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Dynamic wireless charging lanes location model in urban networks considering route choices

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Abstract

Wireless charging technologies have now made it possible to charge while driving, which offers the opportunity to stimulate the market penetration of electric vehicles. This paper aims to support the system planner in optimally deploying the wireless charging lanes on the network, considering traffic dynamics and congestion under multiple vehicle classes. The overall objective is to maximise network performance while providing insights into traffic propagation patterns over the network. A multi-class dynamic system optimal model is adopted to compute an approximate representation of the dynamic traffic flow. As a result, the problem is formulated as a mixed-integer linear program by integrating the dynamic routing behaviour into the charging location problem. Finally, the proposed framework has been tested on different sized test-bed networks to examine the solution quality and illustrate the model's efficacy.

Keywords: Electric vehicles, wireless charging lanes, location model, bi-level optimisation, dynamic traffic assignment

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1. Introduction

In an effort to achieve a sustainable transportation system, Electric Vehicles (EVs) have been promoted by governments worldwide. However, the adoption of this environmentally friendly and economically efficient means of transportation is still limited compared to Internal Combustion Vehicles (ICVs). The relatively short driving range, limited charging availability and long

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charging time are significant hurdles for accelerating transport electrification. In order to overcome these shortcomings, wireless charging technology has been developed and already tested on many platforms, including Tesla, BMW, Nissan, Honda and Renault (Mubarak et al., 2021).

Wireless charging includes stationary wireless charging in which EVs are charged while parking at charging facilities and charging-while-driving with the charging mechanism installed under the road surface (Jang, 2018). Charging-while-driving can also be classified as (i) quasi-dynamic charging when an EV accelerates or decelerates from a resting position and (ii) dynamic charging when an EV is in motion. Although stationary wireless charging is safer and less burdensome, it is not significantly different from conventional plug-in conductive charging in terms of charging time, frequency, vehicle operation, and charging station allocation (Cirimele et al., 2018). However, charging-while-driving, especially dynamic charging, enables EVs to be charged while in operation and offers more opportunities for widespread availability.

In the present paper, we focus on the dynamic wireless charging infrastructure or wireless charging lanes (WCL). In addition to making charging more convenient and increasing EV market penetration, WCL also offers advantages, including the ability to reduce the size of batteries and lighten vehicles and facilitate charging with renewable energy sources (Bi et al., 2019). However, the new technology has raised new issues for network design and operations due to the change of traveller's routing and charging behaviours. This study aims to support the system planner to optimally deploy WCL in the manner of maximising social welfare while considering traffic dynamics and route choice behaviours.

1.1. Related literature

Based on the demand pattern and route choice behaviours, EV charging infrastructure location problems can be formulated and solved by different approaches, including node-based, flow-based and equilibrium-based models (Shen et al., 2019). The node-based models were initially proposed by Church and Meadows (1979) based on the covering demand nodes concept. It is assumed that the demands are generated at individual nodes and can be used to locate on-site low-power charging facilities (level 1 and level 2 modes) where the users can park their vehicles for several hours or overnight to fully recharge (Nozick, 2001; He et al., 2016). On the other hand, the flow-based approach is preferable for fast charging facilities (level 3 charging model) due to its ability to consider the demands in the form of traffic flow and capture travellers' route choice (Hodgson, 1990; Kuby and Lim, 2005; Xu and Meng, 2020). In order to capture the mutual interaction of re-routing behaviours and the charging locations decision and avoid the deterioration in network performance, the equilibrium-based approach can be adopted in a bi-level optimisation program (He et al., 2018; Miralinaghi et al., 2020; Tran et al., 2021b).

In terms of wireless charging infrastructure, most existing literature focuses on the technical side of dynamic in-motion charging. So far, limited studies have explored the transportation network design problem. Riemann et al. (2015) adopted a flow-capturing location model to locate a fixed number of WCL to maximise the total captured flow. The authors used the logit-based stochastic user equilibrium to determine the traffic flow pattern and formulate the problem as a mixed-integer nonlinear program. Then the original problem was reformulated into a mixed-integer linear program which can be solved efficiently by CPLEX solver. All vehicles were considered EVs in the study, and travellers are assumed to start their trips with a full battery. Besides, the authors assumed an EV will be fully recharged when it uses a charging link.

Fuller (2016) approached the WCL location problem by utilising the flow-based set covering model. The model aimed to minimise the investment cost and was solved by a Branch and Bound technique implemented in CPLEX. Besides, Chen et al. (2017) and Chen et al. (2018) investigated the competitiveness of the WCL considering one traffic corridor and electric bus operations, respectively. More recently, Mubarak et al. (2021) studied a WCL deployment network design problem to minimise the instalment cost and electricity cost while satisfying the charging demands of all EVs in the network. The model did not consider travellers' route choice behaviour and suggested deploying chargers to the easy-access locations where there was a high volume of EVs traffic. However, that approach of locating the charging infrastructure might suffer from traffic congestion, especially during rush hours (Yan and Shen, 2021). In addition, neglecting the re-routing behaviour of EVs' travellers while deploying charging infrastructure may sometimes lead to an unreliable solution or a deterioration in the network performance (Ashkrof et al., 2020; Tran et al., 2021b).

Concerning travellers' routing behaviours, Liu and Wang (2017) adopted traditional user equilibrium (UE) and formulate the problem as a tri-level problem. The model aimed to minimise the public social cost and was solved by a heuristic algorithm. Considering two different recharging facilities, i.e. traditional charging stations and modern charging lanes, Zhang et al. (2018) also proposed a bi-level framework in which the upper-level aims to minimise travel time and greenhouse emissions while the lower-level captures UE condition. Similarly, Ngo et al. (2020)

proposed a bi-level program with traditional UE at the lower level to deploy the WCL in a network with minimum total travel time and energy consumption. The model also was solved using a heuristic algorithm. On the other end of the spectrum, He et al. (2020) used the traditional UE to estimate the link travel time and link flow, then proposed a WCL model to serve as many EVs as possible considering the adverse effect of WCL on road capacity. The model was linearised and solved by CPLEX solver.

Recently, Tran et al. (2021b) proposed an equilibrium-based optimisation framework to solve the fast-charging station location problem considering multi-class vehicles with route choice and increasing EVs' penetration. In the study, travellers were assumed to choose the feasible path under the static user equilibrium principle. Subsequently, to capture the stochasticity and environmental impact of the fast-charging location problem, Tran et al. (2021a) developed a systematic framework to simultaneously consider the investment and environment costs as well as the en-route congestion and charging congestion. Specifically, the study jointly considered stochastic travelling demands, stochastic EVs driving range limitations, charging congestion at charging stations and stochastic traveller route choice behaviour (probit-based stochastic user equilibrium). Meta-heuristics were proposed to solve these two frameworks. Meanwhile, by focusing on wireless charging lanes (WCL), the present study proposes an optimisation framework to solve the WCL location problem considering dynamic travellers' route choice and providing insights into flow dynamics over the network.

Along this research stream, a bi-level programming model is typically proposed to formulate the problem in which the route choice is taken into account. In particular, the upper-level problem is to determine the optimal charging locations, and the travellers' route choice behaviour is described in the lower level by static traffic assignment models based on steady-state flows and the time-dependent relationship between link flow and link travel time (Sheffi, 1985). Nevertheless, these relationships in static traffic assignment models might be too coarse even for planning purposes. In other words, they might lead to systematic bias on a broad scale, not just a matter of details (Balijepalli et al., 2014). In order to overcome the deficiencies of static models, dynamic traffic assignment (DTA) models have been developed to reflect the propagation of flow over time and space (Peeta and Ziliaskopoulos, 2001; Szeto and Wong, 2012; Chakraborty et al., 2018). However, the network design problem with DTA models considering the presence of EVs and charging infrastructures remains undeveloped; so do the corresponding solution algorithms

(Wang et al., 2018).

1.2. Objectives and contributions

With the above concerns, this paper aims to fill the gap by addressing the WCL location problem considering the dynamic of traffic flows on urban networks. The overall objective of this paper is to propose a decision-making framework to support the planner to deploy charging lanes to maximise the system performance, i.e. total outflow, during the peak period while tracking the traffic state and considering congestion propagation over the network. The contributions of this study can be highlighted in the following aspects. From the modelling perspective, the present study contributes to the recent and still relatively limited literature in EV charging infrastructure planning by proposing a new mathematical formulation for WCL deployment in urban networks. Compared to previous models, this paper aims to optimally deploy WCL to optimise the system-wide performance and provide more insights into the flow dynamics for regional planning purposes. From the methodological perspective, this study proposes a system-level approach to solving the dynamic WCL location problem. In this study, we scope the problem when travellers cooperatively achieve the minimum system-wide travel time. As a result, a multi-class dynamic system optimal (DSO) model with the presence of EVs and WCL is adopted to compute an approximate representation of the dynamic traffic flows. In this way, the problem is formulated as a mixed-integer linear program (MILP) by integrating the multi-class DSO into the charging location problem.

For the sake of model formulation, the following assumptions are made in this study:

1. In our numerical tests, all EVs are assumed to have the homogeneous battery size and the initial state of energy (B_{max} and B_0, respectively). Nonetheless, since the proposed multi-class model is capable of capturing different vehicle types, EVs can be classified by their initial energy state and battery size. Besides, all paths between the origin and destination are feasible for ICVs due to their relatively long driving range (Jiang et al., 2014).

2. Since the electricity cost is insignificant compared to the travel cost (He et al., 2014), EVs' drivers are assumed to select paths to minimise the total travel cost while ensuring to complete their trips without running out of charge.

3. An EV will be recharged when driving on the WCL with the amount of energy proportional to the traverse time on that link (e.g., $\omega \in [0.33; 3.33]$ kwh/min) (Fuller, 2016). Mean-

while, the energy consumption rate of an EV is proportional to the travelled distance and homogeneous for all EVs (e.g, $\epsilon = 0.29$ kwh/mile or $1.8\text{x}10^{-4}$ kwh/m) (He et al., 2020). However, the energy consumption of an EV may depend on many factors, such as velocity, mass, road gradient and the use of accessory devices. One can deploy energy consumption models developed by Liu and Song (2017) to calculate the amount of energy consumption precisely.

This study focuses on the model formulation thus other calibrated parameters are assumed to be readily available from the literature.

1.3. Nomenclature

For the proof of concept, we are limited to single origin-destination (O-D) networks. However, the proposed framework for deploying WCL lanes under dynamic traffic in the current paper will still contribute to the state-of-the-art. Considering a traffic network as a single O-D directed graph, Table 1 below summarizes the notations used to define the model in this paper.

Table 1: Table of notation

Symbol	**Definition**
Sets and parameters	
$\mathscr{N}$	Set of nodes, including source nodes ($\mathscr{N}_R$), sink nodes ($\mathscr{N}_S$) and normal nodes ($\mathscr{N}_A$)
$\mathscr{E}$	Set of links, including source links ($\mathscr{E}_R$), sink links ($\mathscr{E}_S$) and normal links ($\mathscr{E}_A$)
Υ_a^-	Set of incoming links to link a, $a \in \mathscr{E}/\mathscr{E}_R$
Υ_a^+	Set of outgoing links from link a, $a \in \mathscr{E}/\mathscr{E}_S$
$\mathscr{P}$	Set of all paths between the origin and the destination
$\mathscr{M}$	Set of vehicle classes
$\mathscr{T}$	Set of discrete time steps
τ	The length of time step
$D^m(i)$	Demand rate of vehicle class m at time step i
$\hat{\alpha}_a^m$	The aggregate link-based share factor of vehicle class m
V_a, W_a, K_a	Set of fundamental diagram for each link a
Q_a	The maximum flow capacity of link a
l_a	Length of link a
I	The investment budget
ι	The unit capital cost of charging lane

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Symbol	Definition
ω	The amount of energy an EV can receive when traversing on charging link per time unit
ϵ	The electricity consumption rate of EVs
B_{max}	The maximum state of energy of EVs
B_0	Initial state of energy of EVs
Decision variables	
x_a	Whether a charging lane is deployed on link a or not
y_p^m	Whether path p is feasible for vehicle class m or not
$B_{a,p}$	The state of energy of an EV after traversing link a on path p
$n_a^m(i)$	The number of vehicles class m on link a at time i
$u_a^m(i)$	Incoming traffic flow of vehicle class m to link a at time i
$v_a^m(i)$	Outgoing traffic flow of vehicle class m from link a at time i
$f_{ab}^m(i)$	Upstream traffic of vehicle class m at link b, coming from downstream traffic at link a

Additional symbols used for dual variables and other symbols with a narrower scope will be defined in the sections where they are used.

The remainder of this paper is organized as follows. The problem is mathematically formulated considering users' behaviour in Section 2. Then, a multi-class Dynamic System Optimum (DSO) model with the presence of EVs and WCL is presented in Section 3. In Section 4, the proposed framework is tested on different size networks and gives insights into the traffic state and congestion propagation over the network. Finally, Section 5 concludes the paper and pavs potential extensions for future research.

2. Model development

The charging location problem is a network design problem (NDP) that aims to place the WCL on the network to maximise the social benefit considering the dynamic flow. In most cases, NDP consists of different and independent decision-makers, e.g. the system planner and network users (travellers). The mutual interaction between deployment decisions and the routing behaviour of travellers can be naturally formulated as a bi-level structure (Figure 1).

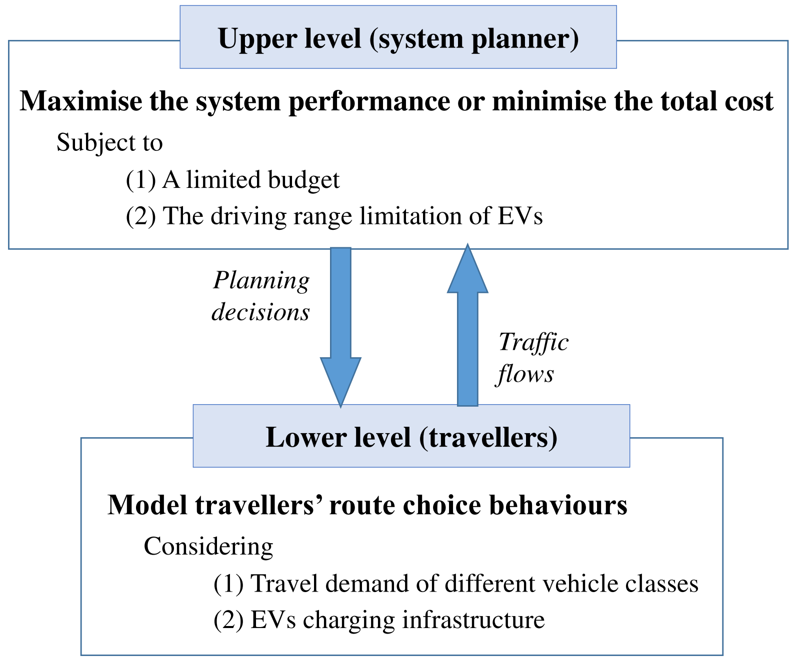

Figure 1: The general bi-level framework for charging location problem

- *The upper level (leader)*: In the NDP with EVs, the objective of the upper level is usually to minimise the system cost to satisfy the specific demands or to maximise the social welfare subject to the limited budget, vehicle driving range limitations, and corresponding traffic flow on the network.
- *The lower level (follower)*: Given the charging infrastructure, the traffic propagation over the network can be modelled as a Traffic Assignment Problem with route choice behaviours and path feasibility constraints.

Compared with previous studies using static assignment models, we model the traffic propagation over space and time as a Multi-class Dynamic Traffic Assignment (DTA) problem to accurately evaluate the infrastructure planning decision on network congestion in the critical period. Two widely accepted travel choice principles can be adopted, i.e. Dynamic User Equilibrium (DUE) and Dynamic System Optimal (DSO). Theoretically, the previous bi-level framework can be extended to capture the dynamics by using Dynamic User Equilibrium (DUE) as a realistic model to present behavioural choices at the lower level. However, it is difficult to set a well-defined objective for the DUE problem. In most literature, the DUE problem is formulated as a variational inequality (VI) problem, which is inefficient in the bi-level framework. Due to the intrinsic complexity, incorporating a multi-class DUE in closed-form into the optimisation formulation has been recognized as one of the most challenging problems in transportation (Lin and Lo, 2000; Hoang et al., 2019). To the best of our knowledge, the problem of solving multi-class DUE in conjunction with network design decisions remains unexamined in the literature (Wang et al., 2018).

Therefore, in the present study, we scope the problem when travellers cooperatively achieve the minimum system-wide travel time. As a result, a multi-class dynamic system optimal (DSO) model with the presence of EVs and WCL is utilised to compute an approximate representation of the dynamic traffic flows. Due to the linear structure of DSO, it assists the development of tractable optimisation formulation and solution approaches and better facilitates investigating the network's traffic dynamics and flow propagation (Ngoduy et al., 2021; Chakraborty et al., 2021). As such, the previous bi-level approach must be reformulated as a single-level program in the case of dynamic traffic assignment. The following reasons further justify the motivation for solving the WCL location problem under DSO route choice.

Although SO Assignment is not a sound behavioural representation of traffic, it still provides a benchmark for performance and demonstrates how best to utilize the transportation system (Mahmassani, 2001). A study by Patil and Ukkusuri (2007) demonstrated that the difference in social costs between SO and UE is less than 5%. Hence, an optimal network design decision with SO traffic assignment can be considered as a good approximate solution under the UE principle (Wang et al., 2013). Furthermore, the WCL location problem under DSO is also helpful for checking the quality of new algorithms to solve a WCL location problem under DUE.

Last but not least, according to the vision of transportation advancements in the future, the framework proposed under the DSO principle can be applied to situations in which technology and strategies can control traffic routing, e.g. transport automation and bottleneck permits (Chakraborty et al., 2021; Fu et al., 2021). Recently, Mansourianfar et al. (2021) have shown that it will also be possible to design a suitable pricing control scheme that aims to incentivize connected and autonomous vehicles (CAVs) to seek SO routing by saving on tolls. As a result, the gap between DSO and DUE becomes less significant. Fu et al. (2021) investigated the equivalence between the queueing delay at a bottleneck in a DUE solution and an optimal toll eliminating the queue in a DSO solution under particular conditions. Meanwhile, Chakraborty et al. (2021) demonstrated the use of DSO in the design of freeway lanes considering both legacy vehicles and automated vehicles. Moreover, the study can be adapted to other networks by appropriately modifying the cost functions, such as fleet management, private carriers, and centralized internet routing (Patil and Ukkusuri, 2007).

In this study, we consider the case where the system planner aims to deploy the WCL to maximise the total outflow subject to the limited budget for WCL deployment and EVs' driving range limitations. This framework targets capturing network congestion considering the worst-case scenario, e.g. rush period, in urban areas. The objective function of maximising the total outflow of the traffic in the network during a peak period $|\mathscr{T}|$ can be formulated as in (1), the second term of the objective function is used as a penalty cost to incentivize travellers to move long instead of waiting. It is worth noting that the objective could be minimising total emissions or total travel times, depending on the planner's priorities. Here, we consider the total travel times, which lead to the maximisation of the total outflow.

$$\max OF = \sum_{m\in\mathscr{M}}\sum_{i\in\mathscr{T}}\left\{\sum_{a\in\mathscr{E}_S}\sum_{b\in\Upsilon_a^-}(|\mathscr{T}|+1-i)f_{ba}^m(i) + o\sum_{a\in\mathscr{E}/\mathscr{E}_S}\sum_{b\in\Upsilon_a^-}(|\mathscr{T}|+1-i)f_{ba}^m(i)\right\} \tag{1}$$

The traffic flow for each vehicle class can be captured by solving the multi-class dynamic traffic assignment problem. In addition, we consider two primary constraints at the planning stage, including the limited budget for deploying the charging infrastructure and EVs driving range limitations that depend on the vehicle state of energy. When commuting between O-D pairs, EV travellers not only choose their routes to minimise the total cost but must also consider the feasibility of the routes due to the state of energy and given WCL. Then, we define an EV feasible path as follows.

Definition 1 (EVs feasible path). *A path can be defined as a feasible path for an EV if the EV's driver can reach the destination through this path without running out of energy.*

According to the above definition, the state of energy after traversing link a on path p, denoted as $B_{a,p}$, can be computed as:

$$B_{a,p} = B_{b,p} - \epsilon l_a + \omega t_a^0 x_a, \quad \forall p \in \mathscr{P}, a \in \mathscr{E}_A, b \in \Upsilon_a^- |(b,a) \in p \tag{2}$$

Equation (2) represents the dependence of EVs' energy state on both the travel distance and travel time. Under assumption (3), we consider the case where the energy consumption is a function of link distance while the amount of recharged energy is a function of travel time. More specifically, we assume the amount of recharged energy when an EV travels on a WCL proportional to the free-flow travel time on that link as it is the lower bound of link travel time, which also enables the proposed model to account for the non-congested period and capture the sense of security of EV drivers.

$B_{a,p}$ can be either positive or negative and x_a is a binary variable presenting whether link a is a charging link or not. Consequently, path p is feasible only if the vehicle can complete all link on that path: $B_{a,p} \geq 0, \forall a \in p$.

The feasible space of the charging location solutions, Ω, can be defined as in (3) - (8).

$$\iota \sum_{a \in \mathscr{E}_A} l_a x_a \leq I \tag{3}$$

$$B_{a,p} = B_0 \qquad \forall p \in \mathscr{P}, a \in \mathscr{E}_R | a \in p \tag{4}$$

$$B_{a,p} = \min\{B_{max}, B_{b,p} - \epsilon l_a + \omega t_a^0 x_a\} \qquad \forall p \in \mathscr{P}, a \in \mathscr{E}_A, b \in \Upsilon_a^- | (b,a) \in p \tag{5}$$

$$M(y_p^{EV} - 1) \leq B_{a,p} \qquad \forall p \in \mathscr{P}, a \in \mathscr{E}_A | a \in p \tag{6}$$

$$\mathbf{x},\ \mathbf{y} : \mathbf{binary} \tag{7}$$

$$\mathbf{B} : \mathbf{unrestricted} \tag{8}$$

Constraint (3) implies the budget limitation for WCL deployment. The energy conservation of EVs is captured by constraints (4) - (5), i.e. the EVs initial state of energy (4), and the state of energy after travelling on link a is no greater than the battery capacity of EVs (5). Constraint (6) represents the relationship between the path feasibility of an EV and its state of energy on this path. Besides, all path between the O-D pair are assumed to be feasible for ICVs due to their relatively long driving range. Finally, constraints (7) and (8) are the variable constraints.

3. Multi-class DSO model with EVs and WCL

The dynamic system optimal (DSO) problem aims to predict the optimal time-dependent routing pattern of users in a network such that the given time-dependent origin-destination demands are satisfied and the total system travel time is minimised, assuming a dynamic network loading. In this paper, we adopt one of the state-of-the art approaches to formulate the DSO as an optimisation problem using the Two-regime Transmission Model (TTM) as the underlying traffic flow model (Ngoduy et al., 2016, 2021).

Considering a single origin-destination network with the presence of EVs and the charging infrastructure, it is assumed that the triangular flow-density relationship with a set of identical fundamental diagram (V_a, W_a, K_a), e.g. free-flow speed, backward speed, and jam density for each link a is used for both EVs and ICVs. This is because it is reasonable to assume that both EV and ICV vehicle type have the same free-flow speed, capacity and wave speed. The only difference between the EV and ICV type is the route choice behaviour as EVs' travellers not only choose their routes to minimise the total cost but must also consider the feasibility of the routes given WCL.

As a result, the maximum flow capacity Q_a and critical density C_a of the feasible link can be identified as in follows.

$$Q_a = \frac{K_a V_a W_a}{V_a + W_a} \tag{9}$$

$$C_a = \frac{K_a W_a}{V_a + W_a} \tag{10}$$

To apply the multi-class DSO framework in Ngoduy et al. (2021), the multi-class DSO as an optimisation problem with TTM-based linear constraints and EVs consideration then can be formulated as (11) - (24). In which the EVs conform to a different route choice behaviour from the ICVs due to the side-constraint (19).

$$\min \quad TTT = \tau \sum_{i \in \mathscr{T}} \sum_{m \in \mathscr{M}} \sum_{a \in \mathscr{E}/\{\mathscr{E}_S\}} n_a^m(i) \tag{11}$$

$$\text{s.t.} \quad n_a^m(i) - \sum_{k=0}^{i} [u_a^m(k) - v_a^m(k)] = 0 \qquad \forall a \in \mathscr{E}, m \in \mathscr{M}, i \in \mathscr{T} \tag{12}$$

$$\sum_{k=i-\frac{l_a}{V_a}+1}^{i} u_a^m(k) - n_a^m(i) \leq 0 \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{13}$$

$$n_a^m(i) + \sum_{k=i-\frac{l_a}{W_a}+1}^{i} v_a^m(k) \leq K_a l_a \hat{\alpha}_a^m \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{14}$$

$$u_a^m(i) = D^m(i) \qquad \forall a \in \mathscr{E}_R, m \in \mathscr{M}, i \in \mathscr{T} \tag{15}$$

$$u_a^m(i) - \sum_{b \in \Upsilon_a^-} f_{ba}^m(i) = 0 \qquad \forall a \in \mathscr{E}/\mathscr{E}_R, m \in \mathscr{M}, i \in \mathscr{T} \tag{16}$$

$$v_a^m(i) - \sum_{b \in \Upsilon_a^+} f_{ab}^m(i) = 0 \qquad \forall a \in \mathscr{E}/\mathscr{E}_S, m \in \mathscr{M}, i \in \mathscr{T} \tag{17}$$

$$v_a^m(i) = 0 \qquad \forall a \in \mathscr{E}_S, m \in \mathscr{M}, i \in \mathscr{T} \tag{18}$$

$$\sum_{b \in \Upsilon_a^-} f_{ba}^m(i) \leq Q_a \sum_{p \in \mathscr{P}} \delta_{a,p} y_p^m \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{19}$$

$$S_a(i) = min \left\{ Q_a; K_a l_a + \sum_{m \in \mathscr{M}} \sum_{k \leq i - \frac{l_a}{W_a}} v_a^m(k) - \sum_{m \in \mathscr{M}} \sum_{k \leq i-1} u_a^m(k) \right\} \qquad \forall a \in \mathscr{E}_A, i \in \mathscr{T} \tag{20}$$

$$D_a(i) = min \left\{ Q_a; \sum_{m \in \mathscr{M}} \sum_{k \leq i - \frac{l_a}{V_a}} u_a^m(k) - \sum_{m \in \mathscr{M}} \sum_{k \leq i-1} v_a^m(k) \right\} \qquad \forall a \in \mathscr{E}_A, i \in \mathscr{T} \tag{21}$$

$$\sum_{m \in \mathscr{M}} u_a^m(i) \leq S_a(i) \qquad \forall a \in \mathscr{E}_A, i \in \mathscr{T} \tag{22}$$

$$\sum_{m \in \mathscr{M}} v_a^m(i) \leq D_a(i) \qquad \forall a \in \mathscr{E}_A, i \in \mathscr{T} \tag{23}$$

$$\mathbf{n},\ \mathbf{u},\ \mathbf{v},\ \mathbf{f} : \textbf{non-negative} \tag{24}$$

Objective (11) aims to minimise the total system travel time ($TSTT$) calculated as the total number of vehicles existing in the network for each time interval $i \in \mathscr{T}$. Constraints (12) - (14) present the link constraints for the TTM, in which constraint (12) defines the number of vehicles in a link at each time step while constraints (13) and (14) provide the lower bound and upper bound of the number of vehicles at each time step, respectively, according to the dynamics of the traffic flow in and out of the link. The constraints (16) and (17) represent the conservation flow at the upstream and downstream of a normal link (i.e. the node model), while constraints (15) and (18) represent the source and sink link constraints. The side constraint on feasible path for each vehicle class is shown as in constraint (19). Constraints (20) - (23) define the supply and demand at the node, and the corresponding entry and exit flows of the link. Constraint (24) guarantees the non-negative number of vehicles and flow on link.

It is worth noting that given the number of vehicles on link $a \in \mathscr{E}_A$, at time step i, $n_a(i)$, the link density k_a and link flow q_a on the link at time step i can be determined as in equations (25) and (26), respectively.

$$k_a(i) = \frac{n_a(i)}{l_a} \tag{25}$$

$$q_a(i) = \begin{cases} V_a k_a(i) & \text{if } 0 \leq k_a(i) < C_a \\ W_a[K_a - k_a(i)] & \text{if } C_a \leq k_a(t) \leq K_a \end{cases} \tag{26}$$

To obtain more insights into the multi-class DSO problem, the dual of the program (11) - (24) has been formulated. The following associated dual variables have been made.

- $\mu_a^m(i), \forall a \in \mathscr{E}, m \in \mathscr{M}, i \in \mathscr{T}$ with the set of constraints (12);
- $\lambda_{1,a}^m(i)$ and $\lambda_{2,a}^m(i), \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T}$ with the set of constraints (13) and (14), respectively;
- $\eta_{1,a}^m(i), \forall a \in \mathscr{E}, m \in \mathscr{M}, i \in \mathscr{T}$ with the set of constraints (15) and (16);
- $\eta_{2,a}^m(i), \forall a \in \mathscr{E}, m \in \mathscr{M}, i \in \mathscr{T}$ with the set of constraints (17) and (18);
- $\zeta_a^m(i), \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T}$ with the set of constraints (19);
- $\theta_{1,a}(i)$ and $\theta_{2,a}(i), \forall a \in \mathscr{E}_A, i \in \mathscr{T}$ with the set of constraints (20) - (23).

Then the dual multi-class DSO can be formulated as (27) - (39).

$$\begin{aligned} \max \quad P_{dual} = \sum_{i \in \mathscr{T}} \Bigg\{ \sum_{m \in \mathscr{M}} \Bigg\{ \sum_{a \in \mathscr{E}_A} \Bigg\{ K_a l_a \hat{\alpha}_a^m \lambda_{2,a}^m(i) + Q_a \sum_{p \in \mathscr{P}} \delta_{a,p} y_p^m \zeta_a^m(i) \Bigg\} \\ + \sum_{a \in \mathscr{E}_R} D^m(i) \eta_{1,a}^m(i) \Bigg\} + \sum_{a \in \mathscr{E}_A} Q_a \left[\theta_{1,a}(i) + \theta_{2,a}(i)\right] \Bigg\} \end{aligned} \tag{27}$$

$$\text{s.t.} \quad \mu_a^m(i) \leq \tau \qquad \forall a \in \mathscr{E}_R, m \in \mathscr{M}, i \in \mathscr{T} \tag{28}$$

$$\mu_a^m(i) - \lambda_{1,a}^m(i) + \lambda_{2,a}^m(i) \leq \tau \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{29}$$

$$\mu_a^m(i) \leq 0 \qquad \forall a \in \mathscr{E}_S, m \in \mathscr{M}, i \in \mathscr{T} \tag{30}$$

$$-\sum_{k=i}^{T} \mu_a^m(k) + \eta_{1,a}^m(i) \leq 0 \qquad \forall a \in \{\mathscr{E}_R \cup \mathscr{E}_S\}, m \in \mathscr{M}, i \in \mathscr{T} \tag{31}$$

$$\sum_{k=i}^{i+\frac{l_a}{V_a}-1} \lambda_{1,a}^m(k) - \sum_{k=i}^{T} \mu_a^m(k) + \eta_{1,a}^m(i) \leq 0 \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{32}$$

$$\sum_{k=i}^{T} \mu_a^m(k) + \eta_{2,a}^m(i) \leq 0 \qquad \forall a \in \{\mathscr{E}_R \cup \mathscr{E}_S\}, m \in \mathscr{M}, i \in \mathscr{T} \tag{33}$$

$$\sum_{k=i}^{i+\frac{l_a}{W_a}-1} \lambda_{2,a}^m(k) + \sum_{k=i}^{T} \mu_a^m(k) + \eta_{2,a}^m(i) \leq 0 \qquad \forall a \in \mathscr{E}_A, m \in \mathscr{M}, i \in \mathscr{T} \tag{34}$$

$$-\eta_{1,a}^m(i) - \eta_{2,b}^m(i) + \theta_{2,b}(i) \leq 0 \qquad \forall a \in \mathscr{E}_S, b \in \Upsilon_a^-, m \in \mathscr{M}, i \in \mathscr{T} \tag{35}$$

$$-\eta_{1,a}^m(i) - \eta_{2,b}^m(i) + \theta_{1,a}(i) + \zeta_a^m(i) \leq 0 \qquad \forall b \in \mathscr{E}_R, a \in \Upsilon_b^+, m \in \mathscr{M}, i \in \mathscr{T} \tag{36}$$

$$-\eta_{1,a}^m(i) - \eta_{2,b}^m(i) + \theta_{1,a}(i) + \theta_{2,b}(i) + \zeta_a^m(i) \leq 0 \qquad \forall a \in \mathscr{E}/\mathscr{E}_S, b \in \Upsilon_a^-/\mathscr{E}_R, m \in \mathscr{M}, i \in \mathscr{T} \tag{37}$$

$$\lambda_1, \lambda_2, \theta_1, \theta_2, \zeta : \textbf{non-positive} \tag{38}$$

$$\mu, \eta_1, \eta_2 : \textbf{unrestricted} \tag{39}$$

It is to note that the dual variable $\eta_{1,a}^m(i)$ can be interpreted as the marginal contribution to the total travel time of an additional unit of demand of vehicle class m on link a at time step i. In other words, the additional unit of demand will increase the total system travel time by $\eta_{1,a}^m(i)$ when the dual and primal programs converge. Furthermore, $\theta_{1,a}(i)$ and $\theta_{2,a}(i)$ can be seen as the objective value change due to the increase in upstream and downstream capacity of link a at time step i, respectively.

Under the multi-class DSO traffic assignment, the total system cost, i.e. total travel times of all vehicle classes, is minimised. The total travel times (TTT) can be measured as the total number of vehicles present in the network for each time step $i \in \mathscr{T}$ as in the objective function (11), which can also be determined as the difference between the cumulative departures from origins and the cumulative arrivals at destinations for time step i. Accordingly,

$$\begin{aligned} TTT &= \tau \sum_{i \in \mathscr{T}} \sum_{m \in \mathscr{M}} \sum_{a \in \mathscr{E}/\{\mathscr{E}_S\}} n_a^m(i) \\ &= \sum_{m \in \mathscr{M}} \sum_{i \in \mathscr{T}} \left(\sum_{k=0}^{i} D^m(k) - \sum_{k=0}^{i} \sum_{a \in \mathscr{E}_S} u_a^m(k) \right) \end{aligned} \tag{40}$$

Besides, the total traffic outflow (TOF) can be defined as in (41).

$$\begin{aligned} TOF &= \sum_{m\in\mathscr{M}} \sum_{i\in\mathscr{T}} \sum_{k=0}^{i} \sum_{a\in\mathscr{E}_S} u_a^m(k) \\ &= \sum_{i\in\mathscr{T}} \sum_{m\in\mathscr{M}} \sum_{a\in\mathscr{E}_S} \sum_{b\in\Upsilon_a^-} \left(|\mathscr{T}|+1-i\right) f_{ba}^m(i) \end{aligned} \tag{41}$$

Given the demand of each vehicle class $D^m(i)$, $i \in \mathscr{T}$, minimising total travel times (40) is equivalent to maximising the total outflow (Shen and Zhang, 2008; Ngoduy et al., 2016), which is aligned with the system planner's objectives. Therefore, we can formulate the problem as a single-level optimisation program by incorporating the DSO traffic assignment at the lower level into the WCL location problem at the upper level.

So far, the overall single-level program for the WCL location problem can be formulated as the following MILP.

$$\begin{aligned} \max \quad & OF \\ \text{s.t.} \quad & (3) - (8), \\ & (12) - (24) \end{aligned} \tag{42}$$

The above MILP (42) can be solved by many optimisation solvers. In this study, we use the Branch and Bound technique by adopting the Python-based open-source software package - PYOMO with the CPLEX solver (Bynum et al., 2021). All instances are solved on a computer with Intel(R) Core(TM) i7-7700 CPU @ 3.60GHz and usable RAM of 15.9 GB, running on Windows 10.

4. Numerical tests

This section provides some numerical results illustrating the efficacy of the proposed framework. To this end, two numerical tests have been conducted: (i) a simple Braess network and (ii) medium- and large-sized grid networks. In the Braess network, we demonstrate the WCL location solution with the system optimal traffic flow and the distribution of congestion over the network. While in grid networks, we aim to show the computational complexity and the potential of the proposed framework for solving larger-scale networks.

4.1. Braess network

Consider the Braess network with link number and link length (m) as shown in Figure 2. Inputs of the problem, i.e. time settings, traffic and design parameters, are given in Table 2. Links 1 and 7 are the virtual source and destination links, respectively. The network is assumed to be empty at $i = 0$ and will be cleared at $i = |\mathscr{T}|$. According to the network topology, there are three paths between the O-D pair (R, S): $(1, 2, 4, 6, 7)$, $(1, 2, 5, 7)$, and $(1, 3, 6, 7)$.

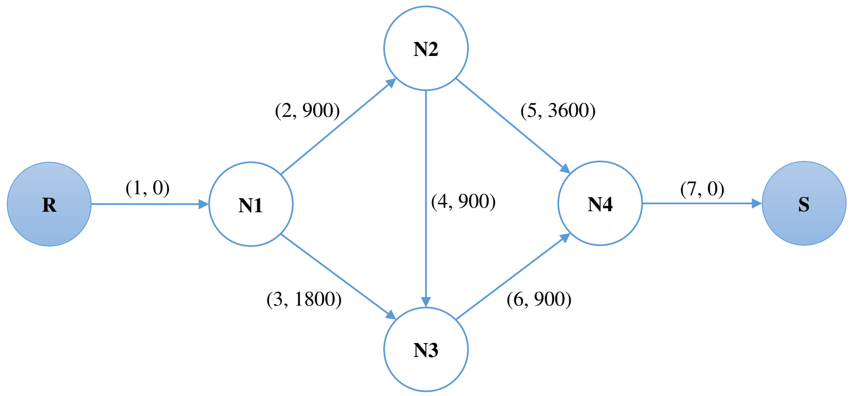

Figure 2: Braess network

Table 2: Braess network - Input parameters

Time settings	
Time horizon $T = 60$ (min),	Time step $\tau = 0.25$ (min)
Demand rates $(D^e, D^g) = (36,\ 36)$ (veh/time step)	
Traffic parameters (for all links)	
$V_a = 900$ (m/min)	$W_a = 450$ (m/min)
$K_a = 0.12$ (veh/m)	$Q_a = 36$ (veh/min)
Design parameters	
$(s_3, c_3) = (1 \times 10^6,\ 500)$	
$(\omega, \epsilon) = (0.33,\ 1.8 \times 10^{-4})$,	$(B_{max}, B_0) = (0.5,\ 0.35)$

The WCL location problem is solved by CPLEX with an optimality gap of 0%, and it is suggested to deploy a WCL on link 6. As a result, there are two feasible paths for EVs' users, i.e. (1,2,4,6,7) and (1,3,6,7). The system outflow is 290,205.0 (veh), and the total travel time is 41,604.75 (min). The computational time is 10.6 (s). The inflow-outflow profiles can be further seen in Figure 3 and Figure 4. The proposed model gives insights into the traffic state and congestion propagation over the network. Figure 3 provides the distinct difference of the inflow and outflow accumulations of each vehicle class in the network, while Figure 4 shows the number of vehicles waiting on each link as the gap of the total inflow and outflow. The shaded area is the total delay on the corresponding link. Besides, Figure 3 also shows the departure choices of each vehicle class at the source link (Link 1). The inflow-outflow profiles indicate that although both ICVs and EVs are generated on the source link at the same time-dependent rates, the ICVs tend to enter the network earlier and reach the destination earlier than EVs.

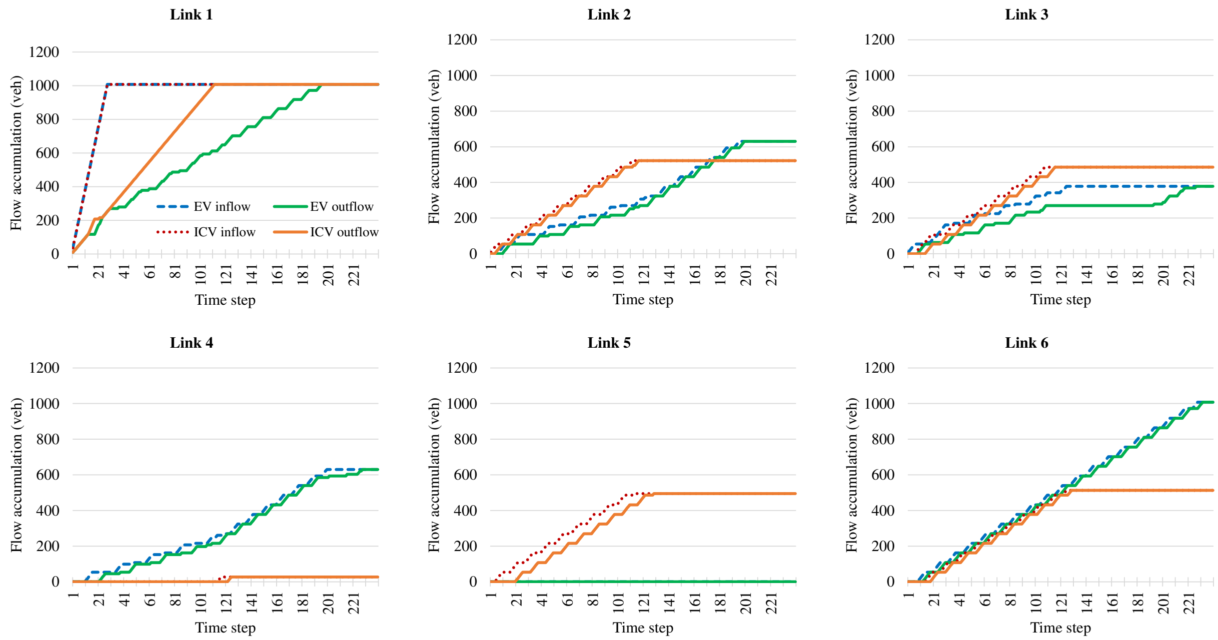

Figure 3: Inflow-outflow profiles of each vehicle class

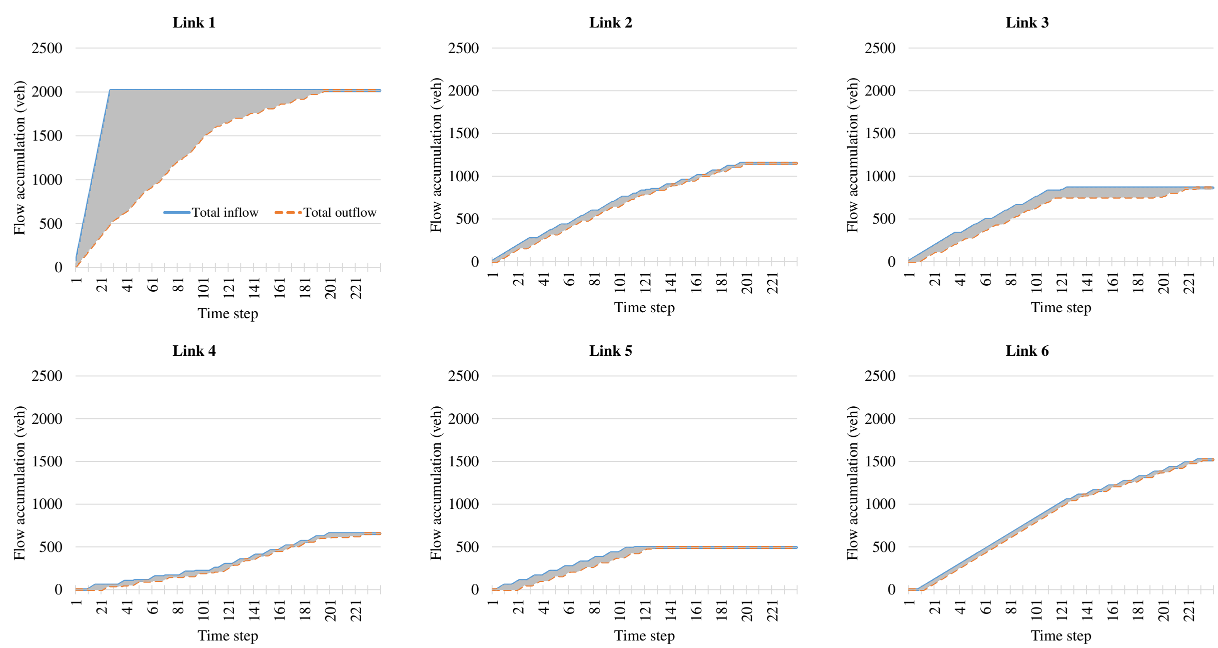

Figure 4: The total inflow-outflow profile

It can be seen that to reduce the traffic congestion, link 4 is used mostly by EVs' users. Meanwhile, there is no EVs on link 5. Moreover, Figure 5 clearly indicates the traffic state and congestion propagation over the network during the time horizon. In the following subsection, we study the computational performance of the proposed model in more realistic size networks, referred to as grid networks.

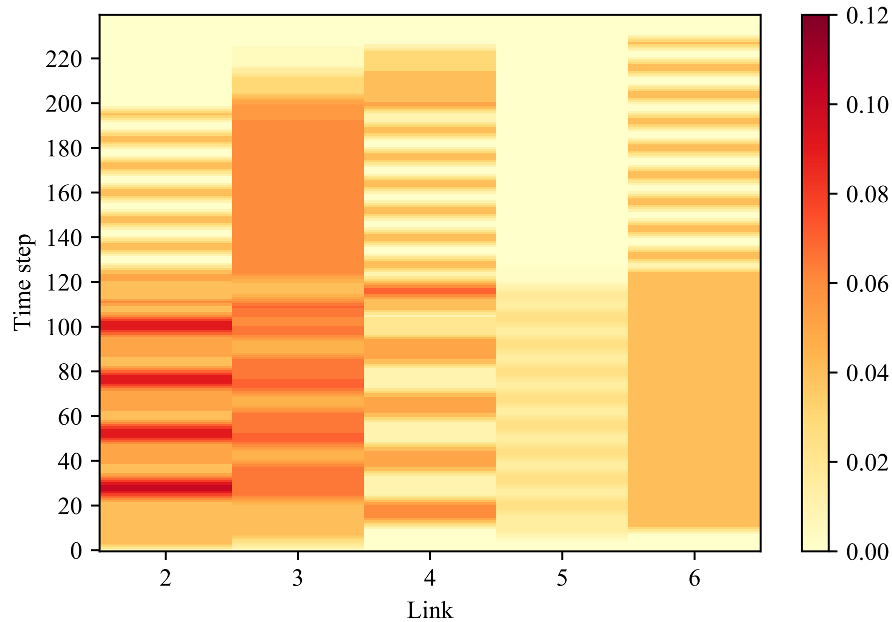

Figure 5: The density on each link over the time horizon (veh/m)

4.2. Grid networks

This section illustrates the efficacy of the proposed framework in more extensive networks. Firstly, a grid network (Grid-42) with 27 nodes and 42 links has been generated as in Figure 6. In Grid-42, link number and link length (m) are shown in the figure. Then, the proposed framework has been tested with a larger test-bed network (Grid-82) of 52 nodes and 82 links with link number shown as in Figure 7. In Grid-82, each link has a 900 (m) length except for the source and sink links. The input parameters for both numerical tests are given in Table 3.

The WCL location problems have been solved by the proposed framework, and the solutions for locating WCL on each test-bed network are presented in Figure 8 and 9, respectively. In Figure 8 and 9, thick green lines indicate the location of WCL, while the dash links show the feasible path for EVs. Furthermore, table 4 summarises WCL locations, capital cost and network performance in each case.

Table 3: Input parameters for grid networks

Time settings	
Time horizon $T = 30$ (min),	Time step $\tau = 0.5$ (min)
Demand rates $(D^e, D^g) = (36,\ 36)$ (veh/time step)	
Traffic parameters (for all links)	
$V_a = 900$ (m/min)	$W_a = 450$ (m/min)
$K_a = 0.12$ (veh/m)	$Q_a = 36$ (veh/min)
Design parameters	
$(s_3, c_3) = (3 \times 10^6,\ 500)$	
$(\omega, \epsilon) = (0.33,\ 1.8 \times 10^{-4})$,	$(B_{max}, B_0) = (1.0,\ 0.35)$

Table 4: Network performances

Grid network	**WCL locations**	**Capital cost** (\$)	**System outflow** (veh)	**Total travel time** (min)
Grid-42	3, 14, 22, 40	2.7×10^6	19,944.0	9,936.0
Grid-82	24, 42, 44, 46, 48, 66	2.7×10^6	13,446.0	9,549.0

Furthermore, the complexity of the proposed framework in terms of the number of variables, constraints, computational time and optimality gap is presented. The complexity and computational performance of the numerical tests are summarised in Table 5.

Table 5: Problem complexity and computational performance

Numerical test	**No. variables** (binary)	**No. linear constraints**	**Run-time** (s)	**Opt. gap** (%)
Braess	18,902 (11)	24,761	10.58	0
Grid-42	31,067 (180)	46,432	283.96	0
Grid-82	63,465 (640)	116,282	28,823.83	13.52

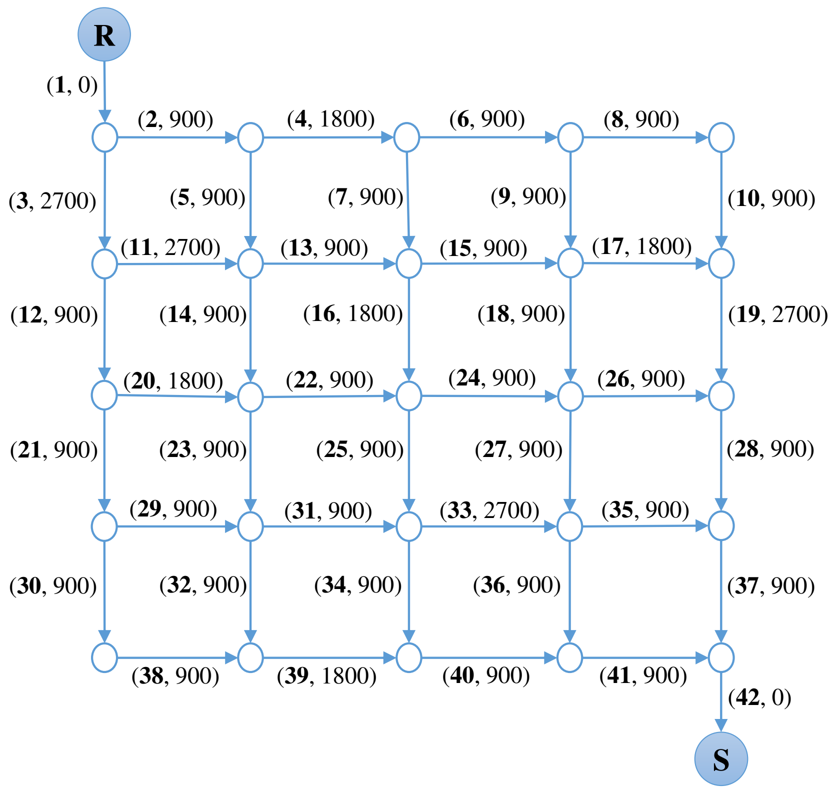

Figure 6: Test-bed network 2 - Grid-42

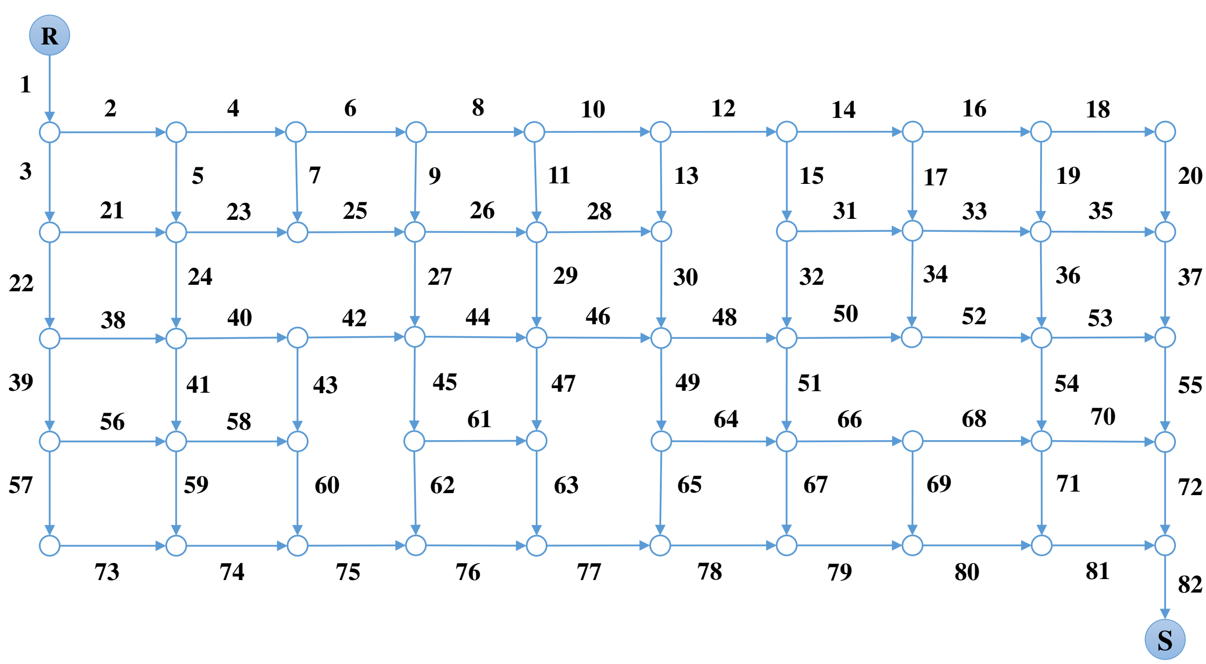

Figure 7: Test-bed network 3 - Grid-82

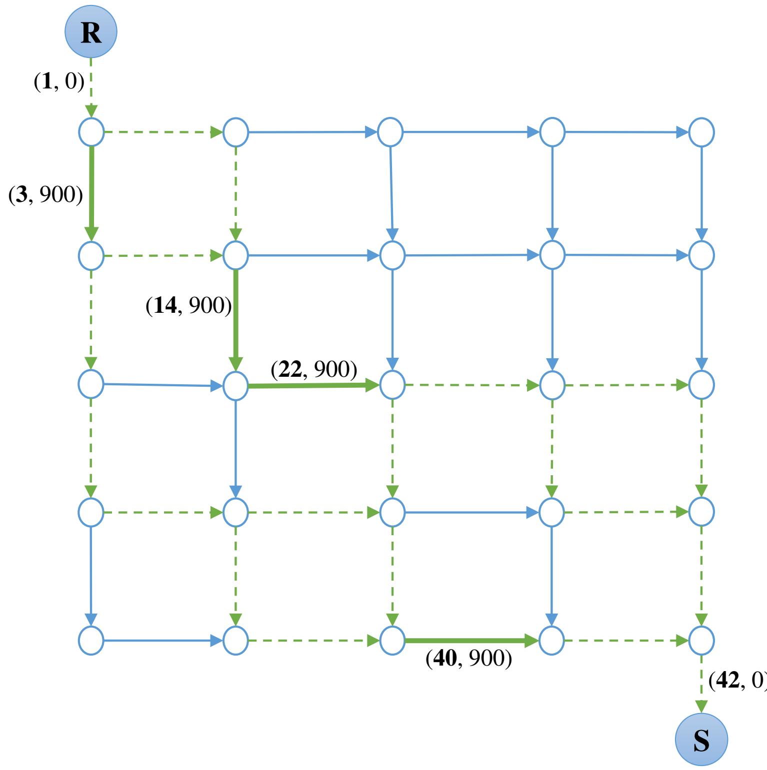

Figure 8: Grid-42 - WCL deployment solution

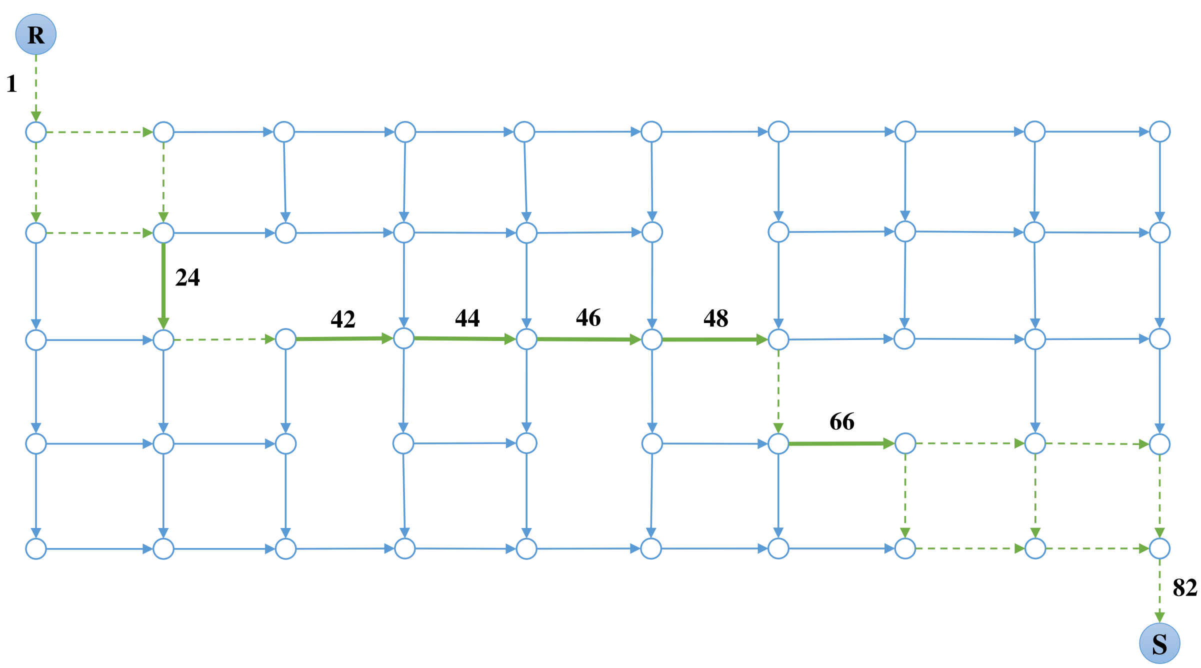

Figure 9: Test-bed network 3 - Grid-82

5. Concluding remarks

Although a few charging location models have been proposed for wireless charging infrastructure, they rarely considered the impacts of it on traffic dynamics and network congestion, especially during peak periods. In this paper, we have proposed a dynamic wireless charging lane (WCL) location model to optimally locate this charging infrastructure on the urban network concerning the dynamic traffic flow and multiple vehicle classes. Instead of using static assignment models, we model the traffic propagation over space and time as a Multi-class Dynamic Traffic Assignment (DTA) problem to accurately evaluate the infrastructure planning decision on network congestion in the critical period. Moreover, due to the intrinsic complexity of incorporating and solving a multi-class Dynamic User Equilibrium (DUE) in the closed-form, a multi-class Dynamic System Optimal (DSO) model with EVs and WCL is utilised to compute an approximate representation of the dynamic traffic flows. As a result, a single-level optimisation program has been formulated by integrating a multi-class DSO into the WCL location problem. Finally, the effectiveness of the proposed model has been verified numerically in different sized test-bed networks.

This research can be extended along the following avenues. Firstly, although the proposed framework is currently limited to multi-class DSO for single O-D networks, it contributes significantly to the state-of-the-art dynamic WCL location problem. This work paves the way for future research exploring more generic networks and traffic demands scenarios. One of the possibilities for relaxing the assumption of a single O-D network is to introduce the O-D index and treat each O-D pair as a vehicle class. For example, the demand and supply capacities of link a at time step i can be formulated as in the following equations.

$$S_a(i) = min\left\{Q_a; K_a l_a + \sum_{w\in\mathscr{W}}\sum_{m\in\mathscr{M}}\sum_{k\le i-\frac{l_a}{W_a}} v_a^{m,w}(k) - \sum_{w\in\mathscr{W}}\sum_{m\in\mathscr{M}}\sum_{k\le i-1} u_a^{m,w}(k)\right\} \quad \forall a\in\mathscr{E}_A, i\in\mathscr{T} \tag{43}$$

$$D_a(i) = min\left\{Q_a; \sum_{w\in\mathscr{W}}\sum_{m\in\mathscr{M}}\sum_{k\le i-\frac{l_a}{V_a}} u_a^{m,w}(k) - \sum_{w\in\mathscr{W}}\sum_{m\in\mathscr{M}}\sum_{k\le i-1} v_a^{m,w}(k)\right\} \quad \forall a\in\mathscr{E}_A, i\in\mathscr{T} \tag{44}$$

where $\mathscr{W}$ is the set of O-D pairs.

Accordingly, the current multi-class DSO model can be utilised to solve the problem of multiple

O-D networks. From a computational perspective, however, the problem size will be increased significantly due to the increase of variables and constraints. Therefore, we leave the problem of developing a more efficient solution method for future study. Moreover, the bi-level framework can be extended to capture the multi-class dynamic user equilibrium as a sound behavioural representation of drivers' routing. Secondly, in reality, the EVs' energy level can be highly stochastic; it is desirable to capture this stochasticity in the model by considering the influence of random factors on either the initial state of energy or the battery capacity. Thirdly, the model can be improved to assess the interaction between WCL infrastructure and the power grid to leverage the peak and off-peak scheme by explicitly tracking the total energy consumption over the network.

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