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Departure time choice behavior in commute problem with stochastic bottleneck capacity: Experiments and modeling

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Abstract: Since environmental uncertainty is usually inevitable in transportation systems, considering its effect on equilibrium patterns is of vital importance to understand travel choice behavior. This paper focuses on stochastic bottleneck capacity, one classical environmental uncertainty, in the morning commute problem. We conducted a laboratory experiment to investigate the effects of stochastic bottleneck capacity on departure time choice behavior based on a variant of Vickrey's single bottleneck model. In the experiment, the bottleneck capacity varied stochastically from round to round, and two different scenarios with different information feedback, i.e., feedback on the subjects' own cost only and feedback on costs of all departure times, were investigated. Our experimental results showed that the relationship between the mean travel cost and the standard deviation of travel cost on each departure time was fitted approximately linearly with a positive slope λ^*, indicating subjects were more likely to minimize their travel cost budget rather than their mean travel cost under environmental uncertainty. Also, we found that the feedback on costs of all departure times resulted in a smaller λ^* than the feedback on the subjects' own travel cost only. We propose a reinforcement learning model to reproduce the main experimental findings. Finally, it is shown that the parameter λ^* obtained from the experiment is within the rational range.

Keywords: departure time choice, bottleneck model, stochastic capacity, laboratory experiment, reinforcement learning model

1. Introduction

Traffic congestion has become a formidable problem in many urban cities, especially during peak hours. Commuting behavior and traffic congestion during the morning peak hours can be described by the classical single bottleneck model proposed by Vickrey (1969). In the model, a fixed number of commuters depart from the same origin (home) to the same destination (workplace) along a single road. There is a potential bottleneck with a fixed capacity on the road, and it will be active when the departure rate exceeds its capacity. A queue develops when the bottleneck is active, and commuters have the travel time cost caused by congestion. Also, commuters are penalized for arriving early or late. Therefore, commuters have to make a tradeoff between travel time and schedule delay to minimize their travel costs. User equilibrium is reached when no commuter can unilaterally alter the departure time to decrease his/her trip costs.

The proposal of the Vickrey's model laid a solid foundation for characterizing the departure time choice behavior of commuters during the morning commute. Since then, many improved models have been proposed to consider more diverse scenarios, such as pricing (Arnott et al., 1990; Laih, 1994, 2004; Lindsey et al., 2012; Wang and Sun, 2014), elastic demand (Arnott et al., 1993; Yang and Huang, 1997), heterogeneous commuters (Arnott et al., 1994; Lindsey, 2004; van den Berg and Verhoef, 2011 and 2014; Yao et al., 2012; Qian et al., 2013; Wang and Xu, 2016), integration of morning and evening peaks (de Palma and Lindsey, 2002; Zhang et al., 2005; Li et al., 2014), modal split (Tabuchi, 1993; Huang, 2002; Lu et al., 2015), rail transit (Hao et al., 2009; Yang and Tang, 2018; Lu et al., 2020), consecutive bottlenecks (Kuwahara, 1990; Lago and Daganzo, 2007), tradable credit scheme (Nie and

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Yin, 2013; Xiao et al., 2013; Tian et al., 2013), car-pooling (Xiao et al., 2016), ride-sharing (Ma and Zhang, 2017; Wang et al., 2019), autonomous vehicles (Liu, 2018; Zhang et al., 2019), time dependent capacity (Zhang et al., 2010), queue dependent capacity (Chen et al., 2019; Fosgerau and Small, 2013; Liu et al., 2015), and parking (Tian et al., 2019; Liu et al., 2016). One can refer to a recent review paper by Li et al. (2020) to comprehensively understand the development of the bottleneck model.

In addition to the literature mentioned above, many studies have investigated commuting behavior under stochastic bottleneck capacity caused by unpredictable events, such as traffic accident, adverse weather, unannounced lane close, and transportation infrastructure malfunction (Lindsey, 1994; Arnott et al., 1999; Xiao et al., 2015; Long et al., 2021; Yu et al., 2021). For example, Lindsey (1994) considered a general distribution of bottleneck capacity and studied the properties of no-toll equilibrium and system optimum of the commuting system. Arnott et al. (1999) considered the case where the demand and capacity are both stochastic and examined the effects of information on the total social cost. Xiao et al. (2015) studied the situation that capacity follows a uniform distribution. They analyzed four possible departure-time situations: always early + always queuing, early or late + always queuing, always late + always queuing, always late + possible queuing. Long et al. (2021) generalized the uniform distribution of capacity to general distribution, and two more departure-time situations: always early + possible queuing, early or late + possible queuing, was pointed out. Yu et al. (2021) studied the value of inaccurate information in the morning commute with binary stochastic bottleneck capacity and investigated the effects of commuters' different actions in response to such inaccurate information.

To better understand the departure time choice principle of commuters and examine the theoretical assumptions, one can use the method of laboratory experiment, which is a powerful tool for studying people's choice behavior and has been widely used to examine equilibrium (Helbing et al., 2002; Gabuthy et al., 2006; Daniel et al., 2009; Rapoport et al., 2014), investigate the effect of information (van Essen et al., 2019; Qi et al., 2019; Xiao and Lo, 2016; Lu et al. 2011, 2014; Ramadurai and Ukkusuri, 2007; Han et al., 2021), study traffic paradoxes (Ramadurai and Ukkusuri, 2007; Morgan et al., 2009; Rapoport et al., 2009, 2014) and assess transportation demand management measures (Hartman, 2012; Aziz et al., 2015; Rey et al., 2016).

Although many previous studies have investigated how commuters made departure time choice by using laboratory experiments (Schneider and Weimann, 2004; Gabuthy et al., 2006; Ramadurai and Ukkusuri, 2007; Daniel et al., 2009; Sun et al., 2017), most of them focused on strategic uncertainty and neglected the influence of environmental uncertainty, such as stochastic bottleneck capacity caused by unpredictable events. To the best of our knowledge, there is only one experiment reported on the departure time choice behavior concerning environmental uncertainty. Rapoport et al. (2010) considered the variability of bottleneck capacity when studying batch queue problems. Two ferries with different capacity would arrive on any particular round with equal probability. In their experiments, the subjects chose whether and when to join the queue. In mixed-strategy equilibrium, the subjects' expected payoff equal to each other. The experimental results showed that subjects' aggregate behavior diverged from mixed-strategy equilibrium in the experiment with stochastic capacity. Subjects were optimistic about obtaining high returns under environmental uncertainty.

Motivated by the fact, this paper conducted a laboratory experiment to study the departure time choice behavior of commuters in the morning commuter under environmental uncertainty. The environmental uncertainty in the experiment is expressed as stochastic bottleneck capacity varying from round to round. We recruited 120 students and assigned them into 6 groups of experiments under two different scenarios of feedback information to make departure time-choice decisions. The experimental results show that the relationship between the mean cost and the standard deviation of cost can be fitted approximately linearly with a positive slope λ^*. This finding indicates that subjects are likely to minimize their travel cost budget (TCB) under environmental uncertainty. Finally, a reinforcement learning model is proposed to simulate the behavior mechanism of the commuters.

The rest of the paper is organized as follows. Section 2 reviews the previous theoretical behavioral assumptions and their relationships and differences. Section 3 introduces the experimental setup. The experimental results are presented in Section 4. In Section 5, a reinforcement learning model is proposed and simulation results are presented. Section 6 studies TCB based user equilibrium and discusses whether the parameter λ^* obtained from the experiment is within the rational range. Section 7 gives a conclusion.

2. Literature Review

In most theoretical works of bottleneck model considering uncertainty, it was assumed that commuters minimize their expected cost

$$E\big(C(t)\big) = E\big[\alpha T(t) + \max\big(\beta(t^* - t - T(t)), 0\big) + \max(\gamma(t+T(t) - t^*), 0)\big], \tag{1}$$

in which $E[\ ]$ is the expectation operator, t is departure time, $T(t)$ is travel time, t^* is work start time, $C(t)$ is trip cost, α, β, and γ denote the unit cost of travel time, the unit cost of schedule delay early (SDE), and the unit cost of schedule delay late (SDL), respectively.

Li et al. (2008, 2009a, 2016, 2017) proposed a different departure time choice principle, in which it is assumed that commuters minimize

$$u(t) = \alpha E\big(T(t)\big) + \max\Big(\beta\left(t^* - t - E\big(T(t)\big)\right), 0\Big) + \max\big(\gamma\big(t+E\big(T(t)\big) - t^*\big), 0\big) + \varepsilon\sigma\big(T(t)\big), \tag{2}$$

in which ε is a parameter, and $\sigma\big(T(t)\big)$ is the standard deviation of travel time.

Note that for commuters always arriving early, the expected travel cost is

$$E\big(C(t)\big) = (\alpha - \beta)E\big(T(t)\big) + \beta(t^* - t) \tag{3}$$

For commuters always arriving late,

$$E\big(C(t)\big) = (\alpha + \gamma)E\big(T(t)\big) + \gamma(t - t^*) \tag{4}$$

For commuters arriving either early or late, one can derive (Li et al., 2009a, 2016; Fosgerau, 2010; Fosgerau and Karlstrom, 2010)

$$E\big(C(t)\big) = \alpha E\big(T(t)\big) + \max\Big(\beta\left(t^* - t - E\big(T(t)\big)\right), 0\Big) + \max\big(\gamma\big(t+E\big(T(t)\big) - t^*\big), 0\big) + \xi_t\sigma\big(T(t)\big), \tag{5}$$

where $0 \le \xi_t \le \frac{\beta+\gamma}{2}$ is an attribute-level dependent parameter. Therefore,

$$u(t) = E\big(C(t)\big) + \begin{cases} \varepsilon\sigma(T(t)) & \text{if commuters always early or late} \\ (\varepsilon - \xi_t)\sigma(T(t)) & \text{if commuters either early or late} \end{cases}. \tag{6}$$

In other words, commuters choose their departure times according to both expected travel cost and the standard deviation of travel time (Li et al., 2008, 2009a, 2016, 2017). However, the weight coefficient of the standard deviation of travel time is situation dependent.

Note that Li et al. (2009b) proposed another cost function consisting of expected travel cost and variability of travel cost

$$\hat{u}(t) = \chi E\big(C(t)\big) + \lambda\sigma\big(C(t)\big) \tag{7}$$

to model commuters' choice behavior under uncertainty. Here $\sigma\big(C(t)\big)$ is the standard deviation of travel cost. However, they only studied the special case that parameter $\lambda = 0$.

Jiang and Lo (2016) also studied the stochastic bottleneck model. They assumed that commuters had to endure an exogenous random delay $\Theta(\mathrm{t}) = \frac{Q(t)}{s}\theta$, in which θ is a random variable with uniform distribution, $Q(t)$ is queue length for travelers departing at time t, and s is bottleneck capacity. They related the influence of travel cost variability on departure time choice and assumed that commuters minimize

$$\bar{u}(t) = E\big(C(t)\big) + \lambda\tilde{\sigma}(t), \tag{8}$$

in which λ is a parameter, $\tilde{\sigma}(t)$ denotes the variability of travel cost and is defined as

$$\tilde{\sigma}(t) = \int_{\theta_{min}}^{\theta_{max}} \big|C(t) - E\big(C(t)\big)\big| f(\theta)d\theta, \tag{9}$$

where $f(\theta)$ is probability density function of θ, θ_{min} and θ_{max} are its lower and upper bound, respectively.

3. Experiment Design

The experiment was carried out in a computer lab in Beijing Jiaotong University. One hundred and twenty undergraduate students participated in the computer-controlled laboratory experiment and received performance-based monetary compensation. The recruited students were assigned into 6 independent groups. Each group had 20 subjects and interacted with each other anonymously. There were only interactions within one group, and subjects belonged to different groups had no interactions. The numbers of male and female subjects were almost equal in each group to avoid the impact of gender on experimental results.

Our experiment was based on a discrete bottleneck model, in which 20 subjects commuted from a single origin (e.g., home) to a single destination (e.g., workplace) along a single road. On the road, there was a potential bottleneck with capacity s, which was constant within day but fluctuated from day-to-day (i.e., round-to-round) following a uniform distribution within $[s_{min}, s_{max}]$. If the flow rate exceeded s, the bottleneck would be activated, and a queue developed. The subjects were informed the distribution of capacity, but they did not know the capacity on each specific day.

The travel time from home to workplace is $T(t) = T^f + T^v(t)$, where T^f is the free travel time, $T^v(t)$ is the queuing time due to congestion, and t is the departure time from home. Without loss of generality, we set $T^f = 0$ as usual. Let $q(t)$ be the queue length. Then, a subject's travel time equals the queuing time $T^v(t) = q(t)/s$, in which

$$q(t) = \max(q(t-1) + n(t) - s, 0). \quad (10)$$

Here $q(t-1)$ is the queue length at previous departure time $t-1$, $n(t)$ is the number of subjects who depart at departure time t.

Given that the working start time is t^*, according to the bottleneck model, if a subject leaves home at departure time t, his/her travel cost is

$$C(t) = \alpha T^v(t) + \beta \cdot \max(t^* - t - T^v(t), 0) + \gamma \cdot \max(t + T^v(t) - t^*, 0), \quad (11)$$

where the first term on the right-hand side is the queuing cost, the second term is the cost for early arrival, and the third term is the cost for late arrival. The coefficients α, β and γ obey $\gamma > \alpha > \beta$, which is in accordance with Small's empirical results (1982).[1]

The primary concern in this experiment was to study the choice behavior of commuters in the morning commute under environmental uncertainty. Since information is widely used in both transport research and practices, examining the role of information can provide some insights that enable us fully understand the travel behavior. Therefore, we conducted 6 groups of experiments under two different scenarios with different information feedback (see Table 1). Note that the unit of parameters (α, β, γ) is Yuan/10 min in experiments.[2] In scenario A, only information related to subjects' own travel costs was provided, including choice in the previous round, the early/late arrival time and cost, the queuing time and cost, travel cost, score in the previous round, and cumulative score in all previous rounds. In scenario B, the travel costs of all departure times in the previous round were feedback to the subjects. Figure 1 shows the screenshots of the two scenarios used in the experiment. The travel cost was the same for the subjects who chose the same departure time in one round, whereas the specific value of the travel cost depended on a subject's own choice and the others' choices. In each round. every subject was given initial points. Here one round corresponds to one day. An individual's score was computed by subtracting travel cost from his/her initial points.

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[1] Note that in the discrete bottleneck model they proposed, Otsubo and Rapoport (2008) assumed that ties, which refer to the order in which commuters receive service, are broken randomly with equal probability among the commuters who arrive at the bottleneck simultaneously.

[2] In some empirical studies, such as Small (1982), the three parameters were calibrated from the data collected from daily commuting behavior. However, in laboratory experiment studies concerning departure time choice, the travel cost function involving the values of the three parameters should be given in advance to guarantee the calculation of travel costs choosing different departure times. After the subjects in one group submitted their decisions in one round, the server would calculate the travel costs on different departure times and provided such information to these subjects in the next round.

Table 1. Experimental designs in each scenario

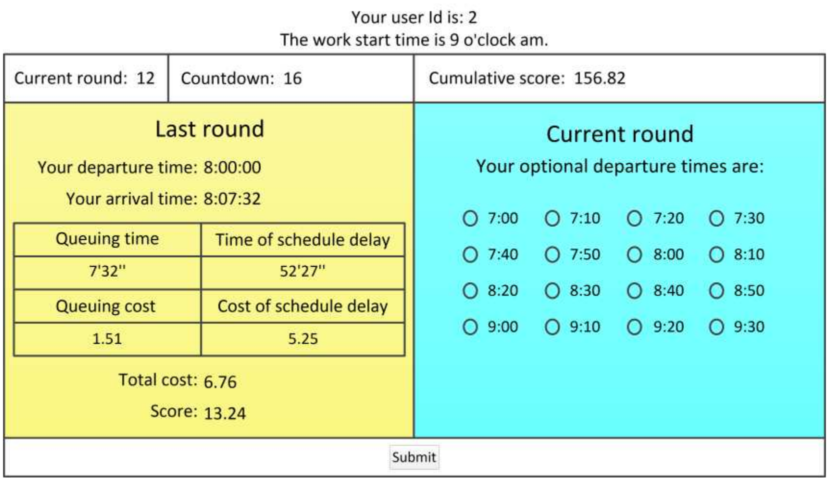
	Number of group	Parameter $\alpha/\beta/\gamma/s$		Initial points	Information provided
		$[\alpha\ \ \beta\ \ \gamma]$	$[s_{min},\ s_{max}]$		
Scenario A	3	[2 1 5]	[1.33, 4.00]	20	Personalized
Scenario B	3	[2 1 5]	[1.33, 4.00]	20	General

In our experiment, the work start time was set to be 9 a.m., and there were 16 discrete departure times available for the subjects to choose from. In each round, each subject was asked to select one departure time and then click the 'submit' button. After the 20 subjects in one group submitted their choices, the server would calculate the travel time cost and early/late arrival cost at each departure time according to the cost function and the capacity of that round.

At the start of the experiment, we took approximately 15 minutes to explain the game to all subjects, followed by a Q & A session. The subjects in each group played the game 150 rounds, lasting approximately 90 minutes. Although each round was not time-limited, subjects were suggested to submit their choices within 20 seconds (there was a 20 second countdown on the screen). The experiment would switch to the next round if and only if all subjects in one group have submitted their decisions.

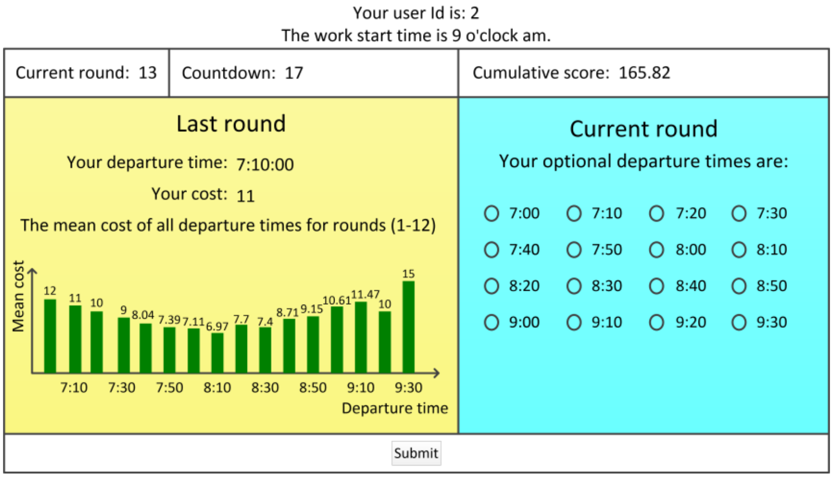
When a group of experiments was over, the cumulative score of each subject was converted to a payoff (in Chinese Yuan) at a ratio of 100: 3.[3] The payoff plus 20 Yuan show-up bonus was their total income. The mean income of all subjects was 72.99 Chinese Yuan.

Note that a subject's payoff on a given round depends on three things: their own choice of departure time, other subjects' departure-time choices, and the realization of capacity. A subject only controls the first factor. The second factor creates strategic uncertainty, and the third factor creates environmental uncertainty.



(a)

[3] Here we convert the cumulative score to payoffs to avoid the possible payoff fluctuations with random selections caused by environmental uncertainty. Another way of calculating payoffs is to randomly select several rounds and convert the score earned in these rounds to payoffs to avoid income effects. We would like to consider this alternative in the future work and compare with the present setup.



(b)

Figure 1. Screenshots of (a) Scenario A, (b) Scenario B. Chinese versions were used in the experiment.

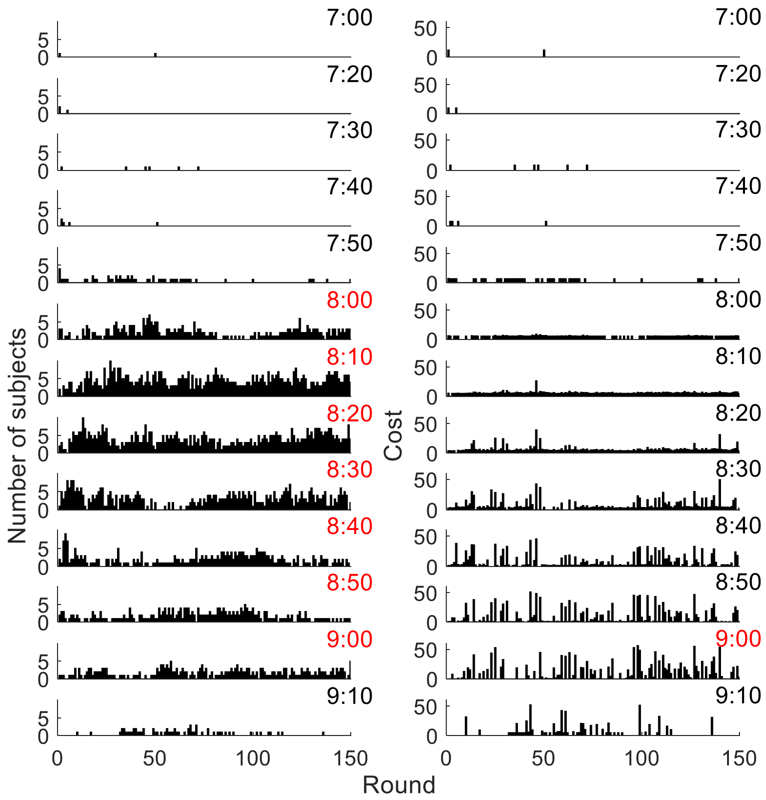
4. Experimental Results

Taking one group of experiments in Scenario A as an example, Figure 2 presents the number of subjects and travel cost at each departure time in each round. As shown in Figure 2(a), most subjects departed between 08:00 and 09:00, and only a small number of subjects departed at 07:50 and 09:10. Very few departed before 07:50, and no subject chose departure time 07:10, 09:20, and 09:30. The number of subjects at each departure time significantly fluctuated throughout the experiment. Figure 2(b) shows the travel cost at each departure time. One can see that with departure times closer to the working start time of 09:00, the travel cost fluctuation became higher, and the fluctuation persisted until the end.

Liu and Szeto (2020) investigated stochastic traffic dynamics in a bi-modal transportation system theoretically, and their results showed that the traffic patterns in the network approached more similar states for consecutive days after a certain period of travelers' learning process. However, convergence can hardly be observed in experiments, even if the route/bottleneck capacity is deterministic. For example, Rapoport et al. (2009) investigated the empirical relevance of the Braess paradox. They showed that the commuting decisions did not converge to the equilibrium after eighty periods. Selten et al. (2004) found no convergence to pure-strategy equilibrium either. They also observed considerable fluctuations around the means. Gabuthy et al. (2006) conducted the experiment where subjects made both departure time choice and route choice. They found that substantial fluctuations persisted until the end of the experiment. Selten et al. (2007) reported a laboratory experiment with a two-route choice scenario, in which every subject had to choose one route in each round. Each group of experiments included 200 rounds and fluctuations persisted until the end of the experiment. These experimental results indicate that strategic uncertainty is ubiquitous in travel choice.

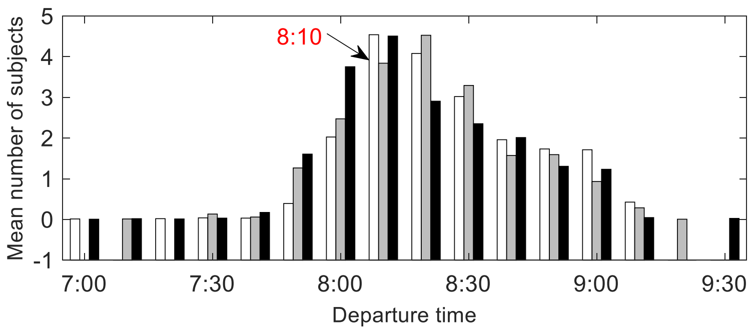
Figure 3 shows the statistical results on the mean number, mean cost and standard deviation of cost, for the 3 groups of experiments in Scenario A. One can see that the number of subjects departing at around 8:10 was the largest, which also corresponded to the lowest mean travel cost. The standard deviation of cost was the largest around 9:00. The late departure times (at and after 09:00) also corresponded to large mean travel cost. For early departure times before 7:30, the travel cost equaled to $\beta(t^* - t)$, since there was always no queue, irrespective of the bottleneck capacity.[4] Accordingly, the standard deviation of travel cost at these early departure times was zero. Similar results were observed in Scenarios B (see Appendix A).

[4] Note that in theoretical equilibrium patterns with continuous-time setting (mean value of parameter λ^* in the experiment is used), no one departs before 7:30. The meaning of λ^* will be introduced later in the main text.

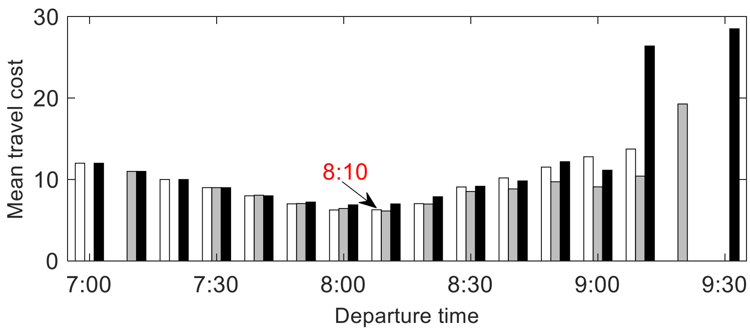


(a) (b)

Figure 2. Number of subjects (a) and cost (b) at each departure time over 150 rounds in one group of experiments in Scenario A.



(a)



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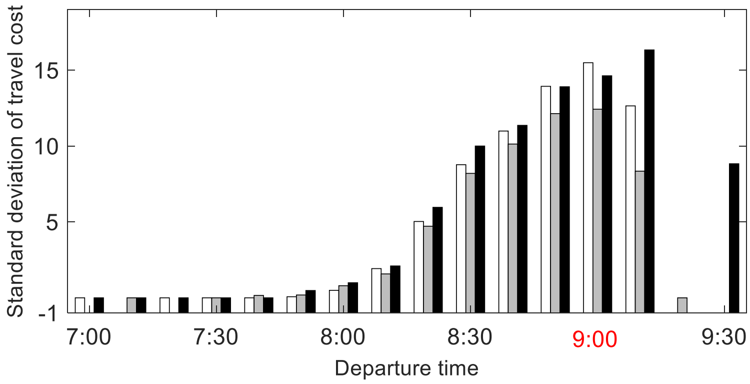


(c)

Figure 3. Statistical results of scenario A. (a) mean number of subjects, (b) mean travel cost, and (c) standard deviation of travel cost. An empty bar indicates no subject chose that departure time.

Figure 4 shows standard deviation of travel cost vs. mean travel cost at each departure time in the experiment[5]. The area of each data point was proportional to the number of times that particular departure time was chosen. We made a weighted linear fit of the data, in which weight was set as the area of the data point. One can see that in both scenarios, the relationship for each group of experiments can be fitted approximately linearly with a positive slope, i.e.

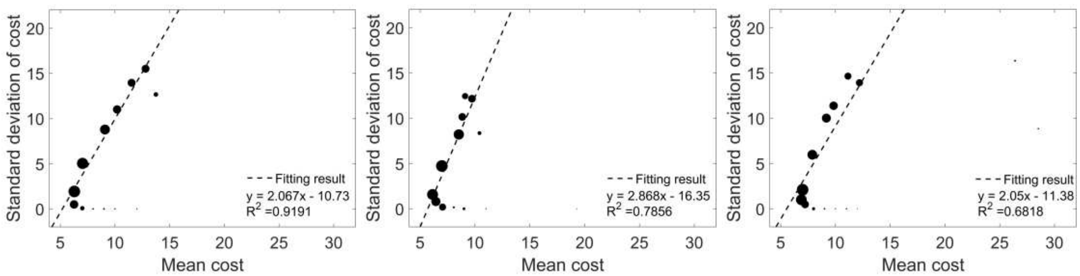
$$\sigma = E(C)/\lambda^* - m, \tag{12}$$

where $E(C)$ is mean travel cost and σ is standard deviation of the travel cost, and parameters $\lambda^* > 0$ and $m > 0$. R-squared results indicate that more than fifty percent of the variance in the standard deviation of the travel cost can be explained by the mean travel cost. The t-test results shown in Table 2 rejected the null hypothesis.

A reformulation thus leads to

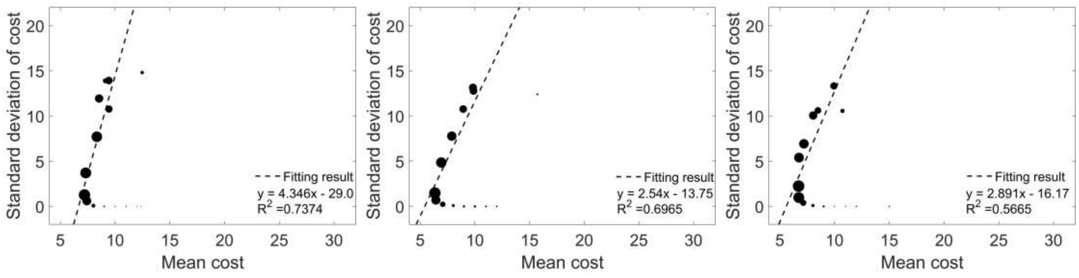
$$\lambda^* \cdot m = E(C) - \lambda^* \cdot \sigma. \tag{13}$$

Similar to the definition of travel time budget proposed by Lo et al. (2006), we can define $E(C) - \lambda^* \cdot \sigma$ as travel cost budget (TCB). Our experiments thus demonstrated that faced with environmental uncertainty (in this case uncertain bottleneck capacity), the subjects were likely to choose the departure time according to their travel cost budget, and they behaved with $\lambda^* > 0$.



(a)

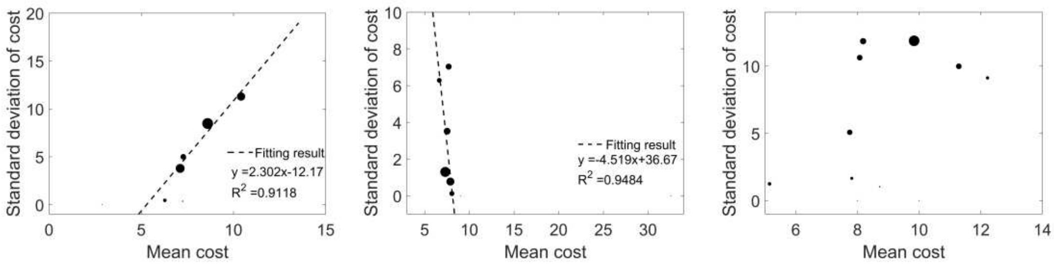
[5] It should be noted that in the second group of experiments in Scenario B, one subject misunderstood the travel cost shown in the interface as payoff. The subject mostly chose very early or very late departure time. Therefore, in the data analysis, the data of this subject were removed.



(b)

Figure 4. The standard deviation of travel cost vs. the mean travel cost in (a) Scenario A and (b) Scenario B. The data points represent the 16 departure times and the size of the data point was in proportion to the number of times that departure time has been chosen.

Until now, we have presented evidence at the aggregate level to reveal the relationship between the mean travel cost and the standard deviation of travel cost. Next, we analyzed such a relationship at the individual level. We investigated the mean travel cost and standard deviation of the travel cost experienced by each subject. Figure 5 shows three typical results. For type I, the mean travel cost and standard deviation of travel cost were positively correlated (P-value<0.001), see panel (a); For type II, they were negatively correlated (P-value=0.026), see panel (b); For type III, the linear relationship between the mean travel cost and standard deviation of travel cost was not significant (P-value=0.116), see panel (c). The number of the three types of relationship was 43, 2 and 15 in Scenario A and 31, 9 and 19 in Scenario B, respectively. One can see that most of subjects belonged to type I although subjects were heterogeneous. Moreover, Scenario B has more type II subjects and less type I subjects than Scenario A.



(a) (b) (c)

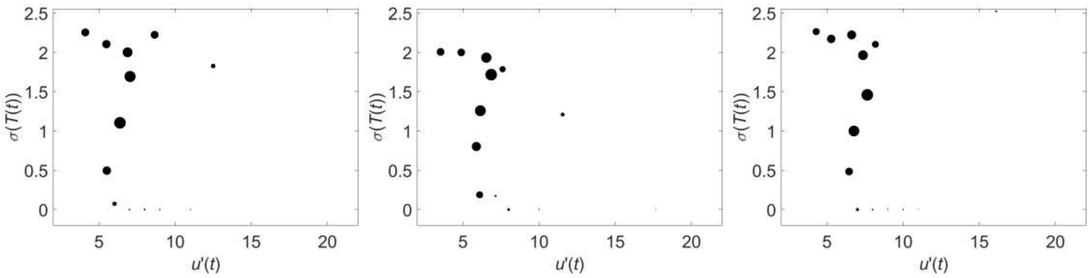
Figure 5. Typical diagrams exhibiting the relationship between the standard deviation of travel cost and the mean travel cost at the individual level. The data points represent departure times a subject chose and the size of the data point was in proportion to the number of times that departure time was chosen by the subject.

We next examined the departure time choice principle in Eq. (2) and Eq. (8). Figure 6 shows standard deviation of travel time (i.e., $\sigma\big(T(t)\big)$) vs.

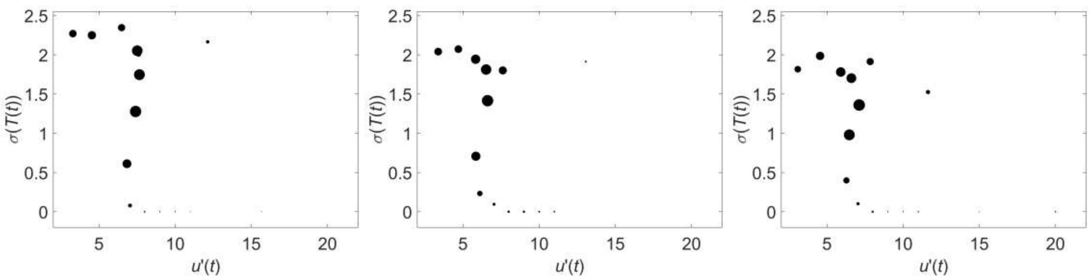
$$u'(t) = \alpha E\big(T(t)\big) + \max\Big(\beta\left(t^* - t - E\big(T(t)\big)\right), 0\Big) + \max(\gamma(t + E\big(T(t)\big) - t^*), 0)$$

for each departure time point in the 6 groups of experiments. Obviously, there was no linear relationship between these $u'(t)$ and $\sigma\big(T(t)\big)$, indicating that the subjects in the experiments did not follow the choice principle in Eq. (2).

However, a linear relationship between $\tilde{\sigma}(t)$ and mean travel cost for each departure time point in the 6 groups of experiments was found, and such relationship was very similar to Figure 4. P-values obtained from the t-test in Table 2 verified the linear correlation.



(a)



(b)

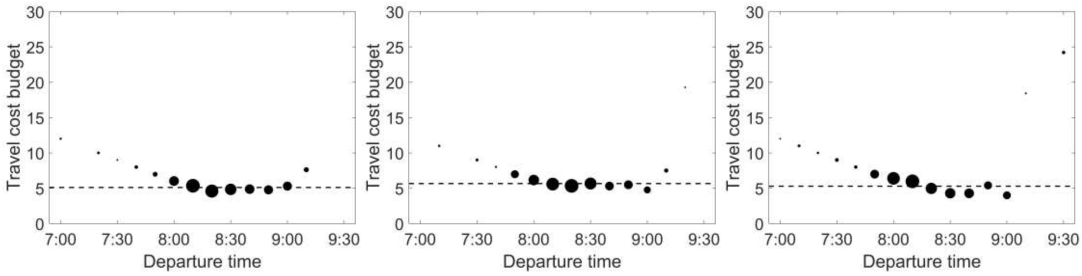
Figure 6. The standard deviation of travel time vs. $u'(t)$ in (a) Scenario A and (b) Scenario B.

Table 2. P-values obtained from the t-test.

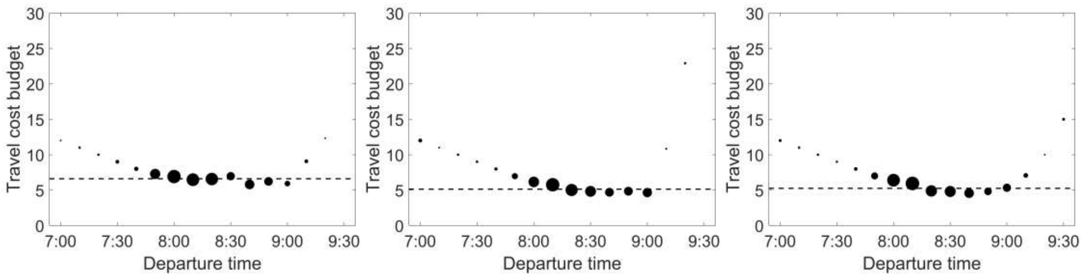
P-value	Standard deviation of cost vs. mean cost			$\tilde{\sigma}(t)$ vs. mean cost		
	Group 1	Group 2	Group 3	Group 1	Group 2	Group 3
Scenario A	2.40×10^{-7}	5.45×10^{-5}	1.50×10^{-4}	2.52×10^{-8}	2.17×10^{-5}	7.54×10^{-5}
Scenario B	4.15×10^{-5}	1.09×10^{-4}	7.66×10^{-4}	2.95×10^{-5}	4.13×10^{-5}	1.06×10^{-4}

It should be noted that the positive correlation between the mean and standard deviation of travel cost might also depend on game dynamics. As shown in Figure 3(b-c), the mean and standard deviation of travel cost both increased with departure time after about 8:10. Presumably, subjects who departed late were more likely to arrive late, which was heavily penalized by the large unit cost of arriving late. They were also exposed to substantial variability in travel times because of the large potential queues that could accumulate during the departure period. As mentioned before, the number of commuters in each departure time significantly fluctuated throughout the experiment. The positive correlation between mean travel cost and the standard deviation of travel cost could also be driven by this inherent property of the bottleneck model.

Based on the above findings, Figure 7 presents the effective travel cost (i.e., travel cost budget) in the experiment. One can see that the effective travel costs were roughly equal from 8:00 ~ 9:00, while they were larger for earlier or later departure times.



(a)



(b)

Figure 1. The travel cost budget (observed value of parameter λ* in the experiment is used) vs. the departure time in (a) Scenario A, (b) Scenario B. The data points represent the 16 departure times and the size of the data point is in proportion to the number of times that departure time has been chosen.

So far, we have comprehensively understood the relationship between the mean travel cost and the standard deviation of the travel cost at both collective and individual level. Next, we investigated the effect of information feedback on the choice behavior. Table 3 compares the mean travel cost of subjects and the value of λ^* between Scenario A and Scenario B. One can see that providing information of all departure times to the subjects slightly decreased the subjects' mean travel cost compared with only providing subjects' own information. Also, the feedback on costs of all departure times decreased the mean of λ^*. One possible reason is that Scenario B has more type II subjects and less type I subjects than Scenario A.

Table 3 A comparison between Scenario A and Scenario B

	Scenario A				Scenario B			
	Group 1	Group 2	Group 3	Average	Group 1	Group 2	Group 3	Average
Mean cost of subjects	8.769	7.782	8.721	8.424	8.377	7.784	7.843	8.001
λ^*	0.455	0.333	0.477	0.422	0.281	0.364	0.358	0.334

Next, we studied the dependence of mean travel cost of all subjects in round r on the bottleneck capacity of round r. As shown in Figure 8, the mean travel cost decreased as the bottleneck capacity increased. For the deterministic bottleneck model with fixed capacity, it is known that the travel cost of each commuter C' depends on the bottleneck capacity s

$$C' = \frac{\beta\gamma N}{(\beta+\gamma)s}, \tag{14}$$

where N is the total number of commuters. We also plotted Eq. (14) in Figure 8. One can see that under both scenarios, when the bottleneck capacity was low, the mean travel cost in the experiment was significantly larger than C'. However, when the bottleneck capacity was large, the mean travel cost was slightly smaller than C'. One possible reason is that subjects under the stochastic bottleneck departed earlier and concentrated than under the deterministic bottleneck, resulting in longer queues and higher mean travel cost under low bottleneck capacity. However, such a departure pattern under stochastic bottleneck might lead shorter queues and lower travel cost when the bottleneck capacity is high.

The mean travel cost in Scenario A and Scenario B was 8.424 and 8.001, respectively. We calculated the mean travel cost of the deterministic bottleneck model as follows:

$$\overline{C'} = \frac{1}{s_{max}-s_{min}}\int_{s_{min}}^{s_{max}} \frac{\beta\gamma N}{(\beta+\gamma)s} ds = \frac{\beta\gamma N}{(s_{max}-s_{min})(\beta+\gamma)} ln\frac{s_{max}}{s_{min}}. \tag{15}$$

The value of $\overline{C'}$ is 6.973, which is much smaller than that in the stochastic capacity situation. This means that if the bottleneck capacity can be predicted and fed back to the commuters in advance, then mean travel cost can be reduced.

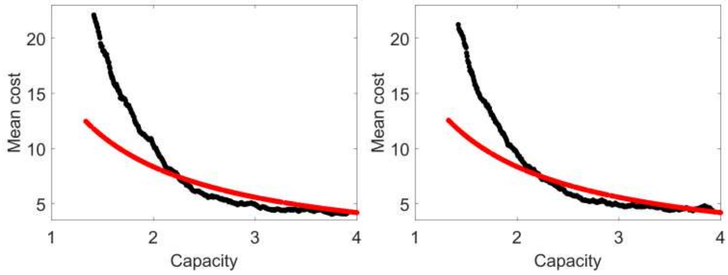
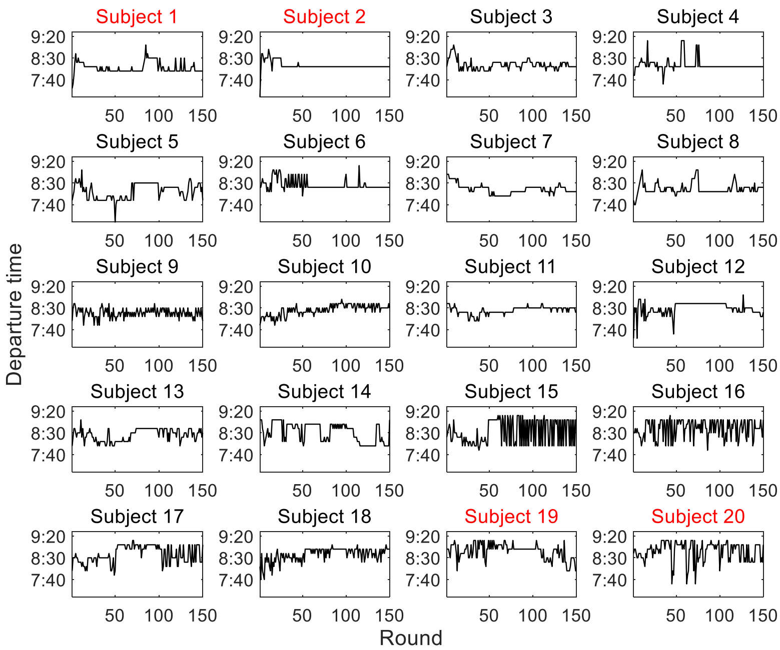


Figure 8. Mean travel cost of all subjects in round r vs. bottleneck capacity of round r (Black lines) in Scenario A (left) and Scenario B (right). The experimental data were averaged over three groups of experiments and processed by moving average in each scenario. The red lines were calculated from Eq. (14).

Now we conducted more analysis to reveal the potential dynamics in the experiment. Still taking one group of experiments in Scenario A as an example, Figure 9(a) presents the departure time choice of each subject and numbers the subjects in ascending order according to their mean travel cost. One can see that subjects with low mean travel cost usually chose early departure times. For example, Subject 1 had the lowest mean travel cost. He/she mainly chose 8:00, 8:10, 8:20 and 8:30. Subject 2 had the second lowest mean travel cost. He/she mainly chose 8:10 and 8:30. In contrast, subjects with high mean travel cost usually chose late departure times. For example, Subject 20 had the highest mean travel cost, and he/she mainly chose 8:40, 8:50, 9:00 and 9:10. Subject 19 had the second highest mean travel cost, and he/she mainly chose 8:50, 9:00 and 9:10. We examined other 5 groups of experiments, and similar results were observed. Figure 9(b) shows the relationship between the mean travel cost and the average departure time, in which a positive correlation was observed. Also, the mean travel cost in Scenario B is more concentrated than that in Scenario A, indicating that providing information of all departure times significantly affects the choice of the subjects.



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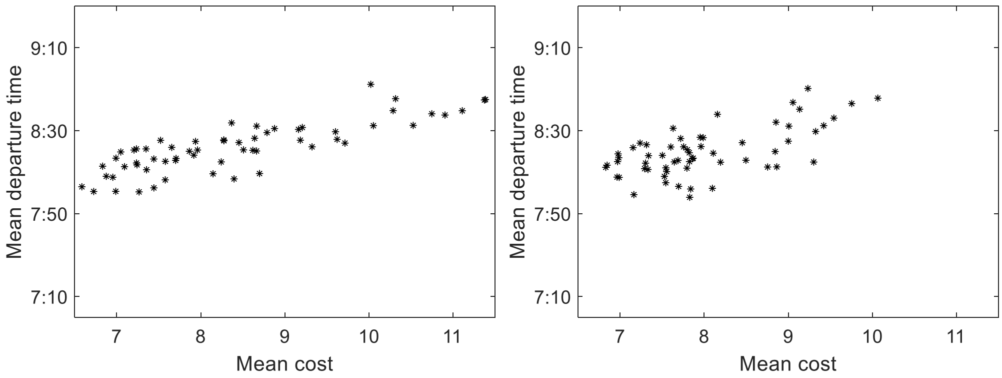


(b)

Figure 9. (a) Evolution of individual behavior in one group of experiments in Scenario A. (b) Mean travel cost vs. the average departure time in Scenario A (left) and Scenario B (right).

Figure 10 presented the impact of capacity and mean cost in round r on the departure time changing ratio from round r to round $r+1$. One can see that the departure time changing ratio was negatively correlated with the bottleneck capacity and positively correlated with the mean travel cost on the previous round. Moreover, there are more subjects shifting their choice in Scenario B than in Scenario A.

In Scenario A, the arrival time information has been fed to the subjects. Therefore, we analyzed the relationship between arrival time in round r and the departure time changing behavior from round r to round $r+1$ in this Scenario. Table 4 shows that after experiencing early arrival in last round (the grey rows), most subjects chose to maintain or delay the departure in the next round. Moreover, the closer the arrival time is to 9:00, the smaller the proportion of people who delay the departure. On the other hand, subjects who were late in the last round tend to depart early. The later the arrival time, the higher the proportion of people who leave early.

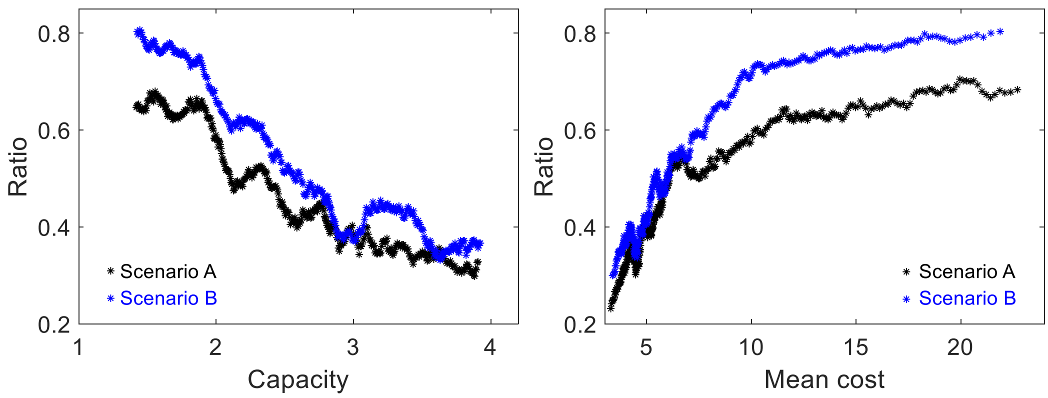


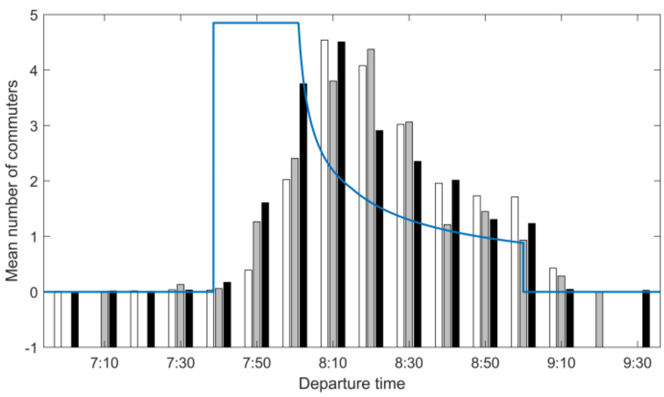
Figure 10. Departure time changing ratio from round r to round r+1 vs. bottleneck capacity (left) and mean travel cost (right) of round r in Scenario A and Scenario B. The data were averaged over three groups of experiments and processed by moving average.

Table 4 Departure time changing behavior from round r to round $r+1$ vs. arrival time in round r in Scenario A. The data were averaged over three groups of experiments.

Arrival time in round r /total number of commuters	Ratio of departing earlier in round $r+1$	Ratio of not changing departure time in round $r+1$	Ratio of departing later in round $r+1$
Earlier than 7:30 / 13	15.38%	0.00%	84.62%
7:30 ~ 7:40 / 31	6.45%	51.61%	41.94%
7:40 ~ 7:50 / 39	2.56%	33.33%	64.10%

7:50 ~8:00 / 441	1.36%	59.18%	39.46%
8:00 ~ 8:10/ 876	2.17%	58.45%	39.38%
8:10 ~ 8:20/ 943	5.62%	65.96%	28.42%
8:20 ~ 8:30 / 1216	10.44%	61.27%	28.29%
8:30 ~ 8:40 / 1143	15.40%	57.39%	27.21%
8:40 ~ 8:50 / 1059	22.00%	54.30%	23.70%
8:50 ~ 9:00 / 943	24.50%	57.58%	17.92%
9:00 ~ 9:10 / 864	42.48%	46.64%	10.88%
9:10 ~ 9:20 / 508	54.33%	31.89%	13.78%
9:20 ~ 9:30 / 309	61.49%	24.27%	14.24%
Later than 9:30 / 555	62.52%	21.44%	16.04%

Many previous studies have investigated the morning commuting behavior under stochastic bottleneck capacity through user equilibrium analysis. For example, Xiao et al. (2015) analyzed one classical departure pattern under stochastic bottleneck capacity with a uniform distribution. Figure 11 compares our experimental results with the equilibrium pattern calculated by Xiao et al. (2015). One can see that there was a significant difference between experimental observations and the theoretical pattern obtained by Xiao et al. (2015). The possible reasons of such difference include (i) Xiao et al. (2015) assumed that commuters minimized the mean travel cost, while we observed that the subjects actually minimized travel cost budget; (ii) A discrete bottleneck model was used in our experiment while the continuous time model was used in Xiao et al. (2015); (iii) The individuals in the experiment were heterogeneous while the theoretical studies assumed homogeneous commuters.



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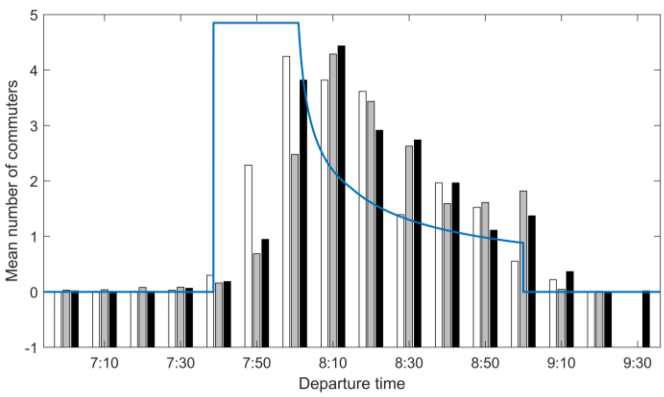


(b)

Figure 11. Comparison between our experimental results and the theoretical result in Xiao et al. (2015). The blue lines denote the results calculated from Xiao et al. (2015). (a) Scenario A. (b) Scenario B.

Finally, we compared the travel cost in the experiment with that in theoretical equilibrium patterns with continuous-time setting (mean value of parameter λ* in Table 3 is used). In the experiment, the mean travel cost of each departure time was 10.238 and 9.7528 in Scenario A and B, respectively. In theoretical equilibrium patterns, the mean travel cost of each departure time was 8.6256 and 8.4956 in Scenario A and B, respectively. One can see that the mean travel cost in the experiment were higher than those in theoretical equilibrium patterns in both Scenario A and B. One possible reason might be that time periods were arranged in 10-minute intervals, which was a coarse discretization. In the future, more experiments are needed to examine the experimental results by decreasing discrete time intervals.

5. Reinforcement Learning Model

Our experiment demonstrated that when the bottleneck capacity was stochastic, commuters were likely to choose the departure time to minimize the travel cost budget instead of the expected travel cost[6]. Based on this finding, a reinforcement learning (REL) model is proposed to reproduce commuters' choice behavior under stochastic bottleneck capacity. The REL model has been widely used to model travel choice behavior, including route choice (Avineri and Prashker, 2005; Selten et al., 2007; Lu et al., 2014), departure time choice (Daniel et al., 2009; Sun et al., 2017) and travel mode choice (Yang et al., 2017). In the REL model, subjects are assumed to have a propensity corresponding to each option of choice, and the propensity is influenced by the choices in previous rounds. Then, the propensity is converted into a probability that controls the choice of people in the next round. In this way, subjects accumulate experience and learn in the decision-making process. The REL model emphasizes that subjects tend to repeatedly choose the strategies which have brought higher payoff in previous rounds.

We propose the REL model as follows

1. Choice in the initial two rounds: it is assumed that in the first and second rounds, all individuals choose each departure time point with equal probability.

2. Update propensities: The propensity of individual i in round $r+1$ $(r>1)$ is updated by

$$q_i^{r+1}(t) = \begin{cases} E[C_i(t)] - \lambda^* \cdot \sigma[C_i(t)], & t \in V_i \\ \frac{q_i^{r+1}(t_i^k) - q_i^{r+1}(t_i^{k-1})}{t_i^k - t_i^{k-1}}\left(t - t_i^{k-1}\right) + q_i^{r+1}\left(t_i^{k-1}\right), & t \notin V_i \text{ and } t_i^{k-1} < t < t_i^k \\ \frac{q_i^{r+1}(t_i^2) - q_i^{r+1}(t_i^1)}{t_i^2 - t_i^1}(t - t_i^1) + q_i^{r+1}(t_i^1), & t = t_i^1 - 1 \\ \frac{q_i^{r+1}(t_i^n) - q_i^{r+1}(t_i^{n-1})}{t_i^n - t_i^{n-1}}(t - t_i^n) + q_i^{r+1}(t_i^n), & t = t_i^n + 1 \\ q_i^{r+1}(t_i^1 - 1), & t < t_i^1 - 1 \\ q_i^{r+1}(t_i^n + 1), & t > t_i^n + 1 \end{cases} \tag{16}$$

where $q_i^{r+1}(t)$ is the propensity of individual i to choose departure time t in the $(r+1)^{th}$ round; $V_i = \left[t_i^1, t_i^2, \cdots, t_i^k, \cdots, t_i^{n-1}, t_i^n\right]$ is the set of departure times that have been chosen by individual i from round 1 to round r, and t_i^1 and t_i^n are the earliest and the latest departure time that have been chosen; $E[C_i(t)]$ and $\sigma[C_i(t)]$ are the mean cost and standard deviation of cost at departure time t, calculated from the rounds that individual i has chosen departure time t; λ^* is a parameter.

[6] Our experiment does not exclude that commuters minimize $\bar{u}(t) = E\big(C(t)\big) + \lambda\tilde{\sigma}(t)$. However, since both $\tilde{\sigma}(t)$ and $\sigma(C(t))$ reflect variability of travel cost, and $\sigma(C(t))$ is much more frequently used than $\tilde{\sigma}(t)$, we use travel cost budget in the modeling.

The basic idea is, for a departure time that has been chosen before, the propensity of individual i is set to equal his/her experienced TCB at that departure time; for departure time between $t_i^1 - 1$ and $t_i^n + 1$ that has not been chosen before, the propensity is obtained from linear interpolation or extrapolation from the propensities of the two nearest chosen departure times; for unchosen departure times earlier than $t_i^1 - 1$ (or later than $t_i^n + 1$), the propensity is set to equal to that on $t_i^1 - 1$ (or $t_i^n + 1$).

3. Update probabilities: The probability of choosing departure time t in round $r+1$ is calculated by

$$p_i^{r+1}(t) = \frac{\exp\left(-\frac{\eta}{\varphi_i} \cdot q_i^{r+1}(t)\right)}{\sum_{k=1}^{T} \exp\left(-\frac{\eta}{\varphi_i} \cdot q_i^{r+1}(k)\right)} \tag{17}$$

where $\eta > 0$ is reinforcement coefficient determining the reinforcement sensitivity, φ_i is mean cost of individual i from round 1 to round r, T is the number of departure times. Here, it is assumed that subjects respond to percentage changes in payoffs rather than absolute amounts.

As in experiment, each simulation run includes 20 participants and lasts 150 rounds, using the same capacity sequence as in the experiment. The value of λ^* in Scenarios A and B and the reinforcement coefficient η need to be calibrated. The calibration result is λ^* =0.2828, 0.2036, respectively, in Scenarios A and B and η =14.7445, by minimizing sum of the difference of commuter number at each departure time between experimental and simulation results.

Figures 12 and 13 compare the experimental and simulation results in Scenario A. Results in Scenario B are shown in Appendix B. As shown in Figure 12, the REL model can capture the general trend of mean number of commuters, which increases with departure times, reaches the maximum at round 8:00-8:10, and then decreases. However, the REL model cannot perfectly reproduce the results observed in the experiment, especially in groups 1 and 3 in Scenario A. One possible reason is that in the three groups, the same value of λ^* was used in the simulation, whereas the observed value of λ^* in the experiment was different.

Figure 13(a) shows the mean travel cost in each round. One can see that the model can well characterize the change of mean travel cost with the round. To further quantify the difference between the simulation results and the experimental ones, Figure 13(b) shows the distribution of the relative error. One can see that the relative errors are mostly smaller than 20%. Here relative error is defined as $\frac{c_{sim} - c_{exp}}{c_{exp}}$, with c_{sim} and c_{exp} denoting mean travel cost in the simulation and in the experiment, respectively. For those relative errors larger than 20%, it is usually because c_{exp} is small. For example, in round 97 in group 1, the relative error is 65.14%, which corresponds to $c_{exp} = 2.5688$.

Table 5 shows the R-squared results between simulation and experimental results. One can see that the model can reproduce the collective choice behavior quite well. Moreover, the REL model also reproduces the linear relationship between standard deviation of travel cost and the mean travel cost, as shown in Figure 14.

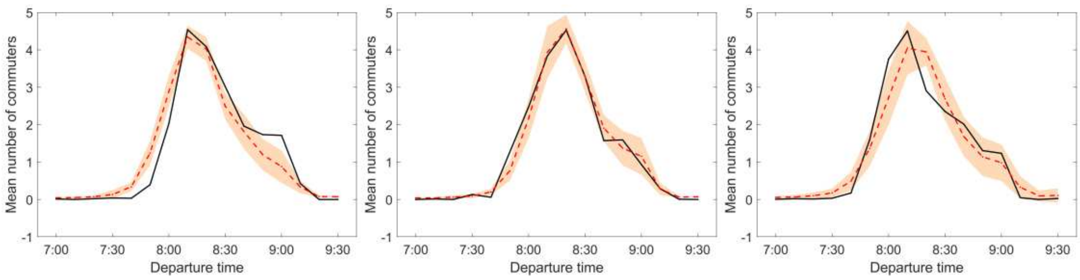
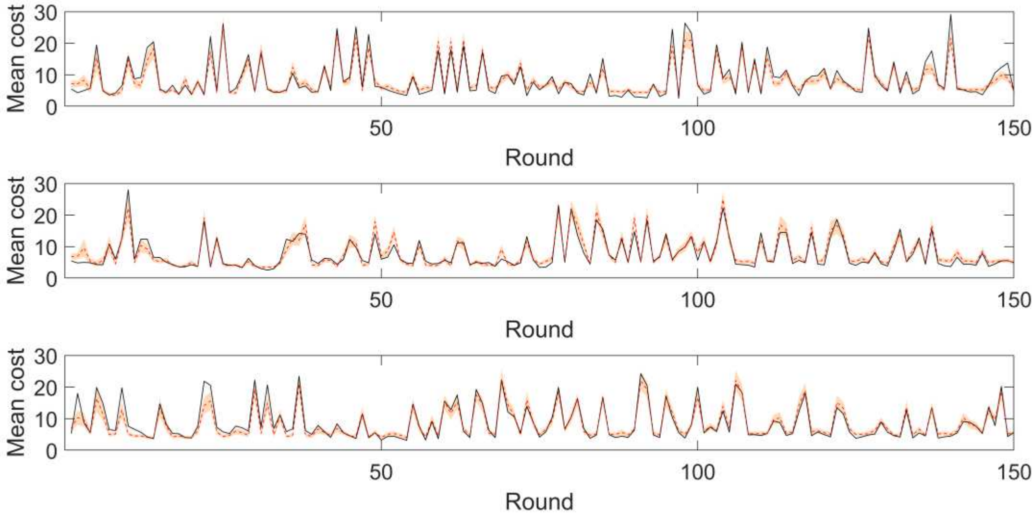


Figure 12. Comparison of mean number of commuters at each departure time between the experiment of Scenario A and the simulation. The black curve is the experimental result, the red dash line is the result of REL model, and the shadow region is the error bar of simulation result.



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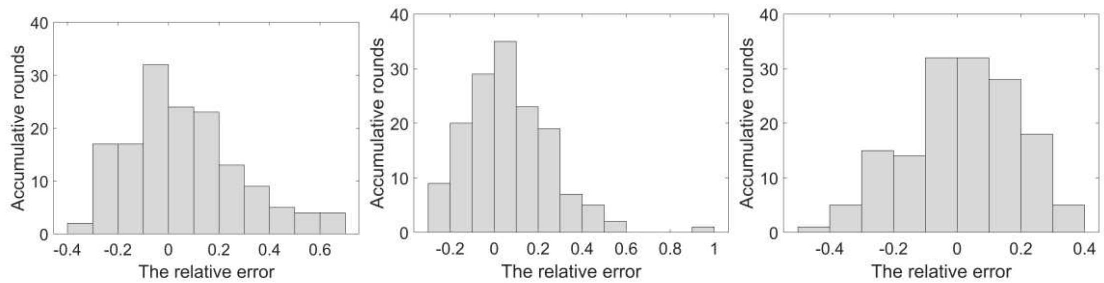
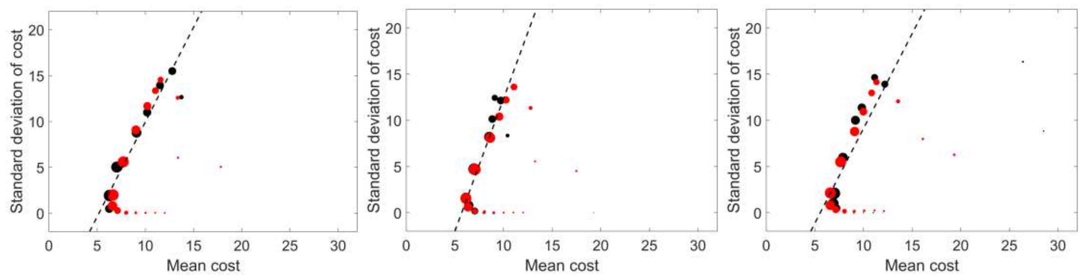


(b)

Figure 13. (a) Comparison of the evolution of mean travel cost between the experiment of Scenario A and the simulation. The black curve is the experimental result, the red dash line is the mean result of REL model, and the shadow region is the error bar. (b) The distribution of the relative error in Scenario A.

Table 5 R-squared results between simulation and experimental results.

	Mean number of commuters			Average cost		
	Group 1	Group 2	Group 3	Group 1	Group 2	Group 3
Scenario A	0.9184	0.9826	0.9115	0.9113	0.9078	0.9055
Scenario B	0.8005	0.9392	0.9082	0.9304	0.9165	0.9058



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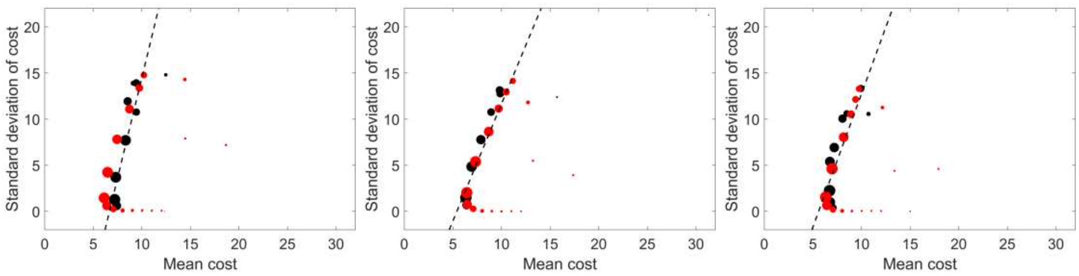

(b)

Figure 14. Standard deviation of travel cost vs. mean travel cost at each departure time in the experiment and the simulation. The area of each point is proportional to the mean number of commuters of choosing the departure time. The red points are the simulation results while the black points represent experimental results. The dash line is the fitted curve of the experimental data. (a) and (b) are results of 3 groups of experiments in Scenario A and B, respectively.

6. Discussion

The experimental findings would inspire us to propose a TCB based user equilibrium for bottleneck model with stochastic capacity[7]. We have studied such a problem (Liu et al., 2020), in which the bottleneck capacity was constant within a day but changes stochastically from day-to-day between a designed value (good condition) with probability $1-\pi$ and a degraded one (bad condition) with probability π. That study revealed that considering variability of travel cost significantly affected departure time choice. Specifically, it was shown that minimizing TCB, which was defined as $\mathrm{E}(\mathrm{C}(\mathrm{t}))+\lambda\sigma(C(t))$, [8] yielded seven possible equilibrium patterns. Closed form solutions to all possible equilibrium patterns and their corresponding parameter ranges were derived. Dependence of travel cost and the duration of peak hours on the commuters' risk attitude has also been derived in each equilibrium pattern.

The rationality of the patterns has been investigated. A pattern is defined as irrational if the low cost of a departure time (that is achieved on good condition) is larger than the high cost of any other departure time (that is achieved on bad condition). It is shown that the parameter λ should be within the range $\frac{-\pi}{\sqrt{\pi(1-\pi)}}<\lambda<\frac{1-\pi}{\sqrt{\pi(1-\pi)}}$. Otherwise, the pattern would be irrational. Moreover, a region in which no equilibrium solution exists is observed. One possible explanation is proposed: the parameter λ of travelers would not be in the region in reality.

This section examines whether the parameter λ^* obtained from the experiments was within the rational range. However, the closed form solutions cannot be derived under the assumption of a uniformly distributed bottleneck capacity. Thus, we use the numerical solutions.

There are still six situations faced by commuters, as mentioned before in Introduction: always early + always queuing (SDE+AQ); early or late + always queuing (SDE/L+AQ); always late + always queuing (SDL+AQ); always early + possible queuing (SDE+PQ); early or late + possible queuing (SDE/L+PQ); always late + possible queuing (SDL+PQ). Here, we only show formulation of departure rate in one of the six situations. Formulation in other situations is shown in Appendix C.

Formulation of departure rate in situation SDE+AQ is given as follows. In this situation, no matter how the bottleneck capacity varies, all commuters always arrive early and always experience queuing. Without loss of generality, the free flow travel time on the highway is set to be zero. Thus, one has

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[7] Note that there are several other equilibrium models based on "effective travel time/cost", including the percentile user equilibrium model, in which all travelers are assumed to choose routes to minimize their own percentile travel time (Nie, 2011); the α-reliable mean excess traffic equilibrium model, in which all travelers are assumed to minimize their travel risk measured by the conditional expectation of the excess travel time for a certain travel time budget (Chen and Zhou, 2010); and the traffic equilibrium model based on the quadratic disutility function (Yin et al., 2004).

[8] Note that λ and λ^* have opposite signs.

$$C(t) = \alpha\left(\frac{R(t)}{s} - t + t_s\right) + \beta\left(t^* - t_s - \frac{R(t)}{s}\right), s \in [\theta\bar{s}, \bar{s}] \tag{18}$$

$$E\big(C(t)\big) = \int_{\theta\bar{s}}^{\bar{s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \beta\left(t^* - t_s - \frac{R(t)}{s}\right)\right] f(s)ds \tag{19}$$

$$E\big(C^2(t)\big) = \int_{\theta\bar{s}}^{\bar{s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \beta\left(t^* - t_s - \frac{R(t)}{s}\right)\right]^2 f(s)ds \tag{20}$$

$$\sigma\big(C(t)\big) = \sqrt{var\big(C(t)\big)} = \sqrt{E\big(C^2(t)\big) - \Big(E\big(C(t)\big)\Big)^2} = (\alpha - \beta)R(t)\sqrt{\frac{1}{\theta\bar{s}^2} - \left(\frac{ln\theta^{-1}}{\bar{s}-\theta\bar{s}}\right)^2} \tag{21}$$

Here $s_{max} = \bar{\mathrm{s}}$, $s_{min} = \theta\bar{\mathrm{s}}$ and $0 < \theta < 1$. $f(s) = 1/(\bar{\mathrm{s}} - \theta\bar{\mathrm{s}})$ indicates the probability density function of the uniform distribution. $R(t) = \int_{t_s}^{t} r(t)dt$ is the cumulative departure from t_s to t. t_s and t^* are the first departure time and the work start time, respectively. $r(t)$ is departure rate at time t. The travel cost budget is still defined as,

$$\mathrm{TCB(t)} = E\big(C(t)\big) + \lambda\sigma\big(C(t)\big) \tag{22}$$

Substituting Eqs. (19) and (21) into Eq. (22), and using the equilibrium condition $d\mathrm{TCB}(t)/dt = 0$, we can obtain the equilibrium departure rate of this situation, given as follows,

$$r(t) = \frac{\alpha}{(\alpha-\beta)\left(\frac{ln\theta^{-1}}{\bar{s}-\theta\bar{s}} + \lambda\sqrt{\frac{1}{\theta\bar{s}^2} - \left(\frac{ln\theta^{-1}}{\bar{s}-\theta\bar{s}}\right)^2}\right)} \tag{23}$$

We would like to mention that except this departure rate and the departure rate in situation SDL+AQ, the departure rate in other 4 situations cannot be obtained analytically.

Having the departure rate in each situation calculated analytically or numerically, we can obtain six possible equilibrium departure patterns. Notice that Patterns 1-3 and Patterns 4-6 are the same as Patterns 1-3 and Patterns 5-7 corresponding to the bivariate capacity (Liu et al., 2020), respectively. But Pattern 4 corresponding to the bivariate capacity does not exist here.

Using the parameters in experiment and changing parameters λ and θ, a diagram exhibiting the six equilibrium patterns can be obtained numerically, as shown in Figure 15. Note that different from that corresponding to bivariate capacity (Liu et al., 2020), there does not exist the region that has no equilibrium solution.

The two red dashed lines represent rationality thresholds. Patterns beyond the two red dashed lines are irrational. Here a pattern is defined as irrational if the lowest cost of a departure time (that is achieved on best condition $s = s_{max}$) is larger than the highest cost of any other departure time (that is achieved on worst condition $s = s_{min}$). Note that individual's travel cost depends not only on what he/she does, but also on what other commuters do and on environmental uncertainty by nature. Here, it is supposed that decisions of the individuals do not change day-to-day, and travel cost changes only due to environmental uncertainty. Rationality of the decisions is judged based on the costs on many days. One can see that Pattern 6 is always irrational, and Patterns 1-4 are partially irrational. It seems that Pattern 5 is always rational. However, unfortunately, we cannot provide rigorous proof due to lack of closed form solutions.

In our experiment, $\theta = 0.3325$, the maximum value of λ^* is 0.477 and the minimum value of λ^* is 0.281 (see Table 3), which correspond to the maximum value $\lambda = -0.281$ and minimum value $\lambda = -0.477$. As indicated by the two dots in Figure 15, the parameter obtained from the experiment is within the rational range.

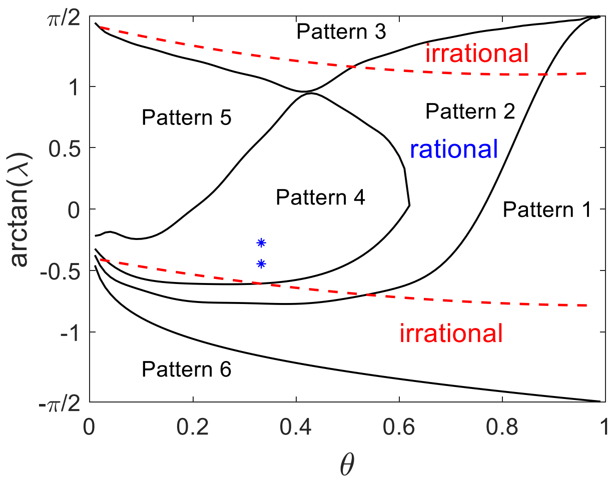

Figure 15. The pattern diagram. Note that the vertical axis is arctan (λ).

7. Conclusion

Environmental uncertainty is an important feature of traffic systems, and it is one of the most important factors affecting commuters' choice behavior. Although theoretical works concerning bottleneck model with stochastic capacity have been reported, the departure time choice behavior has not been validated before.

Laboratory experiments provide us with an efficient way to examine behavior characteristic. This method is controllable, repeatable, and low in experimental cost, and thus becomes the preferred method to reveal the potential mechanism of decision-making behavior.

This paper designed and conducted a laboratory experiment to investigate the effects of stochastic bottleneck capacity on commuter departure time choice behavior. The most important finding is that the relationship between standard deviation of travel cost and mean travel cost can be fitted approximately linearly with a positive slope λ^*. The individual behavior has also been studied. It is found that while the individuals were heterogeneous, standard deviation of travel cost and mean travel cost exhibited positive correlations for most individuals. This suggests that subjects were likely to minimize their travel cost budget rather than mean travel cost.

Other findings include: (i) Providing information of all departure times to the subjects slightly decreased the commuters' travel costs and the slope λ^* compared with providing subjects' own information only. (ii) The mean travel cost under the stochastic bottleneck capacity was higher than that under the deterministic bottleneck capacity. (iii) Subjects with low (high) mean travel cost were more likely to choose early (late) departure times. (iv) The departure time changing ratio from round r to round $r+1$ was negatively correlated with the capacity and positively correlated with the mean travel cost on the previous round. Also, providing information of all departure times increases changing ratio of subjects compared with providing subjects' own information only.

We have proposed a reinforcement learning model. The main characteristic of the model is that for some departure times that have not been chosen before, the propensity is obtained from linear interpolation or extrapolation from the propensities of the two nearest chosen departure times. Simulation shows that the model can reproduce the main experimental findings.

Finally, the rationality of the parameter λ^* obtained from the experiments has been investigated. It is shown that six possible equilibrium patterns can be obtained numerically when the capacity is uniformly distributed. The parameter λ^* in experiment is within the rational range.

Two insights can be provided for traffic management. Firstly, traffic management departments need to have a deep understanding of commuters' behavior so that peak hour traffic congestion can be better managed, since the peak period and the travel cost of commuters depend on their choice behavior. Secondly, information plays an important role in shaping commuters' choice behavior, thus affecting their travel costs. In particular, if the bottleneck capacity can be predicted accurately, and commuters have timely access to the forecast, their travel costs can be reduced significantly. Thus, traffic management departments can utilize information as an effective tool to manage traffic congestion.

It should be noted that the study has some limitations. Firstly, the free-flow travel time is assumed to be zero in the experiment. The free-flow travel time may influence the experimental results by increasing the travel cost of subjects. Secondly, a biased sample only involving students and limited number of participants in each experimental group may influence the validity of the experimental findings. To address the concerns regarding the bias of a specific sample, one needs to use additional samples drawn from a different pool to replicate the results (McGrath, 1982). Thirdly, stochastic capacity is assumed to follow the uniform distribution in the experiment. However, how capacity changes with uncertain environment would be more complex in reality. In the future work, more experiments are needed to address these limitations, by considering free flow time, by recruiting different number of subjects with different background, and by setting different distributions of the capacity. Fourthly, we assume that commuters incur the same travel costs and face the same arrival constraints following the literature about departure time choice using laboratory experiments (Schneider and Weimann, 2004; Gabuthy et al., 2006; Ramadurai and Ukkusuri, 2007; Daniel et al., 2009; Sun et al., 2017). However, some empirical studies have found that commuters' travel costs and arrival constraints were associated with their sociodemographic information, such as age, education, and gender (Small, 1982; Li et al., 2010). Many studies have investigated departure time choice in consideration of heterogeneous commuters through theoretical modeling (Arnott et al., 1987; Li et al., 2020), and these theoretical results considering heterogeneous commuters should be verified through further laboratory experiments. Also, our experiment can be extended in many ways to investigate the impact of, e.g., staggered working hours, ridesharing, and parking management.

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References

Arnott, R., de Palma, A., Lindsey, R., 1989. Schedule delay and departure time decisions with heterogeneous commuters. Transportation Research Record Journal of the Transportation Research Board. 1197, 56-67.

Arnott, R., de Palma, A., Lindsey, R., 1999. Information and time-of-usage decisions in the bottleneck model with stochastic capacity and demand. European Economic Review. 43 (3), 525-548.

Arnott, R., de Palma, A., Lindsey, R., 1990. Economics of a bottleneck. Journal of Urban Economics. 27(1), 111-130.

Arnott, R., de Palma, A., Lindsey, R., 1993. A structural model of peak-period congestion - a traffic bottleneck with elastic demand. American Economic Review. 83 (1), 161-179.

Arnott, R., de Palma, A., Lindsey, R., 1994. The welfare effects of congestion tolls with heterogeneous commuters. Journal of Transport Economics and Policy. 28 (2), 139-161.

Avineri, E., Prashker, J. N., 2005. Sensitivity to travel time variability: Travelers' learning perspective. Transportation Research Part C: Emerging Technologies. 13 (2), 157-183.

Aziz, H.M.A., Ukkusuri, S.V., Romero, J., 2015. Understanding short-term travel behavior under personal mobility credit allowance scheme using experimental economics. Transportation Research Part D: Transport and Environment. 36, 121-137.

Chen, A., Zhou, Z., 2010. The α-reliable mean-excess traffic equilibrium model with stochastic travel times. Transportation Research Part B: Methodological. 44 (4), 493-513.

Chen, J.Y., Jiang, R., Li, X.G., Hu, M.B., Jia, B., Gao, Z.Y., 2019. Morning commute problem with queue-length-dependent bottleneck capacity. Transportation Research Part B: Methodological. 121, 184-215.

Daniel, T.E., Gisches, E.J., Rapoport, A., 2009. Departure times in Y-Shaped traffic networks with multiple bottlenecks. American Economic Review. 99, 2149-2176.

de Palma, A., Lindsey, R., 2002. Comparison of morning and evening commutes in the Vickrey bottleneck model. Transportation Research Record Journal of the Transportation Research Board. 1807, 26-33.

Fosgerau, M., 2010. On the relation between the mean and variance of delay in dynamic queues with random capacity and demand. Journal of Economic Dynamics & Control. 34 (4), 598-603.

Fosgerau, M., Karlstrom, A., 2010. The value of reliability. Transportation Research Part B: Methodological. 44 (1), 38-49.

Fosgerau, M., Small, K.A., 2013. Hypercongestion in downtown metropolis. Journal of Urban Economics. 76 (1), 122-134.

Gabuthy, Y., Neveu, M., Denant-Boemont, L., 2006. The coordination problem in a structural model of Peak-Period congestion: An experimental study. Review of Network Economics. 5(2), 273-298.

Hao, J.X., Zhou, W., Peng, H., Wang, Y.Q., 2009. Elastic bottleneck equilibrium model for rail transit passengers at rush hours. Journal of Transportation Systems Engineering and Information Technology. 9 (3), 93-97.

Han, X., Yu, Y., Jia, B., Gao, Z.Y., Jiang, R., Zhang, H.M., 2021. Coordination behavior in mode choice: laboratory study of equilibrium transformation and selection. Production and Operations Management. Doi: 10.1111/poms.13454.

Hartman, J.L., 2012. Special issue on transport infrastructure: A route choice experiment with an efficient toll. Networks & Spatial Economics. 12 (2), 205-222.

Helbing, D., Schonhof, M., Kern, D., 2002. Volatile decision dynamics: Experiments, stochastic description, intermittency control and traffic optimization. New Journal of Physics. 4, 33-33.

Huang, H.J., 2002. Pricing and logit-based mode choice models of a transit and highway system with elastic demand. European Journal of Operational Research. 140 (3), 562-570.

Jiang, G., Lo, H.K., 2016, The impact of cost variability in a bottleneck model with heterogeneous random delay., Smart Transportation: the 21st International Conference of Hong Kong Society for Transportation Studies (HKSTS), Hong Kong.

Kuwahara, M., 1990. Equilibrium queuing patterns at a 2-tandem bottleneck during the morning peak. Transportation Science. 24 (3), 217-229.

Lago, A., Daganzo, C.F., 2007. Spillovers, merging traffic and the morning commute. Transportation Research Part B: Methodological. 41 (6), 670-683.

Laih, C.H., 2004. Effects of the optimal step toll scheme on equilibrium commuter behaviour. Applied Economics. 36 (1), 59-81.

Laih, C.H., 1994. Queuing at a bottleneck with single-step and multistep tolls. Transportation Research Part A: Policy and Practice. 28 (3), 197-208.

Li, H., Bovy, P.H., C, B.M., 2008. Departure time distribution in the stochastic bottleneck model. International Journal of ITS Research. 6 (2), 79-86.

Li, H., Bliemer, M.C.J., Bovy, P.H.L., 2009a, Reliability-based dynamic discrete network design with stochastic networks., Proceeding of 18th International Symposium on Transportation and Traffic Theory 2009: Golden Jubilee, Hong Kong, 651-673.

Li, H., Bliemer, M.C.J., Bovy, P.H.L., 2009b. Modeling departure time choice with stochastic networks involved in network design. Transportation Research. Record: Journal of the Transportation Research Board. 2091, 61-69.

Li, H., Tu, H., Hensher, D.A., 2016. Integrating the mean-variance and scheduling approaches to allow for schedule delay and trip time variability under uncertainty. Transportation Research Part A: Policy and Practice. 89, 151-163.

Li, H., Tu, H., Zhang, X., 2017. Travel time variations over time and routes: Endogenous congestion with degradable capacities. Transportmetrica B: Transport Dynamics. 5 (1), 60-81.

Li, Z., Hensher, D.A., Rose, J.M., 2010. Willingness to pay for travel time reliability in passenger transport: A review and some new empirical evidence. Transportation research part E: logistics and transportation review, 46 (3), 384-403.

Li, Z.C., Lam, W.H.K., Wong, S.C., 2014. Bottleneck model revisited: An activity-based perspective. Transportation Research Part B: Methodological. 68, 262-287.

Li, Z.C, Huang, H.J., Yang, H., 2020. Fifty years of the bottleneck model: A bibliometric review and future research directions. Transportation Research Part B: Methodological. 139, 311-342.

Lindsey, C.R., van den Berg, V.A.C., Verhoef, E.T., 2012. Step tolling with bottleneck queuing congestion. Journal of Urban Economics. 72 (1), 46-59.

Lindsey, R., 2004. Existence, uniqueness, and trip cost function properties of user equilibrium in the bottleneck model with multiple user classes. Transportation Science. 38 (3), 293-314.

Lindsey, R., 1994. Optimal departure scheduling in the morning rush hour when capacity is uncertain. Presented at the 41st North America Meeting of the Regional Science Association, Niagara Falls, Ontario.

Liu, Q.M., Jiang, R., Liu, R.H., Zhao, H., Gao, Z.Y., 2020. Travel cost budget based user equilibrium in a bottleneck model with stochastic capacity. Transportation Research Part B: Methodological. 139, 1-37.

Liu, W., Yin, Y., Yang, H., 2015. Effectiveness of variable speed limits considering commuters' long-term response. Transportation Research Part B: Methodological. 81, 498-519.

Liu, W., Zhang, F., Yang, H., 2016. Managing morning commute with parking space constraints in the case of a bi-modal many-to-one network. Transportmetrica A: Transport Science. 12 (2), 116-141.

Liu, W., 2018. An equilibrium analysis of commuter parking in the era of autonomous vehicles. Transportation Research Part C: Emerging Technologies. 92, 191-207.

Lo, H.K., Luo, X.W., Siu, B.W.Y., 2006. Degradable transport network: Travel time budget of travelers with heterogeneous risk aversion. Transportation Research Part B: Methodological. 40 (9), 792-806.

Long, J.C., Yang, H., Szeto, W.Y., 2021. Departure time equilibrium and tolling strategies of a bottleneck with stochastic capacity. Transportation Science.

Lu, X. S., Zhang, X., Long, J., Li, Y., 2020. Managing rail transit peak-hour congestion with step fare schemes. Transportmetrica A: Transport Science. 16 (3), 1490-1511.

Lu, X. S., Liu, T. L., Huang, H. J., 2015. Pricing and mode choice based on nested logit model with trip-chain costs. Transport Policy. 44, 76-88.

Lu, X., Gao, S., Ben-Elia, E. 2011. Information impacts on route choice and learning behavior in a congested network experimental approach. Transportation Research Record: Journal of the Transportation Research Board. 2243, 89-98.

Lu, X., Gao, S., Ben-Elia, E., Pothering, R., 2014. Travelers' Day-to-Day route choice behavior with Real-Time information in a congested risky network. Mathematical Population Studies. 21 (4), 205-219.

Ma, R., Zhang, H.M., 2017. The morning commute problem with ridesharing and dynamic parking charges. Transportation Research Part B: Methodological. 106, 345-374.

McGrath, J.E., 1982. Dilemmatics: The study of research choices and dilemmas. In J. E. McGrath, J. Martin, & R. A. Kulka (Eds.), Judgment calls in research, Beverly Hills, CA: Sage publication.

Morgan, J., Orzen, H., Sefton, M., 2009. Network architecture and traffic flows: Experiments on the Pigou-Knight-Downs and Braess Paradoxes. Games and Economic Behavior. 66 (1), 348-372.

Nie, Y., 2011. Multi-class percentile user equilibrium with flow-dependent stochasticity. Transportation Research Part B: Methodological. 45 (10), 1641-1659.

Nie, Y., Yin, Y. F., 2013. Managing rush hour travel choices with tradable credit scheme. Transportation Research Part B: Methodological. 50, 1-19.

Otsubo H., Rapoport A., 2008. Vickrey's model of traffic congestion discretized. Transportation Research Part B: Methodological, 42 (10), 837-889.

Qi, H., Ma, S., Jia, N., Wang, G.C., 2019. Individual response modes to pre-trip information in congestible networks: laboratory experiment. Transportmetrica A: Transport Science. 15 (2), 376-395.

Qian, Z., Zhang, H.M., 2013. The morning commute problem with heterogeneous travelers: the case of continuously distributed parameters. Transportmetrica A: Transport Science. 9 (2), 178-203

Ramadurai, G., Ukkusuri, S.V., 2007. Dynamic traffic equilibrium - Theoretical and experimental network game results in single-bottleneck model. Transportation Research Record: Journal of the Transportation Research Board. 2090, 1-13.

Rapoport, A., Gisches, E.J., Mak, V., 2014. Distributed decisions in networks: Laboratory study of routing splittable flow. Production and Operations Management. 23 (2), 314-331.

Rapoport, A., Kugler, T., Dugar, S., Gisches, E. J., 2009. Choice of routes in congested traffic networks: Experimental tests of the Braess Paradox. Games and Economic Behavior. 65 (2), 538-571.

Rapoport, A., Stein, W.E., Makd, V., Zwick, R., Seale, D.A., 2010. Endogenous arrivals in batch queues with constant or variable capacity. Transportation Research Part B: Methodological. 44 (10), 1166-1185.

Rey, D., Dixit, V.V., Ygnace, J., Waller, S. T., 2016. An endogenous lottery-based incentive mechanism to promote off-peak usage in congested transit systems. Transport Policy. 46, 46-55.

Schneider, K., Weimann, J., 2004. Against All Odds: Nash Equilibria in a Road Pricing Experiment. Human behavior and Traffic Network, 133-153. M. Schreckenberg et al. (Eds.), Springer-Verlag Berlin Heidelberg.

Selten, R. Schreckenberg, M., Chmura, T., Pitz, T., Kube, S., Hafstein, SF., Chrobok, R., Pottmeier, A Wahle, J., 2004. Experimental investigation of day-to-day route-choice behaviour and network simulations of autobahn traffic in North Rhine-Westphalia. In: Schreckenberg, M., Selten, R. (Eds.), Human Behaviour and Traffic Networks. Springer, Berlin, 1-21.

Selten, R., Chmura, T., Pitz, T., Kube, S., Schreckenberg, M., 2007. Commuters route choice behavior. Games and Economic Behavior. 58 (2), 394-406.

Small, K.A., 1982. The scheduling of consumer activities-work trips. American Economic Review. 72 (3), 467-479.

Sun, X.Y., Han, X., Bao, J.Z., Jiang, R., Jia, B., Yan, X., Zhang B., Wang, W.X., Gao, Z.Y., 2017. Decision dynamics of departure times: Experiments and modeling. Physica A-Statistical Mechanics and its Applications. 483, 74-82.

Tabuchi, T., 1993. Bottleneck congestion and modal split. Journal of Urban Economics. 34, 414-431.

Tian, L.J., Sheu, J., Huang, H., 2019. The morning commute problem with endogenous shared autonomous vehicle penetration and parking space constraint. Transportation Research Part B: Methodological. 123, 258-278.

Tian, L.J., Yang, H., Huang, H.J., 2013. Tradable credit schemes for managing bottleneck congestion and modal split with heterogeneous users. Transportation Research Part E: Logistics and Transportation Review. 54, 1-13.

van den Berg, V.A.C., Verhoef, E.T., 2011. Congestion tolling in the bottleneck model with heterogeneous values of time. Transportation Research Part B: Methodological. 45 (1), 60-78.

van den Berg, V.A.C., Verhoef, E.T., 2014. Congestion pricing in a road and rail network with heterogeneous values of time and schedule delay. Transportmetrica A: Transport Science. 10 (5), 377-400.

van Essen, M., Thomas, T., Chorus, C., van Berkum, E., 2019. The effect of travel time information on day-to-day route choice behaviour: evidence from a real-world experiment. Transportmetrica B: Transport Dynamics. 7 (1), 1719-1742.

Vickrey, W.S., 1969. Congestion theory and transport investment. American Economic Review. 59 (2), 251-260.

Wang, J., Ban, X.J., Huang, H., 2019. Dynamic ridesharing with variable-ratio charging-compensation scheme for morning commute. Transportation Research Part B: Methodological. 122, 390-415.

Wang, W., Sun, H.J., 2014. Pricing models in a highway with bottleneck based on reference-dependent theory. Journal of Transportation Systems Engineering and Information Technology. 14, 180-186.

Wang, D.Z.W., Xu, L., 2016. Equilibrium trip scheduling in single bottleneck traffic flows considering multi-class travelers and uncertainty: a complementarity formulation. Transportmetrica A: Transport Science. 12 (4), 297-312.

Xiao, Y., Lo, H.K., 2016. Day-to-day departure time modeling under social network influence. Transportation Research Part B: Methodological. 92, 54-72.

Xiao, F., Qian, Z. S., Zhang, H.M., 2013. Managing bottleneck congestion with tradable credits. Transportation Research Part B: Methodological. 56, 1-14.

Xiao, L. L., Huang, H. J., Liu, R.H., 2015. Congestion behavior and tolls in a bottleneck model with stochastic capacity. Transportation Science. 49 (1), 46-65.

Xiao, L., Liu, T., Huang, H., 2016. On the morning commute problem with carpooling behavior under parking space constraint. Transportation Research Part B: Methodological. 91, 383-407.

Yang, C., Liu, B., Zhao, L.Y., Xu, X. D., 2017. An experimental study on cumulative prospect theory learning model of travelers' dynamic mode choice under uncertainty. International Journal of Transportation Science and Technology. 6 (2), 143-158.

Yang, H., Huang, H.J., 1997. Analysis of the time-varying pricing of a bottleneck with elastic demand using optimal control theory. Transportation Research Part B: Methodological. 31 (6), 425-440.

Yang, H., Tang, Y., 2018. Managing rail transit peak-hour congestion with a fare-reward scheme. Transportation Research Part B: Methodological. 110, 122-136.

Yao, T., Wei, M.M., Zhang, B., Friesz, T., 2012. Congestion derivatives for a traffic bottleneck with heterogeneous commuters. Transportation Research Part B: Methodological. 46, (10) 1454-1473.

Yin, Y.F., Lam, W.H.K., Ieda, H., 2004. New technology and the modeling of risk-taking behavior in congested road networks. Transportation Research Part C: Emerging Technologies. 12 (3-4), 171-192.

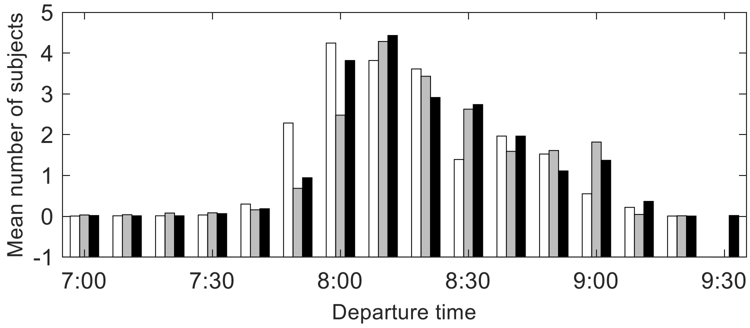
Yu, Y., Han, X., Jia, B., Jiang, R., Gao, Z.Y., Zhang, H.M., 2021. Is providing inaccurate pre-trip information better than providing no information in the morning commute under stochastic bottleneck capacity? Transportation Research Part C: Emerging Technologies. 126, 103085.

Zhang, X.N., Yang, H., Huang, H.J., Zhang, H.M., 2005. Integrated scheduling of daily work activities and morning-evening commutes with bottleneck congestion. Transportation Research Part A: Policy and Practice. 39 (1), 41-60.

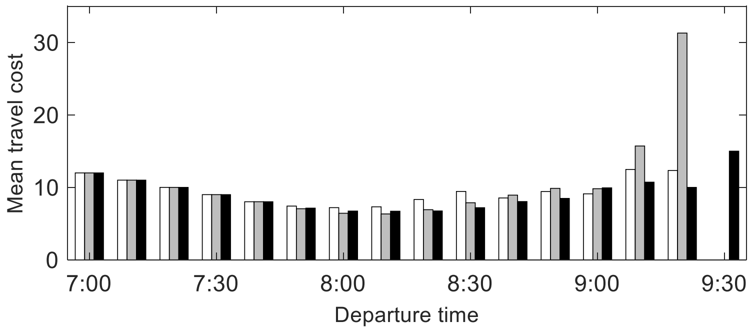
Zhang, X., Liu, W., Waller, S.T., Yin, Y., 2019. Modelling and managing the integrated morning-evening commuting and parking patterns under the fully autonomous vehicle environment. Transportation Research Part B: Methodological. 128, 380-407.

Zhang, X., Zhang, H.M., Li, L., 2010. Analysis of user equilibrium traffic patterns on bottlenecks with Time-Varying capacities and their applications. International Journal of Sustainable Transportation. 4 (1), 56-74.

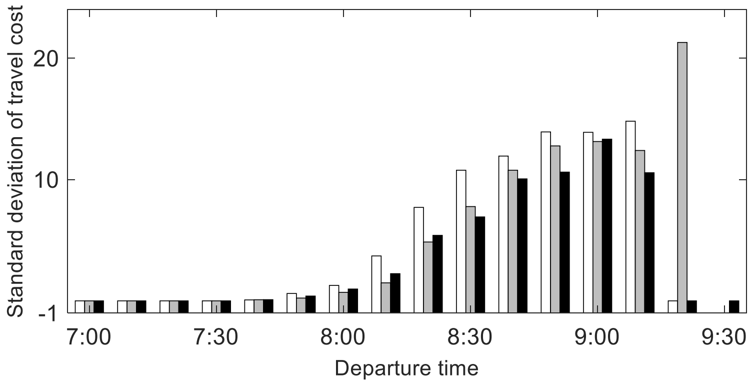
Appendix A：Experimental Results in Scenario B



(a)



(b)



(c)

Figure A1. (a) mean number of subjects, (b) mean travel cost, and (c) standard deviation of travel cost in the three groups of experiments in Scenario B. The histograms in the figure represent three groups of experiments in this Scenario.

Appendix B Simulation results in Scenario B

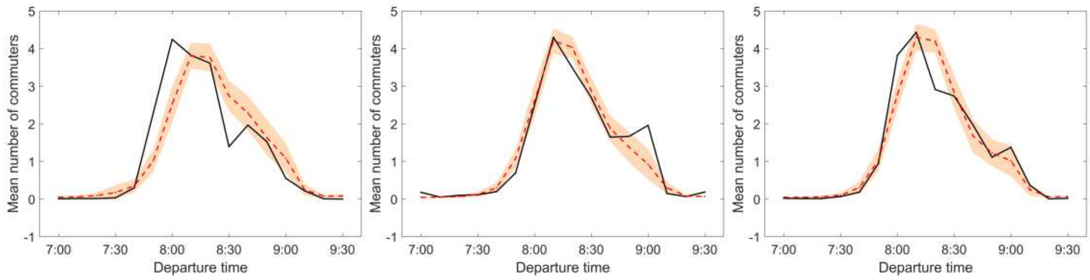
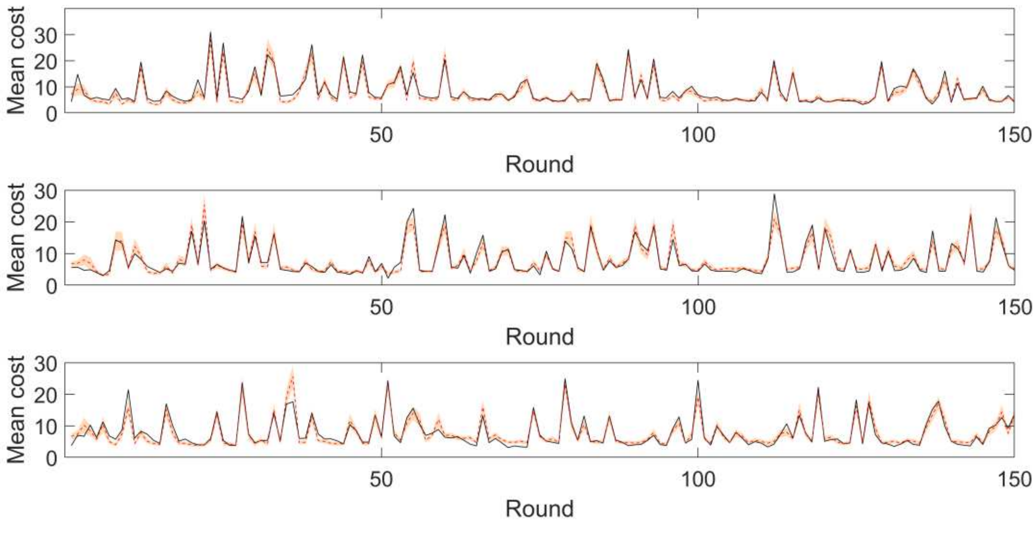
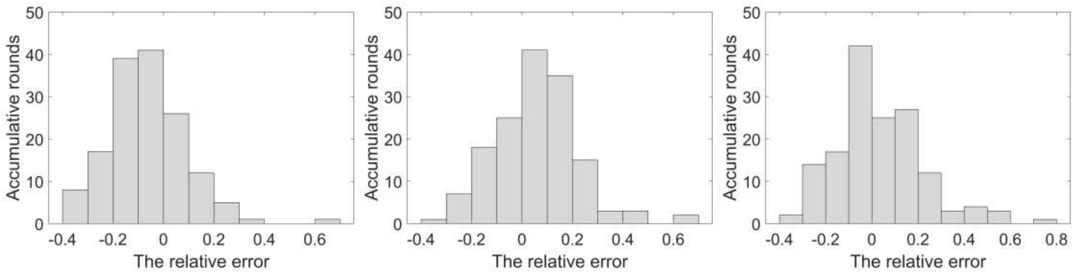


Figure B1. Mean number of commuters at each departure time point in experiment and simulation in Scenario B. The black curve is the experimental result, the red dash line is the result of REL model, and the shadow region is the error bar of simulation result.



(a)



(b)

Figure B2. (a) Mean cost in each round in experiment and simulation in Scenario B. The black curve is the experimental result, the red dash line is the mean result of REL model, and the shadow region is the error bar. (b) The distribution of the relative error in this Scenario.

Appendix C Formulation in other situations

In Section 5, we have provided derivation of departure rate in situation SDE+AQ. The formulations in other 5 situations are provided here.

(i) Early or late + Always queuing (SDE/L+AQ)

In this situation, whether commuters arrive at destination early or late depends on the capacity of bottleneck, but they always experience queuing. Commuters experience schedule delay early if $\frac{R(t)}{s} - (t - t_s) + t \le t^*$, i.e., $s \ge \frac{R(t)}{(t^*-t_s)}$. Otherwise, they experience schedule delay late. Thus, one has

$$C(t) = \begin{cases} \alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right), s \in \left[\theta\bar{s}, \frac{R(t)}{t^*-t_s}\right] \\ \alpha\left(\frac{R(t)}{s} - t + t_s\right) + \beta\left(t^* - t_s - \frac{R(t)}{s}\right), s \in \left(\frac{R(t)}{t^*-t_s}, \bar{s}\right] \end{cases} \tag{C.1}$$

$$E(C(t)) = \int_{\theta\bar{s}}^{\frac{R(t)}{t^*-t_s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right)\right] f(s)ds + \int_{\frac{R(t)}{t^*-t_s}}^{\bar{s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \beta\left(t^* - t_s - \frac{R(t)}{s}\right)\right] f(s)ds \tag{C.2}$$

$$E(C^2(t)) = \int_{\theta\bar{s}}^{\frac{R(t)}{t^*-t_s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right)\right]^2 f(s)ds + \int_{\frac{R(t)}{t^*-t_s}}^{\bar{s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \beta\left(t^* - t_s - \frac{R(t)}{s}\right)\right]^2 f(s)ds \tag{C.3}$$

$$\sigma(C(t)) = \sqrt{var(C(t))} = \sqrt{E(C^2(t)) - \left(E(C(t))\right)^2} \tag{C.4}$$

In this situation, we can only obtain the departure rate numerically.

(ii) Always late + Always queuing (SDL+AQ)

In this situation, no matter how the capacity varies, all commuters experience schedule delay late and always queuing. Thus, one has

$$C(t) = \alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right), s \in [\theta\bar{s}, \bar{s}] \quad \text{(C.5)}$$

$$E\big(C(t)\big) = \int_{\theta\bar{s}}^{\bar{s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right)\right] f(s)ds \quad \text{(C.6)}$$

$$E\big(C^2(t)\big) = \int_{\theta\bar{s}}^{\bar{s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right)\right]^2 f(s)ds \quad \text{(C.7)}$$

$$\sigma\big(C(t)\big) = (\alpha + \gamma)R(t)\sqrt{\frac{1}{\theta\bar{s}^2} - \left(\frac{ln\theta^{-1}}{\bar{s} - \theta\bar{s}}\right)^2} \quad \text{(C.8)}$$

Substituting (C.6) and (C.8) into Eq. (21) and using the equilibrium condition, we can obtain the departure rate as follows,

$$r(t) = \frac{\alpha}{(\alpha+\gamma)\left(\frac{ln\theta^{-1}}{\bar{s}-\theta\bar{s}} + \lambda\sqrt{\frac{1}{\theta\bar{s}^2} - \left(\frac{ln\theta^{-1}}{\bar{s}-\theta\bar{s}}\right)^2}\right)} \quad \text{(C.9)}$$

(iii) Always late + Possible queuing (SDL+PQ)

In this situation, commuters arrive at the destination always late, but may experience queuing depending on the capacity of bottleneck. Commuters experience queuing if $\frac{R(t)}{s} - (t - t_s) > 0$, i.e., $s \leq \frac{R(t)}{(t-t_s)}$. Otherwise, they do not experience queuing. Thus, one has

$$C(t) = \begin{cases} \alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right), s \in \left[\theta\bar{s}, \frac{R(t)}{t-t_s}\right] \\ \gamma(t - t^*), \qquad s \in \left(\frac{R(t)}{t-t_s}, \bar{s}\right] \end{cases} \quad \text{(C.10)}$$

$$E\big(C(t)\big) = \int_{\theta\bar{s}}^{\frac{R(t)}{t-t_s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right)\right] f(s)\, ds + \int_{\frac{R(t)}{t-t_s}}^{\bar{s}} \gamma(t - t^*) f(s)\, ds$$
(C.11)

$$E\big(C^2(t)\big) = \int_{\theta\bar{s}}^{\frac{R(t)}{t-t_s}} \left[\alpha\left(\frac{R(t)}{s} - t + t_s\right) + \gamma\left(\frac{R(t)}{s} + t_s - t^*\right)\right]^2 f(s)\, ds + \int_{\frac{R(t)}{t-t_s}}^{\bar{s}} [\gamma(t - t^*)]^2 f(s)\, ds$$
(C.12)

In this situation, we can only obtain the departure rate numerically.

(iv) Early or late + Possible queuing (SDE/L+PQ)

In this situation, commuters may experience schedule delay early or late, and may experience queuing depending on the capacity of bottleneck. If the capacity $s \geq \frac{R(t)}{(t-t_s)}$, commuters arrive at the destination early and experience no queuing. If the capacity $s < \frac{R(t)}{(t^*-t_s)}$, commuters arrive at the destination late and always experience queuing. Otherwise, commuters experience schedule delay early and experience queuing. Thus, one has

$$C(t)=\begin{cases}\alpha\left(\frac{R(t)}{s}-t+t_s\right)+\gamma\left(\frac{R(t)}{s}+t_s-t^*\right), s\in\left[\theta\bar{s},\frac{R(t)}{t^*-t_s}\right]\\ \alpha\left(\frac{R(t)}{s}-t+t_s\right)+\beta\left(t^*-t_s-\frac{R(t)}{s}\right), s\in\left(\frac{R(t)}{t^*-t_s},\frac{R(t)}{t-t_s}\right]\\ \beta(t^*-t), \qquad s\in\left(\frac{R(t)}{t-t_s},\bar{s}\right]\end{cases} \tag{C.13}$$

$$E\big(C(t)\big)=\int_{\theta\bar{s}}^{\frac{R(t)}{t^*-t_s}}\left[\alpha\left(\frac{R(t)}{s}-t+t_s\right)+\gamma\left(\frac{R(t)}{s}+t_s-t^*\right)\right]f(s)ds+\int_{\frac{R(t)}{t^*-t_s}}^{\frac{R(t)}{t-t_s}}\left[\alpha\left(\frac{R(t)}{s}-t+t_s\right)+\beta\left(t^*-t_s-\frac{R(t)}{s}\right)\right]f(s)ds+\int_{\frac{R(t)}{t-t_s}}^{\bar{s}}\beta(t^*-t)f(s)ds \tag{C.14}$$

$$E\big(C^2(t)\big)=\int_{\theta\bar{s}}^{\frac{R(t)}{t^*-t_s}}\left[\alpha\left(\frac{R(t)}{s}-t+t_s\right)+\gamma\left(\frac{R(t)}{s}+t_s-t^*\right)\right]^2f(s)ds+\int_{\frac{R(t)}{t^*-t_s}}^{\frac{R(t)}{t-t_s}}\left[\alpha\left(\frac{R(t)}{s}-t+t_s\right)+\beta\left(t^*-t_s-\frac{R(t)}{s}\right)\right]^2f(s)ds+\int_{\frac{R(t)}{t-t_s}}^{\bar{s}}[\beta(t^*-t)]^2f(s)ds \tag{C.15}$$

In this situation, we can only obtain the departure rate numerically.

(v) Always early + Possible queuing (SDE+PQ)

In this situation, commuters always experience schedule delay early, but may experience queuing depending on the capacity of bottleneck. Commuters experience queuing if $\frac{R(t)}{s}-(t-t_s)\geq 0$, i.e., $s\leq\frac{R(t)}{(t-t_s)}$. Otherwise, they do not experience queuing. Thus, one has

$$C(t)=\begin{cases}\alpha\left(\frac{R(t)}{s}-t+t_s\right)+\beta\left(t^*-t_s-\frac{R(t)}{s}\right), s\in\left[\theta\bar{s},\frac{R(t)}{t-t_s}\right]\\ \beta(t^*-t), \qquad s\in\left(\frac{R(t)}{t-t_s},\bar{s}\right]\end{cases} \tag{C.16}$$

$$E\big(C(t)\big)=\int_{\theta\bar{s}}^{\frac{R(t)}{t-t_s}}\left[\alpha\left(\frac{R(t)}{s}-t+t_s\right)+\beta\left(t^*-t_s-\frac{R(t)}{s}\right)\right]f(s)ds+\int_{\frac{R(t)}{t-t_s}}^{\bar{s}}\beta(t^*-t)f(s)\,ds \tag{C.17}$$

$$E\big(C^2(t)\big)=\int_{\theta\bar{s}}^{\frac{R(t)}{t-t_s}}\left[\alpha\left(\frac{R(t)}{s}-t+t_s\right)+\beta\left(t^*-t_s-\frac{R(t)}{s}\right)\right]^2f(s)ds+\int_{\frac{R(t)}{t-t_s}}^{\bar{s}}[\beta(t^*-t)]^2f(s)\,ds \tag{C.18}$$

In this situation, we can only obtain the departure rate numerically.