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Robust Iterative Learning Control for Pnematic Muscle with Constraints and Unertainties

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***Abstract*—In this brief, we propose a new iterative learning control (ILC) scheme to deal with trajectory tracking problems for pneumatic muscle (PM) actuators. Two critical issues are discussed: 1) actuator constraints, and 2) model uncertainties. To realize PM dynamics, a three-element model is constructed that takes both parametric and nonparametric uncertainties into consideration. By introducing the composite energy function (CEF) approach incorporated with a barrier Lyapunov function (BLF), the constraint requirements on PM is fulfilled and uncertainties can be effectively handled. Through rigorous analysis, we show that under proposed ILC scheme, uniform convergence of PM state tracking errors are guaranteed and state constraints are always satisfied. In the end, an illustrative example is simulated to demonstrate the efficacy of the proposed ILC scheme.**

I. Introduction

Pneumatic muscles (PMs) have been actively studied in the last decades, as they posses advantages such as light weight, compliance and power efficiency over conventional electric motors [1]. Due to its muscle-like properties, researches have led to many applications in robotic manipulators [2]–[4] and rehabilitation devices [5]–[7]. However, the strong nonlinear nature and time-varying characteristic bring difficulties to precise PM control.

There are several challenging issues regarding the research of PM control, two of which will be discussed in this work. First issue is about actuator constraints. One strict physical constraint is that the finite PM inflation leads to limited contraction range. Recent results can be found in [8] and [9] that treat the constraint as a saturated input problem. However, the input saturation is resolved in the closed-loop analysis as a nonlinear term and no validation for optimizing the control performance is given. In addition, the contraction velocity of PM is also a major concern in many practical environment for guaranteeing safe operation. With predefined threshold [10], [11] and saturation function [12], the PM contraction velocity is restricted before the actual control action begins. Nevertheless, how to tackle PM constraints in controller design while maintaining system stability has not be well addressed.

The second challenge facing the PM control is system uncertainties. The three-element form [13] is a common method to model PM behavior and design controllers [8], [14]–[19]. However, all elements contain uncertain parameter which are hard to compensate accurately. To deal with uncertainties, numerous of nonlinear control methods [8], [17]–[19] have been proposed such as sliding-mode [16], dynamic surface [17], adaptive servomechanism [18], integral of sign of error [8] and proxy-based approach [19]. Nevertheless, a common assumption in these methods is that the state-dependent nonlinearities are bounded by some known value which are hard to justify. In addition, only tracking performance with small contraction range are considered in existing studies. For a large PM contraction range, the convergence of controller while not violate aforementioned constraints is still unclear.

Among lots of robotic application scenarios, repetitiveness is a vital feature. Effectively using the repetitive process in the controller design to improve the tracking performance, is of practical importance and interests. In addition, uncertainties caused by different pressure input of PM periodically vary with the inflation/deflation process, thus have repetitive characteristics when same tasks are conducted repeatedly. Since the pioneering work [20] by Arimoto, iterative learning control (ILC) is known to be effective in handling repetitive control process [21]. P-type [22] and PID-type [23] ILC was applied on PM control by contraction mapping (CM) methods. Recently, dynamic linearization technique was employed that transforms the nonlinearities of PM into a data-driven model [24], with model-free adaptive iterative control (MFAILC) scheme, improved tracking performance was achieved. However, it is hard for CM-based ILC and MFAILC to incorporate available system knowledge, whether parametric or structural, into the learning controller design. Furthermore, strict conditions such as identical initial resetting [22], [23] and global Lipschitz continuous (GLC) [24] are required for controller design.

In this brief, by considering both PM control issues, a new ILC scheme for PM repetitive tracking is presented. The barrier Lyapunov function (BLF) is introduced to satisfy both constraints on PM contraction length and velocity. By constructing PM model upon three-element form, uncertainties are composed of time-varying parametric part and state-dependent nonparametric part. The identical initial condition (i.i.c) is replaced with more practical alignment condition, and the nonparametric term only need to be local Lipschitz continuous (LLC). With designed composite energy function (CEF) incorporated with BLF, we show that uniform convergence of PM state tracking errors are guaranteed under the proposed ILC scheme, whereas both constraints will not be violated. An illustrative example with published PM model parameter is simulated to demonstrate the efficacy of the proposed ILC scheme.

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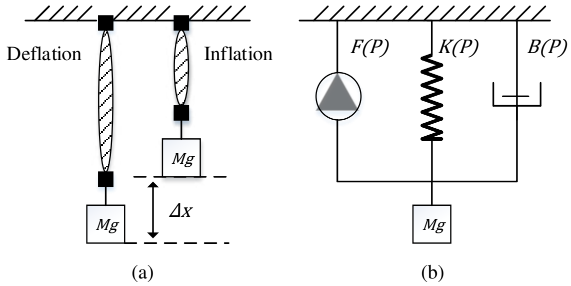

Fig. 1: (a) Operational principle of a PM. (b) Three-element model of PM.

II. PM Model and Problem formulation

A. PM Model

Consider a PM that vertically drives a mass as shown in Fig. 1. The simplified system model [13] is described as

$$\begin{cases} M\ddot{x}_s + B(P)\dot{x}_s + K(P)x_s = F(P) - Mg \\ B(P) = B_1P + B_0 = \begin{cases} B_{i1}P + B_{i0} & inflation \\ B_{d1}P + B_{d0} & deflation \end{cases} \\ K(P) = K_1P + K_0 \\ F(P) = F_1P + F_0 \end{cases} \tag{1}$$

where M, P and g are the mass, pressure and gravitational acceleration, respectively. Contraction length, velocity and acceleration are denoted as $x_s, \dot{x}_s$ and $\ddot{x}_s$. $B(P)\dot{x}_s, K(P)x_s, F(P)$ are the pressure-dependent damping, spring and contractile force elements. Define an equilibrium point at $x_s = x_0$ under internal pressure P_0, we have $K(P_0)x_0 = F(P_0) - Mg$. Let $u = P - P_0$ and $x = x_s - x_0$, then (1) becomes

$$\ddot{x} + B\dot{x} + Kx = (a\dot{x} + bx + c)u \tag{2}$$

where $B = (B_1P_0 + B_0)/M$, $K = (K_1P_0 + K_0)/M$, $a = -B_1/M$, $b = -K_1/M$ and $c = (F_1 + K_1x_0)/M$.
There are four main issues we should tackle in (2),
1) PM is required to perform repetitive task,
2) Parameters are periodically time-varying,
3) Contraction length and velocity are constrained,
4) Other uncertainty such as friction is not considered.
Formulation of these problems will be given as follow.

B. Problem Formulation

For PM works in a repetitive manner, we rewrite (2) as

$$\begin{aligned} x_{2,i}(t) &= \dot{x}_{1,i}(t) \\ \dot{x}_{2,i}(t) &= \theta^T(t)f(x_i,t) + g(x_i,t)u_i(t) + d(x_i,t) \end{aligned} \tag{3}$$

where $x_{1,i}(t)$, $x_{2,i}(t)$ and $\dot{x}_{2,i}(t)$ are the contraction length, velocity and acceleration of PM, respectively. $t \in [0,T], T > 0$ is the time interval during each iteration $i \in \mathbb{Z}^+$. $\theta^T(t) = [-K(t), -B(t)]$ is parametric uncertainty which is iteratively time-varying. $f(x_i,t) = [x_{1,i}(t), x_{2,i}(t)]^T$ is a known state-dependent function, $g(x_i,t) = a(t)x_{2,i}(t) + b(t)x_{1,i}(t) + c(t)$ is a state-dependent uncertain gain for the control input $u_i(t)$ and define the uncertain friction as $d(x_i,t)$. Moreover, the contraction length and velocity are constrained, that is, for any iteration, $|x_{1,i}(t)| < k_{s,1}$ and $|x_{2,i}(t)| < k_{s,2}, \forall t \in [0,T]$ hold, where $k_{s,1}$ and $k_{s,2}$ are positive number.

Property 1: The desired trajectories $x_{1,r}$, $x_{2,r}$ are iteration-invariant with upper bounds $|x_{1,r}|_{sup}$, $|x_{2,r}|_{sup}$, and spatially closed, that is, $x_{1,r}(0) = x_{1,r}(T), x_{2,r}(0) = x_{2,r}(T)$. The actual states are under alignment condition during operation, that is, $x_{1,i}(0) = x_{1,i-1}(T)$, $x_{2,i}(0) = x_{2,i-1}(T)$.

Property 2: For a general PM, $c > 0$ and $|F_1| \gg |B_1|$ always hold that make $g(x_i,t) > 0$ (see [13] and [16]). Thus, we define a lower bound g_{min} for $g(x_i,t)$.

Assumption 1: The state-dependent uncertainties $g(x_i,t)$ and $d(x_i,t)$ are locally Lipschitz continuous (LLC), that is,

$$\begin{aligned} |g(x_1,t) - g(x_2,t)| &< \alpha(x_1,x_2,t)\|x_1 - x_2\| \\ |d(x_1,t) - d(x_2,t)| &< \beta(x_1,x_2,t)\|x_1 - x_2\| \end{aligned}$$

where $\alpha(x_1,x_2,t)$ and $\beta(x_1,x_2,t)$ are continuous bounding functions, and $\|\cdot\|$ is the Euclidean norm for a vector.

Define the state tracking error $e_{1,i} = x_{1,i} - x_{1,r}$ and $e_{2,i} = x_{2,i} - x_{2,r}$. Under Property (1), we have $e_{1,i}(0) = e_{1,i-1}(T)$, $e_{2,i}(0) = e_{2,i-1}(T)$. The control objective is to realize full system states tracking over the iteration domain without violation of constraints on both states.
To facilitate our subsequent discussion, we introduce a fictitious velocity tracking error as $\gamma_{2,i} = x_{2,i} - \sigma_i$, where σ_i is used in some literature as a stabilizing function [25], [26], is defined as $\sigma_i = \dot{x}_{1,r} - \cos^2(\frac{\pi e_{1,i}^2}{2k_{b,1}^2})\kappa_1 e_{1,i}$, where κ_1 is a positive constant to be designed, $k_{b,1}$ is the bound defined for $e_{1,i}$ at any iteration. Since $\gamma_{2,i}$ is a function of $e_{1,i}$, it is also under alignment condition, e.g., $\gamma_{2,i}(0) = \gamma_{2,i-1}(T)$.

Remark 1: Instead of GLC condition that is commonly assumed in PM controller designs [24], [27]–[29], LLC condition is considered in this study. The i.i.c is a general assumption in ILC theory [21] which requires a perfect resetting i.e., $e_i(0) = 0$. From a practical point of view, it can hardly be met and replaced with more realistic alignment condition.

Remark 2: Since $k_{s,1} < |x_{1,r}(t)|_{sup}$ and $k_{s,2} < |x_{2,r}(t)|_{sup}$ hold for complete tracking. Therefore, for PM have constraints that $|x_{1,i}(t)| < k_{s,1}$, $|x_{2,i}(t)| < k_{s,2}$, since $|x_{1,i}(t)| < |e_{1,i}(t)| + |x_{1,r}(t)|$, we can select $k_{b,1}$ such that $0 < k_{b,1} < k_{s,1} - |x_{1,r}(t)|_{sup}$. Similarly, we have $|x_{2,i}(t)| < |\gamma_{2,i}(t)| + |\sigma_i(t)| < |\gamma_{2,i}(t)| + |x_{2,r}(t)| + \kappa_1|e_{1,i}(t)|$. The bound $k_{b,2}$ defined for $\gamma_{2,i}$ can be selected by relation $0 < k_{b,2} < k_{s,2} - |x_{2,r}(t)|_{sup} - \kappa_1 k_{b,1}$ with proper value of κ_1.

III. Robust Controller Design with PM Constraints

A. Robust Controller Design

In order to achieve state tracking error convergence, eliminate the impact due to composite uncertainties and fulfil output constraints, we present the controller and parameter adaption laws first, with rigorous analysis coming later in the detailed proof,

$$\begin{aligned} u_i &= u_i^{ilc} + u_i^r \\ u_i^r &= -\frac{\alpha_i|u_i^{ilc}|\text{sgn}(\gamma_{2,i})\|e_i\|}{g_{min}} - \frac{\beta_i\text{sgn}(\gamma_{2,i})\|e_i\|}{g_{min}} \\ &\quad - \frac{|\hat{\theta}_i^T|\Delta f_i\text{sgn}(\gamma_{2,i})}{g_{min}} - \frac{(\dot{x}_{2,r}-\dot{\sigma}_i)\text{sgn}(\gamma_{2,i})}{g_{min}} \\ &\quad - \frac{\kappa_2\gamma_{2,i}\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})}{g_{min}} - \frac{e_{1,i}\text{sgn}(e_{1,i}\gamma_{2,i})\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})}{g_{min}\cos^2(\frac{\pi e_{1,i}^2}{2k_{b,1}^2})} \end{aligned} \tag{4}$$

$$u_i^{ilc} = proj(u_{i-1}^{ilc}) - \frac{p\gamma_{2,i}}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \tag{5}$$

$$\hat{\theta}_i = proj(\hat{\theta}_{i-1}) + \frac{q\gamma_{2,i}\Delta f_i}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \tag{6}$$

where the control signal u_i consists of a pure iterative leaning part u_i^{ilc} and a robust part u_i^r. $\alpha_i \triangleq \alpha(x_i, x_r, t)$ and $\beta_i \triangleq \beta(x_i, x_r, t)$ are two LLC functions, $\Delta f_i = f_i - f_r$ is a continuous state-dependent function represents the error of regressor f_i. p and q are two ILC gains, sgn is the signum function [30] and κ_2 is a positive constant to be designed. $\dot{\sigma}_i$ can be expressed as

$$\dot{\sigma}_i = \dot{x}_{2,r} - \kappa_1\cos^2(\frac{\pi e_{1,i}^2}{2k_{b,1}^2})e_{2,i} + \kappa_1\frac{e_{1,i}^2}{k_{b,i}^2}\sin(\frac{\pi e_{1,i}^2}{2k_{b,1}^2})e_{2,i}. \tag{7}$$

The projection operators $proj(\cdot)$ are defined as

$$proj(u_{i-1}^{ilc}) = \begin{cases} u_{i-1}^{ilc} & \text{if } u_{i-1}^{ilc} \leq \bar{u}^{ilc} \\ \text{sgn}(u_{i-1}^{ilc})\bar{u}^{ilc} & \text{if } u_{i-1}^{ilc} > \bar{u}^{ilc} \end{cases}$$

and

$$\begin{aligned} proj(\hat{\theta}_{i-1}) &= [proj(\hat{\theta}_{1,i-1}), proj(\hat{\theta}_{2,i-1}), ..., proj(\hat{\theta}_{l,i-1})]^T \\ proj(\hat{\theta}_{j,i-1}) &= \begin{cases} \hat{\theta}_{j,i-1} & \text{if } \hat{\theta}_{j,i-1} \leq \bar{\theta}_j \\ \text{sgn}(\hat{\theta}_{j,i-1}) & \text{if } \hat{\theta}_{j,i-1} > \bar{\theta}_j \end{cases} \\ j &= 1,2,3,...,l \end{aligned}$$

where $\bar{u}^{ilc} \geq |u_r(t)|$ and $\bar{\theta}_j \geq |\theta_j(t)|$, $\forall j = 1,2,...,l$.

Remark 3: The upper bound $\bar{u}^{ilc}$, $\bar{\theta}_j$ can be either selected from hardware limits or just using sufficiently large bounds. Such large bounds will not degrade the control performance while retaining the uniform convergence.

B. Composite Energy Function with Tangent Barrier Lyapunov Function

To guarantee the constraints on PM are not violated, we introduce the following tangent BLF [30], [31]:

$$V = \frac{k_b^2}{\pi}\tan(\frac{\pi\gamma^2}{2k_b^2}), \qquad |\gamma(0)| < k_b \tag{8}$$

which is positive and will approach infinite as $|\gamma| \to k_b$, where k_b is a predefined bound. By selecting proper bounds $k_{b,1}$, $k_{b,2}$, one can guarantee that both PM contraction length and velocity tracking errors are restricted in bound. Consequently, the state constraints are guaranteed for any iteration.

Then, the following CEF is design that incorporates with the BLF at the i-th iteration

$$\begin{aligned} E_i(t) &= V_i^1(t) + V_i^2(t) + V_i^3(t) \\ V_i^1(t) &= V_{1,i}^1(t) + V_{2,i}^1(t) \\ V_{1,i}^1(t) &= \frac{k_{b,1}^2}{\pi}\tan(\frac{\pi e_{1,i}^2}{2k_{b,1}^2}) \\ V_{2,i}^1(t) &= \frac{k_{b,2}^2}{\pi}\tan(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2}) \\ V_i^2(t) &= \frac{1}{2p}\int_0^t g_r(u_i^{ilc} - u_r)^2 d\tau \\ V_i^3(t) &= \frac{1}{2q}\int_0^t (\theta - \hat{\theta}_i)^T(\theta - \hat{\theta}_i) d\tau. \end{aligned} \tag{9}$$

Remark 4: As $k_b \to \infty$, we have $V \to \frac{1}{2}\gamma^2$ which is a quadratic Lyapunov candidate for system without constraint. Therefore, either position constraint or velocity constraint or both on PM can be satisfied by selecting proper k_b. Without loss of generality, in this paper, we take both constraints into consideration. The CEF approach [32] extends the standard Lyapunov function to consecutive learning cycles, thus, the asymptotical convergence of CEF along the learning repetition horizon guarantees the boundedness and pointwise convergence of tracking error over the entire learning cycle.

IV. Analysis of Convergence Property

Theorem 1: For the repetitive PM system (3) under Property (1)-(2), suppose Assumption (1) hold, the proposed control law (4) and two ILC laws (5) and (6) guarantee that

(1) The state tracking errors $e_{1,i}, e_{2,i}$ uniformly converge to zero as $i \to \infty$.
(2) Constraints on the both states will not be violated, that is, $|x_{1,i}| < k_{s,1}$ and $|x_{1,i}| < k_{s,1}$ will always be guaranteed for any iteration.

Proof: The proof consists of three parts. Firstly, the finiteness of $E_i(t)$ is shown. Then, we investigate the non-increasing property of the $E_i(t)$ in the iteration domain at $t = T$ and the asympototical convergence of contraction length tracking error $e_{1,i}$ and fictitious velocity tracking error $\gamma_{2,i}$ in the sense of L^2-norm. Last, the boundedness of involved quantities is given, after which we draw a conclusion regarding the uniform convergence of the real state tracking errors.

Part.I Finiteness of $E_i(t)$

The time derivative of $E_i(t)$ is $\dot{E}_i(t) = \dot{V}_i^1(t) + \dot{V}_i^2(t) + \dot{V}_i^3(t)$. We will exam $\dot{V}_i^1(t)$, $\dot{V}_i^2(t)$ and $\dot{V}_i^3(t)$ in sequence. Start from $\dot{V}_i^1(t)$, $\dot{e}_{1,i} = \gamma_{2,i} - \cos^2(\frac{\pi e_{1,i}^2}{2k_{b,1}^2})\kappa_1 e_{1,i}$ gives

$$\dot{V}_{1,i}^1(t) = \frac{e_{1,i}\gamma_{2,i}}{\cos^2(\frac{\pi e_{2,i}^2}{2k_{b,2}^2})} - \kappa_1 e_{1.i}^2. \tag{10}$$

Similar for $\dot{V}_{2,i}^1(t)$, since $\dot{\gamma}_{2,i} = \dot{e}_{2,i} + \dot{x}_{2,r} - \dot{\sigma}_i$, we have

$$\dot{V}_{2,i}^1(t) = \frac{\gamma_{2,i}\dot{e}_{2,i}}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} + \frac{\gamma_{2,i}}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})}(\dot{x}_{2,r} - \dot{\sigma}_i) \tag{11}$$

where the coupling term $\gamma_{2,i}\dot{e}_{2,i} = \gamma_{2,i}(\dot{x}_{2,i} - \dot{x}_{2,r}) = \gamma_{2,i}\theta^T\Delta f_i + \gamma_{2,i}(g_i u_i - g_r u_r) + \gamma_{2,i}(d_i - dr)$. The control signal u_i is designed to tackle the nonparametric term $g(x_i,t)$ and $d(x_i,t)$, and the parametric term θ^T. Besides, the first term remaining in (10) and the second term in (11) also need to be considered. From Assumption (1), we have

$$\begin{aligned} &\gamma_{2,i}u_i^{ilc}(g_i - g_r) \le \alpha_i|\gamma_{2,i}||u_i^{ilc}|\ \|e_i\| \\ &\gamma_{2,i}(d_i - dr) \le \beta_i|\gamma_{2,i}|\ \|e_i\|. \end{aligned} \tag{12}$$

By considering the controller u_i in form of (4), we have

$$\begin{aligned} \gamma_{2,i}\dot{e}_{2,i} \le\ & \gamma_{2,i}\theta^T\Delta f_i + \gamma_{2,i}(d_i - dr) + \gamma_{2,i}u_i^{ilc}(g_i - g_r) \\ & + \gamma_{2,i}g_r(u_i^{ilc} - u_r) - \alpha_i|\gamma_{2,i}||u_i^{ilc}|\ \|e_i\| \\ & - \beta_i|\gamma_{2,i}|\ \|e_i\| - |\hat{\theta}_i^T\Delta f_i||\gamma_{2,i}| - (\dot{x}_{2,r} - \dot{\tau}_i)|\gamma_{2,i}| \\ & - \kappa_2\gamma_{2,i}^2\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2}) - \frac{|e_{1,i}\gamma_{2,i}|\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})}{\cos^2(\frac{\pi e_{1,i}^2}{2k_{b,1}^2})}. \end{aligned} \tag{13}$$

Substituting (12) and (13) into (11) leads to

$$\begin{aligned} \dot{V}_{2,i}^1(t) \le\ & \frac{\gamma_{2,i}\theta^T\Delta f_i}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} + \frac{\gamma_{2,i}g_r(u_i^{ilc} - u_r)}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} - \frac{|\hat{\theta}_i^T\Delta f_i||\gamma_{2,i}|}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \\ & - \frac{|e_{1,i}\gamma_{2,i}|}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} - \kappa_2\gamma_{2,i}^2. \end{aligned} \tag{14}$$

Combining (10) and (14), we obtain

$$\begin{aligned} \dot{V}_i^1(t) \le & \frac{\gamma_{2,i}\theta^T\Delta f_i}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} + \frac{\gamma_{2,i}g_r(u_i^{ilc} - u_r)}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} - \frac{|\hat{\theta}_i^T\Delta f_i||\gamma_{2,i}|}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \\ & - \kappa_1 e_{1,i}^2 - \kappa_2\gamma_{2,i}^2. \end{aligned} \tag{15}$$

Next we exam $\dot{V}_i^2(t)$, from ILC law (5), we can derive that

$$\dot{V}_i^2(t) = C_1 + \frac{g_r\gamma_{2,i}(u_r - proj(u_{i-1}^{ilc}))}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} + \frac{pg_r\gamma_{2,i}^2}{2\cos^4(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \tag{16}$$

where $C_1 = \frac{1}{2p}g_r u_r^2 - \frac{1}{p}g_r u_r proj(u_{i-1}^{ilc}) + \frac{1}{2p}g_r proj(u_{i-1}^{ilc})^2$ is a finite term guaranteed by projection definition.
For $\dot{V}_i^3(t)$, from ILC law (6), we can derive that

$$\dot{V}_i^3(t) = C_2 - \frac{\gamma_{2,i}\theta^T\Delta f_i}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} + \frac{\gamma_{2,i}\hat{\theta}_i^T\Delta f_i}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} - \frac{q\gamma_{2,i}^2\Delta f_i^T\Delta f_i}{2\cos^4(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \tag{17}$$

where $C_2 = \frac{1}{2q}proj(\hat{\theta}_{i-1})^T proj(\hat{\theta}_{i-1}) - \frac{1}{q}proj(\hat{\theta}_{i-1})^T\theta + \frac{1}{2q}\theta^T\theta$ is a finite term guaranteed by projection definition. Combining (15), (16) and (17), we obtain

$$\begin{aligned} \dot{E}_i(t) \le & C_1 + C_2 + \frac{\gamma_{2,i}g_r(u_i^{ilc} - u_r)}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} + \frac{pg_r\gamma_{2,i}^2}{2\cos^4(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \\ & + \frac{g_r\gamma_{2,i}\{u_r - proj(u_{i-1}^{ilc})\}}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} - \frac{q\gamma_{2,i}^2\Delta f_i^T\Delta f_i}{2\cos^4(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \\ & - \kappa_1\gamma_{1,i}^2 - \kappa_2\gamma_{2,i}^2 \\ = & C_1 + C_2 - \frac{pg_r\gamma_{2,i}^2}{2\cos^4(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} - \frac{q\gamma_{2,i}^2\Delta f_i^T\Delta f_i}{2\cos^4(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \\ & - \kappa_1 e_{1,i}^2 - \kappa_2\gamma_{2,i}^2 < \infty. \end{aligned} \tag{18}$$

For each iteration, the initial CEF is given by

$$E_i(0) = \frac{k_{b,1}^2}{\pi}\tan(\frac{\pi e_{1,i}^2(0)}{2k_{b,1}^2}) + \frac{k_{b,2}^2}{\pi}\tan(\frac{\pi\gamma_{2,i}^2(0)}{2k_{b,2}^2}). \tag{19}$$

When $i = 1$, $e_{1,i}^2(0) < k_{b,1}^2$ and $\gamma_{2,i}^2(0) < k_{b,2}^2$ imply that $E_1(0)$ is finite and $E_1(t)$ finite for $t \in [0,T]$. With finite CEF and alignment condition, $E_i(t)$ is finite for $i \ge 2$.

Part.II Non-increasing $E_i(t)$

The difference of $E_i(t)$ between two consecutive iterations is $\Delta E_i(T) = \Delta V_i^1(T) + \Delta V_i^2(T) + \Delta V_i^3(T)$. We also look at them in sequence. Start from $\Delta V_i^1(T)$, we have

$$\begin{aligned} \Delta V_{1,i}^1(T) = & V_{1,i}^1(T) - V_{1,i-1}^1(T) \\ = & \frac{k_{b,1}^2}{\pi}\tan(\frac{\pi e_{1,i}(0)^2}{2k_{b,1}^2}) - \frac{k_{b,1}^2}{\pi}\tan(\frac{\pi e_{1,i-1}(T)^2}{2k_{b,1}^2}) \\ & + \int_0^T \cos^2(\frac{\pi e_{1,i}(\tau)\dot{e}_{1,i}(\tau)}{2k_{b,2}^2})d\tau. \end{aligned} \tag{20}$$

We will omit τ in the subsequent analysis for convenience. Note that $e_{1,i}(0) = e_{1,i-1}(T)$, we then derive

$$\Delta V_{1,i}^1(T) = \int_0^T \cos^2(\frac{\pi e_{1,i}\dot{e}_{1,i}}{2k_{b,1}^2})d\tau. \tag{21}$$

Similarly for $\Delta V_{2,i}^1(T)$, we have

$$\Delta V_{2,i}^1(T) = \int_0^T \cos^2(\frac{\pi\gamma_{2,i}\dot{\gamma}_{2,i}}{2k_{b,2}^2})d\tau. \tag{22}$$

Using same procedure from (12) to (15), we have

$$\begin{aligned} \Delta V_i^1(T) \le \int_0^T & \frac{\gamma_{2,i}\theta^T\Delta f_i}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} + \frac{\gamma_{2,i}g_r(u_i^{ilc} - u_r)}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} \\ & - \frac{|\hat{\theta}_i^T\Delta f_i||\gamma_{2,i}|}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})} - \kappa_1 e_{1,i}^2 - \kappa_2\gamma_{2,i}^2 d\tau. \end{aligned} \tag{23}$$

For $\Delta V_i^2(T)$, $(u_{i-1}^{ilc}-u_r)^2 \geq (proj(u_{i-1}^{ilc})-u_r)^2$ is guaranteed by projector, hence

$$\begin{aligned}\Delta V_i^2(T) =& \frac{1}{2p}\int_0^T g_r\{(u_i^{ilc}-u_r)^2-(u_{i-1}^{ilc}-u_r)^2\}d\tau \\ \leq& \frac{1}{2p}\int_0^T g_r\{(u_i^{ilc}-u_r)^2-(proj(u_{i-1}^{ilc})-u_r)^2\}d\tau \\ \leq& \int_0^T \frac{\gamma_{2,i}g_r(u_r-u_i^{ilc})}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})}d\tau. \end{aligned} \tag{24}$$

Similarly for $\Delta V_i^3(T)$ and apply the property $(a-b)^T(a-b)-(a-c)^T(a-c)=(b-c)^T(b+c-2a)$, we have

$$\Delta V_i^3(T) \leq \int_0^T \frac{\gamma_{2,i}(\hat{\theta}_i-\theta)^T\Delta f_i}{\cos^2(\frac{\pi\gamma_{2,i}^2}{2k_{b,2}^2})}d\tau. \tag{25}$$

Note that the integral terms in (24) and (25) have different signs in (23). By summing these up, we have

$$\Delta E_i(T) \leq \int_0^T -\kappa_1 e_{1,i}^2-\kappa_2\gamma_{2,i}^2 d\tau \leq 0 \tag{26}$$

hence

$$\begin{aligned}\lim_{i\to\infty} E_i(T) &= E_1(T)+\lim_{i\to\infty}\sum_{j=2}^{i}\Delta E_j(T) \\ &\leq E_1(T)-\lim_{i\to\infty}\sum_{j=2}^{i}\int_0^T -\kappa_1 e_{1,i}^2-\kappa_2\gamma_{2,i}^2 d\tau.\end{aligned}$$

Since $E_i(T)$ is positive and $E_1(T)$ is bounded, we conclude that $e_{1,i}$ and $\gamma_{2,i}$ tend to zero asymptotically in the sense of L^2-norm, as $i\to\infty$, namely

$$\lim_{i\to\infty}\int_0^T e_{1,i}^2 d\tau = 0 \qquad \lim_{i\to\infty}\int_0^T \gamma_{2,i}^2 d\tau = 0. \tag{27}$$

Part.III Uniform convergence of real state tracking errors

By showing that the designed CEF is always bounded in Part.II, while error constraints $|e_{1,i}| \leq k_{b,1}$, $|\gamma_{2,i}| \leq k_{b,2}$ are guaranteed. The boundedness of $x_{1,i}(t)$ and $x_{2,i}(t)$ are derived (see Remark (2)). With bounded states, $e_{2,i}$ is bounded and $\dot{\sigma}_i$ is also bounded. The state-dependent nonlinearity $f(x_i,t)$, $g(x_i,t)$, $d(x_i,t)$ are bounded and θ contains two originally bounded parameters. Hence, the boundedness of $u_i^{ilc}, \hat{\theta}_i$ can be seen and the robustness part u_i^r is also bounded which implies that u_i is bounded. Then, we can derive that $\dot{x}_{2,i}$ is bounded that makes $\dot{e}_{1,i}=\gamma_{2,i}+\sigma_i-\dot{x}_{1,r}$ and $\dot{\gamma}_{2,i}=\dot{x}_{2,i}-\dot{\sigma}_i$ finite. Since $e_{1,i},\gamma_{2.i}$ are uniformly continuous, by convergence property given in Part.II, $e_{1,i},\gamma_{2.i}$ uniformly converge to zero within the close set $[0,T]$, that is

$$\lim_{i\to\infty} e_{1,i}(t)=0 \qquad \lim_{i\to\infty}\gamma_{2,i}(t)=0 \qquad t\in[0,T]. \tag{28}$$

Notice that $\sigma_i\to 0$ as $e_{1,i}\to 0$, so that $\gamma_{2,i}=x_{2,i}-\sigma_i\to x_{2,i}-x_{2,r}=e_{2,i}$. Therefore, we can conclude that the real velocity tracking error also uniformly converge to zero, that is

$$\lim_{i\to\infty} e_{2,i}(t)=0 \qquad t\in[0,T]. \tag{29}$$

■

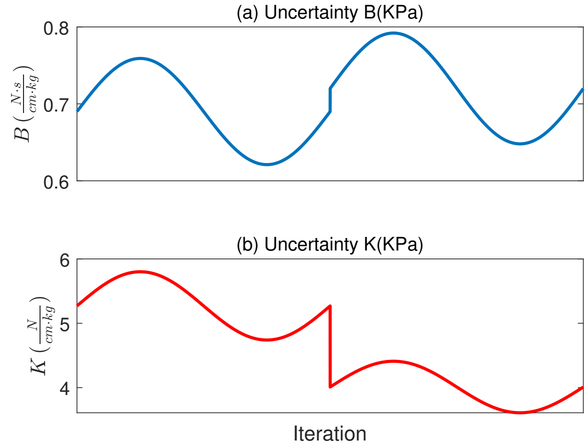

Fig. 2: Time-varying parametric uncertainties for each iteration. (a) $B_{in}(t)=0.69+0.069\sin(4\pi t)$, $B_{de}(t)=0.72+0.072\sin(4\pi t)$. (b) $K_{P\leq 330kPa}(t)=5.27+0.53\sin(4\pi t)$, $K_{P>330kPa}(t)=4.01+0.4\sin(4\pi t)$.

V. SIMULATION RESULTS

In this section, simulation is conducted to demonstrate the efficacy of proposed ILC scheme. Based on the identification results in [18], the coefficients in (1) is given by

$$B(P)=\begin{cases}2.27\times10^{-4}P(Pa)+2435 & inflation\\ 3.2\times10^{-3}P(Pa)+2522 & deflation\end{cases}$$

$$K(P)=\begin{cases}-0.21P(Pa)+9\times10^4 & P\leq 3.3\times10^5(Pa)\\ 0.011P(Pa)+1.8\times10^4 & P>3.3\times10^5(Pa)\end{cases}$$

$$F(P)=0.0022P(Pa)-202.32$$

under 35 N-load and nominal pressure $P_0=338.536$ kPa, while the PM stabilises at $x_0=0.03$ m. This position is set as the new origin such that the control action is still active when the displacement in the new coordinate is negative. The parameters in (2) are calculated as $a=0.5\times10^{-4}$, $b=0.6\times10^{-2}$ and $c=0.05$, while $B(t)$, $K(t)$ are set to be piecewise and time-varying as shown in Fig.2. The maximum contraction range of PM in [18] is 0.05 m, therefore, the control objective is to track the reference trajectory $x_{1,r}=0.015\sin(2\pi t)$, $x_{2,r}=0.03\pi\cos(2\pi t)$, subject to constraints $|x_1|<0.02$ and $|x_2|<0.15$ for all iterations. ILC gains are selected to be $p=100$, $q=100$ in the updating law (5)-(6). We choose $\kappa_1=10$ and $\kappa_2=50$. First, we select the bounds on position error $k_{b,1}=0.0048$ and fictitious velocity tracking error $k_{b,2}=0.006$. For the output constraints, $|x_1|\leq|x_{1,r}|_{sup}+|e_1|\leq 0.015+0.0048<0.02$, $|x_2|\leq|x_{2,r}|_{sup}+\kappa_1 k_{b,1}+|\gamma_2|\leq 0.03\pi+10\times0.0048+0.006<0.15$.

In Fig. 3(a), the tracking error of x_1 approaches zero as iteration increases. In fig. 3(b), both fictitious tracking error and real state tracking error of x_2 are shown. As discussed in convergence analysis, the fictitious term γ_2 approaches e_2

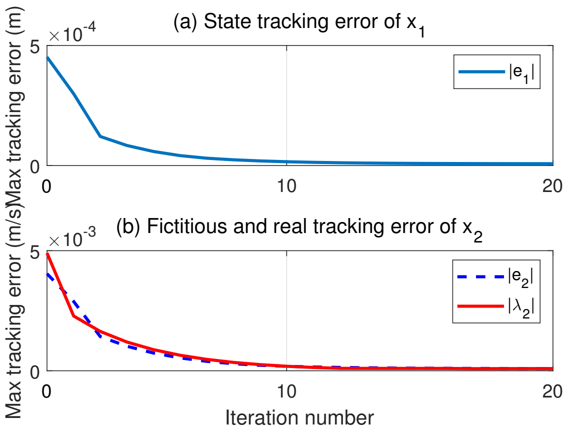

Fig. 3: (a) The state tracking error of x_1, (b) The profile of fictitious tracking error γ_2 and state tracking error e_2, when $k_{b,1} = 0.0048$, $k_{b,2} = 0.006$.

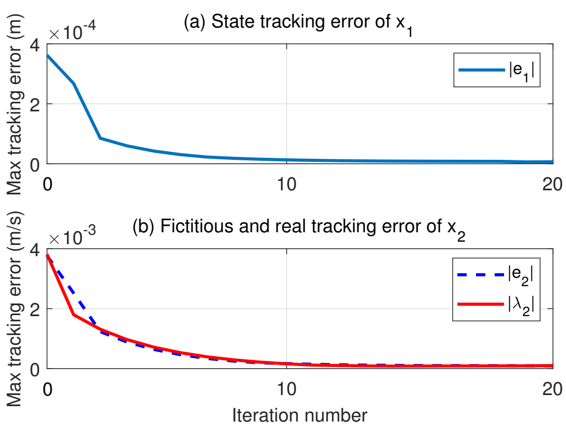

Fig. 4: (a) The state tracking error of x_1, (b) The profile of fictitious tracking error γ_2 and state tracking error e_2, when $k_{b,1} = 0.0038$, $k_{b,2} = 0.0045$.

since the tracking error e_1 converge to zero. Notice that both e_1 and γ_2 are well within the bound due to the use of BLF. Next, we select $k_{b,1} = 0.0038$, $k_{b,2} = 0.0045$ to further illustrate the efficacy of using CEF incorporated with BLF. For the output constraints, $|x_1| \leq 0.015+0.0038 < 0.02$, $|x_2| \leq 0.03\pi+10\times 0.0038+0.0045 < 0.15$. In Fig. 4, the state tracking errors of x_1 and x_2 approach zero as iteration increases, respectively. With the reduced bound $k_{b,1}$ and $k_{b,2}$, the maximum tracking error in the first iteration has been reduced accordingly that implies the BLF works as expected. With bounded error in first iteration, the uniform convergence of our ILC scheme can then successfully fulfil the constraints on PM contraction length and velocity.

Control input signals at the first and last iteration are shown in Fig. 5 and 6. During the first iteration, the robust part has significant contribution to control effort since there are certain amount of state tracking errors. As the iteration increases, the convergences of state tracking errors imply that

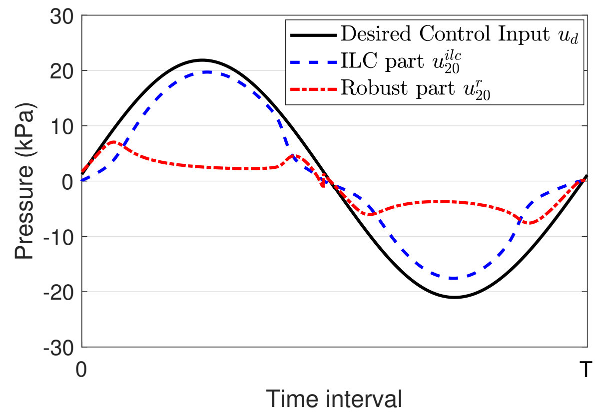

Fig. 5: The control profile at the first iteration, where the ILC part and robust part refer to control law (4) and (5).

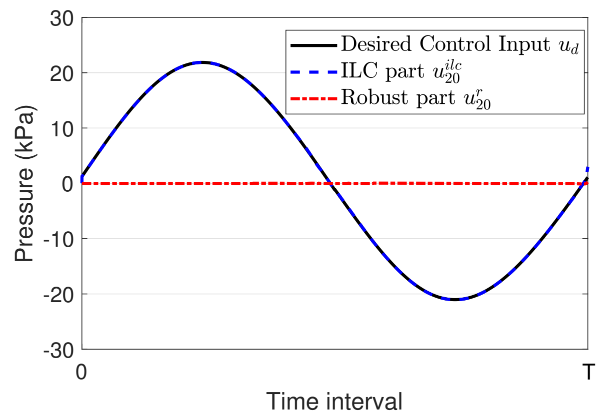

Fig. 6: The control profile at the last iteration, where the difference between u_{20}^{ilc} and u_d is invisible.

no robust compensation is required, while the ILC part u^{ilc} will dominate the control effort.

VI. Conclusion

In this brief, actuator constraints and uncertainties of PM are investigated using ILC approach. Considering the practical application, i.i.c and GLC conditions in most ILC schemes are replaced by alignment and LLC conditions. To deal with PM contraction length and velocity constraints, a tangent BLF is introduced. With designed robust control term, PM uncertainties contain both parametric and nonparametric part can be effectively handled. Under proposed scheme, analysis based on our designed CEF incorporated with BLF guarantees the uniform convergence of PM state tracking errors, whereas both state constraints are not violated.

Parameters in simulation study were taken from the published literature, while the dynamics of PM are widely divergent for different size. Future works will be devoted to construct a self-made PM platform, carrying out accurate modelling results and further validate the efficacy of the proposed scheme.

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