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Equivalence principle in Reissner-Nordström geometry

Rafael P. Bernar*, Luís C. B. Crispino*, Haroldo C. D. Lima Junior*

Faculdade de Física, Universidade Federal do Pará, Belém, Pará, 66075-110, Brazil

Atsushi Higuchi*

Department of Mathematics, University of York, Heslington, York, YO10 5DD, United Kingdom

Abstract

The Equivalence Principle is a key element in the development of General Relativity. In one of its formulations, the Equivalence Principle states that a reference frame at rest in a uniform gravitational field is equivalent to a reference frame in uniformly accelerated motion in the absence of any gravitation field. We analyze the spacetime surrounding a non-rotating spherically symmetric charged body, known as Reissner-Nordström geometry, and exhibit a coordinate transformation, which makes explicit its compatibility with the Equivalence Principle. We revisit the Schwarzschild case, previously analyzed in the literature. We also consider second order terms of the relevant expansion parameters in the approximate metric, which is needed for the computed curvature quantities to be correct at zeroth order.

Keywords: equivalence principle, Schwarzschild spacetime, Reissner-Nordström spacetime

1. Introduction

The Equivalence Principle (EP) is one of the most fundamental pieces in the groundwork of General Relativity (GR), as well as in other theories of the

*Corresponding Author

Email addresses: rbernar@ufpa.br (Rafael P. Bernar), crispino@ufpa.br (Luís C. B. Crispino), haroldolima@ufpa.br (Haroldo C. D. Lima Junior), atsushi.higuchi@york.ac.uk (Atsushi Higuchi)

gravitational interaction. In its weak form, the EP states the universality of free fall [1]. The fact that test particles[1] moving only under the action of gravity fall independently of their internal properties is well tested experimentally [2, 3, 4] and this can be quantified, in a Newtonian approximation, by the equality of their inertial and passive gravitational masses, a fact which was empirically verified by Isaac Newton [5]. The most recent experimental results [6] establish an upper bound of the order of 10^{-14} on the relative acceleration between bodies of different compositions, subjected to the same gravitational field. Theoretically, the role of the EP in the construction of a gravitational theory cannot be ignored, as it was crucial to the birth of GR, as well as being a possible selection criterion for other gravitational theories [7]. In fact, the hint at universality of the gravitational interaction was one of the basis for Albert Einstein to consider gravity as a theory for the geometry of spacetime.

Despite the EP being very important for GR, its manifestation is usually not explicit. An important solution of Einstein's equations is the Schwarzschild geometry, which can describe the spacetime outside a spherically symmetric nonrotating body, with zero total charge[2] [8]. This solution can be shown to explicitly obey the EP, as displayed in Ref. [9], using appropriate approximations. Another important solution of Einstein's equations is the Reissner-Nordström (RN) geometry [8], which is the spacetime outside a charged non-rotating spherically symmetric body. This solution can also describe a static charged black hole. Even if it seems unlikely for any relevant amount of net electric charge acquired by a black hole not to be lost [10], RN geometry exhibits interesting properties to be investigated from the theoretical point of view. These properties lead to non-trivial consequences, for instance, in the study of absorption [11, 12, 13, 14, 15, 16, 17, 18], scattering [19, 20, 21] and radiation

[1]By test particles we mean particles whose effect on the background gravitational field is negligible.

[2]The Schwarzschild solution can also describe the spacetime around a static black hole with zero electric charge.

emission processes [22]. The RN spacetime, although not as simple as the Schwarzschild one, also allows a slightly wider analysis in regard to the EP, especially due to the additional complications of the nonzero electric charge, both of the source of gravitation as well as of the test particle. In this paper, we show that the RN geometry provides another example of the EP's manifestation in GR. We also note some minor differences between the RN and the simpler Schwarzschild cases.

There are many different formulations of the EP [23, 24]. Thus, one has to be careful to which version of the EP one is referring, since they may not be equivalent [25]. Here we are going to consider the EP in the same form as Einstein did [26], which can be formulated in a modern language in the following manner: an uniformly accelerated reference frame in flat spacetime is locally equivalent to a reference frame at rest in a uniform gravitational field.

Since there is no such thing as a perfectly uniform gravitational field in nature [27, 28], this version is an idealization, and therefore can be tested, at best, only in an approximate manner. This version of the EP, as an heuristic tool, works better for our purposes of showing its implementation in representative spacetimes of GR, such as Schwarzschild and RN spacetimes.

In this work on the RN geometry, the origin of the local reference frame will be placed at a coordinate distance R from the center of the spherically symmetric, charged and non-rotating body. Performing a coordinate transformation on the RN line element to Cartesian coordinates and making certain assumptions, we can show that the resulting expression for this geometry, after the transformation to these displaced Cartesian coordinates, can be approximated by the line element of flat spacetime in a uniformly accelerated reference frame.

The rest of this paper is organized as follows. In Sec. 2 we present the Reissner-Nordström geometry and write its line element in a displaced Cartesian coordinate system, which, within certain approximations, represents a local frame at rest, placed at a coordinate distance R from the spherically symmetric body. In Sec. 3 the reference frame in uniformly accelerated motion in flat spacetime is presented and compared with the approximate form of the line element of the RN geometry near the origin of the displaced Cartesian system. In Sec. 4 we show that the Riemann tensor and other curvature-related quantities are nonvanishing in the approximate RN geometry. We consider second order terms in the approximate metric in order for the computed curvature quantities to have meaningful values. In Sec. 5 we compute the local form of the electric field so that it is constant in our approximate description and show that it has the expected form. We discuss our results and end with some concluding remarks in Sec. 6. In the Appendix Appendix A, we exhibit the form of the approximate metric components in the second order expansion. Throughout this paper we adopt metric signature $(-,+,+,+)$, the electric charge is expressed in Gaussian units, and we set the speed of light to unity ($c=1$).

2. Reissner-Nordström Geometry

The Reissner-Nordström geometry is described by the following line element:

$$\mathrm{d}s^2 = -\Delta(r)\,\mathrm{d}t^2 + \Delta(r)^{-1}\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right), \tag{1}$$

where

$$\Delta(r) \equiv 1 - \frac{2\,G\,M}{r} + \frac{G\,Q^2}{r^2} \tag{2}$$

and G is the universal gravitational constant. This is the spacetime outside a spherically symmetric body of total mass M and total electric charge Q. Since the total charge is nonzero, there is a spherically symmetric electric field present in this spacetime. We apply a coordinate transformation in Eq. (1) to write the line element in displaced Cartesian coordinates (x,y,z), with the origin of this new system located at radial distance R, in the old system (r,θ,ϕ), from the source of the gravitational field. We can write the new coordinates (x,y,z) in terms of (r,θ,ϕ) as follows:

$$x = r\,\sin\theta\,\cos\phi, \tag{3}$$

$$y = r\,\sin\theta\,\sin\phi, \tag{4}$$

$$z = r\,\cos\theta - R. \tag{5}$$

Solving Eqs. (3)-(5) for (r, θ, ϕ), we obtain

$$
\begin{aligned}
r(x,y,z) &= R\left(1+\frac{x^2+y^2+z^2}{R^2}+\frac{2\,z}{R}\right)^{\frac{1}{2}}, & (6)\\
\theta(x,y,z) &= \arctan\left(\frac{\sqrt{x^2+y^2}}{z+R}\right), & (7)\\
\phi(x,y,z) &= \arctan\left(\frac{y}{x}\right). & (8)
\end{aligned}
$$

We can use Eq. (6), and the fact that

$$
r^2(\mathrm{d}\theta^2+\sin^2\theta\ \mathrm{d}\phi^2)=\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2-\mathrm{d}r^2, \tag{9}
$$

to rewrite the line element (1) in displaced Cartesian coordinates, namely

$$
\begin{aligned}
\mathrm{d}s^2 &= -\Delta(r)\mathrm{d}t^2+\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2+\left[\Delta(r)^{-1}-1\right]\left(\frac{R}{r}\right)^2\\
&\times\left[\frac{x^2}{R^2}\mathrm{d}x^2+2\frac{xy}{R^2}\mathrm{d}x\,\mathrm{d}y+\frac{y^2}{R^2}\mathrm{d}y^2+2\frac{x}{R}\left(\frac{z}{R}+1\right)\mathrm{d}x\,\mathrm{d}z\right.\\
&\left.+2\frac{y}{R}\left(\frac{z}{R}+1\right)\mathrm{d}y\,\mathrm{d}z+\left(\frac{z}{R}+1\right)^2\mathrm{d}z^2\right],
\end{aligned} \tag{10}
$$

where, in Eq. (10), r should be seen as the function $r(x,y,z)$ given by Eq. (6). We shall assume that $|\frac{x}{R}| \ll 1$; $|\frac{y}{R}| \ll 1$; $|\frac{z}{R}| \ll 1$. Expanding Eq. (10) and retaining only first order terms in x/R, y/R and z/R, we obtain

$$
\begin{aligned}
\mathrm{d}s^2 &= -f(z)\mathrm{d}t^2+\frac{\mathrm{d}z^2}{f(z)}+\mathrm{d}x^2+\mathrm{d}y^2\\
&+\frac{1}{f(z)}\left[\frac{4\,G\,M}{R}-\frac{2G\,Q^2}{R^2}\right]\left(\frac{x}{R}\mathrm{d}x\,\mathrm{d}z+\frac{y}{R}\mathrm{d}y\,\mathrm{d}z\right),
\end{aligned} \tag{11}
$$

in which

$$
f(z) \equiv \left[1-\frac{2\,G\,M}{R}+\frac{G\,Q^2}{R^2}+\left(\frac{2\,G\,M}{R^2}-\frac{2\,G\,Q^2}{R^3}\right)z\right]. \tag{12}
$$

Note that we do not impose any restriction on $\frac{M}{R}$ or $\frac{Q}{R}$, so that no weak field approximation is being assumed. In the next section we will show that part of the line element given by Eq. (11) is related to a uniformly accelerated frame in flat spacetime.

3. Line Element of a Uniformly Accelerated Reference Frame

We begin with the following ansatz for the metric of a uniformly accelerated reference frame [9, 23]:

$$\mathrm{d}s^2_{\text{accel.}} = -\alpha(z)^2\,\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \beta(z)^2\,\mathrm{d}z^2. \tag{13}$$

In order to establish the relation between the line element above and the line element of the Reissner-Nordström geometry in Eq. (10), let us investigate the geodesic equation in both spacetimes. Following Ref. [9], we would like for a free (chargeless) test particle in the uniformly accelerated frame in flat spacetime to have the same acceleration as a (chargeless) test particle freely falling in radial motion in the RN spacetime at the position given by R. We note here that the word "free" implies that the particle is neutral (zero electric charge). We recall that, since we do have a nonzero electric field present in Reissner-Nordström spacetime, a uniformly accelerated frame in flat spacetime without an electric field is not actually equivalent (even locally), for obvious reasons, to the approximately uniform gravitational field in the vicinity of the location with radius R of the Reissner-Nordström spacetime.

However, if there is a uniform electric field in the flat spacetime in such a way as to maintain the (local) equivalence between the electric fields in both situations, the comparison of the behavior of charged test particles in both frames would pose no additional challenge from the conceptual point of view, albeit not necessarily so from the computational point of view.

Assuming that the particle is only moving along the z direction, we can use the geodesic equation in the spacetime described by Eq. (13) to obtain:

$$\frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2} + \Gamma^3_{\rho\,\nu}\frac{\mathrm{d}x^\rho}{\mathrm{d}\tau}\frac{\mathrm{d}x^\nu}{\mathrm{d}\tau} = 0, \tag{14}$$

where τ is the proper time along the timelike geodesics. The condition for the trajectory to be timelike is given by

$$g_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\frac{\mathrm{d}x^\nu}{\mathrm{d}\tau} = -1. \tag{15}$$

We can then use Eqs. (14) and (15) to obtain

$$\frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2} + \left[\frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}z} + \frac{1}{\beta}\frac{\mathrm{d}\beta}{\mathrm{d}z}\right]\left(\frac{\mathrm{d}z}{\mathrm{d}\tau}\right)^2 + \frac{1}{\beta^2\,\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}z} = 0. \tag{16}$$

We can also consider the radial geodesics in the Reissner-Nordström geometry, for which the equation of motion can be written as [29]

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 + \left(1 - \frac{2\,G\,M}{r} + \frac{G\,Q^2}{r^2}\right) = \kappa^2, \tag{17}$$

where κ is a constant. Assuming that the radial movement occurs in the z-axis and differentiating Eq. (17) with respect to τ, we obtain

$$\left.\frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2}\right|_{r=R} + \left(\frac{G\,M}{R^2} - \frac{G\,Q^2}{R^3}\right) = 0. \tag{18}$$

The comparison between Eqs. (16) and (18), imposing that both particles have the same proper acceleration, allows us to obtain, for the functions α and β, the following

$$\alpha^2 = \frac{1}{\beta^2} = \left(\frac{2\,G\,M}{R^2} - \frac{2\,G\,Q^2}{R^3}\right) z + K, \tag{19}$$

where K is an integration constant. Substituting the values of α and β in the line element (13), we get

$$\begin{aligned} \mathrm{d}s^2_{\text{accel.}} = & -\left[K + \left(\frac{2\,G\,M}{R^2} - \frac{2\,G\,Q^2}{R^3}\right) z\right]\mathrm{d}t^2 + \mathrm{d}x^2 \\ & + \mathrm{d}y^2 + \left[K + \left(\frac{2\,G\,M}{R^2} - \frac{2\,G\,Q^2}{R^3}\right) z\right]^{-1}\mathrm{d}z^2. \end{aligned} \tag{20}$$

We can choose the following value for integration constant:

$$K = 1 - \frac{2\,G\,M}{R} + \frac{G\,Q^2}{R^2}. \tag{21}$$

This choice implies that the coordinate time in the accelerated frame is the same as the one in line element (1) of the Reissner-Nordström spacetime. Thus, the line element (20) can be rewritten as

$$\begin{aligned} \mathrm{d}s^2_{\text{accel.}} = & -\left[1 - \frac{2\,G\,M}{R} + \frac{G\,Q^2}{R^2} + \left(\frac{2\,G\,M}{R^2} - \frac{2\,G\,Q^2}{R^3}\right) z\right]\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 \\ & + \left[1 - \frac{2\,G\,M}{R} + \frac{G\,Q^2}{R^2} + \left(\frac{2\,G\,M}{R^2} - \frac{2\,G\,Q^2}{R^3}\right) z\right]^{-1}\mathrm{d}z^2. \end{aligned} \tag{22}$$

Eq. (22) differs from Eq. (11) due to the off-diagonal terms. The Riemann tensor obtained from the line element given by Eq. (22) is zero, since this line element is associated to a flat spacetime, written in an unusual coordinate system. The Reissner-Nordström geometry has nonzero curvature, even when written in the approximate form given by Eq. (11). The off-diagonal terms are the only linear terms in the metric that contribute to the curvature quantities. However, the off-diagonal terms in the line element (11) do not tell the whole story. By considering the terms only up to linear order as in Eq. (11), we do not have enough information to compute even the zeroth order terms in curvature quantities (such as the Riemann tensor components), since they all depend on second derivatives of the metric. Nevertheless, we note that the off-diagonal terms in this case are related to the geodesic deviation equation and hence to tidal effects – we can think of displacements in the x and y directions being associated with the components of the separation vector between two neighboring geodesics in the congruence directed towards the source of gravity. (See [30] for a study of the geodesic deviation equation and tidal forces in Reissner-Nordström spacetime). Moreover, although we could try to eliminate the linear terms using an appropriate coordinate system, hence eliminate the off-diagonal terms as well, the transformation equations would need to contain second order terms in x/R, y/R and z/R [31]. However, we should emphasize that only in a second or higher order approximation we can truly analyze curvature-related quantities. In summary, although the off-diagonal terms are related to tidal effects, they cannot be considered as representing nonzero curvature. We are considering motion only in the z-direction, to which the off-diagonal terms do not contribute. Hence the RN spacetime is indeed locally equivalent to a uniformly accelerated frame, provided the word "locally" means a spacetime region defined within the approximations made in this section, that is motion occurring only in the z-direction and that only linear terms in x/R, y/R and z/R are considered in the metric.

If we choose the case where the total electric charge is zero, that is $Q = 0$, then equations (11) and (22) become, respectively,

$$\mathrm{d}s^2 = -g(z)\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \frac{1}{g(z)}\mathrm{d}z^2 + \frac{4GM}{R}\frac{1}{g(z)}\left(\frac{x}{R}\mathrm{d}x\mathrm{d}z + \frac{y}{R}\mathrm{d}y\mathrm{d}z\right) \quad (23)$$

and

$$\mathrm{d}s^2_{\text{accel.}} = -g(z)\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \frac{1}{g(z)}\,\mathrm{d}z^2, \quad (24)$$

in which

$$g(z) \equiv 1 - \frac{2\,G\,M}{R} + \frac{2\,G\,M}{R^2}z. \quad (25)$$

Eq. (23) is equivalent to Eq. (24) except for the off-diagonal terms. This result agrees with the one in Ref. [9].

When $Q \neq 0$, a caveat is in order due to the presence of the electric field. As we noted earlier, to claim local equivalency between the local frame in RN spacetime and the uniformly accelerated local frame in flat spacetime, it is necessary to endow the local frame in flat spacetime with a uniform electric field. Otherwise we could, in principle, distinguish between these frames, e.g. by investigating the behavior of point particles with different values of electric charge.

4. Curvature

As we have seen, the line element in the vicinity of the origin of the displaced Cartesian system in the Reissner-Nordström geometry (located at the position with radius R), given by Eq. (11), is equivalent to the accelerated reference frame in flat spacetime, as written in Eq. (22), except for the off-diagonal terms. Although these terms contribute to the Riemann tensor and thus are linked to tidal effects, the zeroth order terms of the Riemann tensor are only computed correctly if second order terms in x/R, y/R and z/R of the metric components are included in the calculation. In the case of the Schwarzschild geometry, the Riemann component $R^0{}_{101}$ was computed using the linear approximation in Ref. [9]. Since they had not considered second order terms, their result for the Riemann component is not correct. The correct results for the Riemann tensor components were computed in Ref. [32]. We also present them here for completeness. We compute some curvature quantities using the second order approximation, that is, considering the expansion of (10) up to second order in x/R, y/R and z/R. The metric components in the second order approximation are presented in Appendix Appendix A.

We calculate, as an example, the zeroth order component $R^{0}{}_{1\,0\,1}$ of the Riemann tensor associated with the line element in Eq. (10), computed up to second order in x/R, y/R and z/R, which is given by[3]

$$R^{0}{}_{1\,0\,1} = G\frac{(MR - Q^2)}{R^4}. \tag{27}$$

We also compute the Kretschmann scalar of this approximate geometry. We obtain

$$K = \frac{8G^2(7Q^4 - 12MQ^2R + 6M^2R^2)}{R^8}. \tag{28}$$

Eqs. (27) and (28) agree with the well-known results. The zeroth order expansion of the Ricci scalar vanishes, as expected for a electro-vacuum solution of Einstein's equation. When computed in the first order approximation given by Eq. (11), as we mentioned, the zeroth order terms of quantities related to curvature are not correct. This is because there is curvature information coming from the second order coefficients in the metric. With the linear approximation for the metric components, the resulting Ricci scalar, for example, is nonvanishing.

The non-vanishing of the curvature-related tensors in Eqs. (27) and (28) confirms a result found in Ref. [24], that is, that there are still local effects due to the gravitational field. In the cases presented here, they explicitly appear in the approximated metric to second order, due to our particular choice of coordinates. We can always eliminate the linear terms from the metric by a suitable choice of coordinates, which physically means passing to the freely falling local frame. However, the second order terms are needed to retain some

[3] By setting $Q = 0$ in Eq.(27), we obtain

$$R^{0}{}_{1\,0\,1} = \frac{GM}{R^3}, \tag{26}$$

which differs from Eq. (17) of Ref. [9].

information about the curvature quantities. This is the case, for example, of Riemann normal coordinates, where the coefficients of the second order terms are the zeroth order components of the Riemann tensor [33]. Tidal effects are linked to second derivatives of the metric which cannot, in general, be arranged[4] to vanish locally, as is well known [31].

5. Electromagnetic Tensor in a Uniformly Accelerated Reference Frame

Let us now compute the electromagnetic tensor at the local reference frame in the RN spacetime and analyze the physical content of this tensor, as seen by this local observer.

The electromagnetic tensor generated by the gravitational source in RN geometry, in the (t, r, θ, ϕ) coordinate system, takes the form given by

$$F_{\mu\nu} = \begin{bmatrix} 0 & -\frac{Q}{r^2} & 0 & 0 \\ \frac{Q}{r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{29}$$

where we are using Gaussian units. In order to calculate the form of $F_{\mu\nu}$ in the displaced Cartesian coordinates (x, y, z), with the origin of the system located at the radial distance R, we use the coordinates transformation given by Eqs. (6)-(8).

Since $F_{\mu\nu}$ is a second rank (anti-symmetric) tensor, the transformation of this quantity is given by

$$F'_{\mu\,\nu} = \frac{\partial x^{\gamma}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} F_{\gamma\,\beta}. \tag{30}$$

Using Eqs. (6)-(8) to obtain the partial derivatives, substituting in Eq. (30), and using the approximation $|\frac{x}{R}| \ll 1$; $|\frac{y}{R}| \ll 1$; $|\frac{z}{R}| \ll 1$, we can compute the

[4]The word "arranged" means that we are making a suitable choice of coordinates or, equivalently, choosing a suitable local reference frame.

linear order approximation of the electromagnetic tensor, namely

$$F'_{\mu\nu} = \begin{bmatrix} 0 & -\frac{Qx}{R^3} & -\frac{Qy}{R^3} & -\frac{Q}{R^2} + \frac{2Qz}{R^3} \\ \frac{Qx}{R^3} & 0 & 0 & 0 \\ \frac{Qy}{R^3} & 0 & 0 & 0 \\ \frac{Q}{R^2} - \frac{2Qz}{R^3} & 0 & 0 & 0 \end{bmatrix}. \qquad (31)$$

We note that, at linear order, the electromagnetic tensor does not lead to a uniform electric field in the z direction. However, at zeroth order,the x and y components of the electric field vanish, thus the electric field associated to the electromagnetic tensor is a constant vector in the z direction, as expected. We note that using the zeroth order expansion to describe the electric field is consistent with the linear expansion for the metric, since they both lead to constant forces in this approximate description[5] (see Eq. (18), for example).

6. Discussion and Conclusion

We have investigated the manifestation of the Equivalence Principle in the Reissner-Nordström (RN) spacetime. We have found that the line element of the RN spacetime can be put in a form which explictly manifests one of the formulations of the Equivalence Principle, specifically, that a frame at rest in a uniform gravitational field is equivalent to a uniformly accelerated frame in flat spacetime. Using a specific coordinate transformation, we were able to show that around the origin of the new coordinate system, which can be thought as a small laboratory, the RN spacetime is equivalent to a uniformly accelerated reference frame in a flat spacetime, except for the off-diagonal terms, and provided that the uniformly accelerated frame is endowed with a uniform electric field

[5]The zeroth order approximation to the electromagnetic tensor is equivalent to the linear order approximation to the electromagnetic four-potential, a quantity that plays, in electrodynamics, the same role as the metric in GR.

equivalent to the electric field of the charged body generating the gravitational field. This result presents a manifestation of the Equivalence Principle, and these off-diagonal terms are related to tidal effects. Thus, because tidal effects are the only local gravitational effects in this case, these frames are equivalent when considering local experiments not sensitive to tidal forces [24, 34]. The main difference between the Schwarzschild and Reissner-Nordström geometries, regarding the Equivalence Principle, is the electric field present in the charged case. In order to have full equivalency between the frame at rest in the Reissner-Nordström geometry and the uniformly accelerated frame in flat spacetime, a uniform electric field is required to be present in the flat spacetime frame. Only in this way we can claim the validity of a stronger version of the Equivalence Principle, that is, all types of local experiments involving test fields and particles, not sensitive to tidal effects, including electromagnetic ones, will have the same outcomes in both settings.

Since the Einstein-Maxwell theory is based on the equivalence principle, it is expected that the RN solution also obeys this principle, although an explicitly demonstration was not given in the literature before the present paper. It is important to state that this will not always be the case for a general theory of gravity. For instance, particular solutions of alternative theories of gravity may violate the equivalence principle (See Ref. [35] where a violation of the equivalence principle is provided in $f(R)$ theories). As the equivalence principle is very well-tested in its weak form, it puts constraints on the viability of these particular solutions, even in approximated form, as astrophysical models.

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Appendix A. Metric components in the second order approximation

In this Appendix, we present the metric components in the second order approximation. We also consider them in a general spherically symmetric spacetime, defined by the following line element:

$$\mathrm{d}s^2 = -F(r)\mathrm{d}t^2 + G(r)\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right), \tag{A.1}$$

in which the RN case can be recovered by writing $F(r) = G(r)^{-1} = \Delta(r)$.

Rewriting the line element given by Eq. (A.1) in displaced Cartesian coordinates, which are given by Eqs. (3)-(5), and expanding the result to second order in x/R, y/R and z/R, the metric components are then given by

$$g_{00} = -F(R) - F'(R)z - \frac{F'(R)}{2R}(x^2+y^2) - \frac{F''(R)}{2}z^2, \tag{A.2}$$

$$g_{11} = 1 + \frac{G(R)-1}{R^2}x^2, \tag{A.3}$$

$$g_{22} = 1 + \frac{G(R)-1}{R^2}y^2, \tag{A.4}$$

$$g_{33} = G(R) + G'(R)z + \frac{H(R)}{2R^2}(x^2+y^2) + \frac{G''(R)}{2}z^2, \tag{A.5}$$

$$g_{12} = \frac{G(R)-1}{R^2}xy, \tag{A.6}$$

$$g_{13} = \frac{G(R)-1}{R}x + \frac{1-G(R)+RG'(R)}{R^2}xz, \tag{A.7}$$

$$g_{23} = \frac{G(R) - 1}{R} y + \frac{1 - G(R) + RG'(R)}{R^2} yz, \tag{A.8}$$

where

$$H(R) = RG'(R) - 2G(R) + 2. \tag{A.9}$$

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