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# Fast compressed sensing reconstruction using the least squares and signal correlation

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**Keywords:** compressed sensing, least squares, correlation, redundant reconstruction

## Abstract

A fast compressed sensing reconstruction using least squares method with the signal correlation is presented in this paper. It is well known that the complexity of  $l_1$ -minimisation is very high and is undesirable for many practical applications. The least squares method, on the other hand, has a much lower complexity. However, least squares does not promote the sparsity of signal and therefore cannot provide acceptable reconstructed results. The main contribution of this paper is to show that by exploiting signal correlation, the reconstruction error of least squares is greatly improved. Moreover, the correlated reference used in this method is very flexible, and can contain many kinds of correlation, such as spatial or temporal correlation. Experimental results show that the performance of this method is comparable to the state-of-the-art algorithms, whilst having a much lower complexity. It also shows that this method can be applied to both sparse and redundant signal reconstruction.

## 1 Introduction

In recent years, compressed sensing is of interest among many signal processing researchers as well as statisticians and engineers. In essence, compressed sensing allows a complete signal to be obtained from an under-sampled measurement, in contrast to the traditional signal acquisition/compression currently in use. Since compressed sensing was introduced in [1], this approach instantaneously has attracted lots of attention because of its potential to reduce the acquisition complexity. Low acquisition complexity is crucial in many applications, such as remote sensing and medical imaging. Therefore, there are many research work on applying compressed sensing to such application [2], [3], [4].

Compressed sensing states that for a signal  $\mathbf{x} \in \mathbb{R}^n$ , it is possible to measure only a small subset of samples and then reconstruct a full signal  $\hat{\mathbf{x}}$  afterward. In order for this scheme to be success, signal  $\mathbf{x}$  is required to be sparse. Signal  $\mathbf{x}$  can be sparse in its natural domain or in any of its transform domain, in which case the basis of the sparse transformation is added into the system. A measurement  $\mathbf{y} \in \mathbb{R}^m$  where  $m \ll n$  is obtained by using an incoherent sensing mechanism, *i.e.*,

$$\mathbf{y} = \mathbf{A}\mathbf{x},$$

where  $\mathbf{A} \in \mathbb{R}^{m \times n}$  is the sensing matrix. It has been shown in [5] that by using a sensing matrix that is incoherent to the signal  $\mathbf{x}$ , the power of under-sampled artefacts is spread out and the distortion to the signal is small. It is also shown that most random matrices are incoherent with most signals. Once the measurement  $\mathbf{y}$  is obtained, the full signal  $\hat{\mathbf{x}}$  can be obtained by using the optimisation process known as  $l_1$ -minimisation ( $l_1$ -min), *i.e.*,

$$\min \|\hat{\mathbf{x}}\|_1 \text{ subject to } \mathbf{A}\hat{\mathbf{x}} = \mathbf{y}$$

$l_1$ -min promotes the sparsity of the solution by minimising  $l_1$ -norm of the reconstructed signal (or the sparse domain of the reconstructed signal). The sparsest solution yielded by this process is the best approximation of  $\mathbf{x}$ . It has been show in [5] that if  $\mathbf{x}$  is sufficiently sparse, the reconstruction can be exact.

The problem with the previously described scheme is that  $l_1$ -min is a non-linear optimisation process. It cannot be solved mathematically and the only way to obtain the result is to use some optimisation algorithms. To date, many algorithms have been proposed to consider both the accuracy of the solution and the complexity of the algorithm [6], [7], including many greedy algorithms [8], [9], [10]. It is suffice to say that there is a trade-off between the accuracy and the complexity. Nevertheless, even with the fast greedy algorithms, it is still quite a challenge to implement this reconstruction process into most real-time applications. In a case of compressed video acquisition/ reconstruction, some might be willing to wait for a few hours to reconstruct a video sequence from its under-sampled data. However, for most people, doing so is undesirable. Scarlett *et al.* [11] states that compressed sensing is nothing more than just a shift of complexity load from the encoder side to the decoder side. It is clear that without a faster, near real-time reconstruction concept, the application of compressed sensing is very limited.

Another issue of  $l_1$ -min is that its sparsity-promoting objective does not always provide the best solution. The reason is that most natural signals are highly complex and not as sparse as expected. To overcome this, many works employ the use of side-information to improve the reconstruction accuracy. The motivation of such approach is the fact that in most applications, some characteristics of the signal can be predetermined or approximated. This can easily be seen in Magnetic Resonance Imaging, sensor networks, and multiview imaging. The use of side-information can reduce the possible space of solution significantly and thus help

improve the reconstruction accuracy. Many kinds of side-information have been employed into the reconstruction process. In [11], use the sparsity pattern of signals as a side-information. Sparsity pattern can either be known in advance or be approximated online. The model-based compressed sensing [12] uses the complete sparsity pattern where both location and structure of sparse supports are known. The Kalman-filtered compressed sensing, on the other hand, estimates the sparse supports online using the assumption that sparse supports change slowly [13]. Other kinds of side-information are also available. The work in [14] uses the signal's upper and lower bound as a prior knowledge. The work in [15] reconstructs a group of several signals which have the same statistical characteristics together to improve overall accuracy. In dynamic MRI, the temporal redundancy between each scan is used as the side-information [16].

Let us return to the complexity problem and the practical consideration of the optimisation process. The most widely used optimisation process is the least squares method, typically to solve over-sampled inverse problems. It is well known, however, least squares method performs poorly for under-sampled problems. However, because least squares is a linear operation, its complexity is far lower than that of the  $l_1$ -min.

Here, we proposed to combine the benefits of both the least squares and the side-information together. By using least squares method, the reconstruction process can be done very rapidly, providing the real-time reconstruction capability. The accuracy of the reconstruction results is improved by using references. References are used as side-information to the optimisation process. This paper also shows that such references are very flexible and can be based on various kinds of signal correlation. The exploitation of spatial correlation and temporal correlation are demonstrated here. It also shows that by using correlated signal as references, the sparsity requirement of the signal is no longer necessary. Thus, the proposed method not only allows the rapid reconstruction but also allows the compressed sensing to be performed in the redundant domain.

## 2 Compressed sensing reconstruction using the least squares and signal correlation

### 2.1 Why use the least squares?

The least squares method is a very popular method to solve over-determined problems, such as, data fitting and regression, because of its simplicity. However, it is well known that the least squares, or more accurately the  $l_2$ -norm minimisation, does not perform well, when applied to underdetermined problems. This is because the least squares does not promote the sparsity of the solution. On the contrary, the result of the least squares, based on the geometry of  $l_2$ -ball, tends to be less sparse as much as possible.

Nevertheless, the least squares method is much simpler than the  $l_1$ -min in term of complexity. Whilst  $l_1$ -min is a highly

non-linear algorithm, which usually takes hundreds on iterations to solve a problem, least squares is a linear operation that can yield a solution instantly. This simplicity is the key that many practical applications, despite its tendency to be affected by outliers, choose least squares as a method of choice.

The motivation of this work is that the least squares should be able to perform fairly reasonably well with non-sparse signals. As pointed out earlier, the least squares does not promote the sparsity of solution. An interesting observation is that most natural signals are not quite sparse. Even though they can be transformed into some sparser domains, their sparsity levels are usually far from being sufficient for a perfect compressed sensing reconstruction using sparsity-promoting function. In these cases, the errors of  $l_1$ -min and least squares are about the same in magnitude, but they are from different sources. The error of  $l_1$ -min is due to the fact that the solution is too sparse, whilst the error of the least squares is due to the solution's sparsity is too small.

It is, however, possible to drop the notion of sparsity entirely and use another objective function instead. Here we are proposing the use of signal correlation as the objective function. The correlation is maximised using the least squares method in order to obtain the best solution, as is shown in Proposition 1.

**Proposition 1.** *If the signal  $\mathbf{x} \in \mathbb{R}^n$  has a correlated reference  $\mathbf{r} \in \mathbb{R}^n$ , the reconstructed signal  $\hat{\mathbf{x}} \in \mathbb{R}^n$  can be obtained from the compressed measurement  $\mathbf{y} \in \mathbb{R}^m$ ,  $\mathbf{y} = \mathbf{A}\mathbf{x}$  by*

$$\hat{\mathbf{x}} = \mathbf{r} + \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} (\mathbf{y} - \mathbf{A}\mathbf{r}). \quad (1)$$

**Proof.** Define an  $l_2$ -minimisation problem as

$$\min \|\hat{\mathbf{x}} - \mathbf{r}\|_2 \quad \text{subject to } \mathbf{A}\hat{\mathbf{x}} = \mathbf{y}. \quad (2)$$

Define a Lagrangian as

$$\mathcal{L}(\hat{\mathbf{x}}) = \|\hat{\mathbf{x}} - \mathbf{r}\|_2^2 + \lambda^T (\mathbf{A}\hat{\mathbf{x}} - \mathbf{y}).$$

Set a derivative of  $\mathcal{L}(\hat{\mathbf{x}})$  to zero, *i.e.*,

$$\frac{\partial}{\partial \hat{\mathbf{x}}} \mathcal{L}(\hat{\mathbf{x}}) = 2\hat{\mathbf{x}} - 2\mathbf{r} + \mathbf{A}^T \lambda = 0,$$

to obtain

$$\hat{\mathbf{x}} = \mathbf{r} - \frac{1}{2} \mathbf{A}^T \lambda. \quad (3)$$

To solve for the Lagrange Multiplier  $\lambda$ , substitute Equation (3) into  $\mathbf{y} = \mathbf{A}\hat{\mathbf{x}}$  to obtain

$$\begin{aligned} \mathbf{y} &= \mathbf{A}\hat{\mathbf{x}} = \mathbf{A}(\mathbf{r} - \frac{1}{2} \mathbf{A}^T \lambda) \\ &= \mathbf{A}\mathbf{r} - \frac{1}{2} \mathbf{A}\mathbf{A}^T \lambda. \end{aligned} \quad (4)$$

From Equation (4), we can get

$$\mathbf{A}\mathbf{A}^T \lambda = -2(\mathbf{y} - \mathbf{A}\mathbf{r})$$

and, finally,

$$\lambda = -2(\mathbf{A}\mathbf{A}^T)^{-1}(\mathbf{y} - \mathbf{A}\mathbf{r}). \quad (5)$$

Substitute Equation (5) back into Equation (3) to obtain

$$\hat{\mathbf{x}} = \mathbf{r} + \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} (\mathbf{y} - \mathbf{A}\mathbf{r}). \quad (6)$$

The importance of the reference signal and its correlation will be discussed further. ■

## 2.2 Why use signal correlation?

As discussed in Section 1, many recent works employ the use of side-information to improve the accuracy of reconstruction. Whilst many kinds of side-information, such as sparse support [13] and statistical model [12], are regularised into the problem, a simple signal correlation is also widely used in many works [17], [18]. The main benefit of the signal correlation is its simplicity. Many practical applications naturally acquire signals that are rich with such correlation, for example, temporal redundancy within a video sequence or spatial redundancy in a multi-view image. Some applications, such as, Magnetic Resonance Imaging (MRI) has both spatial and temporal correlation within its data. These correlations can easily be exploited in the reconstruction method.

It is possible to maximise the correlation between a reconstructed signal and its correlated signal, which we call a *reference*, during the reconstruction from the compressed measurement. Moreover, this correlation maximisation can replace the minimisation of the sparsity as the objective function. It is shown in Proposition 2 that the error from this reconstruction scheme is limited to no more than twice the distance between the signal and the reference.

**Proposition 2.** *Given a known reference  $\mathbf{r} \in \mathbb{R}^n$ , a solution  $\hat{\mathbf{x}}_p$  of the problem*

$$\min \|\hat{\mathbf{x}} - \mathbf{r}\|_p \text{ subject to } \mathbf{A}\hat{\mathbf{x}} = \mathbf{y}, \quad (7)$$

where  $p > 0$  and  $\mathbf{y} = \mathbf{A}\mathbf{x}$  is a measurement of  $\mathbf{x} \in \mathbf{X}_p(R)$ , where

$$\mathbf{X}_p(R) = \{\mathbf{x} \mid \|\mathbf{x} - \mathbf{r}\|_p \leq R, \mathbf{x} \in \mathbb{R}^n\}, \quad (8)$$

must satisfy

$$\sup \|\mathbf{x} - \hat{\mathbf{x}}_p\|_2 \leq 2 \|\mathbf{x} - \mathbf{r}\|_2. \quad (9)$$

**Proof.** Consider a set of possible solution from  $= \mathbf{A}\mathbf{x}$  :

$$\hat{\mathbf{X}}_p(\mathbf{y}) = \{\mathbf{x} \mid \mathbf{y} = \mathbf{A}\mathbf{x}, \mathbf{x} \in \mathbf{X}_p(R)\}.$$

According to the theory of Optimal Recovery/ Information-based Complexity [19], the central algorithm  $A_c$  yields the central solution  $\hat{\mathbf{x}}^*$  of the set. This make

$$\text{radius}(\hat{\mathbf{X}}_p(\mathbf{y})) = \sup\{\|\mathbf{x} - \hat{\mathbf{x}}^*\|_2 \mid \mathbf{x} \in \hat{\mathbf{X}}_p(\mathbf{y})\}. \quad (10)$$

Because the least-norm solution  $\hat{\mathbf{x}}_p \in \hat{\mathbf{X}}_p(\mathbf{y})$ , therefore

$$\|\hat{\mathbf{x}}_p - \hat{\mathbf{x}}^*\|_2 \leq \text{radius}(\hat{\mathbf{X}}_p(\mathbf{y})). \quad (11)$$

Since the triangle inequality gives

$$\|\mathbf{x} - \hat{\mathbf{x}}_p\|_2 \leq \|\mathbf{x} - \hat{\mathbf{x}}^*\|_2 + \|\hat{\mathbf{x}}^* - \hat{\mathbf{x}}_p\|, \quad (12)$$

put Equation (10) and (11) into Equation (12) gives

$$\begin{aligned} \|\mathbf{x} - \hat{\mathbf{x}}_p\|_2 &= 2 \text{radius}(\hat{\mathbf{X}}_p(\mathbf{y})) \\ &\leq 2 \sup\{\|\mathbf{x} - \hat{\mathbf{x}}^*\|_2 \mid \mathbf{x} \in \hat{\mathbf{X}}_p(\mathbf{y})\}. \end{aligned} \quad (13)$$

Finally, because the central solution  $\hat{\mathbf{x}}^*$  is at the centre of  $\mathbf{X}_p(R)$ , from Equation (8), therefore  $\hat{\mathbf{x}}^* = \mathbf{r}$ . This makes Equation (13) to become

$$\sup \|\mathbf{x} - \hat{\mathbf{x}}_p\|_2 \leq 2 \|\mathbf{x} - \mathbf{r}\|_2. \quad \blacksquare$$

Proposition 2 shows that the error limit is based purely on the distance from the signal to its reference, and therefore the notion of signal sparsity is no longer important. This enables this reconstruction method to work very well with non-sparse signals, particularly when the correlation of the reference is high.

Since this method does not use the signal sparsity, it is capable of reconstructing the signal both in the sparse domain as well as the redundant domain. Moreover, there is no special characteristic of reference required. The detail about the reference signal will be discussed in the next section.

## 3 Experimental results

There are two experiments in this paper, which are devised to demonstrate the performance of the proposed method both in term of quality and complexity. The first experiment demonstrates the exploitation of spatial correlation in images whilst the second experiment demonstrates the exploitation of temporal correlation in video sequences. In every experiment, the sampling and reconstruction is done in both sparse and redundant representation of the signals. The quality and complexity of the proposed method are discussed and compared with state-of-the-art reconstruction algorithms; these algorithms include both  $l_1$ -min methods, namely ISAL1 [6] and  $l_1$ -Homotopy [7], and greedy methods, namely Subspace Pursuit [8], CoSaMP [9], and Regularised Orthogonal Matching Pursuit [10].

### 3.1 References with spatial correlation

The first experiment demonstrates the use of references that contain the spatial correlation to the signals. In this experiment, an image is sampled and reconstructed in row-by-row basis. Each row in an image usually correlated to nearby rows spatially; hence some rows are selected as references to reconstruct the other rows.

This paragraph explains the setting of this experiment in precise details. Under-sampled measurements  $\mathbf{y}_i$  correspond to each row  $\mathbf{x}_i$  from an image  $\mathbf{x}$  are acquired as random linear combinations of  $\mathbf{x}_i$  using random Gaussian matrices. Each  $\mathbf{y}_i$  is then reconstructed into  $\hat{\mathbf{x}}_i$  individually using reconstruction algorithms. A set of 8 random images is used as a test set in this experiment. Each image is under-sampled at the factors of 0.25, 0.5, and 0.75. For the proposed method, rows  $\mathbf{x}_L$  at every  $L$  interval are used as references, and are conventionally, uncompressed sampled. The value of  $L = 4, 8$  and 12, chosen arbitrarily, are shown here.

Firstly, let us look at the reconstruction quality in sparse domain. The sparse representation used in this experiment is the wavelet domain. Figure 1 shows the peak signal-to-noise ratio (PSNR) of the reconstruction results using each algorithm. It can clearly be seen that the proposed algorithm outperformed every algorithm presented. It can also be seen

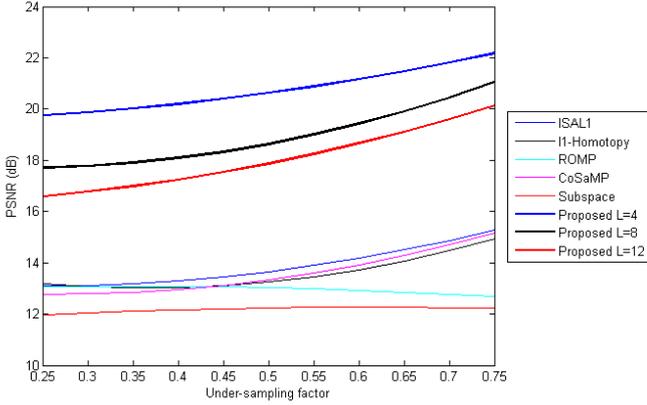


Figure 1: PSNR of reconstructed results from various algorithms exploiting spatial correlation. Sampling and reconstruction is done in wavelet domain.

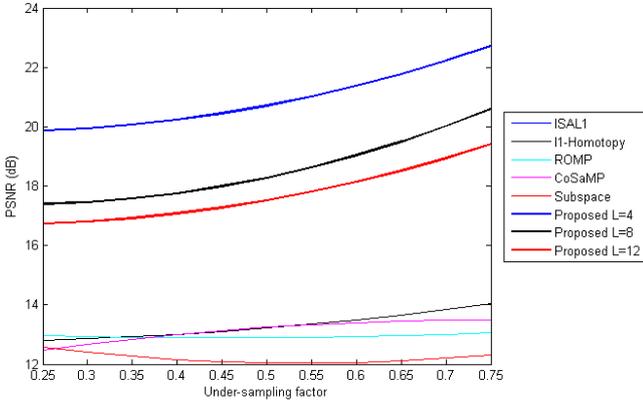


Figure 2: PSNR of reconstructed results from various algorithms exploiting spatial correlation. Sampling and reconstruction is done in spatial domain.

that the performance of the proposed method decreases as the reference interval  $L$  increases. This is because once the interval between the signal and the reference is higher, the distance between them also increases and so the error. Nevertheless, the proposed method clearly performs better under this setting.

Secondly, the same test is now sampled and reconstructed in the redundant domain; which is the spatial domain in this case. Figure 2 shows the PSNR of the results of this setting. In this setting, whilst the performance of every other algorithm decreases since the signal is no longer sparse, the performance of the proposed method is surprisingly unaffected. This demonstrates the fact that the proposed method can work as good in redundant domain as in sparse domain.

Lastly, Table 1 shows the complexity, measured as computation time in seconds per frame, of each algorithm. It can be noticed that in this small problem setting, the complexity of the proposed method is comparable to the greedy algorithms. This is many folds faster than  $l_1$ -min algorithms despite the better reconstruction quality.

Algorithms	Computation time per image		
	R = 0.25	R = 0.50	R = 0.75
ISAL1	8.96	11.13	13.19
l1-Homotopy	4.70	18.39	32.39
ROMP	0.98	0.69	1.24
CoSaMP	70.68	193.43	320.95
Subspace Pursuit	0.27	0.33	0.27
Proposed L=4	0.025	0.042	0.12
Proposed L=8	0.050	0.024	0.296
Proposed L=12	0.066	0.028	0.377

Table 1: Average computation time per image in seconds of each algorithm when using spatial correlation, computed at sampling rate  $R=0.25, 0.5$  and  $0.75$ .

### 3.2 References with temporal correlation

The second experiment demonstrates the use of temporal correlated references. In this experiment, a video sequence is sampled and reconstructed frame-by-frame. Assuming that each frame is fairly similar to its neighbour, it is possible to select some frames as references to reconstruct the other frames.

For each frame  $\mathbf{x}_t$  at time  $t$ , an under-sampled measurement  $\mathbf{y}_t$  is reconstructed into a full frame  $\hat{\mathbf{x}}_t$ . The measurement  $\mathbf{y}_t$  is obtained from a linear random combination using Gaussian random matrices. Each  $\hat{\mathbf{x}}_t$  is reconstructed individually from each other using the same reconstruction algorithms used in previous experiment. The proposed method uses frames  $\mathbf{x}_R$  at every  $R$  frame interval as references. These frames are assumed to be uncompressed sampled. In this experiment, the arbitrary  $R = 5, 10$  and  $15$  are chosen. The dataset used in this experiment is a set of 14 video sequences chosen randomly, which all have different scenes, motions, and other characteristics. The dataset includes situations where there are only small amount of motions, such as surveillance sequences, and high amount motions such as sport sequences.

Figure 3 shows the PSNR of each reconstruction algorithms when the sampling and reconstruction of video sequences is done in a sparse domain. The sparse domain used in this experiment is the discrete cosine transform (DCT) domain widely used in many video encoders. In this setting, it can be seen that the performance of the proposed method is not a candidate to the l1-min algorithms. Whilst the reconstructions using  $R=10$ , and  $R=15$  are generally bad, the performance of  $R=5$  is comparable to that of the greedy algorithms. The main reason for this level of performance is the effect of motion in video sequences. This is particularly clear in sequences that contain lots of motions, making the distance between the references and frames higher, therefore increases the reconstruction error. The larger the interval  $R$  is, the more severe the effect of motions will be.

However, let us consider the other setting where the sampling and reconstruction of video sequences is done in spatial (redundant) domain. Figure 4 shows that whilst the performance of every other algorithm decreases significantly in spatial domain, the performance of the proposed method is roughly maintained at the same level. This demonstrates

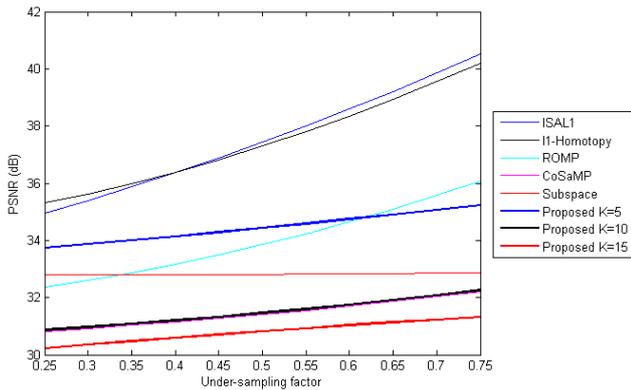


Figure 3: PSNR of reconstructed results from various algorithms exploiting temporal correlation. Sampling and reconstruction is done in DCT domain.

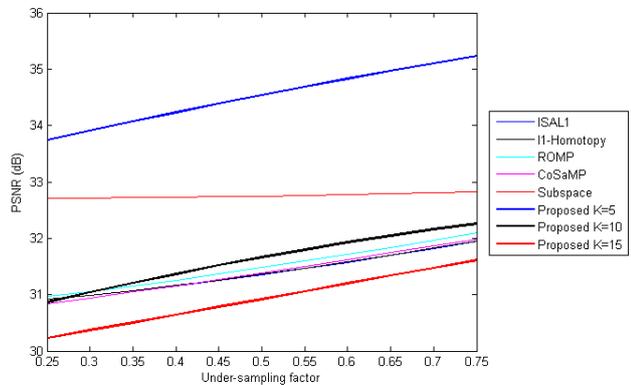


Figure 4: PSNR of reconstructed results from various algorithms exploiting temporal correlation. Sampling and reconstruction is done in spatial domain.

the potential of sampling and reconstructing video sequences directly in spatial domain.

It should be noted that the references used in both experiments are chosen arbitrarily. If the references selecting process is more sophisticate, such as including inter-frame motion prediction or intra-frame prediction, the performance of the proposed method is expected to be improved. However, since this paper aims to emphasise the use of least squares with a support from signal correlation rather than to introduce the advance side-information, the references are chosen naïvely at a constant interval under the assumption of uncompressed sampling.

Nevertheless, Table 2 shows that a computation time per frame of the proposed method is much lower than the other algorithms. This allows a near real-time reconstruction of a compressed video sequence. For a sequence of 1000 frames, the proposed method can reconstruct the whole sequence in around 6 minutes, whereas some algorithms require up to few hours.

Algorithms	Computation time per image		
	R = 0.25	R = 0.50	R = 0.75
ISAL1	21.08	31.79	32.78
l1-Homotopy	5.09	12.76	23.11
ROMP	0.87	3.08	4.12
CoSaMP	60.24	134.46	203.50
Subspace Pursuit	8.03	8.18	8.22
Proposed L=4	0.12	0.39	0.70
Proposed L=8	0.14	0.45	0.79
Proposed L=12	0.14	0.47	0.81

Table 2: Average computation time per image in seconds of each algorithm when using temporal correlation, computed at sampling rate  $R=0.25, 0.5$  and  $0.75$ .

## 4 Conclusions

This paper has introduced a fast compressed sensing reconstruction method using the least squares. The proposed method enables compressed sensing in real-time applications. The reconstruction quality of the least squares is improved to the level comparable to  $l_1$ -min algorithms and greedy algorithms by exploiting signal correlation. By using a correlated signal as references, the reconstruction is done by promoting the correlation between the signal and its reference instead of promoting the sparsity of signal. This enables the proposed method to work equally good in both sparse and redundant domains. This paper also showed that the references can be chosen very flexibly. The results of two naïve reference choices, the spatial correlation between image rows and the temporal correlation between video frames, were demonstrated. It is evident from results, that the closer the reference to the signal, the better the reconstruction result.

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