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Late-transition vs smooth $H(z)$ deformation models for the resolution of the Hubble crisis

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Gravitational transitions at low redshifts ($z_t < 0.1$) have been recently proposed as a solution to the Hubble and growth tensions. Such transitions would naturally lead to a transition in the absolute magnitude M of type Ia supernovae (SnIa) at z_t (Late M Transitions - LMT) and possibly in the dark energy equation of state parameter w (Late $w - M$ Transitions - $LwMT$). Here, we compare the quality of fit of this class of models to cosmological data, with the corresponding quality of fit of the cosmological constant model (Λ CDM) and some of the best smooth $H(z)$ deformation models (w CDM, CPL, PEDE). We also perform model selection via the Akaike Information Criterion and the Bayes factor. We use the full CMB temperature anisotropy spectrum data, the baryon acoustic oscillations (BAO) data, the Pantheon SnIa data, the SnIa absolute magnitude M as determined by Cepheid calibrators and the value of the Hubble constant H_0 as determined by local SnIa calibrated using Cepheids. We find that smooth $H(z)$ deformation models perform worse than transition models for the following reasons: 1) They have a worse fit to low- z geometric probes (BAO and SnIa data); 2) They favor values of the SnIa absolute magnitude M that are lower as compared to the value M_c obtained with local Cepheid calibrators at $z < 0.01$; 3) They tend to worsen the $\Omega_{m,0} - \sigma_{8,0}$ growth tension. We also find that the $w - M$ transition model ($LwMT$) does not provide a better quality of fit to cosmological data than a pure M transition model (LMT) where w is fixed to the Λ CDM value $w = -1$ at all redshifts. We conclude that the LMT model has significant statistical advantages over smooth late-time $H(z)$ deformation models in addressing the Hubble crisis.

I. INTRODUCTION

The scenario considered as the standard model in cosmology is the cosmological constant Λ and cold dark matter (CDM) model, hereafter denoted as Λ CDM, as it is remarkably successful in fitting cosmological and astrophysical observations on a vast range of scales. However, this scenario is not a first principles theory, and it is based on unknown quantities (dark matter, dark energy and inflation), therefore can be considered as a low energy and large scales approximation to a physical law, which has yet to be discovered. In this context, the observational problems in the estimates of the main cosmological parameters, see Refs. [1–4], can hint towards the presence of deviations from the Λ CDM scenario [5, 6].

In particular, the most statistically significant inconsistency is the well known Hubble constant H_0 tension, currently above the 4σ level (see [5, 7–11] and references therein). This tension refers to the disagreement between the value of H_0 estimated from the Planck satellite data [12], assuming a Λ CDM model, and the H_0 measured by the SH0ES collaboration [13]. However, there are many ways to obtain the Hubble constant value, and most of the early indirect estimates agree with Planck, as the Cosmic Microwave Background (CMB) ground telescopes [14, 15] or the Baryon Acoustic Oscillations (BAO) measurements [16], while most of the late time measurements agree with SH0ES, even if obtained with different teams, methods or geometric calibrators [17–21]. Finally, there are a few measurements that are in agreement with both sides, as the Tip of the Red Giant Branch [22], even if the re-analysis of [23] shows a better consistency with the SH0ES value, or those based on the time delay [24].

An additional challenge for the standard model is the *growth tension*. Dynamical cosmological probes favor weaker growth of perturbations than geometric probes in the context of general relativity and the Planck18/ Λ CDM standard model at a level of about 3σ

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[25–31]. It would therefore be of particular interest to construct theoretical models that have the potential to simultaneously address both the H_0 and growth tensions.

A wide range of theoretical models have been proposed as possible resolutions of the Hubble tension [10, 30]. They can be divided in three broad classes:

- “Early time” models that recalibrate the scale of the sound horizon at recombination by modifying physics during the pre-recombination epoch. These models deform the Hubble expansion rate before recombination at $z > 1100$ by introducing early dark energy [32–53], extra neutrinos or relativistic species at recombination [54–75], features in the primordial power spectrum [76, 77], or evaporating primordial black holes [78] etc. These models however, can alleviate but not fully solve the Hubble tension [79–81], and they tend to predict stronger growth of perturbations than implied by dynamical probes like redshift space distortion (RSD) and weak lensing (WL) data and thus they worsen the growth tension [82]. This issue however is still under debate [83].
- Late time deformations of the Hubble expansion rate $H(z)$ that assume a deformation of the best fit Planck18/ Λ CDM $H(z)$ at late times. With the term “deformation” we refer to a modification of the Planck/ Λ CDM best fit form of $H(z)$ such that the new form of $H(z)$ not only tends to satisfy the local measurements of H_0 instead of the CMB best fit value, but also leads to an angular scale of the sound horizon that is consistent with the observed CMB peaks. The analytical method for the construction of such “deformed” $H(z)$ is described in Ref. [84]. In this context, $H(z)$ retains its consistency with the observed CMB anisotropy spectrum while reaching the locally measured value of $H(z = 0)$. In this class of models we can find both interacting dark matter [85–90] or dark energy cosmologies [9, 71, 91–113], or extended and exotic dark energy models [54, 114–148]. While the interacting dark energy models need further investigations,¹ smooth $H(z)$ deformations due to the extended dark energy cosmologies have difficulty in fitting low z cosmological distance measurements obtained by BAO and SnIa data [150]. In addition, this class of models tends to imply a lower value of SnIa absolute magnitude M than the value implied by Cepheid calibrators [151]. Thus, this class of models cannot fully resolve the Hubble problem [123, 152–154], as demonstrated also in the present analysis for a few extended dark energy cosmologies.

- Late time transitions at a redshift $z_t \lesssim 0.01$ of the SnIa absolute magnitude M have also been proposed as possible models that have the potential to resolve the Hubble tension [84, 155]. These models assume an abrupt transition of M to a lower value (brighter SnIa at $z > z_t$) by $\Delta M \simeq -0.2$ mag. Such a reduction of M could have been induced by a fundamental physics transition of the effective gravitational constant G_{eff} . This type of transition² could coexist with a transition of the dark energy equation of state w from $w = -1$ at $z > z_t$ to a lower value at $z < z_t$ (phantom transition). This class of models could fully resolve the Hubble problem while at the same time address the growth tension by reducing the growth rate of cosmological perturbation due to the lower value of G_{eff} at $z > z_t$ [155]. Such models are highly predictive and have been challenged by existing, see Ref. [157], and upcoming (e.g. Gravitational Waves Standard Sirens and Tully-Fisher data [160]) cosmological and astrophysical [161] data. Observationally, viable theoretical models that can support this transition include scalar-tensor theories with potentials where a first order late phase transition takes place [159, 162, 163].

Most previous studies usually marginalize over the SnIa absolute magnitude, treating it as a nuisance parameter [164–167]. In particular, they consider the M -independent $\bar{\chi}^2 \equiv -2 \log \int dM \exp(-\chi^2/2)$ function instead of the full χ^2 function which explicitly depends on M . In our analysis the parameter M is not marginalized over and is included in the MCMC exploration along with the cosmological parameters. This allows us to compare our results with the corresponding inverse distance ladder constraints of [168, 169], even though in the context of SnIa data its degeneracy with H_0 is acknowledged. However, in Refs. [168, 169] the case of a transition in M is not considered and this is one important difference from our approach along with the types of data considered in the fit. Thus, in view of the latter class of models, the designation M tension/crisis might be more suitable to describe the problem [170, 171]. At its core, the issue is due to the fact that the supernova absolute magnitude M used to derive the local H_0 constraint by the SH0ES collaboration is at a mismatch with the value of M that is necessary to fit SnIa, CMB and BAO data.

Note that the local distance ladder methodology of SH0ES considers SnIa data in the redshift range of $0.023 < z < 0.15$. This makes the overall method oblivious to any transitions in the value of M at very low redshifts [84]. In particular, the distance ladder methodology makes the crucial assumption that M is the same at all redshifts. If this assumption is withdrawn and a

¹ See, for example, Ref. [149] for a study of the IDE models with SnIa data.

² A possible evolution of the absolute magnitude M has also been recently investigated in Refs. [156–159]

transition is allowed at $z < 0.01$ then the inferred value of H_0 may change significantly. For example if the transition occurs at $z = 0.01$ then the calibrated value of M correctly obtained at $z < 0.01$ will not be the same as the value of M at $z > 0.01$ even though the value is assumed to be the same in the distance ladder methodology. Also, if the M transition takes place at $z < 0.01$ and the calibration analysis does not allow for such a transition then an incorrect value for M will be obtained by the calibration analysis [172]. Since in the Hubble flow ($z > 0.01$), the SnIa absolute magnitude M is degenerate with H_0 through the observable $\mathcal{M} = M - 5 \log_{10}(h) + 42.38$, where $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, it becomes clear that if the true value of M in the Hubble flow was lower than the value of M for $z < 0.01$ then correspondingly the true value of H_0 would also be lower and would become consistent with the CMB inferred value. If the transition occurs at $z < 0.01$, SnIa in the Hubble flow ($0.023 < z < 0.15$) will naturally follow the calibration provided by CMB+BAO leading to a lower value on the Hubble constant.

Hints for such a late time transition may be seen in a recent re-analysis of the Cepheid SnIa calibration data where the Cepheid color-luminosity parameter is allowed to vary among galaxies [172, 173]. More specifically, in Refs. [172, 173] hints were found for a transition of this parameter or at least for it having a different value for the anchor galaxies compared to the SnIa host galaxies. Even though the errors of the individual R_E parameters for each host are consistent with the corresponding anchor (low distance) values (see Figs. 4 and 5 of Ref. [173]), when binning is implemented, the hints for a consistently different value (transition) becomes statistically more significant as shown in Ref. [172]. It is important to note here that the binning performed in Ref. [172] is not an anchor-calibrator binning, but one based on distance (low distance bin vs high distance bin separated by a critical distance D_c). The latter type of binning is justified and in this case led to a 2σ difference regarding the best fit values of R_W and M_W between the low distance bin and the high distance bin, at $D_c \approx 18 \text{ Mpc}$. This 2σ level of mismatch is clearly not statistically significant enough, but it is of interest to consider that if this degree of freedom is allowed (different values of R_W and/or M_W between high and low distance bins) the favored value of the best fit Hubble parameter becomes consistent with the CMB inferred value. This issue however is currently under debate and needs to be carefully interpreted.

In the present analysis we focus on late time M transition models (LMT), possibly featuring also a transition in the dark energy equation of state parameter w ($LwMT$), and compare their quality of fit to cosmological data with $H(z)$ deformation models. In particular, we address the following questions:

- How much does the quality of fit to low- z cosmological data improve for LMT models as compared to smooth $H(z)$ deformation models?

- What is the level of M transition favored by data?
- What is the value of M favored by smooth $H(z)$ deformation models and how does it compare with the value of M favored by Cepheid calibrators?
- Does the addition of a w transition on top of the M transition significantly improve the quality of fit to the data?

Previous studies [84, 151, 155] have indicated that LMT models have improved quality of fit to cosmological data. However, those studies did not make use of the full CMB anisotropy spectrum but only effective parameters (shift parameter). The present analysis improves on those studies by implementing a more complete and accurate approach using the full Planck18 CMB anisotropy spectrum in the context of a Boltzmann code and a Monte Carlo Markov Chain (MCMC) analysis.

The structure of our paper is the following: in the next Section II we focus on transition models ($LwMT$ and LMT) and present the constraints on their parameters using up-to-date cosmological data. In Section III we compare the quality of fit to cosmological data of transition models with the corresponding quality of fit of $H(z)$ deformation models; we also perform model selection. Finally, in Section IV we summarize our results, discuss possible interpretations and present possible extensions of the present analysis.

II. TRANSITION MODELS CONFRONTED BY OBSERVATIONAL DATA

The LMT model includes a sharp transition in the SnIa absolute magnitude M of the form

$$M(z) = M_{<} + \Delta M \Theta(z - z_t), \quad (2.1)$$

where z_t is the transition redshift, $M_{<} \equiv M_c = -19.24 \text{ mag}$ is the local Cepheid-calibrated value from SH0ES as reconstructed in Refs. [170, 175] (in this Section we neglect uncertainties on M_c), ΔM is the parameter that quantifies the shift from the M_c value, and Θ is the Heaviside step function. The $LwMT$ was first introduced in Ref. [84] and has, in addition to the M transition, a dark energy equation of state w transition of the form

$$w(z) = -1 + \Delta w \Theta(z_t - z), \quad (2.2)$$

where Δw describes the shift from the Λ CDM value ($w = -1$) for $z < z_t$. Both Δw and ΔM are parameters to be determined by the data.

Such transitions in the dark energy equation of state $w(z)$ are in principle well-motivated and can easily happen within the context of a minimally coupled scalar field in general relativity (GR), either of the quintessence or phantom type. For example, in Fig. 1 we show a transition in the dark energy equation of state w (left panel)

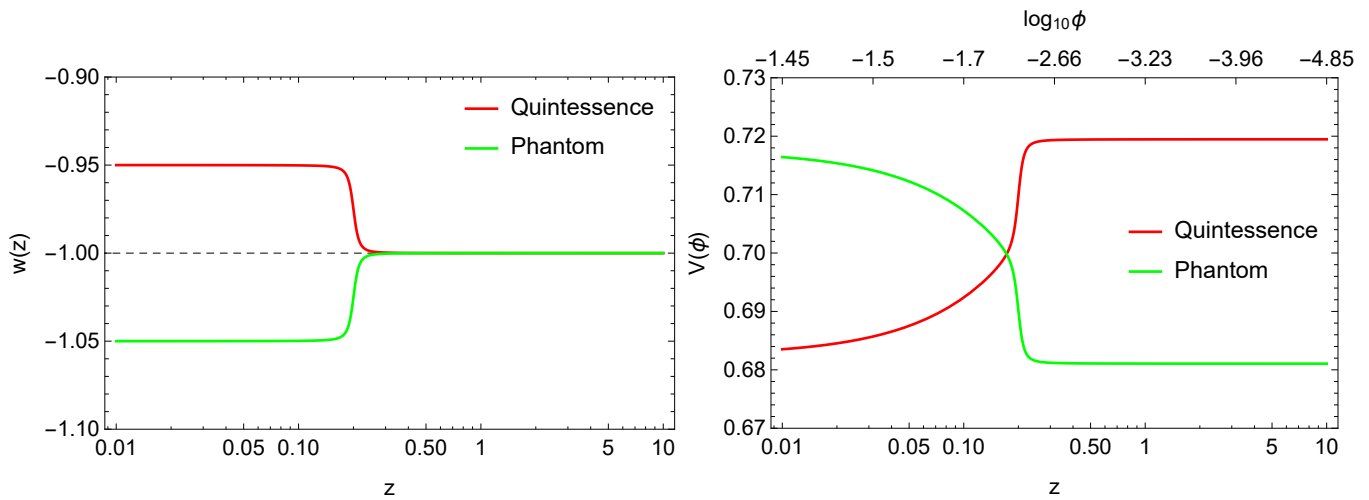


FIG. 1. An example of a transition in the dark energy equation of state w (left panel) and how it can be caused by a sharp transition in a quintessence (red line) or phantom (green line) potential (right panel), with the scalar field running down/up the potential respectively. Here we assumed a smooth transition and reconstructed the potentials following Ref. [174], assuming $\Omega_{m,0} = 0.3$, $z_t = 0.2$, $\Delta w = \pm 0.05$ for quintessence/phantom fields.

and how it can be caused by a sharp transition in a quintessence (red line) or phantom (green line) potential (right panel), with the scalar field running down/up the potential respectively. For this plot we assumed, as an example, a smooth transition and reconstructed the potentials following the procedure of Ref. [174], assuming $\Omega_{m,0} = 0.3$, $z_t = 0.2$, $\Delta w = \pm 0.05$ for quintessence (+) and phantom (-) fields. By adjusting the aforementioned parameters, one may tune both the steepness and the redshift of the transition.

In order to constrain these transition models we use the following data combination:

- The Planck18 temperature CMB data, including the TTTEEE likelihoods for high- l ($l > 30$), the temperature data TT and EE power spectra data for low- l ($2 < l < 30$) [176, 177], as well as the CMB lensing likelihood [178].
- The BAO data presented in Refs. [179–181] as well as the Ly α BAO data of Refs. [182, 183].
- The latest SnIa dataset (Pantheon) presented in Ref. [184].
- A robust redshift space distortion (RSD) compilation discussed in Ref. [185], using the likelihood presented in Ref. [186].

To analyze the data and obtain the best fit parameters we modify the publicly available CLASS code³ and perform the Monte Carlo Markov Chain (MCMC) analysis using the publicly available MontePython code [187–189].

³ For a step-by-step guide for the modifications implemented in CLASS, see [this file](#).

These models by construction provide a great amount of flexibility in fitting the observational data since they can mimic Λ CDM for $z > z_t$, while being fully consistent with local measurements of M . In the case of the transition occurring at very low redshifts where there are almost no available data, *i.e.* at $z_t < 0.01$, we would normally anticipate a fit even better to that of Λ CDM due to the extra parameter Δw in the context of $LwMT$. However, then there would be no H_0 tension, since the local measurement of H_0 should coincide with the measurement of Planck if the M transition is taken into account (a shift of M implies a shift of H_0 since the two parameters are degenerate).

Interestingly there are some works that use data with $z < 0.01$, such as the extended Pantheon dataset of the latest SH0ES analysis (Pantheon+) [190] as well as the analyses of Refs. [172, 191], that can be used to search self-consistently for a transition in M at $z < 0.01$ using the combined Cepheid and SnIa data. Regarding the Pantheon+ dataset however, not only the data are not publicly available yet but also in our analysis we are simultaneously marginalizing over M and H_0 . We also stress that we are not including the full covariance between calibrators and supernovae as in the latest SH0ES analysis. Regarding Ref. [191], the analysis makes no attempt to investigate an M transition or to constrain variations of $H(z)$ for $z < 0.01$ since this redshift region is not in the Hubble flow and thus it can not be reliably constrained. In contrast, it demonstrates that variations of the $H(z)$ parametrization in the Hubble flow (for $z > 0.01$) do not affect the best fit value of H_0 . This result could have been anticipated by the fact that (almost) all $H(z)$ parametrizations reduce to a cosmographic expansion in the range $0.01 < z < 0.1$ where the fit for H_0 is performed.

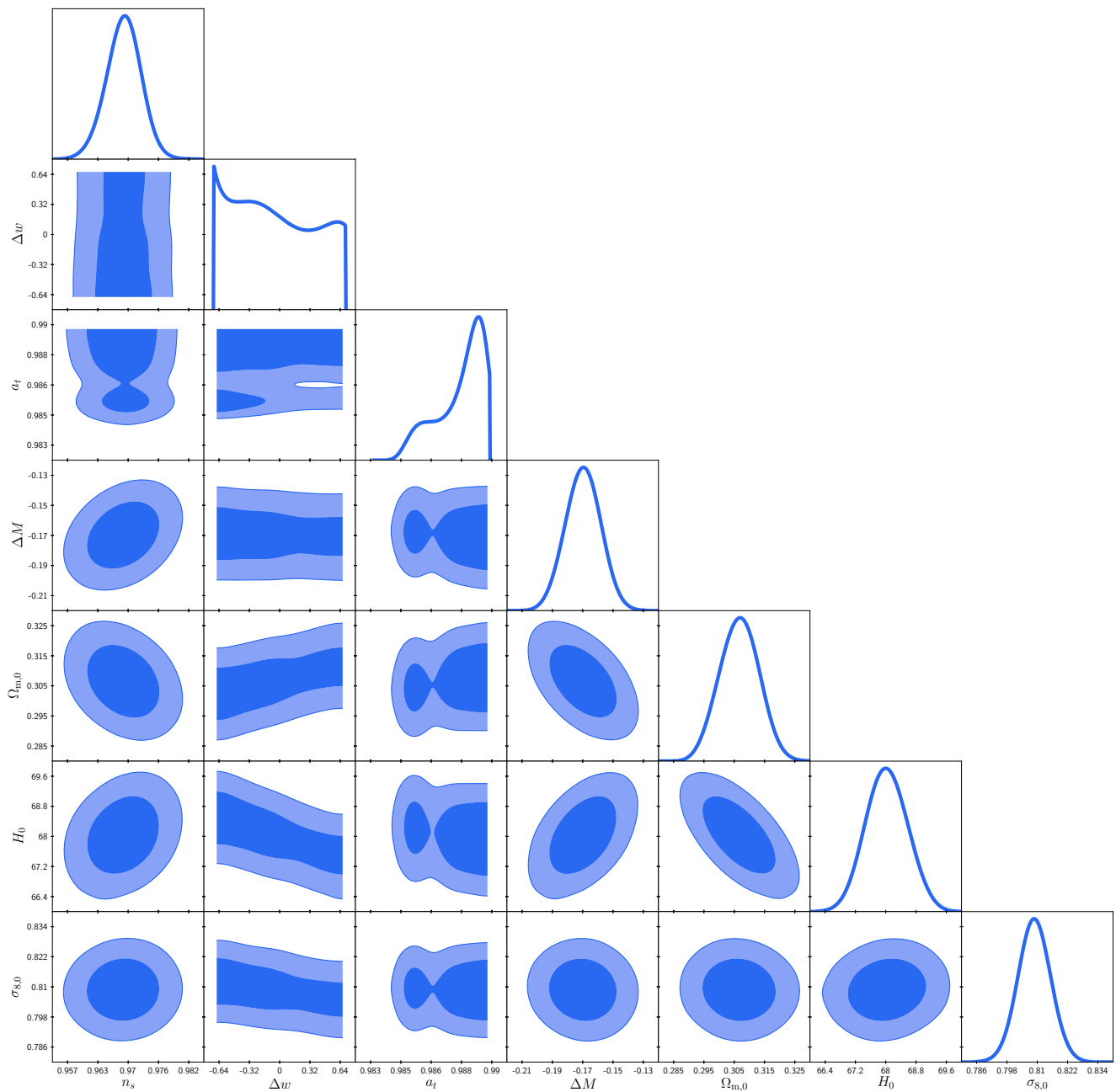


FIG. 2. The 68.3%–95.5% confidence contours for the parameters of the *LwMT* model with $z_t \geq 0.01$, using the CMB+BAO+Pantheon+RSD likelihoods.

In addition, a separate analysis of Ref. [172], focusing on the Cepheid+SnIa data for $z < 0.01$, has found hints for a transition in the Cepheid absolute magnitude M_W and in the color luminosity parameter R_W which, if taken into account, make the value of absolute magnitude M of SnIa consistent with its inverse distance ladder value, thus resolving the Hubble tension.

Hence, in what follows we impose a prior of $z_t \geq 0.01$ that corresponds to $a_t \leq 0.99$, since any lower value of z_t cannot be probed via the considered Hubble flow data. Moreover, we use a prior of $\Delta w \in [-0.7, 0.7]$. The best

fit values of the *LwMT* model with $z_t \geq 0.01$ are shown in Table I, while the $1\sigma - 2\sigma$ corresponding contours are shown in Fig. 2. In Table I we also include the parameter $M_{>} \equiv M_c + \Delta M$ that arises for $z > z_t$ with M_c corresponding to the Cepheid-calibrated value of the SnIa absolute magnitude.

From Table I, we see that the parameter a_t (or equivalently z_t) approaches the highest (lowest) value imposed by the data in order to achieve the best possible quality of fit favoring a transition at very low redshifts. Moreover, the posterior probability of a_t appears to be bimodal.

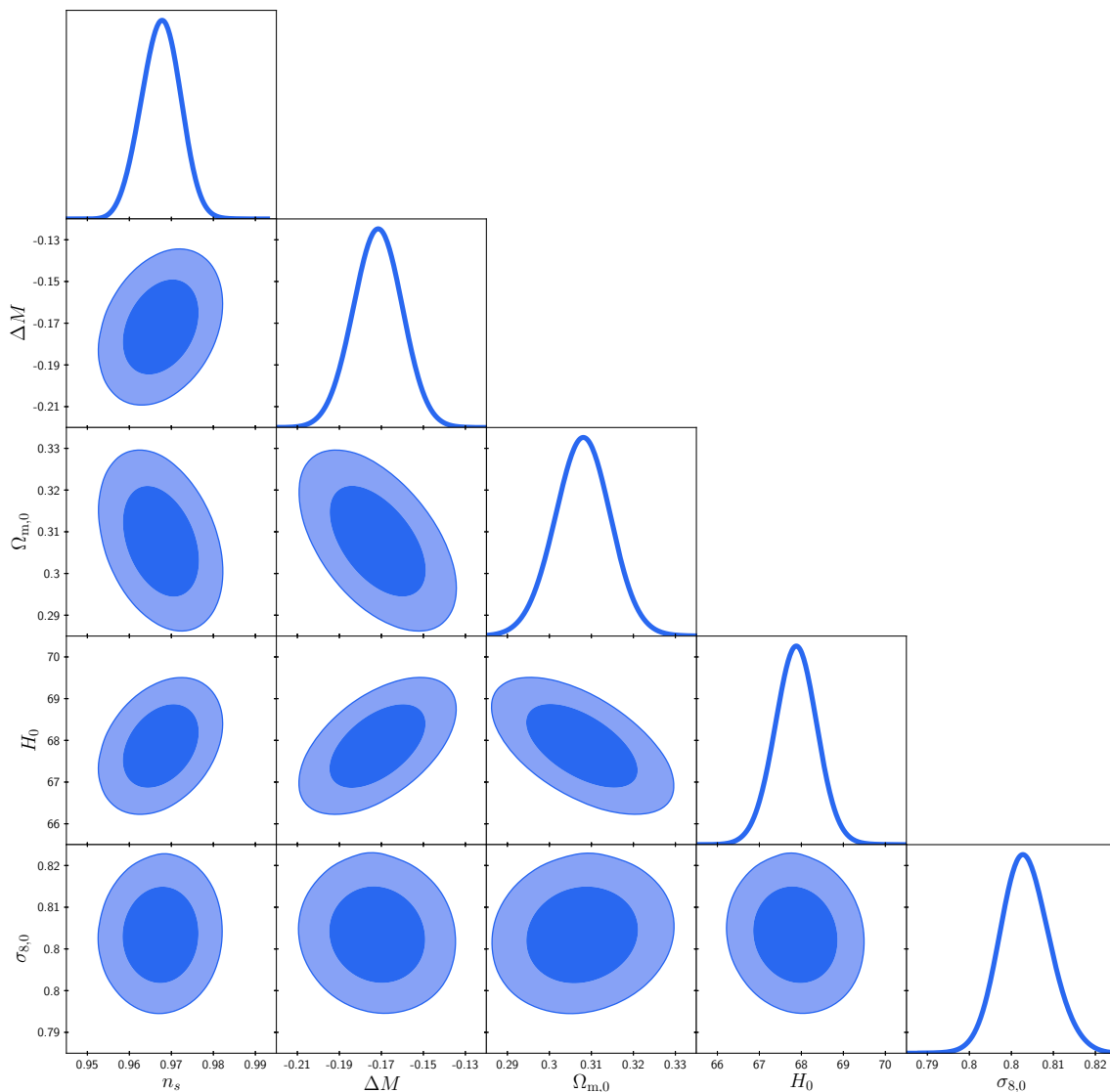


FIG. 3. The 68.3%–95.5% confidence contours for the LMT model with $z_t = 0.01$, using the CMB+BAO+Pantheon+RSD likelihoods.

TABLE I. The best-fit values and constraints at 68.3% CL and 95.5% CL of the parameters for the $LwMT$ model and $z_t \geq 0.01$ (or equivalently $a_t \leq 0.99$) using the CMB+BAO+Pantheon+RSD likelihoods described above.

Parameter	best-fit	mean $\pm\sigma$	95.5% lower	95.5% upper
$\Omega_{m,0}$	0.3018	$0.3066^{+0.0064}_{-0.0065}$	0.2939	0.3196
n_s	0.9708	$0.9685^{+0.0038}_{-0.0037}$	0.9608	0.9759
H_0	68.56	$68.03^{+0.9937}_{-0.58}$	66.94	69.15
$\sigma_{8,0}$	0.8141	0.8089 ± 0.0065	0.7957	0.8219
ΔM	-0.1676	-0.1698 ± 0.012	-0.1933	-0.1467
Δw	unconstrained	unconstrained	unconstrained	unconstrained
a_t	0.9856	> 0.985	> 0.984	> 0.984
$M_{>} \equiv M_c + \Delta M$	-19.408	-19.410 ± 0.012	-19.433	-19.387
$-\ln \mathcal{L}_{\min}$	1917.02			
χ^2_{\min}	3834			

TABLE II. The best-fit values and constraints at 68.3% CL and 95.5% CL of the parameters for the LMT model with $z_t = 0.01$ (or equivalently $a_t = 0.99$) using the CMB+BAO+Pantheon+RSD likelihoods.

Parameter	best-fit	mean $\pm\sigma$	95.5% lower	95.5% upper
$\Omega_{m,0}$	0.3088	$0.3082^{+0.0052}_{-0.0058}$	0.2976	0.3193
n_s	0.9697	$0.968^{+0.0038}_{-0.0037}$	0.9606	0.9754
H_0	67.88	$67.89^{+0.42}_{-0.40}$	67.06	68.71
$\sigma_{8,0}$	0.8085	$0.8084^{+0.0058}_{-0.0061}$	0.7963	0.8205
ΔM	-0.170	-0.172 ± 0.012	-0.195	-0.149
$M_{>} \equiv M_c + \Delta M$	-19.410	-19.412 ± 0.012	-19.435	-19.389
$-\ln \mathcal{L}_{\min}$	1917.52			
χ^2_{\min}	3835			

The reason for this behavior may be seen *e.g.* in Fig. 9 of Ref. [84] or in Fig. 3 of Ref. [170] where the lowest z

bin for the SNIa absolute magnitude M shows a rise that may be interpreted as a hint for a transition at $z \simeq 0.015$

(this is expressed by the first plateau-peak for the likelihood of a_t at $a \simeq 0.0986$). The higher peak however occurs at $a_t = 0.99$ indicating that the minimum χ^2 for the transition redshift is at or below $z = 0.01$ (no clear hint for a transition in the Hubble flow). However, given that the data do not extend to more recent times than $a = 0.99$, the best fit is at $a_t = 0.9856$ as it is explicitly written in Table I and the higher peak at $a_t = 0.99$ can only be interpreted as a lower bound for the value a_t if the transition occurs at more recent times. We thus chose to neglect this second peak.

The timing of the transition is not particularly fine-tuned due to the fact that at very low redshifts dark energy has started to dominate in the Universe. Since at that time $\Omega_\Lambda > 0.5$, new physics could possibly emerge. Furthermore, in previous analyses by some of the authors of the current work [156, 158] a tomographic analysis of the Pantheon dataset has been performed. In both of these references, it has been shown that for $z > 0.01$ the redshift binned best fit Λ CDM parameter values for the parameter M (as well as $\Omega_{m,0}$) vary around the full dataset fit value (assumed constant) by up to $\Delta M = 0.08 \pm 0.06$. This variation is significantly smaller than the variation $\Delta M \simeq 0.2$ required for the resolution of the Hubble tension (see *e.g.* the left panel of Fig. 1 of [158]). Most importantly, however, we observe that despite allowing for an extra degree of freedom induced by having $\Delta w \neq 0$, this parameter seems to be ignored by the data.

Since $z_t \approx 0.01$ is favored by the data, the parameter Δw becomes irrelevant due to the fact that for $z_t = 0.01$, Δw would modify the expansion rate $H(z)$ only in a region where there are no data available ($z < 0.01$). This carries the implication that a w transition is perhaps not needed in order to obtain the best quality of fit to the data⁴. We thus repeat the analysis considering only an M transition (“Late M Transition” - LMT), setting $\Delta w = 0$ and $a_t = 0.99$ (or equivalently $z_t = 0.01$), which is basically the maximum of the posterior of a_t for the $LwMT$ model. We obtain the best fit and mean values as indicated in Table II; the contours are shown in Fig. 3.

As we can see comparing Tables I, II and Figs. 2, 3 the introduction of Δw has practically no effect on the quality of fit, *i.e.* on the χ^2 value. Moreover, the mismatch between the local calibration of the SnIa absolute magnitude and the value inferred from the other probes is very significant, suggesting that the designation M tension/crisis is suitable to describe the H_0 crisis [170, 171]. Finally, it is interesting to note that the inferred value of $M_{>} = -19.41$ mag agrees well with the constraint $M = -19.40$ mag that was obtained using the parametric-free inverse distance ladder of Ref. [192].

⁴ The w transition however may be required for theoretical reasons. In scalar tensor theories a gravitational transition to weaker gravity at early times may require a simultaneous transition to $w < -1$ at late times.

So, some natural questions that arise are the following: “How do the transition models $LwMT$ and LMT compare to some other popular dark energy models in the literature that also try to address the Hubble tension?” and “Can these models provide an M value that is consistent with the Cepheid measurement M_c as the transition models that we discussed?” These questions will be addressed in the following section, where we perform a comparison between some popular dark energy parametrizations (smooth deformation dark energy models) with the transition models $LwMT$ and LMT .

III. COMPARISON OF DARK ENERGY MODELS

In order to truly resolve the H_0 tension, a dark energy model should not only provide a consistent measurement for M , but also maintain a quality of fit comparable (or even better) to Λ CDM with low- z data (BAO and SnIa), as discussed earlier. In this section, we consider some popular dark energy models, that have been suggested as being capable of addressing the H_0 tension, following three different methods:

1. Force all the models to be consistent with the Cepheid absolute magnitude measurement [170, 175] at the 1σ level by imposing a flat prior $M \in [-19.28, -19.2]$ mag.
 2. Analyze all models including the local Cepheid-calibrated prior by SH0ES [170]:
- $$M_c = -19.24 \pm 0.04 \text{ mag.} \quad (3.1)$$
3. Include the SH0ES determination of H_0 [13], allowing at the same time the absolute magnitude M to vary freely (flat prior). This is illustrated in Appendix A, as a complementary analysis.

The $H(z)$ deformation dark energy models that we consider in this work include the w CDM model, *i.e.* a model with a constant equation of state w , assuming a flat Universe and cold dark matter, that is described by a Hubble parameter of the form (neglecting radiation and neutrinos at late times)

$$H(z) = H_0 \sqrt{\Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0})(1+z)^{3(1+w)}}, \quad (3.2)$$

which for $w = -1$ reduces to the usual Hubble parameter for the Λ CDM model. Moreover, we consider the Chevallier-Polarski-Linder (CPL) parametrization, with a dark energy equation of state [193, 194]

$$w(z) = w_0 + w_a \left(\frac{z}{1+z} \right), \quad (3.3)$$

where w_0 and w_a are free parameters. The corresponding Hubble parameter for the CPL model is the following

TABLE III. Constraints at 68.3% CL of the cosmological parameters for all the dark energy models explored in this work when a narrow flat prior $M \in [-19.28, -19.2]$ mag is assumed, forcing the agreement with Cepheid calibration [170, 175] at the 1σ level. Note that this prior is artificial and the correct prior is the Gaussian one of Eq. (3.1) which is adopted in Table IV. For the transition models, M is fixed to -19.24 mag ($M_{<}$). $\Delta\chi^2$ corresponds to the χ^2_{\min} difference of each model with the Λ CDM case. All models provide a much better overall fit as compared to Λ CDM, and the $LwMT$ and LMT models fair considerably better than the rest.

Parameters	Λ CDM	w CDM	CPL	$LwMT$ ($z_t \geq 0.01$)	PEDE	LMT ($z_t = 0.01$)
$\Omega_{m,0}$	$0.2564^{+0.0018}_{-0.0019}$	$0.2571^{+0.0019}_{-0.0020}$	$0.2719^{+0.0041}_{-0.0044}$	0.3066 ± 0.0063	0.2582 ± 0.0020	0.3082 ± 0.0053
n_s	0.992 ± 0.003	0.972 ± 0.004	0.967 ± 0.004	0.968 ± 0.004	0.971 ± 0.003	0.968 ± 0.004
H_0	72.40 ± 0.16	$73.99^{+0.26}_{-0.27}$	72.38 ± 0.48	68.03 ± 0.55	$73.90^{+0.17}_{-0.19}$	67.89 ± 0.40
$\sigma_{8,0}$	$0.8045^{+0.0072}_{-0.0081}$	$0.8507^{+0.0084}_{-0.0083}$	$0.8511^{+0.0084}_{-0.0081}$	0.8088 ± 0.0063	0.8517 ± 0.0059	0.8084 ± 0.0059
S_8	0.7437 ± 0.0077	0.7876 ± 0.0084	0.8103 ± 0.0100	$0.8177^{+0.0101}_{-0.0103}$	0.7901 ± 0.0065	0.8194 ± 0.0100
M	~ -19.28	~ -19.28	~ -19.28	$-19.24 (M_{<})$	~ -19.28	$-19.24 (M_{<})$
ΔM	-	-	-	-0.170 ± 0.011	-	-0.172 ± 0.011
$M_{>} \equiv M_c + \Delta M$	-	-	-	-19.410 ± 0.011	-	-19.412 ± 0.011
Δw	-	-	-	unconstrained	-	-
a_t	-	-	-	> 0.987	-	-
w_0	-	$-1.162^{+0.021}_{-0.019}$	$-0.844^{+0.077}_{-0.089}$	-	-	-
w_a	-	-	$-1.27^{+0.38}_{-0.31}$	-	-	-
χ^2_{\min}	3964	3889	3875	3834	3886	3835
$\Delta\chi^2_M$	-	-75	-89	-130	-78	-129

$$H(z) = H_0 \sqrt{\frac{\Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0}) \times (1+z)^{3(1+w_0+w_a)} e^{-3\frac{w_a z}{1+z}}}{(1+z)^{3(1+w_0+w_a)} e^{-3\frac{w_a z}{1+z}}}} \quad (3.4)$$

Furthermore, we consider the phenomenologically emergent dark energy (PEDE) model which shows significant promise in resolving the H_0 problem. This model was introduced in Ref. [126] and has an equation of state of the form

$$w(z) = -\frac{1}{3 \ln 10} (1 + \tanh[\log_{10}(1+z)]) - 1, \quad (3.5)$$

with a corresponding Hubble parameter of the form

$$H(z) = H_0 \sqrt{\frac{(1 - \Omega_{m,0}) \times [1 - \tanh(\log_{10}(1+z))] + \Omega_{m,0}(1+z)^3}{(1+z)^3}} \quad (3.6)$$

The main advantage of the aforementioned parametrization is that it has the same number of degrees of freedom as Λ CDM. Finally, we consider the transition models $LwMT$ with $z_t > 0.01$ and LMT with $z_t = 0.01$ described in Sec. II, as well as the Λ CDM model itself, thus having a total of six different models.

III.1. Dark energy models comparison using a narrow flat prior on $M \in [-19.28, -19.2]$ mag

We perform the MCMC analysis using the likelihoods described in Section II and imposing a narrow flat prior on the SnIa absolute magnitude $M \in$

$[-19.28, -19.2]$ mag, that is, forcing all models to be consistent with the Cepheid measurement M_c . Rigorously, this prior is artificial as the correct prior is the Gaussian one of Eq. (3.1). However, the use of this narrow prior will be useful to understand the impact of the local calibration on the quality of fit of the various models.

For the transition models $LwMT$ and LMT , we use Eq. (2.1) for the SnIa absolute magnitude and leave ΔM as a free variable. Thus, these are the only models that can, by construction, escape from the imposed M prior. The constraints on the cosmological parameters as well as the 68.3%–95.5% confidence contours of the corresponding parameters of the models are shown in Table III and Fig. 4 respectively. For the sake of clarity, contour plots for the LMT and $LwMT$ models are displayed separately. In this case, the analysis with a narrow flat prior on M produces same constraints as those showed as Fig. 3 and Fig. 2 for the LMT and $LwMT$ models, respectively. A wider flat prior would lead (as in the case of a Gaussian prior of Table IV that follows) the best fit value of M for most models to be very close to the CMB inferred value of $M = -19.4$ with an error bar which makes it inconsistent with the Cepheid inferred value $M = -19.24$. In view of the degeneracy of M with H_0 , this M tension is closely related with the H_0 tension. A similar (but milder) tension occurs also in the case of a Gaussian prior on M as indicated in Table IV.

All models, except the $LwMT$ with $z_t \geq 0.01$ and LMT with $z_t = 0.01$, give an H_0 value that is consistent with the SH0ES determination of H_0 [13] and $M \sim -19.28$ mag, *i.e.* the lowest eligible value of the prior that we imposed, displaying their tendency to provide a significantly lower value for M . On the other hand,

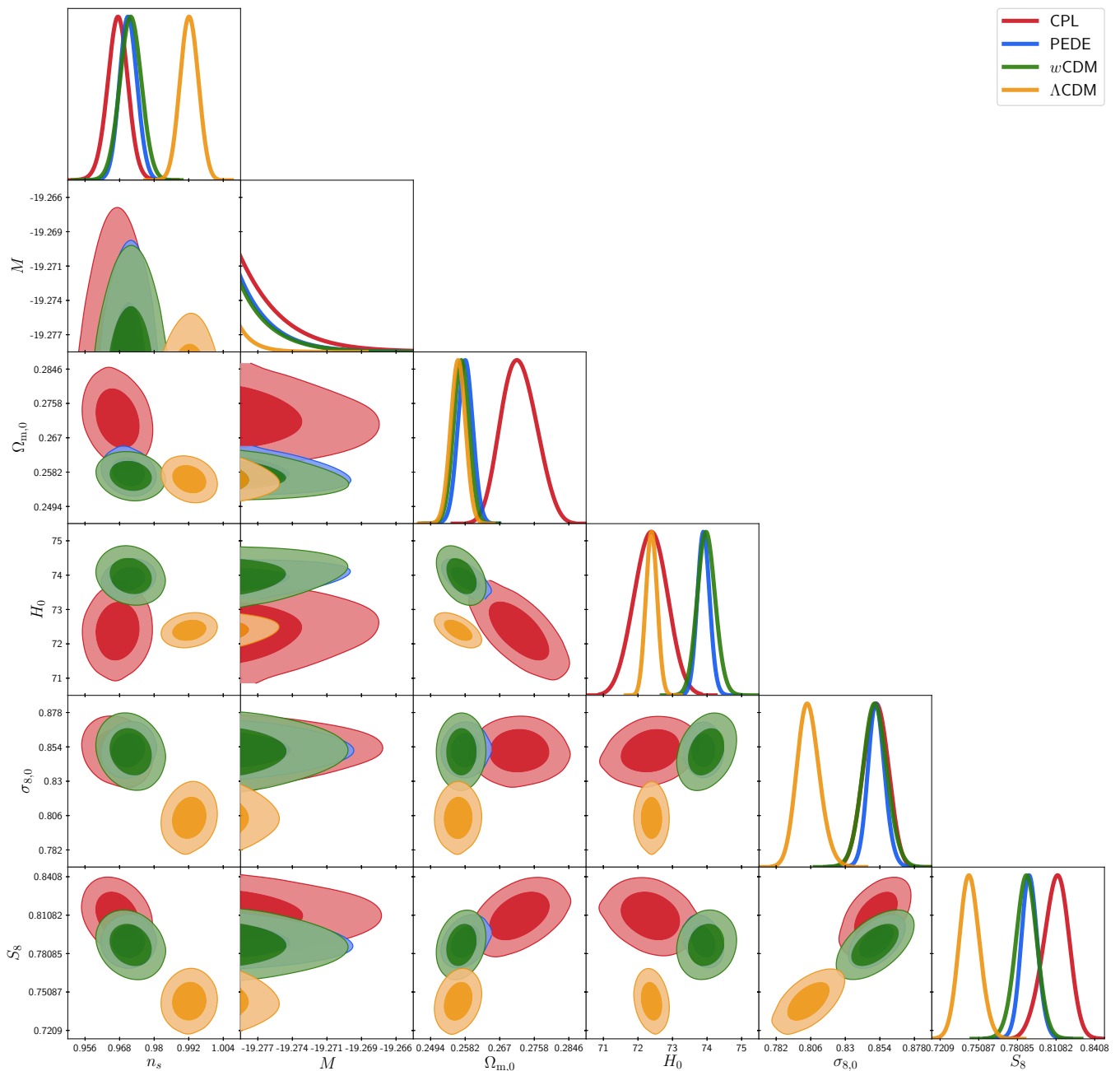


FIG. 4. The 68.3%–95.5% confidence contours for the common parameters of the Λ CDM, CPL, w CDM and PEDE dark energy models corresponding to the bounds illustrated in Table III. We used the CMB+BAO+Pantheon+RSD likelihoods, imposing the narrow flat prior $M \in [-19.28, -19.2]$ mag. The M prior severely constrains the best fit of M to the lowest possible value, displaying their tendency to provide a significantly lower value for M .

the transition models provide a H_0 value close (within the 1σ level) to the typical Planck18/ Λ CDM value, providing at the same time $M \approx -19.4$ mag as expected.

Note that the Λ CDM model has a very bad fit to the data as compared to w CDM, CPL and PEDE. This is due to the fact that, having fixed M to the local M_c value, supernova data constrain the Λ CDM model's luminosity distance to values that are at odds with CMB and BAO.

This clearly shows how the Λ CDM model cannot possibly solve the M crisis [170, 171]. The more flexible w CDM, CPL and PEDE models fare much better but still much worse than the $LwMT$ and LMT models which can fit all observables well.

All the models are forced to be consistent with the local Cepheid-calibrated value M_c at the 1σ level. As a result, in order to achieve consistency with M_c the values of

TABLE IV. Constraints at 68.3% CL of the cosmological parameters for the dark energy models explored in this work when the prior $M = -19.24 \pm 0.04$ mag of Eq. (3.1) from SH0ES is adopted. $\Delta\chi^2$ corresponds to the χ^2_{\min} difference of each model with the Λ CDM case. Only transitions models provide a competitive fit to data as compared to Λ CDM.

Parameters	Λ CDM	w CDM	CPL	$LwMT$ ($z_t \geq 0.01$)	PEDE	LMT ($z_t = 0.01$)
$\Omega_{m,0}$	$0.3022^{+0.0051}_{-0.0052}$	0.2943 ± 0.0065	$0.2974^{+0.0067}_{-0.0068}$	$0.3073^{+0.0063}_{-0.0062}$	0.2789 ± 0.0049	0.3082 ± 0.0053
n_s	0.9704 ± 0.004	0.968 ± 0.004	0.967 ± 0.004	0.968 ± 0.004	0.963 ± 0.003	0.968 ± 0.004
H_0	68.36 ± 0.4	69.47 ± 0.72	69.25 ± 0.73	67.96 ± 0.55	71.85 ± 0.45	67.89 ± 0.40
$\sigma_{8,0}$	$0.8076^{+0.0058}_{-0.0062}$	$0.8215^{+0.0095}_{-0.0097}$	$0.8248^{+0.0096}_{-0.0097}$	$0.8084^{+0.0064}_{-0.0065}$	0.8531 ± 0.0059	0.8085 ± 0.0057
S_8	$0.8105^{+0.0097}_{-0.01}$	0.8135 ± 0.0098	$0.8210^{+0.0107}_{-0.0106}$	0.8181 ± 0.0100	0.8226 ± 0.0095	0.8194 ± 0.0099
M	-19.40 ± 0.01	-19.38 ± 0.02	-19.37 ± 0.02	-19.26 ± 0.04	-19.33 ± 0.01	-19.24 ± 0.04
ΔM	-	-	-	$-0.145^{+0.038}_{-0.035}$	-	-0.168 ± 0.039
$M_{>}$	-	-	-	-19.410 ± 0.011	-	-19.411 ± 0.011
Δw	-	-	-	unconstrained	-	-
a_t	-	-	-	> 0.986	-	-
w_0	-	-1.050 ± 0.027	-0.917 ± 0.078	-	-	-
w_a	-	-	$-0.53^{+0.33}_{-0.28}$	-	-	-
χ^2_{\min}	3854	3851	3848	3833	3867	3835
$\Delta\chi^2$	-	-3	-6	-21	+13	-19

TABLE V. $\Delta\chi^2$ and corresponding ΔAIC and $\ln B$ values for all models of Table IV with respect to Λ CDM. Negative values of $\Delta\chi^2$ and ΔAIC and positive values of $\ln B$ signal that a model is favored with respect to Λ CDM.

Gaussian M Prior Case	$\Delta\chi^2$	ΔAIC	$\ln B$
Λ CDM	-	-	-
$LMT(z_t = 0.01)$	-19	-17	+9.1
$LwMT(z_t \geq 0.01)$	-21	-15	+6.2
w CDM	-3	-1	+2.2
CPL	-6	-2	-2.4
PEDE	+13	+13	-6.5

the other parameters differ significantly from the relevant Λ CDM values. Also, the imposed significantly higher ($M > -19.28$) than the best fit inverse distance ladder value ($M = -19.4$) forces the MCMC process to restrict the rest of the parameters, thus explaining the extremely low uncertainties in order to achieve the best possible quality of fit to the data. We also stress that the peak structure of the CMB in the damping tail constrains very well the combination $\Omega_m h^2$. Therefore, once we force M - H_0 to be in agreement with SH0ES, we need a lower value of Ω_m different from Λ CDM to compensate for the higher H_0 value and keep the peak structure unaltered. Once we relax the M prior (Table IV) Ω_m can go back to the Λ CDM value.

III.2. Dark energy models comparison using the local Cepheid prior on M

Here, we adopt the local Gaussian prior of Eq. (3.1). The constraints on cosmological parameters are given in Table IV, while the corresponding 68.3%–95.5% confidence contours are shown in Fig. 5. The transition $LwMT/LMT$ models fare significantly better than the other models, providing an absolute magnitude that is consistent with the Cepheid calibration of Eq. (3.1). The w CDM and CPL models achieve a slightly better fit to data as compared to Λ CDM, while the PEDE model has a significantly worse fit to data, in agreement with previous findings [127]. Note that constraints for the LMT and $LwMT$ models have been not included in Fig. 5 for the sake of clarity. Constraints for these model are instead showed in Fig. 8 of the Appendix B.

III.3. Model selection

To select the best model one cannot just look at the quality of fit but it is essential to include the information on the number of parameters and their priors. Here, we only consider the case of Section III.2 as it uses the actual Cepheid prior of Eq. (3.1) from SH0ES. We adopt two approaches. First, we consider the Akaike Information Criterion (AIC) [195, 196], defined as

$$AIC \equiv -2 \ln \mathcal{L}_{\max} + 2 N_{\text{tot}} = \chi^2_{\min} + 2 N_{\text{tot}}, \quad (3.7)$$

where N_{tot} corresponds to the total number of free parameters of the considered model and \mathcal{L}_{\max} corresponds to the maximum likelihood. This criterion penalizes a model for any extra parameters. Using Eq. (3.7) we calculate the AIC values for all the models of Table IV

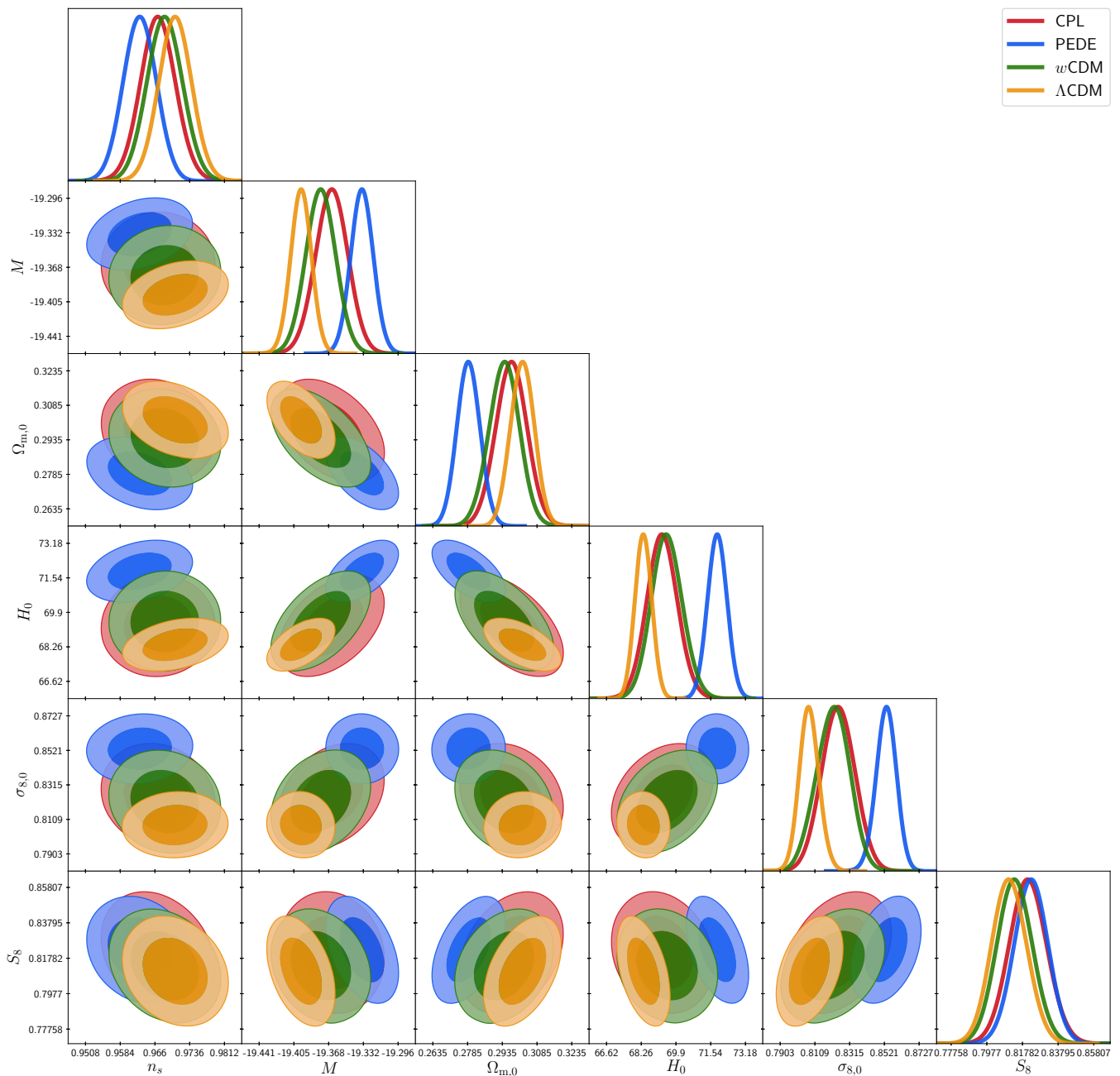


FIG. 5. The 68.3%–95.5% confidence contours for the common parameters of the CPL, w CDM and PEDE dark energy models with the prior $M = -19.24 \pm 0.04$ mag of Eq. (3.1) from SH0ES (corresponding bounds in Table IV).

and construct the corresponding differences $\Delta AIC \equiv AIC_{\text{model}} - AIC_{\Lambda\text{CDM}}$, see Table V. If $|\Delta AIC| \leq 2$, then the compared models can be interpreted as consistent with each other, while if $|\Delta AIC| \geq 4$ it is an indication that the model with the larger AIC value is disfavored [196]. We can see that the $LwMT/LMT$ models are strongly favored over ΛCDM , and that the PEDE model is strongly disfavored.

We also use the `MCEvidence` package [197] in order to compute the Bayesian evidences (marginal like-

lihoods) of each model of Table IV using their respective MCMC chains. This algorithm obtains the posterior for the marginal likelihood, using the k -th nearest-neighbour Mahalanobis distances [198] in the parameter space. In our analysis we have adopted the $k = 1$ case to minimize the effects of the inaccuracies associated with larger dimensions of the parameter space and smaller sample sizes. The strength of the evidence presented in favor or against a model in a comparison, can be found using the revised Jeffreys' scale [199]. Specifi-

cally, in a comparison between two models via the Bayes factor B (ratio of evidences), if $|\ln B| < 1$ the models are comparable with none of them being favored, for $1 < |\ln B| < 2.5$ one model shows weak evidence in its favor, if $2.5 < |\ln B| < 5$ the model in question has moderate evidence on its side, and lastly in the case of $|\ln B| > 5$ one model is strongly favored over the other. From Table V one can see that PEDE is strongly disfavored, w CDM and CPL weakly favored and disfavored, respectively, and that the $LwMT/LMT$ models are strongly favored over Λ CDM.

IV. DISCUSSION AND CONCLUSIONS

We have investigated the quality of fit to cosmological data of five models that attempt to solve the H_0 crisis. Besides the standard Λ CDM model, we considered three smooth $H(z)$ deformation models (w CDM, CPL and PEDE) and two models that allow for a sudden transition of the SnIa absolute magnitude M at a recent cosmological redshift z_t . We performed model selection via the Akaike Information Criterion and the Bayes factor. This is a more detailed and extended fit to the data that includes the full CMB angular power spectrum, instead of just the peak locations, discussed in the previous studies [84, 155] that introduced the ultra-late transition idea. We have also included additional cosmological models to compare the fit with the M transition models and implemented different priors and model selection criteria.

We found that the transition models are strongly favored with respect to the Λ CDM model. We also found that PEDE is strongly disfavored and that w CDM and CPL are weakly favored and disfavored, respectively. Specifically, only M -transition models are able to maintain consistency with the SnIa absolute magnitude M_c measured by Cepheid calibrators while at the same time maintaining a quality of fit to the cosmological data at $z > 0.01$ that is identical with that of Λ CDM.

The required transition with magnitude ΔM can be induced by a corresponding transition of the effective gravitational constant G_{eff} which determines the strength of the gravitational interactions [155]. The corresponding magnitude of the G_{eff} transition depends on the power value b of the expression that connects the evolving Newton's constant G_{eff} with the absolute luminosity L of a SnIa:

$$L \sim G_{\text{eff}}^b. \quad (4.1)$$

In the case of the $LwMT$ and LMT transition models the transition in M implies a transition in $\mu \equiv G_{\text{eff}}/G_{\text{N}}$. In particular, for $z > z_t$, it is:

$$\mu = 1 + \frac{\Delta G_{\text{eff}}}{G_{\text{N}}} \equiv 1 + \Delta\mu, \quad (4.2)$$

while for $z < z_t$ we have $\mu = 1$. Since, $\Delta\mu \ll 1$, we can assume without loss of generality that $\ln(1 + \Delta\mu) \simeq \Delta\mu$,

so it is straightforward to show that Eq. (4.1) corresponds to

$$\Delta M = -\frac{5b}{2} \frac{\ln \mu}{\ln 10}. \quad (4.3)$$

Using the definition (4.2), for $z > z_t$, we have

$$\ln \mu \simeq \Delta M. \quad (4.4)$$

Therefore, substituting (4.4) in (4.3) and solving with respect to b , we derive

$$b = -\frac{2 \ln 10}{5} \frac{\Delta M}{\Delta \mu}. \quad (4.5)$$

We can constrain b based on Eq. (4.5) and the fact that it obeys the general bounds $|b| \in [b_{\text{min}}, +\infty)$ (the $+\infty$ corresponds to the Λ CDM/GR case where $\Delta\mu = 0$). Taking the absolute value of Eq. (4.5) and setting from Table II the $2\text{-}\sigma$ upper bound $|\Delta M|_{\text{min}} = -0.172 + 2 \times 0.012 = -0.148$ mag and $|\Delta\mu|_{\text{min}} = 0.05$ [200], a measurement obtained using up to date primitive element abundances, cosmic microwave background as well as nuclear and weak reaction rates, b_{min} assumes the following 2σ range

$$b_{\text{min},0.05} = (-\infty, -2.7] \cup [2.7, +\infty). \quad (4.6)$$

Similarly, if we consider the constraint from the Hubble diagram SnIa [201], a measurement derived using luminous red galaxies, as well as from Paleontology [202], a measurement obtained using the age of bacteria and algae, that indicate $|\Delta\mu|_{\text{min}} = 0.1$, we derive

$$b_{\text{min},0.1} = (-\infty, -1.4] \cup [1.4, +\infty). \quad (4.7)$$

This range includes the simple expectation that emerges if we assume that the SnIa absolute luminosity is proportional to the Chandrasekhar mass $L \sim M_{\text{Ch}} \sim G_{\text{eff}}^{-3/2}$ which leads to $b = -3/2$.

If the M transition is due to a gravitational transition with a lower value of G_{eff} at $z > z_t$ then this class of models also has the potential to address the growth tension as discussed in previous studies [155]. It should also be stressed that such a gravitational transition would be consistent with solar system tests of modified gravities without the need for screening since the value of G_{eff} is predicted to be constant at $z < z_t$ and therefore no modification of the planetary orbits is expected since the time these orbits have been monitored. However, at the time of the gravitational transition (about 100Myrs ago) a disruption of the planetary orbits and comets is expected [203]. Such a prediction may be consistent with the observational fact that the rate of comets that hit the Earth and the Moon has increased by a factor of 2-3 during the past 100Myrs [204–208].

As discussed in, *e.g.*, Refs. [209–211], SnIa progenitors are not necessarily Chandrasekhar-mass white dwarfs and a significant fraction can arise from sub-Chandrasekhar explosions. While this surely calls for a

TABLE VI. Constraints at 68% CL of the basic parameters for all the considered dark energy models, including the SH0ES measurement $H_0 = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [13]. Clearly all the considered dark energy models except the $LwMT$ model with $z_t \geq 0.01$ and LMT with $z_t = 0.01$ give a SnIa absolute magnitude M that is inconsistent with the local calibration $M_c = -19.24 \pm 0.04 \text{ mag}$ of Eq. (3.1). However, this statistical inconsistency is not included in the χ^2 that is used to interpret the results, see discussion in [170].

Parameters	ΛCDM	$w\text{CDM}$	CPL	$LwMT$ ($z_t \geq 0.01$)	PEDE	LMT ($z_t = 0.01$)
$\Omega_{m,0}$	$0.3022^{+0.0050}_{-0.0052}$	$0.2967^{+0.0067}_{-0.0064}$	$0.2951^{+0.0063}_{-0.0067}$	$0.2989^{+0.0055}_{-0.0060}$	0.281 ± 0.005	$0.3021^{+0.0053}_{-0.0052}$
n_s	0.9705 ± 0.0037	0.9684 ± 0.004	0.9668 ± 0.0040	0.9706 ± 0.0037	$0.9621^{+0.0036}_{-0.0034}$	0.9705 ± 0.0038
H_0	68.36 ± 0.4	$69.17^{+0.65}_{-0.76}$	69.50 ± 0.71	68.71 ± 0.5	$71.69^{+0.45}_{-0.46}$	$68.36^{+0.40}_{-0.41}$
$\sigma_{8,0}$	$0.8075^{+0.0058}_{-0.0064}$	$0.8183^{+0.0089}_{-0.01}$	0.8258 ± 0.0099	0.8098 ± 0.0064	$0.8531^{+0.0064}_{-0.0058}$	$0.8086^{+0.0058}_{-0.0064}$
M	-19.40 ± 0.01	-19.38 ± 0.02	$-19.37^{+0.017}_{-0.018}$	-19.24	-19.34 ± 0.01	-19.24
ΔM	-	-	-	-0.1652 ± 0.011	-	-0.159 ± 0.011
$M_{>} \equiv M_c + \Delta M$	-	-	-	-19.405 ± 0.011	-	-19.40 ± 0.011
Δw	-	-	-	> -0.7	-	-
a_t	-	-	-	> 0.98	-	-
w_0	-	$-1.038^{+0.031}_{-0.018}$	$-0.9576^{+0.075}_{-0.078}$	-	-	-
w_a	-	-	$-0.38^{+0.32}_{-0.27}$	-	-	-
χ^2	3849	3846	3845	3846	3862	3850
$\Delta\chi^2$	-	-3	-4	-3	+13	+1

more detailed analysis of the dependence of the SnIa luminosity on G_{eff} , one must note that the Chandrasekhar mass scale is a fundamental reference scale that plays an important role in all SnIa explosions. However, a non-trivial relation between progenitors and SnIa could again imply that the relation of Eq. (4.1) could feature a value of b different from $-3/2$ corresponding to the simplest case where $L \sim M_{\text{Chandra}}$.

Therefore, interesting extensions of our analysis include the following:

- The search for traces or constraints of a gravitational transition in geological, solar system and astrophysical data.
- The construction of simple theoretical modified gravity models that can naturally induce the required transition of the effective Newton’s G_{eff} at low redshifts ($z_t < 0.01$) perhaps avoiding fine tuning issues.
- The possible identification of alternative non-gravitational physical mechanisms that could induce the transition of SnIa at low redshifts.
- The search for systematic effects in the Cepheid data and parameters that could mimic such a transition and/or induce a higher value of M for SnIa than the one currently accepted.

In conclusion the M -transition class of models is an interesting new approach to the Hubble and possibly to the growth tension that deserves further investigation.

Numerical Analysis Files: The numerical files for the reproduction of the figures can be found in

the GitHub repository [H0_Model_Comparison](#) under the MIT license.

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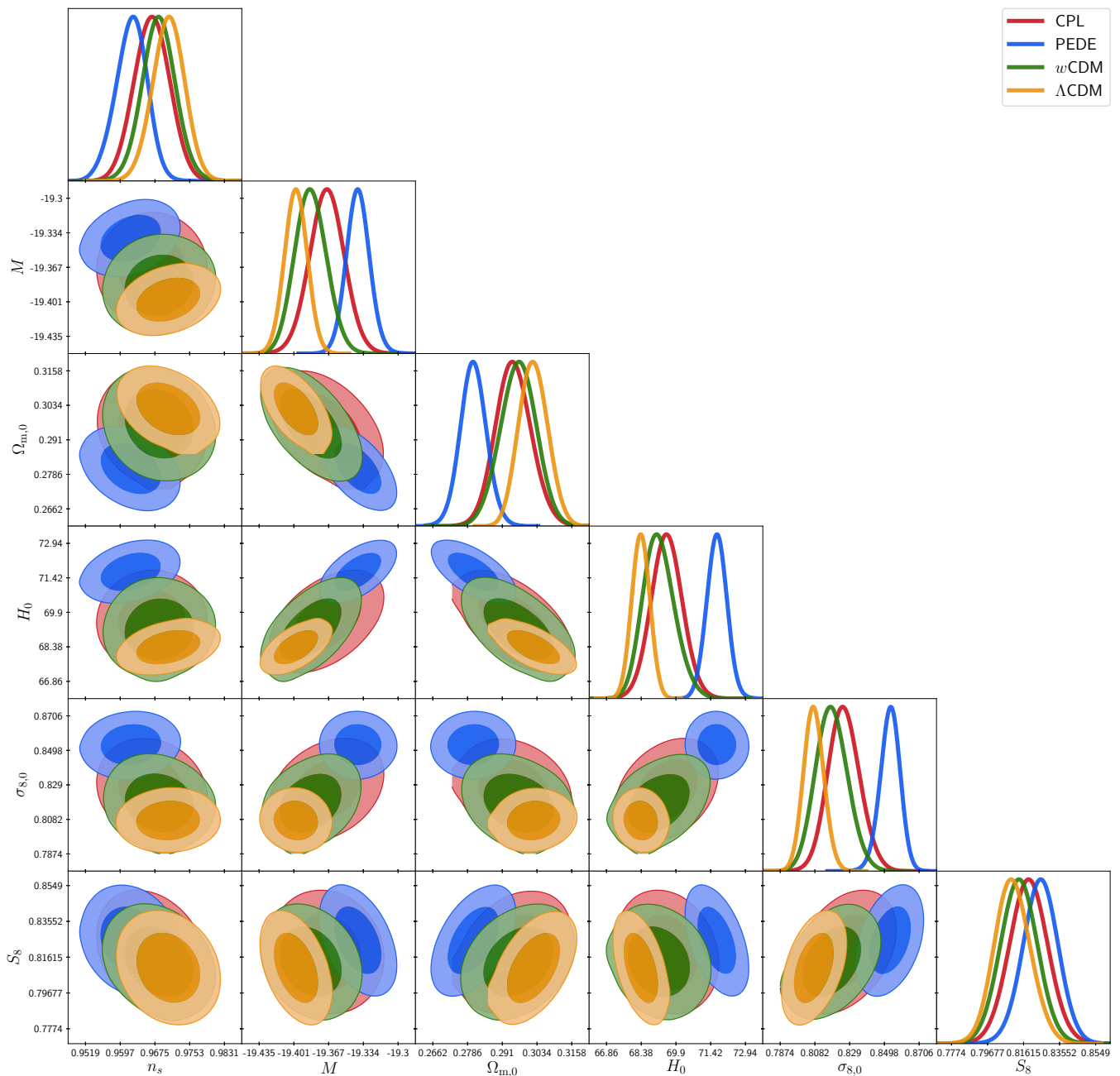


FIG. 6. The 68.3%–95.5% confidence contours for the common parameters of the Λ CDM, CPL, w CDM and PEDE dark energy models corresponding to the constraints given in Table VI. We used the CMB+BAO+Pantheon+RSD likelihoods, including the SH0ES measurement $H_0 = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [13]. The M value of the models is inconsistent with the local calibration of Eq. (3.1) ($M_c = -19.24 \pm 0.04 \text{ mag}$).

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Appendix A: Analysis of the Dark Energy Models including the Local H_0 Measurement

We repeat the analysis for the models in question including the latest SH0ES measurement, $H_0 = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [13], instead of the local prior on M of Eq. (3.1) that was adopted in Section III.2. This is done in order to show that, despite the strong constraining na-

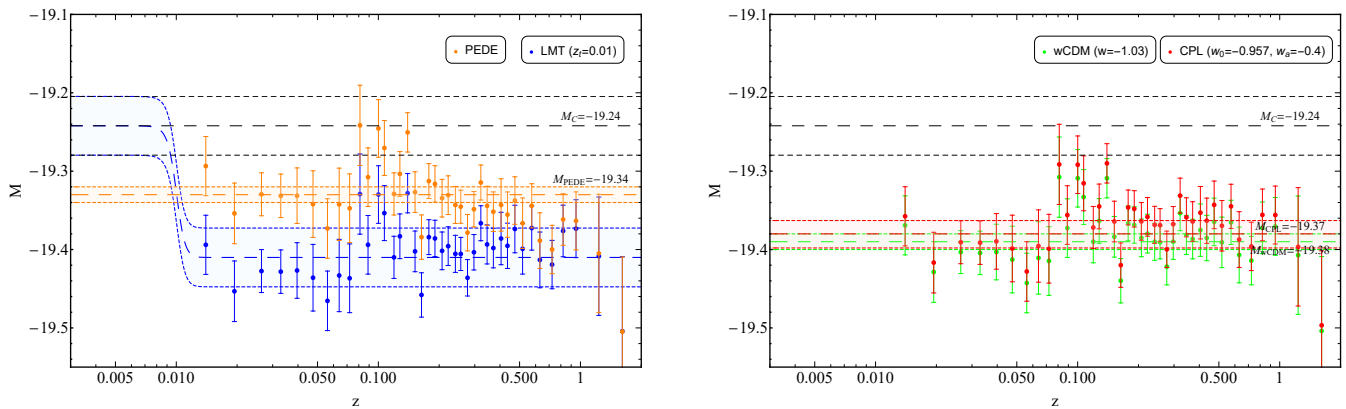


FIG. 7. The best fit absolute magnitude M of the binned Pantheon data as a function of the redshift z . In the left panel we show the corresponding best fit data for the LMT model with $z_t = 0.01$ (blue points) and PEDE models (orange points). In the right panel we show the corresponding best fit data for the $wCDM$ model with $w = -1.03$ (green points) and CPL models (red points). Clearly, all the models provide a value that is inconsistent with the measured Cepheid absolute magnitude M_c (straight dashed lines), unless a model with a transition on M (such as LMT) is considered.

ture of the SH0ES measurement, the obtained absolute magnitude M for smooth $H(z)$ deformation models are inconsistent with the measured Cepheid absolute magnitude $M_c = -19.24 \pm 0.04$ mag of Eq. (3.1). It is worth stressing that it is preferable to adopt the local prior on M for the following reasons [170]: i) one avoids double counting low- z supernova, ii) the statistical information on M is included in the analysis, iii) one avoids adopting a low- z cosmography, with possibly wrong parameters, in the analysis.

Repeating the MCMC analysis and using the same likelihoods described in Section II, we obtain the constraints on cosmological parameters for all the models as shown in Table VI. The corresponding 68.3%–95.5% confidence contours of the common parameters of the models are illustrated in Fig. 6. Constraints for the LMT and $LwMT$ models have been not included in Fig. 6, instead we show those constraints in Fig. 9.

Clearly, all of the considered models (except the models with transitions) tend to prefer a significantly lower value for M (which is considered to be constant) compared to M_c . This is also evident in Fig. 7, where the best fit absolute magnitude M of the binned Pantheon data is shown. In particular, in the case where no prior on M is imposed, two of the considered models, *i.e.* $wCDM$ and CPL, produce a H_0 best fit value that is inconsistent with

the SH0ES measurement [13] at more than 2.4σ . Regarding the $LwMT$ with ($z_t \geq 0.01$), even with the SH0ES measurement, the best fit value of a_t parameter remains unaffected, continuing to favor a transition at very low redshifts. Conclusively, even though the majority of dark energy models discussed in this work (except PEDE and LMT) display a better quality of fit to the data than that of Λ CDM (cyan row of Table VI), they fail to give an M value consistent with the M_c measurement (except the $LwMT$ and LMT models) despite the fact that some of them (such as PEDE) provide a H_0 measurement, that is consistent with the SH0ES measurement at the 1σ level.

Appendix B: Contours plots for the LMT and $LwMT$ models

Here, we show contours plots relative to analyses with the models LMT and $LwMT$. Fig. 8 shows the 68.3%–95.5% confidence contours of cosmological parameters of models LMT and $LwMT$ for the analysis with the Gaussian prior on M . On the other hand, Fig. 9 shows the 68.3%–95.5% confidence contours of cosmological parameters of models LMT and $LwMT$ for the analysis that includes a Gaussian prior on H_0 .

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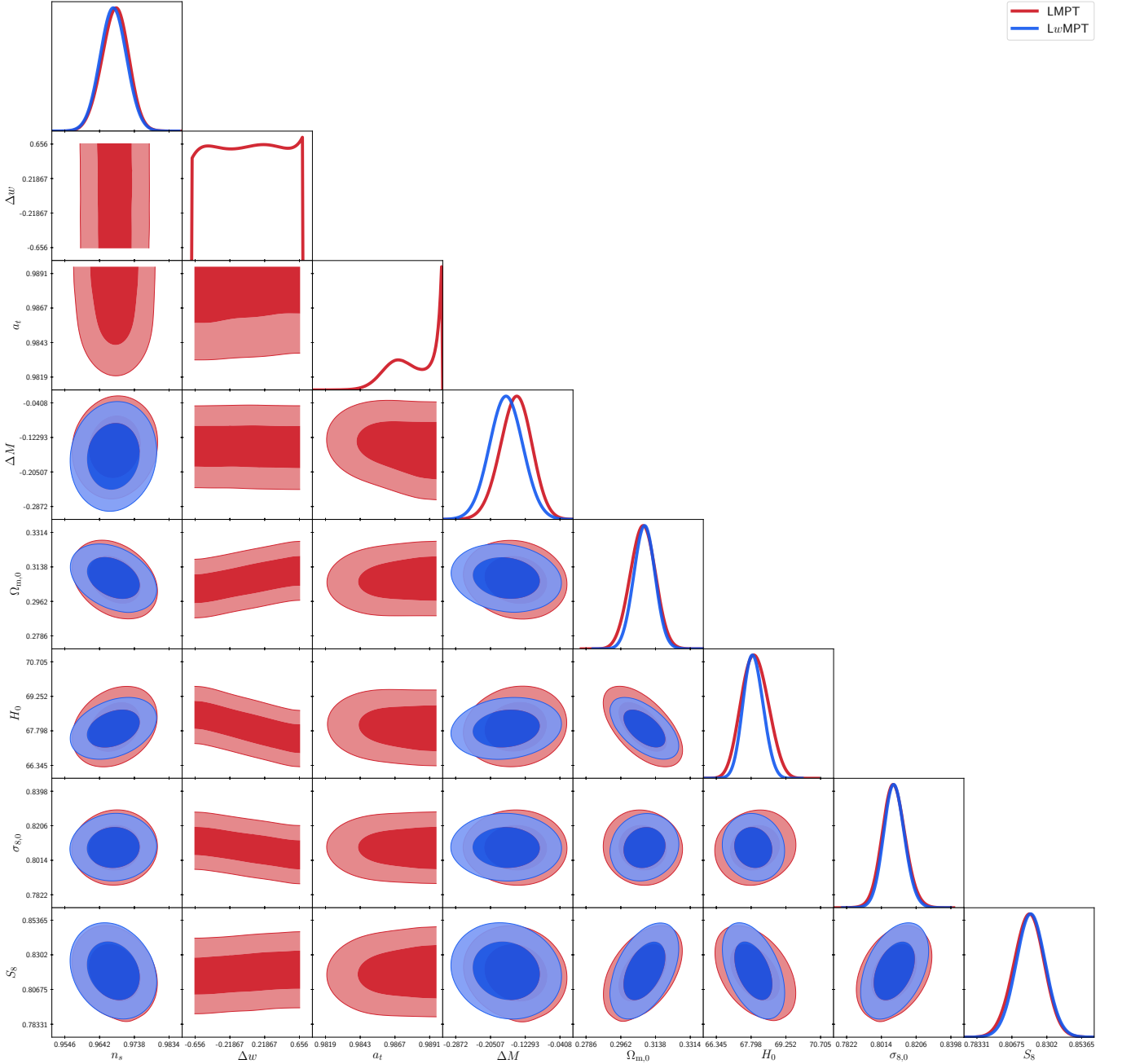


FIG. 8. The 68.3%–95.5% confidence contours of the *LMT*, and *LwMPT* dark energy models with the prior $M = -19.24 \pm 0.04$ mag of Eq. (3.1) from SH0ES (corresponding bounds in Table IV).

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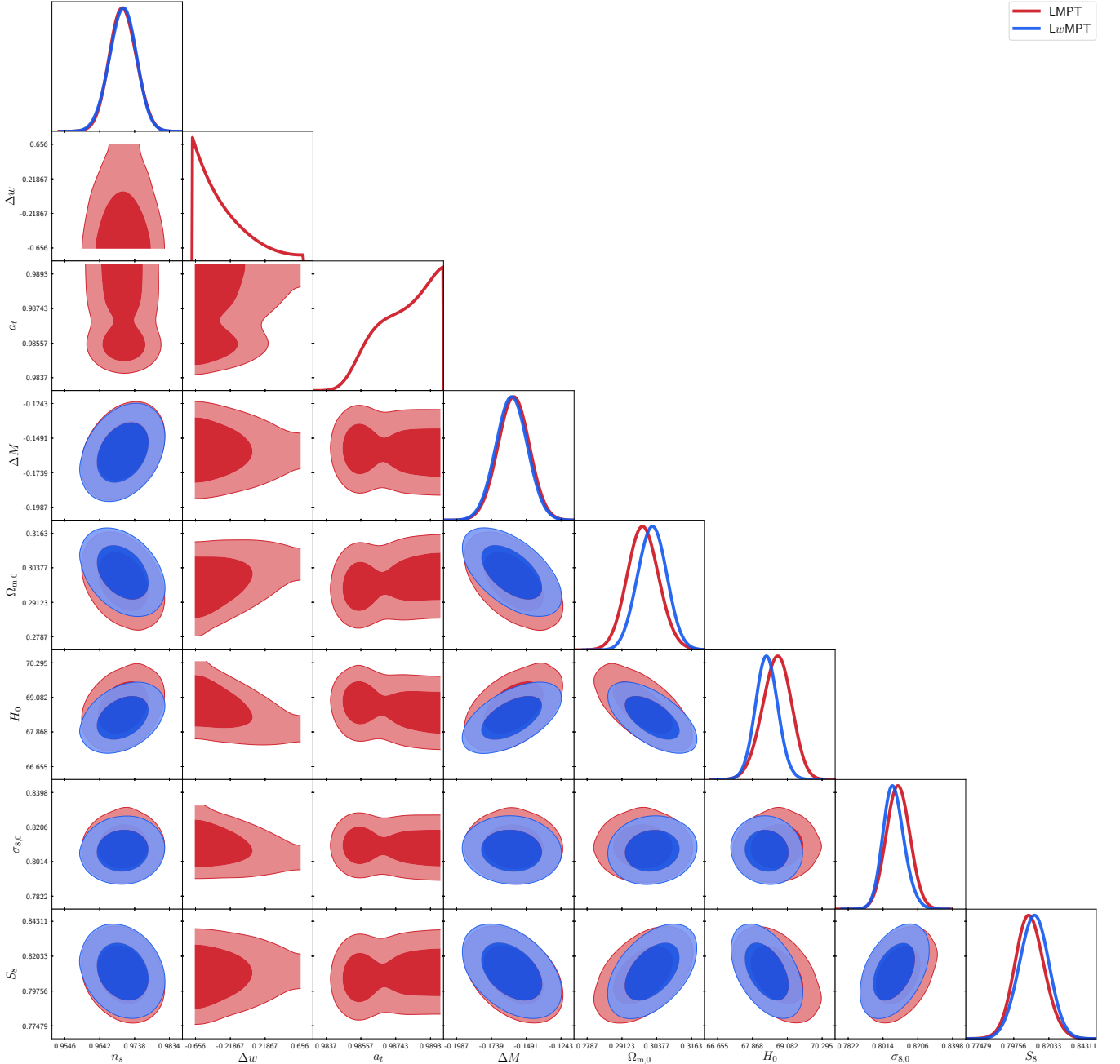


FIG. 9. The 68.3%–95.5% confidence contours for the common parameters of the *LMT*, and *LwMT* dark energy models corresponding to the constraints given in Table VI. We used the CMB+BAO+Pantheon+RSD likelihoods, including the SHOES measurement $H_0 = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [13].

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