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An alternative wheel regenerative mechanism in surface grinding: distributed grit dullness captured by specific energy waves

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ABSTRACT

Regenerative chatter is a serious problem in machining. It is an unstable relative vibration between the workpiece and the cutting tool that adversely affects virtually all chip formation processes. This paper addresses regenerative chatter in grinding, which is one of the most widely used abrasive processes today. As a result of significant tool wear in grinding, surface regeneration (which is a prerequisite for regenerative chatter) can occur not only on the workpiece but also on the grinding wheel. This article is concerned with the regenerative mechanism by which wheel-related instability develops.

In the present study, the role of distributed grit dullness alone is explored. A new chatter model is formulated and validated by both numerical simulations and experimental data. The new theory accurately predicts the existence of stable regimes in grinding, for the first time. This is in contrast to the published literature where the consensus has been that grinding cannot be stable with respect to wheel regeneration. Consequently, the present contribution enables a novel opportunity to increase the productivity of industrial grinding operations.

1. Introduction

Grinding is the oldest machining operation [1], and now represents 20–25% of the manufacturing sector in developed countries [2]. Owing to its unique cutting tool, grinding has a number of advantages and disadvantages compared to conventional machining operations like turning and milling. Abrasive processes are known for producing excellent surface quality and dimensional accuracy, cutting difficult-to-machine materials with relative ease, and achieving high material removal rates. However, the downside of machining with a grinding wheel instead of a conventional cutting tool is excessive tool wear, significant heat generation, and – from a theoretical point of view – the complexity of process modelling and prediction due to the inherent randomness of the wheel geometry [3].

This last disadvantage makes it especially complicated to accurately capture an already intricate phenomenon in machining, which harmfully affects virtually all chip formation processes, namely regenerative chatter. This is a self-excited relative vibration between the workpiece and the cutting tool, the consequences of which are seriously adverse. It deteriorates the surface quality and dimensional accuracy of the workpiece, reduces the lifetime of the cutting tool, generates unpleasant

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noise, and limits productivity in practice. Therefore, predicting and avoiding regenerative chatter is of critical importance when it comes to ensuring the stability, efficiency and productivity of machining operations [4].

Although regenerative chatter has been a topic of extensive research since the beginning of the 20th century [5], grinding has always been lagging behind the results of conventional theories [3]. However, understanding grinding chatter is a particularly critical task, because abrasive processes are often finishing operations responsible for the final surface quality and dimensional accuracy of the machined part. Therefore, unstable relative vibration between the wheel and the workpiece can be especially detrimental in grinding, because it can destroy a product upon which a number of costly machining operations have already been performed.

The aim of this paper is to describe a novel formulation of the theory of grinding stability, which offers new insight into the potential for chatter-free grinding regimes. The contribution is structured as follows. First, the following section presents an overview of the most relevant literature. The next section defines the objective of the present study from a theoretical perspective, describing how the work builds upon previous contributions. The model is then formulated in detail, starting with a schematic explanation of the parameters involved, and then proceeding with a mathematical formulation. Following a stability analysis and a numerical simulation, the theoretical model is validated experimentally. The results are discussed, and conclusions are drawn regarding the validity potential use of the proposed approach.

2. Literature review

The second half of the 20th century brought significant advancements in grinding chatter research. One of the first contributions to the field was made by Hahn, who proposed an initial, relatively simple model of self-excited vibration in grinding, in which he considered surface regeneration on the workpiece but not on the grinding wheel [6]. His theory predicted a distribution of stable and unstable parameter regions, allowing the machinist to select a chatter-free machining scenario. The experiments of Landberg indicated that grinding wheels do not necessarily wear evenly, but can develop surface waves similarly to a rotating workpiece [7]. This was a very important discovery for grinding dynamics, because it demonstrated that surface regeneration can occur not only on the workpiece but on the grinding wheel as well, introducing the possibility of the two phenomena happening simultaneously and influencing one another in real time. The complexity of surface waves developing on both the wheel and the workpiece was addressed by Snoeys and Brown, who formulated potentially the first ever experimentally validated grinding chatter theory considering both wheel and workpiece regeneration in the same model [8]. This idea of double regeneration or doubly regenerative stability was taken further on by Thompson, who confirmed and deepened the scientific understanding of grinding chatter by several of his works [9–13]. The literature was thoroughly reviewed by Inasaki et al., whose paper is recognised as one of the most fundamental and most often-cited publications in the history of grinding chatter research [14].

Prior to 2006, virtually all grinding chatter theories worked with the following modelling assumption and practical observation [14]:

1. The wheel regenerative mechanism by which wheel-related instability occurs can be modelled exclusively as distributed radial wear around the circumference of the grinding wheel.
2. Most grinding processes are unstable in practice with regard to wheel regeneration.

As a consequence of this second point, the growth rate of chatter becomes more important to analyse than absolute stability itself. Due to its practical utility in designing grinding processes (e.g. predicting the available grinding time before chatter develops beyond tolerance), this aspect of grinding chatter has been studied by several authors in the context of not only wheel regeneration but workpiece regeneration as well [11,12,14]. It has also been observed that the growth rate of wheel regeneration is significantly lower than that corresponding to workpiece regeneration, which means that wheel-related instability develops much more slowly than workpiece-related instability [8,14]. Therefore, it is possible to operate under unstable grinding conditions for a reasonable amount of time before wheel-related chatter grows to truly detrimental proportions.

In 2006, the above modelling assumption was challenged by Li and Shin, who presented an alternative way of describing the wheel regenerative mechanism in surface grinding [15]. They proposed that considering distributed radial wear as the sole mechanism responsible for wheel regeneration (and thus wheel-related instability) is an incomplete theory, as previous grinding chatter models adopting such an approach had not been able to account for a number of experimental observations reported in the literature [8, 14,16, 17,18]. Therefore, they formulated a new theory based on an alternative regenerative mechanism that combines distributed radial wear with distributed grit dullness (i.e. the distribution of the dullness of the cutting edges around the circumference of the grinding wheel). The authors characterised grit dullness by the specific energy, which quantifies the amount of grinding energy required to remove a unit volume of workpiece material, or equivalently, the amount of grinding power necessary to sustain a unit material removal rate. That is because a sharper/duller grain corresponds to a lower/higher specific energy, respectively. Therefore, Li and Shin considered not only physical surface waves but also specific energy waves on the grinding wheel. This is a fundamental shift from the general research trend that was predominant in the literature before 2006. By going back to the basics of grinding chatter (i.e. to the regenerative mechanism itself) and reconsidering it in a more realistic way, the authors managed to shed some light on a number of experimental observations that chatter models before 2006 were unable to explain. However, the practical assertion highlighted in the previous paragraph was not touched upon by Li and Shin. They did not consider any potential boundaries between stable and unstable parameter regions, but focused only on unstable processes and the grinding forces and chatter frequencies that arise.

Modern grinding chatter theories often consider complex phenomena alongside self-excited vibration, such as non-linear behaviour [19–22], workpiece imbalance [23] and parallel grinding [24]. Although some of these studies have identified areas of theoretical stability [25], to the authors' knowledge the general consensus has been that grinding is an unstable process with respect to wheel regeneration, and that the instability mechanism is dominated by physical surface waves on the grinding wheel.

3. Objective

The primary aim of this work is to investigate the stability properties of single-pass surface grinding as predicted by the alternative wheel regenerative mechanism of distributed grit dullness. This concept is shown schematically in Fig. 1.

Here, Fig. 1(a) illustrates traditional grinding stability models, which assume that the onset of chatter causes fluctuations in the geometry of the grinding wheel. Fig. 1(b) illustrates the concept proposed by Li and Shin [15] whereby the chatter also introduces fluctuations in the specific cutting energy at different locations on the wheel surface. In contrast, the present study assumes that only variations in specific energy are possible, as shown in Fig. 1(c).

In order to study the effect of distributed grit dullness as a wheel regenerative mechanism, the problem will be isolated by neglecting distributed radial wear, which was virtually the only approach to describing wheel regeneration prior to (and even after) Li and Shin's new theory. Whilst this introduces a significant assumption, and approximation, into the resulting stability model, it will be shown that this contrarian approach introduces new regions of stability that are experimentally validated.

The circumference of the grinding wheel is assumed to remain perfectly circular throughout the entire grinding process. Wheel wear is accounted for only by the dullness of individual cutting edges quantified by the specific energy, which will be considered as a continuous function distributed around the circumference of the grinding wheel.

To isolate the problem even further, the wheel regenerative mechanism of distributed grit dullness will be considered in a grinding operation where workpiece regenerative chatter cannot develop. That is to limit the possible sources of instability to one, namely to wheel regenerative chatter caused by an uneven distribution of grit dullness around the circumference of the grinding wheel. Theoretically speaking, this mechanism has the potential to generate instability on its own, because a varying specific energy distribution (corresponding to a varying cutting-force coefficient in conventional machining) results in a varying grinding force even if the chip thickness is constant, which – under unfavourable circumstances – can generate self-excited vibrations. The question is whether such instability can develop for a practical set of system parameters. The grinding process suitable for eliminating workpiece regeneration will be single-pass surface grinding, because the same workpiece surface is never cut twice in this configuration. In this case, the possibility of rapid, milling-like surface regeneration in the grinding zone is negligible due to the very high number of cutting edges.

4. Model

This work is a direct continuation and a significant expansion of the authors' analytical study of wheel regeneration in surface grinding [26], which showed some numerical examples of the basic concept in the time domain, with only a brief introduction to frequency-domain analysis and without any experimental validation. The model presented in [26] is now

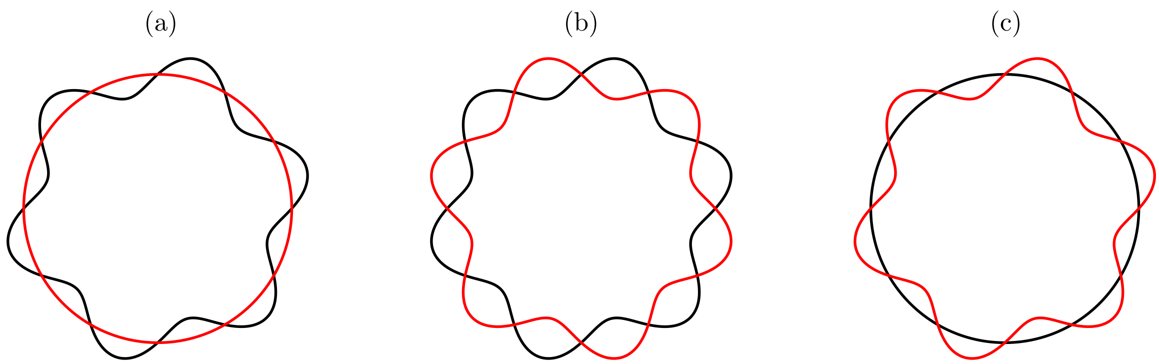


Fig. 1. Three approaches describing the regenerative mechanism leading to chatter: physical wheel surface (black line), specific energy distribution (red line). (a) Classical approach, (b) Li and Shin's approach, (c) Authors' approach.

described in more detail, along with an in-depth stability analysis in the frequency domain, so that later sections can provide new validation results from experimental data.

The two-dimensional, single-degree-of-freedom mechanical model of single-pass surface grinding is presented in Fig. 2. It can be seen that the wheel of radius R_g is allowed to oscillate relative to the workpiece only in the direction of the depth of cut δ, which is described by the general coordinate x. The modal mass, modal damping and modal stiffness of the grinding wheel are denoted by m, c and k, respectively. The rotational speed of the wheel is ω_g (i.e., its circumferential speed is $v_g = \omega_g R_g$), and the feed rate is v_w. The total grinding force is denoted by F, and its individual components are F_n (normal), F_t (tangential) and F_x (x-directional). Figure 2 also highlights two important angles in the grinding zone: the contact angle φ_c, and the grinding force angle α that defines the angular position of the resultant grinding force in the cutting zone.

The new chatter theory consists of three submodels: the grinding force model, the wheel wear model and the wheel vibration model. This section presents each of these in detail, before moving on to the mathematical formulation of the vibration problem.

4.1. Grinding force model

In order to keep the focus on the alternative wheel regenerative mechanism (proposed in Section 3) and the subsequent formulation of a new chatter theory, the applied grinding force model will be relatively simple. One of the most basic ways to describe the grinding force is given by Malkin and Guo [2] in the form

$$F_t = wu\frac{v_w}{v_g}\delta, \tag{1}$$

where w is the grinding width, and u is the specific energy. It can be seen that the three grinding mechanisms are not treated separately by this model, i.e., the specific energy (which is typically responsible for taking such a separation into account) makes no distinction between sliding, ploughing and chip formation. Due to the fact that the x-directional oscillations of the grinding wheel are of interest, the x-component of the total grinding force has to be calculated. It can be written as

$$F_x = \mu_x wu\frac{v_w}{v_g}\delta, \tag{2}$$

where μ_x is the grinding force ratio between the x-directional and tangential grinding force components ($\mu_x = F_x/F_t$), which is a function of the grinding force ratio between the tangential and normal grinding force components ($\mu = F_t/F_n$), and the grinding force angle α (see Fig. 2).

4.2. Wheel wear model

Due to the fact that grit dullness is the sole mechanism responsible for the regenerative effect in this model, the variation of the specific energy needs to be quantified both in space (around the circumference of the grinding wheel) and in time (as it

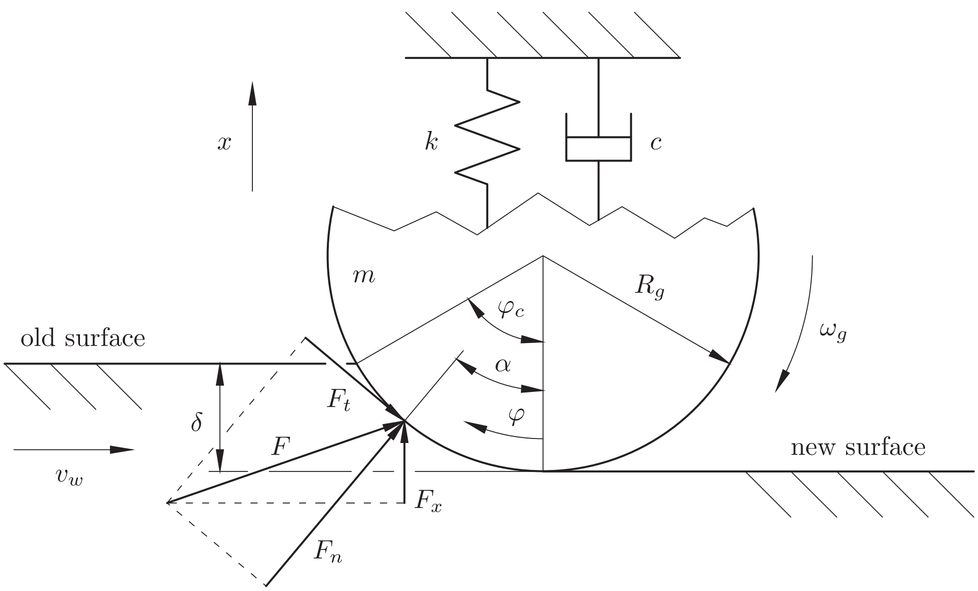


Fig. 2. A two-dimensional, single-degree-of-freedom model of single-pass surface grinding (up-grinding configuration).

develops during grinding). This results in a two-variable description of the specific energy, which constitutes a rather complex mathematical problem when coupled with the stability analysis of a delay differential equation typical of chatter theories. However, the system can be simplified, because the spatial and temporal variables of the specific energy function are not independent of one another. Time is the only truly independent variable in the model, because space (i.e. the angular coordinate running along the circumference of the grinding wheel) depends on time through the constant rotational speed of the wheel. Therefore, the two variables of the specific energy can be reduced to one. This can be achieved by losing the spatial coordinate altogether and specifying that the specific energy u at time t corresponds to the point (or grit) on the grinding wheel that is just leaving the cutting zone. This convention is illustrated in Fig. 3, where T_g is the wheel rotation period, $\tau_g = T_g/Z$ is the grit-passing period (assuming a regular grain distribution), and Z is the number of cutting edges around the circumference of the grinding wheel.

Regarding the specific energy, the following assumptions are made. First, due to the high density of cutting edges on the wheel and the rapid rate at which they leave the grinding zone, the specific energy is assumed to be a continuous function of time. Second, as far as the axial direction is concerned, the specific energy is assumed to be constant along the width of the wheel. And third, the specific energy is assumed to increase inside the grinding zone as a result of wear, and remain constant outside. In order words, no grain gets sharper during grinding, which means that the self-sharpening property of grinding wheels is not considered in the proposed model.

4.3. Wheel vibration model

The wheel vibration is assumed to affect the depth of cut without influencing the chip thickness. In other words, the vibration of the grinding wheel is equivalent to the oscillation of the old workpiece surface, as defined on Fig. 2. Therefore, with reference to Fig. 4, the wheel vibration causes a change in the length of contact between the wheel and the workpiece. For up-grinding this changes the entering angle, whereas for down-grinding this changes the exit angle.

Consequently, the regenerative effect occurs in this model as follows. As a result of some external disturbance, the grinding wheel begins to oscillate relative to the workpiece. This changes the nominal depth of cut and thus the material removal rate in such a way that both of these parameters will vary in time. Since the material removal rate is not constant anymore, different parts of the grinding wheel will remove different amounts of workpiece material. This leads to different levels of wear around the circumference of the wheel, which is quantified by the specific energy. Due to the fact that the grinding force depends on the specific energy, a varying specific energy results in a varying grinding force, which leads to a time-dependent variation in wheel displacement (i.e. wheel vibration) according to the dynamics of the structure. This creates a closed loop: some initial disturbance in the displacement of the grinding wheel causes it to vibrate again. Depending on the magnitude and phase difference between the oscillations of the wheel, the grinding process can be either self-attenuating (stable) or self-amplifying (unstable).

5. Mathematical formulation

This section establishes five mathematical relationships between (1) the wheel vibration and the depth of cut, (2) the depth of cut and the material removed, (3) the material removed and the specific energy, (4) the specific energy and the grinding force, and (5) the grinding force and the wheel vibration. Then the equation of motion is derived in order to assess the dynamic stability of the system.

The relationship between the displacement of the grinding wheel (x) and the variation of the depth of cut (δ) can be formulated as

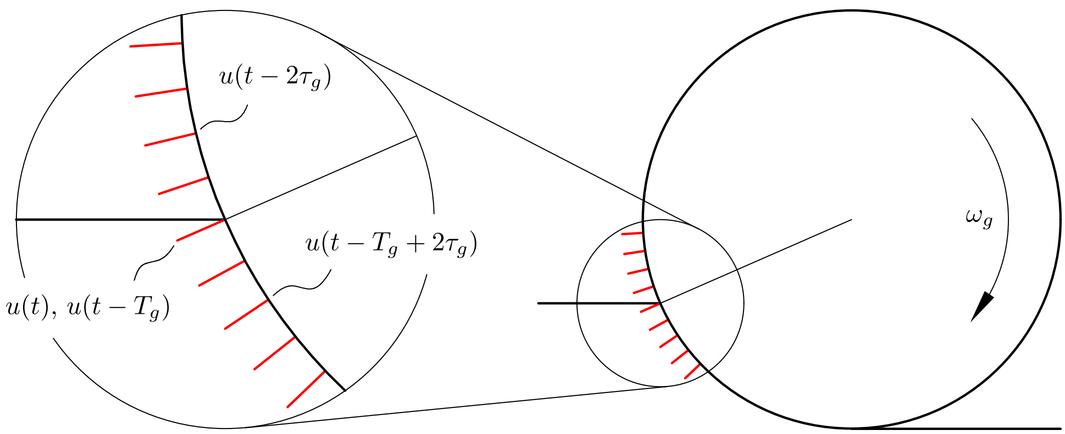


Fig. 3. Defining the specific energy u around the circumference of the grinding wheel.

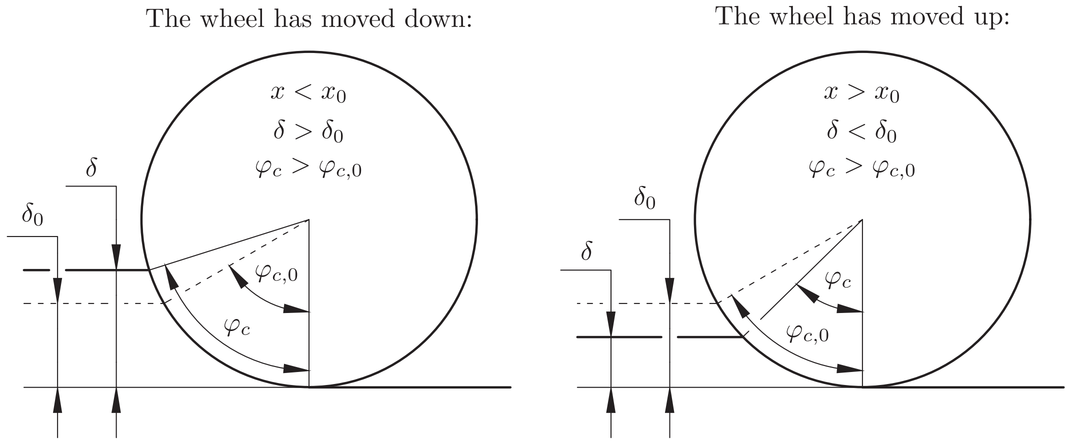


Fig. 4. Depth-of-cut-based wheel vibration model (x, δ and φ_c are instantaneous parameters, while x_0, δ_0 and $\varphi_{c,0}$ correspond to steady-state grinding).

$$\delta(t) = \delta_0 - x(t), \tag{3}$$

where the constant δ_0 is the desired or nominal depth of cut. It can be seen that the origin of the general coordinate x is set in such a way that $x(t) \equiv 0$ corresponds to steady-state grinding.

The relationship between the depth of cut and the amount of material removed by a single grain over one grinding wheel revolution can be calculated as

$$V'_w(t) = v_w \tau_g \delta(t), \tag{4}$$

where V'_w is the specific material removed (material volume per unit grinding width).

According to Fig. 3, the relationship between the specific energy u, which quantifies grit wear, at the current time (t) and one wheel rotation period earlier $(t - T_g)$ can be defined through the specific material removed, which produces grit wear, such that

$$u(t) = u(t - T_g) + C_d V'_w(t), \tag{5}$$

where C_d is the coefficient of dulling introduced by the authors, which translates material removed into specific energy through the concept of grit wear.

The relationship between the specific energy and the x-directional grinding force F_x can be determined by averaging the specific energy distribution in the cutting zone and applying the grinding force model presented in Eq. (2):

$$F_x(t) = \frac{\mu_x w v_w \delta_0}{v_g \tau_{c,0}} \int_0^{\tau_{c,0}} u(t - T_g + \tau_{c,0} - \tau) d\tau + C, \tag{6}$$

where τ is a local time coordinate fixed to the grinding zone ($\varphi = \omega_g \tau$, see Fig. 2), $\tau_{c,0}$ is the contact time of a single grain under steady-state cutting conditions, and C is the time-independent part of the grinding force.

The relationship between the grinding force and the wheel vibration can be established through the classical second-order differential equation

$$\ddot{x}(t) + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 x(t) = \frac{1}{m} F_x(t), \tag{7}$$

where $\zeta = c/(2m\omega_n)$ and $\omega_n = \sqrt{k/m}$ are the damping ratio and natural angular frequency of the wheel, respectively. The natural frequency of the wheel in Hz will be denoted by f_n.

Combining Eqs. (3) to (7) together by eliminating the variables x, δ, V'_w and F_x, and keeping the specific energy as the general coordinate of the system, the equation of motion becomes

$$\ddot{u}(t) + 2\zeta\omega_n \dot{u}(t) + \omega_n^2 u(t) = \ddot{u}(t - T_g) + 2\zeta\omega_n \dot{u}(t - T_g) + \omega_n^2 u(t - T_g) - \frac{\mu_x C_d w v_w^2 \tau_g \delta_0}{m v_g \tau_{c,0}} \int_0^{\tau_{c,0}} u(t - T_g + \tau_{c,0} - \tau) d\tau, \tag{8}$$

where the force constant C has been dropped, since it only offsets and does not change the dynamics (and thus the stability) of a linear system. It can be seen that two time delays appear in the governing equation of motion of the process: a point delay (T_g) corresponding to the rotational speed of the grinding wheel, and a distributed delay ($\tau_{c,0}$) measuring the amount of time that a single grain takes to pass through the cutting zone. It is noteworthy that the point delay is significantly longer than the distributed delay – depending on the radius of the grinding wheel and the applied depth of cut, the two time periods can be orders of magnitude apart.

Therefore, the mathematical model of the problem has been formulated. The dynamics of the system is captured by the governing equation of motion displayed in Eq. (8). The following section is dedicated to the stability analysis of the grinding process based on this delay differential equation.

6. Chatter stability

The stability of the system will be assessed by the Nyquist criterion [27–31], which requires a frequency-domain representation of the system equations. The Laplace transforms of the five state variables are denoted by $\mathscr{L}\{x\}(s) = X(s), \mathscr{L}\{\delta\}(s) = D(s), \mathscr{L}\{V\prime_w\}(s) = W\prime_w(s), \mathscr{L}\{u\}(s) = U(s)$ and $\mathscr{L}\{F_x\}(s) = \Phi_x(s)$, where s is the complex Laplace frequency. Taking the Laplace transforms of Eqs. (3) to (7), and rearranging, allows the definition of the following frequency-domain transfer functions:

$$J(s) = \frac{U(s)}{D(s)} = \frac{C_d v_w \tau_g}{1 - e^{-T_g s}}, \tag{9}$$

$$H(s) = \frac{\Phi_x(s) - \frac{C}{s}}{U(s)} = \frac{\mu_x w v_w \delta_0 e^{-T_g s}(e^{\tau_{c,0} s} - 1)}{v_g \tau_{c,0} s}, \tag{10}$$

$$G(s) = \frac{X(s)}{\Phi_x(s)} = \frac{1}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}, \tag{11}$$

where the transfer functions J, H and G define three frequency-domain relationships between the depth of cut and the specific energy (J), between the specific energy and the grinding force (H), and between the grinding force and the wheel vibration (G). This enables a block diagram representation of the of the system as shown in Fig. 5.

The open-loop transfer function of the negative feedback system, which the Nyquist criterion uses to determine stability, can be calculated by multiplying the three transfer functions to give:

$$T_o(s) = \frac{\mu_x C_d w v_w^2 \tau_g \delta_0 e^{-T_g s}(e^{\tau_{c,0} s} - 1)}{m v_g \tau_{c,0} s(1 - e^{-T_g s})(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \tag{12}$$

Since the pole-zero structure of the open-loop transfer function is necessary for the Nyquist criterion, it is presented in Fig. 6 along with the corresponding Nyquist contour in Fig. 7. It can be seen that the poles of the open-loop transfer function have been bypassed according to the definition of the Nyquist criterion. As the detours around them are infinitesimally small ($\varepsilon \to 0$), there will be sections of infinite magnitude in the Nyquist plot corresponding to these detours. This means that the $(-1, 0)$ position can be encircled by infinite as well as finite sections of the Nyquist plot. The reason this is important to point out is that there is a qualitative difference between the two cases. Encirclements by finite sections (resulting in a "finitely unstable" process) can be theoretically stabilised by appropriately changing any system parameter. However, encirclements by infinite sections (resulting in an "infinitely unstable" process) can be theoretically stabilised only by a certain set of system parameters. This is because some process variables only scale the Nyquist plot and therefore have no effect on its infinite sections. With reference to Eq. (12), parameters can be classified based upon whether they can only scale, rather than distort, the open loop transfer function. This classification is summarised in Tab. 1.

The following subsections present the absolute and relative stability predictions of the proposed model, and also demonstrate the internal consistency of the new theory by validating the frequency-domain results through numerical simulations in the time domain. The analysis will be based upon the arbitrary parameters listed in Tab. 2.

6.1. Stability analysis

The absolute stability of the system is presented in Fig. 8 as a function of the wheel speed (ω_g) and the nominal depth of cut (δ_0), along with the chatter frequencies (ω_c) displayed relative to the natural frequency of the grinding wheel (ω_n). Figure 8 captures three kinds of stability areas.

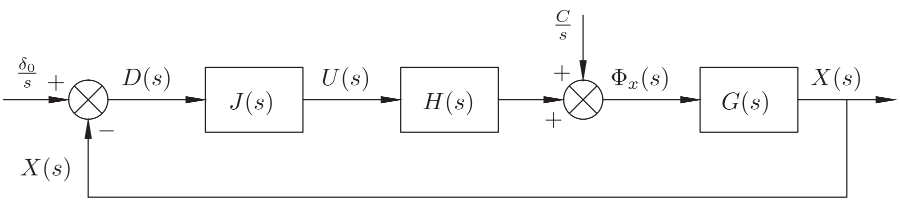


Fig. 5. Block diagram describing wheel regeneration in single-pass surface grinding.

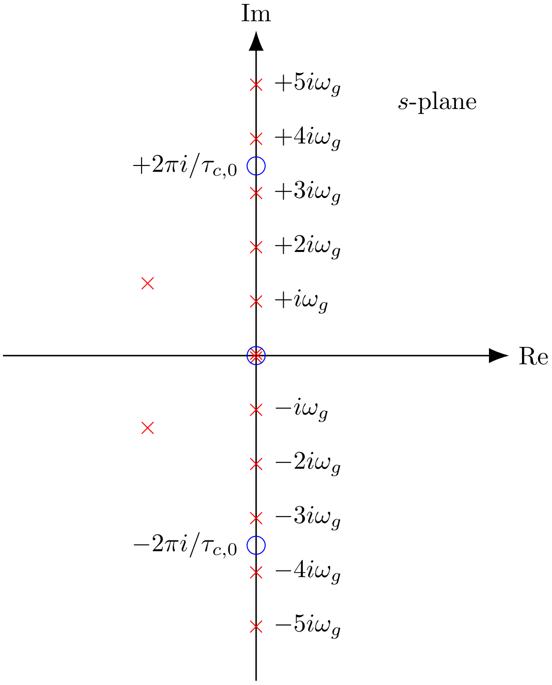


Fig. 6. Poles (×) and zeros (○) of $T_o(s)$.

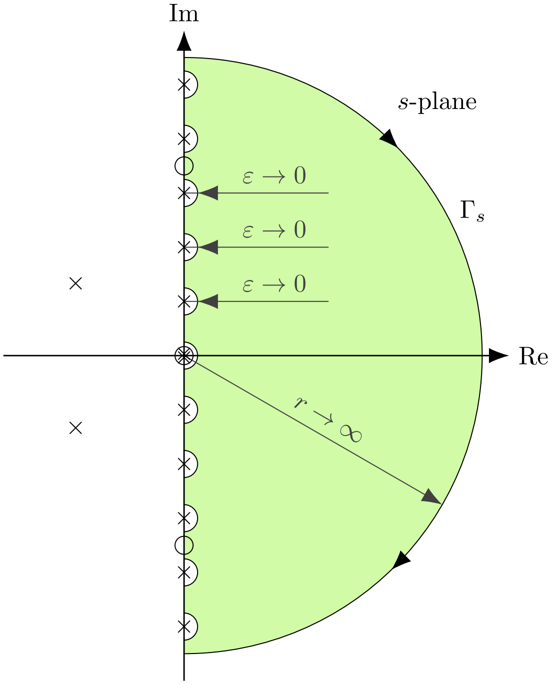


Fig. 7. Definition of the Nyquist contour Γ_s.

Table 1
Classification of system parameters. Parameters that scale T_o cannot always influence stability, depending upon the chatter frequency.

Parameter	Scales T_o	Distorts T_o
Nominal depth of cut (δ_0)	x	x
Damping ratio (ζ)	x	x
Grinding force ratio (μ_x)	x	
Grit contact time ($\tau_{c,0}$)	x	x
Grit-passing period (τ_g)	x	x
Natural frequency (ω_n)	x	x
Coefficient of dulling (C_d)	x	
Modal mass (m)	x	
Grinding wheel period (T_g)	x	x
Wheel surface speed (v_g)	x	x
Feed rate (v_w)	x	
Width of cut (w)	x	

Table 2
Numerical parameters used to illustrate process stability in this section.

m	[kg]	1
ζ	[%]	1
f_n	[Hz]	300
R_g	[mm]	100
w	[mm]	20
Z	[–]	10 000
C_d	[J/mm^3/mm^2]	4×10^6
μ	[–]	0.4
v_w	[mm/min]	56

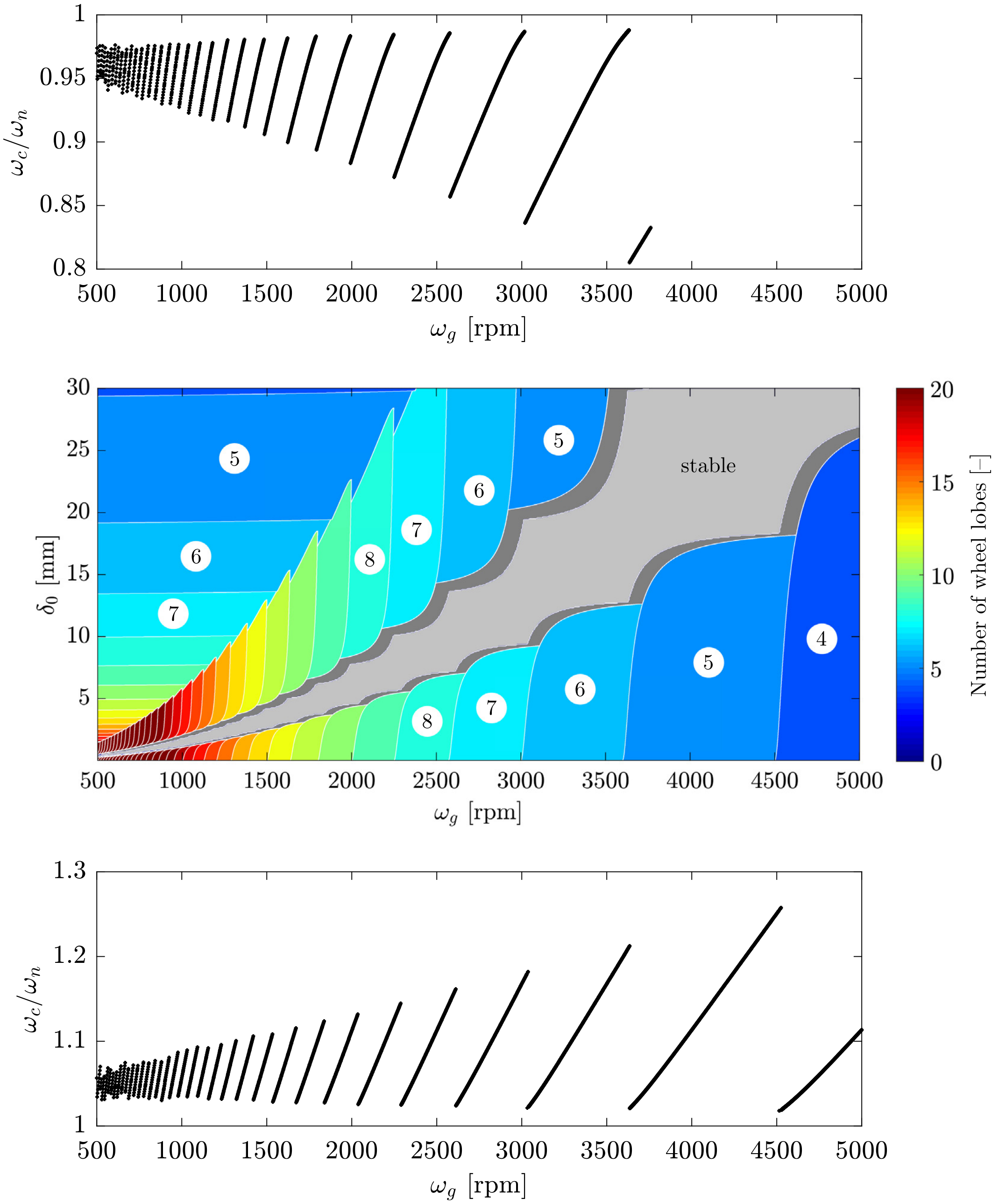


Fig. 8. Stability chart presenting three types of stability regions: infinitely unstable (colour map), finitely unstable (dark grey), and stable (light grey). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

1. A colour map of infinitely unstable grinding operations. In this region, the parameters that scale the open loop transfer function (as listed in Tab. 1) cannot stabilise the system, unlike those that distort the open loop transfer function.
2. A region of finitely unstable processes in dark grey. In this region, the parameters that scale the open loop transfer function (as listed in Tab. 1) can also stabilise the system.
3. A stable zone in light grey.

Due to the fact that the infinite encirclements of the $(-1, 0)$ position always happen at integer multiples of the wheel speed (see Figs. 6 and 7), the chatter frequencies of the infinitely unstable system are integer multiples of the wheel speed as well. Since the ratio between the chatter frequency and the wheel speed clearly defines the number of specific energy waves (or lobes) around the circumference of the grinding wheel, these numbers are highlighted in the infinitely unstable region. The chatter frequencies of the system corresponding to the upper and lower absolute stability boundaries are presented above and below the stability chart, respectively. It can be seen that a loss of stability through the upper/lower stability boundary results in a chatter vibration slightly below/above the natural frequency of the wheel.

Regarding the stability diagram and the overall aim of this paper, two important observations can be made: (1) the wheel regenerative mechanism of distributed grit dullness alone is capable of capturing the phenomenon of chatter on its own, and (2) the new theory also predicts stable operation, which is a very promising result considering the predominant stance of the literature reviewed in Section 2. This stability is restricted to high depths of cut (i.e. the typical instability of grinding observed in practice with respect to wheel regeneration still holds for finishing operations with low depths of cut), but other processes such as creep-feed grinding (CFG) and high-efficiency deep grinding (HEDG) could benefit greatly from chatter-free machining in the high depth of cut region.

6.2. Numerical validation

In order to confirm the theoretical validity of the frequency-domain solution, a number of time-domain simulations have been performed. Applying the central difference scheme to the equation of motion, Eq. (8) can be solved for a given set of grinding parameters. As shown in Fig. 9(a), the two types of results are in agreement.

Furthermore, the time-domain signal of the specific energy corresponding to a particular set of grinding parameters ($\omega_g = 3100$ rpm, $\delta_0 = 21$ mm) is singled out for presentation in Fig. 9(b) and (c). Two observations can be made regarding this diagram: (1) the number of wheel lobes is indeed five, according to the analytical prediction shown in Fig. 8, and (2) the

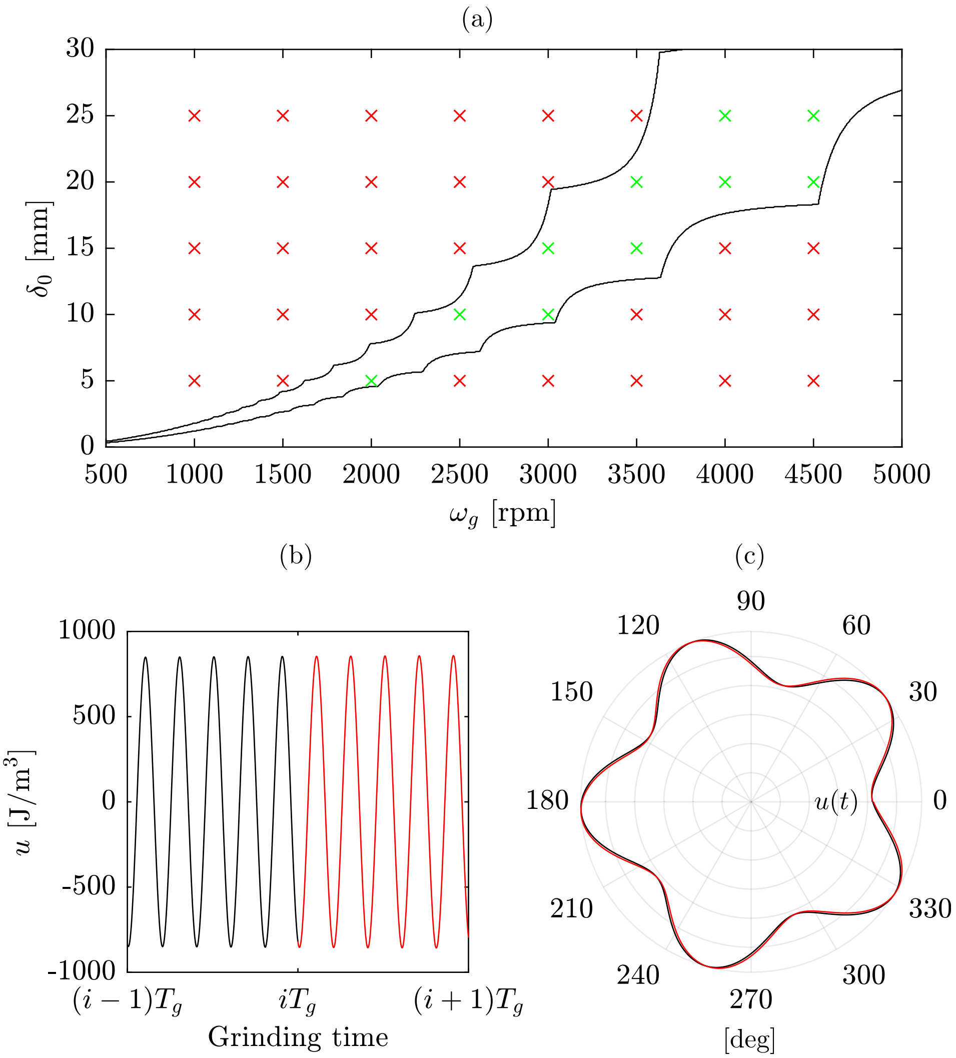


Fig. 9. Numerical validation. (a) Comparison between analytical and numerical results: analytical stability boundary (black line), × stable numerical simulation, × unstable numerical simulation; Linear (b) and polar (c) plots of the specific energy variation for two successive grinding wheel rotations depicted in different colours, for $\omega_g = 3100$ rpm and $\delta_0 = 21$ mm.

specific energy distribution on the grinding wheel is not identical for two successive wheel rotations. In other words, the specific energy waves do not remain stationary, but travel around the circumference of the grinding wheel. This phenomenon is well known in the literature as 'wheel lobe precession' [16,32,33].

7. Grinding experiments

In this section the predicted chatter stability is experimentally validated. The following subsections summarise the impact testing results of the grinding wheel, and discuss the equipment, the methodology, and the signal processing used to capture the cutting forces and assess the stability of each experiment.

7.1. Impact testing

The grinding wheel fixed in the tool holding mechanism was impact tested according to the setup presented in Fig. 10, leading to the frequency response function shown in Fig. 11. One dominant mode of vibration is clearly visible, and its modal parameters (natural frequency (f_n), damping ratio (ζ), modal stiffness (k), and modal mass (m)) are summarised in Tab. 3.

7.2. Equipment

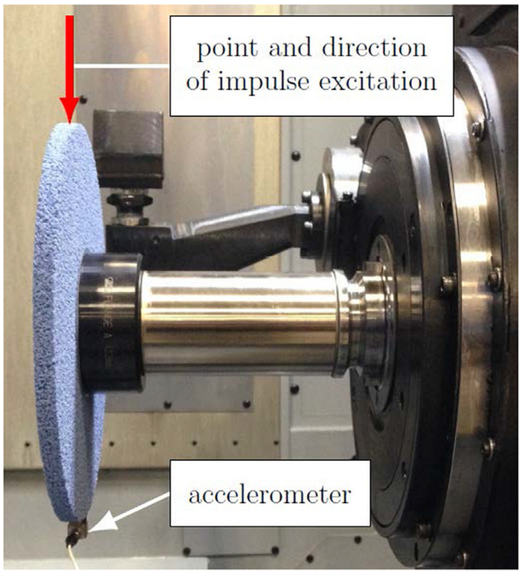
The grinding experiments were performed on a Makino G7 5-axis horizontal machining centre. The grinding force data, was recorded in order to rigorously assess the process stability. This was recorded by a workpiece dynamometer (Kistler Type 9129AA) and corresponding software (DynoWare).

The cutting tool employed in the experiments was a 3M™ Cubitron™ II grinding wheel (specification code: 93DA60/80 F15VPHH901W) with the following physical parameters: outer diameter = 220 mm, inner diameter = 32 mm, and width = 15 mm.

The workpiece material was Custom 465 (stainless steel), and the test pieces were prepared as 100 × 50 × 15 mm blocks to stay within the measurement range of the dynamometer.

The Makino G7 was equipped with RBM's Intelligent Fluid Delivery and Recycling system (IFDR). The programmable coolant nozzle (35 mm wide with an aperture width of 1.5 mm) allowed the machine to make full use of its VIPER grinding capabilities. The application of the coolant was available at different pump pressures (from 30 to 70 bar) through a range of nozzle configurations. The coolant temperature was kept between 18 and 22 °C. For the experiments discussed in this chapter, Master Chemical's TRIM C272 coolant emulsion was used and held at a concentration between 6% and 8%. The filtration of the coolant was integrated into the RBM system itself by means of hydrocyclone technology.

The dressing configuration was unidirectional, i.e., the circumferential speed of the grinding wheel was in the same direction as that of the dressing roll. The wheel was dressed on a flat diamond crush roll (150 mm in diameter, manufactured by Tyrolit) at a dressing roll speed of 3000 rpm, with a dressing speed ratio of 0.8 (roll to wheel), using a dressing feed rate of 0.0012 mm/rev.



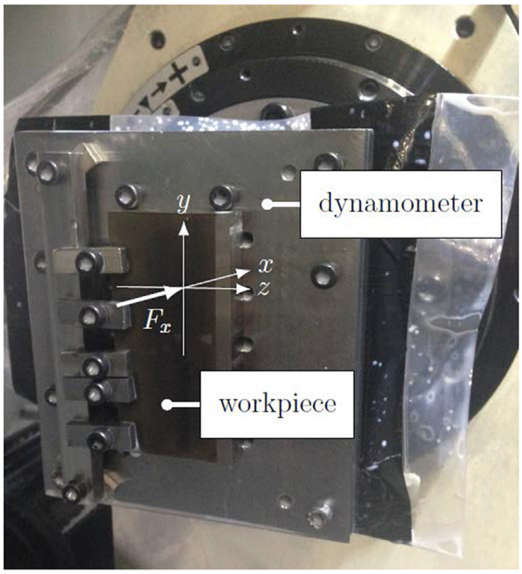


Fig. 10. Impact testing setup (left) and experimental setup for recording the grinding force (right).

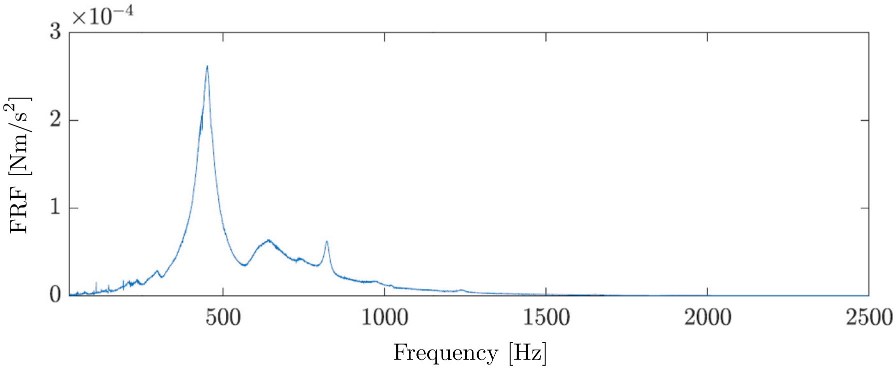


Fig. 11. Frequency response function (FRF) of the impact testing.

Table 3
Modal parameters of the grinding wheel.

f_n [Hz]	ζ [%]	k [N/m]	m [kg]
443.34	4.62	3.60×10^7	4.633

All the equipment used in this work, for both the impact testing and the grinding experiments themselves, is summarised in Tab. 4.

7.3. Methodology

Ideally, a single pass should be taken on the workpiece that is long enough for chatter to become detectable, however, due to the practical limitations of the dynamometer range, multiple passes had to be employed. Nevertheless, in order to approximate the ideal scenario as best as possible, each pass was performed on a fresh workpiece surface. Furthermore, for the purposes of reducing the grinding force and saving more workpiece material, only a third of the total wheel width was used for cutting. These considerations called for a step-like structure to be prepared on the workpiece before testing, in order to avoid the engagement of the passive two thirds of the wheel width with the rest of the test piece. The schematic representation of the prepared workpiece and the experimental setup are shown in Fig. 12.

It can be seen that eight steps were machined onto the workpiece as preparation for the experiments. This was the maximum number of lengthwise steps available, given the total width of the workpiece (50 mm), one third of the wheel width (5 mm), and leaving enough space for clamping (10 mm). Therefore, the maximum number of wheel passes without recutting the same surface twice or swapping the workpiece was also eight.

7.4. Signal processing

This section illustrates an example experimental data set, in order to explain the signal processing steps that were used to diagnose chatter. As mentioned previously, each machining scenario involved multiple passes over a smoothly prepared workpiece, with the cutting force recorded for each case. In the case of an unstable cut, the resulting instability can be observed by forced response at the so-called chatter frequency. To observe this phenomenon, the time-history of the dynamometer signal was first plotted as a waterfall diagram, i.e. a Fast Fourier Transform of each workpiece pass, in order to illustrate the evolution of the force spectra as time progressed. This is illustrated for one scenario in Fig. 13.

Table 4
Summary of all the equipment used in this work.

Function	Equipment
Impact hammer	Kistler 9722A500 + soft PVC tip (Sensitivity: 11.63 mV/N)
Impact sensor	Kistler 8776A50M1 (Sensitivity: 101 mV/g)
Data acquisition system	National Instruments 9234
Impact testing software	CutPro Tap Testing and Modal Analysis Modules
Grinding machine	Makino G7 5-axis horizontal machining centre
Force dynamometer	Kistler Type 9129AA + DynoWare software
Grinding wheel	3M™ Cubitron™ II (specification code: 93DA60/80 F15VPHH901W)
Workpiece material	Custom 465 (stainless steel)

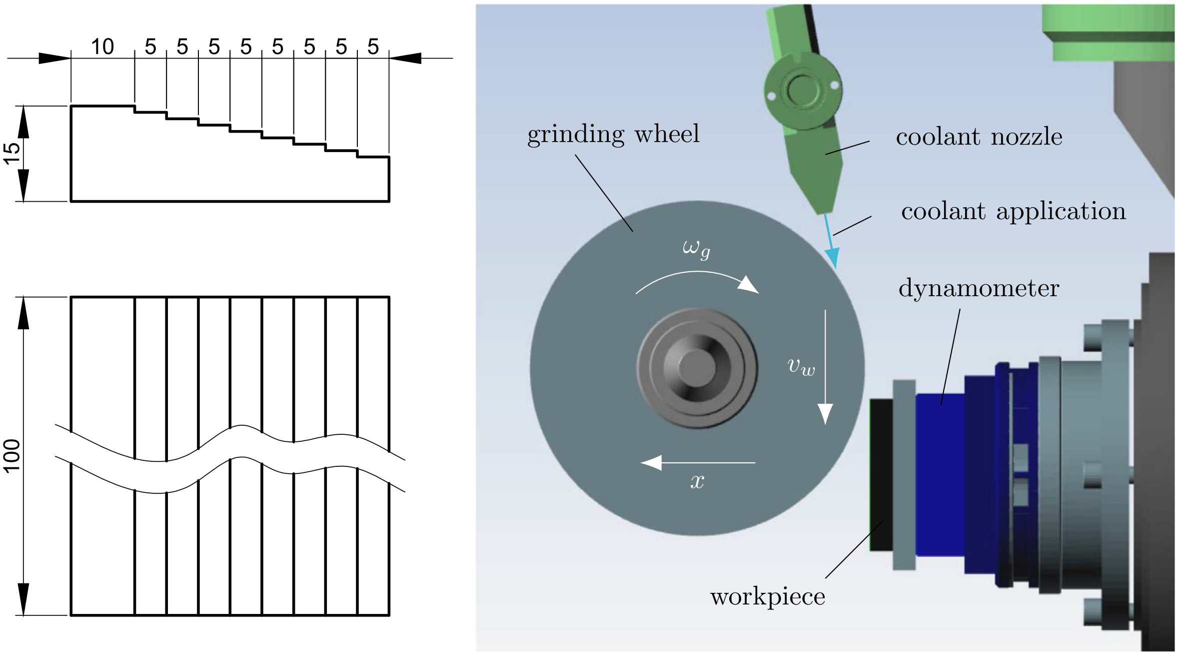


Fig. 12. A schematic representation of the workpiece (left) and the experimental setup (right) in an up-grinding configuration, where the feed motion is performed by the grinding wheel.

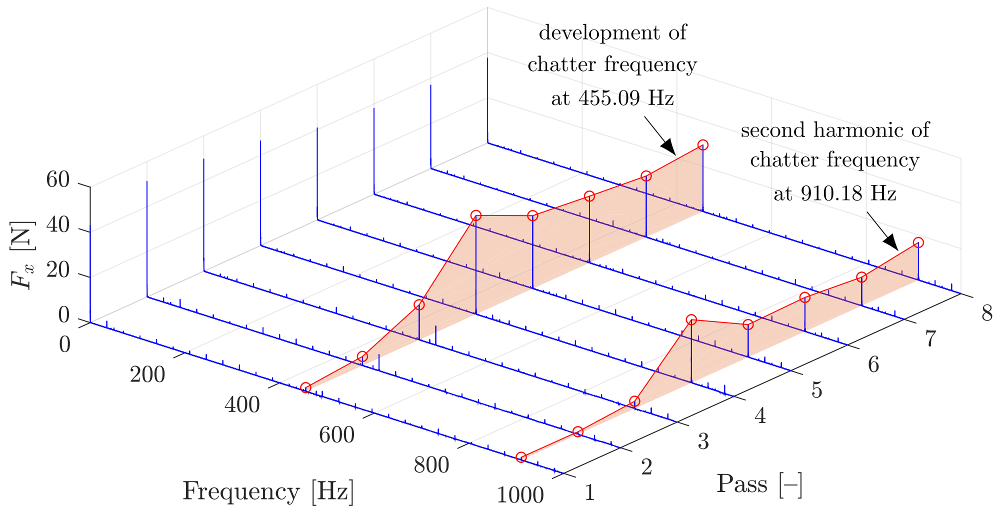


Fig. 13. Evolution of the grinding force in the frequency domain for $v_g = 23$ m/s, $\delta_0 = 0.25$ mm, $R_{g,\text{before}} = 104.492$ mm, $R_{g,\text{after}} = 104.266$ mm, $\omega_g = 2102$ rpm, and $G = 1.349$. The theoretical chatter frequency is 460.07 Hz.

It can be seen that two frequency components stand out of the rest (at 455.09 and 910.18 Hz), the lower of which corresponds to larger grinding force amplitudes than the other, which is the second harmonic of the first. Both of them appear to be growing exponentially before breaking off and settling at an approximately constant value. In order to demonstrate this trend, the evolution of the grinding force amplitude at 455.09 Hz is plotted at a much higher resolution in Fig. 14, as a function of the specific material removed, which is directly related to time or the number of wheel passes. It is clear that an exponential curve can be fitted very accurately to the increasing section of the grinding force, which strongly supports the idea that this frequency component is indeed the chatter frequency of the unstable grinding process.

In order to gain more insight into the time scale of chatter development in the case of wheel-related instability, Fig. 15 presents the time history of the seventh wheel pass in Fig. 13, along with the spectrogram of the same scenario, which provides a visual illustration of the development of the different frequency components in the force signal over time. It can be

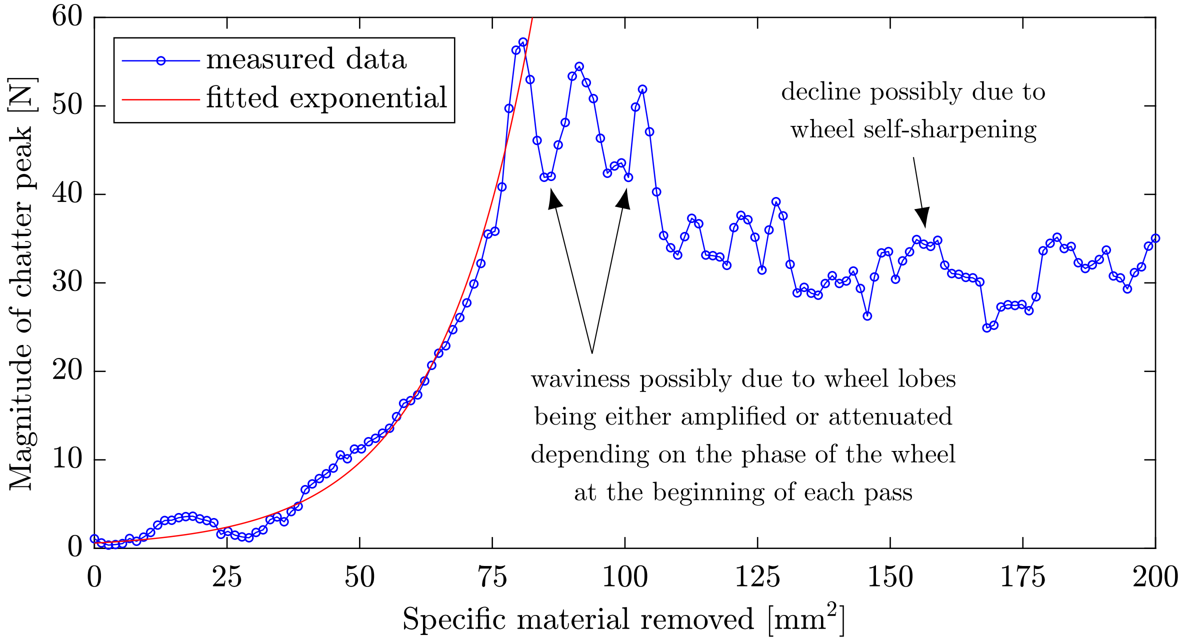


Fig. 14. Variation in the magnitude of the chatter frequency component for $v_g = 23$ m/s, $\delta_0 = 0.25$ mm, $R_{g,\text{before}} = 104.492$ mm, $R_{g,\text{after}} = 104.266$ mm, $\omega_g = 2102$ rpm, and $G = 1.349$.

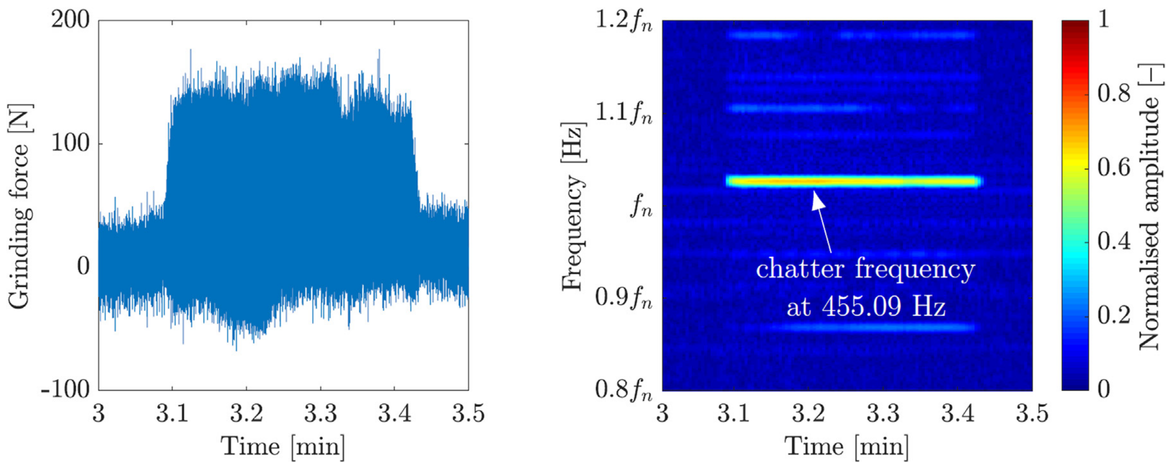


Fig. 15. Time history (left) and spectrogram (right) of the grinding force during the seventh wheel pass in Fig. 13.

seen that the grinding time is in the order of minutes, confirming the well-established view of the literature according to which wheel-related chatter develops much more slowly than workpiece-related instability, or chatter in conventional machining operations such as milling [14].

The stability of each machining scenario was assessed using the approach described above. The chatter vibrations captured by the experiments – although clearly detectable in the force signals – were not visible on the workpiece surfaces. Considering the fact stated earlier in this work that wheel-related vibrations develop very slowly in grinding [14], presumably the level of chatter was still at such an initial stage that chatter marks were not yet visible on the surface of the workpiece. Li and Shin also confirmed that the amount of time necessary for wheel-related chatter to develop is in the order of minutes [15] – as opposed to turning or milling, for instance, where chatter marks on the workpiece surface become visible virtually immediately [4].

8. Results

Figure 16 summarises the stability of each experiment and presents the theoretical boundaries predicted by the new model. Due to the fact that the stability boundaries are rather sensitive to low grinding ratios, which varied quite a lot in the experiments, ideally, every test point should be presented with its own set of stability boundaries, which would be an impractical way of summarising the stability results. Therefore, the authors plot only three sets of stability boundaries corresponding to the minimum, average and maximum grinding ratios, in order to consider the non-negligible variation of this parameter, yet present the results as concisely as possible.

As a result of an unexpected situation that happened during testing, the experiments performed at $\delta_0 = 2.75$ mm were unable to provide reliable information on stability. Due to the large depth of cut, the material removal rate was high, and thermal energy became dominant in the process. In the absence of an effective scrubbing or wheel cleaning solution to manage this factor, the chips were slightly melted and adhered to the surface of the grinding wheel. Such a scenario is highly unfavourable and to be avoided in practice [2], as the resulting surface finish is entirely unacceptable.

Considering the stable and unstable grinding experiments, two lines of transition can be identified, which are represented in Fig. 17 for greater clarity. It can be seen that they mark out a practical stability boundary that is in very good

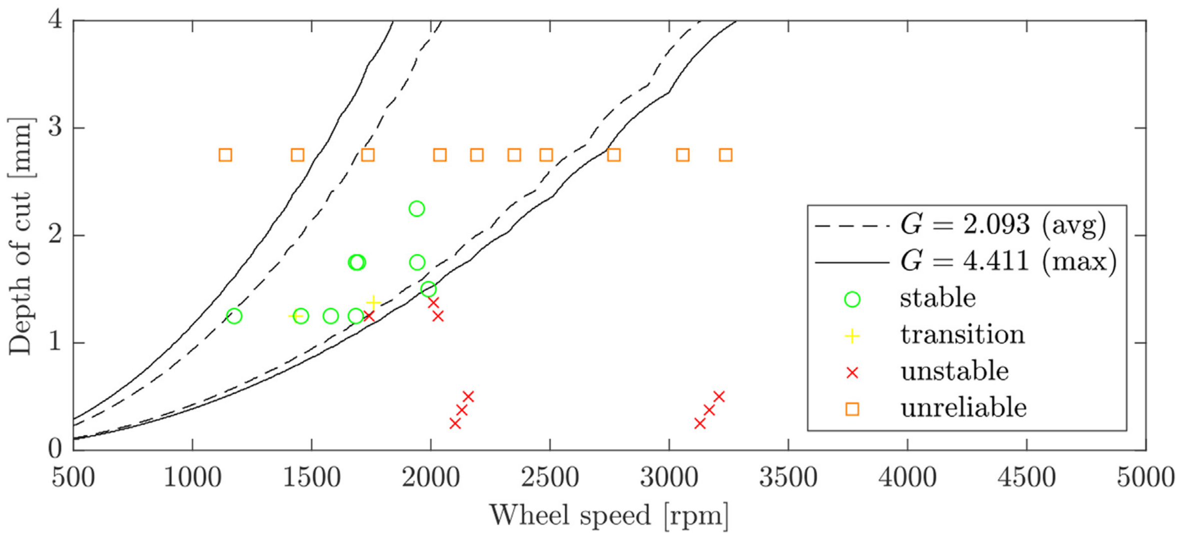


Fig. 16. Theoretical (lines) and experimental (circles) stability properties of all grinding tests. No stability boundary is visible for $G = 0.842$ (min), because the entire parameter region presented above is unstable for this grinding ratio.

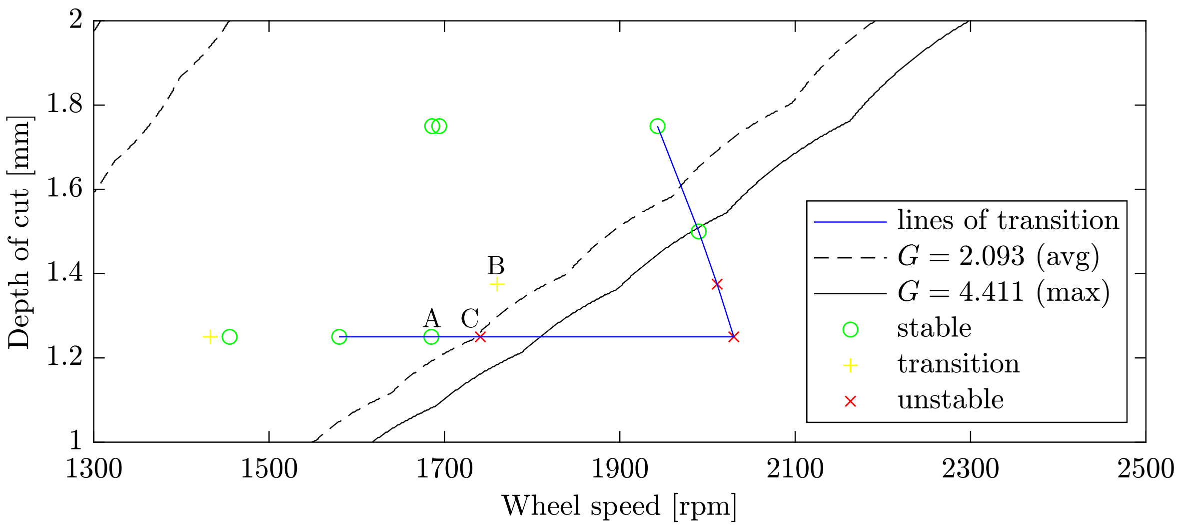


Fig. 17. Two lines of transition between stable and unstable grinding, indicating the existence of a practical stability boundary.

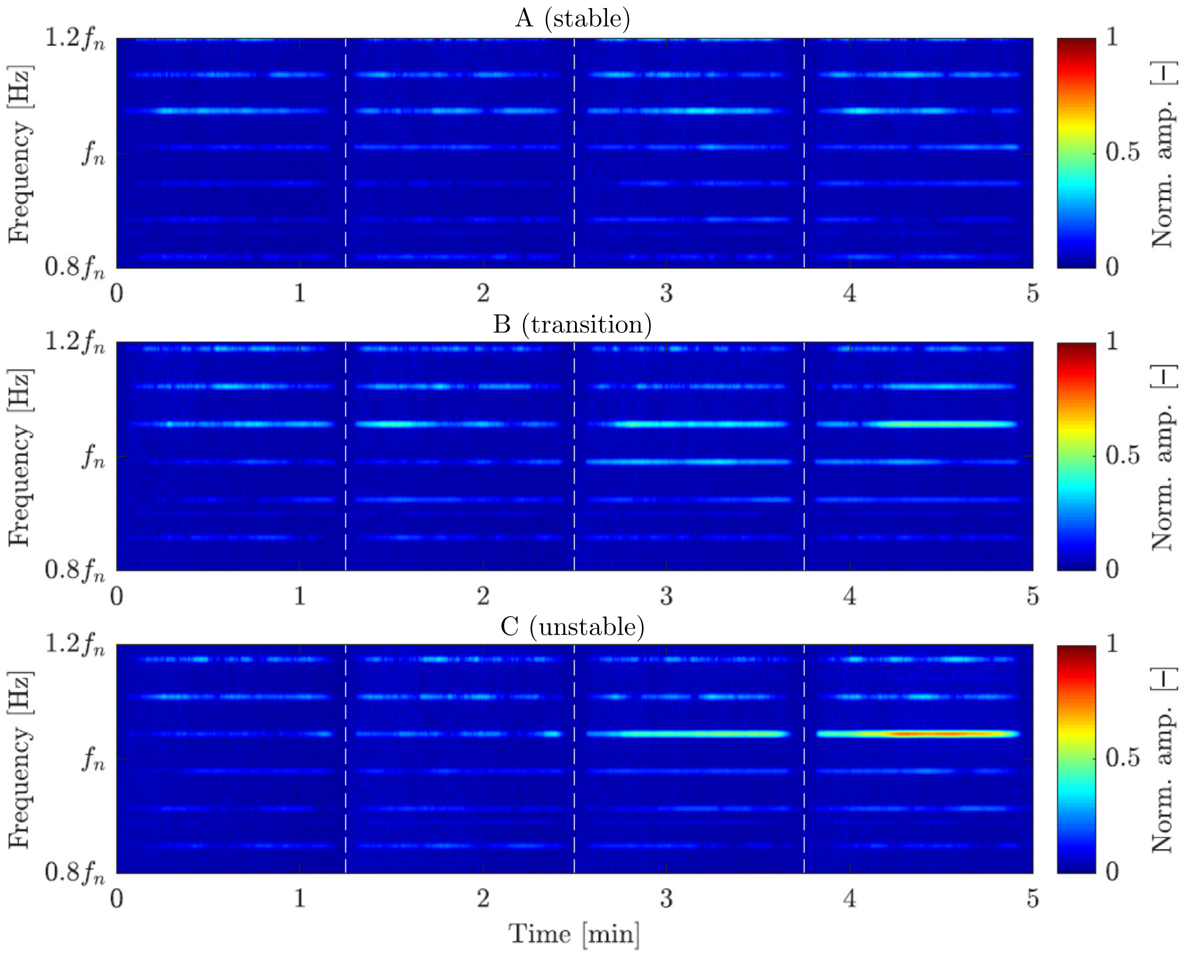


Fig. 18. Spectrograms of stable, transitional and unstable grinding processes from Fig. 17. The three dashed lines separate four wheel passes in each case.

agreement with the theoretical prediction corresponding to the average and maximum grinding ratios. It is important to note that not only the general direction but also the precise location of the practical boundary has been calculated remarkably accurately by the new model.

In order to examine the behaviour of the system across these lines of transition, Fig. 18 is presented, showing the spectrogram for three individual scenarios: one stable (A), one transitional (B), and one unstable (C). These markers are also highlighted in Fig. 17 for clarity and comparison. Apart from the clear differences between these cases in terms of stability, it is noteworthy again how long it takes for chatter to develop, especially when the experiment takes place at a set of grinding parameters close to the stability boundary.

9. Discussion

Given the general consensus of the literature that grinding is typically unstable with respect to wheel regeneration, the discovery of stable parameter spaces is a very significant result. On the one hand, the authors' new theory predicts instability for low depths of cut (pertaining to finishing operations) and therefore reflects conformity to previous chatter models in the literature. On the other hand, the new theory has revealed previously unexplored regions of stable grinding in the high depth of cut domain. This is the main novelty of this work. And since grinding today is no longer restricted to finishing operations only, but is widely used for stock removal as well with very high material removal rates, the practical implications of this new result are very promising indeed. Chatter-free grinding at high depths of cut means increased productivity, reduced cycle times, a lower wheel dressing frequency, and an overall increase in competitiveness for industry.

Furthermore, the new theory suggests that increasing the grinding ratio to that of superabrasives results in stable machining not only for deep cuts, but for finishing operations as well. More work is needed to experimentally validate the stable zones arising at low depths of cut (presented in Fig. 19), as these parameter spaces can be of much benefit to industry. That is because process stability is crucial when it comes to taking shallow cuts in grinding, which are typically

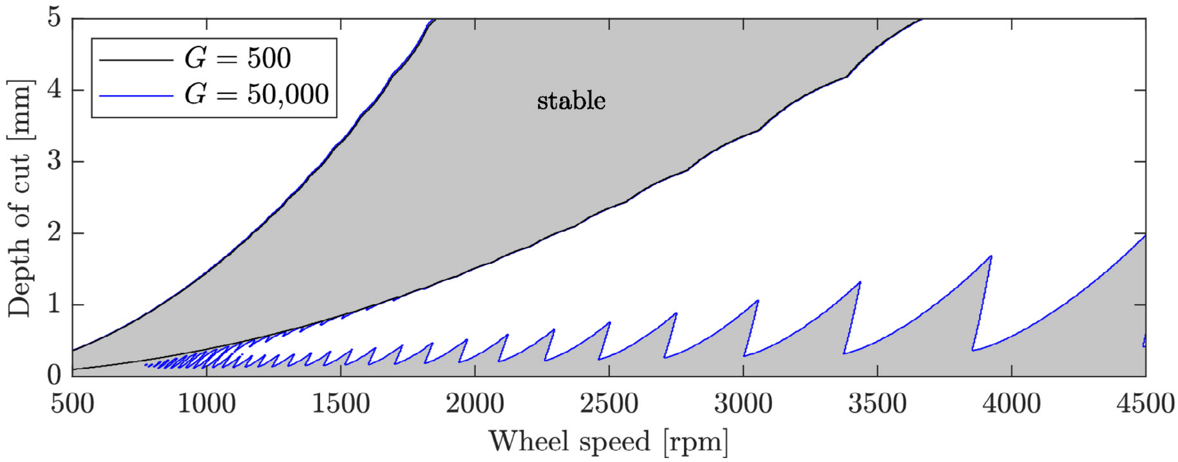


Fig. 19. Stability boundaries corresponding to an extremely high grinding ratio – stable machining is predicted for finishing with superabrasives.

responsible for the final state of the workpiece. Moreover, these stable regions are of interest to academia as well, indicating a closer relationship between superabrasive wheels and conventional cutting tools than previously thought.

It is possible that material removal events introduce a fluctuation in the angular speed of the grinding wheel, and this merits further discussion. In this scenario, the mathematical formulation of the theory changes. Considering Eq. (8), the most important difference compared to the scenario investigated in this paper is that certain parameters (such as T_g and $\tau_{c,0}$ which are the point and distributed delays in the governing equation of motion respectively) become time dependent. And while well-established techniques clearly exist to deal with the stability analysis of such an intricate equation [34], this would be a significantly more complex problem. However, the authors believe that – due to the high specification of the grinding machine used – a practically considerable fluctuation in the angular speed of the wheel is unlikely and therefore negligible in this particular investigation.

The authors ensured that surface regeneration caused by the workpiece was negligible by setting up the experiments in such a way that a fresh workpiece surface was cut at each wheel pass instead of recutting the same surface that had been cut during the previous wheel pass. However, the presence of distributed radial wear on the wheel cannot be excluded, in fact, it was most likely present in the experiments. The value of the proposed theoretical model lies in the fact that it is capable of predicting accurate practical results even by neglecting this effect and considering distributed grit dullness alone. Therefore, it can be concluded that distributed grit dullness has a dominant influence on grinding chatter.

One potential way to continue this research is to compare its results not only with experimental measurements, but also with those of other theoretical models discussed in Section 2. While this type of work is out of the scope of this particular investigation, it would certainly be advantageous in order to further explore the contribution of the current model. In terms of Li and Shin's approach [15], their theory could be used not only to determine the frequency content of force signals corresponding to unstable grinding processes, but also to calculate the actual stability boundaries, which could then be compared with the model presented in this paper. Concerning the classical approach, it is important to reiterate that the general consensus of the literature remains that most grinding processes are unstable in practice with regard to wheel regeneration, which Li and Shin did not explicitly challenge, however, their theory contains such potential. The current model, building on Li and Shin's work, has shown that stable zones do in fact exist in grinding with respect to wheel regeneration, which not only challenges the general consensus of the literature, but also holds great potential with regard to more efficient grinding practice.

10. Conclusion

It can be concluded that grit dullness alone, in the absence of radial wear, can lead to wheel-related instability in single-pass surface grinding. The new chatter theory presented in this paper has been experimentally validated using extensive machining experiments, signal processing and data analysis. This conclusion raises a couple of important issues from the perspective of both industrial users and academic researchers.

First, although the previous literature puts forth grinding as typically unstable with respect to wheel regeneration, this research has shown that stable parameter zones do in fact exist for large depths of cut. This is an unprecedented result not only in terms of academic novelty, but also in terms of industrial impact.

Second, the model provides valuable information on chatter avoidance as well, i.e., which grinding parameters can be changed in order to stabilise an unstable system. This is very important, because depending on the nature of instability, certain grinding parameters have no stabilising effect on the process.

Further work is needed to explore and refine the parameter spaces in which stability occurs, as it could pave the way for enhanced efficiency and productivity in industrial practice. More research is also needed to understand how the predictions of this new theory are influenced by the underpinning modelling assumptions, and the extent to which these assumptions are valid.

CRediT authorship contribution statement

Máté Tóth: Methodology, Investigation, Writing - original draft. **Neil D. Sims:** Conceptualization, Supervision, Writing - review & editing, Funding acquisition. **David Curtis:** Conceptualization, Supervision, Writing - review & editing, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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