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Liu, F, Xun, J, Liu, R orcid.org/0000-0003-0627-3184 et al. (2 more authors) (2022) A Real-Time Rescheduling Approach Using Loop Iteration for High-Speed Railway Traffic. IEEE Intelligent Transportation Systems Magazine, 15 (1). pp. 318-332. ISSN 1939-1390

https://doi.org/10.1109/MITS.2021.3137478

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A Real-time Rescheduling Approach by Using Loop Iteration for High-speed Railway Traffic

Fan Liu¹, Jing Xun¹, Ronghui Liu², Jiateng Yin¹, Hairong Dong¹

Abstract-With the increase of train density on the line in high-speed railways (HSR), delay propagation becomes easy to occur. In this paper, we investigated a real-time rescheduling problem to restore the HSR operation from the delay caused by disturbance. A real-time rescheduling model is proposed with considering the relationships between running time and departure time at the disturbance area, which makes the model more precise. The objective of the proposed model is to minimize the total delay when the disturbance occurred. A loop iterative architecture is proposed to reduce the constraints scale of the proposed rescheduling model. Three experiments were presented to demonstrate the validity of the proposed model and the effectiveness of the proposed algorithm. By using the proposed method, a rescheduling problem with 10 trains and 248 block sections in the 6 stations and 5 inter-station areas can be solved within 60 seconds.

Index Terms—Real-time timetable rescheduling, mixed integrate programming, train speed trajectory

I. INTRODUCTION

A. Motivation

Igh speed railway(HSR) plays a key role in China's comprehensive transportation system. The Bulletin of China Railway shows that by the end of 2020, 2.203 billion passengers were sent by China Railway, of which more than 60 % was sent by HSR. To meet the increasing travel demand, China Railway has made 4 adjustments to the timetable/diagram in 2020, and the number of operating trains has increased gradually.

With the increase of train density on the line, delay propagation becomes easy to occur. On May 1, 2021, an equipment failure occurred near Beijing west railway station. This led to train congestion on Beijing-Guangzhou intercity line firstly, then spread to Guangzhou South station, and further affected other trains to Guangzhou South station. Tens of trains were delayed and the most delay for one train is more than 3 hours. The large-scale train delays have caused passengers detention, affected the normal operation of trains and passenger travel, and even affected the operation of the entire HSR transportation network.

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To operate the HSR system safely and efficiently, China has developed the Chinese Train Control System (CTCS-3). One of the most important features of CTCS-3 is the bi-directional train-to-ground communication technology that enables the frequent exchange of information between the way-side control center and trains [1]. Recently, high speed automatic train operation system (CTCS-3+ATO), which is independently developed by China Railway Signal Communication Corporation Limited(CRSC), has passed the trial and appraisal of the China Railway Corporation by 350 kilometers. It marks the smooth passing of the technology and will officially enter the trial operation stage of the passenger train operation of the Beijing Shenyang passenger dedicated line. In CTCS-3+ATO, when the train departs from the station and passes through the balise group of departure section, ATO will receive the operation plan. According to the operation plan information, train operation status, ATO adopts traction, braking, coasting, and other control strategies, as well as the line information provided by temporary speed restriction servers(TSRS) to automatically control the train operation in the inter-station. Building upon these communication technologies, the quasimoving block system, as a kind of fixed block system, can be implemented to enhance the system capacity, in which the target of the following train emergency braking speed trajectory is the starting point of the block section occupied by the preceding train [2].

B. Literature Review

The research on the real-time rescheduling problem in the case of delay originated from the operation and management of single-track railways in the main railway network. In a singletrack railway, all overtaking and meeting operations of trains can only be carried out on the side track (or side line) of the fixed platform. Therefore, if a train on the line is delayed, it will affect other trains passing by or overtaking it, resulting in subsequent delays of a large number of trains, and ultimately invalidating the current train order. In this case, it is usually necessary for dispatchers to recompile the operation order to ensure the safe overtaking and meeting of the trains on the line by adjusting the stopping and running time of the trains, so as to minimize the negative impact of delays. Based on the characteristics of the infrastructure of railways, such as singletrack, double-track and junctions, the commonly used methods consist of rerouting, reordering and re-timing of trains [3]. The current train real-time rescheduling model solution methods include the following categories: operation research planning method [4] [5] [6] [7] [8] [9] [10], heuristic search method

^{*}This work is supported by the National Natural Science Foundation of China (No.61790573, No.61925302 and No.62073026), the State Key Laboratory of Rail Traffic Control and Safety(Contract No.RCS2021ZT007),Beijing Jiaotong University, Guangxi Science and Technology Planning Project: Construction of China-ASEAN International Joint Laboratory for Comprehensive Transportation (No.GUIKE AD20297125)

In recent years, the modernization of train control equipment has made it possible to integrate train management and operation control technically. Many scholars have begun to pay more attention to integrated train rescheduling and train operation based on the precious rescheduling method. So as to maximize the overall system performance, minimize the negative impact of delays on system operation.

Su et al. [21] proposed an energy-efficient design of train operation diagram and speed trajectory integration based on the feedback idea, and used the energy-efficient operation results of the bottom train as the input of the upper train operation time minutes to minimize the system energy consumption by iterative calculation. Li and Lo [22] constructed a nonlinear programming model to optimize the train traction energy consumption of the whole system from the perspective of the integration of the planned operation diagram and train speed curve and regenerative braking energy utilization for the whole system and solved offline using a genetic algorithm. Yang et al. [23] constructed a two-stage model, in which the first stage is the train operating time and the second stage is the train speed trajectory in each zone and designed a genetic algorithm to optimize the solution, which can save the total energy consumption of the system by 9% - 10% based on the actual example of Beijing Metro Yizhuang Line. Later, they develops an energy-efficient rescheduling approach under delay perturbations for metro trains, which aims to minimize the net energy consumption under the premise of reducing or eliminating the delay altogether. [24]

Corman et al. [25] proposed a train operation adjustment framework based on closed-loop feedback (Closed-Loop Control). Under this framework, the operation control center(OCC) predicts the train's running status, delays, and potential running conflicts in real-time based on the train status feedback information (speed trajectory, position, etc.), and uses rolling optimization to adjust the timetable on the predicted information, which can reduce the delay. The author clearly pointed out in the research that this closed-loop feedback method has an integrated idea to a certain extent, but it has not fully realized the overall optimization of the timetable and speed trajectory. It requires further in-depth research and is also very useful.

Wang et al. [26] studied the integrated adjustment of the speed trajectory and timetable in the case of delay, the purpose of which is to reduce the delay time and running energy consumption of the train, and to achieve the Pareto optimal. Starting from the rail transit train operation control technology, the research took into account practical factors such as the microscopic train dynamics model, line slope, speed limit, and signal block section, and transformed the train operation process into a multi-stage decision-making process. Combining the pseudo-spectrum optimization method, three heuristic train operation adjustment strategies are designed, which are used to reduce the delay of a certain train, improve the overall punctuality rate and energy saving. However, the study can only calculate small-scale cases. In the case of calculating four trains in three operating intervals, the calculation time of the algorithm has reached more than 10min.

Under the constraint of the fixed block, Luan et al. [27] studied the integration of real-time traffic management and train control by using mixed-integer nonlinear programming (MINLP) and mixed-integer linear programming (MILP) approaches. They proposed three innovative integrated optimization approaches to optimize train dispatching (including train routes, orders, departure, and arrival times at passing stations) and train control(i.e., train speed trajectories). Train speed is considered variable, and the blocking time of a train on a block section dynamically depends on its real speed.

Xu et al. [2] integrated the modeling of efficient traffic management measures and the supervision of speed, braking and headway in one general job shop model. A good solution can be found by the proposed model within the computation time of 10 minutes on a commercial solver, by employing a two-step search procedure. The train speed trajectory they got is the average speed at each block section. And the speed change is too fast to satisfy real train operation.

C. The Focus of This Paper

To the best of our knowledge, the existing literature for timetable rescheduling problems considers the train speed trajectory is extremely complex and difficult to solve. To fill the gap between the practical application and theoretical research of the HSR rescheduling problem, this paper proposes a real-time rescheduling approach based on the real train speed trajectory, which can integrate train operation and train scheduling efficiently.

More precisely, the key contributions of our approach are:

- A rescheduling model is proposed with considering the relationships between running time and departure time at disturbance area, which makes the model more precise.
- 2) A loop iterative optimization architecture is proposed to reduce the constraints scale of the proposed rescheduling model. By using the proposed method, a rescheduling problem with 10 trains and 248 block sections in the 6 stations and 5 inter-station areas can be solved within 60 seconds.

The remainder of this paper is organized as follows: Detailed descriptions of the considered problem are described in Section II. The optimization model for solving the realtime rescheduling problem is introduced in Section III. In Section IV, A loop iterative architecture is proposed to reduce the constraints scale of the proposed rescheduling model.In Section V, three experiments were presented to demonstrate the validity of the proposed model and the effectiveness of the proposed algorithm. The paper ends with conclusions and suggestions for future research.

II. PROBLEM DESCRIPTION

In order to conveniently describe the process of HSR regulation, the relevant parameters and decision variables adopted in the mathematical model are listed in Table I and Table II.

TABLE I PARAMETERS AND NOTATION

N^{s}	Number of stations
n	Index of station $n = 1, 2,, N^s$
N^t	Number of trains
i, j	Index of trains, $i, j = 1, 2,, N^t$
S	Number of inter-station, $S = N^s - 1$
s	Index of inter-station $s = 1, 2,, S$
N _L	Numbers of block sections in inter-station s
m	Index of block section $m = 1.2$ N_1
L	Sneed limit of inter-section e
Ls b	Set of train speed levels $h = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix} h$ also denotes
0	Set of train speed levels, $b = \{1, 2, 3, 4, 5\}, b$ also denotes the emount of free sections required between two
	the amount of free sections required between two
DT	consecutive trains by signalling system.
RT_b	Minimum running time at speed level b in a block section
sc_n	Station capacity of station n
e	Minimum dwell time
H_{min}	Minimum departure headway
CO	Minimum arrival headway
$[D_1, D_2]$	Time window of disturbance, D_1, D_2 represent the
	start and end time respectively
$[w_1, w_2]$	Block sections with disturbance, w_1, w_2 represent the
	start and end block section respectively
O_{i}^{a}	Arrival time of train i at station n in the original timetable
O^{d}_{d}	Departure time of train i at station n in the original timetable
$\mathcal{O}_{i,n}$	A infinitesimal positive number
M	A sufficiently large positive number
111	A sufficiently large positive number $h = a_1 + a_2$
K ND	Number of disturbance block sections, $k = w_2 - w_1 + 1$
INP	Number of train speed limits generate by disturbance area,
	$N^p = 2 * k - 1$
$L_s(r)$	The set of speed limit of inter-section s, which is
	corresponded to the number of block section affected by
	disturbance. $r = 1, 2N^p$
0	index of piece-wise function, $o = 1, 2,, N^p + 2$
$d^d is_{i,n}(o)$	Departure time of train i at station n when disturbance occur
v_{a}^{i}	Maximum speed of train i arriving at block section m of
5,111	inter-station s
+i	Time of train <i>i</i> to reach maximum speed at block section m
$v_{s,m}$	of inter stations a
T_{\cdot}	Bunning time of train <i>i</i> at interstation <i>a</i> without disturbance
$\widehat{T}_{i,n}$	Running time of train <i>i</i> at inter-station <i>s</i> without disturbance
$T_{i,n}$	Running time of train i at inter-station s with disturbance
$\varepsilon_{s,m}$	Speed limit of block section m at inter-station $s, \varepsilon_{s,m} \in b$
$\varepsilon_{s,m}$	Speed limit of block section m at at inter-station s
	at disturbance area $\widehat{\varepsilon}_{s,m} \in b$
$\varepsilon_{s,m}^{i}$	Speed level of train i at block section m of inter-station s
·	$\varepsilon^i_{s\ m} \in b$
Piism	Position of train i when train i to reach the maximum speed
0,9,0,110	at block section m of inter-station s
$dt^{i,j}$	The number of block section between train i and train i
$av_{s,m}$	when train <i>i</i> to reach the maximum sneed at block section m
	when that i to reach the maximum speed at block section m
0	of inter-station s .
$Q_{i,n}$	The number of trains stop at station π when train i
	arrive at station n

A. Scenario Analyze

Fig.1 shows a time-space diagram of the timetable change caused by a disturbance in an area with 6 stations and 5 interstations. The horizontal axis represents time, the vertical axis represents space, and the train runs in the upward direction. The gray area represents the space-time area affected by the disturbance. The black solid line is the unaffected operation line, the red solid line is the disturbing operation line. The red dash line is the train speed trajectory affect by disturbance. D_1 and D_2 are the start time and end time of the disturbance respectively. w_1 and w_2 is the start block section and end section of the disturbance. According to the ATO control strategy, we first generate train trajectories for each train at each inter-station based on whether the train will be disturbed,

TABLE II DECISION VARIABLES

	$d_{i,n}$	Time of train i departure from station n
	$a_{i,n}$	Time of train i arrival at station n
	$x_{i,o}$	A binary variable: Indicating the time range of train i departure from station to disturbance area
	$\mu_n^{i,j}$	A binary variable: The order of train i and train j departure
	•	from station <i>n</i> . if train <i>i</i> is later than train <i>j</i> , $\mu_n^{i,j} = 1$,
	, i i	otherwise, $\mu_n = 0$.
	$\lambda_n^{i,j}$	A binary variable: The order of train i and train j arrive
denotes		at station n . If train i is later than train j ,
		$\lambda_n^{i,j} = 1$, otherwise, $\lambda_n^{i,j} = 0$.
	$\gamma_n^{i,j}$	A binary variable: The order of train i arrival at station n
c section		and train j departure from station n . if train i is later than train j
		$\gamma_n^{i,j} = 1$, otherwise, $\gamma_n^{i,j} = 0$.
	$\zeta_n^{i,j}$	A binary variable: When train i arrive at station n , train j
		do not leave station n. if $\lambda_n^{i,j} = 1$ and $\gamma_n^{i,j} = 1$,
9		$\zeta_n^{i,j} = 1$, otherwise $\zeta_n^{i,j} = 0$.
C	$H_{i,n}$	Departure time interval of train i and its former train at station n
the		*
timetable		
al timetable		
		Disturbance speed Disturbance timetable Position
$w_1 + 1$	Station	



Fig. 1. A rescheduling problem when setting speed limit in a section

or not; Then, based on the train speed trajectory, the research problem is to adjust the train arrival time, departure time and headway, which can reduce total delay time when a disturbance occurs.

The following assumptions are made with regard to the proposed model.

- 1) The scope and duration time of disturbance can be predicted.
- 2) The safety distance between two consecutive trains is decided by the quasi-moving block rules.
- 3) The train speed trajectories with different speed limits are calculated based on the ATO control strategy. Under the fixed control strategy, the train speed trajectory can be approximated as a fixed trajectory.
- 4) The timetable is designed by the train speed trajectories, which means that the train running time in each interstation are all corresponded to the train speed trajectories.
- 5) The upward and downward trains do not interfere with each other during operation.

B. Infrastructure

In this paper, we consider the railway network in two levels: Train rescheduling level Fig.2(a) and Train operation level Fig.2(b). The train rescheduling level considers the train order, arrival and departure time, station capacity. The train operation level is to ensure trains keep safe headway when they running in inter-station.

1) Train Rescheduling Level: In the era of high-speed railway, with the significant improvement of domestic economic level, science and technology, population and regional exchanges, double track railway has become the main form of the railway in China. The arrival and departure tracks of high-speed railway are divided into an upward direction and downward direction. The upward and downward trains do not interfere with each other during operation. In this paper, we only consider one direction real-time rescheduling problems. The station we considered has main track and siding tracks which can offer to pass and overtake functionality.(See Fig.2(a))

2) Train Safety Headway at Inter-station: The traditional safety headway time is the time interval between two following trains. The minimum headway depends on blocking time. The blocking time is the time interval in which a block section is exclusively allocated to a train and therefore blocked for other trains. And it consists of the setup time, sight and reaction time of the driver, approach time, running time, clearing time, and release time [28]. The blocking time of a block section is usually longer than the train real running time in this block section.

In CTCS-3, the real-time position and speed of the train can be transmitted to the RBC through GSM-R, the transponder can transmit track information and dynamic speed limit to the on-board system, and the track circuit can sense the number of free zones ahead for calculation of movement authorization. Then, the on-board system generates an emergency braking curve based on the existing information of the RBC and the transponder. The target of the emergency braking speed trajectory is the starting point of the block section occupied by the preceding train. The signaling system can be regarded as a quasi-moving block systems [2].

The quasi-moving block system can be implemented to enhance the system capacity. The number of block sections between two adjacent trains depends on the speed of the following train. Fig.2(b) is a schematic diagram of quasimoving block rules. When the max speed of Train 1 in block section 1 is speed level 1, the minimum number of block sections between Train 1 and Train 2 is one block section. The Train 2 maximum speed in block section 3 is speed level 2, the minimum distance between two trains is two block sections. The maximum speed of Train 3 in block section 6 is speed level 4, the distance between Train 4 and Train 3 can not be less than 4 block sections. Table III expresses the relationship between the speed level, speed and running time range of each speed level [2].

3) Train Running time at disturbance area: We also need to consider the train running time of disturbance area. Once we get the disturbance area, a series of real speed limit trajectories

 TABLE III

 Speed, Running time related to speed level

b	Speed	Running time
1	$v \leq 120 km/h$	[RT(1), M)
2	$120 < v \leq 160 km/h$	[RT(2), RT(1))
3	$160 < v \leq 200 km/h$	[RT(3), RT(2))
4	$200 < v \leq 250 km/h$	[RT(4), RT(3))
5	$250 < v \leq 300 km/h$	[RT(5), RT(4))

can be generated according to the number of block sections affected by the disturbance.

The speed limit trajectories for each block sections without disturbance is:

$$L_s = [\varepsilon_{s,1} \ \varepsilon_{s,2} \ \dots \ \dots \ \varepsilon_{s,N_{b,s}}]. \tag{1}$$

The number of block section affected by the disturbance is:

$$k = w_2 - w_1 + 1, (2)$$

The number of different speed limit trajectories we can generate is:

$$\mathbf{N}^p = 2k - 1,\tag{3}$$

The speed limit trajectories we generate based on the block section affected by the disturbance is:

Fig.3 shows the relationship between speed limit trajectories and disturbance area. The shadow area is the disturbance area. There are three scenarios here. The dark blue area and light blue area denotes that the train is running in the disturbance area before disturbance occurs and the train does not leave the disturbance area when disturbance end, respectively. The yellow area means that the train was disturbed throughout the whole disturbance area.

The train speed trajectories can be easily calculated under the constraints of \hat{L}_s . In this paper, the ATO control strategy we choose is the minimum time strategy. The trajectory of minimum running time is combined by the minimum value of corresponding positions in forwarding traction trajectory and backward braking trajectory, which are computed by the maximum traction and braking characteristics of trains under the constraints of the speed limit and gradient along a line. We discrete the lines by 10 meters, the algorithm can calculate the train speed trajectory in 0.5s.

Meanwhile, Each train speed trajectory in the disturbance area corresponds to a departure time $d_{i,n}^{dis}(o)$ and an running

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Fig. 2. The train rescheduling and train operation levels of the railway network.



Fig. 3. Different train speed limit trajectories at disturbance area



Fig. 4. Running time relate to different departure time

time $\hat{T}_{i,n}$. Fig.4 shows the relationships between running time and departure time at disturbance area. In Fig.4(a), the Y-axis is the position of a inter-station. The shadow area in Fig.4(a) is the disturbance area. The start and end disturbance block section is w_1 and w_2 . The Y-axis in Fig.4(b) is the train running time. The number of block section affected by disturbance is determined by the different departure time($d_{i,n}^{dis}(o)$) at disturbance inter-station. The train running time($\hat{T}_{i,n}(o)$) in the disturbance area is determined by the number of block sections affected by the disturbance. We can fit the train running time into a piece-wise function according to different departure times.

$$\widehat{T}_{i,n}(1) \qquad 0 \leq d_{i,n} \leq d_{i,n}^{dis}(1) \\
\widehat{T}_{i,n}(2) \qquad d_{i,n}^{dis}(1) \leq d_{i,n} \leq d_{i,n}^{dis}(2) \\
\dots \qquad \dots \\
\widehat{T}_{i,n}(k+1) \qquad d_{i,n}^{dis}(k) \leq d_{i,n} \leq d_{i,n}^{dis}(k+1) \\
\dots \qquad \dots \\
\widehat{T}_{i,n}(2k) \qquad d_{i,n}^{dis}(2k-1) \leq d_{i,n} \leq d_{i,n}^{dis}(2*k) \\
\widehat{T}_{i,n}(2k+1) \qquad d_{i,n}^{dis}(2k) \leq d_{i,n} \leq M$$
(5)

The train running time at the disturbance area can be linearized to the piece-wise function

$$\begin{split} \widehat{T}_{i,n} &= \sum_{o=1}^{2k+1} \widehat{T}_{i,n}(o) * x_{i,o} \\ d_{i,n} &\geq 0 * x_{i,1} + \sum_{o=2}^{2k+1} d_{i,n}^{dis}(o-1) * x_{i,o} \\ d_{i,n} &\leq \sum_{o=1}^{2k} d_{i,n}^{dis}(o) * x_{i,o} + M * x_{i,2k+1} \\ \sum_{o=1}^{2k+1} x_{i,o} &= 1, \end{split}$$
(6)

III. MODEL FORMULATION

A. Train Rescheduling Model

1) Objective Function: In this paper, we try to minimize the deviation between the original timetable and the rescheduled timetable at terminal station, which can reduce the total delay when disturbance occurred.

$$Obj: \min \sum_{i=1}^{N^{t}} (a_{i,N^{s}} - O_{i,N^{s}}^{a})$$
(7)

2) Departure Time and Dwell Time Constraints: In order to make passengers catch the train in time, train cannot departure station before its planned departure time(8).

$$d_{i,n} \ge O_{i,n}^d. \tag{8}$$

(9) impose a minimum dwelling time W at stations where a stop is planned, so that passengers have enough time to board or alight.

$$d_{i,n} - a_{i,n} \ge e. \tag{9}$$

Meanwhile, for trains without stop plan at station n, We regard the time for the train to pass through the station as 0.

$$d_{i,n} - a_{i,n} = 0. (10)$$

3) Running Time Constraints: In many previous studies, the train running time in an inter-station is restricted with lower and upper bounds. In this paper, the train running time at each inter-station is related to the train speed trajectory. The train speed trajectory in each inter-station is calculated based on the speed limit and the ATO control strategy. The timetable is also designed by the train speed trajectory. Therefore, the train running time in each inter-station can also be calculated according to the train speed trajectory.

$$a_{i,n+1} - d_{i,n} = T_{i,n}.$$
(11)

The train running time at disturbance inter-station can be formulated as the following equation:

$$a_{i,n+1} - d_{i,n} = T_{i,n}.$$
(12)

4) Station Capacity Constraints: We consider the station as a fixed block section. The minimum arrival headway and departure headway we set is CO and H_{min} . A binary variable $(u_n^{i,j})$ is introduced to model the order of trains at station n. If train i departure from station n later than train jdeparture from station n, $\mu_n^{i,j} = 1$.

$$\mu_n^{i,j} = \begin{cases} 1 & a_{i,n} > a_{j,n} \\ 0 & otherwise, \end{cases}$$
(13)

The order of trains at station n is decided by the following equations.

$$d_{i,n} - d_{j,n} - max(H_{min}, H_n^i(k)) \ge (\mu_n^{i,j} - 1) * M, \quad (14)$$

$$d_{i,n} - d_{j,n} + max(H_{min}, H_n^j(k)) \le \mu_n^{i,j} * M,$$
 (15)

k is the number of iterations. $H_n^i(k)$ is a parameter that is calculated based on the train speed trajectory which can ensure trains keep safe headway when they running in inter-station

We also need to consider the station capacity on the basis of the side lines owned by the station(sc_n). Only one train is allowed to occupy the same siding line at the same time. However, the number of lines in each station is limited, and the operation of all trains must meet the limit of the line capacity of the station.

When train *i* arrives at the station, it is necessary to determine how many trains stop at station $n(Q_{i,n})$.

Step 1: Judgment basis: the time when train i arrives at station n is greater than the time when train j arrives at station n, and the time when train i arrives at the station n is less than the time when train j depart from the station n

Step 2: Here, we introduce two binary variables, $\lambda_n^{i,j}$, $\gamma_n^{i,j}$ represent the following two necessary and sufficient conditions respectively

$$\lambda_n^{i,j} = \begin{cases} 1 & a_{i,n} > a_{j,n} \\ 0 & otherwise, \end{cases}$$
(16)

$$\gamma_n^{i,j} = \begin{cases} 1 & a_{i,n} < d_{j,n} \\ 0 & otherwise, \end{cases}$$
(17)

Step 3: Linearization:

$$a_{i,n} - a_{j,n} - CO \ge (\lambda_n^{i,j} - 1) * M,$$
 (18)

$$a_{i,n} - a_{j,n} + CO \le \lambda_n^{i,j} * M, \tag{19}$$

$$d_{j,n} - a_{i,n} - \tau \ge (\gamma_n^{i,j} - 1) * M,$$
 (20)

$$d_{i,n} - a_{j,n} + \tau \le \gamma_n^{i,j} * M, \tag{21}$$

Step 4: Introducing a binary variable $\zeta_n^{i,j}$, which is used to determine whether train j leaves the station n when train i arrives at the station n:

$$\zeta_n^{i,j} = \begin{cases} 1 & \gamma_n^{i,j} = 1 \text{ and } \lambda_n^{i,j} = 1 \\ 0 & otherwise, \end{cases}$$
(22)

The linearization relationship between γ, λ, ζ is as follows

$$-\gamma_n^{i,j} + \zeta_n^{i,j} \le 0, \tag{23}$$

$$-\lambda_n^{i,j} + \zeta_n^{i,j} \le 0, \tag{24}$$

$$\lambda_n^{i,j} + \gamma_n^{i,j} - \zeta_n^{i,j} \le 1, \tag{25}$$

Step 5: Calculate the number of trains that occupy the station $n(Q_{i,n})$ at the time when train *i* arrives.

$$Q_{i,n} = \sum_{j=1, j \neq i}^{N^t} \zeta_n^{i,j},$$
 (26)

We need to ensure that at least one siding line is reserved for train i,

$$Q_{i,n} \le sc_n - 1. \tag{27}$$

B. Train Operation at Inter-station

The train operation level is to ensure trains keep safe headway when they running in inter-station. We can use the train speed trajectories to judge whether the distance of two adjacent trains is satisfies the safe headway.

When train *i* to reach maximum speed at block section *m* of inter-stations *s*, if $(t_{s,m}^i > t_{s,n}^j) \cup (t_{s,m}^i < t_{s,n+1}^j)$, $P_{i,j,s,m} = n$. The number of block sections between train *i* and train *j* is:

$$dt_{s,m}^{i,j} = P_{i,j,s,m} - m - 1, (28)$$

Train j is in front of train i.

The time interval(TI) between two trains is:

$$TI_{s,m}^{i} = d_{s,m}^{i,j} * RT(\varepsilon_{s,m}^{i}).$$
⁽²⁹⁾

The quasi-moving rules request the number of block sections between two trains is more than the later train speed level. The minimum distance interval between two trains when train *i* arrive at block section *m* is the speed level $\varepsilon_{s,m}^i$. The minimum time interval(MTI) between two trains in block section *m* is:

$$MTI_{s,m}^{i} = \varepsilon_{s,m}^{i} * RT(\varepsilon_{s,m}^{i}).$$
(30)

Hence, all trains at each block section should satisfied the following equation:

$$\varphi_{s,m}^i = TI_{s,m}^i - MTI_{s,m}^i, \tag{31}$$

$$\varphi_{s,m}^i \ge 0. \tag{32}$$

And, the minimum headway of two trains at station n is:

$$\varphi_n^i = \min(\varphi_{s,m}^i). \tag{33}$$

C. Algorithm

Here, we adopt a loop iterative optimization architecture to solve this problem. First, we calculate the train speed trajectory and running time between the disturbance stations based on the disturbance area and line data(Algorithms 1). Second,the CPLEX is used to solve the train rescheduling model. Then we use train speed trajectories we got from Algorithms 1 to determine whether all trains satisfy the safety constrains(Algorithms 2). Finally, we use Algorithms 3 to adjust distance between trains to improve feasibility. Algorithms 2 and 3 are repeated in the loop. The loop iterative optimization architecture can reduce the constraints scale of the proposed rescheduling model in case of solving the job shop model on each block section. 1) Train Running Time at Disturbance Area: The first step we have to do is to calculate train trajectories and train running time at each inter-station under the constraints with different speed limits caused by disturbance.

Algorithm 1	Generate	train	running	time	
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- **Input:** The original timetable, the disturbance area and duration time, line data, train speed trajectories
- 1: Generate train speed limit trajectories based on different disturbance block section $\widehat{L_s}$
- 2: Calculate the disturbance train speed trajectories base on the disturbance train speed limit trajectory and ATO control strategy $[\hat{v}_{s,1}^i, \hat{v}_{s,2}^i, ..., \hat{v}_{s,m}^i], [\hat{t}_{s,m}^1, \hat{t}_{s,2}^i, ..., \hat{t}_{s,m}^i].$
- 3: Calculate equ.(1)-(6) based on train speed trajectories with disturbance at disturbance inter-station

2) Evaluation Function: The target of our research is to minimize the total delay and make sure that all trains should follow the quasi-moving block rules. So, the evaluation function we set is:

$$E = Obj + g * M, \tag{34}$$

where g is the number of times that the quasi-moving block rule is not satisfied. χ is a penalty factor.

Algorithm 2 Evaluation Function

- **Input:** The original timetable, the disturbance area and duration time, Train speed trajectories.
- 1: Use CPLEX to solve (7)-(27), get the rescheduling timetable.
- 2: Judge whether the rescheduling timetable meets the safe headway((28)-(33)).
- 3: Calculate the number of times that trains do not satisfy safe headway: g.
- 4: for i = 1 to N^t do

5: for
$$n = 1$$
 to N^s do

6: if
$$\varphi_n^i(k) < 0, \ g = g + 1$$

7: end for

8: end for

9: Calculate evaluation function, $E = Obj + \chi * g$

3) Headway Adjustment: In this section, we would like to introduce how to adjust the headway. First, we can use quasimoving block rules to generate $\varphi_n^i(k)$ for trains at each station. Then, we will adjust the train interval around $\varphi_n^i(k)$.

Algorithm 3 Headway adjustment
Input: Timetable calculated by Algorithm 2.
1: Generate $\varphi_n^i(k)$ based on (25)-(30).
2: for $i = 1$ to N^t do
3: for $n = 1$ to N^s do
4: $H_n^i(k+1) = H_n^i(k) - \varphi_n^i(k)$
5:
6: end for
7: end for
8: Use neighborhood search to generate new $H_n^i(k+1)$

4) The Loop Iterative Optimization Architecture for the Real-time Rescheduling model: Here, we use a loop iterative optimization architecture to solve our model. The algorithm flowchart is shown in Fig.5.



Fig. 5. The loop iterative optimization architecture

IV. CASE STUDY

In this section, three experiments are presented to verify the effectiveness of the proposed model in solving the problem of HSR timetable rescheduling. The simulation is coded in MATLAB R2018b and run on a computer with Windows 10, 2.4 GHz Intel Core i5 processor and 16 GB RAM. We adopt the commercial optimization software CPLEX 12.9 to solve the proposed model, and the YALMIP toolbox is used to connect CPLEX and MATLAB.

We test the proposed model on a timetable considering the HSR traffic. More precisely, a time horizon of one hour, planning 10 trains timetable is considered. The original timetable is plotted in Fig.6. The maximum train speed is $300 \ km/h$. The infrastructure is a rail line with 6 stations and 5 sections. The block section number of each station are given in Table IV. The length of a block section is $1360 \ m$, which is commonly used in a typical HSR design.

In Beijing-Shanghai HSR, a type of electric multiple unit (EMU) named CRH3 is widely used, and its Davis parameters are listed as follows: $c_0 = 0.7550$, $c_1 = 0.00636$,

 $c_2 = 0.000115$ (unit: N/kN) with respect to the train velocity unit Km/h. In addition, the considered CRH3 consists of 16 vehicles, and the total weight is set as 980t. The maximum accelerating rate and the minimum braking rate are set as $0.5m/s^2$ and $-0.4m/s^2$, respectively. The emergence braking deceleration is $-1m/s^2$. The train speed trajectory are calculated based on these parameters.

TABLE IV The Line Data

Fig. 6. Original Timetable

Several typical delay scenarios are considered where the disturbance area and duration time are different. Delay analysis is performed for 12 scenarios and delays with disturbance duration of 0.5 hours and 1 hour, respectively. Then the First-Scheduling-First Served(FSFS) solution and FSFS with fixed headway(FSFS+FH) solution are calculated to compare with our approach.

A. Experiment 1: Complexity Analysis

Xu et al. [2] integrated the modeling of efficient traffic management measures and the supervision of speed, braking and headway in one general job-shop model under the constraints of quasi-moving block constraints. We discussed the total numbers of variables and critical constraints of the job-shop model in detail. There are 10 trains and 248 block sections in the 6 station and 5 inter-station areas. Table V is the complexity analysis of the JS model and our model.

The job-shop model considered the arrival time for all block sections, in which the number of variables is far more than our research. This paper only considers the departure time and arrival time at each station. Meanwhile, with the increase of variables, the number of running time constraints also increase significantly. As the train speed trajectory has

TABLE V Complexity analysis

Variables or constraints	JS model	Our model
Numbers of variables	2480	96
Timetable constraints	48	48
Dwell time constraints	38	38
Running time constraints	4554	75
Speed limitation constraints	630	None
Speed trajectory constraints	30225	None
Quasi-moving block constraints	187722	None

already been calculated according to the disturbance area and duration time, we don't have speed trajectory constraints. In this paper, the quasi-moving block constraints are set to generate feedback parameters to the upper-level model to adjust headway. Even though this paper adopts an iteration architecture, the calculation time is much less than the jobshop model. In this paper, we can solve our model within 60 seconds. Fig.7 is the solution we get from our approach and Fig.8 is the mapping of speed trajectory in time and space.

Fig. 7. Final rescheduling solution of case 1h and 40 disturbance block sections with speed trajectory

B. Experiment 2: Comprehensive Influence of Speed Limit, Disturbance Area and Disturbance Duration Time

Fig.9 shows the relation between the speed limit impacted by the speed level of disturbance, disturbance area and disturbance duration time. The blue bar, yellow bar and gray bar indicate the total delay time which the disturbance area is from block section 60 to block section 80, block section 60 to block section 90, block section 60 to block section 100, respectively. The left square and right square indicate the 1 hour disturbance duration time and 0.5 hour disturbance duration time, respectively.

From the analysis in Fig.9, we can see how the delay time increases with the speed limit and the number of affected block sections. For the timetable we given in this paper, under the same duration time of disturbance, the relationship between the speed limit and delay time is that the stricter the speed limit is, the faster the delay time increases. Meanwhile, the

Fig. 8. Final rescheduling timetable of case 1h and 40 disturbance block sections

Fig. 9. Delay caused by different speed limits, impacted sections and disturbance duration time

total delay time in b = 4 is not changed, whether it's 1 hour or 0.5 hours, which means that the timetable can absorb the disturbance when it occurs between 0.5h and 1h. The delay time in b = 4 also means that this level of disturbance can rarely influence the timetable.

C. Experiment 3: Compare with Different Rescheduling Rules

In this section, we compare our approach with FSFS with the constrain of quasi-moving block rules and FSFS with fixed headway. The purple line in Fig.10 is our approach solution. The green line is the solution of FSFS with the constrain of quasi-moving block rules and the red line is FSFS with fixed headway solution. Table VII shows the disturbance indices.

We set the FSFS with fixed headway as the benchmark. It can easily find that the solution by the proposed approach in this paper is always better than the benchmark. Also, when the speed limit is 3 or 4, most value of the blue line is equal to the orange line value, which means that the speed limit of 3 and 4 can hardly change the train order.

TABLE VI Delay caused by different speed limit, impacted block sections and disturbance duration time

D	h	w_1, w_2			
D	U	60,80	60,90	60,100	
1h	1	3388	4781	6310	
	2	1603	2356	3367	
	3	403	1085	1379	
	4	86	154	222	
0.5h	1	1801	2628	3741	
	2	897	1049	1927	
	3	403	567	875	
	4	86	154	222	

TABLE VII DISTURBANCE INDICES

	b=1	b=2	b=3	b=4
60-80 1h	1	2	3	4
60-90 1h	5	6	7	8
60-100 1h	9	10	11	12
60-80 0.5h	13	14	15	16
60-90 0.5h	17	18	19	20
60-100 0.5h	21	22	23	24

Fig. 10. Analysis of the difference between FSFS/FSFS+FH solutions and our approach solution

V. CONCLUSION

In this paper, we investigated a real-time timetable rescheduling problem to restore the HSR operation from the delay caused by disturbance. A real-time rescheduling model considering the quasi-moving block rules and the train speed trajectories was constructed. The objective of the proposed model was to minimize the total delay when the disturbance occurred. First, we calculate the train speed trajectory and running time between the disturbance stations based on the disturbance area and line data(Algorithms 1). Second, we build a MILP model without consider safety constraints based to solve this model. Then we use train speed trajectories we got from first stage to determine whether all trains satisfy the safety constraints(Algorithms 2). Finally, we use Algorithms 3 to adjust distance between trains to improve feasibility.

Algorithms 2 and 3 are repeated in the loop. The loop iterative optimization architecture can reduce the constraints scale of the proposed rescheduling model in case of solving the job shop model on each block section. Three experiments were presented to demonstrate the validity of the proposed model and the effectiveness of the proposed algorithm. Compare with using CPLEX to solve the job shop model directly, the solving speed is greatly improved.

Future research directions can consider a more detailed characterization of the station. In this paper, we only consider the number of station lines but there are more infrastructures in the station. Meanwhile, the more complicated disturbance situation can also be considered in future work. The disturbance in this paper is the temporary speed limit caused by disturbance and assumes we can predict the duration time and disturbance area. The situation of we can not predict the disturbance area is an interesting research as well.

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