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Portfolio Allocation and Borrowing Constraints*

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Abstract

We explore the empirical relationship between borrowing constraints and household financial portfolio allocation. To motivate our analysis we develop a mean-variance model of portfolio allocation with three tradable asset classes defined by increasing risk, and establish a link between borrowing restrictions and financial portfolio allocation at the household level. Under non-restrictive assumptions the proportion of wealth allocated to the medium-risk asset is ambiguous. We also demonstrate that in the presence of both correlated background risk and borrowing constraints the domain of the non-binding risk-return space will be a function of background risk. We then analyse the US Survey of Consumer Finances with a view to empirically exploring the predictions of our theoretical framework. The distribution of medium-risk assets in US households is remarkably similar to that for high-risk assets, and suggests the presence of a more general 'risk puzzle', which our proxies for borrowing constraints partially explain. Our findings indicate that such constraints are inversely related to the proportion of financial wealth allocated to both high-risk and medium-risk assets, but are positively related to low-risk asset holdings. In light of our findings, further work aimed at accounting for the allocation of medium-risk assets in US households is considered expedient.

Keywords: Asset Allocation; Borrowing Constraints; Fractional Models. JEL Classification: G11; D11; D14; C35

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1 Introduction

The borrowing constraints faced by governments, firms, and households can play a significant role in shaping economic behaviour and influencing economic outcomes, both at the microeconomic and macroeconomic level. Significantly, the presence of such constraints has been shown to hold implications for the sustainability of government deficits (Wilcox 1989), the behavior of output and prices (Scheinkman and Weiss 1986), and consumption behavior (Carroll 2001). Borrowing constraints have also been shown to affect the composition of a household's financial portfolio (see, for instance: Haliassos and Hassapis 1999; Bucciol, Miniaci, and Pastorello 2017; and Guiso, Jappelli, and Terlizzese 1996).

Using data from the United States Survey of Consumer Finances (SCF) from 1995 to 2019, this contribution investigates the role that borrowing constraints play in determining the composition of US households' financial portfolios. From a public policy perspective, investigating this type of relationship is not just of academic interest. Since the early 1990s, the United States has witnessed increased efforts to provide additional borrowing opportunities for households that were traditionally constrained by credit markets (Lyons 2003). The effectiveness of such efforts takes on additional significance when considered against the backdrop of a more recent report published by the Federal Reserve Bank of New York (Hamdani et al. 2019), which found that for the fourth quarter of 2018, over sixty million US adults were unlikely to access credit at choice, and noted "...wide heterogeneity of credit conditions at the state and county levels in America" (Hamdani et al. 2019, p.12).¹ Given the high incidence of borrowing-constrained US households that persists to this day, exploring the extent to which access to borrowing affects household portfolios is thus particularly apposite.²

¹These authors also report that the number of Americans who cannot easily access loans may be twice as many as previously estimated, when people who cannot easily qualify for loans because of blemishes in their credit histories are taken into account.

²This is of course, notwithstanding the detrimental effect that the inability to borrow plays in exacerbating social inequalities, and its implications for household consumption smoothing.

To motivate our empirical analysis, we initially develop a model of portfolio choice with three tradable financial assets and non-tradable background wealth within a mean-variance expected utility maximization framework. Our model, in which households face short-selling constraints as in Jappelli, Julliard, and Pagano (2010), provides a useful starting point for our subsequent empirical analysis by establishing a link between borrowing constraints, background risk, and asset allocation at the household level. A notable feature of our theoretical framework is that we explicitly model the behavior of a medium-risk asset class, which is motivated by the existing literature's taxonomy of a household's financial assets into three distinct risk-based categories: *low-risk*; *medium-risk*; and *high-risk*.³ Whereas our theoretical model often has clear behavioural implications for the optimal allocation of the household's low-risk and high-risk assets, the implied behavior of the medium-risk asset is more nuanced. Numerical predictions of our model using plausible values of the first and second moments of our three asset categories, are indicative of lower levels of risk aversion being associated with higher shares of both medium-risk and high-risk assets. Support for this finding is found in our ensuing econometric analysis. Our numerical exercise also suggests that when modelling the impact of borrowing constraints empirically, it is appropriate to account for the possible presence of background risk.

The emphasis placed on asset shares in our theoretical model is reflected in our econometric strategy, in which the impact of borrowing constraints on our three empirically observable asset shares is directly estimated. Specifically, we use the ordered fractional model (hereafter 'OF') model of Kawasaki and Lichtenberg (2014), which enables us to impose an inherent ordering on the fraction of assets held in each risk category. This estimation approach contrasts with the majority of contributions, where the propensity to hold only a single asset type is modelled using, for example, censored or fractional regression techniques (see *inter alia*, Cardak and Wilkins 2009, Rosen and Wu 2004, and Edwards 2008). We thus consider

 $^{^{3}}$ See for instance Carroll (2002) and Hurd (2002). Our motivating theoretical model may have implications for other empirical contributions where financial portfolios are treated as being comprised of low, medium, and high-risk assets. From an empirical perspective, many asset management companies describe the portfolio structure of their investment products using this taxonomy.

the impact of borrowing constraints on household financial risk taking in a broader sense, which extends beyond accounting for the propensity of households to directly invest in stocks. In doing so we build on contributions to the 'stockholding puzzle' literature in which assets are categorised as being either 'safe' or 'risky' (see for instance: Fratantoni 2001; Haliassos and Bertaut 1995; Bertaut 1998) and explore the extent to which an avoidance of stockholding is part of a more general 'risk puzzle' that characterises household portfolio allocation. Here, exploring the determinants of medium-risk asset holding is of particular interest given that this asset class is classified as including retirement funds and life insurance policies. Holding such financial products may help to avoid shortfalls in retirement income, or shield household members' from catastrophic financial shocks – such as the inability to maintain mortgage payments due to the death of a breadwinner. In this regard, the consequences of shunning medium-risk assets may in practice be far more damaging to households than not directly holding stocks. We now turn to the literature.

2 Literature

Our contribution builds on an established empirical literature, of which notable early contributions include Jappelli (1990), Jappelli, Pischke, and Souleles (1998) and Haliassos and Michaelides (2003). Jappelli (1990) exploits information contained in the 1983 SCF to construct a liquidity constraint measure based on whether the household has been rejected for a request for credit from a financial institution. The study explores the type of households who are credit constrained and finds race, wealth, income and family composition all impact on the probability of being constrained. Jappelli, Pischke, and Souleles (1998) exploit data from both the SCF and the United States *Panel Study of Income Dynamics* (PSID) and use a two-stage two-sample least squares approach. Using information in the SCF, liquidity constraint measures are constructed; these measures are used to impute the probability of liquidity constraints in the PSID data in order to estimate Euler equations. Haliassos and Michaelides (2003) theoretically explore the household portfolio allocations in the presence of liquidity constraints. The authors study an infinite-horizon model of household portfolio allocation under liquidity constraints to explore the portfolio specialisation puzzle, and report that relatively small fixed stock market entry costs are sufficient to deter households from participating in the stock market.

More recent studies that are related to our contribution include Carroll, Holm, and Kimball (2021), Toussaint-Comeau (2021) and Insler, Compton, and Schmitt (2016). Carroll, Holm, and Kimball (2021) explore the interactions between precautionary saving and liquidity constraints in a theoretical setting, where they allow liquidity constraints and risk to influence consumption concavity. The results suggest that the introduction of risks or liquidity constraints influences the concavity of the consumption function, in turn, changing the precautionary saving motive. Moreover, their results show that the effect of introducing an additional constraint or risk on precautionary saving depends on whether it is correlated with pre-existing constraints or risks. Dependent on how an additional constraint interacts with existing constraints, it may serve to reduce the precautionary motive in earlier periods at some levels of wealth. In addition, Toussaint-Comeau (2021) aims to ascertain how liquidity constraints affect the propensity to save for a range reasons. Analysing the US SCF, the results suggest that liquidity constraints influence a range of saving decisions, and the effects of liquidity constraints are amplified by high-debt payments and high-interest-rate credit cards and serve to hinder savings. In a related study, Insler, Compton, and Schmitt (2016) explore the consumption and investment behaviour of a unique sample of borrowers that are not selected based on their credit history. The paper explores survey data relating to students at the US Naval Academy, given that these individuals have the opportunity to take a Career Starter Loan (CSL); these are significant personal loans offered at low interest rates to all students. As a result, these young adults are not bound by liquidity constraints, which are usually experienced by individuals in this age range. Overall, cognitive ability, those with prior investment experience and those who view themselves as financially literate, on average, invest more and in riskier options.

In exploring the role of credit constraints, our study also controls for the impact of (risky) background wealth on portfolio allocation.⁴ Significantly, Canner, Mankiw, and Weil (1997, p.188) note that, "Human capital – in the form of future labor earnings – is probably the most important non-traded asset".⁵ In this setting, Bertaut (1998) and Haliassos and Bertaut (1995) find that labour income risk is inversely related to holding risky assets. Related work by Fratantoni (2001) reports that in addition to labour income risk, expenditure risk associated with housing has an inverse relationship with household stock holding. Koo (1998) finds that investors will take a smaller fraction of risky assets in their portfolio when they have liquidity constraints and uninsurable income risk. Heaton and Lucas (2000) calibrate labour income risk with personal income data, finding that both labour income risk and the positive correlation between labour income and risky asset returns tend to reduce investments in risky assets. We incorporate some of these features into our motivating theoretical model.

The remainder of the paper is organised as follows. Section 3 presents the theoretical framework which links household borrowing constraints and portfolio allocation. Section 4 outlines the data analysed and Section 5 presents the empirical modelling strategy. Section 6 discusses the empirical results. Finally, Section 7 concludes. To help motivate our empirical analysis, we now turn to a simple model of household portfolio allocation.

3 Theoretical motivation

To motivate our empirical analysis, we develop a model which has a basis in the work of Markowitz (1952), and is a variant of Eichner and Wagener (2012), who analyse risktaking behaviour in a linear portfolio selection problem with non-tradable background wealth.

⁴As argued below, the exogenous and directly uninsurable nature of background risk amounts to the assumption of incomplete markets. The presence of such risk can lead a decision-maker to "reduce exposure to another risk even if the two risks are statistically independent" (Kimball 1992), a concept described as 'temperance' (Kimball 1991). For related theoretical contributions see Pratt and Zeckhauser (1987), Kimball (1991), Gollier and Pratt (1996), and Heaton and Lucas (2000)

⁵Canner, Mankiw, and Weil (1997) explore portfolio allocation among cash, bonds, and stocks, with a view to reconciling the household stockholding puzzle in light of the financial advice received by households.

Assume that households face a linear portfolio choice problem such that the final wealth of a household is given by

$$W = q_L L + q_M M + q_H H + \lambda B, \tag{1}$$

where L, M and H are random variables representing the return from allocating all of wealth to these tradable assets, which are defined as being low-risk (L), medium-risk (M), or highrisk (H).⁶ The three tradable assets are assumed to be increasing in risk and return, and capture the structure of US households' financial portfolios as defined in the existing literature, see for example, Carroll (2002) and Hurd (2002), where financial assets are classified as one of three distinct risk-based categories. The weights that households choose to allocate to these assets are denoted q_i , $i \in \{L, M, H\}$, such that $\Sigma q_i = 1$.⁷ B denotes non-tradable background wealth which can be viewed as the 'stream of household income arising due to labour or entrepreneurial activity, human capital, and government transfers' (Eichner and Wagener 2012, p.424).

An exogenous parameter $\lambda \geq 0$ is used to scale background wealth B up or down; the non-negativity constraint on background wealth reflects the fact that households cannot easily borrow against their human capital in the way they can against financial assets.⁸ Our model is also characterised by restrictions on short-selling which lead to the portfolio weights of the household's tradable assets being subject to non-negativity constraints. This entails that the individual asset shares are characterised by $0 \leq q_L, q_M, q_H \leq 1$. Although short-selling is potentially attractive for households – it markedly increases the total return of a portfolio albeit at the cost of a greatly increased risk – the presence of constraints on

⁶Although 'safe' instruments such as government bonds may be riskless in nominal terms, inflation means that their return may be uncertain in real terms.

⁷Eichner and Wagener (2012) consider the case of two tradable assets whose weights sum to unity. We extend this approach to the case of three tradable assets. Unlike our contribution, Eichner and Wagener (2012) do not consider the impact of short-sale constraints on the tradable assets, which in the context of our contribution, are an important model feature.

⁸Whilst this admits a natural, albeit simplistic, interpretation as a borrowing constraint, Kaplow (1994, p.1505) notes that: 'Individuals' ability to borrow against human capital is limited, but some such borrowing occurs'. For instance, residential mortgages are based in part on predictions of future earnings; further, in the United States, the largest student loans are available to law and medical school students, due to the high and stable expected future income stream associated with jobs in the legal and medical professions.

this form of borrowing is a highly plausible assumption in the context of household finance.⁹ Households are unlike institutional investors, in that the former cannot fund stock market investments through borrowing from financial intermediaries such as banks. Here, Perraudin and Sorensen (2000) suggest that in the context of US households, it is highly unlikely that the majority of households are characterised by optimal, unconstrained portfolios, which involve short-selling stocks; similarly, Poterba (2002, p.141) notes that the assumption of short-selling is "…probably not appropriate for a large set of households".¹⁰

The expected return on total wealth, W, is given by

$$\mu_W = q_L \mu_L + q_M \mu_M + q_H \mu_H + \lambda \mu_B \tag{2}$$

where μ_J is the mean of variable $J \in \{W, L, M, H, B\}$. The corresponding variance, v_W , is given by

$$v_{W} = \begin{bmatrix} q_{L}^{2}v_{L} + q_{M}^{2}v_{M} + q_{H}^{2}v_{H} + \lambda^{2}v_{B} \\ +2q_{L}q_{M}Cov(L,M) + 2q_{L}q_{H}Cov(L,H) + 2q_{M}q_{H}Cov(M,H) \\ +2q_{L}\lambda Cov(L,B) + 2q_{M}\lambda Cov(M,B) + 2q_{H}\lambda Cov(H,B) \end{bmatrix}.$$
 (3)

In (3), the correlation coefficient between any two assets Y and Z is given by $\rho_{YZ} = \frac{\sigma_{YZ}}{\sigma_Y \sigma_Z} \Leftrightarrow \rho_{YZ} \sigma_Y \sigma_Z = \sigma_{YZ}$, such that $\sqrt{v_i} = \sigma_i$, i = Y, Z denotes the standard deviation of a variable, $\sigma_{YZ} = Cov(Y, Z)$ is the covariance, and $-1 \leq \rho_{YZ} \leq 1$. Given space constaints, and to facilitate a more focussed analysis of the interplay between background risk and borrowing

⁹Restricting the portfolio selection by only having positive weights of the assets limits the amounts of possible portfolios and introduces complexity that cannot be handled by closed-form mathematics, and requires computation of the corresponding Kuhn-Tucker conditions.

¹⁰In the context of Italian households, Jappelli, Julliard, and Pagano (2010) also suggest that whereas institutional investors are able to borrow at low-risk to invest in high-risk assets with a greater expected return, households cannot act the same way, for example by funding stock market investments through borrowing from financial intermediaries such as banks. Using household data, the authors show that explicitly considering the no short-selling constraint helps significantly in reconciling individual portfolio choices with the efficient ones implied by financial theory (Markowitz 1952). Borrowing constraints which prevent households from short-selling are also explored in Bucciol, Miniaci, and Pastorello (2017), who also develop a model of household portfolio choice with borrowing restrictions in a mean-variance expected utility maximization framework.

constraints, our subsequent analysis permits background wealth B to be correlated with L, M, and H, but we assume that the tradable assets are not correlated with each other. Although this does not detract from the main thrust of our analysis, it requires setting $\rho_{LM} = \rho_{LH} = \rho_{MH} = 0.$

Now suppose the household specifies μ_W ; by the efficient set theorem (Markowitz 1952) it sets out to minimize the variance of wealth, v_W . Appendix A derives the efficient frontier and asset share allocations consistent with portfolio variance minimization, from which we make the following claim:

Claim 1 In the absence of short-sale constraints, the optimal shares of the tradable assets are related to μ_W on the efficient frontier according to

$$\frac{\partial q_L^*}{\partial \mu_W} < 0; \ \frac{\partial q_M^*}{\partial \mu_W} \stackrel{\leq}{=} 0; \ \frac{\partial q_H^*}{\partial \mu_W} > 0, \tag{4}$$

where q_L^* , q_M^* , and q_H^* denote the variance minimizing shares for low, medium and high-risk assets.

Proof. See Appendix A. \blacksquare

That is, on the efficient frontier, a higher expected portfolio return μ_W is associated with an increase (decrease) in the share of assets in the high-risk (low-risk) category. Significantly, equation (4) shows that for the medium-risk asset, the direction of this effect on q_M^* is ambiguous. From an empirical perspective, this finding is of interest: even in a simple tractable framework like ours, the conditions governing the behaviour of the medium risk asset may be complex, even when the efficient frontier is unconstrained. Specifically, we find that,

$$\frac{\partial q_M^*}{\partial \mu_W} \stackrel{\leq}{=} 0 \quad iff \quad \frac{\upsilon_L}{\upsilon_H} \stackrel{\geq}{=} \frac{\left(\mu_L - \mu_M\right)^2}{\left[\left(\mu_L - \mu_H\right)\left(\mu_M - \mu_H\right) - \left(\mu_M - \mu_H\right)^2\right]}.$$
(5)

It is straightforward to verify that the condition in expression (5) also holds in the presence of independent background risk when the tradable assets are not correlated with each other. The results in Appendix A also imply that independent background risk has no impact on the composition of the minimum variance portfolio on the efficient frontier, a finding consistent with Lusk and Coble (2008) and Jiang, Ma, and An (2010).¹¹ This finding is overturned when the assumption of independent background risk is relaxed.¹² Moreover, Appendix A also demonstrates that under correlated background risk the q_i^* , i = L, M, H are all linear functions of background risk, σ_B . This finding enables us to make the following claim:

Claim 2 In the presence of both correlated background risk and short-sale constraints $q_L^* \ge 0$, $q_M^* \ge 0$, and $q_H^* \ge 0$, the domain of the non-binding risk-return space will also be a function of background risk.

Proof. See Appendix B.

As a prelude to our econometric analysis, and to explore the implications of the above results for US households, it is informative to conduct simple numerical experiments. This approach aligns with Jappelli, Julliard, and Pagano (2010), who construct an efficient frontier for Italian households both with and without short-sale constraints based on historical returns data, although unlike our contribution, the impact of background risk is not considered.¹³ We also note here that as our low-risk category includes wealth held in savings accounts, the short-sale constraint in this category could be interpreted as a more conventional borrowing constraint in which the household is not permitted to hold negative

¹¹Jiang, Ma, and An (2010) also explore the impact of background risk on the efficient frontier, and similarly find that an increase in background risk will shift the frontier to the right. Unlike our contribution, however, these authors do not focus on the implications of shifts in the frontier on household welfare or the optimal composition of a household's portfolio given mean-variance preferences.

¹²As is standard in financial portfolio theory our interest is with the part of the efficient frontier in the $\mu_W - v_W$ space such that $\mu_W^* \ge \mu_W^g$, where μ_W^g denotes the value of μ_W at the global minimum portfolio and μ_W^* is a value of μ_W on the efficient fronter, as captured by expression (A.6) in Appendix A. A discussion of μ_W^g is provided in Appendix C. An important corollary of the results in expressions (4) and (5) is that whilst in an unconstrained setting, increasing risk aversion is associated with a decrease (increase) in the share of assets in the high-risk (low-risk) category, for the medium-risk asset share q_M , the effect will be ambiguous. Under certain conditions, risk-loving households may hold a smaller share of their wealth in medium risk assets relative to risk-averse households.

¹³Although our empirical examples are highly stylised and tied to an explicit formal model, similarities in the profiles of the efficient frontiers for US and Italian households can be observed. See Figure 2 in Jappelli, Julliard, and Pagano (2010), and the associated discussion.

balances.¹⁴

Figure 1 plots efficiency frontiers in the $\mu_W - \sigma_W$ space, and demonstrates how the presence of correlated background risk can exacerbate the impact of short-sale conditions by further restricting the domain of the non-binding return-risk space available to households.¹⁵ For the tradable assets, we choose parameterisations based on historical data for the annualised returns and volatility of the S&P 500 ('high-risk'), the Bloomberg Barclays Municipal Bond Index ('medium-risk'), and average effective fed funds rate ('low-risk'), for the years 1995-2019, which matches the sample period used in our empirical analysis.¹⁶ Although these proxies are arguably crude, they tentatively capture the differences in the nature of returns and volatilities associated with the SCF asset categories, which are discussed in more detail in Section 3. Our parameterisations are: $\mu_L = 2.36$, $\sigma_L = 2.25$; $\mu_M = 5.43$, $\sigma_M = 4.96$; and $\mu_H = 11.89, \sigma_H = 18.44$. For background wealth B we set $\mu_B = 4$ and $\lambda = 1$, and consider two scenarios: one without background risk ($\sigma_H = 0$); and another where background risk is assumed to be positively correlated with the high-risk asset ($\sigma_B = 5, \rho_{LB} = \rho_{MB} = 0$, $\rho_{HB} = 0.4$). Using expression (5) our parameterisations imply that $\partial q_M^* / \partial \mu_W > 0$, which entails that lower levels of risk aversion (and hence a higher μ_W) will be associated with a higher share of medium-risk assets.

To explore how background risk may affect the composition of the household's optimal portfolio, we assume that household preferences are given by the mean-variance utility function

$$G(\mu_W, \upsilon_W) = \mu_W - \gamma \upsilon_W, \tag{6}$$

 $^{^{14}}$ In practice, prohibitively high overdraft charges may act as an impediment to sustaining a negative balance for a considerable period of time.

¹⁵We stress here that in the presence of a short-sale constraint, the non-binding return-risk space available to households satisfying $q_L^* \ge 0$, $q_M^* \ge 0$, and $q_H^* \ge 0$ may be limited, even when background risk is independent of the tradable assets. Under independent background risk, neither the optimal asset allocations associated with different points on the efficient frontier nor the regions of the non-binding risk-return space are functions of background risk.

¹⁶Interest rates in US checking and savings accounts are closely tied to the fed funds rate, which is set by the Federal Open Market Committee (FOMC). Using expression (5) our parameterisations imply that $\partial q_M^*/\partial \mu_W > 0$, which entails that lower levels of risk aversion (and hence a higher μ_W) will be associated with a higher share of medium-risk assets, as well as high-risk assets.

where $\gamma > 0$ is a risk aversion parameter such that greater values imply lower levels of risk tolerance, and set $\gamma = 0.5$.^{17,18} Curves corresponding to the case of no background risk are labelled using a '*' (orange / red lines), whereas under positively correlated background risk, '**' is used (light blue / dark blue lines). The dashed lines show the unconstrained efficient frontiers under each scenario, and the thicker solid lines in red / dark blue capture the impact of the corresponding binding constraints on the respective frontiers.¹⁹ Table 1 reports the corresponding optimal asset allocations and the effects on household welfare.

In the absence of background risk ($\sigma_B = 0$), the tangency point of household preferences with the efficient frontier corresponds to an optimal portfolio allocation spread across all three assets. Introducing background risk has the effect of shifting the efficient frontier to the right,²⁰ and when *B* is positively correlated with *H* ($\rho_{HB} = 0.4$), holdings of high-risk assets fall to zero and holdings of *L* and *M* rise. Of special interest here is the impact of correlated background risk on the model's binding constraints. Figure 1 shows that in the absence of background risk, the most risk averse households are not subject to binding constraints: for instance, at the global minimum, the optimal asset allocation is $q_L^* = 0.819$, $q_M^* = 0.169$, and $q_H^* = 0.012$. As in Jappelli, Julliard, and Pagano (2010), risk-loving households suffer the greatest welfare loss due to the presence of binding constraints (solid red line),²¹ and riskaverse households are unaffected. This finding contrasts with the case of $\rho_{HB} = 0.4$, where unlike in Jappelli, Julliard, and Pagano (2010), the introduction of correlated background

¹⁷Our choice of mean-variance utility is commonplace in standard financial economics texts (Bailey 2005), and is appealing due to its analytical tractability. In our empirical analysis, we use measures of risk attitudes to capture a household's attitude towards risk. We stress here that these measures only proxy for risk preferences, and may not be tightly linked to the shape of a household's utility function.

¹⁸Preference functions for which households are decreasingly absolute risk averse (DARA) do not lead to tractable results, and require solving by linear quadratic programming techniques. The expected log utility function given by $E(U(W)) = \ln \mu_W - v_W/2\mu_W^2$, where W denotes wealth, would fall into this class of function (see Pulley 1993, eqn (2), p.686 and the corresponding discussion for further details).

¹⁹The extent to which a change in σ_B impacts on the value of μ_W^* at the point of tangency is provided in Appendix D, as is the corresponding impact of a change in risk-aversion γ . The Kuhn-Tucker conditions for our model are set out in Appendix E.

²⁰This effect arises irrespective of the nature of correlation between background risk and the tradable assets.

²¹Although not shown in Figure 1, the constraint becomes binding at approximately $\gamma = 0.0761$. Under background risk, the constraint becomes binding at approximately $\gamma = 0.0688$.

risk entails that the most risk averse households will also suffer welfare loss due to binding constraints coming into effect. Under positive correlation of the form $\rho_{HB} = 0.4$ the point of tangency between the unconstrained efficient frontier and household preferences shown in Figure 1 is infeasible due to $q_H^* < 0$, and the optimal allocation associated with the constrained frontier is associated with zero holdings of high-risk assets.²² Households situated at *both* ends of the risk aversion spectrum are observed to be impacted by binding constraints.

Table 1 also reports the results under our model parameterizations of scenarios that are not depicted in Figure 1, namely the cases of independent background risk, and, of negative correlation, where $\rho_{HB} = -0.4$. In the former case, the optimal portfolio allocation is identical to that under no background risk, albeit household welfare is markedly lower. Significantly, Table 1 shows that the presence of background risk reduces household welfare relative to the zero background risk case. Under $\rho_{HB} = -0.4$, households are characterised by riskier portfolios in that a much higher (lower) proportion of wealth is held in the form of high-risk (low-risk) assets.

More generally, the findings reported above are consistent with Arrondel and Calvo-Pardo (2002) who find that "...the sign and magnitude of the correlation may exacerbate or counterbalance the optimal portfolio response to the introduction of a background risk."Notwithstanding our focus on the behaviour of the medium-risk asset, the confluence of borrowing constraints with background risk holds non-negligible ramifications for portfolio allocation in US households. As US households are subject to short-sale constraints in practice (Perraudin and Sorensen 2000, Poterba 2002), this suggests that when econometrically modelling the determinants of household portfolio selection, controlling for the potential effects of background risk is advisable. In this regard, our results align with a wider literature (e.g., Heaton and Lucas 2000) which predicts that the dual presence of borrowing constraints and background risk will affect household portfolio selection. We now turn to our empirical contribution.

²²Under independent background risk an increase in σ_B will, *ceteris paribus*, reduce household welfare. The composition of the household's optimal portfolio will be identical to the no background risk case.

4 Data

The US SCF has been used extensively in the existing literature relating to household finances, see for example, Haliassos and Bertaut (1995), Browning and Lusardi (1996), Bertaut (1998), Spaenjers and Spira (2015), and Fulford (2015), amongst many others. The SCF is a repeated cross-sectional triennial survey of US households conducted on behalf of the Federal Reserve System's Board of Governors. We exploit data from the 1995 – 2019 waves of the SCF.²³

The SCF contains detailed information on household assets and liabilities, in addition to a range of income, socioeconomic and demographic characteristics. It is notable for containing household level information on a wide range of financial assets including: transaction accounts; certificates of deposit; bonds; stocks; pooled investment funds; and retirement funds amongst others. This feature permits us to explore the household's portfolio allocation decisions. To do so, we allocate assets into three distinct risk-based categories: high-risk; medium-risk; and low-risk. This taxonomy corresponds closely to those used in the contributions of Carroll (2002) and Hurd (2002). In line with the existing literature, we construct a measure of high-risk assets that comprises of both direct and indirect risky asset holding, through for example, retirement accounts. Low-risk assets include, for example, checking accounts and certificates of deposit, while medium-risk assets include for example, non-risky pension accounts, tax mutual funds and tax free bonds. Full descriptions of these asset classes are presented in Table 2.²⁴ As is standard in the empirical literature, we explore the ratio of these asset classes to total financial wealth in the household, that is the proportion

²³Given the high rate of non-response associated with micro-data relating to wealth information, the SCF provides five imputations which give a distribution of outcomes. These multiple imputations increase the efficiency of the estimation, whilst also providing uncertainty surrounding the imputed values. Our summary statistics are based on taking the average of the corresponding five implicates for each cross-section. However, the econometric estimates are based on implementing the repeated imputation inference (RII) method, as described by Little and Rubin (1987), Rubin (1987) and Montalto and Sung (1996). The RII uses the average of the corresponding standard errors accordingly.

²⁴We have also conducted the analysis where high-risk assets are defined to be those directly held by the household. Generally, the results are consistent with those presented in this paper.

of financial wealth allocated to each asset class.

In our empirical specifications, we control for a wide range of demographic and socioeconomic characteristics which are based on the existing literature. These include: age; gender; race; children present in the household; education and labour market status. We also control for measures capturing risk attitudes, income (and wider economic) expectations in the coming years, home ownership and the household's saving horizon. A full description of these variables and corresponding summary statistics are provided in Table 2.

Moreover, in line with our theoretical motivation, we control for a set of variables which capture the household's exposure to a range of *background risks*. These draw on the existing literature and include risks associated with income, health, expenditure, business ownership, wealth shocks associated with future inheritance and whether there are a multiple earners in the household. Income risk is captured by two variables: (*i*) if the household has a reasonable idea of what the household's income will be in the future; and (*ii*) if the household's previous annual income was above or below what was expected.²⁵ Health risk is captured by the insurance cover of the household, whilst expenditure risk is captured by whether major financial expenses are expected in the next five years. A full description of these variables is also given in Table 2.

4.1 Measuring Borrowing Constraints

The key aim of this paper is to explore the impact of borrowing constraints on portfolio allocation decisions. In the literature more broadly, the term liquidity constrained is used to refer to households who are credit or borrowing constrained, see for example, Guiso and Paiella (2008), Le Blanc, Porpiglia, Zhu, and Ziegelmeyer (2015) and Campbell and Hercowitz (2019). Generally, households are defined to be borrowing constrained if they are unable to borrow the amount they asked for or they have been charged a higher interest rate

²⁵These measures are similar to those used by Fulford (2015). However, we allow a differential impact of being above or below a household's normal income by entering positive and negative values as separate variables.

than the 'market clearing' interest rate.

Previous studies have exploited a range of constraint measures to test the validity of the Life-Cycle/Permanent Income Hypothesis models (see for example, Hall and Mishkin 1982; Zeldes 1989; and Hayashi 1985). A major limitation of these studies is that constrained households are identified via indirect evidence, such as the household's wealth to income ratio or the household's saving rate. Subsequent studies have used more direct measures of such constraints, which are revealed directly from survey questions.

The SCF includes information which enables us to directly observe constrained households, in line with Jappelli (1990). A full description of these measures is given in Table 3, whilst the corresponding summary statistics are presented in Table 4. We use information on whether households have been turned down for credit at a financial institution or if they did not apply because they believed they would be turned down for credit in order to define a range of borrowing constraint measures. We incorporate information based on whether the household did not apply for credit due to a belief they would be turned down, in addition to actually being turned down for credit, since, as argued in Jappelli (1990), if there is a cost to apply, this subjective likelihood of being turned down may not apply initially due to the likelihood they will be refused credit.

Initially, we exploit information on whether the household chose not to apply for credit based on a belief that it might be rejected (*Constraint 1*). Specifically, this form of constraint is associated with the response to the question: *"Was there any time in the past 12 months that you thought of applying for credit at a particular place, but changed your mind because you thought you might be turned down?"* We construct a binary variable which assumes a value of 1 if the response was "yes", and 0 if "no". Table 4 shows that 14% of the households in our sample did not apply for credit in the past 12 months for this reason.

Our second constraint measure (*Constraint 2*) is based on the question: "In the past twelve months, has a particular lender or creditor turned down any request you (or your husband/wife/partner) made for credit, or not given you as much credit as you applied for?"²⁶ Respondents can respond in three ways: Yes, turned down; Yes, not as much credit; or No. A household is defined to be constrained if they respond either "turned down" or did not receive as much credit as applied for. Households are not constrained if they respond "no". In our sample, 25% are defined to be constrained according to this measure.

Our next measure augments the above with information contained in the following question: "Were you later able to obtain the full amount you requested by reapplying to the same institution or by applying elsewhere?" If a household was subsequently able to obtain the full amount applied for they are defined as not being constrained in our third borrowing constraint measure (*Constraint 3*). As presented in Table 4, once we account for households who were able to obtain credit by reapplying (either at the same institution or else where), the proportion of constrained households falls to 16%, as compared to *Constraint 2*.

Finally, in the spirit of Jappelli (1990) we combine the above questions relating to being turned down and not applying due to a belief of being turned down. Specifically, we construct binary variables which capture if a household has not obtained credit due to either not applying for credit or if they have been unsuccessful in their credit application (*Constraints 4 and 5*). The proportions of households defined to be constrained when we account for both being discouraged and turned down are 29% and 20% for Constraints 4 and 5, respectively.

We restrict our analysis to the sample of households who have made, or thought about making, a credit application in recent years. We argue that, if a household has not applied for credit, then we do not observe whether they are constrained or not. This results in a sample of 27,618 households across the time period 1995-2019.

 $^{^{26}}$ The question changed the time period from 5 years to 12 months in the 2016 and 2019 waves of the survey. If we restrict our analysis to the consistent questions (i.e. between 1995 and 2013) we find no differences in the results.

4.2 Asset Share Distributions

Figure 2 presents the distributions of the dependent variables considered in our analysis, that is, the ratios of high, medium and low-risk assets to total financial wealth. The distributions are clearly non-normal suggesting that linear regression is not an appropriate modelling approach. It is also apparent that there are spikes at various parts of the distribution, particularly at 0 and 1. Considering the distributions shown in Panel A in Figure 2, it is clear that approximately 65% of households hold some sort of high-risk asset either directly or indirectly. The summary statistics presented in Table 4 reveal that households hold around 33% of their financial wealth in high-risk assets, either directly or indirectly, whilst around 41% of financial assets are held in safe assets. Furthermore, the majority of households hold some form of safe asset (which is not surprising as this asset category includes checking and current accounts), with only 0.8% of households not holding any safe assets.

5 Empirical Strategy

For the purposes of comparison to the existing literature, we initially consider univariate models of portfolio allocation, where, in line with the existing literature, a single asset (high-risk) is the focus. In these specifications, the share of high-risk assets in a household portfolio is modelled using a univariate fractional response (FRM) (Papke and Wooldridge 1996) framework.²⁷ For brevity, we do not present the formulations of these standard econometric techniques. In our setting, given the different underlying assumptions of the FRM approach, consideration of the FRM serves as a robustness check. We then go on to implement the OF model, which simultaneously models the three asset categories which have an inherent risk ordering. This approach is outlined below.

 $^{^{27}}$ We also use Tobit analysis (Tobin 1958) to model high-risk asset shares in line with the existing literature. The coefficients and associated marginal effects are discussed in Appendix F.1. Generally these results are in line with those for the FRM approach.

5.1 Ordered fractional model

We are concerned with modeling the household's financial portfolio, where a household can allocate their financial assets to three distinct categories based on their risk exposure, namely, high, medium and low. Our interest lies in modelling the share of the household's financial portfolio allocated to each type of financial asset and the partial effects of observed covariates on these. Consequently, we employ the ordered fractional model of Kawasaki and Lichtenberg (2014). In our set up, we label the shares j = 0, 1, 2, such that they decrease with risk as j increases, that is, j = 0 represents the high-risk category, j = 1 represents medium-risk assets, and j = 2 represents the low-risk assets.²⁸

In order to motivate the OF model, it is intuitive to relate it to the standard ordered probit (OP) model (Greene 2012). Here, agents are assumed to have an underlying latent variable, y_i^* , related to observed characteristics with unknown weights (β) , and a random, normally distributed error term u_i , such that

$$y_i^* = \mathbf{x}_i' \beta + u_i. \tag{7}$$

Denoting the total number of outcomes available as J (here J = 3, such that j = 0, 1, 2), the outcome j that household i chooses will depend on the relationship between y_i^* and the inherent boundary parameters in the OP model (Greene and Hensher 2010), μ , according

²⁸It would be possible to model the effects of drivers on each of the s_{ij} shares as a linear system, such that $s_{ij} = \mathbf{x}'_i \beta_j + u_{ij}$, where \mathbf{x}_i is a matrix of (household) covariates and u_{ij} is a random error. This would effectively be an extension of the linear probability model, the shortcomings of which are well-known (Gujarati 1995; Gujarati 1995; Gujarati 1995). Indeed, such an approach would not be ideal: it would not guarantee that $0 \leq E(s_{ij} | \mathbf{x}_i) = \mathbf{x}'_i \beta_j \leq 1$; would have repercussions on ensuring the adding-up constraint that $\sum_j = E(s_{ij} | \mathbf{x}_i) \equiv 1$; it would also be unable to handle boundary observations of 0 or 1 shares; and would likely embody heteroskedasticity in u_{ij} . We have also explored the use of a multi-nominal fractional response model, see for example, Becker (2014). In this setting, the inherent risk ordering of asset classes is not accounted for in the estimation strategy, instead the multi-nominal probit model is used as the foundation of the estimation strategy. We obtain similar results to those presented when we adopt this alternative modelling strategy.

 to

$$y_{i} = \begin{cases} 0 & if \qquad y_{it}^{*} < \mu_{0} \\ 1 & if \qquad \mu_{0} \le y_{it}^{*} < \mu_{1}. \\ 2 & if \qquad y_{it}^{*} \ge \mu_{1} \end{cases}$$
(8)

Household *i*'s corresponding likelihood when J = 3 is

$$\ell_{i} = \prod_{j=0}^{J-1=2} \left(\Phi \left[\mu_{0} - \mathbf{x}_{i}^{\prime} \beta \right] \right)^{d_{i0}} \left(\left[\Phi \left(\mu_{1} - \mathbf{x}_{i}^{\prime} \beta \right) - \Phi \left(\mu_{0} - \mathbf{x}_{i}^{\prime} \beta \right) \right] \right)^{d_{i1}} \left(1 - \Phi \left(\mu_{1} - \mathbf{x}_{i}^{\prime} \beta \right) \right)^{d_{2}}$$
(9)

where $\Phi(.)$ is the standard normal cumulative distribution function. The indicator function d_{ij} is such that $d_{ij} = 1$ ($y_j = j$), where unlike an OF model, the household can be in only one of the j = 0, 1, 2 outcomes. As such, equations (7) and (9) are not sufficient to model ordered fractional data.

What is required is the effect of covariates on

$$E\left(s_{ij} \left| \mathbf{x}_{i}\right), j = 0, 1, 2\right.$$

$$(10)$$

where E denotes the expected value of the term in parentheses. Replacing $d_{ij} = 1$ $(y_j = j)$ with s_{ij} yields the likelihood function as in Kawasaki and Lichtenberg (2014):

$$\ell_{i} = \prod_{j} \left(\Phi \left[\mu_{0} - \mathbf{x}_{i}^{\prime} \beta \right] \right)^{s_{i,j=0}} \left(\left[\Phi \left(\mu_{1} - \mathbf{x}_{i}^{\prime} \beta \right) - \Phi \left(\mu_{0} - \mathbf{x}_{i}^{\prime} \beta \right) \right] \right)^{s_{i,j=1}} \left(1 - \Phi \left(\mu_{1} - \mathbf{x}_{i}^{\prime} \beta \right) \right)^{s_{i,j=2}}.$$
(11)

Note that here the OF model is consistent with the inherent ordering, in risk, of the asset bundles (and not of the value of the shares themselves). This allocation equation is now characterised by

$$E\left(s_{i,j=0} | \mathbf{x}_{i}\right) = \Phi\left(\mu_{0} - \mathbf{x}_{i}^{\prime}\beta\right)$$

$$E\left(s_{i,j=1} | \mathbf{x}_{i}\right) = \Phi\left(\mu_{1} - \mathbf{x}_{i}^{\prime}\beta\right) - \Phi\left(\mu_{0} - \mathbf{x}_{i}^{\prime}\beta\right)$$

$$E\left(s_{i,j=2} | \mathbf{x}_{i}\right) = 1 - \Phi\left(\mu_{1} - \mathbf{x}_{i}^{\prime}\beta\right)$$
(12)

which by construction satisfy $0 \leq E(s_{i,j} | \mathbf{x}_i) \leq 1$, and are consistent with the risk ordering of the *j* asset bundles in the household's portfolio. Moreover, boundary observations of zero or unity shares are also easily dealt with here. Such a model provides a means of modelling the household's portfolio allocations, which takes into account the inherent risk ordering of the asset categories. This empirical strategy mirrors the theoretical setup presented in Section 3.

6 Results

Initially, prior to discussing the results relating to the OF model, we consider the univariate models of high-risk asset holding. As reported earlier, high-risk assets are defined to include both direct and indirect risky asset holding. In these specifications, we model the share of high-risk assets, relative to the household's total financial assets, using a univariate specification. We then move on to discuss the results relating to the OF model.

6.1 Univariate Model

Table 5 reports the coefficients relating to the FRM model of the proportion of high risk assets, whilst, Table 6 presents the associated marginal effects. The results are generally consistent with the existing literature, see for example, Rosen and Wu 2004, Fratantoni 2001, Haliassos and Bertaut 1995, and Bertaut 1998. For example, white headed households report holding a higher proportion of high-risk assets in their portfolio relative to non-white

headed households. In addition, the share of high-risk assets is increasing in the level of education. For example, as presented in Table 6, compared to having below high school education, possessing a college degree is associated with holding approximately 13% more high-risk assets, holding all other factors constant. Similarly, having children present in the household is inversely related to the share of high-risk assets, whilst, households with retired heads and those who are home owners, hold a higher proportion of high-risk assets, compared to those who report being out of the labour force. Turning our attention to the variables which capture the household's financial position reveal the expected results. Both household income and household net wealth are positively related to the share of high-risk assets. For example, according to Table 6, a 1% increase in household income is associated with an increase in the share of high-risk assets of 0.04%.

The results relating to the head of household's risk attitudes also show the expected results. Having a household head with a higher value of the risk attitudes index is associated with a higher proportion of high-risk assets in the financial portfolio. In addition, households with heads who report their saving horizons to be further in the future and those who have a positive economic outlook both hold a higher proportion of high-risk assets.

We now explore the impact of the variables which capture the household's exposure to background risk. We find that the head of household knowing their income for the following year has a positive association with the proportion of high-risk assets held. In addition, we find that households who have no health insurance cover hold a lower level of high-risk assets, arguably capturing the household's potential exposure to health cost uncertainty. Moreover, the households who expect a major financial expense in the next five years or those who anticipate a substantial inheritance, hold a higher proportion of high-risk assets.

Turning our attention to the relationship between our borrowing constraint measures and risky asset holdings reveals that, across all borrowing constraint measures, there exists an inverse relationship between being constrained and the share of high-risk assets held. For example, considering the results presented in Table 6, *Constraint 5*, that is believing that they would be turned down or were turned down and did not reapply for credit is associated with holding a lower level of high-risk assets in the portfolio. Specifically, being constrained by this measure is associated with a reduction in the share of risky assets by 1.6%. The magnitudes across all five measures of borrowing constraints are similar, irrespective of how we define being borrowing constrained or the modelling approach adopted.

6.2 Ordered fractional model

We now discuss the results relating to the OF model, which jointly models the household's financial portfolio, that is, the proportion of financial wealth allocated to high, medium and low-risk asset categories. Table 7 presents the coefficients and the marginal effects for the OF model. Recall here that a negative coefficient is interpreted as being positively related to holding a higher proportion of high-risk asset categories, whilst the marginal effects are interpreted in the usual way.

It is clear from Table 7 that the impact of the standard variables on the household's portfolio allocations are generally in accordance with the univariate models. For example, white headed households hold a higher proportion of high-risk assets, whilst having children in the household induces households to hold safer portfolios. Specifically, relative to being a non-white headed household, white headed households hold 4.5% and 0.5% higher high-risk and medium-risk asset shares, respectively, whilst having a white head of household is associated with holding a 5.0% lower low-risk asset share. Similarly, education is strongly related to the household's portfolio composition. Relative to having below high school level education, having a college education is associated with a 14.7% higher high-risk asset share, and a 16.2% lower low-risk asset share. Moreover, households with retired heads are associated with holding a higher proportion of high-risk assets relative to not being in the labour force.

With regards to the head of household's attitudes towards risk, saving horizons and

subjective economic expectations, the expected impacts are found. If the head of household has a positive economic outlook and if the head of household is more risk tolerant, they are associated with a riskier portfolio allocation. For example, a one-unit increase in the risk attitudes index is associated with an increase in the shares of high-risk assets by 7.2% and a reduction in the share of low-risk assets 7.9%.

Considering the impact of the background risk measures reveals that expecting a major financial expenditure, health insurance cover, inheritance and having income below that expected in the previous year all have statistically significant impacts on the household's portfolio composition. For example, households who do not have health insurance hold, on average, a safer portfolio than those households who have health insurance cover. For example, having no health insurance cover is associated with holding a 10.9% higher low-risk asset share, whilst it is associated with holding 9.9% and 1.0% lower high and medium-risk asset shares, respectively.

Turning our attention to the borrowing constraint measures reveals broadly similar results to the univariate specifications, that is, households who are defined to be constrained hold, on average, a safer financial portfolio. Interestingly, these results reveal that the borrowing constraints, in addition to being associated with a lower share of high-risk assets, are associated with a reduction in the share of medium-risk assets held. Specifically, across all measures, constrained households hold a lower proportion of high and medium-risk assets, and hold a larger proportion of low-risk assets in their financial portfolio, in line with the theoretical model presented in Section 3. For example, *Constraint 5* is associated with holding 1.3% and 0.1% lower shares of high and medium-risk assets, respectively, and is associated with holding a 1.5% higher share of low-risk assets. Similar findings are observed for the other borrowing constraint measures thereby endorsing the robustness of our empirical findings related to the influence of borrowing constraints on the composition of the household financial portfolio.

7 Conclusion

This paper has contributed to the household finance literature in a number of ways. For motivation, we developed a tractable theoretical framework with a basis in standard portfolio theory, which is structured around three tradable assets differentiated by increasing rates of risk and return. The rationale for the presence of three tradable assets explicitly reflects of the nature of our empirically observable data, in which household financial wealth is classified as being either low-risk, medium-risk, or high-risk. Under non-restrictive assumptions the behavior of the medium-risk asset was found to be ambiguous. Further, the confluence of correlated background risk and borrowing constraints was found to impact on the domain of the non-binding risk-return space, which in turn holds implications for the structure of the household's optimal portfolio. This suggests that when empirically modelling the impact of borrowing constraints on household portfolio allocation, it is important to control for the impact of background risk.

Our empirical findings suggest that households that are defined as being borrowing constrained, and further, that are exposed to higher levels of background risk, are inclined to allocate their financial wealth towards less risky asset categories. This result is important, given the theoretical ambiguity which we showed characterises the behavior of the mediumrisk asset class. In particular, both our theoretical and empirical modeling approaches reveal that borrowing constraints may have significant ramifications for the behavior of the medium-risk category, and not just assets that are classified as being high-risk. This may be important, as our sample data suggests that in addition to there being a 'stockholding puzzle' – which affects high-risk assets – US households also shun holding medium-risk assets, which our empirical proxies for borrowing constraints appear to partially explain. Significantly, a lack of access to borrowing is not only negatively associated with a stockholding, but, the holding of medium-risk assets. This finding has important public policy implications, which are potentially more far-reaching for US households than their propensity to shun holding stocks. For example, medium-risk assets include retirement funds and life insurance polices. Non-participation in the markets for these financial products may substantially increase the likelihood being in poverty in old age, or increase the likelihood of family members suffering a catastrophic financial shock—such as the inability to maintain mortgage payments—in the event of early death. In this regard, public policies aimed at ensuring greater household access to borrowing and credit may be essential to increasing the uptake of some medium-risk financial products, which in turn may mitigate the financial impacts of events such as retirement and death across the lifecycle of a household. In this sense, our contribution suggests that further work directed towards accounting for the allocation of medium-risk assets in US households should be considered expedient.

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Figures and Tables



Figure 1: The efficient frontier and household preferences under short-sale constraints, with (**) and without (*) background risk

Notes: The following parameterisations are used: $\mu_L = 2.36$, $\sigma_L = 2.25$; $\mu_M = 5.43$, $\sigma_M = 4.96$; $\mu_H = 11.89$, $\sigma_H = 18.44$; $\mu_B = 4$, $\lambda = 1$; $\gamma = 0.5$. Orange/red curves capture the absence of background risk ($\sigma_H = 0$). Light blue/dark blue curves correspond to the case of positively correlated background risk ($\sigma_B = 5$, $\rho_{LB} = \rho_{MB} = 0$, $\rho_{HB} = 0.4$).





 $\it Notes:$ The above figure shows the distributions of the three assets classifications in the SCF.
Table 1: Optimal portfolio composition predicted by the model under alternative background risk scenarios

	Optima	l portfolio	composition	
Background risk scenarios	q_L^*	q_M^*	q_H^*	Welfare
No background risk ($\sigma_B = 0$)	0.694	0.268	0.038	5.191
Positive correlation $(\rho_{LB} = \rho_{MB} = 0; \rho_{HB} = 0.4)$	0.726	0.274	0	-6.749
Independent background risk $(\rho_{LB} = \rho_{MB} = \rho_{HB} = 0)$	0.694	0.268	0.038	-7.309
Negative correlation $(\rho_{LB}=\rho_{MB}=0; \rho_{HB}=-0.4)$	0.605	0.249	0.146	-3.917

Labic 2. variable definitions	Table 2:	Variable	definitions
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Dependent Variables	Definition
High-risk	Directly held stocks; Stock mutual funds; Risky retirement accounts; Risky annuities; trust
Medium-risk	funds; mortgage bonds; corporate and foreign bonds Safe retirement funds; State and local bonds; Tax-free bonds; Life insurance policy
Low-risk	Checking and saving accounts; Certificates of deposit; Money market deposit accounts; US savings bonds; Call accounts
Independent Variables	Definition
Age Age Squared Male White Child College High school Married Employed Self-Employed Retired	 Age of household head in years. Age of household head in years squared divided by 100. = 1 if head of household is male; 0 if female. = 1 if the head of household is white, 0 otherwise. = 1 if there are children under the age of 16 present in the household, 0 otherwise. = 1 if head of household has college degree as highest educational qualification, 0 otherwise = 1 if head of household is married or in a relationship, 0 otherwise. = 1 if head of household reports being employed, 0 otherwise. = 1 if head of household reports being self-employed, 0 otherwise. = 1 if head of household reports being retired, 0 otherwise.
Economic Exp.	Over the next five years, do you expect the US economy as a whole to perform better, worse, or about the same as it has over the past five years? $0 =$ Worse, $1 =$ About the same, $2 =$ Better.
Saving horizon.	Index based on responses to the question, "In planning or budgeting your (family's) saving and spending which period is most important to you? 1 is next few months, 2 next year, 3 next few years, 4 5-10 years, 5 longer than 10 years.
Risk Attitudes	An index, increasing in risk tolerance, based on responses to the question "Which of the following statements comes closest to describing the amount of financial risk that you are willing take when you save or make investments? $0 = Not$ willing to take any financial risks, $1 = Take$ average financial risks expecting to earn average returns, $3 = Take$ above average financial risks expecting to earn above average returns, $4 = Take$ substantial financial risks expecting to earn substantial returns."
Total Income	Natural Logarithm transformation of total household income divided by 10. Inverse hyperbolic sine transformation of net wealth, that is, total assets minus total debt,
Net Wealth	divided by 10.
Homeowner	= 1 if the household owns or is buying/land contract on property; 0 otherwise. Dummy variables for the 1995 (omitted), 1998, 2001, 2004, 2007, 2010, 2013, 2016 and 2019
Year controls	years are included.
Background Risk	Definition
Own Business Multi earner	=1 if owns business which was not started by the household, 0 otherwise =1 if there are more than one earner in the household, 0 otherwise.
Major Fin. Exp.	=1 if household head reports "Yes" to the question "In the next five to ten years, are there any foreseeable major expenses that you (and your family) expect to have to pay for (your-self/yourselves), such as educational expenses, purchase of a new home, health care costs, support for other family members, or anything else?"
Health Insurance Inheritance Know Income	 = 1 if not all individuals in the household are covered by some type of health insurance; 0 otherwise. = 1 if expect inheritance or significant transfer of assets in the next five years, 0 otherwise. = 1 if household head has "a good idea of what your (family's) income for next year will be", 0 otherwise.
Income Above Norm.	Takes the values of the difference between "normal" income and previous years income, if previous years income exceeded "normal" income.
Income Below Norm.	Takes the values of the difference between "normal" income and previous years income, if previous years income was below "normal" income.

Table 3: Borrowing constraint measures

Panel A: The questions used in constructing the borrowing constraint measures

(1): In the past twelve months, has a particular lender or creditor turned down any request you or your (husband/wife/partner) made for credit, or not given you as much credit as you applied for?

(2): Were you later able to obtain the full amount you requested by reapplying to the same institution or by applying elsewhere?

(3): Was there any time in the past twelve months that you (or your husband/wife/partner) thought of applying for credit at a particular place, but changed your mind because you thought you might be turned down?

Panel B: Borrowing constraint measures definitions						
Constraint 1	=1 if the household did not apply for credit due to a belief that the application would be rejected, 0 otherwise					
Constraint 2	=1 if the household responds they were turned down or did not obtain the full amount of credit applied for, 0 otherwise					
Constraint 3	= 1 if the household responds they were turned down or did not obtain the full amount of credit applied for and they did not reapply or were able to obtain credit elsewhere, 0 otherwise					
Constraint 4	= 1 if constraint 1 or constraint 2 are satisfied, 0 otherwise					
Constraint 5	= 1 if constraint 1 and constraint 3 is satisfied, 0 otherwise.					

The table shows how the borrowing constraint status of the household can be constructed using the information in the SCF.

 Table 4: Summary statistics

Dependent Variables:		
Proportion High Risk Assets (Direct and Indirect)	0.333	(0.002)
Proportion Medium Risk Assets	0.258	(0.002)
Proportion Low Risk Assets	0.409	(0.002)
Independent Variables:		
Age	48.048	(0.086)
Age Squared	25.146	(0.087)
Male	0.821	(0.002)
White	0.790	(0.002)
Education:		
Below High School	0.064	(0.001)
High School	0.407	(0.003)
College Degree	0.529	(0.003)
Married	0.705	(0.003)
Child	0.491	(0.003)
Employment:		
Employed	0.585	(0.003)
Self-Employed	0.243	(0.003)
Retired	0.103	(0.002)
Other	0.069	(0.002)
Economic Exp.	1.157	(0.005)
Saving Horizon	2.183	(0.008)
Risk Attitudes	1.085	(0.005)
Total Income	11.648	(0.009)
Net Worth	9.923	(0.052)
Homeowner	0.671	(0.003)
Background Risks:		. ,
Business Own.	0.282	(0.003)
Multi Earners	0.389	(0.003)
Major Fin. Exp.	0.596	(0.003)
No Health insur.	0.083	(0.002)
Inheritance	0.174	(0.002)
Know Income	0.706	(0.003)
Income Above Norm.	1.396	(0.024)
Income Below Norm.	-1.746	(0.025)
Borrowing Constraints:		
Constraint 1	0.141	(0.002)
Constraint 2	0.254	(0.003)
Constraint 3	0.158	(0.002)
Constraint 4	0.293	(0.003)
Constraint 5	0.197	(0.002)
Observations	27,618	<u> </u>
	.,	

[1] The table shows summary statistics of the variables. [2] Standard errors in parentheses.

$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$				~		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		1		Specification 3		5
Age Squared -0.017^{4***} -0.017^{2***} -0.017^{3***} -0.017^{3***} -0.017^{3***} -0.017^{3***} -0.0002^{3} Male (0.0029) (0.0029) (0.0029) (0.0029) (0.0029) (0.0029) (0.0029) (0.0029) (0.0027) (0.0171) (0.0133) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0334) (0.0290) (0.0290) (0.0290) (0.0271	Age			/ · · · · · · ·	2 · · · · · · ·	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Age Squared	(0.0029) - 0.0174^{***}				
		(0.0029)	(0.0029)	(0.0029)	(0.0029)	(0.0029)
	Male	(0.0228)	(0.0228)	(0.0228)	(0.0227)	(0.0227)
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	White					
			× /	· · · ·	. ,	· /
$\begin{array}{cccc} {\rm College Degree} & 0.3961^{**} & 0.3976^{**} & 0.3959^{**} & 0.3957^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.3970^{**} & 0.2335 & 0.02333 & 0.03333 & 0.03333 & 0.03333 & 0.03333 & 0.03333 & 0.03333 & 0.03333 & 0.03333 & 0.03333 & 0.0325 & 0.0224 & 0.2225 & 0.0224 & 0.2225 & 0.02224 & 0.0225 & 0.0224 & 0.0225 & 0.0224 & 0.0225 & 0.0224 & 0.0235 & 0.0387^{**} & 0.0387^{**} & 0.0387^{**} & 0.0387^{**} & 0.0387^{**} & 0.0387^{**} & 0.0387^{**} & 0.0390 & 0.0316 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0336 & 0.0328 & 0.0278^{**} & 0.0$	High School	/ · · · · · · · · · · · · · · · · · · ·			2 · · · · · · ·	
$\begin{array}{llllllllllllllllllllllllllllllllllll$	College Degree	0.3961***	0.3976***	0.3989* ^{**}	0.3954^{***}	0.3970***
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Married					
	Child			(0.0224)		
Self-Employed (0.0290) (0.0290) (0.0290) (0.030) (0.030) (0.030) Retired 0.1274^{***} 0.1289^{***} 0.1290^{***} 0.1273^{***} $0.0335)$ (0.0336) Retired 0.1274^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0278^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0292^{***} 0.0283^{***} 0.0293^{***} 0.0283^{***} 0.0293^{***} 0.0283^{***} <		0.2306***	0.2321***	0.2315^{***}	0.2312***	0.2305***
Retired (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0336) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0333) (0.0033) (0.0033) (0.0083) (0.0083) (0.0083) (0.0083) (0.0083) (0.0083) (0.0083) (0.0051) (0.0051) (0.0051) (0.0051) (0.0051) (0.0051) (0.0051) (0.0051) (0.0051) (0.0051) (0.0051) (0.0057) <th< td=""><td></td><td></td><td></td><td></td><td></td><td>(0.0290)</td></th<>						(0.0290)
	Self-Employed	(0.0336)	(0.0336)	(0.0336)	(0.0335)	(0.0336)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Retired					
$\begin{array}{llllllllllllllllllllllllllllllllllll$	Economic Exp.					
Risk Attitudes (0.0051) (0.0051) (0.0051) (0.0052) (0.0051) (0.0052) Risk Attitudes 0.2324^{***} 0.2319^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.2326^{***} 0.119^{****} 0.119^{****} 0.119^{****} 0.119^{****} 0.119^{****} 0.01076) (0.0076) (0.0011) (0.0113) (0.0113) (0.0163) (0.0163) (0.0163) (0.0163) (0.0163) (0.0163) (0.0163) (0.0163) (0.0163) (0.0145) (0.0125) (0.0125) (0.0125) (0.0125) (0.0125) (0.0125) (0.0125)	Saving Horizon				$(0.0083) \\ 0.0287^{***}$	$(0.0083) \\ 0.0293^{***}$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		(0.0051)	(0.0051)	(0.0052)	(0.0051)	(0.0052)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Risk Attitudes					(
Net Worth $\dot{0}.0160^{***}$ $\dot{0}.0159^{***}$ $\dot{0}.0161^{***}$ $\dot{0}.0158^{***}$ $\dot{0}.0161^{***}$ Home owner 0.1415^{***} 0.1423^{***} 0.1444^{***} 0.1413^{***} 0.1413^{***} 0.1426^{***} Business Own. -0.0158 -0.0157 (0.0157) (0.0163) (0.0163) (0.0163) Multi Earners 0.0327^{**} 0.0330^{**} 0.0326^{**} 0.0280^{**} 0.0215^{**} 0.0145^{*} 0.0145^{*} 0.0125^{*} 0.0125^{*} 0.0125^{*} 0.0125^{*} 0.022^{**} 0.022^{**} 0.022^{**} 0.022^{**} 0.022^{**} 0.022^{**} 0.022^{**} 0.022^{**} 0.022^{**} 0.022^{**} 0.022^{*}	Total Income	2 · · · · · · · · · · · · · · · · · · ·	· · · · · · ·			
$\begin{array}{llllllllllllllllllllllllllllllllllll$	Net Worth	0.0160***	0.0159^{***}	0.0161***	0.0158^{***}	0.0161***
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Home owner	0.1415^{***}	0.1432^{***}	0.1444^{***}	0.1413^{***}	0.1426^{***}
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			(0.0137)			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Business Own.					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Multi Earners	0.0327^{**}	0.0330**	0.0332**	0.0326**	0.0329**
No Health insur. -0.2119^{***} -0.2134^{***} -0.2142^{***} -0.2110^{***} -0.2120^{***} Inheritance (0.0280) (0.0281) (0.0280) (0.0282) (0.0281) Inheritance 0.0522^{***} 0.0522^{***} 0.0522^{***} 0.0521^{***} Know Income (0.0144) (0.0144) (0.0144) (0.0144) Income Above Norm. 0.0513^{***} 0.0512^{***} 0.0507^{***} 0.0510^{***} Income Below Norm. -0.0000 0.0000 -0.0000 0.0001 -0.0000 (0.0015) (0.0015) (0.0015) (0.0015) (0.0015) Income Below Norm. 0.0000 0.0001 -0.0000 0.0001 (0.0015) (0.0015) (0.0015) (0.0015) (0.0015) Borrowing Constraints: -0.0713^{***} (0.0172) (0.0176) Constraint 1 -0.0713^{***} (0.0172) -0.0595^{***} Constraint 5 -3.7819^{***} -3.7778^{***} -3.7952^{***} -3.7603^{***} Constraint 5 -3.7819^{***} -3.7778^{***} -3.7603^{***} -3.7807^{***} (0.0952) (0.0964) (0.0950) (0.0974) (0.0957) Year Fixed EffectsYesYesYesYesYesYes	Major Fin. Exp.	(0.0145) 0.0557^{***}	(0.0145) 0.0554^{***}	(0.0145) 0.0544^{***}	(0.0145) 0.0561^{***}	(0.0145) 0.0549^{***}
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	No Health insur.	-0.2119***	-0.2134***	-0.2142***	-0.2110***	-0.2120***
Know Income $\dot{0}.0513^{***}$ $\dot{0}.0512^{***}$ $\dot{0}.0515^{***}$ $\dot{0}.0507^{***}$ $\dot{0}.0510^{***}$ Income Above Norm. -0.0000 0.0000 -0.0000 0.0001 -0.0000 Income Below Norm. 0.0015 (0.0015) (0.0015) (0.0015) Income Below Norm. 0.0001 0.0001 -0.0000 0.0001 Income Below Norm. 0.0001 0.0001 0.0001 -0.0000 0.0015 (0.0015) (0.0015) (0.0015) (0.0015) Borrowing Constraints: Constraint 1 -0.0713^{***} (0.0210) (0.0172) (0.0176) Constraint 2 -0.0486^{***} (0.0172) -0.0595^{***} (0.0197) Constraint 4 -0.0595^{***} (0.0197) -0.0514^{***} Constraint 5 -3.7819^{***} -3.7778^{***} -3.7603^{***} Constant -3.7819^{***} -3.7778^{***} -3.7603^{***} -3.7807^{***} Year Fixed EffectsYesYesYesYesYesYes	Inheritance	(0.0280) 0.0522^{***}	(0.0281) 0.0525^{***}	(0.0280) 0.0522^{***}	(0.0282) 0.0525^{***}	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Know Income		(0.0144) 0.0512^{***}	(0.0144) 0.0515^{***}		(0.0144) 0.0510^{***}
Income Below Norm. $\dot{0}.0000'$ $\dot{0}.0001'$ $\dot{0}.0001'$ $\dot{0}.0001'$ $\dot{0}.0000'$ $\dot{0}.0015'$ $\dot{0}.0$	Income Above Norm.	-0.0000	Ò.0000 ´	$(0.0134) \\ -0.0000$	Ò.0001 ´	$(0.0134) \\ -0.0000$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Income Below Norm.	0.0000	Ò.0001	0.0001	-0.0000	Ò.0000 ´
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		-0.0713***	(0.0010)	(0.0010)	(0.0010)	(0.0013)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Constraint 2	(0.0210)				
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Constraint 3		(0.0172)			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Constraint 4			(0.0176)		
$\begin{array}{ccccc} \text{Constant} & -3.7819^{***} & -3.7778^{***} & -3.7952^{***} & -3.7603^{***} & -3.7807^{***} \\ (0.0952) & (0.0964) & (0.0950) & (0.0974) & (0.0957) \\ \text{Year Fixed Effects} & \text{Yes} & \text{Yes} & \text{Yes} & \text{Yes} & \text{Yes} \\ \end{array}$	Constraint 5				(0.0197)	
Year Fixed Effects Yes Yes Yes Yes Yes	Constant					-3.7807***
Observations 27,618 27,618 27,618 27,618 27,618	Year Fixed Effects		(/			
	Observations	27,618	27,618	27,618	27,618	27,618

 Table 5:
 Fractional Probit Model - Coefficients

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[1] Dependent variable is the proportion of high risk assets. [2] Fractional probit regression model parameters [3] Standard errors in parentheses. [4] *denotes significance at the 10% level **denotes significance at the 5% level and ***denotes significance at the 1% level.

			Specificatior	1	
	1	2	3	4	5
Age	0.0063***	0.0062***	0.0062***	0.0062***	0.0062***
A ma Command	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)
Age Squared	-0.0055^{***} (0.0009)	-0.0055^{***} (0.0009)	-0.0055^{***} (0.0009)	-0.0055^{***} (0.0009)	-0.0055^{***} (0.0009)
Male	0.0159**	0.0165**	0.0162**	0.0163**	0.016**
XX71. : 4 -	(0.0072)	(0.0072)	(0.0072)	(0.0072)	(0.0072) 0.0553^{***}
White	0.0547^{***} (0.0054)	0.0554^{***} (0.0054)	0.0559*** (0.0054)	0.0547^{***} (0.0055)	(0.0053)
Education:	· · · ·	. ,	· /	. ,	. ,
High School	0.0624^{***} (0.0107)	0.0627^{***} (0.0107)	0.0626^{***} (0.0107)	0.0624^{***} (0.0108)	$\begin{array}{c} 0.0623^{***} \\ (0.0107) \end{array}$
College Degree	0.1258^{***}	0.1263^{***}	0.1268^{***}	0.1256^{***}	0.1261^{***}
	(0.0106)	(0.0106)	(0.0106)	(0.0106)	(0.0106)
Married	-0.0088 (0.0071)	-0.0086 (0.0071)	-0.0084 (0.0071)	-0.0088 (0.0071)	-0.0085 (0.0071)
Child	-0.0121***	-0.0122***	-0.0125***	-0.012***	-0.0123***
	(0.0046)	(0.0046)	(0.0045)	(0.0046)	(0.0046)
Employment: Employed	0.0733***	0.0738***	0.0736***	0.0735***	0.0732***
	(0.0092)	(0.0092)	(0.0092)	(0.0092)	(0.0092)
Self-Employed	0.0095	0.01	0.0098	0.0098	0.0096
Retired	(0.0107) 0.0405^{***}	$(0.0107) \\ 0.0409^{***}$	$(0.0107) \\ 0.041^{***}$	$(0.0106) \\ 0.0404^{***}$	$(0.0107) \\ 0.0405^{***}$
	(0.0109)	(0.0109)	(0.0109)	(0.0109)	(0.0109)
Economic Exp.	$0.0089^{\star \star \star}$	0.0088^{***}	0.0088^{***}	$0.0088^{\star * *}$	0.0088^{***}
Saving Horizon	(0.0026) 0.0093^{***}	$(0.0026) \\ 0.0093^{***}$	(0.0026) 0.0094^{***}	(0.0026) 0.0091^{***}	(0.0026) 0.0093^{***}
	(0.0016)	(0.0016)	(0.0016)	(0.0016)	(0.0016)
Risk Attitudes	0.0738^{***} (0.0023)	0.0738^{***} (0.0023)	0.0737^{***} (0.0023)	0.0739^{***} (0.0023)	0.0737^{***} (0.0023)
Total Income	(0.0023) 0.0382^{***}	(0.0023) 0.0381^{***}	(0.0023) 0.0382^{***}	(0.0023) 0.038^{***}	(0.0023) 0.0381^{***}
	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)
Net Worth	0.0051^{***} (0.0003)	0.0051* ^{***} (0.0003)	0.0051* ^{***} (0.0003)	0.005^{***} (0.0003)	0.0051* [*] ** (0.0003)
Home owner	0.045^{***}	(0.0003) 0.0455^{***}	0.0459^{***}	(0.0003) 0.0449^{***}	(0.0003) 0.0453^{***}
Dechamour d. Distan	(0.005)	(0.005)	(0.0049)	(0.005)	(0.005)
Background Risks: Business Own.	-0.005	-0.0049	-0.005	-0.0049	-0.005
	(0.0052)	(0.0052)	(0.0052)	(0.0052)	(0.0052)
Multi Earners	0.0104^{**} (0.0046)	0.0105^{**} (0.0046)	0.0106^{**} (0.0046)	0.0104^{**} (0.0046)	0.0105^{**}
Major Fin. Exp.	(0.0046) 0.0177^{***}	(0.0046) 0.0176^{***}	(0.0046) 0.0173^{***}	(0.0046) 0.0178^{***}	(0.0046) 0.0174^{***}
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
No Health insur.	-0.0673^{***} (0.0089)	-0.0678^{***} (0.0089)	-0.0681^{***} (0.0089)	-0.067^{***} (0.0089)	-0.0673^{**} (0.0089)
Inheritance	(0.0089) 0.0166^{***}	(0.0089) 0.0167^{***}	(0.0089) 0.0166^{***}	(0.0089) 0.0167^{***}	(0.0089) 0.0166^{***}
	(0.0046)	(0.0046)	(0.0046)	(0.0046)	(0.0046)
Know Income	0.0163^{***} (0.0042)	0.0163^{***} (0.0042)	0.0164^{***} (0.0042)	0.0161^{***} (0.0042)	0.0162^{***} (0.0042)
Income Above Norm.	0.0000	0.0000	0.0000	0.0000	(0.0042) 0.0000
In come Deless New	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)
Income Below Norm.	0.0000 (0.0005)	0.0000 (0.0005)	0.0000 (0.0005)	0.0000 (0.0005)	0.0000 (0.0005)
Borrowing Constraints:	· · · ·	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Constraint 1	-0.0226^{***}				
Constraint 2	(0.0067)	-0.0154***			
		(0.0055)			
Constraint 3		-	-0.0124^{**}		
Constraint 4			(0.0063)	-0.0189***	
				(0.0056)	
Constraint 5					-0.0163^{**}
Year Fixed Effects	Yes	Yes	Yes	Yes	(0.0058) Yes

Table 6: Fractional Probit Model - Marginal Effects

[1] Dependent variable is the proportion of high risk assets. [2] Fractional probit regression model marginal effects [3] Standard errors in parentheses. [4] *denotes significance at the 10% level **denotes significance at the 5% level and ***denotes significance at the 1% level.

	Coeffic	ients	High-	risk	Medium	n-risk	Low-	risk
	Coef.	SE	Coef.	SE	Coef.	SE	Coef.	SE
Age	-0.0351***	(0.0026)	0.0123***	(0.0009)	0.0013***	(0.0001)	-0.0135***	(0.001)
Age Squared	0.0286^{***}	(0.0026)	-0.0100***	(0.0009)	-0.001***	(0.0001)	0.011^{***}	(0.001)
Male	0.0263	(0.0211)	-0.0092	(0.0074)	-0.0010	(0.0008)	0.0102	(0.0081)
White	-0.1295***	(0.015)	0.0453^{***}	(0.0052)	0.0047^{***}	(0.0006)	-0.0500***	(0.0058)
Education		· · · · ·				· · · · ·		· · · · ·
High School	-0.2400***	(0.0279)	0.0839^{***}	(0.0098)	0.0088^{***}	(0.0011)	-0.0927***	(0.0108)
College Degree	-0.4196***	(0.0281)	0.1467^{***}	(0.0098)	0.0154^{***}	(0.0012)	-0.1621***	(0.0109)
Married	-0.0352*	(0.0203)	0.0123*	(0.0071)	0.0013^{*}	(0.0008)	-0.0136*	(0.0078)
Child	0.0288^{**}	(0.0121)	-0.0101**	(0.0042)	-0.0011**	(0.0004)	0.0111**	(0.0047)
Employment		× /		· /		· /		````
Employed	-0.2630***	(0.0262)	0.0920***	(0.0091)	0.0096^{***}	(0.0011)	-0.1016***	(0.0101)
Self-Employed	-0.0160	(0.03)	0.0056	(0.0105)	0.0006	(0.0011)	-0.0062	(0.0116)
Retired	-0.1370***	(0.0308)	0.0479***	(0.0108)	0.0050***	(0.0012)	-0.0529***	(0.0120)
Economic Exp.	-0.0183**	(0.0071)	0.0064^{**}	(0.0025)	0.0007**	(0.0003)	-0.0071**	(0.0027)
Important Period	-0.0285***	(0.0045)	0.0100***	(0.0016)	0.0010***	(0.0002)	-0.0110***	(0.0017)
Risk Attitudes	-0.2046***	(0.0071)	0.0715^{***}	(0.0025)	0.0075***	(0.0004)	-0.0790***	(0.0027)
Total Income	-0.1287***	(0.0057)	0.0450***	(0.0020)	0.0047***	(0.0003)	-0.0497***	(0.0022)
Net Worth	-0.0190***	(0.0009)	0.0067***	(0.0003)	0.0007***	(0.0000)	-0.0073***	(0.0004)
Home owner	-0.1413***	(0.0142)	0.0494***	(0.0050)	0.0052***	(0.0006)	-0.0546***	(0.0055)
Background Risk		()		()		()		()
Business Owner	0.04^{***}	(0.0154)	-0.014***	(0.0054)	-0.0015***	(0.0006)	0.0155***	(0.006)
Multi Earners	-0.0599***	(0.0136)	0.0209***	(0.0048)	0.0022***	(0.0005)	-0.0231***	(0.0053)
Major Fin. Exp.	-0.0533***	(0.0114)	0.0186***	(0.004)	0.0020***	(0.0004)	-0.0206***	(0.0044)
No Health insur.	0.2823***	(0.0246)	-0.0987***	(0.0086)	-0.0103***	(0.001)	0.109***	(0.0095)
Inheritance	-0.0507***	(0.0135)	0.0177***	(0.0047)	0.0019***	(0.0005)	-0.0196***	(0.0052)
Know Income	-0.0584***	(0.0125)	0.0204***	(0.0044)	0.0021***	(0.0005)	-0.0226***	(0.0048)
Income Above Norm.	0.0012	(0.0014)	-0.0004	(0.0005)	0.0000	(0.0001)	0.0005	(0.0005)
Income Below Norm.	-0.0020	(0.0014)	0.0007	(0.0005)	0.0001	(0.0001)	-0.0008	(0.0005)
1998	-0.1204***	(0.023)	0.0421***	(0.0081)	0.0044***	(0.0009)	-0.0465***	(0.009)
2001	-0.1339***	(0.023)	0.0468***	(0.0081)	0.0049***	(0.0009)	-0.0517***	(0.0089)
2004	-0.0171	(0.0222)	0.0060	(0.0078)	0.0006	(0.0008)	-0.0066	(0.0087)
2007	-0.0286	(0.0225)	0.0100	(0.0079)	0.0010	(0.0008)	-0.0110	(0.0087)
2010	-0.0755***	(0.0213)	0.0264***	(0.0075)	0.0028***	(0.0008)	-0.0292***	(0.0082)
2013	-0.0421*	(0.0218)	0.0147*	(0.0077)	0.0015*	(0.0008)	-0.0163*	(0.0085)
2016	-0.0161	(0.0223)	0.0056	(0.0078)	0.0006	(0.0008)	-0.0062	(0.0087)
2019	0.0337	(0.0231)	-0.0118	(0.0081)	-0.0012	(0.0008)	0.0130	(0.0090)
Boundary parameters:		(0.0-0-)	0.00	(01000-)	0.00	(0.0000)	0.0200	(0.0000)
μ_0	-4.2512***	(0.0873)						
μ_1	-3.4836***	(0.0870)						
Borrowing Constraints:		(0.0010)						
Constraint 1	0.0318^{*}	(0.0189)	-0.0111*	(0.0066)	-0.0012*	(0.0007)	0.0123*	(0.0073)
Constraint 2	0.0302**	(0.0100) (0.0145)	-0.0106**	(0.0051)	-0.0011**	(0.0001) (0.0005)	0.0117**	(0.0056)
Constraint 3	0.0334^{*}	(0.0110) (0.0175)	-0.0117*	(0.0061) (0.0061)	-0.0012^{*}	(0.0006)	0.0129^{*}	(0.0068)
Constraint 4	0.0358**	(0.0110) (0.0144)	-0.0125**	(0.0051)	-0.0012	(0.0005)	0.0128**	(0.0056)
Constraint 5	0.0378**	(0.0111) (0.0166)	-0.0132**	(0.0051) (0.0058)	-0.0014**	(0.0006)	0.0146**	(0.0064)

 Table 7: Ordered fractional model

[1] Fractional Ordered Probit Parameters [2] Standard errors in parentheses. [3] *denotes significance at the 10% level **denotes significance at the 5% level and ***denotes significance at the 1% level. [4] Coefficients of the independent variables are those from the specification including Constraint 5.

Appendix

A Deriving explicit expressions for the tradable asset shares

Using expression (2) subject to the constraint that $q_L + q_M + q_H = 1$, Gauss–Jordan elimination can be applied to demonstrate that the tradable asset shares are given by

$$q_L = \alpha + \beta \Gamma;$$

$$q_M = 1 - \alpha - (1 + \beta)\Gamma; and$$

$$q_H = \Gamma,$$

(A.1)

where

$$= \frac{\mu_W - \lambda \mu_B - \mu_M}{\mu_L - \mu_M} \quad and \quad \beta = \frac{\mu_M - \mu_H}{\mu_L - \mu_M},$$
(A.2)

and Γ is a scalar that varies over the real line.²⁹ Using the solutions for q_L , q_M , and q_H derived in expression (A.1) entails that expression (3) can instead be expressed as

$$v_{W} = \begin{cases} (\alpha + \beta \Gamma)^{2} v_{L} + (1 - \alpha - (1 + \beta)\Gamma)^{2} v_{M} + \Gamma^{2} v_{H} + \lambda^{2} v_{B} \\ + 2 (\alpha + \beta \Gamma) (1 - \alpha - (1 + \beta)\Gamma) \rho_{LM} \sigma_{L} \sigma_{M} \\ + 2 (\alpha + \beta \Gamma) \Gamma \rho_{LH} \sigma_{L} \sigma_{H} + 2 (1 - \alpha - (1 + \beta)\Gamma) \Gamma \rho_{MH} \sigma_{M} \sigma_{H} \\ + 2 (\alpha + \beta \Gamma) \lambda \rho_{LB} \sigma_{L} \sigma_{B} + 2 (1 - \alpha - (1 + \beta)\Gamma) \lambda \rho_{MB} \sigma_{M} \sigma_{B} + 2 \Gamma \lambda \rho_{HB} \sigma_{H} \sigma_{B} \end{cases}$$
(A.3)

Now suppose the household is assumed to specify μ_W ; by the efficient set theorem it sets out to minimize the variance of wealth, v_W . Differentiating expression (A.3) with respect to Γ and solving for Γ constitutes a first step in deriving asset share allocations consistent with portfolio variance minimization. This yields

$$\Gamma^* = \frac{\begin{pmatrix} (1-\alpha)(1+\beta)v_M - \alpha\beta v_L \\ + (\alpha - \beta + 2\alpha\beta)\rho_{LM}\sigma_L\sigma_M - \alpha\rho_{LH}\sigma_L\sigma_H - (1-\alpha)\rho_{MH}\sigma_M\sigma_H \\ -\lambda(\beta\rho_{LB}\sigma_L\sigma_B - (1+\beta)\rho_{MB}\sigma_M\sigma_B + \rho_{HB}\sigma_H\sigma_B) \end{pmatrix}}{\begin{cases} \beta^2 v_L + (1+\beta)^2 v_M + v_H - 2\beta(1+\beta)\rho_{LM}\sigma_L\sigma_M \\ + 2\beta\rho_{LH}\sigma_L\sigma_H - 2(1+\beta)\rho_{MH}\sigma_M\sigma_H \end{cases}}.$$
(A.4)

The variance minimizing shares, q_L^* , q_M^* and q_H^* , are given by

 α

$$\begin{aligned} q_L^* &= \alpha + \beta \Gamma^*; \\ q_M^* &= 1 - \alpha - (1 + \beta) \Gamma^*; \text{ and} \\ q_H^* &= \Gamma^*. \end{aligned} \tag{A.5}$$

The efficient frontier, v_W^* , is thus given by

$$v_W^* = \begin{bmatrix} q_L^{*2} v_L + q_M^{*2} v_M + q_H^{*2} v_H + \lambda^2 v_B \\ + 2q_L^* q_M^* Cov(L, M) + 2q_L^* q_H^* Cov(L, H) + 2q_M^* q_H^* Cov(M, H) \\ + 2q_L^* \lambda Cov(L, B) + 2q_M^* \lambda Cov(M, B) + 2q_H^* \lambda Cov(H, B) \end{bmatrix}.$$
(A.6)

Here, it is useful to note that the solution in expression (A.4) is equivalent to $\Gamma^* = C^{\rho}/D^{\rho}$ such that

$$C^{\rho} = \begin{cases} \nu_{M} \left\{ (\mu_{L} - \mu_{W} + \lambda\mu_{B}) (\mu_{L} - \mu_{H}) \right\} \\ -\nu_{L} (\mu_{W} - \lambda\mu_{B} - \mu_{M}) (\mu_{M} - \mu_{H}) \\ + \left\{ \mu_{L} \mu_{H} + \mu_{M} \mu_{H} - 2\mu_{L} \mu_{M} \\ + (\mu_{W} - \lambda\mu_{B}) (\mu_{L} + \mu_{M} - 2\mu_{H}) \right\} \rho_{LM} \sigma_{L} \sigma_{M} \\ - (\mu_{W} - \lambda\mu_{B} - \mu_{M}) (\mu_{L} - \mu_{M}) \rho_{LH} \sigma_{L} \sigma_{H} \\ - (\mu_{L} + \lambda\mu_{B} - \mu_{W}) (\mu_{L} - \mu_{M}) \rho_{LB} \sigma_{L} \sigma_{B} \\ -\lambda \left\{ \mu_{M} - \mu_{H} (\mu_{L} - \mu_{M}) (\mu_{L} - \mu_{M}) + \rho_{HB} \sigma_{H} \sigma_{B} \right\} \end{cases}$$
(A.7)

²⁹Deriving (A.1) requires solving a two equation system with three unknowns. As there are an infinite number of solutions it is necessary to pin down the value of the scalar Γ . It is also possible to specify $q_M = 1 - \alpha - (1 + \beta)\Gamma$ in (A.1) as $1 - \Gamma - q_L$.

and

$$D^{\rho} = \begin{cases} \upsilon_{L} (\mu_{M} - \mu_{H})^{2} + \upsilon_{M} (\mu_{L} - \mu_{H})^{2} + \upsilon_{H} (\mu_{L} - \mu_{M})^{2} \\ -2 (\mu_{L} - \mu_{H}) (\mu_{M} - \mu_{H}) \rho_{LM} \sigma_{L} \sigma_{M} \\ +2 (\mu_{M} - \mu_{H}) (\mu_{L} - \mu_{M}) \rho_{LH} \sigma_{L} \sigma_{H} \\ -2 (\mu_{L} - \mu_{H}) (\mu_{L} - \mu_{M}) \rho_{MH} \sigma_{M} \sigma_{H} \end{cases}$$
(A.8)

Now, for simplicity, assume that the tradable assets are not correlated with each other, and that background wealth B is correlated with all of the tradable assets L, M and H. Using the results in expressions (A.2), (A.4) and (A.5)–(A.8), it is straightforward to demonstrate that

$$\frac{\partial q_L^*}{\partial \mu_W} < 0; \ \frac{\partial q_M^*}{\partial \mu_W} \stackrel{\leq}{=} 0; \ \frac{\partial q_H^*}{\partial \mu_W} > 0, \tag{A.9}$$

such that

$$\frac{\partial q_M^*}{\partial \mu_W} \stackrel{\leq}{\equiv} 0 \quad iff \quad \frac{\upsilon_S}{\upsilon_H} \stackrel{\geq}{\equiv} \frac{\left(\mu_S - \mu_M\right)^2}{\left[\left(\mu_S - \mu_H\right)\left(\mu_M - \mu_H\right) - \left(\mu_M - \mu_H\right)^2\right]}.$$
(A.10)

It is also straightforward to verify that the condition in expression (A.10) also holds in the presence of independent background risk when the tradable assets are not correlated with each other. Further, inspection of expressions (A.2), (A.4), (A.5)–(A.8) indicates that independent background risk will not influence asset allocations along the efficient frontier.³⁰

As is noted below in Appendix B, the presence of σ_B in the numerator of Γ^* implies that the domain of the non-binding risk-return space will also be a function of background risk when it is correlated with the model's tradable assets. Moreover, from expression (A.4) it is evident that in the presence of correlated background risk, when the short-sale constraint is not binding, a change in σ_B will impact on the three individual asset shares on the efficient frontier. Specifically, we have that:

$$\frac{dq_L^*}{l\sigma_B} = -\frac{(\mu_M - \mu_H)}{(\mu_L - \mu_M)} \cdot \frac{\lambda \left\{ \begin{array}{c} (\mu_M - \mu_H)(\mu_L - \mu_M)\rho_{LB}\sigma_L \\ -\rho_{MB}\sigma_M(\mu_L - \mu_H)(\mu_L - \mu_M) + \rho_{HB}\sigma_H \end{array} \right\}}{v_L (\mu_M - \mu_H)^2 + v_M(\mu_L - \mu_H)^2 + v_H(\mu_L - \mu_M)^2};$$
(A.11)

$$\frac{dq_M^*}{d\tau} = \frac{(\mu_L - \mu_H)}{(\mu_L - \mu_H)} \cdot \frac{\lambda \left\{ \begin{array}{c} (\mu_M - \mu_H)(\mu_L - \mu_M)\rho_{LB}\sigma_L \\ -\rho_{MB}\sigma_M(\mu_L - \mu_H)(\mu_L - \mu_M) + \rho_{HB}\sigma_H \end{array} \right\}}{(\mu_M - \mu_M)^2 + \mu_M (\mu_M - \mu_M)(\mu_M - \mu_M)};$$
(A.12)

$$\frac{1}{d\sigma_B} = \frac{1}{(\mu_L - \mu_M)} \cdot \frac{1}{v_L (\mu_M - \mu_H)^2 + v_M (\mu_L - \mu_H)^2 + v_H (\mu_L - \mu_M)^2};$$
(A.12)

$$\frac{dq_{H}^{*}}{d\sigma_{B}} = -\frac{\lambda \left\{ \begin{array}{c} (\mu_{M} - \mu_{H})(\mu_{L} - \mu_{M})\rho_{LB}\sigma_{L} \\ -\rho_{MB}\sigma_{M}(\mu_{L} - \mu_{H})(\mu_{L} - \mu_{M}) + \rho_{HB}\sigma_{H} \end{array} \right\}}{\upsilon_{L} (\mu_{M} - \mu_{H})^{2} + \upsilon_{M}(\mu_{L} - \mu_{H})^{2} + \upsilon_{H}(\mu_{L} - \mu_{M})^{2}},$$
(A.13)

where $dq_L^*/d\sigma_B + dq_M^*/d\sigma_B + dq_H^*/d\sigma_B = 0.$

The impact of background risk of a change in asset shares in expressions (A.11)–(A.13), is summarised in Table A.1, for which we assume that background risk is positively correlated with all respective tradable assets. We denote this restriction using the notation ρ_{LB}^+ , ρ_{MB}^+ and ρ_{HB}^+ . For each row, the positive and negative signs capture the direction of the background risk effect on the derivative $dq_i^*/d\sigma_B$, i = L, M, Bof the ρ_{iB} , i = L, M, B. For instance, the impact of ρ_{LB}^+ is to reduce the share of low-risk assets in the household's perfolio, whereas the impact of σ_{LB}^+ and σ_{LB}^+ is to reduce the share of low-risk assets in the household's portfolio, whereas the impact of ρ_{MB}^+ and ρ_{HB}^+ is to increase and reduce the share, respectively. The overall effect on the low-risk asset share $dq_L^*/d\sigma_B$ is therefore ambiguous, and determined by the values of $\mu_i, \sigma_i, i = L, M, B$. The presence of negative correlation between B and L, M, and H will be associated with a reversal of the signs in Table A.1.

³⁰Alternatively, we can express (A.10) as $v_S(\mu_S - \mu_H)(\mu_M - \mu_H) - v_S(\mu_M - \mu_H)^2 \stackrel{\geq}{\leq} v_H(\mu_S - \mu_M)^2$. Once a household's optimal portfolio allocation is bound by the no short-sale constraint, this condition will not hold.

	Trada	ble asset	correlation
1 4	$ ho_{LB}^+$	ρ_{MB}^+	$ ho_{HB}^+$
$\frac{dq_L^*}{d\sigma_B}$	—	+	_
$\frac{dq_M^*}{d\sigma_B}$	+	_	+
$\frac{dq_{H}^{*}}{d\sigma_{B}}$	_	+	_

Table A.1: The impact of background risk on optimal asset allocations when B is positively correlated with L, M, and H

B Conditions under which the non-negativity of q_L^* , q_M^* , and q_H^* are satisfied when the short-sale constraint is non-binding

It is possible to use the results in expression (A.5) in Appendix A to determine the conditions under which $q_L^* \ge 0$, $q_M^* \ge 0$, and $q_H \ge 0$ are jointly satisfied when the short-sale constraint is *not* binding. From (A.5), imposing $q_H^* \ge 0$ is equivalent to the constraint that $\Gamma^* \ge 0$. For $q_L^* \ge 0$ we note that $\alpha + \Gamma^*\beta \ge 0$ can equivalently be written as $\Gamma^* \ge -\frac{\alpha}{\beta}$. Imposing $q_M^* \ge 0$ implies that $1 - \alpha - (1 + \beta)\Gamma^* \ge 0$, or equivalently, $\Gamma^* \le \frac{(1-\alpha)}{(1+\beta)}$. Together, these results imply that q_L^* , q_M^* , and q_H^* will be positive – *that is, the short-sale constraint will not be binding* – when Γ^* lies in the interval

$$-\frac{\alpha}{\beta} \le \Gamma^* \le \frac{(1-\alpha)}{(1+\beta)} \text{ where } \Gamma^* \ge 0.$$
(B.1)

Drawing on the fact that $\alpha = \frac{\mu_W - \lambda \mu_B - \mu_M}{\mu_L - \mu_M}$ and $\beta = \frac{\mu_M - \mu_H}{\mu_L - \mu_M}$ in Appendix A, means that the condition in expression (B.1) can be re-expressed in terms of the *expected returns*, viz.,

$$\frac{\mu_M + \lambda \mu_B - \mu_W}{\mu_H - \mu_M} \le \Gamma^* \le \frac{\mu_L + \lambda \mu_B - \mu_W}{\mu_L - \mu_H} \quad where \ \Gamma^* \ge 0.$$
(B.2)

Condition (B.2) suggests that in the presence of a short-sale constraint, the unconstrained space of returns and risks available to households satisfying $q_L^* \ge 0$, $q_M^* \ge 0$, and $q_H^* \ge 0$ may be limited. As shown in Figure 1 of Section 3, this may lead to the no-short-sale constraint becoming binding for households with low-risk tolerance.³¹ Moreover, expression (A.7) of Appendix A implies that for the fully correlated model, the domain of the non-binding risk-return space governed by condition (B.2) will also be a function of background risk due to the presence of σ_B in Γ^* .

³¹In our simple static model, borrowing restrictions assume the form of short-selling constraints. However, in a dynamic setting, Dybvig and Huang (1988) note that imposing a non-negativity constraint (or in fact any negative lower bound) on wealth is a plausible economic assumption, and admits a natural interpretation as a credit constraint. Another way is to prevent borrowing from future income. There are many institutional restrictions on the amount of credit an individual can obtain.

C The global minimum portfolio and the optimal household portfolio with mean-variance preferences

C.1 Zero-correlation between assets

The point of tangency between the household's mean-variance utility function and the efficient frontier in the risk-return space is obtained by differentiating the efficient frontier v_W^* in expression (A.6) with respect to μ_W^* , setting the resulting expression equal to $1/\gamma$ and solving for μ_W^* . The absence of correlation requires setting all of the covariance terms in expression (3) to zero prior to the model being differentiated. Given the mean-variance preferences of the household captured by expression (6), namely $G(\mu_W, v_W) = \mu_W - \gamma v_W$, the point of tangency between a household's indifference curve will be given by the solution to the equation

$$\begin{bmatrix} \left(\frac{\partial q_L^*}{\partial \alpha} \frac{\partial \alpha}{\partial \mu_W^*} + \frac{\partial q_L^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*}\right) 2q_L^* \upsilon_S + \left(\frac{\partial q_M^*}{\partial \alpha} \frac{\partial \alpha}{\partial \mu_W^*} + \frac{\partial q_M^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*}\right) 2q_M^* \upsilon_M \\ + \frac{\partial q_H^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*} 2q_H^* \upsilon_H \end{bmatrix} = \frac{1}{\gamma},$$
(C.1)

and will satisfy the constraint

$$v_W^* = q_L^{*2} v_S + q_M^{*2} v_M + q_H^{*2} v_H + \lambda^2 v_B.$$
(C.2)

Solving (C.1) for μ_W^* yields

$$\mu_{W}^{\text{tan}} = -\frac{\begin{bmatrix} ((A - (\mu_{M} - \mu_{H}) B) (AC + (\mu_{M} - \mu_{H}) D)) v_{L} \\ + ((\mu_{L} - \mu_{H}) B - A) (AE - (\mu_{L} - \mu_{H}) D) v_{M} - (\mu_{L} - \mu_{M})^{2} BDv_{H} \\ - \frac{(A(\mu_{L} - \mu_{H}))^{2}}{2\gamma} \\ \hline \left[(A - (\mu_{M} - \mu_{H}) B)^{2} v_{S} + ((\mu_{L} - \mu_{H}) B - A)^{2} v_{M} + (\mu_{L} - \mu_{M})^{2} B^{2} v_{H} \right]$$
(C.3)

where μ_W^{tan} denotes the value of μ_W on the frontier at the point of tangency with expression (6), such that terms A to E are defined as:

$$A = v_L (\mu_M - \mu_H)^2 + v_M (\mu_L - \mu_H)^2 + v_H (\mu_L - \mu_M)^2;$$
(C.4)

$$B = v_M (\mu_L - \mu_H) + v_L (\mu_M - \mu_H);$$
 (C.5)

$$C = -\lambda \mu_B - \mu_M; \tag{C.6}$$

$$D = \begin{pmatrix} \upsilon_M \left\{ (\mu_L + \lambda \mu_B) (\mu_L - \mu_H) \right\} \\ + \upsilon_L (\lambda \mu_B + \mu_M) (\mu_M - \mu_H) \end{pmatrix};$$
(C.7)

and

$$E = \mu_L + \lambda \mu_B. \tag{C.8}$$

The value of v_W at the point of tangency, which we denote $v_W^{\tan}(\mu_W^{\tan})$, can be obtained by substituting equation (C.3) into expression (3). The global minimum portfolio is obtained by differentiating the efficient frontier v_W^* in expression (A.6) with respect to μ_W^* , setting the resulting expression to zero, and solving for μ_W^* . This yields a solution as in expression (C.3), albeit without the $(A(\mu_L - \mu_M))^2/2\gamma$ term in the numerator. Finally, from expressions (C.3)–(C.8) it is evident that under independent background risk μ_W^{tan} will be unaffected by change in v_B : v_B (and by implication $\sqrt{v_B} = \sigma_B$) is absent from (C.3).

C.2 Correlated background risk

Now consider the restricted case of the correlated model where background risk is correlated with each of the tradable assets, but the tradable assets are not correlated with each other,³² so that the equation for the efficient frontier is given by

$$v_W^* = q_L^{*2} v_L + q_M^{*2} v_M + q_H^{*2} v_H + \lambda^2 v_B + 2q_L^* \lambda \rho_{LB} \sigma_L \sigma_B + 2q_M^* \lambda \rho_{MB} \sigma_M \sigma_B + 2q_H^* \lambda \rho_{HB} \sigma_H \sigma_B.$$
(C.9)

 $^{^{32}}$ The impact on the efficient frontier when tradable assets are correlated is well explored in the literature. We therefore refrain from focusing on this area.

Assuming mean-variance preferences as in expression (6), the point of tangency with the portfolio frontier is given by solving for μ_W^* the expression

$$\begin{pmatrix} \frac{\partial q_L^*}{\partial \alpha} \frac{\partial \alpha}{\partial \mu_W^*} + \frac{\partial q_L^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*} \end{pmatrix} 2q_L^* \upsilon_L + \begin{pmatrix} \frac{\partial q_M^*}{\partial \alpha} \frac{\partial \alpha}{\partial \mu_W^*} + \frac{\partial q_M^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*} \end{pmatrix} 2q_M^* \upsilon_M + \frac{\partial q_H^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*} 2q_H^* \upsilon_H \\
+ \begin{pmatrix} \frac{\partial q_L^*}{\partial \alpha} \frac{\partial \alpha}{\partial \mu_W^*} + \frac{\partial q_L^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*} \end{pmatrix} 2\lambda \rho_{LB} \sigma_L \sigma_B \\
+ \begin{pmatrix} \frac{\partial q_M^*}{\partial \alpha} \frac{\partial \alpha}{\partial \mu_W^*} + \frac{\partial q_M^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*} \end{pmatrix} 2\lambda \rho_{MB} \sigma_M \sigma_B + \frac{\partial q_H^*}{\partial \Gamma^*} \frac{\partial \Gamma^*}{\partial \mu_W^*} 2\lambda \rho_{HB} \sigma_H \sigma_B = \frac{1}{\gamma}.$$
(C.10)

Although somewhat cumbersome, this yields

$$\mu_{W}^{\text{tan}} = -\frac{\begin{pmatrix} ((A - (\mu_{M} - \mu_{H})B)(AC + (\mu_{M} - \mu_{H})D))v_{L} \\ + ((\mu_{L} - \mu_{H})B - A)(AE - (\mu_{L} - \mu_{H})D)v_{M} - (\mu_{L} - \mu_{M})^{2}BDv_{H} \\ + (\mu_{L} - \mu_{M})A(A - (\mu_{M} - \mu_{H})B)\lambda\rho_{LB}\sigma_{L}\sigma_{B} \\ + (\mu_{L} - \mu_{M})A((\mu_{L} - \mu_{H})B - A)\lambda\rho_{MB}\sigma_{M}\sigma_{B} \\ - (\mu_{L} - \mu_{M})^{2}BA\lambda\rho_{HB}\sigma_{H}\sigma_{B} - \frac{(A(\mu_{L} - \mu_{M}))^{2}}{2\gamma} \\ \hline \left[(A - (\mu_{M} - \mu_{H})B)^{2}v_{L} + ((\mu_{L} - \mu_{H})B - A)^{2}v_{M} + (\mu_{L} - \mu_{M})^{2}B^{2}v_{H} \right]$$
(C.11)

where μ_W^{tan} denotes the value of μ_W on the frontier at the point of tangency in the risk-return space. Here, we note that the value of μ_W at the global minimum variance can be obtained by omitting the term $(A(\mu_L - \mu_M))^2/2\gamma$ which appears in the numerator of expression (C.11). Significantly, expression (C.11) implies that μ_W^{tan} will be partially determined by the presence of background risk, σ_B , when it is correlated with the tradable asset shares.

D The impact of background risk and risk preferences on μ_W^{tan}

In the correlated model, the additional presence in expression (C.11) of terms containing σ_B , namely

$$(\mu_L - \mu_M) A (A - (\mu_M - \mu_H) B) \lambda \rho_{LB} \sigma_L \sigma_B,$$
(D.1)

$$(\mu_L - \mu_M) A((\mu_L - \mu_H) B - A) \lambda \rho_{MB} \sigma_M \sigma_B, \qquad (D.2)$$

and

$$2\left(\mu_L - \mu_M\right)^2 BA\lambda\rho_{HB}\sigma_H\sigma_B,\tag{D.3}$$

implies that the presence of background risk will affect the point of tangency, μ_W^{tan} . In particular, the sign and magnitude of the correlation coefficients ρ_{LB} , ρ_{MB} , and ρ_{HB} are central to determining how background risk, which appears as $\sqrt{v_B} = \sigma_B$, affects the point of tangency with household preferences. Consider the impact of correlation between background wealth *B* and the high-risk tradable asset *H*, whilst assuming $\rho_{LB} = \rho_{MB} = 0$. For an increase in σ_B , a negative (positive) value of ρ_{HB} , will lead to a fall (rise) in the value of μ_W^{tan} , which using expression (D.3) in (C.11) is captured by

$$\frac{\partial \mu_W^{\text{tan}}}{\partial \sigma_B} = \frac{2(\mu_L - \mu_M)^2 BA\lambda \rho_{HB} \sigma_H}{\left[(A - (\mu_M - \mu_H) B)^2 \upsilon_L + ((\mu_L - \mu_H) B - A)^2 \upsilon_M + (\mu_L - \mu_M)^2 B^2 \upsilon_H \right]}.$$
 (D.4)

Inspection of expression (C.11) also indicates that a change in v_B will lead to a change in the value of μ_W corresponding to the global minimum variance, μ_W^g . This is unlike the case under independent background risk, in which the global minimum variance is unaffected by such a change.

For any given expected return in the upward sloping region of the frontier space where $\mu_W^{\text{tan}} > \mu_W^{\text{g}}$, the impact on μ_W^{tan} of a change in risk aversion $\gamma > 0$ will be given by

$$\frac{\partial \mu_W^{\text{tan}}}{\partial \gamma} = -\frac{\left(A(\mu_L - \mu_M)\right)^2}{2\gamma^2 \left[\begin{array}{c} \left(A - \left(\mu_M - \mu_H\right)B\right)^2 v_S \\ + \left(\left(\mu_L - \mu_H\right)B - A\right)^2 v_M + \left(\mu_L - \mu_M\right)^2 B^2 v_H \end{array} \right]}.$$
(D.5)

Expression (D.5) says that an increase (decrease) in risk aversion γ will be associated with a decrease (increase) in the value of μ_W at the new tangency point. Based on the results implied by the expressions for $\partial q_i^*/\partial \mu_W$, i = L, M, H in (4), the associated increase in μ_W^* will be associated with a fall (rise) in the

proportion of high-risk assets held by the household, and a rise (fall) in the proportion of low-risk assets held at the new point of tangency. However, the impact on q_M^* will be ambiguous. We emphasise here that the change in μ_W^{tan} implied by expression (D.5) is realised irrespective of whether background risk is assumed to be correlated with, or independent of, the tradable assets.

E Short-selling restrictions and Kuhn-Tucker conditions

In the presence of short-selling restrictions, Kuhn-Tucker conditions are used so that the (inequality constrained) minimum variance portfolio can be determined. For our minimization problem, we first convert the problem into a *maximization* one by multiplying the objective function, v_W , by -1. For the fully correlated model in Appendix A we wish to calculate

$$\max_{\Gamma} \left[-v_W \right] = - \begin{cases} \left(\alpha + \beta \Gamma \right)^2 v_L + \left(1 - \alpha - (1 + \beta) \Gamma \right)^2 v_M + \Gamma^2 v_H + \lambda^2 v_B \\ + 2 \left(\alpha + \beta \Gamma \right) \left(1 - \alpha - (1 + \beta) \Gamma \right) \rho_{LM} \sigma_L \sigma_M \\ + 2 \left(\alpha + \beta \Gamma \right) \Gamma \rho_{LH} \sigma_L \sigma_H + 2 \left(1 - \alpha - (1 + \beta) \Gamma \right) \Gamma \rho_{MH} \sigma_M \sigma_H \\ + 2 \left(\alpha + \beta \Gamma \right) \lambda \rho_{LB} \sigma_L \sigma_B + 2 \left(1 - \alpha - (1 + \beta) \Gamma \right) \lambda \rho_{MB} \sigma_M \sigma_B \\ + 2 \Gamma \lambda \rho_{HB} \sigma_H \sigma_B \end{cases}$$
(E.1)

whilst ensuring the non-negativity of our assets shares, namely

$$q_L \ge 0; \ q_M \ge 0; \ q_H \ge 0.$$
 (E.2)

Using the results in (A.1), it is useful to restate the inequalities in (E.2) as

$$\begin{cases} g^0(\Gamma) = \alpha + \Gamma\beta \ge 0; \\ g^1(\Gamma) = 1 - \alpha - (1+\beta)\Gamma \ge 0; \\ \Gamma \ge 0. \end{cases}$$
(E.3)

Forming the Lagrangian gives

$$\mathbb{L} = -v_W + \phi_0 \left(\alpha + \Gamma\beta + s_0\right) + \phi_1 \left(1 - \alpha - (1 + \beta)\Gamma + s_1\right)$$
(E.4)

which we wish to evaluate subject to

$$-g^{i}(\Gamma) + s_{i} = 0, \quad i = 0, 1$$
(E.5)

and the following *non-negativity* constraints

$$\Gamma, s_0, s_1 \ge 0. \tag{E.6}$$

Setting up the Lagrangian, \mathbb{L} , thus yields

$$\mathbb{L} = -v_W + \sum_{i=0}^{1} \left(\phi_i g^i(\Gamma) - s_i \right) \qquad i = 0, 1.$$
(E.7)

Since Γ , s_0 and s_1 must be non-negative, the first order conditions on our variable set implies that for a maximum, the following set of conditions must hold,

$$\frac{\partial \mathbb{L}}{\partial \Gamma} \leq 0 \qquad \Gamma \geq 0 \qquad and \qquad \Gamma \frac{\partial L}{\partial T} = 0 \\
\frac{\partial \mathbb{L}}{\partial s_i} \leq 0 \qquad s_i \geq 0 \qquad and \qquad s_i \frac{\partial L}{\partial s_i} = 0 \\
\frac{\partial \mathbb{L}}{\partial \phi_i} = 0 \qquad \qquad i = 0, 1$$
(E.8)

From expressions (E.7) and (E.8), it follows that if $\frac{\partial \mathbb{L}}{\partial s_i} = -\phi_i$, then $-\phi_i \leq 0$, $s_i \geq 0$ and $-\phi_i s_i = 0$, or equivalently,

$$\phi_i \ge 0; \ s_i \ge 0; \ \text{and} \ \phi_i s_i = 0.$$
 (E.9)

However, the third line of (E.8) – a restatement of the modified constraints in (E.5) – implies that $s_i = g^i(\Gamma)$. By substituting the latter into (E.9), means that the second and third lines of (E.8) can be combined to yield

$$g^{i}(\Gamma) \ge 0 \quad \phi_{i} \ge 0 \quad \text{and} \quad \phi_{i}\left(g^{i}(\Gamma)\right) = 0.$$
 (E.10)

This enables us to express the first order conditions (E.8) in an equivalent form without dummy variables. Using the symbol g_{Γ}^{i} to denote $\frac{\partial g^{i}}{\partial \Gamma}$ the Kuhn-Tucker conditions for our maximization problem can be expressed as

$$\frac{\partial \mathbb{L}}{\partial \Gamma} = \frac{\partial \left[-\upsilon_W\right]}{\partial \Gamma} + \sum_{i=0}^{1} \phi_i g_{\Gamma}^i \le 0 \qquad \Gamma \ge 0 \quad \text{and} \quad \Gamma \frac{\partial L}{\partial \Gamma} = 0; \tag{E.11}$$
$$g^i(\Gamma) \ge 0 \quad \phi_i \ge 0 \quad \text{and} \quad \phi_i[g^i(\Gamma)] = 0, \quad i = 0, 1.$$

Writing these using the explicit function form for $g^i(\Gamma)$, i = 0, 1 yields

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial \Gamma} &= \frac{\partial \left[-\upsilon_W \right]}{\partial \Gamma} + \phi_0 \beta - \phi_1 (1+\beta) \le 0 \quad \Gamma \ge 0 \quad \text{and} \quad \Gamma \frac{\partial \mathbb{L}}{\partial \Gamma} = 0; \\ \alpha + \Gamma \beta \ge 0 \quad \phi_0 \ge 0 \quad \text{and} \quad \phi_0 [\alpha + \Gamma \beta] = 0; \\ 1 - \alpha - (1+\beta)\Gamma \ge 0 \quad \phi_1 \ge 0 \quad \text{and} \quad \phi_1 [1 - \alpha - (1+\beta)\Gamma] = 0. \end{aligned}$$
(E.12)

Assuming $\phi_1 = 0, \phi_0 \ge 0$ yields

$$\frac{\partial \mathbb{L}}{\partial \Gamma} = \frac{\partial \left[-\upsilon_W\right]}{\partial \Gamma} + \phi_0 \beta \le 0 \quad \Gamma \ge 0 \quad \text{and} \quad \Gamma \frac{\partial \mathbb{L}}{\partial \Gamma} = 0;$$

$$\alpha + \Gamma \beta \ge 0 \quad \phi_0 \ge 0 \quad \text{and} \quad \phi_0 [\alpha + \Gamma \beta] = 0;$$

$$1 - \alpha - (1 + \beta)\Gamma \ge 0 \quad \phi_1 \ge 0 \quad \text{and} \quad \phi_1 [1 - \alpha - (1 + \beta)\Gamma] = 0.$$

(E.13)

F Tobit Results: Coefficients and Marginal Effects

		2	Specification	ı ,	
	1	2	3	4	5
Age	0.0100^{***} (0.0012)	0.0099^{***} (0.0012)	0.0099^{***} (0.0012)	0.0099^{***} (0.0012)	0.0100^{***} (0.0012)
Age Squared	(0.0012) -0.0091^{***} (0.0012)	-0.0090^{***} (0.0012)	-0.0090^{***} (0.0012)	(0.0012) -0.0091^{***} (0.0012)	-0.0091^{***} (0.0012)
Male	0.0157	0.0168*	0.0162*	0.0164*	0.0158
White	(0.0097) 0.0877^{***} (0.0074)	(0.0097) 0.0888^{***} (0.0074)	(0.0097) 0.0892^{***} (0.0074)	(0.0097) 0.0876^{***} (0.0074)	(0.0097) 0.0881^{***} (0.0074)
Education:		· /	· /	· /	· /
High School College Degree	$\begin{array}{c} 0.1173^{***} \\ (0.0136) \\ 0.2206^{***} \end{array}$	$\begin{array}{c} 0.1178^{***} \\ (0.0136) \\ 0.2212^{***} \end{array}$	$\begin{array}{c} 0.1175^{***} \\ (0.0136) \\ 0.2216^{***} \end{array}$	$\begin{array}{c} 0.1175^{***} \\ (0.0136) \\ 0.2200^{***} \end{array}$	$\begin{array}{c} 0.1171^{***} \\ (0.0136) \\ 0.2206^{***} \end{array}$
Married	(0.0133) -0.0014	(0.0134) -0.0012	(0.0133) -0.0007	(0.0134) -0.0014	(0.0133) -0.0008
	(0.0098)	(0.0098)	(0.0098)	(0.0098)	(0.0098)
Child	-0.0243^{***} (0.0063)	-0.0244^{***} (0.0063)	-0.0248^{***} (0.0063)	-0.0240^{***} (0.0063)	-0.0244^{***} (0.0063)
Employment: Employed	0.1317^{***}	0.1325^{***}	0.1320^{***}	0.1320^{***}	0.1313^{***}
Self-Employed	(0.0119) 0.0274^{*} (0.0142)	(0.0119) 0.0283^{**} (0.0142)	(0.0119) 0.0277^{*} (0.0142)	(0.0119) 0.0278^{*} (0.0142)	(0.0119) 0.0272^{*} (0.0142)
Retired	(0.0142) 0.0794^{***} (0.0151)	(0.0142) 0.0801^{***} (0.0151)	(0.0142) 0.0799^{***} (0.0151)	(0.0142) 0.0792^{***} (0.0151)	(0.0142) 0.0789^{***} (0.0151)
Economic Exp.	0.0133^{***}	0.0131^{***}	$(0.0131)^{(0.0131)}$ (0.0131^{***}) (0.0036)	0.0131***	$(0.0131)^{***}$ (0.0036)
Saving Horizon	(0.0036) 0.0163^{***}	(0.0036) 0.0162^{***} (0.0022)	0.0164* ^{**}	(0.0036) 0.0159^{***} (0.0022)	Ò.0162* ^{**} *
Risk Attitudes	(0.0023) 0.1111^{***} (0.0034)	(0.0023) 0.1112^{***} (0.0034)	(0.0023) 0.1109^{***} (0.0034)	(0.0023) 0.1113^{***} (0.0034)	(0.0023) 0.1110^{***} (0.0034)
Total Income	0.0610^{***}	0.0609* ^{**} *	(0.0034) (0.0610^{***}) (0.0026)	(0.0034) (0.0606^{***}) (0.0026)	0.0608* ^{**} *
Net Worth	(0.0026) 0.0080^{***} (0.0004)	(0.0026) 0.0080^{***} (0.0004)	Ò.0080* [*] *	0.0079* ^{**} *	(0.0026) 0.0080^{***} (0.0004)
Home owner	(0.0004) 0.0790^{***} (0.0068)	(0.0004) 0.0798^{***} (0.0068)	$(0.0004) \\ 0.0800^{***} \\ (0.0068)$	(0.0004) 0.0786^{***} (0.0069)	(0.0004) 0.0790^{***} (0.0068)
Background Risks: Business Own.	-0.0044	-0.0042	-0.0044	-0.0042	-0.0045
Multi Earners	(0.0080) 0.0273^{***}	(0.0080) 0.0274^{***}	(0.0080) 0.0275^{***}	(0.0080) 0.0272^{***}	(0.0080) 0.0273^{***}
Major Fin. Exp.	(0.0067) 0.0324^{***}	(0.0067) 0.0323^{***}	(0.0067) 0.0317^{***}	(0.0067) 0.0326^{***}	(0.0067) 0.0320^{***}
No Health insur.	$(0.0056) \\ -0.1217^{***} \\ (0.0111)$	$(0.0056) \\ -0.1224^{***} \\ (0.0111)$	$(0.0056) \\ -0.1224^{***} \\ (0.0111)$	$(0.0056) \\ -0.1211^{***} \\ (0.0112)$	$(0.0056) \\ -0.1211^{***} \\ (0.0112)$
Inheritance	(0.0111) 0.0284^{***} (0.0070)	(0.0111) 0.0286^{***} (0.0070)	(0.0111) 0.0284^{***} (0.0070)	(0.0112) 0.0286^{***} (0.0070)	(0.0112) 0.0283^{***} (0.0070)
Know Income	(0.0070) 0.0329^{***} (0.0060)	(0.0010) 0.0327^{***} (0.0060)	(0.0010) 0.0328^{***} (0.0060)	(0.0070) 0.0324^{***} (0.0060)	(0.0070) (0.0325^{***}) (0.0060)
Income Above Norm.	(0.0000) (0.0001) (0.0007)	(0.0000) (0.0001) (0.0007)	(0.0000) (0.0001) (0.0007)	(0.0000) (0.0001) (0.0007)	(0.0000) (0.0001) (0.0007)
Income Below Norm.	(0.0007) (0.0000 (0.0007)	(0.0007) (0.0000 (0.0007)	(0.0007) (0.0000 (0.0007)	(0.0007) -0.0000 (0.0007)	(0.0007) (0.0000 (0.0007)
Borrowing Constraints: Constraint 1	-0.0391***	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Constraint 2	(0.0086)	-0.0296^{***}			
Constraint 3		(0.0071)	-0.0306^{***} (0.0082)		
Constraint 4			(0.0082)	-0.0350^{***} (0.0073)	
Constraint 5				(0.0013)	-0.0354^{***}
Constant	-1.4759^{***}	-1.4714^{***}	-1.4781^{***}	-1.4620^{***}	(0.0075) -1.4704*** (0.0412)
Year Fixed Effects	$\begin{array}{c} (0.0413) \\ \text{Yes} \end{array}$	$\begin{array}{c} (0.0416) \\ \text{Yes} \end{array}$	(0.0411) Yes	$\begin{array}{c} (0.0419) \\ \text{Yes} \end{array}$	(0.0413) Yes
Observations	$27,\!618$	27,618	27,618	27,618	27,618

Table F.1: Tobit mode: High risk asset share - Coefficients

[1] Tobit regression marginal effects. [2] Standard errors in parentheses. [3] *denotes significance at the 10% level **denotes significance at the 5% level and ***denotes significance at the 1% level.

	_		Specification		
	1	2	3	4	5
Age Age Squared	0.0068*** (0.0008) -0.0062***	$\begin{array}{c} 0.0068^{***} \\ (0.0008) \\ -0.0062^{***} \end{array}$	0.0068*** (0.0008) -0.0062***	0.0068*** (0.0008) -0.0062***	0.0068^{***} (0.0008) -0.0062^{***}
Male	(0.0002) (0.0008) 0.0107	(0.0008) 0.0114^*	(0.0002) (0.0008) 0.0111^*	(0.0008) (0.0112^*)	(0.0002) (0.0008) 0.0107
White	(0.0066) 0.0598^{***}	(0.0066) 0.0605^{***}	(0.0066) 0.0608^{***}	(0.0066) 0.0597^{***}	(0.0066) 0.0601^{***}
Education:	(0.0050)	(0.0050)	(0.0050)	(0.0051)	(0.0050)
High School	0.0800^{***} (0.0093)	0.0804^{***} (0.0093)	0.0802^{***} (0.0093)	0.0801^{***} (0.0093)	0.0799^{***} (0.0093)
College Degree	(0.1505^{***}) (0.0091)	(0.1509^{***}) (0.0091)	(0.1511^{***}) (0.0091)	(0.1501^{***}) (0.0091)	0.1504^{***} (0.0091)
Married	-0.0009' (0.0067)	-0.0008' (0.0067)	-0.0005' (0.0067)	-0.0009' (0.0067)	-0.0006 (0.0067)
Child	(0.0007) -0.0166^{***} (0.0043)	(0.0001) -0.0166^{***} (0.0043)	(0.0001) -0.0169^{***} (0.0043)	(0.0001) -0.0164^{***} (0.0043)	(0.0007) -0.0167^{***} (0.0043)
Employment: Employed	0.0898***	0.0904***	0.0900***	0.0900***	0.0896***
Self-Employed	$(0.0081) \\ 0.0187^* \\ (0.0097)$	(0.0081) 0.0193^{**} (0.0097)	$(0.0081) \\ 0.0189^* \\ (0.0097)$	$(0.0081) \\ 0.0190^{*} \\ (0.0097)$	$(0.0081) \\ 0.0185^{*} \\ (0.0097)$
Retired	0.0542^{***} (0.0103)	0.0547^{***} (0.0103)	0.0545^{***} (0.0103)	0.0540^{***} (0.0103)	0.0538^{***} (0.0103)
Economic Exp.	0.0091* [*] ** (0.0025)	0.0089^{***} (0.0025)	0.0089* ^{**} * (0.0025)	0.0089* ^{***} (0.0025)	0.0089* ^{′**} (0.0025)
Saving Horizon	0.0111^{***} (0.0016)	0.0111^{***} (0.0016)	0.0112* ^{***} (0.0016)	0.0109^{***} (0.0016)	0.0110^{***} (0.0016)
Risk Attitudes	0.0758^{***} (0.0023)	0.0758^{***} (0.0023)	0.0757^{***} (0.0023)	0.0759^{***} (0.0023)	(0.0757^{***}) (0.0023)
Total Income	(0.0018) (0.0018)	(0.0025) 0.0415^{***} (0.0018)	(0.0018) (0.0018)	(0.00128) (0.0414^{***}) (0.0018)	$(0.0015)^{0.0415***}$ $(0.0018)^{0.0018}$
Net Worth	(0.0013) 0.0055^{***} (0.0003)	(0.0013) 0.0054^{***} (0.0003)	(0.0013) (0.0055^{***}) (0.0003)	(0.0013) 0.0054^{***} (0.0003)	(0.0013) (0.0055^{***}) (0.0003)
Home owner	(0.0003) (0.0539^{***}) (0.0046)	(0.0003) 0.0544^{***} (0.0046)	(0.0003) 0.0546^{***} (0.0046)	(0.0003) 0.0536^{***} (0.0047)	(0.0003) (0.0539^{***}) (0.0046)
Background Risks: Business Own.	-0.0030	-0.0029	-0.0030	-0.0029	-0.0030
Multi Earners	(0.0054) 0.0186^{***} (0.0046)	(0.0054) 0.0187^{***} (0.0046)	$(0.0054) \\ 0.0188^{***} \\ (0.0046)$	(0.0054) 0.0185^{***} (0.0046)	(0.0054) 0.0186^{***} (0.0046)
Major Fin. Exp.	(0.0046) 0.0221^{***} (0.0038)	(0.0040) 0.0220^{***} (0.0038)	(0.0040) 0.0216^{***} (0.0038)	(0.0040) 0.0222^{***} (0.0038)	(0.0040) 0.0218^{***} (0.0038)
No Health insur.	(0.0033) -0.0830^{***} (0.0076)	(0.0038) -0.0835^{***} (0.0076)	(0.0036) -0.0835^{***} (0.0076)	(0.0038) -0.0826^{***} (0.0076)	(0.0038) -0.0826^{***} (0.0076)
Inheritance	(0.0070) 0.0193^{***} (0.0047)	(0.0010) (0.0195^{***}) (0.0047)	(0.0070) 0.0194^{***} (0.0047)	(0.00195^{***}) (0.0047)	(0.0070) (0.0193^{***}) (0.0047)
Know Income	(0.0041) 0.0224^{***} (0.0041)	(0.0041) (0.0223^{***}) (0.0041)	(0.0041) 0.0224^{***} (0.0041)	(0.0041) (0.0221^{***}) (0.0041)	(0.0041) (0.0222^{***}) (0.0041)
Income Above Norm.	Ò.0000 (0.0001	0.0001	0.0001	0.0001
Income Below Norm.	(0.0005) 0.0000 (0.0005)	(0.0005) 0.0000 (0.0005)	(0.0005) 0.0000 (0.0005)	(0.0005) -0.0000 (0.0005)	(0.0005) 0.0000 (0.0005)
Borrowing Constraints: Constraint 1	(0.0005) - 0.0267^{***} (0.0058)	(0.0005)	(0.0005)	(0.0005)	(0.0005)
Constraint 2	(0.0058)	-0.0202^{***}			
Constraint 3		(0.0048)	-0.0209^{***}		
Constraint 4			(0.0056)	-0.0238^{***} (0.0050)	
Constraint 5				(0.000)	-0.0242^{***} (0.0051)
Year Fixed Effects	Yes	Yes	Yes	Yes	(0.0051) Yes
Observations	27,618	27,618	27,618	27,618	27,618

Table F.2: Tobit model: High risk asset share - Marginal effects

[1] Tobit regression marginal effects [2] Standard errors in parentheses. [3] *denotes significance at the 10% level **denotes significance at the 5% level and ***denotes significance at the 1% level.