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# Time-dependent pricing strategies for metro lines considering peak avoidance behaviour of commuters 

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## Time-dependent pricing strategies for metro lines considering peak avoidance behaviour of commuters


#### Abstract

With growing concerns about travel demand management practices in overcrowded metro systems, it is considered that time-dependent pricing strategies are effective for dealing with the crowding occurring during peak commuting hours. In this study, a bi-level optimisation framework is used to consider the peak avoidance behaviour of commuters in the development of timedependent pricing strategies. The behavioural sensitivity of commuters to pricing factors is investigated in terms of departure time and mode shift decisions based on a stated preference survey conducted in Beijing, China. The proposed bi-level programming model comprises a multi-objective optimisation model at the upper level and a nested logit-based stochastic user equilibrium model at the lower level. Based on an empirical case study of the Batong line in Beijing metro, nine optimal time-dependent pricing strategies are tailored by representative decision preferences, yielding an optimal off-peak discount of approximately $30 \%$ and best extra peak charge of $135 \%$. Accordingly, the peak ridership is reduced by up to $13.97 \%$ during rush hours.


Keywords: differential pricing; bi-level optimisation; departure time choice; mode shift; nested logit model

## 1. Introduction

Owing to its massive capacity, high efficiency, and low energy consumption, urban rail transit has been commonly considered as a preferred mode choice for the public and has played a crucial role in alleviating traffic congestion and promoting sustainable mobility in many megacities. As the most reliable method for commuting daily, the continuous expansion of metro networks is leading to a significant increase in the number of commuters during peak hours. Consequently, there is an inevitable imbalance between transport supply and travel demand.

As an example, the Beijing metro has 23 lines and 394 stations in operation, with a total mileage of 678 km by the end of 2019, ranking the system as the second
largest in the world. On average, approximately 5 million passengers use this metro network during the morning peak from 7:00 a.m. to 10:00 a.m. The broad application of the automated fare collection (AFC) system provides access to data, allowing the ridership patterns to be statistically analysed. Figure 1 presents the hourly ridership variation of the entire network on a weekday in 2019. Clearly, the hourly ridership is distributed nonuniformly throughout the day, with two distinct peaks, which are most probably governed by commuting activities.

Figure 2 presents the crowdedness degree of line 4 in terms of the section load rate (SLR). For each section between two stations, the SLR represents the ratio of the number of passengers transported to the train carrying capacity. The temporal and spatial distributions of the SLRs in the southbound and northbound directions present different characteristics. The southbound direction departs from the suburb to the city centre, leading to SLR peaks in the first few sections during the morning peak. The bottleneck of the northbound direction is located in the city centre during the evening rush hours. From the perspective of the operator, a high SLR generally implies highly crowded carriages and leads to operational risks in the localised network.

To alleviate the operation pressure during peak times, travel demand management (TDM) strategies involving both regulatory and incentivised approaches have been proposed. Following the definition in (Roby 2014), TDM refers to the various strategies for alleviating the effects of recurrent congestion by redistributing the travel demand in space and time. In metro operation management, passenger flow control is the most commonly used regulatory strategy, aiming to avoid crowd accumulation on a platform by implementing batch release at security points, e-gates, or even transfer channels. However, regulatory strategies inevitably negatively impact the
travel experiences of passengers and probably lead to a loss of ridership owing to the low service quality.

Concurrently, incentivisation strategies are aimed at promoting voluntary peak avoidance behaviour. Typically, time-dependent pricing, also known as time-based pricing, is a pricing strategy that allows flexible prices for products or services determined by the current market demands. Time-dependent pricing has been successfully applied in varied industries, including retailing, energy, tourism, and public transport. In the context of metro operation management, time-dependent pricing strategies, also called fare incentive strategies (Zhou et al. 2020), are commonly recognised as practical approaches for incentivising possible adjustment of unnecessary or flexible travel demand. They are currently used in metro systems of many megacities worldwide, such as London, Singapore, and Sydney.

In Beijing, a time-dependent pricing scheme was first introduced on the Batong (BT) and Changping lines in December 2015. During the first year of trial, the passengers who tapped in at any station of the specific lines before 7:00 a.m. automatically received a $30 \%$ decrease in their entire trip fare. The reduced fare was deducted from their e-card when tapping out at any station. Subsequently, the concerned authority raised the off-peak discount to $50 \%$ for strengthening the incentivising effects. Despite the above strategies, the daily ridership continued to grow drastically and surpassed 11 million in 2019. Discussions on future implementation plans of timedependent pricing are ongoing. To develop effective strategies that balance the interests of all involved stakeholders and overcome the problems of experience-based decisionmaking, quantitative analysis and comprehensive optimisation methods are required.

The remainder of this paper is organised as follows. Section 2 reviews the related research on time-dependent pricing strategies. Section 3 presents a bi-level
optimisation model for developing time-dependent pricing strategies. Section 4 reports the results of the empirical case study conducted considering the BT line in the Beijing metro network. Section 5 summarises the findings, policy implications, and directions for future studies.

## 2. Literature review

This section reviews the past studies in regard to commuter behaviour modelling, effects of pricing strategies on commuter behaviour, and existing methods for developing pricing strategies.

Commuter behaviour has been extensively studied in the field of choice modelling. Based on collected revealed preference (RP) or stated preference (SP) data, commuter behaviour is typically modelled in terms of departure time choice, mode choice, or both. Hess et al. (2007) employed a nested logit (NL) model to interpret trip timing and mode choice using SP data. Habib et al. (2009) used a joint discretecontinuous model to understand trip timing and mode choice using RP data. Lemp, Kockelman, and Damien (2010) formulated a continuous cross-NL model to obtain departure time decisions based on RP data. The above studies suggested the importance of including the sensitivity of an individual to pricing for understanding the commuting behaviour of commuters. In this respect, Kouwenhoven and de Jong (2018) and Kou et al. (2017) explored the key factors that affect the value of travel time and its reliability.

With pricing strategies becoming increasingly common in mass transit systems, commuter behaviour is receiving significant attention. Ho, Hensher, and Wang (2020) used SP/RP data to calibrate the NL-based mode and time of a day choice model considering the differential pricing of a train. Li et al. (2018) used SP data to understand departure time choice employing a smart corrected mixed logit model considering metro fare discount.

Furthermore, from an empirical perspective, numerous real-world cases of pricing strategies are available in the literature for analysing their effects on the behavioural changes of commuters. Ben-Elia and Ettema (2011a; 2011b) conducted a 13-week field survey to determine the rush hour avoidance intentions after the introduction of the Spitsmijden project in the Netherlands, which set rewards of different levels and types, either monetary or in-kind, for driving commuters. The itineraries of the participants were tracked using state-of-the-art detection equipment and modelled by a discrete mixed model. The results revealed that the rewards promoted shifts to off-peak periods, public transport, and working from home. Peer, Knockaert, and Verhoef (2015) targeted an annual train pass holder in the Netherlands to explore trip scheduling preferences in a peak avoidance experiment. A customised application was installed on the smartphones of the participants in advance to record their commuting activities. The collected data indicated that the rewards motivated a $22 \%$ decrease in the number of peak trips.

Using smart card data recorded before and after the implementation of a pricing strategy, Lovrić et al. (2016) developed a schedule-based demand framework for evaluating a free pre-peak travel policy. They found that the policy led to a $3.48 \%$ decrease in the peak hour ridership to central business districts and a $2.16 \%$ loss in the revenue of the operator. Graham et al. (2020) studied the impact of an early bird discount policy on trip scheduling preferences. They suggested that similar levels of pre-peak discounts had very limited effects on relieving peak crowding, which is in agreement with the experiences in cities such as Melbourne and London. In relation to the Beijing metro, Lin and Feng (2013) measured the loss aversion of different groups of metro travellers in response to a ticketing reform in Beijing, China. The feasibility of duration-, distance-, and zone-based pricing strategies was also discussed in their study.

Zhang, Fujii, and Managi (2014) explored the key factors for motivating peak avoidance behaviour and presented the effects of pre-peak fare discounts, free Wi-Fi, breakfast coupons, and flexible work time schedules.

Summarizing, in the above studies, the validity of various pricing strategies was assessed based on behavioural models or empirical cases. However, the discussion on the difference between traditional mode choice behaviour and mode shift behaviour in the context of time-dependent pricing is still insufficient. Some of the related studies obtained the departure time and mode choice preferences of commuters as an approximate reference to support the development of time-dependent pricing strategies. Inevitably, the habitual factors of regular metro commuters were observed to be restricted and resulted in biases when evaluating behavioural effects. Thus, it is crucial to provide various ranges of the departure time and mode shift alternatives in survey design and choice modelling, given that commuters show different peak avoidance intentions for the same pricing strategy. Additionally, some studies used smart card data before and after the implementation of pricing strategies to capture well the peak avoidance behaviour of metro commuters (Lovrić et al. 2016; Graham et al. 2020). Alternatively, they conducted SP surveys to collect the departure time decision preferences of commuters towards reduced fares (Li et al. 2018). However, there is still a lack of comprehensive analysis of the effects of different pricing strategies, particularly the combined effects of an extra peak charge and an off-peak discount.

Regarding the development of pricing strategies for transport systems, the previous studies mostly focused on Ramsey-based pricing models (a deviation from the standard socially optimal pricing principle aimed at maximising social welfare), mathematical models with various optimisation objectives, or proposing specific design criteria. Hamdouch and Lawphongpanich (2010) aimed to achieve the least overall
delay under an equilibrium condition by adjusting metro fares. Gong and Jin (2014) applied the trilateral game theory to pricing modelling to achieve a balance between the interests of the government, transport operator, and passengers. The most important contribution of the above research can be considered as the provision of relevant guidance for pricing scheme adjustment in specific development stages. In recent years, the methodologies for optimising time- or area-dependent pricing strategies have been receiving increasing attention not only for metro systems but also for other mass transit systems. Regarding the latter case, Sabounchi et al. (2014) and Zheng, Rérat, and Geroliminis (2016) proposed simulation-based methods for tailoring area-dependent pricing schemes for multimodal transport systems considering a mode share between private cars and public transit. Wu et al. (2019) presented a two-stage optimisation model for the dynamic pricing of a high-speed rail in the Beijing-Shanghai corridor. Tang, Ge, and Lam (2019) evaluated time-, area-, and quality-dependent pricing strategies for a bus transit system. Further progress has been achieved regarding the time-dependent pricing problem in a metro network. Aiming to rebuild the mode share patterns in a multimodal transport system, Liu and Wang (2017) modelled a timedependent pricing problem with the objective of minimising the total travel time of passengers. Yang and Tang (2018) presented a fare-reward scheme to incentivise departure time shift by offering a free off-peak trip after a certain number of paid peak trips. Furthermore, Tang et al. (2020) combined a fare-reward scheme with a nonrewarding uniform fare scheme. Using the proposed transit bottleneck model, they determined the free fare intervals, reward ratio, and new fares of the sub-schemes. Using a bi-level programming framework, Tang et al. (2020) optimised a surchargereward scheme to incentivise departure time shift from a central period to shoulder periods. Specifically, the upper-level problem maximised the total equilibrium cost of
the commuters, and the lower-level problem specified the equilibrium condition for the departure time choice.

Although methods for developing time-dependent pricing strategies have been investigated for a wide range of research scenarios, the past studies are still not sufficiently comprehensive in the following aspects, which is also the main motivation of the present study. First, departure time and mode shift behaviours should be considered simultaneously to capture the changes in metro demand patterns, which is essential for optimising desirable pricing strategies. Second, the combined effects of an extra peak charge and an off-peak discount need to be further explored. Third, it is necessary to discuss various decision preferences that either favour the interests of one stakeholder or balance the interests of all. Fourth, the above three elements should be collectively incorporated in optimisation models for developing demand-responsive time-dependent pricing strategies. In this study, the peak avoidance behaviour of metro commuters in response to time-dependent pricing strategies is investigated and modelled using an NL structure. Moreover, an elastic demand is incorporated into the optimisation process based on a bi-level programming framework. Additionally, timedependent pricing schemes optimised by representative decision preferences-operator and commuter benefits preferred, ridership peak-cutting preferred, and balanced schemes-are evaluated to provide references to policymakers.

## 3. Methodology

When a time-dependent pricing strategy is scheduled to be implemented, the impacts of the policy elements, such as the affected periods and stations, level of off-peak discount, and extra peak charge, need to be understood a priori to ensure the intended effects. Thus, an SP survey is conducted to obtain commuter responses to time-dependent
pricing strategies. Based on the estimated NL model, a bi-level optimisation model comprising a multi-objective optimisation model in the upper level and a stochastic user equilibrium (SUE) model in the lower level is formulated, together with a solution algorithm for the models.

### 3.1. Modelling peak-avoidance behaviour of commuters

### 3.1.1. SP survey

The SP survey is designed to identify the factors that influence the spontaneous behaviour changes of commuters in response to time-dependent pricing strategies. Specifically, we consider the peak avoidance behaviour of commuters in terms of their departure time and mode shift decisions. Face-to-face interviews were conducted in the areas within the Fifth Ring Road in Beijing in May 2018. Because only regular metro commuters are qualified to respond, the interviews are mostly conducted in shopping centres around metro stations during after-work hours.

Each interview comprises three steps. The interviewee is asked about his/her commute mode choice at the beginning. Only qualified metro commuters move to the second step, in which a brief introduction is provided to ensure that the respondent has the basic knowledge of time-dependent pricing. Subsequently, the interviewer guides the respondent to complete the SP choice tasks sequentially and provides assistance where needed. In the actual survey, each SP task shows ten alternatives of metro departures covering a wide range of departure times and three alternatives involving shifts to other travel modes. We base the scenarios on a regular commuting plan, given that previous travel habits of a commuter might influence the intention of adjusting previous behaviour. Figure 3 displays as an example a set of SP choice tasks developed
based on a real commuting route between the Huilongguan community and the Xizhimen centre area in Beijing.

The regular departure time of a commuter directly determines the difficulty in accessing the off-peak discount. For example, if a commuter typically leaves home at 7:10 a.m. and the off-peak discount is planned to end by 7:00 a.m., then the reduced fare is available to the commuter subject to a shift to an earlier departure by a minimum of 10 min . In contrast, for the commuters departing at 8:00 a.m., the off-peak discount will be less attractive as a significant change (an hour) is required in their regular schedule. Therefore, it is necessary to include regular departure times in an SP scenario, providing respondents with a highly realistic decision-making environment. As forms of timedependent pricing, apart from the most commonly used off-peak discount, an extra peak charge is also considered for further strengthening policy effects.

By on-site interviews, we collected a total of 2,467 SP choice observations from 135 respondents, of which 1,245 valid choices were answered in the morning peak scenario and the remaining 1,222 in the evening peak scenario. Each respondent was invited to complete ten SP choice tasks in both morning and evening peak scenarios. Each scenario comprised a current pricing scheme and two hypothetical pricing schemes (Schemes I and II in Figure 3). The respondents made five SP choices in Scheme I and another five SP choices in Scheme II. Figure 4 presents the descriptive statistics of the responses.

Based on the preliminary statistics of the socio-demographics, although there is a high proportion of male interviewees, most of the indicators are distributed uniformly across the respondents, which is in line with the gender distribution of metro passengers. To preliminarily test the elasticity of the peak avoidance behaviour, Figure 5 summarises the statistical results of the SP responses to the shift in the departure time.

The intention of the commuters for departing earlier is relatively stronger than that for departing later, which is particularly evident from the higher proportions of accepting the $5-\mathrm{min}$ and $20-\mathrm{min}$ ahead options. This is in line with the aim to arrive on time for work and also indicates a loss of metro ridership caused by the mode shift behaviour when the pricing strategies take effect.

### 3.1.2. NL model of departure times and mode shift decisions of commuters

Based on the random utility theory, the utility function for alternative $i$ and decision maker $n$ consists of the following two parts:

$$
\begin{equation*}
U_{n i}=V_{n i}+\varepsilon_{n i}, \tag{1}
\end{equation*}
$$

where $V_{n i}$ and $\varepsilon_{n i}$ are the deterministic and stochastic terms of the utility function, respectively.

The assumption of a type I extreme value distribution yields a logit model, with the probability of commuter $n$ choosing alternative $i$ out of $J=1,2, \ldots, J$ given by

$$
\begin{equation*}
p_{n i}=\frac{e^{V_{n i}}}{\sum_{j=1}^{J} e^{V_{n i}}}, \tag{2}
\end{equation*}
$$

where $0 \leq p_{n j} \leq 1, \forall n, j$ and $\sum_{j=1}^{J} p_{n j}=1, \forall n$.

Under the assumption of a generalised extreme value distribution, an NL structure is used instead, in which the alternatives are grouped into mutually exclusive nests. The probability of a commuter choosing alternative $i$, where $i \in m$, with $m$ being one of $R=1,2, \ldots, R$ different nests, is given by

$$
\begin{equation*}
p_{i m}=p_{i \mid m} \times p_{m}, \tag{3}
\end{equation*}
$$

where $p_{i \mid m}^{n}$ is the conditional probability of alternative $i$ being chosen in nest $m$ and $p_{m}^{n}$ is the marginal probability of nest $m$ being chosen.

$$
p_{i \mid m} \text { and } p_{m} \text { can be calculated as }
$$

$$
\begin{align*}
& p_{i \mid m}=\frac{e^{\mu_{2} V_{i m}}}{\sum_{j=1}^{J} e^{\mu_{2} V_{j p m}}},  \tag{4}\\
& p_{m}=\frac{e^{\mu_{1}\left(V_{m}+V_{m}^{*}\right)}}{\sum_{r=1}^{R} e^{\mu_{1}\left(V_{r}+V_{r}^{*}\right)}},  \tag{5}\\
& V_{m}^{*}=\frac{1}{\mu_{2}} \ln \sum_{j=1}^{J} e^{\mu_{2} V_{j m}}, \tag{6}
\end{align*}
$$

where $V_{m}^{*}$ is the logsum term reflecting the impact of the lower level on the upper level and $\mu_{1}$ and $\mu_{2}$ are the scale parameters of the upper and lower levels, respectively. Because the normalisation is performed at the top, $\mu_{1}=1$ and $\mu_{2}>1 . V_{i \mid m}$ is the lowerlevel utility of choosing alternative $i$ in nest $m$ perceived by a commuter and $V_{m}$ is the upper-level utility of choosing nest ${ }^{m}$ perceived by a commuter.

Intuitively, there might be differences in the peak avoidance intentions of commuters in the morning and evening peak times. Commuters frequently have more choices for after-hour activities, instead of returning home immediately, which probably leads to a difference in the value of time (VOT). Thus, we establish choice models for morning and evening peaks separately. The NL model structure is illustrated in Figure 6.

The upper level shows whether a commuter keeps travelling by metro or shifts to other travel modes, such as buses, taxis, and private cars. The lower level describes the departure time adjustment of a commuter, including maintaining the regular schedule and departing earlier or later in various time ranges.

There are four nests referring to the nests of metro, bus, taxi, and private car, respectively. $A_{j}$ denotes the alternative in the first nest of metro in Table 1, where the attributes used in the utility function are listed.

The entire sample set is divided into two subsets in terms of the peak types. Subsequently, we use Guass 16.0 to calibrate the NL models with the classified SP data. The model estimation results are summarized in Table 2.

All estimates for the situational attributes listed in Table 2 have absolute t -values greater than 1.96, indicating significant effects on the commuter behaviour. The variables of travel time, travel cost, shift in departure time, and crowding degree have significant negative impacts on the departure time and the mode shift decisions, as expected. The adjusted $\rho^{2}$ values of the two models are 0.417 and 0.388 , respectively, suggesting they perform well in terms of model fitting. Additionally, the VOTs reflected by the time-related attributes are calculated based on Equation (7). The VOT estimation results are summarized in Table 3.

$$
\begin{equation*}
\eta=\beta_{\mathrm{T}} / \beta_{\mathrm{F}}, \tag{7}
\end{equation*}
$$

where $\eta$ is the VOT of a time-related variable, $\mathrm{CNY} / \mathrm{h}$; and $\beta_{\mathrm{T}}$ and $\beta_{\mathrm{F}}$ are the estimated coefficients of a time-related variable and the travel cost, respectively.

Based on the data released by the Beijing Municipal Bureau of Statistics, the per capita disposable income of Beijing was 62,361 CNY in 2018. As a reference for the obtained VOTs, the mean working VOT of Beijing residents is estimated as 29.52 CNY/h considering 8 working hours a day, 22 working days a month, and a total of 2,112 working hours per year.

The VOTs in the morning peak are higher than those in the evening peak because the former is typically more crowded and less comfortable than the latter. We
also speculate that commuters generally prefer to have flexibility in adjusting the afterwork schedule. In most cases, the primary task of a commuter in the morning is to avoid being late for work. However, in the evening peak, numerous people tend to spend time on after-work activities, which also helps motivate peak avoidance behaviour. Therefore, time-dependent pricing strategies typically perform better in the evening peak times and are lesser likely to lead to a loss of ridership than those in the morning peak times, which is also in line with the statistical survey results displayed in Figure 5.

We also notice that the sensitivity to shifting the departure time is less strong than that to the travel time. This is closely related to the natural differences between these two attributes; specifically, travel time is the actual time consumed of time by the commuters to complete a daily round-trip. However, when commuters reschedule their commuting plans to access a reduced fare, a scheduling disutility occurs; concurrently, the time can still be used for other activities.

### 3.2. Bi-level optimisation model

Based on the estimated choice model, this section presents the bi-level programming conducted for developing time-dependent pricing strategies. The upper-level optimisation model generates candidate strategies that balance the interests of the metro operator and the commuters with multiple objectives of maximising the sum of the operator and commuter surplus, and minimising the peak ridership. The lower level utilizes an NL-based SUE model to estimate the variations in the travel demand under the impact of the candidate strategies. The feedback of the SUE demand patterns from the lower-level model enables the upper-level model to evaluate the candidate strategies accurately and helps obtaining the optimal solution for the entire bi-level model.

### 3.2.1. Upper-level multi-objective optimisation model

The upper-level model aims to ensure an appropriate optimisation direction to avoid a local optimum solution. Regarding the optimisation objectives, the primary goal of implementing a time-dependent pricing strategy during rush hours is to motivate the peak avoidance behaviour of passengers to the maximum extent to alleviate the overcrowding on platforms and trains. At most crowded stations, the arrival of numerous passengers in a short period overloads the metro system. Hence, reducing the peak ridership is crucial for operators to ensure safe operation and provide high-quality commuter service. Concurrently, metro operators intend to maintain or even improve the current profits from ticket sales. From a commuter perspective, the preferences are for better service and lower prices. Based on the above considerations, two objectives are used in the upper-level optimisation model: maximising the sum of the operator and commuter surplus and minimising the peak ridership.

For the operator surplus, we assume that the cost of the metro operator is relatively fixed in our research context; it mainly consists of the construction cost, operation cost (largely depending on the train timetable and the rolling stock schedule), and labour cost. Although pricing strategies impact the perceived service quality of the commuters, and thus, lead to changes in the travel demand, they have less effect on the overall cost of the operator. However, the ticket revenue of the operator is directly affected by the adopted pricing strategy. Given that the cost of the operator is assumed to be constant in this study, the revenue changes can be regarded as equivalent to the surplus changes. Thus, we use the difference between the ticket revenue before and after implementing a pricing strategy to represent the operator surplus.

Let $K$ denote the set of research periods, $k \in K ; K_{1}, K_{2}$, and $K_{3}$ denote the sets of pre-off-peak, peak, and after-off-peak periods, respectively; and $W$ denote the set of origin-destination (O-D) pairs, $w \in W$. Thus, the operator surplus can be expressed as

$$
\begin{equation*}
E=\sum_{w \in W} \sum_{k \in K} p_{w}\left[\delta_{w}(k) q_{m}^{w}(k)-\hat{q}_{w}(k)\right], m=1, \tag{8}
\end{equation*}
$$

where $E$ is the operator surplus, $\mathrm{CNY} ; p_{w}$ is the base fare between $\mathrm{O}-\mathrm{D}$ pair $w . \delta_{w}(k)$ is the time-dependent fare rate between $\mathrm{O}-\mathrm{D}$ pair $w$ during period $k$, which is also the decision variable in the optimisation model. $\hat{q}_{w}(k)$ is the number of commuters travelling by the metro between $\mathrm{O}-\mathrm{D}$ pair $w$ during period $k$ before implementing a time-dependent pricing strategy. $q_{m}^{w}(k)$ is the number of commuters travelling by mode $m$ between $\mathrm{O}-\mathrm{D}$ pair $w$ during period $k$ after implementing the time-dependent pricing strategy.

Given that travel cost of a commuter comprises both time and monetary costs, we calculate the commuter surplus in terms of time and monetary components. Taking the monetary component as an example, we consider the current ticket price (i.e., the base fare) as the amount that commuters are willing to pay and consider the timedependent ticket price as the amount that a commuter actually spends. It should be noted that the monetary component of the commuter surplus (MCCS) should cover all commuters either still taking the metro or shifting to other travel modes. For the minority commuters shifting to other travel modes, we take the fare of the new travel mode as the amount they spent. Hence, there is a difference between the metro ticket revenue of the operator and the total monetary cost of the commuters. The equations for calculating the MCCS are

$$
\begin{equation*}
C_{\mathrm{M}}=\sum_{w \in W} \sum_{k \in K} \sum_{m \in M}\left[p_{w} \hat{q}_{w}(k)-p_{m}^{w}(k) q_{m}^{w}(k)\right], \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
p_{m}^{w}(k)=p_{w} \delta_{w}(k), m=1, \tag{10}
\end{equation*}
$$

where $C_{\mathrm{m}}$ is the MCCS, CNY; $M$ is the set of the nests in the NL model, $m \in M$; and $p_{m}^{w}(k)$ is the travel cost of mode $m$ between O-D pair $w$ during period $k$ CNY. Note that except when $m=1, p_{m}^{w}(k)$ is related only to O-D pair $w$ and does not vary with period $k$, given that time-dependent pricing only applies to the metro.

The time component of the commuter surplus (TCCS) is calculated similarly as above. We use the difference between the original and current travel times to represent the TCCS. Using the VOT coefficient obtained from the choice model, the unit of the TCCS can be further converted from seconds to CNY as follows:

$$
\begin{equation*}
C_{\mathrm{T}}=\eta \sum_{w \in W} \sum_{k \in K} \sum_{m \in M}\left[\hat{t}_{w}(k) \hat{q}_{w}(k)-t_{m}^{w}(k) q_{m}^{w}(k)\right], \tag{11}
\end{equation*}
$$

where $C_{\mathrm{T}}$ is the generalised TCCS, $\mathrm{CNY} ; \hat{t}_{m}^{w}(k)$ is the metro travel time between $\mathrm{O}-\mathrm{D}$ pair $w$ during period $k$ before implementing a time-dependent pricing strategy; and $t_{m}^{w}(k)$ is the time travelling by mode $m$ between O-D pair $w$ during period $k$ after implementing the time-dependent pricing strategy.

Based on the above specifications, the upper-level optimisation model can be formulated as follows:

$$
\begin{gather*}
Z_{1}=\max \left(E+C_{\mathrm{M}}+C_{\mathrm{T}}\right),  \tag{12}\\
Z_{2}=\min \left[\max q_{m}^{w}(k)\right], m=1, \tag{13}
\end{gather*}
$$

subject to

$$
\begin{equation*}
\delta_{\text {min }} \leq \delta_{w}(k) \leq 1, \forall w \in W, \forall k \in K_{1} \mid K_{3}, \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
1 \leq \delta_{w}(k) \leq \delta_{\max }, \forall w \in W, \forall k \in K_{2} \tag{15}
\end{equation*}
$$

where $Z_{1}$ and $Z_{2}$ are the two objectives of the optimisation model, representing the decision preferences for improving the benefits of operator and commuters, and reducing the peak ridership in the network during the rush hours, respectively. $\delta_{\text {min }}$ and $\delta_{\text {max }}$ are the lower and upper limits of the decision variable, $\delta_{w}(k)$.

Equations (15) and (16) are set to ensure reasonable ranges of the solutions, i.e., time-dependent fare rates $\delta_{w}(k)$. Considering realistic scenarios, we assume that the extra peak charge only applies to the peak hours and the off-peak discount only to the off-peak hours. Accordingly, we set an upper limit for the extra peak charge and a lower limit for the off-peak discount to protect the basic interests of the commuters and the operator, respectively.

### 3.2.2. Lower-level NL-SUE model

Instead of simply assuming that all commuters follow the least-cost choice as in the deterministic user equilibrium model, the SUE model describes the decision-making of the users as a dynamic process. This is because the behaviour of a commuter, which affects the behaviour of other commuters as one of the determinants (i.e., crowding degree), varies with his/her peak avoidance behaviour (i.e., the shift in the departure time and the mode choice).

The SUE model defines a scenario in which no user can improve the perceived travel cost by changing the alternatives unilaterally (Prashker and Bekhor 2004). Under this condition, each alternative in the NL choice model has a non-zero probability of being chosen (Zhang, Yao, and Pan 2019). Thus, the travel demand of the commuters
can be measured by their choice results under the SUE condition, i.e., the flow of choosing the alternatives in the NL model.

In a two-level NL model, two SUE conditions are derived from each level of the choice model. The first condition is the equilibria of the mode choice decisions, and the second condition is the equilibria of the departure time decisions in the nest of the metro. The generalised cost of the alternatives can be expressed as

$$
\begin{gather*}
C_{i \mid m}^{w}(k)=-V_{i \mid m}^{w}(k), m=1,  \tag{16}\\
C_{m}^{w}(k)=\left\{\begin{array}{l}
-\frac{1}{\lambda_{2}} \ln \sum_{i \in I} \exp \left[\lambda_{2} V_{i \mid m}^{w}(k)\right], m=1, \\
-V_{m}^{w}(k), m \neq 1
\end{array}\right. \tag{17}
\end{gather*}
$$

where $C_{m}^{w}(k)$ is the generalised cost of choosing nest $m$ between $\mathrm{O}-\mathrm{D}$ pair $w$ during period $k ; C_{m}^{w}(k)$ is the generalised cost of choosing alternative $i$ in nest $m$ during period $k$; and $I$ is a set of alternatives in the NL model, $i \in I$.

Extensive studies have suggested that the lower-level SUE model can be expressed as a minimisation problem (Prashker and Bekhor 2004). However, entropybased formulations have no economic interpretation (Liu et al. 2018; Tang et al. 2020). To illustrate well the SUE conditions regarding the departure time and mode joint choice problem, we formulate the lower-level NL-SUE model as a fixed-point problem.

Regarding the upper-level mode choice, $E\left\{\min \left[C_{m}^{w}(k)\right]\right\}$ denotes the minimum expected cost of choosing mode-related nest $m$. The perceived cost of the commuters of choosing nest $m$ can be expressed as

$$
\begin{equation*}
S\left[C_{m}^{w}(k)\right]=E\left\{\min \left[C_{m}^{w}(k)\right]\right\}=-\frac{1}{\lambda_{1}} \ln \sum_{m \in M} \exp \left[-\lambda_{1} C_{m}^{w}(k)\right] . \tag{18}
\end{equation*}
$$

Similarly, with respect to the lower-level departure time choice, $E\left\{\min \left[C_{i \mid m}^{w}(k)\right]\right\}$ denotes the minimum expected cost of choosing departure time-related alternative $i$. The perceived cost of the commuters for choosing alternative $i$ can be expressed as

$$
\begin{equation*}
S\left[C_{i \mid m}^{w}(k)\right]=E\left\{\min \left[C_{i \mid m}^{w}(k)\right]\right\}=-\frac{1}{\lambda_{2}} \ln \sum_{i \in I} \exp \left[-\lambda_{2} C_{i \mid m}^{w}(k)\right], m=1 \tag{19}
\end{equation*}
$$

There are two SUE conditions in this problem: equilibria of the mode choice and departure time choice, respectively. When the SUE conditions are reached, the generalised costs of choosing different alternatives perceived by the commuters are equal and minimised, which are applied to each O-D pair $w$ and period $t$ as follows:

$$
\begin{gather*}
\sum_{w} \sum_{k} q_{m}^{w}(k)\left\{C_{m}^{w}(k)-S\left[C_{m}^{w}(k)\right]\right\}=0,  \tag{20}\\
\sum_{w} \sum_{k} q_{i \mid m}^{w}(k)\left\{C_{i \mid m}^{w}(k)-S\left[C_{i \mid m}^{w}(k)\right]\right\}=0, m=1,  \tag{21}\\
q_{m}^{w}(k)=\hat{q}_{w}(k) \frac{e^{\mu / V_{m}^{w}(k)}}{\sum_{r=1}^{R} e^{\mu_{1} V_{V}^{w}(k)}}, \text { and }  \tag{22}\\
q_{i \mid m}^{w}(k)=q_{m}^{w}(k) \frac{e^{\mu V_{2} V_{i m}^{w}(k)}}{\sum_{j=1}^{J} e^{\mu_{2} V_{j m}^{w}(k)}}, m=1 \tag{23}
\end{gather*}
$$

where $q_{i j m}^{w}(k)$ is the number of commuters choosing alternative $i$ between $\mathrm{O}-\mathrm{D}$ pair $w$ during period $k$, i.e., the alternative flow.

Additionally, the SUE conditions specified in Equations (20)-(23) are subject to the following constraints:

$$
\begin{equation*}
\hat{q}_{w}(k)=\sum_{m} q_{m}^{w}(k), \forall w, k, \tag{24}
\end{equation*}
$$

$$
\begin{gather*}
q_{m}^{w}(k)=\sum_{i} q_{i \mid m}^{w}(k), \forall w, k, m=1,  \tag{25}\\
q_{i \mid m}^{w}(k), q_{m}^{w}(k), \hat{q}_{w}(k)>0, \forall w, k,  \tag{26}\\
C_{m}^{w}(k) \geq S\left[C_{m}^{w}(k)\right], \forall w, k, \text { and }  \tag{27}\\
\quad C_{i \mid m}^{w}(k) \geq S\left[C_{i \mid m}^{w}(k)\right], \forall w, k \tag{28}
\end{gather*}
$$

Equations (24) and (25) express the conservation conditions of the flows of the mode-related nests and the departure time-related alternatives, respectively. Equation (26) is a non-negative constraint. Equations (27) and (28) illustrate the relationship between the perceived cost of the commuters and the minimum expected cost in the SUE problem. The flow-related constraints of the feasible region ensure a non-empty convex set. In addition, the utility functions in Equations (16) and (17) as well as the functions for calculating the probability of the alternatives being chosen in Equations (22) and (23) are continuous. The Brouwer fixed-point theorem ensures the existence of a fixed point, which also indicates the existence of a solution of the equivalent NL-SUE problem.

### 3.3. Solution algorithm

The bi-level programming is an NP-hard problem that involves high computational complexity. Following the method of successive weighted averages (MSWA) proposed by (Liu, He , and He 2009 ), a genetic algorithm-based MSWA algorithm is proposed to solve the formulated bi-level optimisation model.

### 3.3.1. Computing optimal time-dependent pricing strategies

Given the two optimisation objectives in the upper-level optimisation model, a normalisation approach is used to standardise their dimensions. The extreme values of both objectives are estimated in advance by solving each single-objective optimisation problem. Subsequently, the multi-objective optimisation problem is converted into a single-objective form using the fuzzy-compromise decision-making method, such that

$$
\begin{gather*}
\max Z=\max \left(\alpha_{1} \mu_{1}-\alpha_{2} \mu_{2}\right),  \tag{29}\\
\mu_{e}=\frac{Z_{e}-Z_{e}^{\min }}{Z_{e}^{\max }-Z_{e}^{\min }}, \tag{30}
\end{gather*}
$$

where $\mu_{e}$ and $\alpha_{e}$ are the allocation and weight coefficients of objective $Z_{e}$, respectively, $\sum \alpha_{e}=1, e \in\{1,2\} ;$ and $Z_{e}^{\max }$ and $Z_{e}^{\text {min }}$ are the maximum and minimum values of objective $Z_{e}$, respectively.

A set of time-dependent fare rates for the pre-off-peak, peak, and after-off-peak hours forms a gene in the genetic algorithm, which is equivalent to the decision variable in the optimisation model. The chromosome is composed of a set of genes, forming an individual in the population. The best individual carries the optimal solution of the timedependent fare rates when convergence is reached. In addition, several essential procedures, such as selection, crossover, and mutation, are included in the algorithm. We use the gambling-wheel disk selection method to support selecting good individuals from the population as well as to avoid falling into a local optimum solution. The algorithm for solving the upper-level model involves the following steps (for detailed steps, see Figure 7):

- Step 1: Parameter initialisation. The related parameters are the binary code of the gene, population size, probabilities of crossover and mutation, and number of iterations. In particular, the first generation of the population is initialised subject to Equations (14) and (15).
- Step 2: Fitness evaluation. The best individual is substituted into the lower-level model to obtain the SUE solution. Furthermore, the fitness of an individual is measured by the objective function based on the obtained SUE solution.
- Step 3: Individual selection. All individuals in the current population are prioritised in terms of the fitness performance. Thus, good individuals are preserved for the next generation using the gambling-wheel disk selection method.
- Step 4: Population evolution and convergence determination. A two-point crossover and a stochastic mutation are applied for the evolution of the population. The above steps are repeated until the convergence condition is satisfied.


### 3.3.2. Computing $N L-S U E$ solution

The main objectives of the lower-level model are to estimate the choice results of the commuters under the influence of the candidate time-dependent pricing strategies generated by the upper-level model and provide accurate criteria for the fitness evaluation of the candidate strategies. An MSWA-based method is used to solve the SUE problem by the following steps:

- Step 1: Parameter initialisation. Let iteration index $g=1$, lower error bound $\varphi=10^{-4}$, period index $k=1$, flow of mode $q_{m, w}^{(g)}(k)=0$, flow of departure time $q_{i \| n, w}^{(g)}(k)=0$, and additional flows $\bar{q}_{m, w}^{(g)}(k)=0$ and $\bar{q}_{i \mid m, w}^{(g)}(k)=0$.
- Step 2: Utility updation of the alternatives. Based on the current status of the situational attributes, utilities $V_{m, w}^{(g)}(k)$ and $V_{i \mid m, w}^{(g)}(k)$ of the alternatives are calculated. Concurrently, probabilities of choosing each alternative $p_{m, w}^{(g)}(k)$ and $p_{i m, w}^{(g)}(k)$ are also updated.
- Step 3: Flow assignment. The formulae for calculating the flow of choosing each alternative are

$$
\begin{gather*}
q_{i m, w}^{(g)}(k)=q_{i j m, w}^{(g-1)}(k)+\frac{g}{1+2+\cdots+g}\left[\bar{q}_{i m, w}^{(g)}(k)-q_{i m, w}^{(g-1)}(k)\right] \text { and }  \tag{31}\\
\bar{q}_{i m, w}^{(g)}(k)=q_{i m, w}^{(g)}(k) p_{i m, w}^{(g-1)}(k+1) . \tag{32}
\end{gather*}
$$

- Step 4: Convergence determination. The algorithm is ended when the convergence condition in Equation (31) is satisfied. Otherwise, Step 2 is repeated, and the iteration is continued.

$$
\begin{equation*}
\frac{\sum_{w} \sum_{j} \sum_{k} q_{i j n, w}^{(k)}(k)-q_{i j n, w}^{(8-1)}(k) \mid}{\sum_{w} \sum_{j} \sum_{k} q_{i m, w}^{(8)}(k)}<\varphi . \tag{33}
\end{equation*}
$$

By combining the above two parts of the solution procedure, the proposed bilevel optimisation model can be solved efficiently. For a more explicit illustration, Figure 7 presents a flowchart of the solution algorithm.

## 4. Case study

In this section, an empirical case study of the BT line in the Beijing metro network conducted to test the proposed model is discussed. The BT line connects the eastern part to the central area of Beijing and currently has 13 stations in operation, as shown in Figure 8.

The eastern end of the BT line is located in a residential area of east Beijing, where numerous commuters take the metro to the CBD area every morning. The BT line plays a crucial role in supporting the west-east commuting corridor of Beijing. The blue dotted lines represent the O-D pair studied in this case. Specifically, the residential area is composed of the Liyuan (LY), Linheli (LHL), and Tuqiao (TQ) stations, and the CDB area is composed of the Yonganli (YAL), Guomao (GM), Dawanglu (DWL), Sihui (SH), and Sihuidong (SHD) stations.

The study period was set between 6:00 am and 9:00 am, in which the peak hour is between 7:00 am and 8:00 am. The periods on either side are defined as the off-peak hours on a weekday morning: a pre-off-peak hour from 6:00 am to 7:00 am and an after-off-peak hour from 8:00 am to 9:00 am.

The data on the travel demand in the study period were extracted from the smart card usage records collected by the AFC system. Additionally, the current pricing scheme, train capacity, and timetable were obtained for use in the upper-level optimisation model. In particular, access to the situational attributes of the substituted travel modes was obtained using the Baidu web application programming interface, a professional tool for developers to use real-time navigation data provided by the Baidu Map. Thus, all attributes involved in the utility functions were derived well to support solving the lower-level NL-SUE model.

The parameters of the solution algorithm were determined by parameter tuning. Based on the computational complexity of the optimisation problem and the available computing resources, the optimisation results were tested under different levels of parameter values and evaluated in terms of the convergence rate and the solution quality. In the genetic algorithm, the population size was set as 30 and the maximum number of iterations was set as 500. The crossover and mutation probabilities were set as $99 \%$ and $5 \%$, respectively. The smallest unit of the fare rate change was set as 0.01 . For the MSWA, the upper error bound for the convergence was set as $10^{-4}$. Additionally, the upper and lower limits of the decision variable, i.e., the time-dependent fare rate, were set as 2.00 and 0.25 for the extra peak charge and the off-peak discount, respectively.

From a practical perspective, time-dependent pricing strategies take responsibility for all interest groups. Generally, although a balanced strategy is preferred, the optimal strategy varies with the priorities of policy-making. Under different weight combinations of the two optimisation objectives (i.e., $Z_{1}$ and $Z_{2}$ ), timedependent pricing schemes under three representative decision preferences-operator and commuter benefits preferred pricing schemes $\left(Z_{1}: Z_{2}=1: 0\right.$, namely, only $Z_{1}$ was retained, and $Z_{2}$ was converted into a constraint), ridership peak-cutting preferred pricing schemes ( $Z_{1}: Z_{2}=1: 2$ ), and balanced pricing schemes $\left(Z_{1}: Z_{2}=1: 1\right)$-were optimised using the proposed model. Table 4 reports the optimal time-dependent fare rates.

As seen from Table 4, there are three pricing measures in each category of the pricing schemes-off-peak discount, extra peak charge, and both—intending to provide comprehensive references to policymakers. A total of nine optimal schemes are obtained and evaluated in this study, along with the current pricing scheme as the reference. In the operator and commuter benefits preferred pricing schemes (Schemes

II, III, and IV), the price increase and decrease are smaller than those in the ridership peak-cutting preferred pricing schemes (Schemes V, VI, and VII), indicating the necessity of including the second objective. The two objectives achieve a balance in the third category of schemes (Schemes VIII, IX, and X), with an optimal off-peak discount of approximately $30 \%$ and the extra peak charge working best at $135 \%$.

To provide more quantitative criteria for choosing appropriate strategies, the performance of the above six optimal strategies were further evaluated. Figure 9 presents the temporal distributions of the metro ridership under the influence of the time-dependent pricing strategies.

Schemes V, VI, and VII reduce the peak ridership most significantly during 7:45-8:00. Commuters who originally depart at the start or the end of the peak hour tend to avoid the extra peak charge or prefer to receive the off-peak discount by adjusting their departure times. Among these three most powerful strategies, the flowregulation effects appear to be excessive as numerous commuters are incentivized to shift to the pre-off-peak period of 6:45-7:00 as well as the after-off-peak period of 8:00-8:15, thereby leading to new ridership peaks. Schemes VIII, IX, and X perform moderately for mitigating crowding during the peak hours. In contrast to the above two categories of the schemes, Schemes II, III, and IV have comparatively weaker flowregulation effects. Furthermore, Figure 10 presents the mode share patterns under the above schemes to provide a complete evaluation.

As seen in Figure 10, Scheme VI results in the maximum loss of metro ridership of approximately $6 \%$, given that only a relatively high level of extra peak charge is adopted. As shown in Figure 9, the peak-cutting effects of Schemes VII (both measures), VI (extra peak charge), and V (off-peak discount) decrease in order, which is basically in line with the loss of the total ridership. It should be noted that Scheme VII leads to
less loss of ridership than Scheme VI, indicating that the inclusion of the off-peak discount helps retain ridership while pursuing more potent incentivizing effects. If only focusing on the performance of ridership retention, Schemes II, VI, and VIII can be deemed as the best three schemes because they are more concerned about the benefits of the commuters by purely offering off-peak discounts. Additionally, the most favourite alternative travel mode of the metro commuters is confirmed to be a bus, transporting approximately three-quarters of the total number of shifted commuters.

Finally, we compare the objective-related indicators of the pricing schemes to evaluate their overall performance. In Table 5, the obtained nine optimal pricing schemes are compared with the current pricing scheme, Scheme I, in terms of the ticket revenue of the operator, generalised travel cost of the commuters (including both travel time cost and monetary cost), and peak ridership after implementing the time-dependent pricing schemes. Clearly, all nine schemes reduce the peak ridership regardless of the amount of change. Schemes II, V, and VIII not only reduce the cost of the commuters but also the revenue of the operator. Schemes III, VI, and IX have positive impacts on the revenue of the operator but increase the cost of the commuters. Only Schemes IV, VII, and X achieve a win-win scenario for the operator and the commuters. Although the benefits for both are small, the decrease in the peak ridership is significant, which is more important for policymakers. Among these three schemes, Scheme IV has the weakest effect on the ridership peak cutting; therefore, we do not recommend its use under realistic conditions. In addition, Scheme VII outperforms Scheme X in terms of the three indicators. It should also be noted that Scheme VII results in greater loss of the metro ridership than Scheme X. Overall, Schemes V and VIII are recommended if the operator has the customer-first mindset or has access to government subsidies or other forms of financial supports. Schemes VI and X are suggested under general
circumstances. Comparatively, the former can maximise the benefits of the system, whereas the latter has less impact on the current travel demand patterns. The fare rate changes in Scheme X are smaller than those in Scheme VI. Scheme X can be deemed as a moderate alternative to Scheme VI to some extent, and therefore, is recommended for policy exploration.

## 5. Conclusions

The present paper proposes a comprehensive framework for optimising time-dependent pricing strategies based on the peak avoidance behaviour of commuters, which enables policymakers to manage the increased travel demand on overcrowded metro lines. Several conclusions can be drawn from this study.

Between the two forms of time-dependent pricing discussed in this paper-offpeak discount and extra peak charge-the former is clearly beneficial for commuters but incapable of reshaping demand patterns based on the results of an empirical case study of the BT Line. In this regard, the off-peak discount is more preferable for metro operators who have particular access to government grants or other sources of revenue. The latter (i.e., extra peak charge) performs effectively for both flow regulation and ticket revenue retention. However, it occasionally leads to a severe loss of the metro ridership and tends to be less commuter-friendly. From the application perspective, the extra peak charge is a reasonable choice for incentivising peak avoidance behaviour only if there is a preference for maintaining the revenue of the operator. Moreover, it should also be based on the premise that the metro overwhelmingly dominates other modes in a specific commuting corridor so that the loss of ridership can be minimized. In contrast, the combined strategy possesses the advantages of both measures. Notably, inclusion of the off-peak discount reduces the aversion of the commuters to shift outside the peak hours, which provides cost-saving choices to them to avoid the expensive peak
trip, and thus, can be a useful supplement to the extra peak charge. Concurrently, the accompanying loss of ticket revenue driven by the off-peak discount can be compensated by the lower reduction in the total ridership.

In addition, there are still limitations requiring further improvements in future studies. The first concern is associated with the implementation of time-dependent pricing in a real-world context. With the fixed boundary points (i.e., starting at 7:00 a.m. and ending at 8:00 a.m.) currently adopted in the time-dependent pricing strategies, commuters who depart much later than the given boundary point are less likely to be affected by the pricing strategy. In this regard, the rapid adoption of e-tickets will create opportunities to track the commuting activities of individuals in the future. There is a possibility for providing a customised reward plan to individuals who intend to adjust their personal commuting behaviour in response to the call for peak avoidance.

Additionally, in the NL model, the mode shift intentions of the metro commuters are measured using the collected SP data. By contrast, the commuters who initially take other travel modes and might be attracted by the metro pricing strategy (mainly by the off-peak discount) are not considered in this research, which is another limitation of this study. In fact, there is a theoretical possibility of inducing demand outside of the metro, although the impact may be minimal and is typically ignored in relevant studies. To fully understand the mode share patterns in a commuting corridor, a large survey involving commuters who travel by modes other than the metro is needed in future research.

Overall, this study provides insights about the peak avoidance behaviour of commuters in response to pricing incentivisation measures and further proposes a methodology for tailoring demand-responsive time-dependent pricing strategies for practical needs. The findings regarding the departure times and mode shift intentions of
commuters indicate that significant attention should be paid to understanding the evolution of travel demand patterns in the context of TDM. This is particularly essential for research on the issues of operation management (e.g., train scheduling, bus bridging service, and passenger flow control strategies) as well as other empirical studies in relation to the supply and demand issues in mass transit systems. The proposed optimal time-dependent pricing strategies meet a wide range of preferences for decision-making, allowing policymakers to adopt steps to reshape demand patterns depending on actual requirements.

## Conflict of interest

On behalf of all authors, the corresponding author states that there are no conflicts of interest.

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Table 1. Composition of utility functions

| $\underbrace{}_{\text {Attributes }}$ Alternatives |  | Metro |  |  | Bus | Taxi | Private car |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{A}_{1} \quad \mathrm{~A}_{2}$ |  | $\mathrm{A}_{J}$ |  |  |  |
| Upper level | Alternative-specific |  | - |  | - | $\sqrt{1}$ | - |
|  | constant (ASC) |  | - |  | - | - | $\checkmark$ |
|  | Travel time |  | $\checkmark$ |  | $\checkmark$ | $\checkmark$ | $\checkmark$ |
|  | Travel cost |  | $\checkmark$ |  | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Lower level | Shift in departure time |  | $\checkmark$ |  | - | - | - |
|  | Crowding degree |  | $\checkmark$ |  | - | - | - |

${ }^{1}$ Tick mark indicates that attribute in horizontal row is included in utility function of alternative in vertical column. Short dash reflects opposite case.

Table 2. Model estimation results

| Attributes | Unit | Morning peak |  | Evening peak |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Est. | t -rat. | Est. | t -rat. |
| Scaled parameter | - | 0.967 | 3.140 | 0.558 | 3.144 |
| ASC for taxi | - | 12.471 | 2.201 | 17.843 | 2.750 |
| ASC for private car | - | 10.564 | 2.046 | 15.546 | 2.632 |
| Travel time | h | -8.068 | -11.198 | -5.651 | -14.828 |
| Travel cost | CNY |  |  |  |  |
| Shift in departure time | h | -0.250 | -2.870 | -0.317 | -3.180 |
| Crowding degree ${ }^{2}$ | $\%$ | -0.333 | -3.238 | -2.446 | -3.204 |
| Sample size | 1245 |  |  |  |  |
| Adjusted $\rho^{2}$ | 0.417 |  |  |  |  |

${ }^{1}$ CNY/USD $\approx 0.145$ during survey period. ${ }^{2}$ Crowding degree is quantified by mean SLR of route.

Table 3. VOT estimation results

| Time-related variables | Morning peak (CNY/h) | Evening peak (CNY/h) |
| :---: | :---: | :---: |
| Travel time | 32.3 | 17.8 |
| Shift in departure time | 22.2 | 7.7 |

Table 4. Optimal time-dependent pricing schemes under various decision preferences

| No. | Scheme description | Pricing measure |  | Optimal fare rate |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Off-peak discount | Extra peak charge | $\begin{gathered} \text { Pre- } \\ \text { off-peak } \end{gathered}$ | Peak | After-off-peak |
| I | Current pricing scheme | $\times$ | $\times$ | 1.00 * | $1.00^{*}$ | 1.00* |
| II | Operator and commuter | $\checkmark$ | $\times$ | 0.93 | $1.00^{*}$ | 0.95 |
| III | benefits preferred pricing | $\times$ | $\checkmark$ | 1.00 * | 1.18 | 1.00 * |
| IV | scheme | $\checkmark$ | $\checkmark$ | 0.87 | 1.13 | 0.90 |
| V |  | $\checkmark$ | $\times$ | 0.46 | 1.00* | 0.49 |
| VI | Ridership peak-cutting preferred pricing scheme | $\times$ | $\checkmark$ | 1.00 * | 1.63 | 1.00 * |
| VII |  | $\checkmark$ | $\checkmark$ | 0.56 | 1.52 | 0.60 |
| VIII |  | $\checkmark$ | $\times$ | 0.73 | 1.00* | 0.74 |
| IX | Balanced pricing scheme | $\times$ | $\checkmark$ | 1.00 * | 1.48 | 1.00 * |
| X |  | $\checkmark$ | $\checkmark$ | 0.68 | 1.35 | 0.70 |

${ }^{*}$ indicates that fare rate is constant in pricing scheme.

Table 5 Overall performance of different pricing schemes

| Scheme <br> No. | Ticket revenue of <br> operator/kCNY | Time cost of <br> commuters/kCNY | Monetary cost of <br> commuters/kCNY | Peak ridership <br> during rush hour |
| :---: | :---: | :---: | :---: | :---: |
| I | 44.32 | 220.52 | 44.32 | 1430 |
| II | $42.93(-3.13 \%)^{1}$ | $217.98(-1.15 \%)$ | $43.45(-1.96 \%)$ | $1384(-3.23 \%)$ |
| III | $47.63(7.47 \%)$ | $219.59(-0.42 \%)$ | $49.53(11.74 \%)$ | $1355(-5.28 \%)$ |
| IV | $44.89(1.29 \%)$ | $217.84(-1.22 \%)$ | $46.08(3.96 \%)$ | $1338(-6.41 \%)$ |
| V | $31.98(-27.85 \%)$ | $219.73(-0.36 \%)$ | $32.32(-27.08 \%)$ | $1245(-12.91 \%)$ |
| VI | $56.89(28.37 \%)$ | $220.63(0.05 \%)$ | $59.73(34.77 \%)$ | $1239(-13.32 \%)$ |
| VII | $45.08(1.71 \%)$ | $218.16(-1.07 \%)$ | $47.27(6.67 \%)$ | $1230(-13.97 \%)$ |
| VIII | $37.75(-14.82 \%)$ | $219.12(-0.64 \%)$ | $38.23(-13.73 \%)$ | $1302(-8.95 \%)$ |
| IX | $54.18(22.25 \%)$ | $220.71(0.09 \%)$ | $56.50(27.49 \%)$ | $1282(-10.33 \%)$ |
| X | $44.68(0.81 \%)$ | $218.35(-0.98 \%)$ | $46.29(4.45 \%)$ | $1267(-11.41 \%)$ |

${ }^{1}$ Value in parenthesis are percent changes in indicator compared to those in Scheme I.

Figure 1. Hourly ridership variation on single weekday


Figure 2. Spatiotemporal distributions of section load rate of line 4


Figure 3．Example of hypothetical scenario in questionnaire

Assuming that you live in Huilongguan community and work near Xizhimen，below are various commuting plans，as well as two time－dependent pricing schemes that may be implemented in the near future．

| Scenario \＃1 |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Plan No． | Travel by | Leave at | Arrive by | Duration | Crowded－ ness | Fare |  |  |
|  |  |  |  |  |  | Current | Scheme I | Scheme II |
| 1 | Metro | 6：35 | 7：40 | 1 h 5 min | $\ddot{\\|}$ | $\ddagger 5$ | $\ddagger 2$ | $\ddagger 2$ |
| 2 |  | 6：40 | 7：55 | 1h20min | 中 | ¥ 5 | ¥ 3.5 | $\ddagger 4$ |
| 3 |  | 6：50 | 8：10 | 1h20min | in | ¥5 | ¥3．5 | ¥4 |
| 4 |  | 6：55 | 8：25 | 1h30min | 新 | $\ddagger 5$ | $\ddagger 5$ | ¥8 |
| 5 |  | 7：10 | 8：40 | 1h30min | 榊新 | ¥5 | \＃5 | ¥8 |
| 6 |  | 7：25 | 8：55 | 1h30min | $\frac{4 i n}{n}$ | $\ddagger 5$ | $\ddagger 5$ | ¥8 |
| 7 |  | 7：50 | 9：10 | 1h20min | 新䡚 | ¥5 | $\ddagger 5$ | $\ddagger 8$ |
| 8 |  | 8：02 | 9：25 | 1h20min | 神新 | $¥ 5$ | ¥ 3.5 | $¥ 4$ |
| 9 |  | 8：35 | 9：40 | 1 h 5 min | in | $\neq 5$ | $¥ 3.5$ | $\ddagger 4$ |
| 10 |  | 8：50 | 9：55 | 1h5min | 中inin | $\not \geqslant 5$ | $\not \geqslant 2$ | $¥ 2$ |
| 11 | Bus | 1h40min |  |  | iif | $¥ 5$ |  |  |
| 12 | Taxi | 1h10min |  |  |  | $\not \approx 71$ |  |  |
| 13 | Car | 1h10min |  |  |  | $¥ 65$ |  |  |

If your regular time to arrival at the workplace is as below，and Scheme I is implemented，please tick the box below the option that you prefer．

| Regular plan | OptionsNote：$\square \leftarrow 10$ refers to departing 10 minutes earlier than usual |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 3－arrive at 8：10 | $\square \leftarrow 15$ | $\square \leftarrow 10$ | $\square$ |  |  |  |  |  |  | $\square \rightarrow \mathbf{1 2 0}$ | $\square$ | $\square$ | $\square$ |
| 4 －arrive at 8：25 | $\square \leftarrow 25$ | $\square \leftarrow 15$ | $\square \leftarrow 5$ | $\square$ |  |  |  | $\square \rightarrow 70$ | $\square \rightarrow \mathbf{1 0 0}$ | $\square \rightarrow 115$ | $\square$ | $\square$ | $\square$ |
| 5－arrive at 8：40 | $\square \leftarrow 35$ | $\square \leftarrow 30$ | $\square \leftarrow 20$ | $\square \leftarrow 15$ | $\square$ |  |  | $\square \rightarrow 55$ | $\square \rightarrow 85$ | $\square \rightarrow 110$ | $\square$ | $\square$ | $\square$ |
| 6－arrive at 8：55 | $\square \leftarrow 50$ | $\square \leftarrow 45$ | $\square \leftarrow 35$ | $\square \leftarrow 30$ |  | $\square$ | $\square \rightarrow 25$ | $\square \rightarrow 40$ | $\square \rightarrow 70$ | $\square \rightarrow 85$ | $\square$ | $\square$ | $\square$ |
| 9－arrive at 9：40 | $\square \leftarrow 120$ | $\square \leftarrow 115$ |  |  |  |  |  |  | $\square$ | $\square \rightarrow 15$ | $\square$ | $\square$ | $\square$ |

If your regular time to arrival at the workplace is as below，and Scheme II is implemented，please tick the box below the option that you prefer．

Figure 4. Descriptive statistics of collected samples


Figure 5. Statistical results of SP choices towards departing earlier


Figure 6. NL model structure


Figure 7. Flowchart of solution algorithm


Figure 8. Schematic of BT line


Figure 9. Metro ridership patterns under different pricing schemes


Figure 10. Mode share patterns under different pricing schemes


