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Robust Distributed Control for DC Microgrids with System Constraints

Grigoris Michos, Pablo R. Baldivieso-Monasterios, George C. Konstantopoulos

***Abstract*—This work proposes a distributed robust control architecture for meshed DC Microgrid networks. Each interlinking converter is modelled as a network node and is connected in parallel to a constant power load representing the network's power consumption. Each node employs a local controller consisting of two parts; current regulation based on a modified version of the state-limiting PI and a distributed MPC driving the system to desired setpoints. We analytically prove each controller's robustness to model variations caused by changes in both the power demand and the transmitted information among the subsystems. The concept of positive invariance sets and the inherent robustness properties of the nominal MPC are used to prove recursive feasibility of the optimal control problem and guarantee constraint satisfaction at all times. The stability proof of the cascaded node dynamics is based upon the emerging properties of both the state limiting PI and the distributed MPC design. Demonstration of the results is given in a simulated scenario.**

***Index Terms*—Network systems, Distributed control, optimal control, Microgrids, optimal control.**

I. Introduction

The concept of a MicroGrid (MG) brought a paradigm change to the architecture of conventional power networks [1]. The traditionally centralised structure shifted to geographically decentralised clusters, that are able to operate both isolated, known as islanded mode, and in a grid-connected setting. The MG structure can be found in both AC and DC architectures, however in many cases the use of a DC structure is often preferred because it provides higher efficiency and reliability, e.g. in High-Voltage-Direct-Current networks, aircrafts and transportation vehicles [2].

In this context, the MG attempts to integrate a variety of different energy sources, consumption and storage units into a network system that can be controlled by a local controller. In order to bring the network operation to desired levels and achieve homogeneous operation, the energy sources are interlinked with converters that regulate their respective output current and voltage, as well as achieve the conversion to DC output in the case of an AC source. The common control strategy that has been widely used in the literature is the adoption of the droop control method. In its most basic form, it operates as a static output resistance that achieves proportional power sharing and stable steady state operation [3]. In many cases, the system is required to operate within a predefined range in order to prevent high transients from damaging the electronic components. This overvoltage and overcurrent protection can be achieved by employing saturation devices, however it has been shown that this leads to performance degradation and may cause loss of stability [4]. Furthermore, droop control methods often result in poor power sharing among the nodes and unnecessary power losses caused by circulating currents.

In light of these issues, MPC and other optimisation techniques have been employed in order to include the operational constrains in the control design procedure and improve the system performance, see [5]. Despite the fact that an MG represents a smaller part of the overall power network, it follows similar control principles with a Large-Scale system. Centralised techniques may reduce the reliability and resilience of the system to faults, as an all-to-all communication network requires transmission of large volume of information in very short times. In order to address these issues, in this paper we investigate the problem of designing a distributed control architecture that reduces the communication network to a neighbour-to-neighbour structure. Distributed MPC architectures have previously been proposed in the literature, see [6] for a detailed survey. Notably, the authors of [7] propose a method that bounds the subsystem interactions influenced by the Tube-MPC approach [8]. The main differences with the approaches presented in the literature is the extension to a nonlinear setting and dealing with strong system coupling; the latter prohibits the use of a Tube-based method to deal with the system interactions. We show that using the proposed control scheme, system robustness to both load and transmitted information variations can be achieved, while asymptotic stability of the cascaded dynamics and constraint satisfaction for the uncertain system are guaranteed. More specifically, Section II presents the problem formulation, Section III proposes a system decomposition of the uncertain dynamics and describes the control design of each dynamical component. We analytically show the recursive feasibility of the optimal control problem (OCP), as well as prove the stability of the cascaded dynamics in Section IV, and finally Section V demonstrates the closed loop system operation in a simulated scenario.

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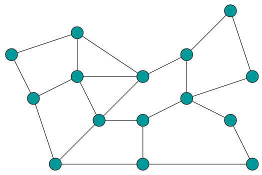

Fig. 1. Network topology of a meshed islanded MicroGrid.

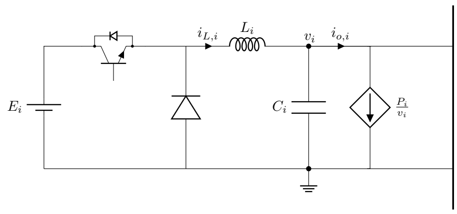

Fig. 2. Node circuit modelled as a DC/DC Buck converter connected in parallel to a local constant power load and the local bus.

A. Notation

A MG can be seen as an undirected graph $\mathcal{G} = (\mathcal{M}, \mathcal{E})$ where the set of nodes $\mathcal{M}$ represent a collection of power converters and local loads; the set of edges $\mathcal{E} \subseteq \mathcal{M} \times \mathcal{M}$ defining the MG topology is characterised by the node-edge matrix $\mathcal{B} \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{M}|}$ which for edge $\varepsilon = (i, j) \in \mathcal{E}$ involving nodes i and j can be defined as $[\mathcal{B}]_{ei} = 1$ if node i is the source of $e \in \mathcal{E}$, and $[\mathcal{B}]_{ej} = -1$ if node j is its sink, and zero otherwise.

II. Problem Formulation

We consider the problem of controlling a meshed DC Micro-Grid with interlinking buck converters, see Fig.1. We model the Micro-Grid as a connected undirected graph with $|\mathcal{M}|$ number of nodes. Each node represents a DC/DC buck converter integrating a distributed energy resource (DER) unit into the network, see Fig.2. The output voltage of the DER unit is modelled as the source of the buck converter. The consumer is modelled as a constant power load (CPL), represented by a current source in parallel to the output capacitor. Utilizing the Kirchoff's laws, the local node dynamics of each subsystem are

$$L_i \frac{di_i}{dt} = u_i E_i - v_i, \tag{1a}$$

$$C_i \frac{dv_i}{dt} = i_i - i_{o,i}, \tag{1b}$$

where L_i is the converter inductance, C_i the output capacitance, u_i the duty ratio of the switching element, E_i the input voltage, $i_{o,i}$ is the output current, and (i_i, v_i) the converter current and node voltage respectively. Let the set $\mathcal{N}_i = \{j \in \mathcal{M} \colon\ \mathcal{L}_{ij} \neq 0,\ i \neq j\}$ denote the neighbours of the i^{th} node, *i.e.* the nodes of the network where there exists a direct line connecting the two nodes. One can define the incidence matrix of the network $\mathcal{B} \in \mathbb{R}^{n\times m}$, where the each column denotes an edge $\varepsilon_{ij} \in \mathcal{E}$. In addition, let $R \in \mathbb{R}^{m\times m}$ be a diagonal matrix denoting the line resistances, *i.e.* the graph's edge weights, then R^{-1} is the line admittance matrix. The topology of the network is then described through the Laplacian matrix $\mathcal{L} = \mathcal{B}^\top R^{-1}\mathcal{B}$. The output current $i_{o,i}$ in (1b) is shared between the local load and the network, given as the sum of the load current and the edge line current connected to the i^{th} node, *i.e.*

$$i_{o,i} = i_{L,i} + i_{T,i} = \frac{P_i}{v_i} + \mathcal{L}_{ii} v_i - \sum_{j\in\mathcal{N}_i} \mathcal{L}_{ij} v_j$$

where P_i is the local power demand. It is evident that there exists a strong coupling between neighbouring subsystems. To adopt an overvoltage and overcurrent protection, each subsystem is subjected to voltage and current constraints represented by the sets $\mathbb{X}_i$ and $\mathbb{X}_{c,i}$ respectively. By decomposing the load current as the summation of a nominal load $\bar{P}_i$ and the deviation from that load, *i.e.* $i_{L,i} = (\bar{P}_i + \delta P_i)/v_i$, the system uncertainty can be reduced to $w_i = \frac{\delta P_i}{v_i}$. Hence, we can re-write (1b) as

$$C_i \frac{dv_i}{dt} = i_i - \frac{\bar{P}_i}{v_i} - \mathcal{L}_{ii} v_i - \sum_{j\in\mathcal{N}_i} \mathcal{L}_{ij} v_j + w_i.$$

where for $D_i(P_i, v_i) = \frac{\bar{P}_i}{v_i}$ and bounded load variations $\delta P_i \in \delta\mathbb{P}_i$, the disturbance is bounded within $\mathbb{W}_i = D_i(\delta\mathbb{P}_i, \mathbb{X}_i)$. In order to ensure regularity of the optimal control problem developed later, we adopt the following assumption.

Assumption 1. *The sets $\mathbb{X}_i$, $\mathbb{X}_{c,i}$, $\delta\mathbb{P}_i$ are compact and $\mathbb{X}_{c,i}$, $\delta\mathbb{P}_i$ include the origin in their nonempty interior.*

III. Control Design

This section describes the control design procedure for each local subsystem of the form (1). In order to simplify the analysis, we invoke a common assumption in the literature; the converter states are operating in different time scales, *i.e.* the converter current has converged and is considered constant in the analysis of the node voltage, see for example [9] or [10]. This enables a separate analysis for each dynamic component.

A. Converter Current Regulation

We start by designing the converter current controller. In order to bound the current trajectories within $\mathbb{X}_{c,i}$, we employ a modified version of the state-limiting PI, first introduced in [11]. We formulate the duty ratio as

$$\begin{aligned} u_i &= (v_i - k_{p,i} i_i + M_i \sigma_i)/E_i \\ M_i \dot{\sigma}_i &= k_{I,i}(1 - \sigma_i^2)(\hat{i}_i - i_i). \end{aligned} \tag{2}$$

where $k_{p,i}, k_{I,i}, M_i$ are tuning parameters, σ is an integrator state, and $\hat{i}_i$ is the reference current. Substituting to (1a) results in the closed loop dynamic behaviour

$$L_i \frac{di_i}{dt} = -k_{p,i} i_i + M_i \sigma_i \tag{3}$$

In the following proposition we show that an appropriate choice of tuning parameters results in the desired property.

Proposition 1 (Boundedness of the node current). *The set* $\mathbb{C}_i = [-\mathrm{I}_{\max,i}, \mathrm{I}_{\max,i}] \times [-1,1] \subseteq \mathbb{X}_{c,i}$ *with* $\mathrm{I}_{\max,i} = \frac{M_i}{k_{p,i}}$ *is control invariant for the converter current dynamics (3).*

Proof. We begin by showing the boundedness of the integrator dynamics. For each individual component $\sigma_i(0) \in [-1,1]$, we assume there is a time τ_2 such that $|\sigma_i(\tau_2)| > 1$. By continuity of the dynamics, there exists a time τ_1 such that $|\sigma(\tau_1)| = 1$. By defining a function $E_{\sigma,i} = \frac{1}{2}\sigma_i^2$ with $\dot{E}_{\sigma,i} = \sigma_i \frac{k_{I,i}}{M_i}(1-\sigma_i^2)(\hat{i}_i - i)$ it is seen that at $|\sigma_i| = 1$ we have $\dot{E}_{\sigma,i} = 0$ and therefore the state trajectory cannot escape the circle $|\sigma_i| = 1$. This leads to a contradiction, and we conclude $|\sigma_i| \leq 1$.

We consider the energy function of the inductor $E_{c,i} = \frac{1}{2}L_i i_i^2$ with time derivative

$$\dot{E}_{c,i} = -k_{p,i} i_i^2 + i_i M_i \sigma_i \leq -k_{p,i}|i_i|^2 + |i_i| M_i.$$

Following [12, Theorem 4.18], given $i_i(0) \leq \frac{M_i}{k_{p,i}}$, then the solution of the system $i_i(t)$ is ultimately bounded with bound $\mathrm{I}_{\max,i} = \frac{M_i}{k_{p,i}}$. □

B. Node Voltage Regulation

The second part of the control design is the voltage regulation. The time-scale separation allows the use of the reference current $\hat{i}_i$ as a control input to the voltage loop. In the following, we propose a procedure to compute $\hat{i}_i$, such that the dynamical behaviour of the voltage is always constrained within the voltage constraint set $\mathbb{X}_i = \{v_i \in \mathbb{R}\colon\ \underline{v}_i \leq v_i \leq \bar{v}_i\}$. In addition, we enforce the reference current to be generated within the bounded range specified in Proposition 1, by adopting an input control set $\mathbb{U}_i \subseteq [-\mathrm{I}_{\max,i}, \mathrm{I}_{\max,i}]$. This prevents possible saturation functions, and thus discontinuities, emerging in the equilibrium point mapping of the closed loop system.

We propose a non-iterative, non-cooperative distributed control architecture that introduces robustness of the system to uncertainties caused by variations in the load δP_i and the interaction term. We begin by writing the uncertain system

$$C_i \frac{dv_i}{dt} = \hat{i}_i - \frac{\bar{P}_i}{v_i} - \mathcal{L}_{ii} v_i - \sum_{j \in \mathcal{N}_i} \mathcal{L}_{ij} v_j + w_i, \tag{4}$$

and define a nominal system

$$C_i \frac{dz_i}{dt} = \eta_i - \frac{\bar{P}_i}{z_i} - \mathcal{L}_{ii} z_i + d_i, \tag{5}$$

where $d_i = -\sum_{j \in \mathcal{N}_i} \mathcal{L}_{ij} z_j$. The evolution of the error between the nominal and the uncertain state $e_i = v_i - z_i$ is formulated as,

$$C_i \dot{e}_i = -\mathcal{L}_{ii} e_i + \frac{\bar{P}_i}{z_i(z_i+e_i)} e_i - \sum_{j \in \mathcal{N}_i} \mathcal{L}_{ij} e_j - K_i e_i + w_i, \tag{6}$$

where we have substituted for $\hat{i}_i = \eta_i - K_i e_i$. The resulting error subsystem has two inputs, the nominal voltage z_i and the disturbance w_i. In the following, we will exploit the boundedness of both external inputs in order to show that the error dynamics are restricted within a robust positive invariant set, when an appropriate choice of an initial state is made.

Proposition 2 (Robust positive invariance of error dynamics). *For a uniformly bounded nominal voltage* $z_i(t)$ *with* $\underline{z}_i \leq z_i \leq \bar{z}_i$ *and bounded disturbance* $w_i \in \mathbb{W}_i$*, if*

$$K_i > \frac{\bar{P}}{\underline{z}^2},$$

then the dynamics (6) are contained in a robustly positive invariant set $\mathcal{S}_i$*.*

Proof. We begin by considering the overall nominal part of the error system in order to exploit the Laplacian properties of the dynamics,

$$C\dot{e} = -\mathcal{L}e + \mathrm{diag}\left\{\frac{\bar{P}_i}{z_i(z_i+e_i)}\right\} e - Ke$$

where C, K are diagonal positive definite matrices. We need to establish conditions such that this system is asymptotically stable, while considering z as a bounded input to the system. Using the Taylor expansion around an equilibrium point $(0, \hat{z})$, the Jacobian matrix is computed as,

$$J = C^{-1}\left(-\mathcal{L} + \mathrm{diag}\left\{\frac{\bar{P}_i}{\hat{z}_i^2}\right\} - K\right)$$

where we need $J \prec 0$ in order to show asymptotic stability of the nominal dynamics. We note that the Laplacian matrix $\mathcal{L}$ eigenvalues satisfy $\lambda_1 = 0 < \lambda_2 \leq \cdots \leq \lambda_n$. By using the fact that $\lambda_{\max}(-\mathcal{L}) = 0$ in combination with the diagonal structure of the latter two summands, we can deduce a sufficient condition for $J \prec 0$ in the scalar form

$$K_i > \frac{\bar{P}_i}{\hat{z}_i^2}.$$

The right hand side of the above inequality is maximised at $\hat{z}_i = \underline{z}_i > 0$, thus a substitution to the above with $\underline{z}_i$ results in the required condition. Since $w \in \mathbb{W}$, the system has a local-ISS property, where there exists a constant $a > 0$ such that $\dot{e} \leq ae + w$ and the error is bounded in $\mathcal{S} = a^{-1}\mathbb{W}$. Hence, for each $i \in \mathcal{M}$, the node error voltage is contained in the respective projection of $\mathcal{S}$, or simply $e_i(t) \in \mathcal{S}_i$. This completes the proof. □

We can now formulate the nominal control policy that guarantees boundedness and asymptotic stability of the nominal voltage dynamics. We adopt a parametrisation of

$$\eta_i = \nu_i^o - K_{\eta,i} z_i. \tag{7}$$

The first term is the first element of a control sequence $\boldsymbol{\nu}_i^o$ generated by solving a finite receding horizon (RC) OCP. This distributed MPC problem $\mathbb{P}_i(z_i, d_i)$ consists of

$$J^o(z_i, \nu_i, z_i^r, \nu_i^r, d_i) = \min_{\boldsymbol{\nu}_i} \int_{t_0}^{t_f} \ell_i(z_i - z_i^r, \nu_i - \nu_i^r)dt + J_{f,i}(z_i(t_f) - z_i^r) \tag{8}$$

subject to the constraints,

$$
\begin{aligned}
& z_i(0) = z_i \\
& C_i \dot{z}_i = \nu_i - K_{\eta,i} z_i - \frac{\bar{P}_i}{z_i} - \mathcal{L}_{ii} z_i + d_i, \\
& (z_i, \nu_i) \in Z_i \times U_i, \;\; z_{f,i} \in Z_{f,i},
\end{aligned} \tag{9}
$$

where (z_i^r, ν_i^r) is a desired setpoint pair. The constraint sets of the nominal problem are "tighter" versions of the original constraint sets $\mathbb{X}_i$ and $\mathbb{U}_i$ respectively. These have been parametrised using the robust invariant set of the error dynamics as $Z_i = \mathbb{X}_i \ominus \mathcal{S}_i$ and $U_i = \mathbb{U}_i \ominus \mathcal{S}_i \oplus K_{\eta,i} Z_i$, thus effectively constraining the uncertain system operation within the respective original constraint sets. Furthermore, we invoke the following standard assumptions, see [13], that will assist in establishing recursive feasibility of (8).

Assumption 2. *For each node $i \in \mathcal{M}$, the sets $\mathbb{X}_i$, $\mathbb{U}_i$ are compact and $\ell_i \colon Z_i \times U_i \to \mathbb{R}_i$ is a positive definite, continuous function.*

Assumption 3. *For each node $i \in \mathcal{M}$, the terminal cost function satisfies*

$$J_{f,i}(z_i^*) + \ell(z^*, \nu^*) \le J_{f,i}(z_i), \; \forall \in \mathbb{Z}_i$$

Assumption 4. *For each node $i \in \mathcal{M}$, the terminal set $Z_{f,i}$ is control invariant for the nominal dynamics (5).*

The second term of (7) counteracts the steady state negative impedance effect of the CPL and assists in establishing asymptotic stability of the terminal dynamics, *i.e.* beyond the horizon $[0, T]$.

Proposition 3 (Stability of the terminal dynamics)**.** *Considering an equilibrium $\hat{z}_i$ of the nominal voltage dynamics (5), if*

$$K_{\eta,i} > \frac{\bar{P}}{\hat{z}_i^2}$$

then $\hat{z}_i$ is stable for the terminal dynamics with $t \in (T, \infty)$.

We omit the proof as it similar to the one of Proposition 2.

IV. Recursive Feasibility and Stability Analysis

This section provides an analysis of the recursive feasibility properties of the RC-OCP, followed by an analysis on the stability properties of the cascaded system dynamics. The state decomposition of the previous section has resulted in the cascaded dynamics

$$L_i \frac{di_i}{dt} = -k_{p,i} i_i + M_i \sigma_i \tag{10a}$$

$$M_i \frac{d\sigma_i}{dt} = k_{I,i}(1 - \sigma_i^2)(\nu_i^0(z) - K_{\eta,i} z_i - K_i e_i - i_i) \tag{10b}$$

$$C_i \frac{dz_i}{dt} = \nu_i^0(z_i) - K_{\eta,i} z_i - \frac{\bar{P}_i}{z_i} - \mathcal{L}_{ii} z_i + d_i, \tag{10c}$$

$$C_i \frac{de_i}{dt} = -\mathcal{L}_{ii} e_i + \frac{P_i}{z_i(z_i + e_i)} e_i - \sum_{j \in \mathcal{N}_i} \mathcal{L}_{ij} e_j - K e_i + w_i, \tag{10d}$$

where e_j, d_i are piecewise constant signals as a result of the communication structure we have adopted. This is outlined in the following assumption.

Assumption 5 (Communication framework)**.** *At each sampling instant t, each node receives the voltages v_j, z_j from its neighbouring nodes $j \in \mathcal{N}_i$ and assumes $v_j, z_j =$ constant until $t + \delta$, with $\delta > 0$ acting as a sampling interval.*

First, we analyse the recursive feasibility properties of $\mathbb{P}_i$, which by applying Assumptions 2-4 meets all the requirements outlined in [13] with a small caveat; the interaction term is included in the optimisation problem, however its dynamic behaviour is restricted to a piecewise change over the sampling intervals. At each time t the local OCP constructs $d_i(t)$ and solves the optimisation problem considering a constant interaction, *i.e.* $d_i(t) = \bar{d}_{i,t}$ over the horizon T, ignoring possible future changes in the interaction signal. Then, at time $t + \delta$, the OCP constructs the new interaction term $d_i(t + \delta) = \bar{d}_{i,t+\delta}$ and solves the problem with prediction horizon $[t + \delta, t + \delta + T]$.

Let the feasibility set of the OCP at time $t + \delta$ be defined as $\mathcal{Z}_i^{T+1}(d_i) := \{v_i \in Z_i \colon \exists \nu_i \in U_i, \; x(t) \in \mathcal{Z}_i^T(d_i)\}$. The fact that $\bar{d}_{i,t} - \bar{d}_{i,t+\delta} \neq 0$ can "brake" the nesting property of the feasibility sets $\mathcal{Z}_i^T \supseteq \mathcal{Z}_i^{T-1} \supseteq \cdots \supseteq \mathcal{Z}_{f,i}$ and thus result in the loss of the recursive feasibility property, see [14]. Nevertheless, similar to [15], we can exploit the inherent robustness properties of the nominal MPC and impose conditions on the variations of the interaction signal between sampling times. This will allow us to preserve the recursive feasibility property of the OCP. We start by the simple case, where the interaction term remains unchanged between sampling times, *i.e.* $\bar{d}_{i,t} - \bar{d}_{i,t+\delta} = 0$.

Proposition 4. *Suppose Assumptions 1-4 hold and $d_i(t) = d_i(t + \delta)$, then $z_i(t) \in \mathcal{Z}_i^T(d_i)$ implies $z_i(t + \delta) \in \mathcal{Z}_i^T(d_i^+)$.*

Proof. Consider the solution of the problem at time t with resulting state and control predicted trajectories $z_i^o\big([t, t+T], d_i\big)$ and $\nu_i^o\big([t, t+T], d_i\big)$ respectively. Due to fact that the terminal set $Z_{f,}$ is control invariant, combined with the fact that $d_i(t) = d_i(t+\delta)$, the resulting predicted trajectories $z_i^o\Big([t+\delta, t+\delta+T], d_i^+\Big)$ and $\nu_i^o\Big([t, t+T], d_i^+\Big)$ are the tails of those at time t with $z_i^o\Big(t + \delta + T, d_i^+\Big) = z_{f,i}$. Therefore, we can conclude that the OCP at time $t + \delta$ is feasible and thus the nesting property holds, *i.e.* $\mathcal{Z}_i^T(d_i) \supseteq \mathcal{Z}_i^{T-\delta}(d_i^+)$. $\square$

The next step is to prove recursive feasibility when the interaction signal is permitted to change over the sampling intervals. To this aim we invoke the following assumption on the continuity of the value function of (8).

Assumption 6 ($\mathcal{K}$-continuity of the value function)**.** *For each node $i \in \mathcal{M}$, the value function $J_i^o(\cdot)$ satisfies,*

$$\big|J_i^o(z_1) - J_i^o(z_2)\big| \le \mathcal{F}(|z_1 - z_2|)$$

where $\mathcal{F}$ is a class-$\mathcal{K}$ function.

We also require to bound the interaction variations over the sampling intervals as in the following assumption.

Assumption 7 (Bounded interaction signal variations)**.** *For each node $i \in \mathcal{M}$, the signal variation $\delta d_i = d_i^+ - d_i$ satisfies,*

$$\delta d_i \leq \mathcal{F}^{-1}((\rho_i - \gamma_i)\beta_i)$$

where $\gamma_i \in (0,1)$, $\rho_i \in (\gamma, 1)$ and $\beta_i > 0$ such that it defines a level set of the value function $\Omega_{\beta,i}(d_i) = \{z_i \in \mathcal{Z}_i^T(d_i)\colon\ J_i^o(d_i) \leq \beta_i\}$.

We are now in place to show the recursive feasibility of the proposed distributed controller using the next theorem.

Theorem 1 (Recursive feasibility of the distributed controller)**.** *Suppose Assumptions 1-7 hold. If $z_i(t) \in \mathcal{Z}_i^T(d_i)$, then this implies that at time $t+\delta$ the state satisfies $z_i(t+\delta) \in \mathcal{Z}_i^T(d_i^+)$.*

Proof. Suppose that $z_i \in \Omega_{\alpha,i}(d_i)$ for any $\alpha_i \geq \beta_i \geq 0$ that satisfies $\Omega_{\alpha,i}(d_i) \subseteq \mathcal{Z}_i^T(d_i)$. Then, using Proposition 4 the value function satisfies the monotonic descend property, *i.e.* ,

$$J_i^o(z_i^+, d_i) \leq \gamma_i J_i^o(z_i, d_i).$$

In addition, using Assumption 6 we obtain

$$J_i^o(z_i^+, d_i^+) - J_i^o(z_i^+, d_i) \leq \mathcal{F}\left(\left|d_i^+ - d_i\right|\right)$$

Using the boundedness of the interaction signal variations from Assumption 7,

$$\begin{aligned} J_i^o(z_i^+, d_i^+) &\leq J_i^o(z_i^+, d_i) + ((\rho_i - \gamma_i)\beta_i) \\ &\leq \gamma J_i^o(z_i, d_i) + ((\rho_i - \gamma_i)\beta_i) \\ &\leq \gamma\alpha_i + ((\rho_i - \gamma_i)\alpha_i) \\ &\leq \rho_i\alpha_i \\ &< \alpha_i. \end{aligned}$$

Therefore, the state at the next sampling interval satisfies $z_i^+ \in \Omega_{\alpha,i}(d_i)$ and as a result $\Omega_{\alpha,i}(d_i)$ is invariant for the nominal dynamics with $z_i^+ \in \mathcal{Z}_i^T(d_i^+)$. □

We conclude this section by proving the stability of the cascaded dynamics using the driving/driven subsystem principle [16]. According to this, asymptotic stability of the cascaded dynamics follows from asymptotic stability of the driving subsystem and boundedness of the driven dynamics. We start by showing asymptotic stability of the current dynamics.

Theorem 2 (Stability of the current dynamics)**.** *For every node $i \in \mathcal{M}$, the $\mathcal{C}^1$ function $\mathcal{W}_i\colon\ \mathbb{R} \times \mathbb{R} \to \mathbb{R}$,*

$$\begin{aligned} \mathcal{W}_i(i_i, \sigma_i) = \frac{1}{2}L_i(i_i - \hat{i}_i)^2 + \frac{M_i^2 - M_i k_{p,i}\hat{i}_i}{k_{I,i}} \ln\frac{1}{1-\sigma_i^2} + \\ + \frac{M_i k_{p,i}\hat{i}_i}{k_{I,i}} \ln\frac{1}{1+\sigma_i}, \end{aligned} \tag{11}$$

is a Lyapunov function for the system (10a), (10b), and the system is asymptotically stable with equilibrium point $\hat{i}_i$.

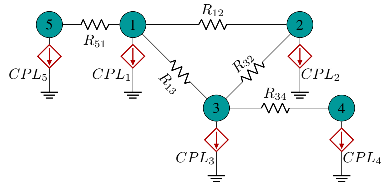

Fig. 3. Network topology of a meshed islanded MicroGrid.

Proof. Assuming time scale separation of the dynamics, *i.e.* $\hat{i}_i =$ constant, the time derivative of $\mathcal{W}_i$ results in

$$\begin{aligned} \dot{\mathcal{W}}_i &= L_i\frac{di_i}{dt}(i_i - \hat{i}_i) + \frac{M_i^2 - M_i k_{p,i}\hat{i}_i}{2k_{I,i}}\frac{2\sigma_i}{1-\sigma_i^2}\frac{d\sigma_i}{dt} \\ &- \frac{M_i k_{p,i}\hat{i}_i}{k_{I,i}(1+\sigma_i)}\frac{d\sigma_i}{dt} \\ &= -k_{p,i}(i_i - \hat{i}_i)^2, \end{aligned}$$

where

$$\dot{\mathcal{W}}_i = -k_{p,i}(i_i - \hat{i}_i)^2 \leq 0$$

for $k_{p,i} > 0$. Following La Salle's invariance theorem and noting that the derivative vanishes only at $i_i = \hat{i}_i$, the driving subsystem is asymptotically stable with respect to the equilibrium point $\hat{i}_i$. □

The asymptotic stability of the cascaded dynamics (10) follows:

Theorem 3 (Asymptotic stability of the cascaded dynamics)**.** *For every $i \in \mathcal{M}$, the cascaded dynamics (10) are asymptotically stable with equilibrium set,*

$$\begin{aligned} \mathbb{B}_i := \{\hat{i}_i, \hat{\sigma}_i, \hat{z}_i, \hat{e}_i \in \mathbb{R}\colon\ \hat{i}_i = \nu_i^o - K_{\eta,i}\hat{z}_i - K_i\hat{e}_i, \\ \hat{\sigma}_i = M_i^{-1}k_{p,i}\hat{i}_i, \hat{z}_i = z_i^r,\ \hat{e}_i \in \mathcal{S}_i\} \end{aligned} \tag{12}$$

Proof. Asymptotic stability of the driving subsystem follows from Theorem 2. In addition, the dynamics described by (10c) are also asymptotically stable, where the proof follows the common approach of exploiting the recursive feasibility property of the optimal control problem and using the cost function as a Lyapunov function for the system [13]. Finally, boundedness of (10d) follows from Proposition 2. Thus, according to [16], the cascaded dynamics are asymptotically stable with equilibrium set $\mathbb{B}_i$. □

V. Simulations

In this section we demonstrate the proposed control scheme in a simulated scenario of a five node network, see Fig. 3. We require the nominal voltage to reach given references, while satisfying the "tighter" constraint sets, hence the true voltage to always remain within $\mathbb{X}_i$. The voltage evolution is depicted in Fig.4. Both the current trajectories and generated references satisfy the current constraint set $X_{c,i}$ as shown in Fig.5, while Fig.6 provides the current load deviations $\delta i_{L,i}$ from the nominal value. The rated voltage is set to $v^* = 100V$. The current loop control parameters are chosen as $k_{p,i} = 600, M_i = 9000$ resulting in a maximum current $I_{\max,i} = 15A$. The nominal power

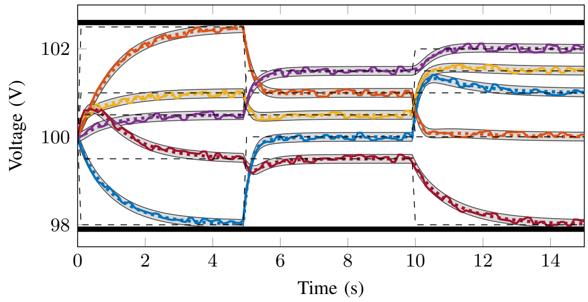

Fig. 4. Voltage and nominal voltage trajectories for Node 1(—,⋯), Node 2(—,⋯), Node 3(—,⋯), Node 4(—,⋯) and Node 5(—,⋯) respectively. The constrained region is represented with black solid lines (▬) and the voltage references by black dashed lines (– – –). The voltage trajectories are within the respective $\mathcal{S}_i$ (▭) at all times.

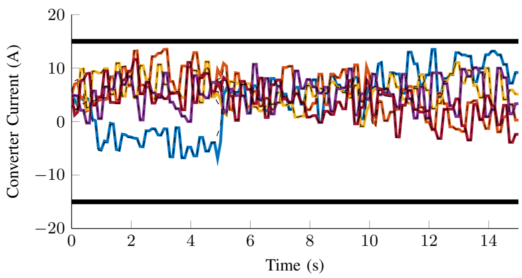

Fig. 5. Converter Current trajectories for Node 1(—), Node 2(—), Node 3(—), Node 4(—) and Node 5(—) respectively.The constrained region $\mathbb{C}$ is represented by black solid lines (▬) and the current references $\hat{i}_i$ by black dashed lines (– – –).

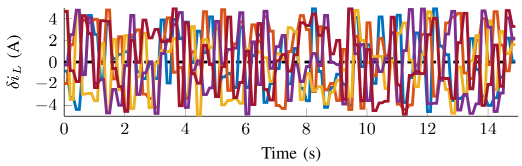

Fig. 6. Load current deviation from nominal value for Node 1(—), Node 2(—), Node 3(—), Node 4(—) and Node 5(—) respectively.

demand is $\bar{P}_i = 500W$ and we bound the maximum deviation at $|\delta P_i| \leq 500$. We furthermore parametrise the voltage constraint set as $\mathbb{X}_i = \{v_i \in \mathbb{R}^n : \ 97.9 \leq v_i \leq 102.6\}$. Finally, we choose the voltage control parameters as $K_i = 50$ and $K_{\eta,i} = 2$.

VI. Conclusions

This study proposed a robust distributed control scheme for meshed DC MGs. Following the conventional approach, we use a bounded controller for the current regulation, however the proposed design proactively guarantees boundedness and smoothness of the current trajectories. We avoid saturated current references by imposing this as a constraint within the design of the voltage regulation. The voltage subsystem is decomposed into a nominal and an error term where it is initially proven that for a bounded nominal voltage the error dynamics are bounded in a positive invariant set parametrised by the disturbance. The nominal voltage is regulated by an MPC scheme in order to optimally drive the system to desired setpoints, while ensuring constraint satisfaction at all times. Using the bounded set of the error dynamics, we include a modification of the original constraint sets into the OCP. This guarantees constraint satisfaction of the uncertain system trajectories at all times. We use the inherent robustness properties to prove recursive feasibility of the OCP and construct a proof of the overall system stability based on the interconnected system theory. Future approaches will aim to reduce the conservativeness in the choice of the robust positive invariant set of the error dynamics and generalise this approach to control affine network systems.

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