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NON-COHERENT DOA ESTIMATION OF OFF-GRID SIGNALS WITH UNIFORM CIRCULAR ARRAYS

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***Abstract*—Recently, some non-coherent DOA estimation methods are presented under a sparse phase retrieval framework, where DOAs of incident signals are assumed to be on the predefined grid points. However, this may not be correct in practice; in order to address this issue, an off-grid model involved with a bias vector is proposed and an efficient two-step method based on this model is developed. In addition, instead of using ULAs, uniform circular arrays (UCAs) are employed in order to overcome the ambiguities arising in non-coherent measurements, as analysed in detail. Numerical simulations show that, compared to on-grid model with a denser grid points, the off-grid model with a coarse grid can achieve a better performance with a lower computational complexity.**

***Index Terms*—uniform circualr array, DOA estimation, non-coherent, phase retrieval, off-grid.**

I. Introduction

Direction of arrival (DOA) estimation has various applications such as radar, sonar and wireless communications [1]. Traditionally, the phase information is assumed to be available at the array of sensors and many proposed high resolution DOA estimation algorithms often rely on this assumption, such as MUSIC [2], ESPRIT [3] and those based on compressive sensing [4]. However, in practice, the phase information may not be reliable due to various reasons and in the extreme case, we may only have the magnitude information.

For such a non-coherent DOA estimation problem, several estimation methods has been proposed. [5]–[9]. It is suggested in [5] that, while applying uniform linear array (ULA), with a high gain reference signal (12 dB over unknown signals), DOA of incident signals can be obtained by estimating the frequency component of the received non-coherent measurements; furthermore, the number of required reference signals with normal gain can be restricted to one by employing a second ULA. However, its accuracy in DOA estimation relies on its frequency resolution, which requires a large number of sensor measurements.

On the other hand, the non-coherent DOA estimation problem can be expressed in a compressing sensing form, which can be solved by sparse phase retrieval algorithms. In the case of one snapshot, sparse phase retrieval has been applied to non-coherent DOA estimation directly since they have a similar signal model [6]–[8]. In the case of multiple snapshots, where all snapshots share the same spatial support, group sparsity based phase retrieval algorithm for non-coherent DOA estimation has been proposed [9]–[11]. Under the sparse framework, the inherent ambiguity issue of non-coherent measurements was resolved using a known reference signal when only one unknown source impinges upon the array and more reference signals are required when there are more incident signals.

Although these sparsity based non-coherent methods are effective, it has two challenges: (1) known reference signal(s) are required to solve the inherent ambiguities of phaseless measurements associated with ULAs; (2) DOAs of incident signals are assumed to fall on the discrete grid points. However, in practice, quite often the true DOAs may not lie on the predefined grid points, which leads to an off-grid problem. One solution to it is applying a denser grid, which significantly increases the computational complexity. Another solution is grid refinement [12], which defines a coarse grid at first and then, based on the initial DOA results, a denser steering matrix is built around the estimated locations of incident signals. However, computational complexity of this method is still high. In coherent-measurement scenario, several off-grid DOA estimation methods has been proposed [13], [14], where the estimated DOAs are no longer assumed to be in the predefined grid points. Apart from applying a denser grid or grid refinement, this off-grid issue for non-coherent DOA estimation has not been addressed yet.

In order to deal with the two challenges, a two-step off-grid non-coherent DOA estimation method employing uniform circular arrays (UCAs) is proposed, where the on-grid DOAs of incident signals and its off-grid terms are estimated separately. In the first step, DOAs are approximated with a coarser steering matrix. In the second step, their off-grid bias is estimated through an iterative process, which has an closed-form solution in each iteration. Moreover, it is shown that due to the unique structure of UCAs, the inherent ambiguities associated with ULAs would not arise in UCAs and reference signals are not necessary when there are two or more incident signals; a reference signal is required in the scenario with only one incident signal, but the DOA of the reference signal can be arbitrary and unknown to the estimator.

The remaining part of this paper is structured as follows. The on-grid and off-grid non-coherent signal models and inherent ambiguities of phase retrieval based DOA estimation are described in Sec. II. The proposed off-grid non-coherent DOA estimation method is presented in Sec. III. Simulation results are provided in Sec. IV and conclusions are drawn in Sec. V.

II. Non-Coherent Signal Model Based on UCAs

A. Signal Model

Assume that there are K narrowband signals s_k with the same wavelength λ impinging from directions θ_k, $k = 1, 2, ..., K$, respectively, on a UCA of N sensors with an adjacent sensor spacing d. The corresponding received signal vector at time index p without noise is expressed as [15], [16]

$$\mathbf{x}[p] = \mathbf{A}(\theta)\mathbf{s}[p], \tag{1}$$

where $p \in \{1, ...P\}$ represents the p-th snapshot, and $\mathbf{s}[p]$ is the source signal vector expressed as

$$\mathbf{s}[p] = \big[s_1[p], s_2[p], .., s_K[p]\big]^T. \tag{2}$$

$\mathbf{A}(\theta)$ is steering matrix with its columns $\mathbf{a}(\theta_k)$, for $k = 1, ..., K$, being the corresponding steering vectors, given by

$$\mathbf{a}(\theta_k) = [e^{j\xi\cos(\theta_k-\gamma_1)}, ..., e^{j\xi\cos(\theta_k-\gamma_N)}]^T, \tag{3}$$

where $\xi = 2\pi r/\lambda$, and $\gamma_n = 2\pi n/N$, $n = 1, ..., N$. Since the phase information is not available, we only have magnitude-only measurements, given by

$$\mathbf{y}[p] = |\mathbf{x}[p]| + \mathbf{n}[p] = |\mathbf{A}(\theta)\mathbf{s}[p]| + \mathbf{n}[p], \tag{4}$$

where $|\cdot|$ is the element-wise absolute value operator and $\mathbf{n}$ is the additive white Gaussian noise. If the admissible DOA range is divided into G grid points with $G \gg N$, an overcomplete steering matrix

$$\tilde{\mathbf{A}} = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_G)] \tag{5}$$

can be formed with each column representing a potential incident angle. Accordingly, the signal vector $\mathbf{s}[p]$ is extended to a $G \times 1$ sparse vector $\tilde{\mathbf{s}}[p] = [s_1[p], ..., s_G[p]]^T$, where only K entries at the corresponding incident angles are supposed to be non-zero.

Then, the array output under the sparse representation framework is given by

$$\mathbf{x}[p] = \tilde{\mathbf{A}}\tilde{\mathbf{s}}[p]. \tag{6}$$

Collecting P snapshots to form $\mathbf{X} = \big[\mathbf{x}[1], ..., \mathbf{x}[P]\big]$, it has

$$\mathbf{X} = \tilde{\mathbf{A}}\tilde{\mathbf{S}}, \tag{7}$$

where $\tilde{\mathbf{S}} = \big[\tilde{\mathbf{s}}[1], ..., \tilde{\mathbf{s}}[P]\big]$ is the sparse signals matrix with the corresponding K rows being non-zero. For multiple snapshots, we have

$$\mathbf{Y} = |\mathbf{X}| + \mathbf{N} = |\tilde{\mathbf{A}}\tilde{\mathbf{S}}| + \mathbf{N}, \tag{8}$$

where $\mathbf{Y} = \big[\mathbf{y}[1], ..., \mathbf{y}[P]\big]$, and $\mathbf{N} = \big[\mathbf{n}[1]], ..., \mathbf{n}[p]\big]$.

B. Ambiguity

Results obtained from phaseless measurements with uniform linear array (ULA) suffers from some inherent ambiguities, which would affect the DOA estimation results: mirroring and spatial shift [6], [11], [17]. Denote the measurement at the n-th sensor of the UCA as

$$x_n = \sum_{k=1}^{K} s_k e^{j\xi\cos(\theta_k-\gamma_n)}. \tag{9}$$

For mirroring ambiguity, it refers to the phenomenon that signals arriving from $-\theta_k$ will generate the measurements with the same magnitude. With UCA, however, we have

$$|\check{x}_n| = |\sum_{k=1}^{K} s_k e^{j\xi\cos(-\theta_k-\gamma_n)}| = |\sum_{k=1}^{K} s_k e^{j\xi\cos(\theta_k+\gamma_n)}|. \tag{10}$$

Obviously, the magnitude of $\check{x}_n$ is in general different from x_n, thus the mirroring ambiguity in ULAs will not appear in UCAs.

For spatial shift ambiguity, it refers to that the received signals at the array are phased shifted by a specific amount ϕ_n,

$$\check{x}_n = e^{j\xi\phi_n}\sum_{k=1}^{K} s_k e^{j\xi\cos(\theta_k-\gamma_n)} = \sum_{k=1}^{K} s_k e^{j\xi\cos(\theta_k-\check{\theta}_{n,k}-\gamma_n)}. \tag{11}$$

Although $\check{x}_n$ would share the same magnitude as with x_n at the n-th sensor, $\check{\theta}_{n,k}$ for the corresponding k-th signal at different sensors are different due to the non-linear property of cos function, which implies that, there is no common shift variable ϕ_n to simultaneously keep the same magnitude as x_n and same shifted angle $\check{\theta}_{n,k}$ for all N sensors.

Thus, we can conclude that the inherent mirroring and spatial shift ambiguities involved in ULAs will not appear in UCAs.

But there is another ambiguity involved in non-coherent measurements of UCAs. For the whole range $[-\pi, \pi]$, K incident signals $\mathbf{s}^*$ from angle $(\theta_k \pm \pi)$ would share the same magnitude as $\mathbf{x}_n$, expressed as

$$\check{x}_n = \sum_{k=1}^{K} s_k^* e^{j\xi\cos(\theta_k\pm\pi-\gamma_n)} = \sum_{k=1}^{K} s_k^* e^{-j\xi\cos(\theta_k-\gamma_n)} = x_n^*. \tag{12}$$

There are two possible solutions to solve this ambiguity. One is to limit the area of interest to $[-90°, 90°]$, since for $-\pi/2 \leq \theta_k \leq \pi/2$, $\theta_k \pm \pi$ will exceed the limit.

Another one is applying a reference signal at the end of interested area $[-\pi]$ and assume no signal come from $0°$ (Generally, define θ_{ref} and remove either column of $[\theta_{ref} \pm \pi]$ as appropriate from $\tilde{\mathbf{A}}$). With this reference signal, either $\theta_k - \pi$ or $\theta_k + \pi$ will be out of the range $[-\pi, \pi]$. In practice, due to influence of noise, a short range of $[\theta_k \pm \pi - u, \theta_k \pm \pi + u]$ should be removed from the area of interest.

Note that, the non-coherent DOA estimation does not work if there is only one incident signal due to only magnitude information can be obtained irrespective to array structure. Therefore, for such a scenario, a reference signal has to be deployed. However, different from existing methods, its DOA does not need to be known in advance.

III. Proposed Two-Step Off-Grid Method

A. Off-Grid Signal Model

Let $\overline{\boldsymbol{\theta}} = [\overline{\theta}_1, ..., \overline{\theta}_k]$ denote the true DOAs of K incident signals and θ_{g_k} represent the nearest grid point for the k-th signal, and (7) can be approximated by

$$\mathbf{X} \approx (\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\tilde{\boldsymbol{\Delta}})\tilde{\mathbf{S}}, \tag{13}$$

with $\tilde{\mathbf{B}} = [\mathbf{b}(\theta_1), ..., \mathbf{b}(\theta_G)]$, $\mathbf{b}(\theta_g) = \frac{\partial \mathbf{a}(\theta_g)}{\partial \theta_g}$, $\tilde{\boldsymbol{\Delta}} = \mathrm{diag}(\tilde{\boldsymbol{\beta}})$ and $\tilde{\boldsymbol{\beta}} = [\beta_1, ..., \beta_G]$, where

$$\beta_g = \begin{cases} \bar{\theta}_k - \theta_{g_k}, & \text{if } g = g_k, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

β_g satisfies $-\frac{e}{2} \leq \beta_g \leq \frac{e}{2}$, and e is grid stepsize. Accordingly, the non-coherent measurements (8) is changed to

$$\mathbf{Y} \approx |(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\tilde{\boldsymbol{\Delta}})\tilde{\mathbf{S}}| + \mathbf{N}. \tag{15}$$

Then, the off-grid non-coherent DOA estimation problem can be solved by the following unconstrained optimisation problem

$$\min_{\tilde{\mathbf{S}}, \tilde{\boldsymbol{\Delta}}} \; \||(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\tilde{\boldsymbol{\Delta}})\tilde{\mathbf{S}}| - \mathbf{Y}\|_F^2 \quad s.\ t.\ |\tilde{\mathbf{S}}\|_{2,0} \quad \leq K. \tag{16}$$

where the $\|\cdot\|_{2,0}$ is the $l_{2,0}$ norm, which enforces row sparsity of $\tilde{\mathbf{S}}$.

B. Proposed Method

Jointly estimate $\tilde{\boldsymbol{\Delta}}$ and $\tilde{\mathbf{S}}$ from (16) is a non-convex optimization problem. In the following, a two-step method is proposed.

In the first step, $\tilde{\boldsymbol{\Delta}}$ is assumed to be zero, and the corresponding optimization problem is formulated as

$$\min_{\tilde{\mathbf{S}}} \; \||\tilde{\mathbf{A}}\tilde{\mathbf{S}}| - \mathbf{Y}\|_F^2 \quad s.\ t.\ \|\tilde{\mathbf{S}}\|_{2,0} \quad \leq K, \tag{17}$$

The objective function (17) can be solved by existing group sparse phase retrieval algorithms [9]–[11].

In the second step, in order to estimate the off-grid bias, the PRIME technique [18] is employed. For a single snapshot of (16), we reformulate the non-convex objective function as

$$\min_{\tilde{s}} \; \|\tilde{\mathbf{A}}\tilde{\mathbf{s}} - \mathbf{c}^q\|_2^2, \quad \text{with} \quad \mathbf{c}^q = \mathbf{y} \odot e^{j\arg(\tilde{\mathbf{A}}\tilde{\mathbf{s}}^q)}, \tag{18}$$

where $\odot$ is the Hadamard product, $\arg(\cdot)$ represents the phase of its variable applied element-wise, and $\mathbf{c}^q$ is a known complex vector.

After applying PRIME column by column to (16), its first term can be replaced by a convex surrogate, and the corresponding objective function is changed to

$$\min_{\tilde{\mathbf{S}}} \; \|(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\tilde{\boldsymbol{\Delta}})\tilde{\mathbf{S}} - \bar{\mathbf{C}}\|_F^2 \quad s.\ t.\ \|\tilde{\mathbf{S}}\|_{2,0} \quad \leq K, \tag{19}$$

with

$$\bar{\mathbf{C}} = \mathbf{Y} \odot e^{j\arg((\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\boldsymbol{\Delta})\tilde{\mathbf{S}}_e)}, \tag{20}$$

where $\tilde{\mathbf{S}}_e$ is estimated signals from step one, which is up to a global phase ambiguity.

After that, similar to [19], an iterative algorithm for estimating dictionary bias $\tilde{\boldsymbol{\beta}}$ is proposed. This method first estimates K non-zero rows of estimated signals $\tilde{\mathbf{S}}_e$ as

$$\bar{\boldsymbol{S}}_K^i = (\bar{\boldsymbol{A}}_K^i)^\dagger \bar{\mathbf{C}}, \tag{21}$$

where $(\cdot)^\dagger$ is the pseudo-inverse operator, $i \in \{1, ..., I\}$ is iteration index, $\bar{\boldsymbol{A}}_K^i = \bar{\boldsymbol{A}}(\theta_K^i)$ is the steering matrix with K columns corresponding the estimated DOAs $\boldsymbol{\theta}^i = [\theta_1^i, ..., \theta_K^i]$, and θ_K^i is updated in each iteration. $\bar{\mathbf{A}}_K^0$ is initialized as K columns of $\tilde{\mathbf{A}}$, which corresponds to DOAs of estimated signals $\tilde{\mathbf{S}}_e$. As the bias $\tilde{\boldsymbol{\beta}}$ shares same support with $\tilde{\mathbf{S}}_e$, $\bar{\mathbf{B}}_K$, is obtained, which is the sub-matrix of $\tilde{\mathbf{B}}$ with corresponding K columns at the support of $\tilde{\mathbf{S}}_e$. By denoting $\bar{\boldsymbol{\Delta}}_K^i = \mathrm{diag}(\bar{\boldsymbol{\beta}}_K^i) = [\beta_1^i, ...\beta_K^i]^T$ as the bias of corresponding DOAs of incident signals, $\bar{\boldsymbol{\Delta}}_K^i$ can be estimated by solving

$$\min_{\bar{\boldsymbol{\beta}}_K} \; \|(\bar{\mathbf{A}}_K + \bar{\mathbf{B}}_K\bar{\boldsymbol{\Delta}}_K^i)\bar{\mathbf{S}}_K^i - \bar{\mathbf{C}}\|_F^2. \tag{22}$$

Dropping index i for simplicity, (22) can be reformulated as [19], [20]

$$\begin{aligned} &\|(\bar{\mathbf{A}}_K + \bar{\mathbf{B}}_K\bar{\boldsymbol{\Delta}}_K)\bar{\mathbf{S}}_K - \bar{\mathbf{C}}\|_F^2 \\ &\approx \mathrm{tr}\big\{\bar{\mathbf{S}}_K^H\bar{\boldsymbol{\Delta}}\bar{\mathbf{B}}_K^H\bar{\mathbf{B}}_K\bar{\boldsymbol{\Delta}}\bar{\mathbf{S}}_K\big\} - 2\mathrm{Re}\big\{(\bar{\mathbf{C}} - \bar{\mathbf{A}}\bar{\mathbf{S}}_K)^H\bar{\mathbf{B}}_K\bar{\boldsymbol{\Delta}}_K\bar{\mathbf{S}}_K\big\} \\ &= \bar{\boldsymbol{\beta}}_K^T\big(\bar{\mathbf{B}}_K^H\bar{\mathbf{B}}_K \odot (\bar{\mathbf{S}}_K\bar{\mathbf{S}}_K^H)^*\big)\bar{\boldsymbol{\beta}}_K \\ &- 2\mathrm{Re}\big\{\mathrm{diag}[\bar{\mathbf{S}}_K(\bar{\mathbf{C}} - \bar{\mathbf{A}}_K\bar{\mathbf{S}}_K)^H\bar{\mathbf{B}}_K]\bar{\boldsymbol{\beta}}_K\big\}, \end{aligned} \tag{23}$$

where $\mathrm{tr}(\cdot)$ and $\mathrm{Re}(\cdot)$ represent the trace and real part of its variable, separately. With the optimal condition of (23), $\bar{\boldsymbol{\beta}}_K^i$ can be obtained by

$$\bar{\boldsymbol{\beta}}_K^i = \mathrm{Re}\{(\mathbf{D}^i)^{-1}\mathbf{h}^i\}, \tag{24}$$

where $(\cdot)^{-1}$ is the inverse operator, and

$$\begin{aligned} \mathbf{D}^i &= \mathbf{B}_K^H\bar{\mathbf{B}}_K \odot (\bar{\mathbf{S}}_K^i\bar{\mathbf{S}}_K^i)^*, \\ \mathbf{h}^i &= \big\{\mathrm{diag}[\bar{\mathbf{S}}_K^i(\bar{\mathbf{C}} - \bar{\mathbf{A}}_K\bar{\mathbf{S}}_K^i)^H\bar{\mathbf{B}}_K]\big\}^T. \end{aligned} \tag{25}$$

Since $-\frac{e}{2} \leq \beta_g \leq \frac{e}{2}$, for $k = 1, ..., K$, it has

$$\beta_k^i = \begin{cases} -e/2, & \text{if } \beta_k^i < e/2, \\ e/2 & \text{if } \beta_k^i > e/2, \\ \beta_k^i, & \text{otherwise.} \end{cases} \tag{26}$$

Note that, the non-coherent DOA estimation results still suffer from the global phase ambiguity, that is

$$\begin{aligned} \bar{\mathbf{C}} &= \mathbf{Y} \odot e^{j\arg(\tilde{\mathbf{A}}\tilde{S}_e)} \approx \mathbf{A}_r\mathbf{S}_r e^{j\phi}, \\ \bar{\mathbf{S}}_K &= (\bar{\boldsymbol{A}}_K^t)^\dagger\bar{\mathbf{C}} \approx \mathbf{S}_r e^{j\phi}, \end{aligned} \tag{27}$$

where $\mathbf{S}_r$ and $\mathbf{A}_r$ represent the real signal and its corresponding real steering matrix, respectively, and ϕ is a global phase factor. It can be seen that, when substituting (27) into (24), the global phase factor cancels, which means that the global phase ambiguity will not affect bias estimation in this step.

With $\bar{\boldsymbol{\beta}}^i$, the steering matrix $\mathbf{A}_K$ is updated as

$$\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^0 + \boldsymbol{\beta}^i, \quad \bar{\boldsymbol{A}}_K^{i+1} = \bar{\boldsymbol{A}}_K(\theta^{i+1}), \tag{28}$$

where θ^0 is the initial DOAs obtained from the first step, i.e corresponding DOA of the non-zero rows of $\tilde{\boldsymbol{S}}_e$. Finally, the output DOA $\boldsymbol{\theta}_e$ is obtained as

$$\boldsymbol{\theta}_e = \boldsymbol{\theta}_0 + \boldsymbol{\beta}^I. \tag{29}$$

The full algorithm is summarized in the algorithm summary.

Algorithm Summary (Two-Step Off-Grid)

Input: $\tilde{\mathbf{A}}$, $\mathbf{Y}$,

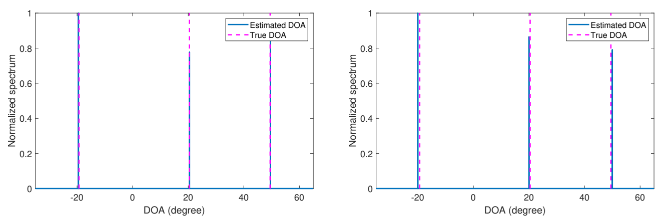
Initializtion: $\bar{\boldsymbol{\beta}}_K^0 = \mathbf{0}$.

Step 1: Estimate $\tilde{\mathbf{S}}_e$ via existing group sparse phase retrieval algorithms,
Obtain $\bar{\boldsymbol{A}}_K^0$ and $\boldsymbol{\theta}^0$,
Calculate $\bar{\mathbf{C}} = \mathbf{Y} \odot e^{j\arg(\tilde{\mathbf{A}}\tilde{\mathbf{S}}_e)}$.

Step 2: for i=1, ..., I

1) Calculate $\bar{\mathbf{S}}_K^i = (\bar{\boldsymbol{A}}_K^i)^\dagger \bar{\mathbf{C}}$.
2) Calculate $\bar{\boldsymbol{\beta}}_K^i = \mathrm{Re}\{(\mathbf{D}^i)^{-1}\mathbf{h}^i\}$ from (25),
3) Restrict elements of $\bar{\boldsymbol{\beta}}^i$ within range $[-\frac{r}{2}, \frac{r}{2}]$.
4) Calculate $\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^0 + \boldsymbol{\beta}^i$,
$\bar{\mathbf{A}}_K^{i+1} = \bar{\mathbf{A}}_K(\theta^{i+1})$.
5) i=i+1, go to 1).

Output estimated DOA: $\boldsymbol{\theta}_e = \boldsymbol{\theta}^0 + \bar{\boldsymbol{\beta}}^I$.



(a) Results by off-grid model. **(b)** Results by on-grid model.

Fig. 1. Estimation results based on the UCA structure.

IV. Simulation Results

In this section, simulation results are provided to show the performance of the proposed off-grid non-coherent DOA estimation method in comparison with the on-grid model, where results of on-grid model are obtained from the first step of the proposed method. The area of interest is considered within $[-\pi/2, \pi/2]$ to avoid ambiguity involved in UCAs. The number of sensors N is set as 19 while the radius r of UCAs is set as $r = Nd/2\pi$ with $d = \lambda/2$, and $P = 500$ snapshots are collected in all simulations. A recently proposed sparse phase retrieval algorithm called ToyBar in [10], [11] is applied in the first step of the proposed method, its iteration number is 500 and 20 random initializations are used in order to find the global minimum of the phase retrieval problem, while the iteration number for the second step is 50.

In the first set of simulations, the steering matrix is formed with a step size of $2°$ and input SNR is set as 20 dB. DOA estimation results of both on-grid and off-grid model are compared and shown in Fig. 1, where dotted lines represent true DOAs and solid lines are estimated ones. We can observe that, although both models can identify DOAs more or less correctly, the off-grid model provides a more accurate result.

In the second set of simulations, RMSE results of the off-grid model and on-grid model with different SNR values ranging from 0 dB to 20 dB are compared, with each point being an average of 200 trials. The Cramer-Rao bound derived in [11] is also provided. In all trials, locations of $K = 3$ signals are defined as $[-40° + u_1, 0° + u_2, 30° + u_3]$, where u_k is randomly generated at each run within $[-1°, 1°]$. The results

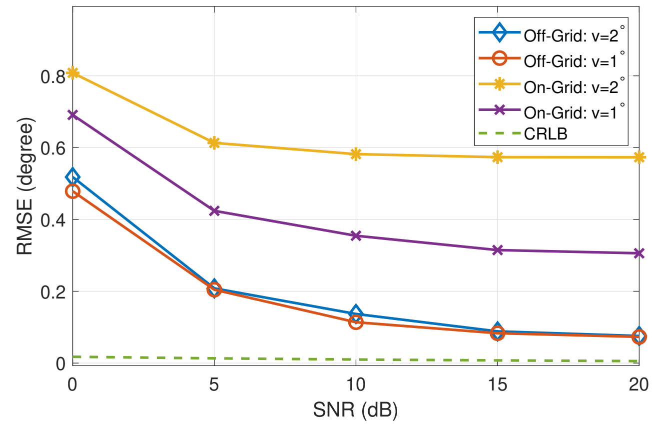


Fig. 2. RMSEs versus SNR.

Table I: Running times of off-grid and on-grid models.

Stepsize	On-grid(s)	Off-grid(s)
2°	25.54	25.56
1°	52.65	52.69

are further parameterized by the grid stepsize v. According to Fig. 2, it can be observed that, although a smaller stepsize can improve the performance of both on-grid and off-grid models, the off-grid model outperforms the on-grid model even when the on-grid model has a denser grid; moreover, the improvement achieved by a denser grid for the proposed off-grid method is very small, which means the second step of the method is working effectively.

Finally, the computational complexity of both on-grid and off-grid models with different stepsizes is compared in terms of running time, and the results are shown in Table 1, based on a computer with 1.8GHz CPU i7-10510U and 16GB RAM. We can see that a smaller stepsize significantly increases the computation time, whereas the extra time cost by the second step of the off-grid model is minimal.

V. Conclusions

In this paper, non-coherent DOA estimation of off-grid signals has been studied and an efficient two-step algorithm was proposed. In the first step, dictionary bias is assumed to be zero and the off-grid problem is considered as a normal group sparse phase retrieval problem, while in the second step, dictionary bias is estimated through an iterative process. Simulation results indicates that, for the same grid, the proposed off-grid non-coherent DOA estimation method has given more accurate results than the on-grid model with very marginal extra time consumption. In addition, although the off-grid model with a larger stepsize requires less CPU time than the on-grid model with a smaller stepszie, the DOA estimation accuracy of the off-grid model is still better than the on-grid one.

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