



UNIVERSITY OF LEEDS

This is a repository copy of *Research on Punctual and Energy-Efficient Train Driving Strategy Based on Two-Stage Allocation of Redundant Running Time*.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/179570/>

Version: Accepted Version

Proceedings Paper:

Liu, G, Li, W, Li, K orcid.org/0000-0001-6657-0522 et al. (3 more authors) (2021) Research on Punctual and Energy-Efficient Train Driving Strategy Based on Two-Stage Allocation of Redundant Running Time. In: Recent Advances in Sustainable Energy and Intelligent Systems: 7th International Conference on Life System Modeling and Simulation, LSMS 2021 and 7th International Conference on Intelligent Computing for Sustainable Energy and Environment, ICSEE 2021, Han. 7th International Conference on Life System Modeling and Simulation, LSMS 2021 and 7th International Conference on Intelligent Computing for Sustainable Energy and Environment, ICSEE 2021, 30 Oct - 01 Nov 2021, Hangzhou, China. Springer, Singapore , pp. 133-146. ISBN 978-981-16-7209-5

https://doi.org/10.1007/978-981-16-7210-1_13

Reuse

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.



eprints@whiterose.ac.uk
<https://eprints.whiterose.ac.uk/>

Research on Punctual and Energy-Efficient Train Driving Strategy Based on Two-stage Allocation of Redundant Running Time

Gaofeng Liu¹, Wei Li¹, Kang Li², Meiqi Yao¹, Sizhe Zhao¹, Shijuan Gao³

¹ School of Traffic and Transportation Engineering, Central South University, Changsha 410075, Hunan, China

² School of Electronic and Electrical Engineering, University of Leeds, UK

³ School of Geosciences and Info-Physics, Central South University, Changsha 410075, Hunan, China

Abstract. The energy-efficient train control can effectively reduce the overall energy consumption of railway network by optimizing train driving strategy. This paper proposed an optimization method for energy-efficient train driving strategy with two-stage allocation of redundant running time, which coordinately optimizes allocation of redundant running time and switching points of driving regimes. The proposed method calculates the shortest running time of the train under the time-optimal driving strategy, and then compares it with the given running time of the section to calculate the redundant running time. In the first stage, part of the redundant running time is allocated to the new-added long coasting regime before maximum braking. In the second stage, the remaining redundant running time is allocated to several maximum acceleration-coasting regime pairs to replace the cruising regime after the maximum acceleration from the start point. In this paper, Genetic Algorithm is used to optimize the allocation of redundant running time and switching points of driving regimes in the two optimization stages. Time complexity of proposed method is linear. A case study was carried out using CRH₃ high-speed train and an actual section of the Shangqiu-Hefei-Hangzhou high-speed railway. The results showed that: after the first-stage optimization, due to the reduction of maximum braking distance, energy consumption increased than the energy consumption under time-optimal driving strategy. After the second-stage optimization, the redundant running time of the train is fully allocated, so the train has realized punctual running, and the running energy consumption has been saved by 10.33%.

Keywords: Energy-Efficient Train Control, Redundant running time, Genetic Algorithm, High-speed train.

1 Introduction

Railway is currently one of the transportations with the lowest energy consumption per passenger. Railway network has contributed a lot of kinetic energy to the economic development, but it inevitably also consumes a lot of energy. From the perspective of train operation, there is a certain room for optimization of the traction energy consumption of trains, for the traction energy consumption accounts for more than 70% of the energy consumption of the train operation [1]. The power of auxiliary energy consumption remains basically constant, so the feasibility of optimizing the traction energy consumption in train operation is high.

Since Ichikawa K. [2] proposed the concept of train energy-efficient driving in 1968, and proposed that an optimal speed profile includes four driving regimes: maximum acceleration, cruising, coasting, and maximum braking. This research field of train energy-efficient control has been developing in the past decades. The complexity of the train energy-efficient driving problem modelling developed from level track to varying gradient track, from simple mechanical braking to regenerative braking which convert kinetic energy into electric energy then feed it back to catenary or on-board energy storage devices. Optimization methods developed from the analytical method such as Pontryain's Maximum Principle to numerical method represented by Dynamic Programming. Intelligent optimization methods represented by swarm intelligence algorithms that has developed rapidly in recent decades. The energy-efficient train control problem has attracted many scholars to actively explore because of its positive economic benefits and promotion of low-carbon society. Scholars represented by Milroy I. P., Howlett P., Pudney P., Khmelnitsky E., Albrecht A. R., Liu R. R., Lu S., Su S., etc., extended the problem of train energy-efficient driving to the optimal train operation control, and played a positive role in promoting this research field.

In 1980, Milroy I. P. [3] studied the diesel locomotives at the time in a doctoral dissertation of Loughborough University. Milroy I. P. used the Pontryain's Maximum Principle to optimized the continuous variable which was the train acceleration a . Considering that the operation handle in the diesel locomotive's cab at that time was set to discrete positions, the research did not match the actual

situation. Howlett P. [4] established a discrete train model considering relatively flat slopes based on the Pontryain's Maximum Principle, and pointed out that the optimal driving strategy for trains on level track include four phases: maximum acceleration, speed holding(cruising), coasting, and braking. Pudney P. and Howlett P. [5] also pointed out in subsequent studies that when the speed limit of line is lower than the optimal cruising speed of the train, the optimal driving strategy is to drive the train at the maximum limited speed. Considering varying gradient, speed limit, and regenerative braking, Khmelnsky E. [6] established continuous train driving model using distance as independent variable rather running time. On basis of previous research, and Jiixin C. and Howlett P. [7] first proposed the discrete control model with discrete throttle settings, considering varying gradient, that made the control model more realistic.

In addition to analytical methods, some scholars tried to use other methods to solve the energy-efficient train control problem. Lu S. et al. [8] summarized that train control can be categorized as coasting control and general control. Coasting control reduces traction energy consumption by optimizing the coasting margin to use up allowable time, and general control generating speed profile to reduce total energy consumption of train. Lu S. et al. respectively adopted Genetic Algorithm, Ant Colony Optimization and Dynamic Programming then designed corresponding solution rules based on the characteristics of different algorithms, calculated the energy-efficient speed profile of train. The paper pointed out that analytical methods could not solve the energy-efficient train control problem. It is recommended that various methods should be adopted to improve the global optimality of the optimization. Haahr J. T. et al. [9] used the time-space graph formulation solved by Dynamic Programming to generate the improved train speed profile considering passage point problem. The paper did not mention the optimization relationship between the running energy consumption and the running time.

Modern intelligent optimization algorithms have gradually been widely used in energy-efficient train control due to their less requirements for objective functions and the ability to perform global stochastic optimization. In the energy-efficient train control problem, the meta-heuristic algorithm is the most widely used algorithm [10]. Multi-Objective Particle Swarm Optimization which had been refined was adopted to optimize the objectives of energy consumption and running time, and taken comfort and operational constraints into consideration, improved speed profiles were generated [11].

In view of the excessive simplifications to get feasible solutions of analytical method, the “*curse of dimensionality*” of numerical methods, and the difficulty of ensuring the global optimality of one-stage optimization of intelligent optimization algorithms, this paper proposed a method based on two-stage allocation of redundant running time. The proposed method allocates redundant running time in two stages, and transfer time-optimal speed profile to punctual and energy-efficient speed profile after two-stage optimizations. The optimization of train speed profile in the existing literature mostly optimizes the switching points of driving regimes, or adjust the allowable running time to reduce energy consumption. The proposed method optimizes two aspects of allocation of redundant running time and switching points of driving regimes. Through the idea of allocating redundant running time as a resource, the time resource can also be recycled in the case of a slight train delays, so as to alleviate the impact of train delays.

2 Energy-Efficient Train Control Problem

2.1 The Train Dynamics

When the train is running in the section, the basic forces experienced by the train include traction force $F(v)$, braking force $B(v)$, basic resistance $W_0(v)$ and additional resistance $W_i(v, x)$.

The traction force $F(v)$ is determined by the traction characteristics of the train, and is used as the maximum traction force constraint $F_{\max}(v)$ in the optimization process. The traction force from the electric motor under traction regime is related to the running speed of the train, that is, the traction force F is presented as a function of speed v .

$$F = F(v) \quad (1)$$

The braking force $B(v)$ is determined by the braking characteristics of the train, and is used as the maximum braking force constraint $B_{\max}(v)$ in the optimization process. The braking force of the train under braking regime is related to the running speed of the train, that is, the braking force B is presented as a function of speed v .

$$B = B(v) \quad (2)$$

The resistance experienced by the train during the section can be divided into basic resistance $W_0(v)$ and additional resistance $W_i(v, x)$.

The basic resistance can be expressed by Davis equation [12] as:

$$w_0(v) = c_1 + c_2v + c_3v^2 \quad (3)$$

$$W_0 = M \cdot g \cdot w_0 \cdot 10^{-3} \quad (4)$$

Among them, w_0 represents the unit basic resistance, and the unit of w_0 is N/kN . Non-negative empirical coefficient c_1 , c_2 , and c_3 is according to different model of trains, and are usually given. The constant coefficient c_1 and the linear coefficient c_2v are the rolling resistance experienced by the train, which is related to the mass of the train. The quadratic coefficient c_3v^2 is the air resistance experienced during the operation of the train, which is mass independent. W_0 is the basic resistance during the actual operation of the train, in kN . M is the mass of the train, in t . g is the acceleration of gravity, and is 9.8 m/s^2 .

In addition to the basic resistance, the train running in the section is also affected by the additional resistance. The additional resistance includes the additional resistance of slope, tunnel and curve. This paper only considers the additional resistance of the slope $W_i(v, x)$.

$$w_i(v, x) = Mg \sin(\theta) \approx \theta \quad (5)$$

$$W_i = M \cdot g \cdot w_i \cdot 10^{-3} \quad (6)$$

The unit slope additional resistance w_i , in N/kN , is a function of the train speed v and the current position x of the train. In practice, w_i can be approximately equal to the train slope value θ in $\%$. W_i is the additional resistance of the slope experienced by the train in operation, and in kN .

In this paper, the gradient of slope is set as a function of distance, and the slope function is called when calculating the train's running states such as speed and position.

2.2 Train Driving Regimes

According to the output characteristics of train traction during train operation, the driving regimes are divided into four types: Maximum Acceleration (MA), Cruising (CR), Coasting (CO) and Maximum Braking (MB).

The traction regime and braking regime described in this paper are MA and MB. In the following text, CR stands for cruising and CO stands for coasting.

The corresponding force conditions are as follows, and the unit is KN .

MA: The train outputs the maximum traction force. This driving regime produces traction energy consumption. The resultant force experienced by the train C_1 is as follows.

$$C_1 = F - W_0 - W_i \quad (7)$$

CR: According to the basic resistance of the train and the additional resistance of the slope, the train outputs partial maximum traction force to offset the reverse force of the basic resistance and the additional resistance of the slope. The resultant force of the train C_2 is zero, and this driving regime produces partial traction energy consumption.

$$C_2 = F - W_0 - W_i = 0 \quad (8)$$

CO: The train does not output traction force, and this driving regime does not produce traction energy consumption. The resultant force on the train C_3 is the sum of the basic resistance and the additional resistance of the slope, and the direction of the resultant force on the train is opposite to the forward direction of the train.

$$C_3 = -W_0 - W_i \quad (9)$$

MB: The train outputs the maximum braking force, and the resultant force received C_4 is the sum of the maximum braking force, the basic resistance and the additional resistance of the slope. The direction of the resultant force is opposite to the direction of the train.

$$C_4 = -B - W_0 - W_i \quad (10)$$

This paper considers regenerative braking in MB regime. During the MB regime, the train converts part of the kinetic energy into electrical energy through a four-quadrant converter and feeds it back to the catenary or on-board energy storage devices.

2.3 Train Energy Consumption Modelling

The single-phase 50Hz 25kV AC power obtained from the catenary, part of which is converted from single-phase AC to DC and finally to three-phase AC power, through the main traction converter to drive the traction motor. The other part is converted by the auxiliary converter to complete the same form conversion of electric energy, is used to supply power to non-traction equipment such as air conditioning, lighting and information system. Therefore, the operating energy consumption of the train section consists of traction energy consumption and auxiliary energy consumption, and the calculation formula is as follows.

$$E_{total} = E_{traction} + E_{auxiliary} \quad (11)$$

$$E_{auxiliary} = P_{auxiliary} T \quad (12)$$

The energy consumption of the train section E_{total} is composed of traction energy consumption $E_{traction}$ and auxiliary energy consumption $E_{auxiliary}$. Traction energy consumption $E_{traction}$ is the integral of the running distance of the train's traction force $F(v)$ from the starting point x_0 to the end point x_{end} . Taking into account the energy consumption efficiency, then dividing $F(v)$ by the energy utilization coefficient η . The value of auxiliary power $P_{auxiliary}$ is generally known. Therefore, auxiliary energy consumption $E_{auxiliary}$ is the product of auxiliary energy consumption power $P_{auxiliary}$ and the actual running time of the train T .

In this paper, the current position of the train x is used as the independent variable to iterate to calculate the energy consumption of the train. The calculation formula of traction energy consumption is as follows:

MA:

$$F_{MA} = F(v) \quad (13)$$

$$E_{traction} = \sum_{i=1}^k \frac{(b_i - a_i)}{6} (F_{MA}(v_{a_i}) + 4F_{MA}(v_{\frac{a_i+b_i}{2}}) + F_{MA}(v_{b_i})) \quad (14)$$

CR:

$$F_{CR} = -W_0(v) - W_i(v, x) \quad (15)$$

$$E_{traction} = \sum_{i=1}^k \frac{b_i - a_i}{6} (F_{CR}(v_{a_i}, x + a_i) + 4F_{CR}(v_{\frac{a_i+b_i}{2}}, x + \frac{a_i+b_i}{2}) + F_{CR}(v_{b_i}, x + b_i)) \quad (16)$$

CO:

$$F_{CO} = 0 \quad (17)$$

$$E_{traction} = 0 \quad (18)$$

MB:

$$F_{MB} = B(v) \quad (19)$$

$$E_{traction} = -\lambda \sum_{i=1}^k \frac{b_i - a_i}{6} (F_{MB}(v_{a_i}) + 4F_{MB}(v_{\frac{a_i+b_i}{2}}) + F_{MB}(v_{b_i})) \quad (20)$$

x is the current position of the train, which means the distance the train has traveled from the start point. a_i is the starting position of the minimum iterative interval i of the train, b_i is the end position of the minimum iterative interval i . The minimum iterative interval i of the train defined as the distance that the train travels for one second under the acceleration of the current speed of the train. k is the number of minimum iterative intervals of the section. $F(v_{a_i})$ is the dependent variable of the starting velocity v_{a_i} in the current minimum iterative interval i , in kN . In the same way, $f_{\frac{a_i+b_i}{2}}$ and f_{b_i} are the dependent variables of the midpoint velocity $v_{\frac{a_i+b_i}{2}}$ and the end velocity v_{b_i} in the current minimum iterative interval i respectively. λ is the regenerative braking energy utilization factor.

2.4 Mathematical Modelling of Energy-Efficient Train Control

By analyzing the force conditions of the train, the four driving regimes of the train operation are divided according to the force conditions. The train energy-efficient control model is described as follows. By establishing this model, the optimal sequence of driving regimes and switching points can be obtained by specific optimization methods.

$$J = \min_F \int_{x_0}^{x_{end}} \frac{F(v)}{\eta} dx + P_{auxiliary} T \quad (21)$$

subject to

$$\frac{dt}{dx} = \frac{1}{v} \quad (22)$$

$$\frac{dv}{dx} = \frac{F(v) - B(v) - W_0(v) - W_i(v, x)}{(1 + \rho)M} \quad (23)$$

$$v(x_0) = 0, v(x_{end}) = 0, v(x) \in [0, v_{\max}(x)] \quad (24)$$

$$F(v) \in [0, F_{\max}(v)], B(v) \in [0, B_{\max}(v)] \quad (25)$$

$$T = T_{given} \quad (26)$$

$$x_{end} - x_0 = x_{section} \quad (27)$$

J is the objective function, that is, the total energy consumption required for the complete operation of the train section, in kWh ; x_0 and x_{end} is the start point and ending point of train operation respectively. $F(v)$, $B(v)$, $W_0(v)$ and $W_i(v, x)$ correspond to the traction force, braking force, basic resistance and slope additional resistance respectively, all in kN ; ρ is the dimensionless rotating mass factor [13]; M is the quality of train, in t ; $v_{\max}(x)$ is the speed limit of the position in position x ; $F_{\max}(v)$, is the maximum traction force constraint. $B_{\max}(v)$ is the maximum braking force constraint; T is the actual train running time while T_{given} is the given running time of the train section which is given by the timetable. $x_{section}$ is the length of the train section.

3 Two-stage Allocation of Redundant Running Time

3.1 Time-optimal Speed Profile

During the train operation in the section, the energy consumption of train traction and the running time of the section are in a trade-off relationship. Time-optimal driving strategy of the section is MA-CR-MB, where the cruising speed is the speed limit. The time required for train operation under this strategy is the shortest, but the traction energy consumption is the highest.

In order to generate the time-optimal driving strategy, three steps need to be taken. Firstly, after the train start running from the starting point, keep the maximum acceleration till the train speed up to speed limit, then keep cruising; Secondly, reversely calculate the maximum braking distance of the train from the end point; Finally, when the train reach the braking point calculated before, switching to maximum braking regime for safe parking. Driving the train under this strategy, the running time of the train section is the shortest.

The redundant running time is defined as the difference between the given running time T_{given} of the shortest train running time $T_{shortest}$ under the time-optimal driving strategy. By generating the time-optimal speed profile, the running time and energy consumption of the train under this strategy can be obtained. Based on this, the two-stage allocation of redundant running time can be carried out, and the switching points of driving regimes can be optimized.

3.2 Energy-Efficient Sequence of Driving Regimes

When the train is running in a section, the switching between driving regimes must meet certain principles. Unreasonable switching will produce unnecessary energy consumption. The principle of train driving regimes switching as follows, see Table 1.

Table 1. Switching principles of train driving regimes.

Current driving regime	Next driving regime	Switchable?
MA	MA	•
	CR	√
	CO	√
	MB	×
CR	MA	√
	CR	•
	CO	√
	MB	×
CO	MA	√
	CR	√
	CO	•
	MB	√
MB	MA	×
	CR	√
	CO	√
	MB	•

In Table 1, “•” means that the next driving regime is consistent with the current driving regime, “√” means that it is allowed to switch from the current driving regime to the next driving regime, and “×” means that it is not allowed to switch from the current driving regime to the next driving regime.

The energy-efficient train driving strategy consists of two parts: the energy-efficient sequence of driving regimes and the optimal switching points of driving regimes. The energy-efficient sequence of driving regimes of train operation is composed of four driving regimes. If an optimization method is used

to find the global energy-efficient sequence of driving regimes directly, it involves the combination and arrangement of the four driving regimes, which is a NP-complete problem. The solving time required for NP-complete problem is unacceptable.

Su S. et al. [14] analyzed the energy-efficient sequence of driving regimes of a section, and pointed out the energy-efficient sequence of driving regimes when the section contains one, two, and multiple switching points, as shown in Table 2.

Table 2. Energy-efficient sequence of driving regimes in a section with various switching points.

Switching points of driving regimes in a section	Energy-efficient sequence of driving regimes
None	MA、CR、CO、MB
One	MA-MB、MA-CO、CO-CR、MA-CR、CO-MB、CR-MB
Two	CO-CR-MB、MA-CO-MB、MA-CO-CR、MA-CR-MB
Multiple	MA-CO、MA-CO-CR-MB、MA-MA-CO-MB

A comprehensive analysis of the above-mentioned four sections of different switching points of driving regimes, it can be concluded that the energy-efficient sequence of driving regimes is the following three types [14] : MA-CO-MB, MA-(MA-CO pairs)-C-MB, MA-CO-CR-MB.

According to the literature [3], the MA-CO-MB sequence is optimal when operating in short sections, which is suitable for metro and suburban railway system; MA-(MA-CO pairs)-C-MB adds several maximum acceleration-coasting regime pairs, in order to reduce the energy consumption of continuous cruising regime; MA-CO-CR-MB driving regime is suitable for long and steep downhill lines, and the CR means speed holding after the CO regime for the steep downhill gradient. According to the *Code for Design of High-speed Railway* (TB10621-2014) [15], the gradient of the main line of Chinese high-speed railway should not exceed 20‰, and it should not exceed 30‰ under difficult conditions. Therefore, the sequence of driving regimes MA-CO-CR-MB is not consistent with the actual line conditions of Chinese high-speed railways, and is more suitable for freight locomotives in heavy-haul railways.

In summary, the optimal sequence of driving regimes for high-speed train in a section is MA-(MA-CO pairs)-C-MB. In other words, the optimal sequence of driving regimes of high-speed train in a section is maximum acceleration, several pairs of maximum acceleration and coasting, coasting, and maximum braking.

3.3 Two-stage Allocation of Redundant Running Time

Under the premise of ensuring that the train arrives on time, the redundant running time can be used as the allocation resource for the energy-efficient driving strategy of the train section. By allocating the redundant running time to energy-free coasting regimes, traction energy consumption can be reduced to the lowest under punctuality.

In the first stage of optimization, the redundant time is allocated to the long coasting regime before the maximum braking regime. From literature [4], it can be known that the energy-efficient sequence of driving regimes is the maximum traction, cruising, coasting, and maximum braking. By allocating the redundant running time to the inert driving regimes between the cruising and the maximum braking before the end point, the energy consumption of train in section can be reduced.

In practice, trains without Automatic Train Operation (ATO) subsystem, which under the supervision of Automatic Train Protection (ATP), are driven by human drivers. High-speed trains in China are driven by human at most of railway lines except Beijing-Zhangjiakou high-speed railway, which ATO was first introduced in December, 2019. In the second stage, MA-CO regime pairs is introduced to replace CR regime in order to further reduce traction energy.

Tian Y. et al. [16] used ten optimization algorithms to optimize the solution of one hundred thousand functions. Genetic Algorithm (GA) achieved the optimal solutions in the most function experiments than other algorithms. In this paper, the Genetic Algorithm is used to optimize the allocation of redundant running time and switching points of driving regimes.

Therefore, this paper replaces the optimized CR regime of the first stage with several MA-CO regime pairs, and used Genetic Algorithm to solve the optimal allocation of redundant running time and optimal coasting points to further reduce the traction energy consumption of train. The second stage of optimization is to allocate redundant time to MA-CO regime pairs.

The flow chart of the proposed two-stage optimization method in this paper is shown in Fig. 1.

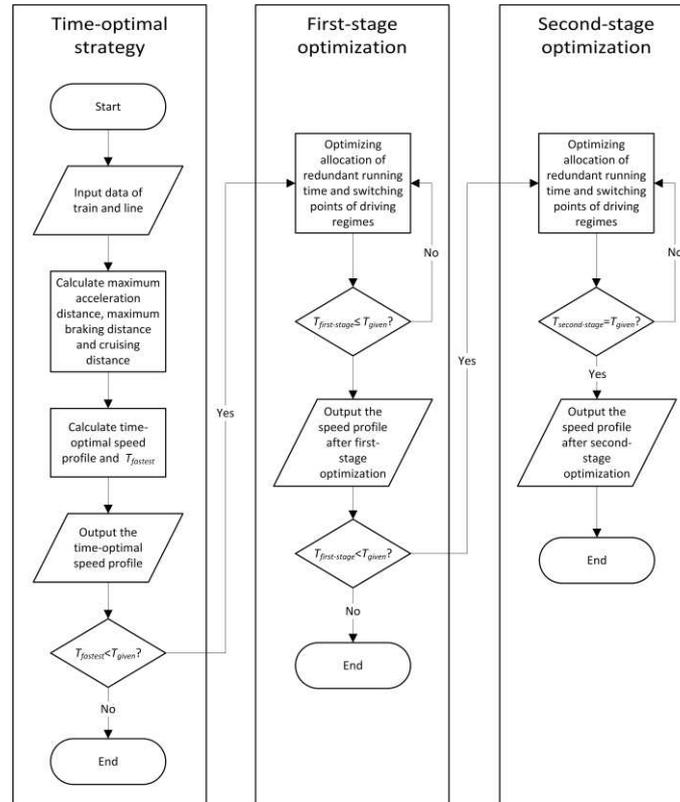


Fig. 1. Flow chart of the proposed two-stage optimization method.

3.4 Relationship between Allocated Redundant Running Time and Saved Energy

According to literature [17], energy consumption and running time are often in conflict with each other in the optimization process, and it is necessary to make a trade-off to achieve the desired overall optimization effect of each objective.

For the punctual and energy-efficient train driving strategy based on allocation of redundant running time, the proposed method is based on the principle of Pareto optimization. The train speed profile generated by the proposed method fulfilled the requirement of punctual operation of the section. As shown in the Fig. 2, “ $+\Delta T$ ” means the allocated redundant running time and “ $-\Delta E$ ” means the saved energy by allocation of redundant running time. Minimum running time is the technical shortest running time constrained by the characteristics of the train and line. Given running time is given by the predesigned timetable. It is obvious to know from the figure that if the traction energy consumption continues to be reduced, the train running time of the section will exceed the given running time which will cause train delay.

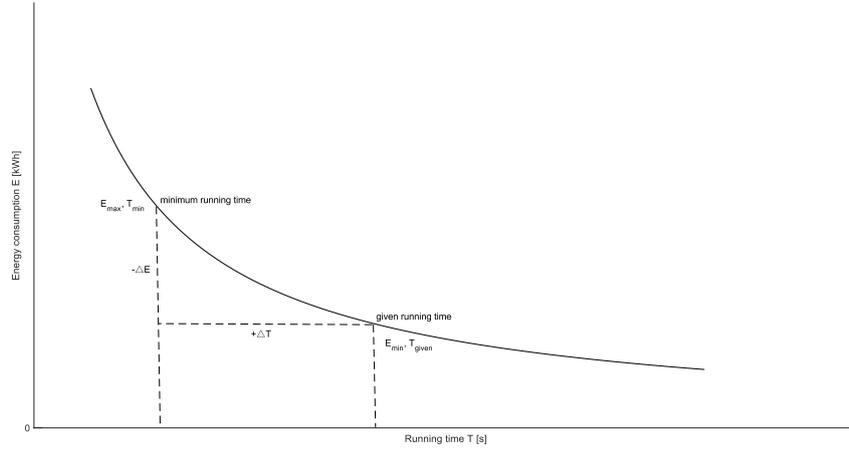


Fig. 2. Relationship between energy consumption and running time.

4 Case Study

4.1 Train Data and Line Data

The train model used in the numerical experiment in this paper is the CRH₃ high-speed EMU train. The main characteristics of the train are shown in Table 3.

Table 3. Main characteristic parameters of CRH₃

Characteristics of CRH ₃	Parameters
Train sets	4M4T
Mass (<i>t</i>)	563
Dimensionless rotating mass factor	0.06
Maximum traction force (<i>kN</i>)	300
Maximum braking force (<i>kN</i>)	300
Electric utilization coefficient	0.9
Auxiliary energy power	0.05
Regenerative energy utilization coefficient	0.85

The line used in the numerical experiment is an actual section of the Shangqiu-Hefei-Hangzhou high-speed railway with a length of 74.8km. The line data is shown in Table 4.

Table 4. Main characteristic parameters of railway line.

Characteristics of line	Parameters
Length of the line (<i>km</i>)	74.8
Speed limit (<i>km/h</i>)	300
Given running time of section (<i>s</i>)	1200

The gradient of the line is shown in Fig. 3.

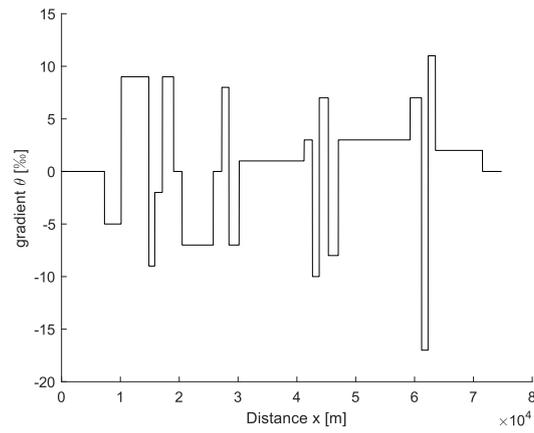


Fig. 3. Varying gradient of an actual section of Shangqiu-Hefei-Hangzhou high-speed railway.

4.2 Time-optimal Speed Profile

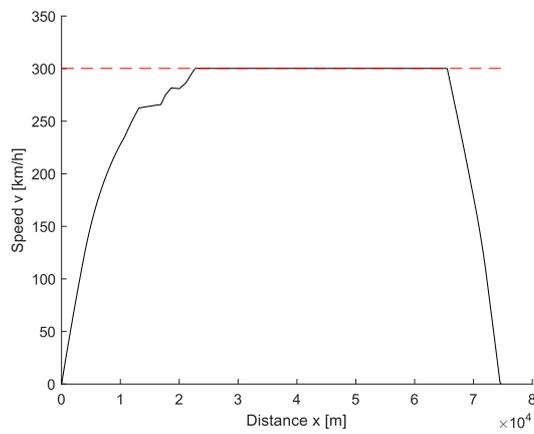


Fig. 4. Time-optimal speed profile considering varying gradient. The running time under this driving strategy is 1105 seconds while the given running time is 1200 seconds.

4.3 First-stage Optimized Speed Profile

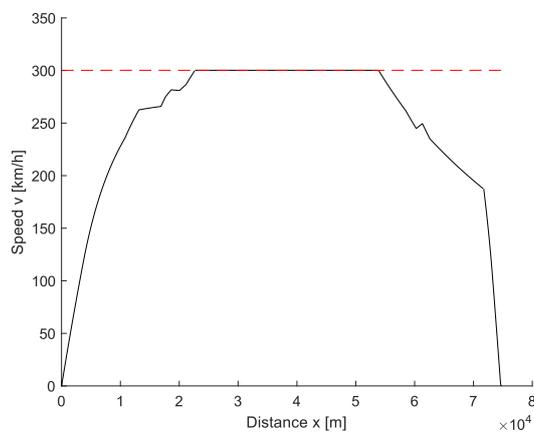


Fig. 5. Speed profile after first-stage optimization. Optimized running time of train after first-stage optimization is 1144 seconds while the energy consumption increased 0.02%.

4.4 Second-stage Optimized Speed Profile

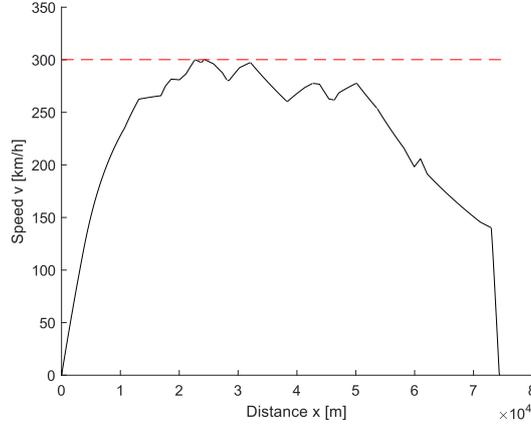


Fig. 6. Speed profile after first-stage optimization. All the redundant running time was allocated to energy-free regimes, so 10.33% energy was saved with the running time consistent with the given running time.

4.5 Performance of The Proposed Method

Table 5. More detailed data about the optimization process. Fig. 4, fig. 5 and fig. 6 in column *Speed profile* stand for time-optimal speed profile, first-stage optimized speed profile and second-stage optimized speed profile of corresponding driving strategy, as shown in figures above.

Speed profile	Energy consumption (<i>kWh</i>)						Energy saving percent (%)	Running time (s)	Solving time (s)	
	MA	CR	MA-CO pairs	CO	MB	Total				
Fig. 4	959.73	955.91	•	•	-	472.22	1443.42	•	1105	0.05
Fig. 5	959.73	694.20	•	0	-	186.56	1467.37	-0.02	1144	2.69
Fig. 6	959.73	•	431.55	0	-97.01	1294.27	1294.27	10.33	1200	31.36

5 Discussion

In Table 5, an interesting result shows that energy consumption increased after first-stage optimization. By analyzing the energy consumptions of different driving regimes, a conclusion comes out that: the saved energy by the shorter CR regime and the new-added CO regime in first-stage optimization, is not enough to offset the regenerative energy that converted from kinetic energy in MB regime. After the second-stage optimization, all redundant running time was used up, and 10.33% energy was saved with the proposed method.

Based on the allocation method of redundant running time proposed in this paper, it can alleviate the slight delay of trains. The restriction condition is that the delay time is less than the difference between the remaining given running time and the remaining running time of train under time-optimal driving strategy. When the train is not running under the time-optimal driving strategy, by recycling the remaining redundant running time of train of the section, the impact of train delay can be alleviated by

sacrificing some energy. Research on alleviating train delays through the proposed methods has yet to be carried out.

6 Conclusion

This paper proposed a two-stage optimization method, which allocates resources of redundant running time to achieve lower energy consumption. Under the premise of ensuring punctual arriving, the conversion from the time-optimal speed profile to the punctual and energy-efficient speed profile is realized. This method combines two optimization ideas of allocation of redundant running time and optimization of switching points. With the given energy-efficient sequence of driving regimes for train interval operation according to the literature, time complexity of the propose method is linear.

Through numerical experiment, the method proposed in this paper can achieve the energy saving effect of 10.33% compared with the time-optimal speed profile. An interesting phenomenon is that energy consumption increased after first-stage optimization. By analyzing the energy consumption data, the reason is that the saved energy by the shorter CR regime and the new-added CO regime in first-stage optimization, is not enough to offset the regenerative energy that converted from kinetic energy in MB regime.

Train delays can be alleviated in some extent when the train is not driving under the time-optimal driving strategy. By recycling redundant running time, train delays can be alleviated with the higher energy consumption.

Acknowledgements. This research was supported by Major Science and Technology Project of Guangxi Province of China under Grant No. Guike AA20302010.

References

1. Gu, Q., Tang, T., Cao, F., Song, Y.-d.: Energy-efficient train operation in urban rail transit using real-time traffic information. *IEEE Transactions on Intelligent Transportation Systems* 15, 1216-1233 (2014)
2. Ichikawa, K.: Application of optimization theory for bounded state variable problems to the operation of train. *Bulletin of JSME* 11, 857-865 (1968)
3. Milroy, I.P.: Aspects of automatic train control. Loughborough University (1980)
4. Howlett, P.: An optimal strategy for the control of a train. *The ANZIAM Journal* 31, 454-471 (1990)
5. Pudney, P., Howlett, P.: Optimal driving strategies for a train journey with speed limits. *The ANZIAM Journal* 36, 38-49 (1994)
6. Khmel'nitsky, E.: On an optimal control problem of train operation. *IEEE transactions on automatic control* 45, 1257-1266 (2000)
7. Jiabin, C., Howlett, P.: Application of critical velocities to the minimisation of fuel consumption in the control of trains. *Automatica* 28, 165-169 (1992)
8. Lu, S., Hillmansen, S., Ho, T.K., Roberts, C.: Single-train trajectory optimization. *IEEE Transactions on Intelligent Transportation Systems* 14, 743-750 (2013)
9. Haahr, J.T., Pisinger, D., Sabbaghian, M.: A dynamic programming approach for optimizing train speed profiles with speed restrictions and passage points. *Transportation Research Part B: Methodological* 99, 167-182 (2017)
10. Fernández, P.M., Sanchís, I.V., Yepes, V., Franco, R.I.: A review of modelling and optimisation methods applied to railways energy consumption. *Journal of Cleaner Production* 222, 153-162 (2019)
11. Domínguez, M., Fernández-Cardador, A., Cucala, A.P., Gonsalves, T., Fernandez, A.: Multi objective particle swarm optimization algorithm for the design of efficient ATO speed profiles in metro lines. *Engineering Applications of Artificial Intelligence* 29, 43-53 (2014)
12. Davis, W.J.: The tractive resistance of electric locomotives and cars. General Electric (1926)
13. Brünger, O., Dahlhaus, E.: Running time estimation. *Railway timetable & traffic* 58-82 (2014)
14. Su, S., Tang, T., Chen, L., Liu, B.: Energy-efficient train control in urban rail transit systems. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 229, 446-454 (2015)
15. Code for Design of High-speed Railway. vol. TB 10621-2014. National Railway Administration of China (2014)
16. Tian, Y., Peng, S., Zhang, X., Rodemann, T., Tan, K.C., Jin, Y.: A Recommender System for

- Metaheuristic Algorithms for Continuous Optimization Based on Deep Recurrent Neural Networks. IEEE Transactions on Artificial Intelligence 1, 5-18 (2020)
17. Scheepmaker, G.M., Goverde, R.M., Kroon, L.G.: Review of energy-efficient train control and timetabling. European Journal of Operational Research 257, 355-376 (2017)