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Dynamics of droplet impacting on a cone

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10 ABSTRACT

11 Droplet rebound dynamics on superhydrophobic surfaces has attracted much attention due to its importance in 12 numerous technical applications, such as anti-icing and fluid transportation. It has been demonstrated that changing 13 the macro-structure of the superhydrophobic surface could result in significant change in droplet morphology and 14 hydrodynamics. Here we conduct both experimental and numerical studies of droplet impacting on a cone, and 15 identify three different dynamic phases by changing the impacting conditions, i.e., the Weber number and the cone 16 angle. The spreading and retracting dynamics are studied for each phase. Particularly, it is found that in Phase 3, where the droplet leaves the surface as a ring, the contact time is reduced by 54% compared with that of a flat 17 18 surface. A theoretical model based on energy analysis is developed to get the rebound point in Phase 3, which agrees 19 well with the simulation result. Besides, the effect of Weber number and cone angle on the contact time is explored. 20 Finally, the phase diagram of the three phases distribution with We and cone angle is given, which can provide 21 guidance to related applications.

22 I. INTRODUCTION

Droplet impacting onto solid surfaces is a unique phenomenon either in nature, e.g., rain falls, or in engineering fields such as anti-icing, and it has attracted great attention for many years.¹⁻⁵ Depending on the surface properties, droplet impact may present different outcomes: deposition, splashing, and rebound.⁶⁻¹⁰ Rebound happens when droplet impacts on superhydrophobic surfaces, on which droplet will go through spreading, retracting, and rebounding due to the small liquid-solid contact area and weak adhesion of the surface.¹¹⁻¹⁷ The duration of this whole process is defined as the contact time, which is important in droplet impacting dynamics and it has been proposed that contact time scales as the inertial-capillary time τ_0 ($\tau_0 = \sqrt{\rho r_0^3 / \sigma}$, where ρ is the liquid density, r_0 is the droplet initial radius, and σ is the liquid surface tension), which is independent of impact velocity (v_0).¹⁸

31 In many engineering applications, it is beneficial to reduce contact time to increase the performance of self-32 cleaning, anti-icing, or anti-frosting. Conventional methods to reduce contact time is to reduce liquid-solid 33 interaction and change the surface wettability. For instance, Li et al. found that the contact time is subjected to the solid fraction of liquid contacted.¹⁹ However, it was also proposed that, even if no liquid-solid adhesion on the 34 35 surface and no liquid penetration into the surface asperities exist, there is still a theoretical contact time limit of $2.2\tau_0$ 36 due to the symmetry in the process of spread and retraction on flat superhydrophobic surfaces.²⁰ Thus, approaches 37 to break the symmetric dynamics of droplet have been carried out. Bird et al. added a macro ridge, whose height is slightly less than the spreading film thickness, to the flat superhydrophobic surfaces. It was found that liquid 38 possessed a faster retraction along the ridge than that on the flat surface, leading to a 37% reduction in contact 39 time.²¹ Following this work, studies focused on contact time reduction by asymmetric dynamics have been carried 40 out, including the inclined surface,^{22, 23} curved surface,²⁴⁻³⁰ surfaces with macro structures of different geometries,³¹⁻ 41 42 ⁴⁰ moving surfaces, ^{41, 42} and off-center impact.^{43, 44}

Besides the aforementioned asymmetric approaches, symmetric bouncing by involving the droplet center to retraction with point-like structure to reduce contact time has also been proposed. Liu et al. fabricated convex surfaces with a height of 0.6 mm and a radius of 2.0 mm and found that such structures can reduce the contact time by 28.5%.⁴⁵ A point-like structure was applied to reduce contact time by Chantelot et al., and it was proposed that droplets can lift up as a ring and a reduction in contact time can be achieved.⁴⁶

48 Based on the widely existence of sharp-edged structures, there is a strong need to investigate the symmetric droplet impact behavior on point-like structures, a field that has seldom studied. The effect of impact conditions on 49 50 the droplet dynamic behavior and contact time still remains unclear. To improve our understanding, we conducted 51 a systematic investigation of droplet impacting on superhydrophobic cone surfaces with focuses on the morphologic 52 evolution and contact time. We reveal three different phases of morphologic evolution and investigate their impacting dynamics respectively, among which a theoretical model to predict the rebound point in Phase 3 is 53 54 established on the basis of energy analysis. Furthermore, the effects of Weber number and cone angle on the contact 55 time are studied and a phase diagram delimiting three phases for impacting dynamics is proposed. We expect that 56 this work could deepen the understanding on the droplet impacting dynamics on pointy structures and provide 57 fundamental supports to related engineering applications such as anti-icing of the aero-engine rotating cone.

58 II. METHOD

59 A. Experimental Setup

60 The experimental system is shown in Fig. 1. During experiments, the droplet is produced by a syringe pump 61 and an injector with a certain diameter determined by the needle size. The droplet falls under gravity and impacts 62 on the cone tip with no offset distance. A high-speed camera (PCO. Dimax HS4, Germany) is applied to capture the 63 droplet impact process and a strong light source is placed opposite the camera to achieve high-contrast image capture. 64 The droplet diameter is measured from the images processed by Matlab and ImageJ; the impact velocity is calculated 65 by the formula $v_0 = \sqrt{2gh}$, where g and h denote gravity acceleration and the height of the needle above cone tip, respectively. The cone is made of copper and is machined with an angle of φ . The whole surface is coated with a 66 superhydrophobic coating fabricated by the chemical deposition-etching method.⁴⁷ As the SEM image in Fig. 1 67 68 shows, the cone surface is covered by moss-like structures. Measured by the equipment (Biolin Theta Lite, Finland), 69 the static, advancing, and receding contact angles are $160\pm1.3^{\circ}$, $164\pm2.3^{\circ}$, and $156\pm2.9^{\circ}$, respectively, indicating 70 excellent superhydrophobicity of the cone surface. In this work, the experiment is mainly used to validate our 71 numerical method, which is introduced below.



72

FIG. 1. Schematic of the experimental setup. The experimental system includes a droplet production module (syringe pump, injector, and needle) and a high-speed camera module (high-speed camera, light source, and computer). The cone has an angle of φ and the cone surface is superhydrophobic with moss-like structures.

76 B. Numerical Method

77 In the present work, Volume of Fluid (VOF) method is used to capture the interface of two phases, which has 78 been successfully applied in many previous studies. In this method, the volume fraction (*α*) is defined to delimit different phases: $\alpha = 0$ represents that the cell is occupied by gas, $\alpha = 1$ represents that the cell is occupied by the liquid phase, and $0 < \alpha < 1$ corresponds to the interface between the two phases. The corresponding mass and momentum conservation equations are as

82
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{U}) = 0, \qquad (1)$$

$$\frac{\partial \rho \boldsymbol{U}}{\partial t} + \nabla \cdot (\rho \boldsymbol{U} \boldsymbol{U}) = \nabla \cdot (\mu (\nabla \boldsymbol{U} + \nabla \boldsymbol{U}^T)) - \nabla p + \rho g + \boldsymbol{F}, \qquad (2)$$

where U is the velocity vector, t is the time step, p is the pressure, and F is the volumetric surface tension force on the fluid at the gas-liquid interface, calculated by the continuum surface force (CSF) model, which is

$$\boldsymbol{F} = \boldsymbol{\sigma} \boldsymbol{\kappa} \nabla \boldsymbol{\alpha} \,, \tag{3}$$

87 in which σ is the surface tension coefficient and κ is the mean curvature of the free surface,

88
$$\kappa = -\nabla \cdot (\frac{\nabla \alpha}{|\nabla \alpha|}). \tag{4}$$

89 In Eqs. (1) and (2), ρ and μ are calculated by the volume fraction:

$$\rho = \alpha \rho_{\text{liquid}} + (1 - \alpha) \rho_{\text{gas}}, \qquad (5)$$

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90

83

86

$$\mu = \alpha \mu_{\text{liquid}} + (1 - \alpha) \mu_{\text{gas}}.$$

(6)

92 The advection equation for α is employed as

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \boldsymbol{U}) = 0.$$
⁽⁷⁾

94 In the present work, the dynamic contact angle model proposed by Kistler is utilized to relate the contact velocity

and dynamic contact angle on the triple line, which has been validated in numerous researches.⁴⁸⁻⁵⁰ During the

simulation, the value of advancing contact angle and receding contact angle are set according to the wettability of

97 the experimental surface.



98

99FIG. 2. Simulation setup. The computation domain is set as $6 \times 6 \times 8 \text{ mm}^3$ with two adjacent surfaces as symmetry plane,100the bottom surface as no-slip boundary, and the other three surfaces as pressure outlet boundaries.

101 In this work, the interFoam solver within the framework of Open Source Field Operation and Manipulation (OpenFOAM) is used to perform numerical simulations. The accuracy and efficiency of the interFoam solver for 102 the simulation of multiphase flows was evaluated approvingly by Deshpande et al.,⁵¹ and has been widely applied 103 to the study of droplet dynamics.⁵²⁻⁵⁴ Due to the rotational symmetry of the simulation, only a quarter of the 104 computation domain of 6×6×8 mm³ is simulated to save computation resources as in Fig. 2. The bottom surface is 105 set as the no-slip boundary with the static contact angle being 160°, two adjacent surfaces are symmetry planes, and 106 107 the other surfaces are pressure outlet boundaries. Here the physical properties of the water and air at 20 °C are 108 chosen. The mesh independence test is conducted with four different grids of minimum sizes of $50 \times 50 \times 50 \ \mu m^3$, 109 $25 \times 25 \times 25 \ \mu\text{m}^3$, $12 \times 12 \times 12 \ \mu\text{m}^3$, and $6 \times 6 \times 6 \ \mu\text{m}^3$, and it shows that the result of $12 \times 12 \times 12 \ \mu\text{m}^3$ is almost the same 110 as that of $6 \times 6 \times 6 \,\mu\text{m}^3$, so the mesh of $12 \times 12 \,\mu\text{m}^3$ is chosen to perform the numerical work.

111 C. Model Validation

112 To support the reliability of the numerical model, the morphology comparison between experiments conducted 113 with the devices shown in Fig. 1 and simulations set as Fig. 2 is conducted. Two series of conditions are validated to ensure the validity of the numerical model. One condition is set as a droplet impacts the flat superhydrophobic 114 115 surface (the static contact angle of the surface θ is 160°, droplet radius r_0 is 1.05 mm, and the impact velocity v_0 is 0.816 m/s), and another condition is that the droplet impacts a cone with a cone angle (φ) of 50° (θ =160°, r_0 = 1.115 116 mm, and $v_0 = 1$ m/s) as shown in Fig. 3 (b) (multimedia view). The results are shown in Fig. 3 and we can see that 117 118 in both conditions, the temporal evolution of morphology obtained by experiment and simulation are in good 119 agreement, which indicates the applicability of the numerical model to predict the droplet impact phenomenon.



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FIG. 3. Comparison between experimental results and numerical results: (a) droplet impacting on flat superhydrophobic surfaces under the condition of $r_0 = 1.05$ mm, $v_0 = 0.816$ m/s; (b) droplet impacting on a cone with an angle of 50° under condition of $r_0 = 1.115$ mm, $v_0 = 1$ m/s. Both results indicate that the numerical model can well predict droplet impact behaviors (multimedia view).

125 III. RESULTS AND DISCUSSION

126 A. Morphologic Evolution and Impact Dynamics

127 1. Morphologic evolution

Droplet morphology is affected by the interaction between liquid and solid. Here, we classify the droplet morphology evolutions into three phases by changing the impacting velocity (v_0) and cone angle (φ). Figure. 4 (multimedia view) shows the droplet morphologic evolutions with time when the droplets impact on the cone of φ = 110°, where Phase 1, 2, and 3 are depicted in Figs. 4 (a), (b), and (c), respectively. In Phase 1 and Phase 2, droplets both rebound from the tip of the cone after impacting; differently, the droplet in Phase 2 has been impaled by the cone (top view at $\tau = 5.5$ ms in Fig. 4 (b)), while the droplet in Phase 1 has not been impaled but rebounds immediately after reaching the maximum spreading. In Phase 3, the droplet is also impaled by the cone, but it does

not rebound from the tip of the cone as in Phase 2; instead, it leaves the surface as a ring with two contact lines

136 retracting towards each other. It should be noted that the initial position of the droplet is 0.1 mm above the cone tip

137 at $\tau = 0.0$ ms.



138

FIG. 4. Three different impact outcomes: (a) Phase 1: droplet impacts on a cone, spreads, recoils and rebounds ($r_0 = 1.0$ mm, $v_0 = 0.2$ m/s, $\varphi = 110^\circ$); (b) Phase 2: droplet is impaled by the cone and then rebounds from the cone tip ($r_0 = 1.0$ mm, $v_0 = 0.6$ m/s, $\varphi = 110^\circ$); (c) Phase 3: droplets is impaled by the cone and leaves the surface as a ring ($r_0 = 1.0$ mm, $v_0 = 1.2$ m/s, $\varphi = 110^\circ$). (multimedia view)

143 To quantitatively describe impact dynamics, variations of the pressure contour, the velocity field, and the 144 spreading factor ($\beta = r/r_0$, where *r* is the spreading radius, which is defined as the parallel distance along the cone 145 surface between the cone tip and the contact line) with time in three phases are investigated.

146 2. Impact dynamics in Phase 1

In Phase 1, the droplet firstly impacts the cone and spreads under inertial force; in this stage, the droplet goes through a deformation driven by pressure gradients ($\tau = 2.5$ ms in Fig. 5 (a)).⁵⁵ Then it reaches the maximum spreading, at which time, velocity inside the droplet is almost zero ($\tau = 6.5$ ms in Fig. 5 (a)). Later the droplet begins

to retract under capillary forces with the velocity vector upward and finally rebounds from the tip of the cone at

- 151 12.5 ms in Fig. 5 (a). Figures 5 (b) and (c) show the effect of cone angle and Weber number (We = $\rho v_0^2 d_0 / \sigma$) on the
- 152 spreading factor variation.





FIG. 5. Pressure distribution, velocity vector, and spreading factor in Phase 1: (a) the distribution map of pressure inside droplet (Left) and velocity vector chart inside the droplet during impact (Right) under the condition of $r_0 = 1$ mm, $v_0 = 0.2$ m/s, and $\varphi = 110^\circ$; (b) spreading factor varying with time at different cone angles ($r_0 = 1$ mm, $v_0 = 0.2$ m/s, $\varphi = 50^\circ$, 70° , 90° , and 110° , respectively); (c) spreading factor varying with time at different Weber numbers ($\varphi = 110^\circ$, We = 0.3, 1.1, 2.5, and 4.4, respectively).

Droplet spreading stage is dominated by inertial force, so the spreading velocity is unrelated to cone angle (Fig. 5 (b)) and increases with impact velocity (Fig. 5 (c)). On the basis of energy analysis, during impact, the initial kinetic energy will transfer to surface energy and viscous dissipation, both of which are positively related to the liquid-solid contact area ($\sim \pi r_{max}^2 \sin(\varphi/2)$). Thus, both decreasing cone angle and increasing We can increase the maximum spreading radius (Figs. 5 (b) and (c)) to guarantee the energy conservation.

As for the contact time, a shorter contact time can be reached under blunter cone angle due to the decreasing maximum spreading radius (Fig. 5 (b)). Contact time under the same cone angle decreases with increasing We, which can be explained by that, for droplet impact with low We, the deformation is small and the droplet, to some extent, could be treated as an elastic sphere, and the time scales decreases with increasing impact velocity.⁵⁶

168 **3.** Impact dynamics in Phase 2

In Phase 2, firstly, the droplet impacts the cone and then spreads to the maximum contact length. However, the droplet film will be impaled because of the high pressure region and outward velocity vector at the center part of the droplet film ($\tau = 4.0$ ms in Fig. 6 (a)). After the impalement, the droplet becomes a ring shape ($\tau = 5.5$ ms in Fig. 6 (a)) with an internal contact line emerging. After reaching the maximum position, both the inner and outer edges start to retract and the liquid ring coalesces at the tip of the cone ($\tau = 9.0$ ms in Fig. 6 (a)). The kinetic energy transformed from the surface energy renders the coalesced liquid an upward motion and finally lifts the droplet up from the cone tip ($\tau = 12.5$ ms in Fig. 6 (a)).

To study the spreading and retracting dynamics in Phase 2, the effect of cone angle and We on the outer contact line spreading factor β are depicted in Figs. 6 (b) and (c), respectively. Similar to Phase 1, the spreading stage is controlled by inertial force and thus the spreading velocity of the leading edge is independent of cone angle and increases with increasing We. Besides, the maximum spreading radius increases with decreasing cone angle and increasing We (Figs. 6 (b) and (c)) as explained above.

In Phase 2, contact time increases with increasing cone angle, which is the same as it in Phase 1. When the cone angle remains unchanged, the contact time presents a similar variation with We to it on the flat superhydrophobic surfaces, which is in the scale of inertial-capillary time scale, $\sqrt{\rho r_0^3 / \sigma}$, and is independent of impact velocity, which is consistent with droplet impact on flat superhydrophobic surfaces.^{18, 56-59}



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FIG. 6. Pressure distribution, velocity vector, and spreading factor in Phase 2: (a) the distribution map of pressure inside droplet (Left) and velocity vector chart inside the droplet during impact (Right) under the condition of $r_0 = 1$ mm, $v_0 = 0.6$ m/s, and $\varphi = 110^\circ$; (b) spreading factor varying with time at different cone angle ($r_0 = 1$ mm, $v_0 = 0.4$ m/s, $\varphi = 50^\circ$, 70°, 80°, and 90°, respectively); (c) spreading factor varying with time at different Weber number ($\varphi = 110^\circ$, We = 6.9, 10.0, 13.6, and 17.8, respectively).

191 4. Impact dynamics in Phase 3

In Phase 3, the stages of spread and impalement are similar to those of Phase 2 ($\tau = 0.5 \sim 3.0$ ms in Fig .7); however, different from Phase 2, the inner contact line moves towards the outer contact line ($\tau = 3.0$ ms in Fig .7) and the two contact lines coincide with each other and the contact region between liquid and solid becomes a line ($\tau = 4.5$ ms in Fig .7). At this moment, there is an upward velocity vector perpendicular to the surface which is similar to the retraction process on flat surfaces, and the droplet rebounds upwards finally.



FIG. 7. Pressure distribution and velocity vector of Phase 3: the distribution map of pressure inside droplet (Left) and velocity vector chart inside the droplet during impact (Right) under the condition of $r_0 = 1$ mm, $v_0 = 1.2$ m/s, and $\varphi = 200$ 110° .

201 The inner and outer contact radius variations with time are shown in Fig. 8 (a) ($r_0 = 1.0$ mm, $v_0 = 1.2$ m/s, $\phi =$ 80°), where r_{in} represents spreading radius of the inner contact line and r_{out} represents spreading radius of the outer 202 203 contact line. The three insert graphs in Fig. 8 (a) are the droplet morphologies at $\tau = 2.0, 3.0, \text{ and } 4.0 \text{ ms}$, respectively. 204 During $\tau = 0.0 \sim 2.0$ ms, the outer contact line spreads along the cone surface; and at $\tau = 2.0$ ms, the droplet is impaled by the cone tip and the inner contact line starts to retract along the cone surface; during $\tau = 2.0 \sim 4.5$ ms, 205 both outer and inner contact lines move along the surface, but because the retract velocity of inner contact length is 206 faster than the outer one for the thinner film thickness of the inner part,⁶⁰ the two contact lines collide with each 207 other and all parts of the droplet rebound from the cone ($\tau = 4.5$ ms). The spreading radius at the collision moment 208 209 is defined as the final contact radius, $r_{\rm f}$.



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FIG. 8. (a) Spreading factor varying with time in Phase 3 ($r_0 = 1.0 \text{ mm}$, $v_0 = 1.2 \text{ m/s}$, $\varphi = 80^\circ$), where square and circle dots refer to outer contact line spread radius (r_{out}) and inner line spread radius after impalement (r_{in}), respectively. r_f is the spreading radius when the two contact lines coincide, which is also the rebound moment. (b) droplet profile at the rebound moment.

The diagram of the droplet morphology at the collision moment is shown in Fig. 8 (b), which is a ring shape characterized by the two radius dimensions: r_1 and r_2 ; besides, the cone angle φ and the final contact radius r_f are also indicated. The value of r_f determines where the drop ring rebounds from the cone surface, so the theoretical value is deduced by energy methods. The energy involved in the droplet impact process mainly includes kinetic energy, surface energy, and viscous dissipation. Before impact, the initial kinetic energy E_{k0} and surface energy E_{s0} are

221
$$E_{k0} = \frac{2}{3}\pi\rho r_0^3 v_0^2, \qquad (8)$$

$$E_{\rm s0} = 4\pi\sigma r_0^2 \,. \tag{9}$$

According to the diagram in Fig. 8 (b), the surface energy at the collision moment is

$$E_{\rm s} = 4\pi^2 \sigma (r_1 + r_2) r_2. \tag{10}$$

The relation between r_1 and r_2 can be obtained by the conservation of liquid volume due to the incompressibility of the liquid. And the liquid volume at initial and collision time is

227
$$V_0 = \frac{4}{3}\pi r_0^3, \tag{11}$$

$$V_{\rm f} = 2\pi^2 (r_1 + r_2) r_2^2 \,. \tag{12}$$

229 By $V_0 = V_f$, the ring characteristic size r_1 can be expressed by r_2 :

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230
$$r_1 = \frac{2r_0^3}{3\pi r_2^2} - r_2.$$
(13)

231 The viscous dissipation involved can be calculated using the formula.⁶¹

232
$$W = \int_0^{\tau_f} \int_{\Omega} \Phi d\Omega dt = \Phi \Omega \tau_f .$$
 (14)

where Φ is the viscous dissipation function and can be expressed as $\Phi = \mu (v_0/\delta)^2 (\delta = 2d_0/\sqrt{Re})$ is the thickness of the boundary layer) and Ω is the volume of the viscous fluid calculated by $\Omega = \pi r_1^2 / \sin(\varphi/2)\delta$, and the time duration τ_f is estimated by $\tau_f = 8d_0/3v_0$.⁶² Thus, the viscous dissipation in Eq. (14) can be changed to

236
$$W = \frac{4\pi\mu v_0 r_1^2 \sqrt{\text{Re}}}{3\sin(\varphi/2)}.$$
 (15)

For the kinetic energy at the collision moment, according to our simulation results ($E_s = 5.3 \times 10^{-7}$, $W = 3.4 \times 10^{-7}$, $E_k = 3.8 \times 10^{-8}$, at the condition of $d_0 = 2.0$ mm, $v_0 = 1.2$ m/s, and $\varphi = 110^{\circ}$), it is assumed that the residual kinetic energy is negligible to simplify the theoretical analysis,

$$E_{\rm k} \approx 0.$$
 (16)

(17)

241 During the droplet impact process, the energy conservation is expressed as

242 $E_{k0} + E_{s0} = E_k + E_s + W \; .$

243 We can obtain the relationship between the characterized size β_2 ($\beta_2 = r_2/r_0$) and the impact conditions (We and 244 φ) by substituting energy terms of Eqs. (8), (9), (10), (15), and (16) into the conservation equation of Eq. (17):

245
$$\left[\frac{1}{3} - \frac{4}{3\sin(\varphi/2)\sqrt{Re}} (\frac{2}{3\pi\beta_2^2} - \beta_2)^2\right] We - \frac{8}{3\beta_2} + 4 = 0.$$
(18)

246 Due to Eq. (13) and $r_f = r_1/\sin(\varphi/2)$, Eq. (18) can be converted to the relationship between r_f and impact conditions (We and φ). Figure 9 compares the droplet contact length ratio $\beta_f = r_f/r_0$ at the rebound moment with 247 248 different We and cone angles obtained from the simulation (scatter point) and theoretical model (line). As shown in 249 Fig. 9, β_f presents the same increasing trend with We at different cone angles, and it decreases with increasing cone 250 angle, which is similar to Phase 2. Both Figs. 9 (a) and (b) show the good agreement between simulation results and 251 theoretical results and indicate the availability of the theoretical model. The discrepancy between the theoretical and 252 numerical value can be explained by the assumption that the residual kinetic energy, E_k , equals zero, which slightly 253 overestimates the energy for the droplet to spread on the cone and makes the theoretical value of the spreading factor 254 larger than the numerical value.



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240

FIG. 9. Comparison between theoretical value and simulation value of spreading factor at collision moment ($\beta_f = r_f/r_0$): (a) β_f varies with Weber number ($\varphi = 50^\circ$, 80° , and 110°); (b) β_f varies with cone angle (We = 16.0, 30.1, and 71.1).

To clearly observe the contact length variation with time in Phase 3, the effect of We and cone angle on the absolute contact length ($\beta_{out-in} = \beta_{out} - \beta_{in}$), the dimensionless expression of the absolute contact length r_{out-in} , is depicted in Fig. 10. In Fig. 10, the maximum value of β_{out-in} emerges at the moment when the droplet is impaled by the cone tip, and after this moment the absolute contact length of the droplet decreases with time due to the movement of inner contact line towards the outer one; finally, the time when the value of β_{out-in} equals zero corresponds the collision of the two contact lines and the rebound of the droplet. The whole impact process in Phase 3 can be divided into two stages, one of which is the stage before impalement and another is the process between the impalement and the rebound moment, and the time durations of these two stages are defined as τ_{impale} and $\tau_{retract}$, respectively.

With increasing cone angle, due to the more blunt cone tip, it is more difficult to impale the liquid film and the time reaching the impalement (τ_{impale}) increases (Fig. 10 (c)); thus, the contact length at the impalement moment (r_{impale}) increases (Fig. 10 (a)) and the corresponding film thickness decreases, which causes the inner contact line posess a higher retraction velocity (v_{in}). We can conclude that droplet possesses a higher absolute shrinking velocity (v_{out-in}) as shown in Fig. 10 (a) and a shorter retraction time duration ($\tau_{retract}$) (Fig. 10 (c)).

With increasing We, the spreading velocity increases and it is easier to impale the liquid film with more initial kinetic energy; thus τ_{impale} decreases and r_{impale} increases (Figs. (b) and (d)). As mentioned above, v_{in} increases with increasing r_{impale} ; however, at high We, the outer contact line still possesses a high kinetic energy which plays a major role, so v_{out-in} presents a decreasing trend (Fig. 10 (b)) and $\tau_{retract}$ increases with increasing We (Fig. 10 (d)).





FIG. 10. Absolute contact line spreading factor (β_{out-in}) varying with time (a) under different cone angles ($\varphi = 80^{\circ}, 90^{\circ}, 100^{\circ}, 110^{\circ}, and 130^{\circ}$) and (b) Weber number (We = 27.7, 40.0, 54.4, and 71.1). Variation of the time reaching the impalment moment (τ_{impale}) and the time duration for retraction ($\tau_{retract}$) (c) under different cone angles ($\varphi = 80^{\circ}, 90^{\circ}, 100^{\circ}, 110^{\circ}, and 130^{\circ}$) and (d) under different Weber number (We = 27.7, 40.0, 54.4, and 71.1).

281 B. Contact Time and Phase Diagram

On the basis of the above analysis for the droplet hydrodynamics in different phases, in the next section, the contact time in different phases is studied, where the contact time is defined as the time duration between the moment when the droplet contacts with the cone tip and the time at which droplet leaves the surface, and it is normalized by the inertial-capillary time, $\tau_0 = (\rho r_0^3 / \sigma)^{1/2}$. The effect of cone angles and We on the contact time is studied, and the diagram of different phases distribution with cone angles and We is obtained.

287 Figure 11 (a) shows the contact time as a function of the cone angle in different phases. For droplet impacts in 288 Phase 1 and 2, contact time decreases with increasing cone angle, which is consistent with the impact dynamics in 289 Fig. 5 (b) and Fig. 6 (b); and this is caused by the increasing time for reaching the maximum contact length. For 290 droplet impacts in Phase 3, the contact time is independent of the cone angle, which can be understood by the 291 analysis in Fig. 10 (a) that droplet possesses a longer spreading time τ_{impale} and a shorter retraction time $\tau_{retract}$. In 292 Fig. 11 (a), it is also found that contact time in Phase 1 and Phase 2 (We = 1.1 and We = 6.9) almost coincide with 293 each other, and this is related to the relationship between contact time and We, which will be discussed in the next 294 section.

295 In Fig. 11 (b), the contact time varying with We under three different cone angles, $\varphi = 80^\circ$, 90° , and 110° , is 296 depicted, which shows that under each cone angle, contact time presents three different trends corresponding to 297 different phases. Firstly, in Phase 1 which is represented by purple background, contact time decreases with 298 increasing We; in Phase 2 which is represented by green background, contact time remains almost constant with 299 varying We; and in Phase 3 which is represented by yellow background, contact time still remains unchanged with 300 varying We at a lower value compared with it in Phase 2. Compared with the dimensionless contact time of droplet 301 impacting on flat surface, $\tau_{\text{flat}}/\tau_0 = 2.6$, the contact time in Phase 1 and Phase 2 are longer, indicating that droplet 302 impacting on cone in Phase 1 and Phase 2 cannot reduce droplet contact time. Conversely, droplet impacting on 303 cone in Phase 3 can reduce the dimensionless contact time from 2.6 to 1.2, which is about 54% reduction for the 304 retraction of both the inner and outer contact line.





FIG. 11. The effect of impact conditions on contact time in different phases. (a) Variation of the contact time with cone angles: purple points with We = 1.1 refer to cases in Phase 1; green points with We = 6.9 refer to cases in Phase 2; and 15/20

308 yellow points with We = 40.0 refer to cases in Phase 3. (b) Variation of the contact time with We under different cones 309 with $\varphi = 80^\circ$, 90° , and 110° .

310 To reveal the corresponding cone angle and We for the occurrence of different impact phases, we propose a phase diagram delimiting the above three phases as in Fig. 12. It could be observed that at the low cone angle (φ = 311 50°), Phase 1 and Phase 2 emerge at low We, and Phase 3 happens in a wide range of We; with increasing cone 312 angle, the range of We corresponding to Phase 2 expands while the range of Phase 1 changes less. For the boundary 313 314 between Phase 1 and 2, with increasing cone angle, phases transition becomes sensitive to We; for the boundary 315 between Phase 2 and 3, the critical We is positively correlated with the cone angle. This diagram reveals impact 316 conditions under which different phases could occur and provides a guidance for relevant applications and 317 requirements.



318

FIG. 12. Phase diagram distinguishing three phases under the conditions in this work ($\theta = 160^{\circ}$, and the fluid used here is water and air at 20 °C): blue points represent Phase 1, green points represent Phase 2, and purple points represent 221 Phase 3.

322 IV. CONCLUSIONS

In summary, we investigate the droplet impacting dynamics on superhydrophobic cones and discuss the 323 324 morphologic evolution characteristics, as well as the variation of contact time. By proper selections of Weber 325 number and cone angle, there are three typical morphologic evolution phases and each phase exhibits different 326 impact dynamics. In Phase 1, the droplet undergoes a rebound without impalement, which is similar to that on flat 327 surfaces; in Phase 2, the droplet is impaled by the cone, but the re-coalescence of the ring morphology lifts the droplet up from the cone tip; in Phase 3, the droplet leaves the cone surface as a ring after impalement, and a 328 theoretical model to predict the rebound point is established, which agrees well with the numerical results. The 329 330 variation of contact time exhibits different tendencies in the above phases. The contact time continuously reduces 331 with the increase of We in Phase 1, while it remains almost constant in Phase 2, both of which are larger than that 332 on flat surfaces. In Phase 3, a sharp reduction in contact time is observed which is about 54% compared with flat 333 surfaces. Besides, the effect of cone angle on the contact time in different phases is studied: in Phase 1 and Phase 2, 334 contact time decreases with increasing cone angle, while in Phase 3, contact time is independent of the cone angle.
335 A phase diagram delimiting three phases for impacting dynamics is finally proposed, and the critical Weber number
336 of different phases increases with increasing cone angle. The above findings pioneer in quantitatively clarifying the
337 droplet impacting dynamics on superhydrophobic cones, which shall further guide related engineering applications.

338

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343 DATA AVAILABILITY

344 The data that support the findings of this study are available within the article.

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