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An Approach to Formally Specifying the Behaviour of Mixed-Criticality Systems

A. Burns ✉

University of York, York, UK

Cliff B. Jones ✉

Newcastle University, Newcastle upon Tyne, UK

Abstract

This paper proposes a formal framework for describing the relationship between a criticality-aware scheduler and a set of application tasks that are assigned different criticality levels. The exposition employs a series of examples starting with scheduling simple jobs and then moving on to mixed-criticality robust and resilient tasks. The proposed formalism extends the rely-guarantee approach, which facilitates formal reasoning about the functional behaviour of concurrent systems, to address *real-time* properties.

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1 Introduction

Since Vestal published his seminal paper in 2007 [61], there have been a wealth of models and protocols published [16, 17] on the topic of Mixed Criticality Systems (MCS). One of the aims of this wide ranging set of techniques is to improve the survivability of systems by providing a variety of degraded behaviours that can take effect if the system experiences overrunning execution times.

Inevitably these techniques require significant support from the underlying operating system. Unfortunately commercially-available, general-purpose, RTOSs do not provide this support. Hence, in order to utilise many of the more advanced scheduling ideas that are to be found in the MCS literature, it is necessary to develop the code for a bespoke scheduler as part of the application. Programming languages such as Ada [11] do provide the primitives necessary for this software to be developed but to deliver a reliable MCS scheduler the MCS protocols and models must be precisely specified. Research papers that focus on the algorithmic properties of protocols tend to give, at best, informal descriptions of the actual required run-time behaviour of the required scheduler.

The objective of the research described in this paper is *to develop a framework for formally specifying and reasoning about timing correctness properties of mixed-criticality systems*. The following paragraphs explain this objective in greater detail. In general, correctness in safety-critical systems can be considered from two perspectives: (i) (pre-run-time) verification, and (ii) (run-time) survivability.

Pre-run-time *verification* of a safety-critical system involves verifying, prior to deployment, that the run-time behaviour of the system will be consistent with expectations. Verification



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44 assumptions are made regarding the kinds of circumstances that will be encountered by the
 45 system during run-time and guarantees are used to specify the required runtime behaviour
 46 of the system (provided that the assumptions hold).

47 In contrast, *survivability* addresses expectations of system behaviour in the event that the
 48 assumptions fail to hold fully (in which case a fault or error is said to have occurred during
 49 run-time). Survivability may further be considered to comprise two notions: robustness and
 50 resilience [14]. Informally, the robustness of a system is a measure of the degree of fault it
 51 can tolerate without compromising the quality of service it offers; resilience refers to the
 52 degree of fault for which it can provide a degraded, yet acceptable, criticality-aware quality
 53 of service.

54 The contribution of this paper is to develop a framework for the formal specification of
 55 MCS; we define a formal approach that:

- 56 ■ Demonstrates that the Rely/Guarantee approach (see Section 2) can be extended to
 57 cover temporal properties (see Section 3) of concurrent systems (in addition to their
 58 functionality).
- 59 ■ Precisely specifies the required behaviour of a run-time scheduler (in normal and degraded
 60 modes of operation).
- 61 ■ Enables proofs to be developed and discharged that employ the contract(s) between the
 62 jobs and tasks comprising an application, and the scheduler.
- 63 ■ Enables, with additional specifications of the functional elements of the scheduler, the
 64 code of the scheduler to be produced as a refinement of these specifications.
- 65 ■ Enables the scheduler to be replaced or modified by verifying that a new version satisfies
 66 the original specification.
- 67 ■ Identifies the assumptions that the analysis (scheduling and execution time) makes such
 68 that the result of the analysis confirms that the system will meet its timing requirements.
- 69 ■ Enables the many approaches to resilience and robustness to be compared – this requires
 70 the formal framework to be sufficiently expressive to capture the semantics of the various
 71 schemes that have been proposed.

72 This initial description of our approach focusses on the specification aspects; future work will
 73 address verification. We do however demonstrate where proof can be used to ensure that,
 74 whenever a degraded mode must be entered, its prerequisites are ensured by the guaranteed
 75 conditions of the mode that has just been abandoned. We also make explicit the proof
 76 obligations on the offline scheduling analysis that must be applied to the application prior to
 77 deployment.

78 We explain the elements of the framework via a series of related, increasingly challenging,
 79 examples. The initial examples are sufficiently straightforward that, arguably, a full formal
 80 specification is not required; however the later examples do show the value of precise
 81 specifications. The examples illustrate the approach with at most two criticality levels, this
 82 helps to explain the framework, but again the full value of a formal approach comes when the
 83 system has increased complexity as happens when there are three or more criticality levels.

84 In this paper an MCS is assumed to consist of a finite set of jobs/tasks and a single specific
 85 Scheduler. Rely and guarantee conditions capture the run-time relationship between the
 86 Scheduler and the jobs/tasks, yielding a specification of the necessary behaviours/properties
 87 of the Scheduler. Note that this process does not delve into the internal structure of the
 88 Scheduler: the scheduling-theoretic issues of how it meets its specification (if indeed it can)
 89 is *not* the focus of this work. Rather, in this paper we are only seeking to provide a clear and
 90 intuitive explanation of the formalism. The history of formal methods (such as Hoare Logic)
 91 leads us to believe that methods can be developed for showing that specific MC-scheduling

92 algorithms can satisfy (or not) the proof obligations that arise from the Rely/Guarantee
93 (R/G) specifications. Related work in this area includes PROSA which addresses mechanised
94 verification of results from scheduling analysis [21, 10]. (Mechanisation of R/G reasoning is
95 on-going [29, 22]).

96 **Organisation.** The paper is organised as follows. After an introduction to R/G conditions
97 (Section 2), the basic properties of the proposed framework are developed in Section 3 via a
98 focus on *jobs* – this allows the approach to be motivated and explained. Mixed-criticality
99 jobs are then covered in Section 4 including the introduction of fault-tolerance via modes of
100 operation each with their own R/G conditions. Extensions of the same ideas to *tasks* are
101 then given in Sections 5 and 6. Conclusions are in Section 7.

102 **2 Introduction to Rely/Guarantee conditions**

103 Hoare’s ‘Axiomatic Approach’ provides the basis of a development method for sequential
104 programs. Although [32] employed post conditions of single states, subsequent development
105 methods such as VDM [39], B [1] and Event-B [2] use relational post conditions that define
106 acceptable final states with respect to their initial values. Crucially, there is a relatively
107 obvious notion of compositionality for sequential programs where a specification can be
108 replaced by anything that satisfies its pre/post condition specification.

109 Finding compositional development methods for the development of concurrent programs
110 proved to be difficult precisely because of the ‘interference’ that comes with (shared-variable)
111 concurrency. One approach is to record and reason about interference using rely and guarantee
112 conditions [37, 38] (a more algebraic presentation of the ideas is covered in [31]). The details
113 and proof obligations of the R/G approach are not the main issue in the current paper. The
114 basic idea is straightforward: just as pre conditions define a subset of possible starting states
115 on which a component is expected to operate, *rely conditions* record interference that the
116 specified component must tolerate; and, just as post conditions abstract from algorithms
117 to achieve the transition from initial to final state, *guarantee conditions* are relations that
118 define the maximum interference that the component may inflict on its environment. It is
119 important to remember that pre and rely conditions are assumptions that a developer is
120 invited to make; in contrast, guarantee and post conditions are obligations on the code to be
121 created. A guarantee condition needs to be satisfied (only) as long as the corresponding rely
122 condition is respected. Stating this negatively, if the environment makes a transition that
123 does not satisfy the rely condition, the developed code is free from further obligations.

124 The R/G idea targeted the design of concurrent programs where the R/G conditions
125 provide a way of decomposing designs. Papers such as [30, 42, 19] show that the R/G idea
126 can be used to tackle the design of fault-tolerant CPS by using rely conditions to describe
127 assumptions about physical system components. Where the physical components exhibit
128 continuous change, the rely conditions record assumptions about the rate of such changes.
129 This work also showed how layered R/G conditions can assist in addressing fault tolerance;
130 *resilience* is represented by hierarchically related R/Gs—strong rely conditions address
131 full functionality, weaker rely conditions are matched with lesser guarantees (perhaps only
132 the safety-critical aspects), even weaker rely conditions might only guarantee safe fail-stop
133 behaviour. *These properties of related R/G conditions are central to the framework developed*
134 *in this paper.*

135 **3 Job-based system model**

136 This section focuses on a system comprising a set of jobs, \mathcal{J} , that are managed by a Scheduler
 137 (denoted by the symbol S). A representative job, $j \in \mathcal{J}$, has a relative deadline of D_j ,
 138 arrives (and is released for execution) at time a_j and thus has an absolute deadline at time
 139 $d_j = a_j + D_j$. Let f_j denote the time at which it completes (finishes) its execution.¹ The set
 140 $act(\mathcal{J}, t)$ is the subset of \mathcal{J} containing the jobs that are active at time t , i.e.

$$141 \quad j \in act(\mathcal{J}, t) \Leftrightarrow j \in \mathcal{J} \wedge (a_j \leq t < f_j)$$

142 A job that is immediately terminated on arrival (as required in specific circumstances by
 143 some MCS protocols) has $f_j = a_j$; it is deemed never to be active and to have missed its
 144 deadline.

145 We assume a discrete time model in which all job parameters are given as non-negative
 146 rational numbers with arbitrary precision. Time is an external physical phenomenon: the
 147 Scheduler has no control over the passage of time.

148 The specification of each job, j , consists of its pre-condition, P_j , post-condition Q_j , rely
 149 condition R_j and guarantee condition G_j . In this paper each of these conditions is expressed
 150 as a predicate over the system state. For an actual system these conditions will capture
 151 both the functional and timing behaviour of the job; here we focus only on the temporal
 152 properties. This requires that system states are indexed by time² and that the rely and
 153 guarantee conditions directly reference time. We write $R_S(t)/G_S(t)$ for the Scheduler and
 154 $R_j(t)/G_j(t)$ for jobs.

155 Properties that should remain true as time progresses are normally classed as *invariants*
 156 but here are represented as rely or guarantee conditions. This is because the jobs (and
 157 Scheduler) must take action in order to maintain correct behaviour – a job will miss its
 158 deadline if it is not scheduled appropriately.

159 The primary concern for each job is its execution time; and hence we define, for each job
 160 j , $e_j(t)$ which is the amount of execution time the job has consumed up to time t . There are
 161 obvious properties (axioms) for e :

$$162 \quad \forall j \in \mathcal{J}, t \bullet e_j(t) \leq WCET_j \tag{1}$$

163 where WCET is the worst-case execution time of the job;

$$164 \quad \forall j \in \mathcal{J}, t_1, t_2, t_1 < t_2 \bullet e_j(t_2) - e_j(t_1) \leq t_2 - t_1 \tag{2}$$

165 no job can execute faster than ‘real time’;

$$166 \quad \forall j \in \mathcal{J}, t_1, t_2, t_1 < t_2 \bullet e_j(t_1) \leq e_j(t_2) \tag{3}$$

167 a job cannot ‘lose’ execution time; and

$$168 \quad \forall j \in \mathcal{J} \bullet \left(\forall t \leq a_j \bullet e_j(t) = 0 \wedge \forall t \geq f_j \bullet e_j(t) = e_j(f) \right) \tag{4}$$

169 a job cannot execute before it arrives or after it has finished.

¹ A job that is yet to finish has $f = \infty$; a job that is permanently suspended but never terminated retains this value.

² A slightly different approach to handling the progress of time was taken in [40]. In that paper a distinction is made between an abstract notion of *Time* and the *ClockValues* stored in a computer.

170 In this section the scheduler is deemed to exist for the entire life-time of the system, it is
171 therefore specified by a single rely condition $R_S(t)$ and a single guarantee condition $G_S(t)$.

172 The following derivations first illustrate the basic approach with a set of single criticality
173 jobs. Note that the role of the formal framework is to represent precisely the relationship
174 between the Scheduler and the client jobs in a range of degraded and partial behaviours. It
175 is not a model of a particular scheduler's run-time behaviour; rather it is a specification of
176 the required properties of any scheduler (and its schedulability test) that is being proposed
177 for the particular problem under investigation.

178 A key feature of mixed-criticality models is that they allow a system to degrade gracefully
179 when faults occur. This leads to the Scheduler's run-time behaviour having different modes
180 of operation. In each mode, different R and G conditions for the jobs and scheduler are
181 defined, as is the transition between R/G contracts.

182 We start by considering a finite set of jobs that each have the same criticality; there is no
183 degraded behaviour and hence only a single mode of operation. A job j is characterised by its
184 Worst-Case Execution Time, $WCET_j$ (this is a value that will not be known with certainty)
185 and C_j an estimate of $WCET_j$. The timely execution of a job *relies* on this estimate of
186 $WCET$ being valid, and the Scheduler can only meet its obligations with a *reliance* of each
187 job executing for no more than C_j . If these rely conditions hold, a *valid* Scheduler *guarantees*
188 to manage the processing capacity so as to ensure that all jobs complete by their deadlines
189 regardless of when the jobs arrive; each job *guarantees* to execute, when active, for no more
190 than C_j .

191 Note that the value C_j plays a number of roles: the job relies on its environment behaving
192 according to whatever model or measuring process was used to derive C_j , but the job also
193 has a contract with the scheduler not to execute for more than C_j . The scheduler is assumed
194 to have used some form of analysis to verify (offline usually) that, if all jobs respect their
195 guarantee conditions, then it will be able to provide the necessary capacity to each job.
196 Hence the job can rely upon being allowed to execute for up to C_j before its deadline.

197 With all four axioms ((1)-(4) above) in force, the rely and guarantee conditions of any
198 valid Scheduler are as follows:

$$199 \quad R_S(t) \stackrel{\text{def}}{=} \forall j \in \text{act}(\mathcal{J}, t) \bullet e_j(t) \leq C_j$$

$$200 \quad G_S(t) \stackrel{\text{def}}{=} \forall j \in \text{act}(\mathcal{J}, t) \bullet t + (C_j - e_j(t)) \leq d_j$$

202 The Scheduler relies on all jobs executing within their estimated WCET and guarantees
203 to provide sufficient resource, following a defined policy, to ensure that each job always
204 has sufficient space to complete before its deadline (i.e. that $t + (C_j - e_j(t)) \leq d_j$).³ The
205 Scheduler's guarantee is an *obligation* that must be achieved by its code – i.e. the Scheduler's
206 offline schedulability test must ensure this property. The conditions $R_S(t)$ and $G_S(t)$ are
207 defined to refer only to jobs that are active at time t .

208 In order to satisfy G_S , the Scheduler must manage the dispatching of jobs in an appropriate
209 manner. If necessary it will allocate to each job up to C_j execution time. It follows that if
210 $WCET_j \leq C_j$ then each job will terminate by its deadline (i.e. $f_j \leq d_j$).

211 The R and G conditions of each active job are therefore:

$$212 \quad R_j(t) \stackrel{\text{def}}{=} WCET_j \leq C_j \wedge t + (C_j - e_j(t)) \leq d_j$$

³ An alternative formulation [12] to the one presented here is for the Scheduler to guarantee a budget (of at least C for each job), and for each job to rely on this budget. Example specifications and further investigations indicated that the method defined in the current paper is the more realistic and effective.

213

$$214 \quad G_j(t) \stackrel{\text{def}}{=} e_j(t) \leq C_j$$

215

216 At run-time, the job does not need to be aware of its deadline or current execution time;
217 although more expressive and flexible behaviours may require this. Once a job (j) terminates
218 the R_j and G_j conditions no longer apply.

218

219 The constraints imposed upon execution time are represented as guarantees and not
220 post-conditions for a number of reasons:

220

221 1. post-conditions are, by definition, required to hold upon termination, but a failure may
222 lead to the job not terminating;

222

223 2. to add fault tolerance (i.e. to cope with jobs whose estimated execution times are not
224 respected) we will need to know the point in time at which a rely condition fails to hold
(and hence a guarantee condition no longer has to hold); and

224

225 3. deadlines may change (or be removed) during the execution of the job (see later examples).

226

227 The semantics of rely/guarantee conditions is that guarantees are required to be met
228 as long as the rely conditions are satisfied. If a job overruns and breaks its guarantee that
229 $e_j(t) \leq C_j$ there must be a rely condition ‘at fault’. For this reason, we explicitly include
230 $\text{WCET}_j \leq C_j$ in the rely condition: in an environment where this assumption does not hold,
231 a job is not obliged to guarantee its temporal properties.

231

232 If the environment (hardware platform including the influence of concurrently executing
233 jobs, preemption effects on cache etc.) behaves such that the WCET estimate of some job
234 k is exceeded, then this job may execute for more than C_k , thus breaking its guarantee
235 condition. As a consequence the rely condition for the Scheduler would not be satisfied and
236 hence it would be under no obligation to provide the necessary capacity to every job — some
237 jobs may still be active at their deadlines. This takes us to the topic of survivability and
how MCS supports graceful degradation.

238 **4 Mixed-criticality jobs**

239

240 To illustrate how a level of resilience can be added, two criticality levels are considered: *HI*-
241 crit and *LO*-crit; with \mathcal{J}_L a set of *LO*-crit jobs, \mathcal{J}_H a set of *HI*-crit jobs, and $\mathcal{J} = \mathcal{J}_L \cup \mathcal{J}_H$.
242 Job h is a representative *HI*-crit job; l is a representative *LO*-crit job; j continues to represent
243 any job. So, for example, $R_h(t)$ is the rely condition for any *HI*-crit job, $h \in \mathcal{J}_H$. With Mixed-
244 Criticality jobs there are two estimates of C_j : $C_j(L)$ and $C_j(H)$; with $C_j(L) \leq C_j(H)$ [61].

244

245 It is initially assumed that the system is either in the Normal mode, in which case all
246 jobs should meet their deadlines, or in the *HI*-crit mode in which only the *HI*-crit jobs are
247 guaranteed to meet their deadlines. For the Normal (N) mode the (R , G) conditions are as
above except that $C_j(L)$ replaces C_j in R_j, G_j, R_S and G_S :

248

$$248 \quad R_S^N(t) \stackrel{\text{def}}{=} \forall j \in \text{act}(\mathcal{J}, t) \bullet e_j(t) \leq C_j(L)$$

249

$$249 \quad G_S^N(t) \stackrel{\text{def}}{=} \forall j \in \text{act}(\mathcal{J}, t) \bullet t + (C_j(L) - e_j(t)) \leq d_j$$

250

$$250 \quad R_j^N(t) \stackrel{\text{def}}{=} \text{WCET}_j \leq C_j(L) \wedge t + (C_j(L) - e_j(t)) \leq d_j$$

251

$$251 \quad G_j^N(t) \stackrel{\text{def}}{=} e_j(t) \leq C_j(L)$$

252

253 The rely and guarantee conditions for the N mode are therefore:

254

$$254 \quad R^N(t) = R_S^N(t) \wedge \bigwedge_{j \in \mathcal{J}} R_j^N(t)$$

$$G^N(t) = G_S^N(t) \wedge \bigwedge_{j \in \mathcal{J}} G_j^N(t)$$

Most of these rely and guarantee conditions are mutually supportive in the sense that they “cancel out” when looking at the whole system. The only rely condition that depends on external compliance is:

$$\forall j \in \mathcal{J} \bullet \text{WCET}_j \leq C_j(L)$$

4.1 Adding resilience to *HI*-crit jobs

Considering *HI*-crit jobs ($h \in \mathcal{J}_H$) and their rely condition:

$$R_h^N(t) \stackrel{\text{def}}{=} \text{WCET}_h \leq C_h(L) \wedge t + (C_h(L) - e_h(t)) \leq d_h$$

We want to give a higher (safer) bound on WCET, so we consider a more conservative value ($C_h(H)$), where $C_h(H) > C_h(L)$. Now for all *HI*-crit jobs (h) we have a new *HI*-crit mode (H) and:

$$R_h^H(t) \stackrel{\text{def}}{=} \text{WCET}_h \leq C_h(H) \wedge t + (C_h(H) - e_h(t)) \leq d_h$$

$$G_h^H(t) \stackrel{\text{def}}{=} e_h(t) \leq C_h(H)$$

The Scheduler’s definition for mode H is

$$R_S^H(t) \stackrel{\text{def}}{=} \forall h \in \text{act}(\mathcal{J}_H, t) \bullet e_h(t) \leq C_h(H) \wedge \forall l \in \text{act}(\mathcal{J}_L, t) \bullet e_l(t) \leq C_l(L)$$

$$G_S^H(t) \stackrel{\text{def}}{=} \forall h \in \text{act}(\mathcal{J}_H, t) \bullet t + (C_h(H) - e_h(t)) \leq d_h$$

In this *HI*-crit mode there is no obligation to provide any level of service to the lower criticality jobs or indeed to prevent these jobs from using resources (perhaps at a background priority in a priority-based scheduler). Hence:

$$R_l^H(t) \stackrel{\text{def}}{=} \text{WCET}_l \leq C_l(L)$$

$$G_l^H(t) \stackrel{\text{def}}{=} e_l(t) \leq C_l(L)$$

The above specification is, however, not sufficient for many of the protocols advocated for mixed-criticality scheduling. The standard ‘mixed-criticality’ mechanism for being able to add more capacity to the *HI*-crit jobs is to take computation time away from the *LO*-crit jobs. Or, more precisely, to no longer execute these jobs. This further adds to the guarantees of the Scheduler.

To facilitate this functionality it is necessary to know the time at which R_S^N became false (i.e. when an active *HI*-crit job has first executed for $C(L)$ without terminating). We refer to this as mode N ’s *deviation time*, η^N ; defined by the following property:

$$\exists \eta^N, h \in \text{act}(\mathcal{J}_H, \eta^N) \bullet e_h(\eta^N) \geq C_h(L) \wedge \forall t, t < \eta^N, g \in \text{act}(\mathcal{J}_H, t) \bullet e_g(t) < C_g(L)$$

At the deviation time R_S^N becomes false, mode N is left and, simultaneously⁴, mode H is entered. The rely and guarantee conditions $R^H(t)$ and $G^H(t)$ apply for $t \geq \eta^N$.

⁴ The notion of simultaneous is taken from the Timebands [18] framework that allows instantaneous actions to be defined at one time band (granularity) but implemented by an activity at a finer time band.

293 We assume here the extreme Vestal behaviour of not executing LO -crit jobs again after
 294 η^N . This leads to a full specification for the guarantee condition for the Scheduler:

$$295 \quad G_S^H(t) \stackrel{\text{def}}{=} \forall h \in \text{act}(\mathcal{J}_H, t) \bullet t + (C_h(H) - e_h(t)) \leq d_h \wedge \forall l \in \text{act}(\mathcal{J}_L, t) \bullet e_l(t) = e_l(\eta^N)$$

296 with a simplified rely condition as the Scheduler no longer relies on the behaviour of LO -crit
 297 jobs as it guarantees that they do not execute:

$$298 \quad R_S^H(t) \stackrel{\text{def}}{=} \forall h \in \text{act}(\mathcal{J}_H, t) \bullet e_h(t) \leq C_h(H)$$

299 and therefore:

$$300 \quad R^H(t) = R_S^H(t) \wedge \bigwedge_{l \in \mathcal{J}_L} R_l^H(t) \bigwedge_{h \in \mathcal{J}_H} R_h^H(t)$$

$$301 \quad G^H(t) = G_S^H(t) \wedge \bigwedge_{l \in \mathcal{J}_L} G_l^H(t) \bigwedge_{h \in \mathcal{J}_H} G_h^H(t)$$

303 This strategy of pausing all LO -crit jobs is not an option that the Scheduler *could* choose,
 304 but a requirement that is part of the specification of the job's behaviour — and hence must
 305 be explicitly contained in G_S^H .

306 With this specification the LO -crit jobs are suspended; but they may execute later in
 307 another mode (perhaps after their deadlines). To abort these and future LO -crit jobs, rather
 308 than preempt them indefinitely, the Scheduler could (if specified to do so) enforce termination:

$$309 \quad \forall t, t > \eta^N \bullet \text{act}(\mathcal{J}_L, t) = \emptyset$$

310 4.2 Transitioning from mode N to mode H

311 The specification above requires a movement from mode N to mode H . To provide useful
 312 fault tolerance, it must be true that, whenever the rely condition for N fails to be satisfied,
 313 the corresponding rely condition for H is satisfied (and remains so) i.e. at time η^N when
 314 $R^N(\eta^N)$ no longer pertains: $R^H(\eta^N)$ is satisfied. If $R^H(\eta^N)$ is true then the guarantee
 315 condition, $G^H(t)$, is delivered for all $t > \eta^N$, and as a consequence $R^H(t)$ must hold.

316 In general a mode change could involve modes with unrelated functionality and hence
 317 the truth of the rely condition in the new mode would need to be asserted independently of
 318 the rely condition in the old mode. This is identical to what is required at system startup
 319 where the rely condition of the initial mode must be established. In this work, however, we
 320 require a more constrained relationship between the modes:

321 ► **Definition 1.** *Mode B is a weakened form of mode A if*

322 1. *for all times (t) before η^A when $R^A(t)$ is true then $R^B(t)$ is true (i.e. $R^A(t) \Rightarrow R^B(t)$);*
 323 *and*

324 2. *at time η^A when some aspect of $R^A(\eta^A)$ is no longer true $R^B(\eta^A)$ remains true.*

325 As $R^B(\eta^A)$ is true, it followed that $G^B(t)$ is true for all $t > \eta^A$.

326 *Counter Example.* We require that mode H is a weakening of mode N . Consider the
 327 first element of the definition of weakening: in two of the three rely conditions, this is indeed
 328 the case as:

$$329 \quad R_S^N(t) \Rightarrow R_S^H(t); \quad R_l^N(t) \Rightarrow R_l^H(t)$$

330 but $R_h^N(t)$ does not have a simple relationship to $R_h^H(t)$. The first conjunct is a weakening of
 331 the 'external' rely condition as $\text{WCET}_h \leq C_h(L) \Rightarrow \text{WCET}_h \leq C_h(H)$. The second conjunct

332 is, however, a strengthening; hence modes N and H do not have the required hierarchical
 333 relationship – H is not a weakened form of N .

334 *A Modified Definition of Mode N (N^*).* In order to assert that mode H is a weakened
 335 form of the initial mode it is necessary to constrain the behaviour of the Scheduler further
 336 in the Normal mode. It must do more than simply guarantee to provide for all jobs $C(L)$
 337 before the deadline d , it must also reserve sufficient slack so that, at any time a switch can
 338 be made, it is possible to guarantee $C(H)$ before d .

339 It follows that, for a HI -crit jobs, h , to be schedulable in both N^* and H modes, there
 340 exists a virtual deadline v_h with

$$341 \quad v_h \leq d_h - (C_h(H) - C_h(L))$$

342 that is defined (and confirmed) by the applicable scheduling analysis, such that: if the
 343 Scheduler in mode N^* guarantees $C(L)$ by v , then the Scheduler in mode H will be able
 344 to guarantee $C(H)$ by d .⁵ To accommodate this constraint the guarantee condition of the
 345 Scheduler in mode N^* must be modified to:

$$346 \quad G_S^{N^*}(t) \stackrel{\text{def}}{=} \forall j \in \text{act}(\mathcal{J}, t) \bullet t + (C_j(L) - e_j(t)) \leq v_j$$

347 and the Rely conditions of HI -crit jobs becomes

$$348 \quad R_h^{N^*}(t) \stackrel{\text{def}}{=} \text{WCET}_h \leq C_h(L) \wedge t + (C_h(L) - e_h(t)) \leq v_h$$

349 For LO -crit jobs (l) $v_l = d_l$ and hence $G_S^{N^*}$ has not changed for these jobs. For HI -crit jobs
 350 (h) there is a proof obligation on the scheduling analysis to demonstrate:

$$351 \quad \forall t, h \in \text{act}(\mathcal{J}_H, t) \bullet G_S^{N^*}(t) \Rightarrow t + (C_h(H) - e_h(t)) \leq d_h \quad (5)$$

352 Such an obligation could be verified using mechanised proof tools such as PROSA [21, 10].

353 **► Lemma 2.** *Mode H is a weakening of mode N^* .*

354 **Proof.** As noted above $\forall t : R_S^N(t) \Rightarrow R_S^H(t)$ and $R_l^N(t) \Rightarrow R_l^H(t)$. The modification to
 355 N^* does not effect these rely conditions. Also $\text{WCET}_h \leq C_h(L) \Rightarrow \text{WCET}_h \leq C_h(H)$
 356 (as $C_h(H) \geq C_h(L)$). Finally $t + (C_h(L) - e_h(t)) \leq v_h \Rightarrow t + (C_h(H) - e_h(t)) \leq d_h$ as
 357 $v_h \leq d_h - (C_h(H) - C_h(L))$.

358 The second step is to show that, at time η^{N^*} (when $R^{N^*}(\eta^{N^*})$ fails), $R^H(\eta^{N^*})$ remains
 359 true. Condition $R^{N^*}(\eta^{N^*})$ is false because the $WCET$, for some HI -crit job k , is not
 360 bounded by $C_k(L)$. Moreover η^{N^*} is the first time instant at which R^{N^*} is false. Hence at
 361 time η^{N^*} , $R_k^{N^*}(\eta^{N^*})$ is false, but $R_k^H(\eta^{N^*})$ is true as $C_k(H) > C_k(L)$.⁶ ◀

362 This weakening property and the proof obligation represented by eqn (5) are therefore
 363 sufficient to ensure that, whenever the Normal mode must be abandoned, the HI -crit mode
 364 can be entered and will deliver its guaranteed behaviour. The final point to note about the
 365 transition from N^* to H is that the Guarantee conditions are also weakened. The system
 366 moves from guaranteeing all job deadlines to just guaranteeing the HI -crit ones. Hence
 367 $G^{N^*}(t) \Rightarrow G^H(t)$.

⁵ This virtual deadline is used directly in the EDF-based scheduling scheme EDF-VD [5] and in fixed-priority scheduling is equivalent to the worst-case (maximum) computed response time of the HI -crit job in the Normal mode [6]. Note whatever scheduling protocol is employed at run-time there is an implicit (if not explicit) virtual deadline in the Normal mode. If this were not the case then there would be insufficient spare capacity in the Normal mode to satisfy the extra demand of the HI -crit mode.

⁶ Strictly, we require $C_k(H) > C_k(L) + \delta$ where δ is the minimum time step that the system can undertake in its discrete model of time.

368 **4.3 Postponing the deviation time**

369 As noted in the introduction, the main focus of this paper is to motivate and define a formal
 370 framework for the specification of mixed criticality systems. In this section we are able to
 371 give an example of how this framework can be utilised.

372 A system is considered to degrade at deviation time η^{N^*} which is defined, above, as the
 373 first time that a *HI*-crit job executes beyond its $C(L)$ constraint. But if this deviation time
 374 could be postponed then the dynamics of the system may alleviate the need to make the
 375 mode change – the *LO*-crit jobs could continue to meet their deadlines. Possible favourable
 376 dynamic behaviours include sporadic jobs not arriving at their maximum rate, and other jobs
 377 executing for less than their maximum $C(L)$ bound. To explore the possibility of delaying
 378 the deviation time consider again the specification of the N^* mode:

$$379 \quad R_S^{N^*}(t) \stackrel{\text{def}}{=} \forall j \in \text{act}(\mathcal{J}, t) \bullet e_j(t) \leq C_j(L)$$

$$380 \quad G_S^{N^*}(t) \stackrel{\text{def}}{=} \forall j \in \text{act}(\mathcal{J}, t) \bullet t + (C_j(L) - e_j(t)) \leq v_j$$

$$381 \quad R_j^{N^*}(t) \stackrel{\text{def}}{=} \text{WCET}_j \leq C_j(L) \wedge t + (C_j(L) - e_j(t)) \leq v_j$$

$$382 \quad G_j^{N^*}(t) \stackrel{\text{def}}{=} e_j(t) \leq C_j(L)$$

383 where $v_j = d_j$ for *LO*-crit jobs and $v_l \leq d_l - (C_l(H) - C_l(L))$ for *HI*-crit jobs.

384 If all jobs behave according to this R/G specification then all virtual deadlines will be
 385 met. This implies there is a weakened form of behaviour (which we denote as mode \hat{N}^*):

$$386 \quad R_S^{\hat{N}^*}(t) \stackrel{\text{def}}{=} \forall j \in \text{act}(\mathcal{J}, t) \bullet t \leq v_j$$

$$387 \quad G_j^{\hat{N}^*}(t) \stackrel{\text{def}}{=} t \leq v_j$$

388 with $G_S^{\hat{N}^*} = G_S^{N^*}$ and $R_j^{\hat{N}^*} = R_j^{N^*}$.

389 From the definition of the virtual deadline we have $R_S^{N^*} \Rightarrow \hat{R}_S^{N^*}$ and $G_j^{N^*} \Rightarrow \hat{G}_j^{N^*}$.

390 The deviation time (when $\hat{R}_S^{N^*}$ becomes false for the first time) is now when a *HI*-crit
 391 job is still executing at its virtual deadline. And this time is likely to be significantly later
 392 than that provided by the earlier definition. Note also that this alternative definition of the
 393 deviation time for the normal mode changes what needs to be monitored – from execution
 394 time to elapsed time. This is likely to reduce the runtime overheads of the MCS scheduler.

395 Again it is straightforward to prove that mode *H* is a weakening of (the modified) mode
 396 \hat{N}^* , and the proof obligation on the offline scheduling analysis (eqn (5)) must again be used
 397 to validate the v values assigned to each *HI*-crit job. Recent scheduling results [8] have
 398 demonstrated that for fixed priority-based scheduling and AMC-rtb analysis the same v
 399 values are valid for the original definition of deviation time and the one derived in this section.
 400 That paper also demonstrated the benefits in terms of run-time performance that is gained
 401 from postponing the mode change.

402 The proposed framework allowed this new protocol to be easily defined and verified.
 403 Further properties can be proven (such as the above definition of deviation time being the
 404 latest possible). In this introductory paper, however, priority is given to extending the
 405 framework to task-based systems.

5 Task-based system model

The above treatment of mixed-criticality jobs has demonstrated that the proposed specification framework has sufficient expressive power to capture the properties commonly required of job-based systems. The scheduling literature typically describes jobs as being organised within tasks — in this section we extend the study to cope with tasks.

A real-time system is deemed to consist of a set of tasks. A single execution of the code of a task is a job. So a task gives rise to a sequence of jobs. The scheduler determines the order in which jobs from different tasks are executed. With a task-based model there is an assumption that the duration of the system is unbounded. This means that any specification framework must cater for the return of the system from any degraded mode back to the initial mode for the system (and to allow these mode changes to occur numerous times). We assume that each task k delivers a potentially unbounded sequence of jobs, k^1, k^2 etc, with job k^m having arrival time a_k^m and completion time f_k^m . This sequence is not ‘reset’ as new modes are entered; it continues to extend indefinitely.

This treatment focuses on issues related to execution time and mixed criticality. It does not directly address the rely and guarantee conditions related to when and how a task is released for execution. For example, time-triggered tasks require their job releases to be guaranteed by some Dispatcher; and event-triggered tasks rely on their releasing events obeying some minimum separation requirement. These issues are covered here by each task guaranteeing that its jobs do not arrive too early — a rely condition for the Scheduler.

The system is again assumed to be defined over two criticality levels, *LO*-crit and *HI*-crit, and to have two modes of behaviour: N^* and H . We however drop, for ease of presentation, the superscript from N in the following. To define a general model, each of the defining temporal parameters of each task (D, T, C, V) has an L and a H value.

We again make use of sets: \mathcal{T} is the set of all tasks, \mathcal{T}_L the set of *LO*-crit tasks, and \mathcal{T}_H the set of *HI*-crit tasks, and $\mathcal{T} = \mathcal{T}_L \cup \mathcal{T}_H$. The axioms defined in Section 3 still apply.

At any time t , each task k has a single *current job*. We let $c(t)$ be the index of this job (for ease of presentation, we just use c for this index as the t value is always implied). Hence the current job of task k is denoted by k^c . This job may have finished, but the next job of this task has not yet arrived ($f_k^c \leq t < a_k^{c+1}$). In task models that allow a job to arrive before the previous job of the same task has finished (i.e. tasks with $D > T$), the ‘current’ job is the one that arrived first.

We modify the definition of ‘active’ to cater for tasks; a task is active if its current job has not yet terminated:

$$k \in act(\mathcal{T}, t) \Leftrightarrow k \in \mathcal{T} \wedge (a_k^c \leq t < f_k^c)$$

In each of the criticality modes the relative parameters (V_k and D_k) are added to the arrival time a_k^c to give the absolute values: $v_k^c, d_k^c(L)$ and $d_k^c(H)$.

5.1 Vestal-inspired example

This section specifies the required behaviour of the system (Scheduler and tasks) for a typical model inspired by the Vestal approach [61]. The properties of this model are, briefly:

- System starts in the mode N in which all jobs of all tasks execute for no more than $C(L)$ and all job deadlines are met.
- All *LO*-crit tasks are assumed (or constrained) to execute for no more than $C(L)$.
- All *HI*-crit tasks are assumed (or constrained) to execute for no more than $C(H)$.
- If any, or indeed all, *HI*-crit tasks execute for more than $C(L)$ then:

- 455 ■ all *HI*-crit tasks must still meet their deadlines;
- 456 ■ all *LO*-crit tasks have their periods and deadlines increased, but must still meet their
- 457 deadlines.
- 458 ■ If there is an idle instant then the system must return to the Normal mode of operation.

459 This extension of the Vestal model is often referred to as the *elastic task model* [20] in which
 460 the periods and deadlines of *LO*-crit tasks are extended from $T_l(L)$ (and $D_l(L)$) to $T_l(H)$
 461 (and $D_l(H)$), but are still guaranteed.

462 The major difference when moving from jobs to tasks is that each task, like the Scheduler,
 463 exists for the full duration of the time spent in each mode. Although individual jobs terminate,
 464 the task does not (in the model being utilised here). So $R_k(t)$ and $G_k(t)$ are the rely and
 465 guarantee conditions of task k , but they refer to the job that is current (and possibly active)
 466 at time t .

467 For the Vestal-inspired model outlined above we have, for all *LO*-crit tasks, $l \in \mathcal{T}_L$, $C_l(L) =$
 468 $C_l(H)$, $T_l(H) > T_l(L)$, $D_l(H) > D_l(L)$ and $V_l = D_l(L)$ and for all *HI*-crit tasks, $h \in \mathcal{T}_H$,
 469 $C_h(L) < C_h(H)$, $T_h(L) = T_h(H)$, $D_h(L) = D_h(H)$ and $V_h < D_h(L) - (C_h(H) - C_H(L))$.

470 The conditions for the normal mode N are:

$$471 \quad R_S^N(t) \stackrel{\text{def}}{=} \forall k \in \text{act}(\mathcal{T}, t) \bullet e_k^c(t) \leq C_k(L) \wedge (c > 1 \Rightarrow a_k^c - a_k^{c-1} \geq T_k(L))$$

$$472 \quad G_S^N(t) \stackrel{\text{def}}{=} \forall k \in \text{act}(\mathcal{T}, t) \bullet t + (C_k(L) - e_k^c(t)) \leq v_k^c$$

$$473 \quad R_k^N(t) \stackrel{\text{def}}{=} \text{WCET}_k \leq C_k(L) \wedge k \in \text{act}(\mathcal{T}, t) \Rightarrow t + (C_k(L) - e_k^c(t)) \leq v_k^c(L)$$

$$474 \quad G_k^N(t) \stackrel{\text{def}}{=} e_k^c(t) \leq C_k(L) \wedge (c > 1 \Rightarrow a_k^c - a_k^{c-1} \geq T_k(L))$$

478 R_S^N contains the separation condition: if the current job is not the first instantiation of the
 479 task then it must arrive at least $T_k(L)$ after the previous job.

480 In the *HI*-crit mode, H , we have a similar formulation but with different parameters:

$$481 \quad R_S^H(t) \stackrel{\text{def}}{=} \forall k \in \text{act}(\mathcal{T}, t) \bullet e_k^c(t) \leq C_k(H) \wedge (c > 1 \Rightarrow a_k^c - a_k^{c-1} \geq T_k(H))$$

$$482 \quad G_S^H(t) \stackrel{\text{def}}{=} \forall k \in \text{act}(\mathcal{T}, t) \bullet t + (C_k(H) - e_k^c(t)) \leq d_k^c(H)$$

$$483 \quad R_k^H(t) \stackrel{\text{def}}{=} \text{WCET}_k \leq C_k(H) \wedge k \in \text{act}(\mathcal{T}, t) \Rightarrow t + (C_k(H) - e_k^c(t)) \leq d_k^c(H)$$

$$484 \quad G_k^H(t) \stackrel{\text{def}}{=} e_k^c(t) \leq C_k(H) \wedge (c > 1 \Rightarrow a_k^c - a_k^{c-1} \geq T_k(H))$$

488 These two formulations can easily be combined into a single specification that is a function
 489 of the mode (N or H) but are separated here to improve readability.

490 5.2 Transitioning from N to H

491 In this and the following section we consider the movement between modes; from Normal,
 492 N , to the *HI*-crit mode, H , and then the return to the Normal mode. In a long-lived
 493 task-based system there may be many such transitions between N and H . Each time a mode
 494 is entered, we consider this to be a new *occurrence* of the mode and therefore there is a new
 495 *occurrence* of the Scheduler for that mode. A move from N to H involves one occurrence of
 496 the N -mode Scheduler terminating and, instantaneously, a new occurrence of the H -mode
 497 Scheduler starting its execution⁷. A natural linkage between Scheduler occurrences is for the

⁷ An implementation may utilise a single Scheduler that modifies its behaviour depending upon which mode is current. Nevertheless, from a modelling point of view we consider each occurrence of the Scheduler to be a distinct execution.

498 post-condition of one mode, say A (Q_S^A), to ensure the pre-condition of the follow-on mode,
 499 B (P_S^B), with $Q_S^A \Rightarrow P_S^B$.

500 We note that the two mode changes contained within this task-based two-level mixed
 501 criticality system are of a quite different nature. The movement from N to H is forced, as
 502 N must be left. But the transition from H back to N is one of preference – both modes are
 503 acceptable, but the functional behaviour of the system is enhanced by being in the N mode.

504 In Section 4.2 we noted that as mode N is left at time η^N , due to $R^N(\eta^N)$ being false,
 505 we must prove that $R^H(\eta^N)$ is true. This involves two steps. First, at any time $t < \eta^N$,
 506 $R^N(t) \Rightarrow R^H(t)$. Second, at time η^N , when $R^N(\eta^N)$ is broken, $R^H(\eta^N)$ remains true.

507 Following the approach in Section 4.2, the task model has again made use of a virtual
 508 deadline for HI -crit jobs; from this we derive the proof obligation:

$$509 \quad \forall t, h \in \text{act}(\mathcal{T}_{\mathcal{H}}, t) \bullet G_S^N(t) \Rightarrow t + (C_h(H) - e_h^c(t)) \leq d_h^c(H) \quad (6)$$

510 *Counter Example.* With this Vestal-inspired example, the periods of the LO -crit tasks are
 511 expanded when the H mode is entered. It is therefore **not** true that $a_i^c - a_i^{c-1} \geq T_i(L) \Rightarrow$
 512 $a_i^c - a_i^{c-1} \geq T_i(H)$ as $T_i(H) > T_i(L)$. Hence R_S^N does **not** imply R_S^H .

513 *A Modified Definition of Mode H (H^*).* We must again modify the specification. However
 514 on this occasion rather than strengthen the rely condition in mode N we weaken the definition
 515 of the rely condition for the Scheduler in the HI -crit mode:

$$516 \quad R_S^{H^*}(t) \stackrel{\text{def}}{=} \forall k \in \text{act}(\mathcal{T}, t) \bullet e_k^c(t) \leq C_k(H) \wedge (c > 1 \wedge t > \eta^N \Rightarrow a_k^c - a_k^{c-1} \geq T_k(H))$$

517 Note the addition of $t > \eta^N$, the constraint on the arrival times of jobs in the new
 518 mode only applies strictly *after* η^N . The Guarantee condition of mode H^* is unchanged
 519 ($G^{H^*}(t) = G^H(t)$) and for the tasks: $R_k^{H^*}(t) = R_k^H(t)$, and $G_k^{H^*}(t) = G_k^H(t)$.

520 ► **Lemma 3.** *Mode H^* is a weakening of mode N .*

521 **Proof.** First, $\forall t < \eta^N$: For LO -crit tasks: $C_l(H) = C_l(L)$ and $v_l^c = d_l^c$ hence $R_l^N = R_l^H$ (so
 522 $R_l^N \Rightarrow R_l^H$). For HI -crit tasks: $\text{WCET}_h \leq C_h(L) \Rightarrow \text{WCET}_h \leq C_h(H)$ (as $C_h(H) \geq C_h(L)$);
 523 and $t + (C_h(L) - e_h^c(t)) \leq v_h^c \Rightarrow t + (C_h(H) - e_h^c(t)) \leq d_h^c$ as $v_h^c \leq d_h^c - (C_h(H) - C_h(L))$. Hence
 524 $R_h^N \Rightarrow R_h^{H^*}$. For the Scheduler, the first conjunct is appropriate as $e_k^c(t) \leq C_k(L) \Rightarrow e_k^c(t) \leq$
 525 $C_k(H)$, the second conjunct does not apply as $t < \eta^N$.

526 The second step (showing R^{H^*} is true at time η^N) follows the proof of Lemma 2; noting
 527 again that the second conjunct of $R_S^{H^*}(t)$ does not apply when $t = \eta^N$. ◀

528 As $R^{H^*}(\eta^N)$ is true, it follows that $G^{H^*}(t)$ is true for all $t > \eta^N$ and hence $R^{H^*}(t)$ is
 529 true for all $t \geq \eta^N$ as long as all task execution times are bounded by $C_k(H)$.

530 The proof obligations on the necessary scheduling analysis must allow for all LO -crit
 531 generated jobs to arrive at the time of the mode change. One of the advantages of this
 532 more formal specification of the Scheduler's behaviour is that it helps identify this constraint
 533 explicitly. We note that many examples of published scheduling algorithms for mixed-
 534 criticality systems (for example [15]) do allow LO -crit jobs to arrive (and subsequently
 535 execute) at the time of the mode change even if that would not be allowed in the new mode.
 536 However this property is often hidden within the analysis (by the use of a 'floor plus one'
 537 rather than a 'ceiling' representation of job arrivals). Within our formal framework the
 538 property is explicit.

539 To summarise, in order to prove that R^H is true whenever a forced mode change can
 540 occur, we note three distinct situations:

541 1. Conjuncts within R^H are weakened forms of those in R^N and remain true.

- 542 2. Conjuncts in R^N must be strengthened so that they then imply the corresponding
 543 conjunctions in R^H .
 544 3. Conjuncts in R^H must be weakened so that they are implied by the corresponding
 545 conjunctions in R^L .
 546 The above example makes use of all three strategies.

547 5.3 Transitioning from H to N

548 As long as the execution times of the HI -crit tasks are bounded by their $C(H)$ estimates,
 549 the system will stay in the H mode. All the rely conditions will remain true. However it is
 550 desirable to return to the Normal mode if possible as this mode provides a better level of
 551 service – i.e. LO -crit tasks will be able to occur more often and have shorter deadlines.

552 Once the over-running HI -crit job that caused the transition to mode H has terminated,
 553 there is the possibility that all new jobs can be released with their LO -crit parameters
 554 and, if they all execute for no more than $C(L)$, all deadlines can be met. But we know
 555 that any scheduling scheme can only guarantee deadlines if there is bounded (indeed often
 556 zero) residual work in the system at the time the Normal mode is (re-)activated [7]. It is
 557 therefore scheduler specific as to when the system is ‘safe’ to return to the Normal N mode
 558 of operation.

559 May/must constraints [19] are useful here. If the system is idle (there are no jobs to
 560 execute), it is usual to state that the scheduler *must* return the system to the Normal mode,
 561 but it *may* make this change earlier if a proof obligation has shown that such a transition is
 562 safe.

563 In terms of the framework presented in this paper a switch back to N mode is allowed
 564 only when the scheduling obligations (as represented by G_S^N) of that mode can be satisfied
 565 by the current Scheduler. If these obligations are satisfied, the move from H to N can be
 566 sanctioned by an appropriate pre-condition on the Normal mode. An example of one such
 567 pre-condition is the commonly used protocol that the Normal mode can only be (re-)entered
 568 at time t if there are no active jobs at time t (other than ones that arrive at time t):

$$569 \quad P_S^N(t) \stackrel{\text{def}}{=} k \in \text{act}(\mathcal{T}, t) \Rightarrow a_k^c = t$$

570 The Scheduler for the Normal mode can therefore assume this property and it is the
 571 responsibility of the Scheduler in the HI -crit mode to enforce it whenever it invokes a mode
 572 change back to Normal. In other words this is a post-condition for the Scheduler in mode H :

$$573 \quad Q_S^H(t) \stackrel{\text{def}}{=} k \in \text{act}(\mathcal{T}, t) \Rightarrow a_k^c = t$$

574 6 Robustness and resilience

575 Here we extend the treatment for tasks to show how we can more systematically specify
 576 levels of robustness and resilience for mixed-criticality systems, the motivation here being to
 577 develop a means of quantifying robustness and resilience. The first step in this process is to
 578 specify the various schemes being proposed.

579 Informal definitions of robustness and resilience are provided in [14] – i.e. the robustness
 580 of a system is a measure of the level of faults it can tolerate without compromising the
 581 quality of service it offers; resilience, by contrast, refers to the level of faults for which it can
 582 provide degraded yet acceptable (e.g. safe) quality of service. It is noted in [14] that there
 583 are a number of standard responses in the fault tolerance literature for systems that suffer
 584 transient faults (equating to one or more concurrent job failures in this work):

- 585 1. Fail (Fully) Operational – all tasks/jobs execute correctly (i.e. meet their deadlines).
- 586 2. Fail Robust – some tasks are allowed to skip a job but all non-skipped jobs execute
587 correctly and complete by their deadlines; the quality of service at all criticality levels is
588 unaffected by job skipping. Many periodic control tasks have this property [62]; there
589 is sufficient inertia in the physical system to allow the occasional control signal to be
590 missed.
- 591 3. Fail Resilient – some lower criticality tasks are given reduced service such as having their
592 periods/deadlines extended, priorities dropped and/or their execution budgets reduced; if
593 the budget is reduced to zero then this is equivalent to subsequent jobs of the task being
594 abandoned.
- 595 4. Fail Safe/Restart – where the level of failure exceeds what Fail Resilient bounds can
596 accommodate, more extreme responses are required including rebooting or system shut-
597 down (if the application has a fail-safe state). If a fail-safe state cannot be achieved then
598 the system may need to rely on best-effort tactics that have no guarantees. This is, of
599 course, the last resort to achieving survivability.

600 6.1 Failure modes

601 The framework developed above has been extended to include a number of more complex
602 behaviours that arise from supporting robust and resilient behaviour. In this section we
603 briefly outline a set of possible failure modes.

604 **Fail operational – FO.** A *HI*-crit job experiences a fault if it executes for more than
605 $C(L)$. One measure of Fail Operational is therefore the number of such job failures that can
606 be accommodated while still meeting all task deadlines. However, if a job from a *HI*-crit
607 task executes for more than $C(L)$, we still assume that the $C(H)$ bound remains operational.

608 One criticism of those models derived from Vestal [61] is that they usually assume that
609 any overrun of $C(L)$ results in an execution time of $C(H)$. In practice this is very unlikely
610 to occur, a minor overrun is more likely. We therefore introduce a parameter, C_O , that
611 represents a unit of overrun (for all jobs). Fail Operational is a measure of how many such
612 overruns can be accommodated. Let O denote this number over all the tasks. A *HI*-crit
613 job that executes for more than $C_h(L)$ but less than $C_h(L) + C_O$ has an O value of 1. In
614 general, a task has an O value of n if its overrun is between $(n - 1) * C_O$ and $n * C_O$.

615 The metric for Fail Operational is therefore the maximum O value allowed (F_O) in a
616 defined interval, I_O . This interval could be of a fixed length (and would usually be much
617 greater than the maximum task period). Alternatively it could be the interval from the
618 current time back to when there was an idle moment, m , defined by:

$$619 \quad \exists m, m < t \bullet \left(\forall k \in \text{act}(\mathcal{T}, m) \bullet a_k^c = m \right) \wedge$$

$$620 \quad \forall n, m < n < t \bullet \left(\exists k \bullet k \in \text{act}(\mathcal{T}, n) \wedge a_k^c < n \right)$$

622 so the only active tasks at time m are those that released a job at that time, and there are
623 active tasks that have not just been released for all times between m and t . Note m must
624 exist as system startup (time 0) matches the definition of m as the only active tasks are
625 those released at time 0. We note that m is a function of t , hence $m(t)$ in the following.

626 To compute O at time t , we need to know how many overruns each job has experienced.
627 This can be computed as follows:

$$628 \quad O = \sum_{\forall h \in \mathcal{T}_H, s \bullet t > a_h^s \geq m(t)} \left\lceil \frac{e_h^s(t) - C_h(L)}{C_O} \right\rceil_0$$

629 where $\lceil \cdot \rceil_0$ constrains the ceiling function to return a value no less than 0.

630 If this value is greater than 1 but no greater than F_O then the system mode should be
 631 Fail Operational (FO) with all tasks meeting their deadlines. It follows that the rely and
 632 guarantee conditions for the Scheduler are as follows. Remember that for LO -crit tasks
 633 $C(H) = C(L)$:

$$634 \quad R_S^{FO}(t) \stackrel{\text{def}}{=} \forall k \in \text{act}(\mathcal{T}, t) \bullet e_k^c(t) \leq C_k(H) \wedge (a_k^c = t \wedge c > 1) \Rightarrow a_k^c - a_k^{c-1} \geq T_k(L) \wedge$$

$$635 \quad \forall h \in \mathcal{T}_{\mathcal{H},s} \bullet t > a_h^s \geq m(t) \left[\frac{e_h^k(t) - C_h(L)}{C_O} \right]_0 \leq F_O$$

$$636 \quad G_S^{FO}(t) \stackrel{\text{def}}{=} \forall k \in \text{act}(\mathcal{T}, t) \bullet t + C_k(L) - e_k^c(t) \leq v_k^c \wedge$$

$$637 \quad \forall h \in \mathcal{T}_{\mathcal{H}} \bullet a_h^c \geq m(t) \wedge e_h^c(t) > C_h(L) \Rightarrow t + C_k(H) - e_k^c(t) \leq d_k^c(H)$$

641 As there are no overruns in the normal mode we can deduce that $R_S^N \Rightarrow R_S^{FO}$.

642 Note this formulation is structurally different from that given earlier for a pure Vestal-like
 643 model. What the Scheduler must rely on is a property of the whole set of HI -crit tasks, not
 644 a specific property of each individual task. The Scheduler can therefore guarantee $C_h(H)$
 645 (by the task's deadline) to any HI -crit tasks that overrun. But this guarantee is subject to
 646 the rely condition remaining true (i.e. there is a bound on the number and extent of these
 647 overruns).

648 The specification of the HI - and LO -crit tasks in the normal mode, and for most tasks
 649 in the FO mode, is simply

$$650 \quad R_k^{FO}(t) \stackrel{\text{def}}{=} \text{WCET}_k \leq C_k(L) \wedge k \in \text{act}(\mathcal{T}, t) \Rightarrow t + C_k(L) - e_k^c(t) \leq v_k^c$$

$$651 \quad G_k^{FO}(t) \stackrel{\text{def}}{=} e_k^c(t) \leq C_k(L) \wedge (a_k^c = t \wedge c > 1) \Rightarrow a_k^c - a_k^{c-1} \geq T_k(L)$$

653 But for the tasks that overrun, they experience a mode change that moves the system to a
 654 variant of FO :

$$655 \quad R_h^{FO^*}(t) \stackrel{\text{def}}{=} \text{WCET}_h \leq C_h(H) \wedge h \in \text{act}(\mathcal{T}_{\mathcal{H}}, t) \Rightarrow t + C_h(H) - e_h^c(t) \leq d_h^c(H)$$

$$656 \quad G_h^{FO^*}(t) \stackrel{\text{def}}{=} e_h^c(t) \leq C_h(H) \wedge (a_h^c = t \wedge c > 1) \Rightarrow a_h^c - a_h^{c-1} \geq T_h(H)$$

658 For the non overrunning tasks and the Scheduler $R^{FO^*} = R^{FO}$, and $G^{FO^*} = G^{FO}$.

659 A small number of tasks experiencing this change will not cause the Scheduler to change
 660 mode, unless its rely condition is invalidated. The proof obligation (6) will again ensure that
 661 $R_h^{FO^*}$ is a weakening of R_h^N and R_h^{FO} .

662 In summary, a system stays in the normal mode until a single HI -crit task executes for
 663 more than $C(L)$. The system then moves to mode FO with the overrunning task behaving
 664 according to mode FO^* . Further HI -crit tasks may overrun and move to mode FO^* .
 665 Eventually either an idle instant occurs and the system will return to the normal mode N ,
 666 or the F_O count is breached and R_S^{FO} is invalidated. The system will now fail unless there is
 667 a further degraded mode it can transition to; such a mode is considered next.

668 **Fail robust – FR.** A *robust task* is one that can safely drop one non-started job in a
 669 defined time interval. Each task (be it HI -crit or LO -crit), as part of its definition, has a
 670 robustness parameter, w . If a task has successfully completed the execution of w consecutive
 671 jobs then the Scheduler can drop the next job (before it has been given any execution time).
 672 As such jobs should only be dropped if they have to be, this requires a new mode: FR (Fail
 673 Robust). This mode will only be entered if the rely condition of the Scheduler in mode

674 FO becomes false (i.e there are more than F_O overruns). Within FR F_R overruns will be
 675 tolerated (with $F_R > F_O$); i.e.

$$676 \quad \sum_{\forall h \in \mathcal{T}_{\mathcal{H}}, s \bullet t > a_h^s \geq m(t)} \left\lceil \frac{e_h^s(t) - C_h(L)}{C_O} \right\rceil_0 \leq F_R$$

677 We introduce a predicate, $req_k(t)$ (short for required) that returns true if the current
 678 job of task k at time t must be executed. Tasks that require all their jobs to execute are
 679 assigned, for ease of presentation, $w = 0$. The conditions for the current job (k^c) of task k to
 680 be required are: (1) $w_k = 0$, or (2) the task has not yet executed w_k jobs, i.e. $c \leq w$, or (3)
 681 one of the previous w_k jobs (before c) had a zero execution time — this is an indication that
 682 the job was dropped. This leads to the following definition:

$$683 \quad req_k(t) \stackrel{\text{def}}{=} w_k = 0 \vee c \leq w_k \vee \exists s, s \in c - w_k..c - 1 \bullet e_k^s(f_k^s) = 0$$

684 In other words, $req_j(t)$ is false only when the last w_j jobs of τ_j (i.e. $j_j^{c-1}, j_j^{c-2}, \dots, j_j^{c-w_j}$)
 685 have completed successfully. A non robust task is always ‘required’ (in that its current job
 686 must always complete). The R/G conditions can again be easily derived for the Fail Robust
 687 mode:

$$688 \quad R_S^{FR}(t) \stackrel{\text{def}}{=} \forall k \in act(\mathcal{T}, t) \bullet e_k^c(t) \leq C_k(H) \wedge (a_k^c = t \wedge c > 1) \Rightarrow a_k^c - a_k^{c-1} \geq T_k(L) \wedge$$

$$689 \quad \sum_{\forall h \in \mathcal{T}_{\mathcal{H}}, s \bullet t > a_h^s \geq m(t)} \left\lceil \frac{e_h^s(t) - C_h(L)}{C_O} \right\rceil_0 \leq F_R$$

691 Note this is a weakening of the rely condition as $R_S^{FO} \Rightarrow R_S^{FR}$ which follows from $F_R > F_O$
 692 i.e. more overruns can be tolerated in the Fail Robust mode.

693 We can now complete the full specification. The Scheduler only guarantees execution
 694 time to those jobs that are required; moreover, if a job is not required the Scheduler ensures
 695 it does not execute.

$$696 \quad G_S^{FR}(t) \stackrel{\text{def}}{=} \forall k \in act(\mathcal{T}, t) \bullet req_k(t) \Rightarrow t + C_k(L) - e_k^c(t) \leq v_k^c \wedge$$

$$697 \quad \forall k \in \mathcal{T} \bullet a_k^c = t \wedge \neg req_k(t) \Rightarrow f_k^c = t \wedge$$

$$698 \quad \forall h \in \mathcal{T}_{\mathcal{H}} \bullet a_h^c \geq m(t) \wedge e_h^k(t) > C_h(L) \Rightarrow t + C_k(H) - e_k^c(t) \leq d_k^c(H)$$

701 The tasks only need execution time if they are required; their guarantee conditions remain
 702 true even if the current job does not execute.

$$703 \quad R_k^{FR}(t) \stackrel{\text{def}}{=} WCET_k \leq C_k(L) \wedge k \in act(\mathcal{T}, t) \wedge req_k(t) \Rightarrow t + C_k(L) - e_k^c(t) \leq v_k^c$$

$$704 \quad G_k^{FR}(t) \stackrel{\text{def}}{=} e_k^c(t) \leq C_k(L) \wedge (a_k^c = t \wedge c > 1) \Rightarrow a_k^c - a_k^{c-1} \geq T_k(L)$$

706 As with mode FO , an individual HI -crit task can fail (rely condition becomes invalid, false)
 707 leading to a weakened specification:

$$708 \quad R_h^{FR^*}(t) \stackrel{\text{def}}{=} WCET_h \leq C_h(H) \wedge h \in act(\mathcal{T}_{\mathcal{H}}, t) \Rightarrow t + C_h(H) - e_h^c(t) \leq d_h^c(H)$$

$$709 \quad G_h^{FR^*}(t) \stackrel{\text{def}}{=} e_h^c(t) \leq C_h(H) \wedge (a_h^c = t \wedge c > 1) \Rightarrow a_h^c - a_h^{c-1} \geq T_h(H)$$

711 Note the req_k condition has been removed from the rely condition as the current job must
 712 be required to execute as it has a non-zero execution time (i.e. a value that exceeded $C(L)$);
 713 also this is another weakening of the rely condition.

714 Again with this specification the Scheduler must rely on a property of the whole set of
715 *HI*-crit tasks, not a specific property of each individual task.

716 **Fail resilient (graceful degradation) – GD.** Once the count of job failures becomes
717 greater than F_R , the *FR* mode must be abandoned as the rely condition of the Scheduler
718 becomes false. To add resilience, a number of different general strategies for graceful
719 degradation have been discussed in the literature [55, 45, 54]. Some strategies are hierarchical,
720 in that they form a natural progression of increasingly severe forms of degradation that
721 are invoked by increasingly severe forms of failure. Others take the form of alternative
722 approaches.

723 All strategies are defined by their level of fault tolerance (the maximum O count they
724 can deal with) and their impact on *LO*-crit tasks. Example strategies include:

- 725 1. Increasing the periods and deadlines of *LO*-crit tasks [60, 59, 36, 58, 57, 53, 25], called
726 *task stretching*, the *elastic task model* or *multi-rate* (also see Section 5.1)
- 727 2. Imposing only a weakly-hard constraint on the *LO*-crit tasks [24, 51]
- 728 3. Decreasing the computation times of the *LO*-crit tasks [13, 4], perhaps by utilising an
729 imprecise mixed-criticality (IMC) model [50, 52, 49, 33] or budget control [26, 27]
- 730 4. Moving some *LO*-crit tasks to a different processor that has not experienced a criticality
731 mode change [63, 64, 35, 3].
- 732 5. Abandoning *LO*-crit work in a disciplined sequence [23, 34, 28, 56, 46, 47].

733 Some example strategies have already been described in the paper. Of course the specific
734 set of schemes that may be applicable will depend on the details of the application. Never-
735 theless, any collection of approaches can be (partially) ordered using preferences and the
736 strengths/weaknesses of the rely conditions of the Scheduler.

737 In general, the full set of modes forms a lattice with the Normal N mode at the top, and
738 the Fail Safe (*FS*) mode at the bottom (see below). Preferences are assigned to reflect the
739 structure of this lattice (N is the most preferred mode, *FS* the least). The least preferred
740 resilient mode is the one that represents the total abandonment of all *LO*-crit jobs. We define
741 this to be the backstop mode (*BM*). In the following *BM* is entered after the failure of *GD*:

$$742 R_S^{BM}(t) \stackrel{\text{def}}{=} \forall h \in \text{act}(\mathcal{T}_{\mathcal{H}}, t) \bullet e_h^c(t) \leq C_h(H) \wedge (a_h^c = t \wedge c > 1) \Rightarrow a_h^c - a_h^{c-1} \geq T_h(H)$$

$$743 G_S^{BM}(t) \stackrel{\text{def}}{=} \forall h \in \text{act}(\mathcal{T}_{\mathcal{H}}, t) \bullet t + C_h(H) - e_h^c(t) \leq d_h^c \wedge \forall l \in \text{act}(\mathcal{T}_{\mathcal{L}}, t) \bullet e_h^c(t) = e_h^c(\eta^{GD})$$

745 where again η^{GD} is the time this mode is entered (i.e. when some graceful degradation mode,
746 *GD* must be abandoned). Now no active *LO*-crit jobs execute.

$$747 R_h^{BM}(t) \stackrel{\text{def}}{=} \text{WCET}_h \leq C_h(H) \wedge h \in \text{act}(\mathcal{T}_{\mathcal{H}}, t) \Rightarrow t + C_h(H) - e_h^c(t) \leq d_h^c(H)$$

$$748 G_h^{BM}(t) \stackrel{\text{def}}{=} e_j^h(t) \leq C_h(H) \wedge (a_h^c = t \wedge c > 1) \Rightarrow a_h^c - a_h^{c-1} \geq T_h(H)$$

$$749 R_l^{BM}(t) \stackrel{\text{def}}{=} \text{true}$$

$$750 G_l^{BM}(t) \stackrel{\text{def}}{=} (a_l^c = t) \Rightarrow (f_l^c = t)$$

754 hence any newly arrived *LO*-crit job is immediately finished (aborted).

755 **Fail safe/restarts – FS.** The final ‘strategy’ is fail safe, perhaps via fail stop, followed
756 by a subsequent restart (which may use a cold, warm or hot standby). It is not the purpose
757 of this paper to review these approaches to fault tolerance. But for completeness we note
758 that wherever possible there should be a mode (*FS*) which guarantees a fail safe outcome.

$$759 P_S^{FS} \stackrel{\text{def}}{=} \text{true}$$

760

761 $R_S^{FS}(t) \stackrel{\text{def}}{=} true$

762

763 $G_S^{FS}(t) \stackrel{\text{def}}{=} t \leq (\eta^{BM} + D_S^{FS})$

764

765 $Q_S^{FS} \stackrel{\text{def}}{=} safe_shut_down$

766 where D_S^{FS} is the (relative) deadline of the scheduler in this mode – there is a bound on how
767 far t can reach.

768 This mode must be the lowest preference mode (i.e. be at the base of the lattice). It can
769 always be entered, but must only be entered when all Schedulers in other modes have rely
770 conditions that are false. Note we give the Scheduler a deadline in this mode to instigate the
771 shut-down activity, but no further functional information can be given as the Scheduler is no
772 longer operational.

773 6.2 Robust and resilient mode changes

774 In the above discussion a number of Scheduler modes have been introduced. They naturally
775 form a sequence based on preference; the inverse of this sequence describes the behaviour of
776 the system as it experiences graceful degradation:

777 $N \rightarrow FO \rightarrow FR \rightarrow GD \rightarrow BM \rightarrow FS$

778 An application could have a number of intermediate modes between FR and BM . In addition
779 there could be a number of ‘best-effort’ (not guaranteed) behaviours/modes between BM
780 and FS .

781 For the set of operational modes it will be necessary to show they form a hierarchy:

782 $R^N \Rightarrow R^{FO} \Rightarrow R^{FR} \Rightarrow R^{GD} \Rightarrow R^{BM} \Rightarrow R^{FS}$

783 Moreover, at the time a rely condition becomes invalid and the next mode is entered (at times
784 $\eta^N, \eta^{FO}, \eta^{FR}, \eta^{BM}$), it can be proven (see Lemmas 2 and 3) that the new rely condition is
785 true and henceforth the guarantee condition holds.

786 In contrast to this gradual decline in functionality, a system that is programmed to
787 recover will move directly from any of the degraded modes back to mode N . This move is
788 driven by preference; but to reenter the Normal mode there will be some prerequisites. As
789 noted in Section 5.3 this could be simply that at the time the Normal mode is re-entered
790 there are no active tasks that had been released prior to this time.

791 7 Conclusions and Future Work

792 There is extensive published work on Mixed-Criticality scheduling and implementation, but
793 not on their formal specification. We believe formalisation is essential since the notion of
794 mixed criticality has subtle semantics: often concepts such as correctness, resilience and
795 robustness are neither straightforward nor intuitive for such systems. The R/G approach
796 has proved a successful formalism for specifying non-real-time safety-critical systems and our
797 main contribution in this paper is to extend R/G to (i) time, and (ii) multiple criticalities.

798 The proposed framework is based on an ordering of modes (in general, this would form
799 a lattice) with the normal mode (N) being at the top and a Fail Stop (FS) mode at the
800 base. Each mode has an R/G coupling with a move down the ordering accompanied by a
801 weakening of the rely and guarantee conditions. Examples were used to show that to obtain

802 a true hierarchical relationship between the rely conditions (e.g. $R^A \Rightarrow R^B$, for modes A
 803 and B), it is often necessary to strengthen the R^A and/or weaken the R^B conditions. A
 804 movement of the system down the ordering (from mode A to B) occurs only when forced by
 805 R^A no longer being true. At this time it is necessary to prove that R^B remains true. The
 806 return of the system back to mode N is sanctioned by the rely and pre conditions of N being
 807 reestablished.

808 The examples presented in this paper have demonstrated that the developed approach
 809 has the expressive power necessary to enable a wide range of possible runtime strategies to be
 810 precisely specified and evaluated (in terms of their internal consistency). Further work will
 811 address the application of the R/G specifications in the development of the necessary run-time
 812 code that will be needed to support these mixed-criticality protocols. This would benefit
 813 from mechanical proof support as undertaken by the PROSA team [21, 10]. Although this
 814 work is not covered in the current paper there is ample evidence that R/G specifications can
 815 form the basis for the formal development of implementations. A useful example is tackled
 816 in [43, 41]: although not scheduling per se, Simpson's 4-slot algorithm is a delicate piece of
 817 intricate code for asynchronous communication mechanisms. A number of other examples of
 818 developments based on R/G specifications are listed and/or tackled in [48, 31, 44, 9].

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