in this time
171 period, see Heaton et al. (2020) for details.

172 Acknowledgements

173 This work was supported by National Science Foundation grant EAR 0112480 and a Leverhulme
174 Trust Fellowship RF-2019-140\9 .

175 Appendix

176 We want to prove that the two expressions (9) and (10) given for the matrix W are equivalent.
 177 The normalising constant obtained in (8) can also be shown to be equivalent to the approach
 178 using the result that the sum of two multivariate Gaussians is Gaussian, although we do not
 179 show this here. In this Appendix, all the symbols represent $N \times N$ matrices, so we omit the
 180 bold face notation. The two forms are:

$$W_A = D^{-1} - D^{-1}(\Sigma^{-1} + D^{-1})^{-1}D^{-1} \quad W_B = (D + \Sigma)^{-1} \quad (16)$$

181 where D and Σ are defined in the main text. W_A is the form given by equation (9), while
 182 W_B is the form given by equation (10). It is sufficient to show that $W_B^{-1}W_A = I$, where I is
 183 the identity matrix. One useful relation is that for two matrices, $(HY)^{-1} = Y^{-1}H^{-1}$. Thus

$$D^{-1} + \Sigma^{-1} = D^{-1}(I + D\Sigma^{-1}) \quad (D^{-1} + \Sigma^{-1})^{-1} = (I + D\Sigma^{-1})^{-1}D \quad (17)$$

184 So we evaluate

$$W_B^{-1}W_A = (D + \Sigma)(D^{-1} - D^{-1}(\Sigma^{-1} + D^{-1})^{-1}D^{-1}) \quad (18)$$

185 or

$$W_B^{-1}W_A = I + \Sigma D^{-1} - (I + D\Sigma^{-1})^{-1} - \Sigma D^{-1}(I + D\Sigma^{-1})^{-1} \quad (19)$$

186 or

$$W_B^{-1}W_A = I + K - (I + K)(I + K^{-1})^{-1} \quad (20)$$

187 where $K = \Sigma D^{-1}$. Note that

$$I + K^{-1} = K^{-1}(I + K) \quad (I + K^{-1})^{-1} = (I + K)^{-1}K \quad (21)$$

188 Finally we have

$$W_B^{-1}W_A = I + K - (I + K)(I + K)^{-1}K = I \quad (22)$$

189 References

- 190 Adolphi, F., Muscheler, R., Friedrich, M., Güttler, D., Wacker, L., Talamo, S., and Kromer,
 191 B. (2017). Radiocarbon calibration uncertainties during the last deglaciation: Insights
 192 from new floating tree-ring chronologies. *Quaternary Science Reviews*, 170:98–108.
- 193 Bard, E., Ménot, G., Rostek, F., Licari, L., Böning, P., Edwards, R., Cheng, H., Wang,
 194 Y., and Heaton, T. (2013). Radiocarbon calibration/comparison records based on marine
 195 sediments from the Pakistan and Iberian margins. *Radiocarbon*, 55(4).
- 196 Blackwell, P. G. and Buck, C. E. (2008). Estimating radiocarbon calibration curves. *Bayesian*
 197 *Analysis*, 3(2):225 – 248.
- 198 Bronk Ramsey, C. (2015). *Mathematics and Archaeology*, chapter Bayesian approaches to
 199 the building of Archaeological Chronologies, pages 272–292. Taylor and Francis.

- 200 Bronk Ramsey, C., Heaton, T. J., Schlolaut, G., Staff, R. A., Bryant, C. L., Brauer, A.,
 201 Lamb, H. F., Marshall, M. H., and Nakagawa, T. (2020). Reanalysis of the Atmospheric
 202 Radiocarbon Calibration Record from Lake Suigetsu, Japan. *Radiocarbon*, 62(4):989–999.
- 203 Bronk Ramsey, C., van der Plicht, J., and Weninger, B. (2001). ‘Wiggle Matching’
 204 Radiocarbon Dates. *Radiocarbon*, 43(2A):381–389.
- 205 Butzin, M., Heaton, T. J., Köhler, P., and Lohmann, G. (2020). A Short Note on Marine
 206 Reservoir Age Simulations Used in IntCal20. *Radiocarbon*, 62(4):865–871.
- 207 Cheng, H., Lawrence Edwards, R., Southon, J., Matsumoto, K., Feinberg, J. M., Sinha, A.,
 208 Zhou, W., Li, H., Li, X., Xu, Y., Chen, S., Tan, M., Wang, Q., Wang, Y., and Ning,
 209 Y. (2018). Atmospheric $^{14}\text{C}/^{12}\text{C}$ changes during the last glacial period from Hulu Cave.
 210 *Science*, 362(6420):1293–1297.
- 211 Christen, J. and Litton, C. (1995). A Bayesian approach to wiggle-matching. *Journal of*
 212 *Archaeological Science*, 22(6):719–725.
- 213 Heaton, T., Bard, E., and Hughen, K. (2013). Elastic tie-pointing-transferring chronologies
 214 between records via a Gaussian process. *Radiocarbon*, 55(4).
- 215 Heaton, T. J., Blaauw, M., Blackwell, P. G., Bronk Ramsey, C., Reimer, P. J., and Scott,
 216 E. M. (2020). The IntCal20 Approach to Radiocarbon Calibration Curve Construction:
 217 A New Methodology Using Bayesian Splines and Errors-in-Variables. *Radiocarbon*,
 218 62(4):821—863.
- 219 Heaton, T. J., Blackwell, P. G., and Buck, C. E. (2009). A Bayesian Approach to the
 220 Estimation of Radiocarbon Calibration Curves: The IntCal09 Methodology. *Radiocarbon*,
 221 51(4):1151–1164.
- 222 Hughen, K. and Heaton, T. (2020). Updated Cariaco Basin ^{14}C Calibration Dataset from
 223 0-60 cal kyr BP. *Radiocarbon*, 62(4).
- 224 Millard, A. R. (2008). Comment on “Estimating radiocarbon calibration curves”. *Bayesian*
 225 *Analysis*, 3(2):255–262.
- 226 Niu, M., Heaton, T. J., Blackwell, P. G., and Buck, C. E. (2013). The Bayesian Approach
 227 to Radiocarbon Calibration Curve Estimation: The IntCal13, Marine13, and SHCal13
 228 Methodologies. *Radiocarbon*, 55(4):1905–1922.
- 229 Pearson, G. W. (1986). Precise Calendrical Dating of Known Growth-Period Samples Using
 230 a ‘Curve Fitting’ Technique. *Radiocarbon*, 28(2A):292–299.
- 231 Reimer, P. J., Austin, W. E. N., Bard, E., Bayliss, A., Blackwell, P. G., Bronk Ramsey, C.,
 232 Butzin, M., Cheng, H., Edwards, R. L., Friedrich, M., Grootes, P. M., Guilderson, T. P.,
 233 Hajdas, I., Heaton, T. J., Hogg, A. G., Hughen, K. A., Kromer, B., Manning, S. W.,
 234 Muscheler, R., Palmer, J. G., Pearson, C., van der Plicht, H., Reimer, R. W., Richards,
 235 D. A., Scott, E. M., Southon, J. R., Turney, C. S. M., Wacker, L., Adolphi, F., Büntgen,
 236 U., Capano, M., Fahrni, S., Fogtmann-Schulz, A., Friedrich, R., Köhler, P., Kudsk, S.,

- 237 Miyake, F., Olsen, J., Reinig, F., Sakamoto, M., Sookdeo, A., and Talamo, S. (2020).
238 The IntCal20 Northern Hemisphere radiocarbon age calibration curve (0–55 cal kBP).
239 *Radiocarbon*, 62(4):725–757.
- 240 Southon, J., Noronha, A. L., Cheng, H., Edwards, R. L., and Wang, Y. (2012). A high-
241 resolution record of atmospheric ^{14}C based on Hulu Cave speleothem H82. *Quaternary*
242 *Science Reviews*, 33:32–41.
- 243 Turney, C. S. M., Fifield, L. K., Hogg, A. G., Palmer, J. G., Hughen, K., Baillie, M. G. L.,
244 Galbraith, R., Ogden, J., Lorrey, A., Tims, S. G., and Jones, R. T. (2010). The potential
245 of New Zealand kauri (*Agathis australis*) for testing the synchronicity of abrupt climate
246 change during the Last Glacial Interval (60,000–11,700 years ago). *Quaternary Science*
247 *Reviews*, 29(27):3677–3682.
- 248 Turney, C. S. M., Palmer, J., Bronk Ramsey, C., Adolphi, F., Muscheler, R., Hughen, K. A.,
249 Staff, R. A., Jones, R. T., Thomas, Z. A., Fogwill, C. J., and Hogg, A. (2016). High-
250 precision dating and correlation of ice, marine and terrestrial sequences spanning Heinrich
251 Event 3: Testing mechanisms of interhemispheric change using New Zealand ancient kauri
252 (*Agathis australis*). *Quaternary Science Reviews*, 137:126–134.
- 253 van der Plicht, J., Bronk Ramsey, C., Heaton, T. J., Scott, E. M., and Talamo, S. (2020).
254 Recent Developments In Calibration For Archaeological And Environmental Samples.
255 *Radiocarbon*, 62(4):1095–1117.