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SAWstitch: exploring self-avoiding walks through hand embroidery

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Abstract

A self-avoiding walk (SAW) is a sequence of moves on a grid that does not visit the same point more than once. SAWs are used to study how networks form, including social networks, biological networks and computer networks, and have provided inspiration to scientists, artists and designers. Here we describe a collaborative project which aims to deliver public engagement activities that embrace creative thinking to explore SAWs through the medium of hand embroidery. We introduce the physics of SAWs and then present an activity which uses materials from hand embroidery to explore these concepts. Specifically, the activity makes use of a Maker Kit which contains all the materials needed to create SAWs on an embroidery hoop. We evaluate the impact of the Maker Kits and reflect on the opportunities provided by a creativity-led engagement activity for physics teaching and research.

Keywords: random walks, networks, creativity, public engagement, art-science interface

Supplementary material for this article is available online

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1. Introduction

A random walk involves moving in steps in an entirely random direction to a new location and repeating this process over and over. Random walks have applications in engineering and many scientific fields including computer science, physics, chemistry and biology. Physicists use random walks to describe diffusion, the random spreading out of molecules in liquids and gases. Random walks are used in biology [1] to study the movement and dispersal of animals and microorganisms [2] and in chemotaxis models [3] of cell signalling and movement in blood vessel formation and cancer cell invasion [4]. More broadly, random walks explain the observed behaviours of many processes in different fields and serve as a fundamental model for the recorded random activity. For example, random walks are used in financial forecasting of stock prices. So-called 'random walk theory' is a financial model which assumes that the future price of each stock is independent of its own historical movement and price of any related stocks [5]. Friend suggestions in social media and keyword searches in databases make use of random walk-based algorithms, with two entities being marked as similar if there are lots of sort paths between them [6]. More recently, random walks have been studied in the context of synchronisation and time dependent social networks [7, 8].

A self-avoiding walk (SAW) is a special case of a random walk in which a sequence of moves on a lattice does not visit the same point more than once. A SAW can be considered as a chain-like path with a fixed step length, s, and the specific property that it does not cross itself. A system of SAWs satisfies the excluded volume condition, wherein a particle has a volume that is inaccessible to other particles in the system as a result of the presence of the first particle. Interestingly, relative to random walks, much less is understood about SAWs from a mathematical perspective, although physicists have provided much insight through the application of numerical simulations, including Monte Carlo methods. Previous studies have used simulations to examine the length [9–11] and shape [12] of SAWs, knots in SAWs [13] and SAWs subject to force [14]. SAWs play a central role in the modelling of the topological and knot theory behaviour of thread and loop-like molecules such as proteins and DNA [15]. SAWs are fractal, i.e. they appear similar at different length scales, a property known as self-similarity [16]. When modelling the growing structure of larger polymer chains, created by the random collision of constituent smaller subunits, studies have examined the properties of self-avoiding random walks. However, solving the theoretical structure of self-avoiding random walks for polymeric systems is difficult and much of what we know has been generated by using numerical simulations to model the progress of this type of random walk [17].

There are many excellent resources for engaging physics students with the topic of random walks. Howard Berg's book on 'Random walks in biology' [1] provides a fascinating introduction to the statistical physics of random walks for undergraduate- and postgraduate-level students of biology and physics, including random motions of molecules or cells and random walks in the context of diffusion, sedimentation, chromatography and cell motility. Resources from the Revised Nuffield Physics Teachers' Guide Year 4 include practical activities for 14–16 year-old physics students and involve drawing a random walk on both squared graph paper and triangular-grid graph paper [18]. These class-based activities can be completed multiple times and the results collated to gather data to, for example, calculate the most likely displacement from the starting point, r. An analysis of all the random walks allows r to be calculated for n steps of length s. In another practical activity for 14–16 year-old physics students, the diffusion and expansion of bromine vapour is measured. The expansion aspect of the activity requires specialist equipment, including vacuum pumps, glassware and tubing and safe handling of bromine, which is toxic and should not be inhaled. Such activities are well placed within a science classroom where there is access to necessary equipment, space for the activity to be completed and appropriate health and safety expertise. However, this limits the reach of these activities to within a science classroom setting and misses opportunities to engage more diverse audiences.

We are interested in developing resources to share SAWs to a wider audience, including beyond a classroom setting and with students that are not studying physics. SAWs have provided inspiration beyond the scientific community. This includes artists such as Antony Gormley who created the Quantum Cloud sculpture, currently displayed in London, which was designed by a computer using a random walk algorithm. The textile artist Marilyn Rathbone was inspired by the patterns formed by networks and created a textile piece entitled ‘Self-avoiding walk’ which was informed by the rules of a SAW. We have explored a creativity-led approach to SAWs as part of a wider ‘Maker kit’ project at the University of Leeds. The Maker Kit project aims to deliver public engagement activities, which embrace creativity and creative thinking, to allow young people and families to explore innovation in science and engineering and, in particular, in material design. The project builds on the culture of collaborative projects we have established which connect scientists, engineers and artists, allowing them to work together and explore innovation in materials design. Such a creativity-led approach has previously proven successful in communicating complex ideas such as in the use of knitted patterns as a model for anisotropy [19]. Creative connections provide a powerful opportunity for scientists to share their research with the public. The project goals are to support connectivity and encourage participation and involvement, to encourage curiosity and learning and enable new ways of thinking and acting. Here we present the activities, how we shared them, lessons learned and next steps.

2. Random walks

2.1. Random walk

A random walk is a stochastic process that describes a path that consists of a succession of random steps in some space. A simple one-dimensional example of a random walk is the path taken on the integer number line where the walker starts at 0 (figure 1). At each time interval, a step is taken either to the left or to the right with equal probability.

This system is mathematically identical to a coin-flipping experiment, with a step to the right being equivalent to getting a head and to the left, a tail. In this case, each step has a length, s, of 1, moving $+1$ or -1 along the integer line and the number of steps taken is n.

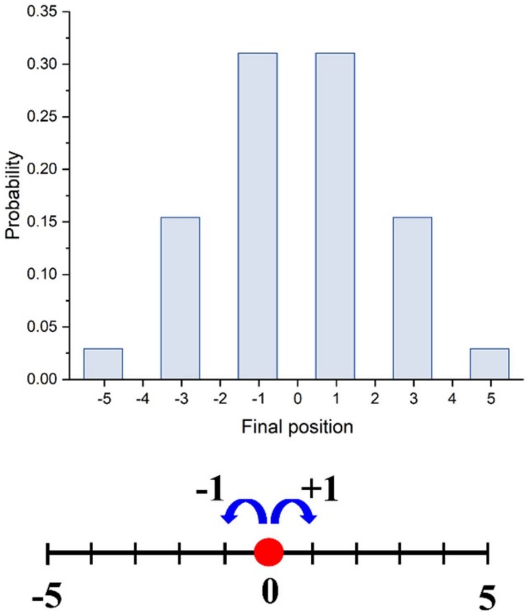

Figure 1. A random walk in one dimension moving a step size of $+1$ or -1. The associated probability distribution and final position after random walks of step number $n = 5$ starting from a position of zero is shown above.

A simple two-dimensional example of a random walk is the path taken on a square lattice, with steps of length one side of the square lattice, in an up, down, left or right direction and all with equal probability (figure 2). In three-dimensions, a random walk describes the path taken by a molecule as it travels in a gas because of Brownian motion.

A random walk can be considered as a series of steps of length s, where each step is in a random direction in space [20]. We can therefore define a series of vectors $\boldsymbol{s}$ that combine to form a random walk, where i indexes the step number from 1 to n in a random walk of n steps. Therefore the end-to-end displacement vector of the random walk, $\boldsymbol{r}$, is:

Therefore the end-to-end displacement vector of the random walk, $\boldsymbol{r}$, is:

$$r = \sum_{i=1}^{n} s_i.$$

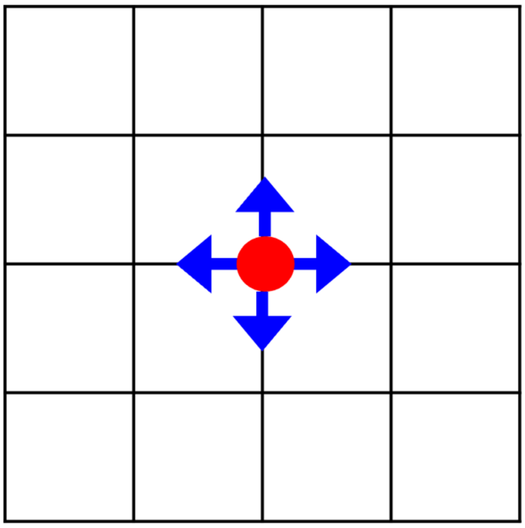

Figure 2. A random walk on a 2D square lattice.

The mean square displacement $<\boldsymbol{r}\cdot\boldsymbol{r}>$ can be written as:

$$\langle r\cdot r\rangle = \left\langle \sum_{i=1}^{n} s_i \cdot \sum_{j=1}^{n} s_j \right\rangle = \left\langle \sum_{i=1}^{n}\sum_{j=1}^{n} s_i\cdot s_j \right\rangle.$$

Mathematically, this can be separated into terms where $i=j$ and $i\neq j$ to give:

$$\langle r\cdot r\rangle = \left\langle \sum_{i=j} s_i\cdot s_j \right\rangle + \left\langle \sum_{i\neq j} s_i\cdot s_j \right\rangle.$$

For the second term, where $i\neq j$ and n is large, the vector dot product can range between $-s^2$ and $+s^2$, therefore this term averages to 0. Hence,

$$\left\langle \boldsymbol{r}^2 \right\rangle = ns^2$$

and the most likely displacement of the random walk is:

$$r=\sqrt{\langle r^2\rangle}=n^{1/2}s.$$

This illustrates how the general solution of a random walk through space leads to an average displacement proportional to the number of steps raised to the power of $^1/_2$. In this general model of a random walk, the trajectory is allowed to visit the same location in space more than once.

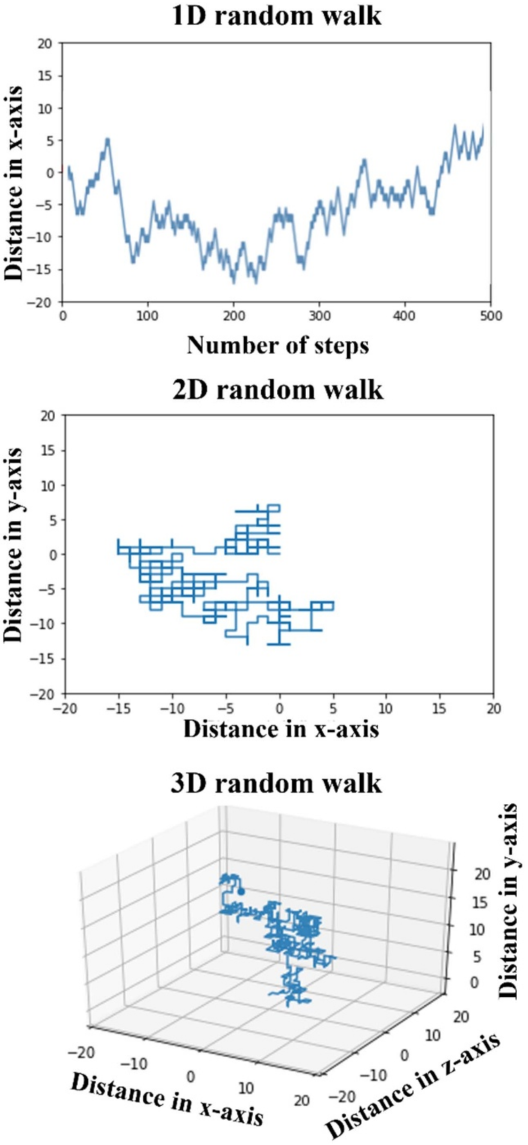

Figure 3. A random walk of 500 steps in 1D (top), 2D (middle) and 3D (lower) created using a python script (source: www.codingem.com/random-walk-in-python/).

Figure 3 shows examples of a random walk of 500 steps in one dimension, two dimensions and three dimensions, created using a python script. However, for some applications it is desirable for the path to not revisit the same points in the network: a condition known as self-avoidance. In this case, the mathematics becomes much more complex.

2.2. Self-avoiding random walks

A SAW is a path from one point to another which never intersects with itself (figure 4). Those familiar with the video game ‘Snake’ may already be familiar with this concept, where the games ends if the snake interacts with itself. In reality, this manifests as a repulsive-like interaction within the network that drives the progression of the random walk away from locations in space that have already been visited. We may still expect the displacement of a random walk to be proportional to the number of steps raised to some power, ν, such that $r \propto n^{\nu}$. Our expectation is that self-avoidance will lead to an exponent in the range $1/2 < \nu < 1$, i.e. the network will be more expanded than a random walk without the self-avoidance condition ($\nu = 1/2$) due to the effective self-repulsion causing the path to expand, but less than a linear path where all the steps are in the same direction ($\nu = 1$).

The polymer physics community have extensively studied the properties of random walks to understand the conformations that polymers adopt in solutions and melts [21]. Perhaps surprisingly, the solution for a non-avoiding random walk, above, is applicable to polymers under some special cases such as concentrated melts, or very specific solution environments (known as theta solvents). However, in general, a polymer in a good solvent can be modelled as a SAW.

A relatively simple approach to this is to consider the segments of the polymer as a gas of unconnected segments within the volume of the polymer coil [22]. Each segment of the polymer contributes to the excluded volume of the polymer and leads to an entropic repulsion in the free energy of the polymer. This entropic swelling effect predicts an exponent of $\nu = 0.6$. For an apparently naïve mathematical treatment that ignores the connectivity of the random walk, this gives a prediction that is close to that found by more rigorous computational modelling approaches, which predict $\nu = 0.588$ [23].

In general, mathematical approaches for SAWs become challenging. Therefore, such paths are usually considered to occur on lattices so that steps are only allowed in a discrete number of steps, n, and of certain lengths, s, with a discrete number of possible directions each step can be taken in. Questions can be asked about the nature of these random walks depending on a number of variables. This includes the number of dimensions through which the random walk takes place and the geometry of the lattice. This determines the number of nearest neighbour sites that the random walk can step to, and the length of the random walk, n, where the mathematical approaches presented above are only valid in the limit of large n, which allows the network to follow a statistically well-defined solution.

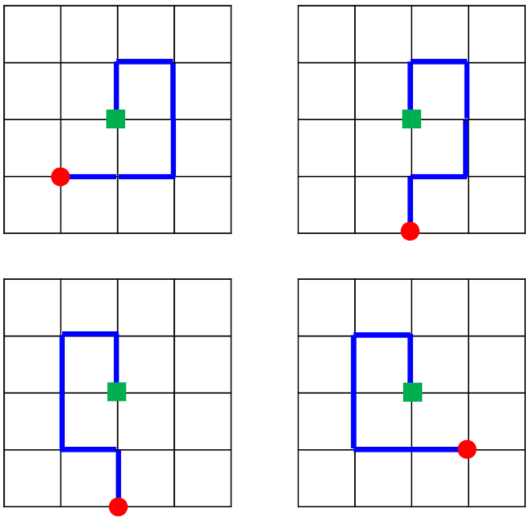

Figure 4. Four examples of SAWs (blue) on a 2D square lattice with steps, $n = 6$, and step length $s = 1$. A green square shows the start of the path and a red circle shows the end of the path.

A further complication comes where SAWs on a lattice can become trapped by taking a path that leads to a dead-end with no allowed move to a neighbouring lattice site (figure 5). This can limit the number of permitted steps, n, in a random walk, prematurely terminating its path. Therefore, an ensemble of these walks may not all be of the same length and statistical approaches assuming a large number of steps may fail.

3. SAWstitch

3.1. The maker kits

Each SAWstitch maker kit included all the materials needed for the activities described below including; a 3 inch embroidery hoop, cross stitch

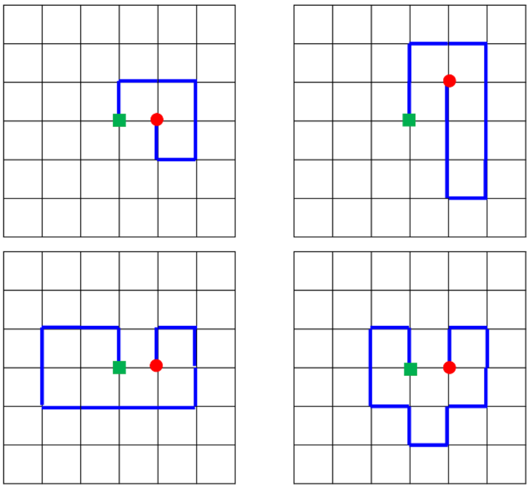

Figure 5. Examples of four SAWs on a 2D square lattice which become trapped because they must avoid intersecting with the path. A green square shows the start of the path and a red circle shows the end of the path.

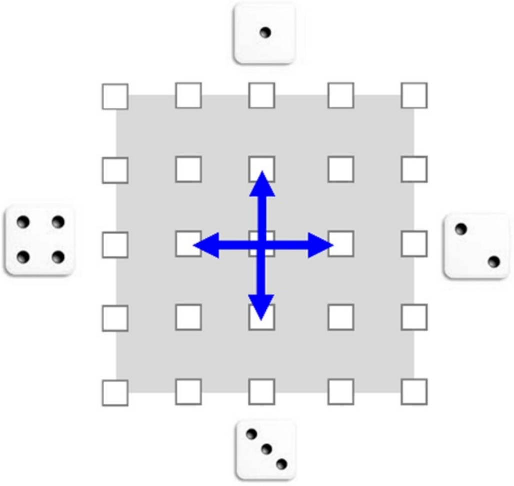

Figure 6. A roll of the die determines the position of each stitch in the SAW, with each number between 1 and 4 allocated to a particular direction on the 2D square lattice.

fabric, a sewing needle, embroidery thread and a die. Each kit contains instructions, providing a step-by-step guide for securing the fabric in the embroidery hoop, threading the needle and making a stitch using 'backstitch' (see SI for the full instructions and links to videos available online at stacks.iop.org/PhysEd/57/045029/mmedia). The Maker Kits were contained in a small, sealed paper bag which could be easily distributed at an in-person event or posted in a small envelope.

3.2. *Creating SAWs through hand embroidery*

A die was used to determine the placement of the stitches on the embroidery walk and the direction of the SAW. The stitching began in the centre of the hoop and the first stitch was made using backstitch (see SI for instructions on stitching). The die was then rolled to determine the direction of the next stitch. Each die number has a specific direction on the fabric.

If the number 1 was rolled on the die, then the next embroidered stitch was made in the upwards direction, i.e. stitched 'up' one square on the cross stitch fabric (figure 6). If the number 2 was rolled, the next stitch will be 'right' one square on the fabric. If the number 3 was rolled, the next stitch will be 'down' one square on the fabric. If the number 4 was rolled, the next stitch will be 'left' one square on the fabric. If a number 5 or 6 are rolled, no stitching is made and the die is rolled again.

To obey the rules of a SAW, if a number is rolled which would result in a new stitch intersecting the already-stitched path, then this number is ignored and the die is rolled again. i.e. the stitched path is avoided. This means there are effectively only three possible moves (at most), and if there is a neighbouring segment of the SAW, this becomes less than three. The die-rolling and stitching is continued until it is no longer possible to make another stitch, i.e. the path has become trapped (figure 7).

3.3. *Exploring SAWs through hand-embroidery*

There are a number of interesting observations which can be made through the process of making a SAW through hand embroidery. Furthermore, because the stitching takes time, it gives time to reflect on the direction the path is making in real-time, allowing for deeper understanding of the underlying physics. For example, we can observe how many stitches (or path steps) are

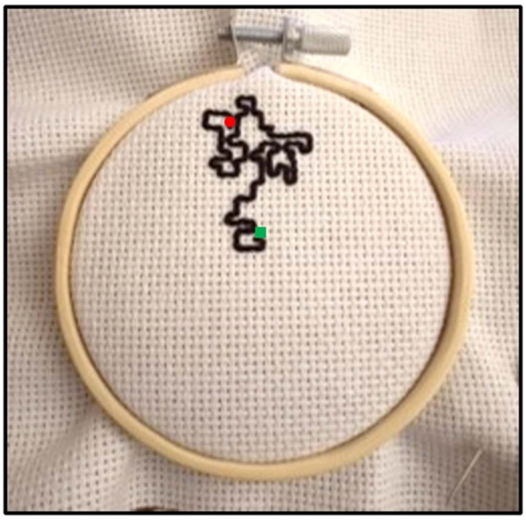

Figure 7. An example of a hand-embroidered SAW on a circular embroidery hoop. The cross-stitch fabric (in white) provides a 2D square grid and the stitches (black) show the path taken by the SAW. A green square shows the start of the path and a red circle shows the end of the path.

Figure 8. Three different examples of SAW embroidered on different hoops. Each SAW has a different number of total steps before it either gets trapped or reaches the edge of the hoop.

made in the SAWstitch. We can observe and compare when the SAWstitch stops and why it has become ‘trapped’ (figure 8).

If a second SAW is made on the sample embroidery hoop, we can observe how the SAWstitches are similar and how they are different (figure 9). If a number of SAWs are made, we can begin to see common features emerge, such as the SAW becoming trapped, and observe unusual outliers, such as very short SAWstitches (a small number of steps) and very long SAWstitches (a large number of steps).

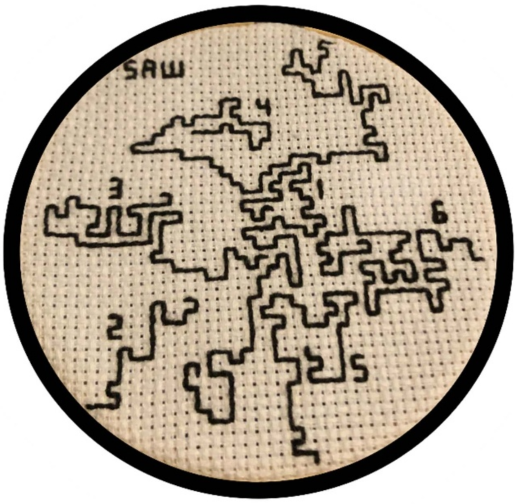

Figure 9. An embroidery hoop with six different SAWs embroidered in back thread and labelled 1–6. Credit: the SAWstitch was created by Naomi McMorn and is published with her permission.

It can be informative to complete a number of SAWs on the same embroidery hoop to illustrate the range of different paths that can be achieved, all from following the same set of simple rules (figure 10). It also reveals inter-random walk repulsion-like behaviour, as well as intra-random walk repulsion which can lead to observations of new phenomena. For example, an apparent attraction between sections of the random walk can be experienced when they come into close proximity due to the reduced number of available directions the path can take, increasing the probability of following the contour of an existing path for several steps. Examples of this apparent ‘stickiness’ in a SAW, which only has short-range repulsive interactions, can be seen in figure 9.

The SAWstitch activity can be easily extended by connecting with research studies which have explored SAWs through more traditional approaches. For example, a video by Nathan Clisby shows a SAW of 100 million steps on a square lattice, generated using a computational model and uses an algorithm to roll the die. In another study, the average number of steps that SAWs take on a square lattice was explored, where a walker may get trapped after any number n steps [24]. In this study, the distribution $t(n)$ of walk

Figure 10. An example of a SAWstitch with 50 different SAWs embroidered on a large 30 cm embroidery hoop. Each SAW is stitched using a different coloured embroidered thread and then labelled with a number between 1 and 50.

lengths was determined by a Monte Carlo calculation using 60 000 walks. The longest SAW encountered had 490 steps, while the average walk length was 70.7 ± 0.2 steps. Such a large number of SAWs would be very challenging to replicate through hand-embroidery, yet this comparison is useful because it highlights the breadth of the SAW distribution. These observations are perhaps more meaningful because of the time taken to create the SAW through hand embroidery i.e. the process of making provides a connection with results of the larger square lattice study.

In the same study [24], the authors identify SAWs which become stuck or 'trapped', as a result of obeying the rule of self-avoidance. Interestingly, they found that the average displacement of trapped walkers is merely 11.9 step lengths, s. Again, this result is interesting in the context of the SAWstitch activity, as lattice units equate to a small number of squares on the cross stitch fabric.

The patterns created by the SAWstitch can also be compared to SAWs created by more traditional approaches. For example, a previous study explored SAWs in 2D, 3D and 4D [25]. By applying numerical simulations the authors determined the multifractality of SAWs on percolation clusters and obtained estimates for the set of critical exponents, ν, that govern scaling laws. Comparison can be made between the 2D SAWs created in their study and the SAWs created through hand-embroidery. This comparison provides a powerful opportunity to connect a craft-based public engagement activity with advanced numerical modelling in physics, an opportunity that might otherwise be very difficult to achieve.

4. SAWstitch and engagement with different audiences

4.1. Physics research environment

We have described a series of activities for communicating the physics of SAWs to a non-expert audience. We developed these activities during the COVID-19 pandemic, at a time when university research laboratories were closed and members of the public were self-isolating at home, globally. Our interest in SAWs stems from our research on network formation of protein hydrogels [26, 27].

We use biomolecules called proteins as our 'thread' and create networks by making them connect through sticky points on their surface. The 'sticky' connections are made through a chemical reaction which is activated by light of a particular wavelength [28–30]. This results in the creation of a network with a defined shape and material properties [31–33]. SAWstitch project development and discussions took place online and the physical materials of the Maker Kits provided an important physical focus for connecting across the project team. We shared the kits with other researchers through a Physics Craft Gathering group, which ran virtually throughout the pandemic. We noted a common response from the researchers. Many people had come across random walks before in their previous studies, but had not considered the topic for some time. The act of embroidering together to create the SAWstitch provided a collective activity in which questions were raised and explored in a non-traditional setting.

4.2. The general public

Our second delivery was at the 'Be Curious' event at the University of Leeds, a research open day with an audience of approximately 1200 people. At the event, a number of different stalls are typically set up, each showcasing areas of research from across the full spectrum of disciplines at the

University. In 2021, Be Curious required a flexible approach to an ever-changing national picture because of the COVID-19 pandemic. The uncertainty surrounding COVID-related restrictions and the health and safety of staff and members of the public led the organisers to focus efforts on offering an enhanced programme of virtual events, which built upon the feedback received (from both researchers and members of the public) from Be Curious 2020. This provided an ideal opportunity to share the SAWstitch Maker kits with members of the public. Facilitated by the University of Leeds Public Engagement Team, the SAWstitch Maker Kits were advertised through their social media platforms and members of the public had the opportunity to request free kits to be posted to their homes. Fifty kits were distributed to locations across the UK, Europe and USA, with kits 'selling out' within 10 days of the programme launching. When asked about their main motivation for requesting a Maker Kit, one evaluation response was: '[I am] always looking for inspiration and trying new ideas to spark further thought and action, my child is home educated, they are autistic and hugely creative, any activity that combines analytical thought with their art practice is usually really well received… they do not easily access "math" or "science" if presented in a typical format.' After using their Maker Kit, 89% of respondents said they had a better understanding of SAWs and 78% said it had inspired them to learn more about the topic. Further feedback suggested that audiences valued the opportunity to take part in a hands-on activity: 'It was amazing, very enjoyable and interesting to do and the result is really great and is proudly hanging on the wall. Thank you so much for organising.' 'Thank you for an enjoyable activity and learning. Really appreciate a free kit to do it practically!'

4.3. Early years children and their families

An important outcome of the engagement activity outline above was a request from adults to adapt the activity for their younger children, including early years (3–5 year old) and primary school (5–11 year old) aged children. This involved a consideration of the materials to make them safe and accessible for this younger age group. This was achieved by adapting the activity to replace the embroidery fabric with a street map of the local area and the thread with a pen. The rules of the die remained the same, as described in section 3.2. Children and their families were encouraged to roll the die and use the rules to create a new path. Families explored the activity in two ways. They followed the instructions to create different paths or they used the local street map provided to create a new path for their family walk. It is worth highlighting that this activity took place at a time when the public were required to follow government rules which included a restriction of one outdoor physical activity per day, in close proximity to home and without interaction with anyone outside your home unit. In the early-years age group, an interesting observation was that some of the children experienced their first opportunity to roll a die and learn the numbers on the sides of the die. It also gave them the opportunity to experience the consequences or random probability or chance in a structured activity. This simple discovery-led activity provided powerful opportunities for adult companions to engage with creative thinking and imagination to support a learning outcome [34].

4.4. The scientific community

Outputs from the SAWstitch project were shared as part of the American Physical Society Gallery of Soft Matter at the their annual meeting in Chicago. Images of the embroidered SAWs were displayed during the meeting, which attracted 12 000 physicists through an in-person and virtual platform.

5. Discussion and conclusions

In 2014, the Institute of Mechanical Engineers reported on the personalisation of engineering education [35]. The study showed that young people divide themselves broadly into five categories (termed 'Tribes'), determined by their values and beliefs, their attitudes to school, family and work, as well as their reactions to STEM as a subject and as a potential career. The five Tribes were (a) STEM devotees, (b) social artists (c) enthused unfocused (d) individualists and (e) less engaged. The report raised important questions about whether we should replace the current 'be like me' engagement approach with programmes that take difference into account and provided specific recommendations for bespoke engagement strategies for each Tribe. The two largest Tribes, ‘STEM devotees’ and ‘Social artists’, are present in similar proportions across all ages, and express very different attitudes and ambitions, yet both appear more focused in their goals than other Tribes. While the ‘STEM devotees’ expressed very high levels of enjoyment of STEM subjects and saw STEM related careers as prestigious and attainable, the ‘Social artists’ tribe, a large, female-dominated, academically highly capable and creative section of the population had relatively little affinity with STEM subjects or careers. The report suggested that the rejection of the ‘Social artist’ Tribe to STEM is mainly driven by absence of interest rather than lack of confidence or ability.

We are interested to explore whether the ‘lack of interest’ in STEM of the Social Artist Tribe can be surmounted by a new approach to education and engagement which embraces creativity. Recent studies have examined the role of creative thinking in the education of young people [36, 37] and questioned whether creative thinking and problem solving across all disciplines, including the sciences, is sufficiently supported by the current education system [38]. Embracing creativity in education has the potential to retain the creative talents and innovative abilities of many young people who do not fit the obvious STEM archetype [39, 40]. Indeed, under-represented groups in STEM, including girls, ethnic minorities and young people from low income families, are deciding that careers in science are ‘not for people like me’ [41]. Parents and the wider family play a key influence in the subject and subsequent career choices for many under-represented groups. Could creativity in education retain the creative talents and innovative abilities of young people who do not fit the STEM archetype? The SAWstitch activities take an alternate, creativity-led approach for exploring the physics of SAWs and provides opportunities to reach new and more diverse audiences.

Recent reports have highlighted that one of the UK’s key economic problems is a shortage of STEM skills in the workforce [42] and emphasised the need to tackle particular shortages of STEM skills. Innovation at the cutting edge of science requires creativity [43]. Unless creative thinking is embedded throughout science education from an early age, the most creative minds are not as likely to follow a STEM career path, which is to the detriment of science, industry and wider society [44, 45]. The SAWstitch project is an example of a creativity-led approach for physics education and may support diversification of the future workforce in STEM subjects such as physics.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgments

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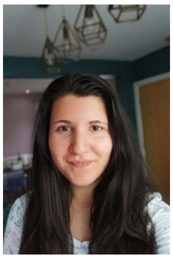

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