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Chen, Yijuan and Sivey, Peter orcid.org/0000-0002-3703-615X (2021) Hospital report cards : Quality competition and patient selection. Journal of health economics. 102484. ISSN 0167-6296

https://doi.org/10.1016/j.jhealeco.2021.102484

Hospital report cards: Quality competition and patient selection*

Yijuan Chen† and Peter Sivey‡

May 6, 2021

Abstract

Hospital 'report cards' policies involve governments publishing information about hospital quality. Such policies often aim to improve hospital quality by stimulating competition between hospitals. Previous empirical literature lacks a comprehensive theoretical framework for analysing the effects of report cards. We model a report card policy in a market where two hospitals compete for patients on quality under regulated prices. The report card policy improves the accuracy of the quality signal observed by patients. Hospitals may improve their published quality scores by costly quality improvement or by selecting healthier patients to treat. We show that increasing information through report cards always increases quality and only sometimes induces selection. Report cards are more likely to increase patient welfare when quality scores are well risk-adjusted, where the cost of selecting patients is high, and the cost of increasing quality is low.

*Acknowledgements: We thank Jason Potts, Duncan McVicar, Hugh Gravelle, two anonymous reviewers, and seminar participants at RMIT University, University of Melbourne, and the Australasian Health Economics Society for their helpful comments. Any errors remain our own.

†Research School of Economics, Australian National University

‡Centre for Health Economics, University of York

1 Introduction

Recent years have seen a proliferation in government policies to increase consumer information in markets for healthcare. These policies involve the introduction of websites publishing indicators of hospital quality. Examples include the NHS service search website in the UK[1], Medicare Hospital Compare in the USA[2], and the MyHospitals website in Australia.[3]

The market context for these policies is usually characterised by regulated prices and profit seeking (or loss-minimising) hospitals who have an incentive to increase the volume of patients treated. The primary motivation for publishing quality indicators is that they will lead to improvements in hospital quality levels through a competitive process. The quality indicators published on websites are often termed 'report cards'. These report cards are outcome indicators for patients who have recently been treated at the hospital such as (risk-adjusted) mortality rates or re-admission rates.

A critical insight is that hospitals may be able to improve their quality indicators through two alternative channels: firstly by investing in genuine quality improvement, or secondly by engaging in 'selecting' patients who have better outcomes on average. In the second case, the selection has to be on attributes that are unobserved to the risk-adjuster, or the government agency who compiles the indicators. Dranove et al (2003) wrote the seminal study suggesting that hospitals in New York and Pennsylvania engaged in 'risk selection', treating healthier patients on average after the introduction of hospital performance reporting in the form of mortality rates for CABG patients.

The aim of this paper is to develop a model to examine the impact of report card policies on both quality investment and patient selection by hospitals. In the model, there are two hospitals, two types of patients and two time periods. In the first period, each hospital has a monopoly on a quantity of high-severity and low-severity patients and choose how many of each to treat. In the second period, the two hospitals compete on quality for pa-

[1] https://www.nhs.uk/service-search [accessed: 7/12/2018]

[2] https://www.medicare.gov/hospitalcompare/search.html? [accessed: 7/12/2018]

[3] https://www.myhospitals.gov.au/ [accessed: 7/12/2018]

tients who observe quality with an imperfect signal that is influenced by the amount of high-severity patients treated in the first period.[4] An increase in the precision of the imperfect signal represents the introduction of a report card policy. The introduction of report cards increases hospitals' incentives to increase quality and to treat fewer high-severity patients in the first period.

This paper relates to a parallel empirical literature on the effects of hospital report cards (Dranove et al 2003, Cutler et al 2004, Dranove and Sfekas 2008, Pope 2009, Chen and Meinecke 2012, Wang et al 2012). Previous empirical papers have tested for both quality improvement and patient selection effects of report cards, but with little theoretical framework for the analyses. Previous theoretical models have concentrated on the quality improvement effects of report cards (Gravelle and Sivey 2010, Huessman and Mimra 2015, Ma and Mak 2014), or patient selection effects (Chen 2011, Mak 2017). In this paper we provide a framework that allows for both quality improvement and patient selection effects.

An important earlier literature considers the incentives for health care providers to engage in risk selection, cream skimming, or dumping, as a result of imperfectly risk-adjusted reimbursement mechanisms (eg Ma 1994, Ellis 1998). Our model abstracts from these issues by assuming reimbursement is perfectly risk-adjusted and considers risk-selection (or dumping) incentives which arise only from hospital quality report cards.

Probably the most relevant previous studies are Gravelle and Sivey (2010), Huessman and Mimra (2015) and Mak (2017) as they all consider imprecise report cards in a context where patients choose between providers based on quality reports. Gravelle and Sivey show how more precise quality reporting increases quality competition, and therefore equilibrium quality levels, except if hospitals have very different quality-production technologies. In this paper we add the possibility of patient selection to Gravelle and Sivey's model, whilst adopting the reference case in their paper of identical quality production technology. Gravelle and Sivey show that if report cards increase quality, they always increase patient welfare. In contrast, we show that even if report

[4]We will elaborate on the definition and interpretation of the two patient types in the next section.

cards increase quality, patient welfare may fall due to risk selection if there are diminishing marginal returns to higher quality. Huessman and Mimra (2015) focuses on report cards in a multi-dimensional quality setting. In common with Gravelle and Sivey (2010), they model quality improvement/investment but not patient selection. As in other multidimensional models (e.g. Dranove and Satterthwaite 1992), they show how excessive focus on reporting one dimension of quality over others may be welfare-reducing. Mak shows how different forms of risk adjustment in providers' quality reports lead to differing incentives to 'dump' vulnerable patients who negatively affect quality reports. This is similar to the modelling of risk selection in our paper where providers may choose not to treat a proportion of high-severity patients. Our approach is very different from Mak in that we do not consider the multidimensional signalling function of quality and quantity reports. But we add substantially to the policy relevance of his approach by allowing hospitals to endogenously choose their quality level and therefore allow for the quality-improving effects of report cards.

Our model uses a patient choice model with uniformly distributed information signals about hospital quality from Gravelle and Sivey (2010). The model has similarities to the literature on horizontal product differentiation (Hotelling 1929, Salop 1979, Anderson et al 1992). In particular the models of Anderson et al (1992) use a discrete choice model where the error term represents horizontal differentiation. In our model, following Dranove and Satterthwaite (1992), the error term represents imprecise information about hospital quality.

Our results show that increasing patient information about hospital quality (report cards) always increases equilibrium quality levels. Report cards also increase the incentive for hospitals to engage in selection, but selection only becomes positive when information is relatively high. So report cards only sometimes induce or increase the level of patient selection.

Hospital report cards will always improve aggregate patient welfare if we assume a linear relationship between hospital quality and patient utility. Some high-severity patients lose out by not being treated if selection occurs but this effect is outweighed by the increase in quality for patients who are treated. Our welfare analysis shows that report cards increase welfare more strongly when the costs of increasing quality are low, when quality scores are well risk-adjusted, and where the costs of selection are high (eg. where hospitals treatment choices are more rigorously audited for appropriateness).

If there are diminishing returns to quality in patients' utility functions, there may be an 'optimum' level of patient information, beyond which report cards harm aggregate patient welfare as the reduction in the number of patients treated (selection) outweighs the diminishing gains from higher quality. In this case, even if just considering patient welfare, it is not always optimal to increase patient information as much as possible

2 Institutional Context

Our model relies on a particular set of institutional features. The defining feature of our approach is that patients choose hospitals only based on perceived quality, and that they receive a noisy signal about the quality of each hospital.[5] The quality signal represents publicly available data, recommendations from a patient's primary care physician, from previous personal experience or from the experience friends or family. Report card policies can increase the accuracy of this quality signal by increasing the amount of objective data available at the hospital level, for example publishing risk-adjusted mortality or readmission rates, patient satisfaction scores or patient reported outcome measures. We assume that publishing these data via websites, apps or newspapers and magazines reduces the degree of 'noise' in the hospital quality signal that patients observe. Prominent examples include Medicare Hospital Compare in the USA which publishes indicators such as mortality rates, infection/complication rates and patient experience surveys for all hospitals treating Medicare patients. In the UK, NHS Choices publishes mortality rates, patient experience surveys and the results of formal inspections by a government agency for all public hospitals.

There is an ongoing debate regarding the effectiveness of these quality

[5]While other factors, notably location and waiting times, may also be important in patient choice of provider, we abstract from them in this model.

reports and around what information should be included in the report cards. For example, the UK government previously published hospital "star ratings" commencing in 2001 which were subsequently phased out in 2005 (Bevan and Hood 2006) and there is continuing disagreement about the merits of publishing mortality rates at the surgeon-level (Bridgewater et al 2013, Jarral et al 2016).

We assume that hospital quality scores are not perfectly risk-adjusted so that the patients' quality signal about hospitals may be biased by the casemix of patients treated. This implies that treating a group of patients that we define as "high-severity" has a negative effect on a hospital's expected quality score, for example treating patients who are sicker and therefore more likely to die or be re-admitted. This group includes patients who *could* be treated with the procedure for which outcomes are being reported, but which hospitals could justifiably choose not to treat, for example due to the risks of surgery being too high. By choosing to avoid treating some of these patients, a hospital can improve their casemix and avoid the negative effect on the hospital's quality score.

The untreated high-severity patients will not be treated with the procedure for which outcomes are being reported but may be treated with substitute treatments for which outcomes are not reported. For example, for surgical procedures such as coronary artery bypass graft (CABG), there are substitute treatments such as percutaneous coronary intervention (PCI or PTCA) or medical management for which outcomes may not be included in report cards. This example is studied by Dranove et al (2003). The process of choosing not to treat a proportion of the high-severity patients is defined as "risk selection" in the model. This risk selection comes at some cost to the hospital, because, for example, auditing processes may make it progressively more difficult for hospitals to choose to treat fewer high-severity patients.

We assume that hospitals are paid a regulated price per patient treated. This reimbursement mechanism is increasingly common in practice, and is known as 'payment by results' in the UK National Health Service or as 'activity-based funding' in Australia and Canada.

We also assume perfect risk-selection of the reimbursement scheme such that high-severity and low-severity patients are equally profitable. This assumption reflects the focus in our model on risk selection due to report cards, not due to reimbursement differences between patient types. Many DRG schemes account for patient severity in their reimbursement rates (see eg Gaughan and Kobel 2014 for DRG adjustments for coronary artery bypass graft procedures).

Hospitals in our model profit maximise (or loss-minimise). In order for our model to be useful this doesn't have to be the only objective of hospitals in the real world, but even in public health care systems, hospitals usually have an incentive to balance their budget, or even to create a surplus. Our model therefore captures these financial incentives that influence hospital behaviour.

A further assumption is that hospitals are not capacity constrained, and will therefore be able to treat more patients if they can attract them. While it is common for public hospital systems to have waiting times, these tend to be substantial for only some types of treatment, and so for other areas of hospital treatment, it may be reasonable to assume no binding capacity constraint. In addition, Chen et al (2016) show that hospitals have incentives to attract more patients even when capacity constrained in the short-run if they are able to invest in increasing their 'service rate' from one period to another.

3 Model

3.1 Set-up

There are two hospitals, 1 and 2 and two periods, 1 and 2. There are two groups of patients, low-severity and high-severity patients. In each period, there is a total measure $2m$ patients, with $m > 1$. Among them, the measure of high-severity patients is 2, and the rest $(2m - 2)$ are low-severity patients. Suppose in period 1, hospital j receives a measure h_j of high-severity patients, and a measure l_j of low-severity patients. Hospital j can "select" patients to treat: it can choose to treat $h_j - s_j$ of high severity patients, and $l_j - x_j$ of low-severity patients, with $0 \leq s_j \leq h_j$ and $0 \leq x_j \leq l_j$.

In period 2, patients receive a quality signal $\hat{q}_j$ from each hospital and can choose which hospital to attend. This set-up separates the patients who are potentially subject to selection (Period 1) and those who receive a quality signal and choose between the hospitals (Period 2).

A summary of the game is:

- Period 1: Hospitals set quality levels q_1, q_2 and select the number of Period 1 high-severity and low-severity patients to be treated $h_1 - s_1, h_2 - s_2, l_1 - x_1, l_2 - x_2$.
- Period 2: Period 2 patients observe the reported $\hat{q}_j$'s and choose which hospital to attend, hospitals treat patients and make profits.

The hospital demands in period 2 are given by a patient choice process similar to that from Gravelle and Sivey (2010), with the addition of a 'selection' term. In period 2, the patients receive the following signal about hospital quality from hospital j:

$$\hat{q}_j = q_j + \alpha \frac{s_j}{h_j} + \varepsilon_j,\ \varepsilon_j \sim U(-\frac{1}{2v}, \frac{1}{2v})\ j = 1, 2, \tag{1}$$

where $U(-\frac{1}{2v}, \frac{1}{2v})$ is a uniform distribution on the support $(-\frac{1}{2v}, \frac{1}{2v})$.

The patient's utility from receiving the treatment at hospital i is $\beta(q_i)^\gamma$, where $\beta > 0$ measures the degree to which the patient values the hospital's quality, and $\gamma \in (0, 1]$ indicates that the utility function exhibits a standard decreasing marginal utility from treatment quality. The level of 'information' given by the parameter $v \in (0, \infty)$ reduces the variance of the error ε_j in the quality signal patients receive. Throughout, a 'report card' policy will be interpreted as an increase in the parameter v.

Selection, reducing the number of high-severity (s_j) patients treated, affects the expected value of the quality signal according to the parameter $\alpha \in (0, 1)$. A hospital's choice of s_j will be influenced by the effect on demand through the quality signal.

The rationale is that the quality signal $\hat{q}$ includes operation failure rates (examples are mortality rates and readmission rates), and high-severity patients are likely to have higher failure rates than other patients. By assumption, the quality signal given in (1) is not perfectly 'risk adjusted' because it increases with the amount of selection, which affects the mix of patients that are treated.

With perfect risk adjustment we have $\alpha = 0$ and the hospitals would have no incentive to select patients. From this point of view, α can be interpreted as the degree of imperfection in the risk-adjustment. The selection that takes place can be regarded as selection of patients on characteristics that are *unobservable to the risk-adjuster*, but are observed by doctors and administrators in the hospital and do influence the quality score.

For example, by meeting patients face-to-face, doctors can ascertain lifestyle factors and their family history of illness, while the risk-adjuster is unlikely to observe this detailed information in hospital records. Through face-to-face observation, doctors can also observe relevant characteristics such as frailty (e.g. walking speed and grip strength) which are strongly associated with 'failure rates' included in report cards such as mortality rates (Mitnitski et al 2005).

We assume that, due to their lack of medical knowledge and understanding of the quality-reporting system patients have an informational disadvantage and are unable to infer the degree of each hospital's selection s_j from the reported information $\hat{q}_j$.

As their utility from quality, $\beta(q_j)^\gamma$, is always increasing in q_j, we assume that patients will respond to the reported information by choosing the hospital with the highest $\hat{q}_j$. A previous study has shown that patients choose hospitals with higher quality scores, even if these scores are not risk-adjusted and may therefore encourage risk selection (Varkevisser et al 2012). Furthermore the general behavioural economics literature shows that consumers often display limited attention and awareness to information disclosure as well as inattention to missing information (Sections 3.2 and 3.3 in Loewenstein et al 2014) lending support to the claim that patients are unlikely to consider the possibility that quality reports may reflect risk selection as well as true quality.[6]

[6] The weaker assumption that patients know about selection affecting the quality re-

Only symmetric Nash equilibria are considered, so the equilibrium selection and quality is equal for each hospital, $q_1 = q_2 = q^*$, $s_1 = s_2 = s^*$, and $x_1 = x_2 = x^*$.

In a symmetric equilibrium one question arises is that will patients equally randomise between the hospitals given there is no difference between them? We argue that choosing the hospital with the highest $\hat{q}_j$ is a more natural assumption as this is the level of quality at hospital j as perceived by the patient.

3.2 Revenue, Costs and Profit

Denote D_t^j the measure of patients that hospital j receives in period t. Revenue is the sum of the number of patients received in each period, minus the level of selection for each type of patient (s_j and x_j), multiplied by the regulated price p paid by the government purchaser.

Note that there is no difference in the profitability of the two types of patients. This implies that we assume DRG pricing is perfectly risk-adjusted, that the price paid fully captures any differential cost of treating high and low-severity patients. We model hospital behaviour in this benchmark case so that the only reason we will observe risk selection is because of imperfectly risk-adjusted report cards, not the reimbursement incentives[7]. An extension of the model to imperfectly risk-adjusted DRG pricing is considered in Appendix A, where we show that the main results carry over.

Profit for hospital j is given by

$$\Pi^j = p(D_1^j - s_j - x_j + D_2^j) - \frac{1}{2}\delta_s(s_j + x_j)^2 - \frac{1}{2}\delta_q q_j^2.$$

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ports, but assume that both hospitals always engage in the same amount of selection (i.e. that $E[q_j|\hat{q}_j] = \hat{q}_j + \alpha\bar{s}; \quad j = 1, 2$ where $s_1 = s_2 = \bar{s}$), produces the same results. This is because only the difference between the two hospitals quality expectations matters in determining demand.

[7] For example, for heart-surgery patients, the assumption may mean that the monetary cost is the same across the two types of patients, but the low-severity patient has a lower mortality rate. The mortality rate is reported in the report card. Thus the provider quality q is contaminated with the selection s in the report card $\hat{q}$.

Where $\delta_q,\ \delta_s \in (0, \infty)$ are the cost parameters associated with quality production and selecting patients.

We assume that quality is a public good for the patients of a hospital, as in Gravelle and Masiero (2000), Brekke et al. (2006) and Gravelle and Sivey (2010). The hospital incurs the same cost to achieve a given level of quality irrespective of the number of patients treated. Examples include investment in staff training, facilities and hospital-wide quality improvement policies. The quality cost parameter δ_q captures technology and input prices which affect the cost of producing quality.

We also model the cost of selection, $s_j + x_j$ in the same way, suggesting there is an increasing marginal cost of engaging in selection. This assumption is plausible if we may consider that for small amounts of selection it is relatively easy for a hospital to justify withholding treatment from this group of patients. An example would be patients in the oldest age groups or with many co-morbidities who may be more 'at risk' of complications from surgery. However, as a hospital looks to increase $s_j + x_j$ further, then it must deny treatment to patients who are still damaging to their quality score (e.g. they have worse than average mortality or readmission outcomes) but for whom the case to deny treatment is less justifiable therefore potentially more costly to the hospital.[8]

3.3 Equilibrium

We focus on the symmetric equilibrium with $(q_1, s_1, x_1) = (q_2, s_2, x_2) = (q^*, s^*, x^*)$.

Since x_j does not enter the report card, a hospital will have no incentive to select the low-severity patients. So in equilibrium it must be $x_j = 0$ for each hospital. Therefore the search for an equilibrium is reduced to finding (q^*, s^*).

In period 1, since the patients have no access to report cards, they will be indifferent between the two hospitals. Therefore in equilibrium in period 1

[8] The idea of an increasing marginal cost of 'cheating' is also found in Gilpatric (2011) for cheating in contests.

the hospitals will equally share the patients: each hospital receives a measure m of patients, among whom a measure 1 are high-severity patients ($h_j = 1$). Then, together with $x_j = 0$, we can further simplify the profit function of hospital j to

$$\Pi^j = pD_2^j + p(m - s_j) - \frac{1}{2}\delta_s s_j^2 - \frac{1}{2}\delta_q q_j^2 \tag{2}$$

As patients choose the hospital with the highest $\hat{q}_j$, hospital j's demand in period 2, $j = 1, 2$, is given by $D_2^j(q_j, q_{-j}, s_j, s_{-j}) = 2m\Pr(\hat{q}_j \geq \hat{q}_{-j})$. As, in equilibrium, the hospitals equally share the patients in period 1, $h_j = 1$,we have $\hat{q}_j = q_j + \alpha s_j + \varepsilon_1$ and so

$$\begin{aligned}\Pr(\hat{q}_j \geq \hat{q}_{-j}) &= \Pr(q_j + \alpha s_j + \varepsilon_j \geq q_{-j} + \alpha s_{-j} + \varepsilon_{-j}) \\ &= \Pr(q_j - q_{-j} + \alpha(s_j - s_{-j}) \geq \varepsilon_{-j} - \varepsilon_j).\end{aligned}$$

Under the assumption that $\varepsilon_j \sim U\left(\frac{-1}{2v}, \frac{1}{2v}\right)$, $\varepsilon_{-j} - \varepsilon_j$ is distributed according to the triangular distribution.[9]

Due to the symmetry between the two hospitals, we now analyse hospital 1 to simplify notation. The demand for hospital 1 is a function in two sections either side of the point where $E\left[\hat{q}_1 - \hat{q}_2\right] = 0$. For convenience we define $E\left[\hat{q}_1 - \hat{q}_2\right] = q_1 - q_2 + \alpha\left(s_1 - s_2\right) = \Delta$.We have

$$\begin{aligned} D_2^1 &= \begin{cases} 2m \cdot \frac{1}{2}\{v[q_1 - q_2 + \alpha(s_1 - s_2)] + 1\}^2 \text{ if } \Delta < 0 \\ 2m \cdot \left[1 - \frac{1}{2}\{v[-q_1 + q_2 + \alpha(-s_1 + s_2)] + 1\}^2\right] \text{ if } \Delta \geq 0 \end{cases} \\ &= \begin{cases} m\{v[q_1 - q_2 + \alpha(s_1 - s_2)] + 1\}^2 \text{ if } \Delta < 0 \\ m\left[2 - \{v[-q_1 + q_2 + \alpha(-s_1 + s_2)] + 1\}^2\right] \text{ if } \Delta \geq 0 \end{cases}. \end{aligned}$$

And the first derivatives of demand with respect to q_1 and s_1 are:

$$\frac{\partial D_2^1}{\partial q_1} = \begin{cases} 2mv \cdot \left[1 + v\left[q_1 - q_2 + \alpha(s_1 - s_2)\right]\right] \text{ if } \Delta < 0 \\ 2mv \cdot \left[1 - v\left[q_1 - q_2 + \alpha(s_1 - s_2)\right]\right] \text{ if } \Delta \geq 0 \end{cases},$$

[9] See Appendix A of Gravelle and Sivey (2010) for a formal derivation of the demand function in an equivalent case.

$$\frac{\partial D_2^1}{\partial s_1} = \begin{cases} 2m\alpha v \left[1 + v\left[q_1 - q_2 + \alpha(s_1 - s_2)\right]\right] \text{ if } \Delta < 0 \\ 2m\alpha v \cdot \left[1 - v\left[q_1 - q_2 + \alpha(s_1 - s_2)\right]\right] \text{ if } \Delta \geq 0 \end{cases}.$$

A feature of the triangular distribution is that the first derivatives of demand with respect to quality and selection are linear in those attributes

3.4 Equilibrium q^* and s^*

Differentiating the profit function for hospital 1 given in equation (2) with respect to q_1 we obtain the first order condition:

$$\begin{aligned} \frac{\partial \Pi^1}{\partial q_1} &= p\frac{\partial D_2^1}{\partial q_1} - \delta_q q_1 \\ &= \begin{cases} 2mpv \cdot \left[1 + v\left[q_1 - q_2 + \alpha(s_1 - s_2)\right]\right] - \delta_q q_1 \text{ if } \Delta < 0 \\ 2mpv \cdot \left[1 - v\left[q_1 - q_2 + \alpha(s_1 - s_2)\right]\right] - \delta_q q_1 \text{ if } \Delta \geq 0 \end{cases} \end{aligned}$$

This first-order condition shows how the quality level of the hospital 2 (q_2) and the selection levels of both hospitals (s_1 and s_2) affect the profitability of increasing hospital 1's quality level (q_1). To find the symmetric equilibrium we set $q_1 = q_2 = q^* > 0$, and $s_1 = s_2 = s^*$, so that

$$\left.\frac{\partial \Pi^1}{\partial q_1}\right|_{q_1=q_2=q^*,\ s_1=s_2=s^*} = 0,$$

which implies

$$2mpv - \delta_q q_1^* = 0,$$

and thus

$$q_1^* = \frac{2mpv}{\delta_q}.$$

The derivation of q_1^* can be shown graphically. Given $s_1 = s_2 = s^*$, we can derive the marginal cost (MC) and marginal revenue (MR) of hospital 1 for quality improvement:

$$MC(q_1) = \delta_q q_1$$

and

$$MR(q_1) = \begin{cases} 2mpv \cdot [1 + v(q_1 - q_2)] & \text{if } \Delta < 0 \\ 2mpv \cdot [1 - v(q_1 - q_2)] & \text{if } \Delta \geq 0 \end{cases}.$$

Figure 1 below illustrates the MC and MR curves.

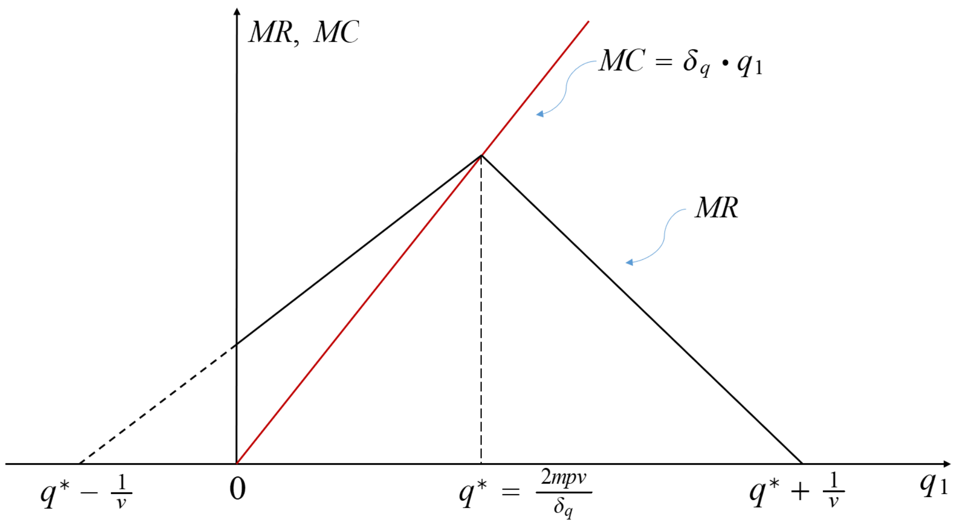

Figure 1: MC and MR of quality improvement.

First, as can be seen from Figure 1, the MR curve has a kink at the point $q_1 = q_2 = q^*$. Therefore, Π^1 is not twice differentiable and thus the second order condition is not applicable.

Second, note that for $q_1 > q^*$, MR is strictly decreasing in q_1 while MC is strictly increasing in q_1. Therefore, given $s_1 = s_2 = s^*$ and $q_2 = q^*$ it is never a best response to choose $q_1 > q_2$.

Third, as can be seen from the figure, a necessary condition for $q^* > 0$ is that it must be that

$$q^* - \frac{1}{v} < 0,$$

which, given $q^* = \frac{2mpv}{\delta_q}$, implies that

$$v < \sqrt{\frac{\delta_q}{2mp}}. \tag{3}$$

Differentiating the profit function for hospital 1 given in equation (2) with respect to s_1 we obtain the first order condition:

$$\frac{\partial \Pi^1}{\partial s_1} = p\frac{\partial D_2^1}{\partial s_1} - p - \delta_s s_1$$
$$= \begin{cases} 2mp\alpha v \cdot [1 + v\,[q_1 - q_2 + \alpha(s_1 - s_2)]] - p - \delta_s s_1 \text{ if } \Delta < 0 \\ 2mp\alpha v \cdot [1 - v\,[q_1 - q_2 + \alpha(s_1 - s_2)]] - p - \delta_s s_1 \text{ if } \Delta \geq 0 \end{cases}$$

At $q_1 = q_2 = q^*$, $s_1 = s_2 = s^*$, for $s^* \in (0, 1)$ it must be

$$\frac{\partial \Pi^1}{\partial s_1} = 0,$$

which implies

$$2mp\alpha v - p - \delta_s s^* = 0,$$

that is,

$$s^* = \frac{p}{\delta_s}(2m\alpha v - 1).$$

For $s^* > 0$ we need $2m\alpha v - 1 > 0$, that is,

$$v > \frac{1}{2m\alpha}.$$

On the other hand, for $s^* < 1$, we need $\frac{p}{\delta_s}(2m\alpha v - 1) < 1$, that is

$$v < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1).$$

The derivation of s^* can be shown graphically. Given $q_1 = q_2 = q^*$, consider the marginal cost (MC) and the marginal revenue (MR) of hospital 1 for selecting patients. We have

$$MC(s_1) = p + \delta_s s_1$$

and

$$MR(s_1) = \begin{cases} 2mp\alpha v \cdot [1 + v\alpha(s_1 - s_2)] \text{ if } \Delta < 0 \\ 2mp\alpha v \cdot [1 - v\alpha(s_1 - s_2)] \text{ if } \Delta \geq 0 \end{cases}.$$

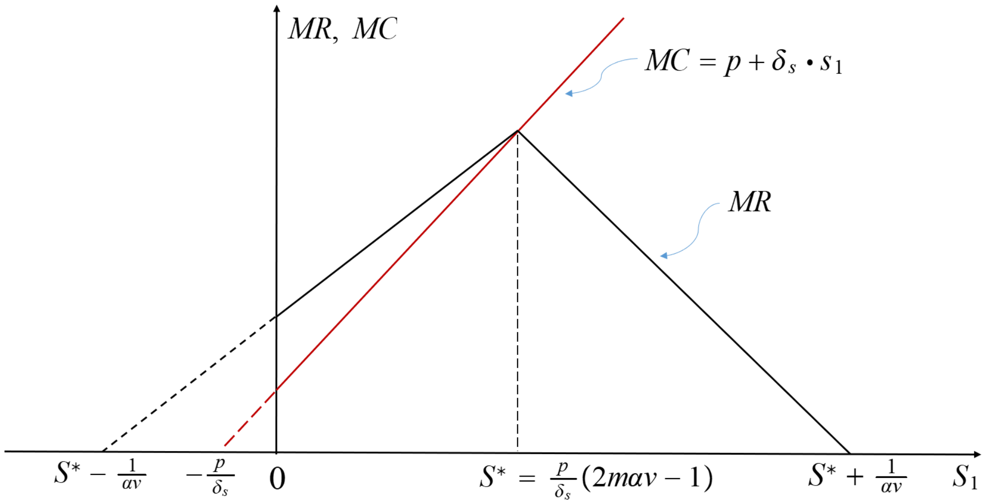

Figure 2: MR and MC of selecting patients

Observing Figure 2, we have the following results.

First, as can be seen from the figure, the MR curve has a kink at the point $s_1 = s_2 = s^*$. This means Π^1 is not twice differentiable and thus the second order condition is not applicable.

Second, for $s_1 > s^*$, the marginal cost of increasing s_1 exceeds the marginal revenue, and therefore given $q_1 = q_2 = q^*$ and $s_2 = s^*$, it is not a best response to set $s_1 > s^*$.

Third, as can be seen from the figure, a necessary condition for $s^* > 0$ is that

$$s^* - \frac{1}{\alpha v} < -\frac{p}{\delta_s},$$

that is,

$$\frac{p}{\delta_s}(2m\alpha v - 1) - \frac{1}{\alpha v} < -\frac{p}{\delta_s},$$

which implies

$$v < \sqrt{\frac{\delta_s}{2mp\alpha^2}}. \tag{4}$$

For ease of notation, denote $\hat{v} = \min\{\sqrt{\frac{\delta_q}{2mp}}, \sqrt{\frac{\delta_s}{2mp\alpha^2}}\}$. Figure 3 summarizes the above results under the assumption that $\frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1) < \hat{v}$.

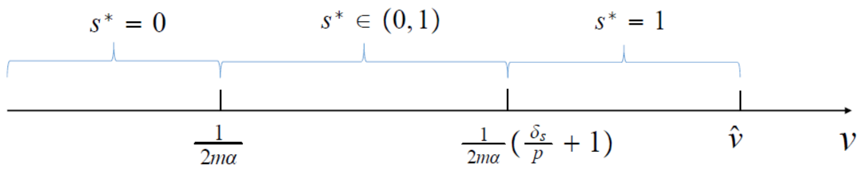

Figure 3: s^* depends on v.

We continue with the analysis assuming that $\hat{v}$, the highest value of v for which there is still an equlibrium q and s as defined by $q_1^* = \frac{2mpv}{\delta_q}$ and $s^* = \frac{p}{\delta_s}(2m\alpha v - 1)$, is the highest possible value of v. Appendix B considers higher possible values of v, demonstrating that a symmetric equilibrium does not exist and proposing a simple additional assumption (an upper limit for q) which will allow for an equlibrium to exist for all values of v.

4 Analysis

We analyse the effects of report card policies which increase patient information about hospital quality. We therefore concern ourselves with comparative statics on v the level of patient information about hospital quality.

The comparative static results are summarised in Figure 4, which plots equilibrium values of q^* and s^* as a function of v for the case where $\frac{1}{2m\alpha} < \hat{v} \leq \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$. Firstly, for the equilibrium quality level, we can see that $q^*(v)$ is linear and increasing over the domain of the function, $(0, \hat{v})$. The interpretation is that increased levels of patient information about hospital quality increases the incentives for hospitals to increase quality to attract more patients. The function has the slope $\frac{2mp}{\delta q}$, demonstrating that a higher price and a higher share of low-severity patients increases the effect of increasing information on the equilibrium quality, and higher costs of increasing quality reduce this effect.

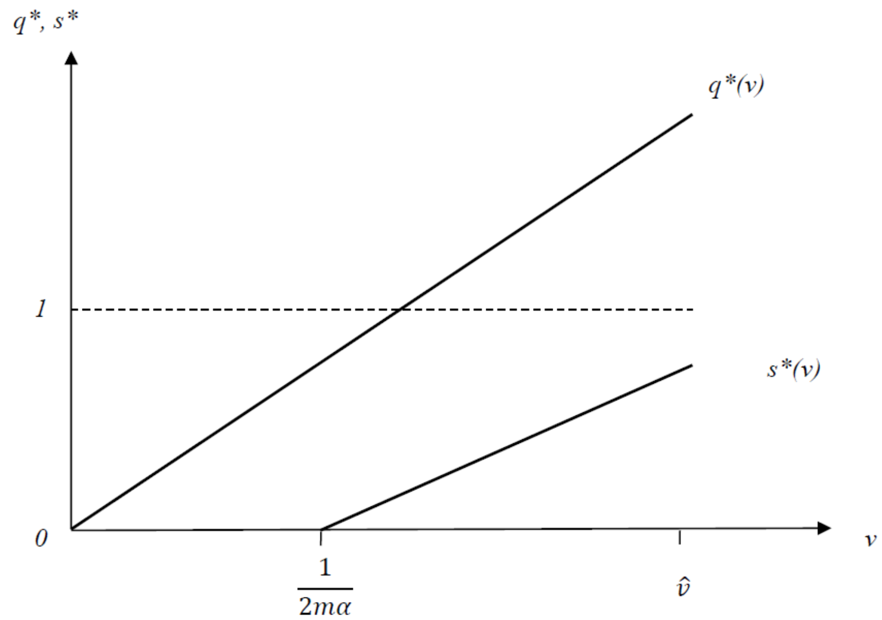

Figure 4: Equilibrium q^* and s^* as a function of v when $\frac{1}{2m\alpha} < \hat{v} \leq \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$.

Secondly, for the equilibrium level of selection (the reduction in the number of high-severity patients treated in period 1), $s^*(v)$, we can characterise two sections of the function over different domains. The first section, with $v \in \left(0, \frac{1}{2m\alpha}\right]$, is a corner solution with $s^* = 0$. We can interpret this section as showing that for sufficiently small levels of patient information (v), there are insufficient incentives for the hospitals to engage in any selection of high-severity patients. Specifically, the benefit of selection in terms of increasing $\hat{q}_j$, does not outweigh the explicit cost of selecting patients, represented by δ_s, and the lost revenue p from forgoing treatment of a high-severity patient. When v is small, the likelihood of a large error term ε_j is high, implying that the selection will not be adequately reflected in the reported information $\hat{q}_j$, and thus it will not attract enough low-severity patients by increasing s above zero to outweigh the costs. As the cost of selection outweighes the benefit, the hospitals refrain from selecting any high-severity patients. Poorer risk selection (higher α) and a higher share of low-severity patients (m) increases the incentive to engage in selection for low levels of v.

In the second section with $v \in \left(\frac{1}{2m\alpha}, \hat{v}\right]$, the equilibrium level of selection is linearly increasing in the level of patient information, v. We can interpret this as showing that increased patient information increases the marginal revenue from treating fewer high-severity patients in period 1 (engaging in selection) to increase the hospital's quality score. The slope of the equilibrium level of selection as a function of v is $\frac{2m\alpha p}{\delta^s}$. The slope is increasing in the price, p, the share of low-severity patients m, the imperfection in risk-adjustment α, and decreasing in the cost of selection, δ^s. The more profitable and the greater in number are period 2 patients, the stronger the incentives to attract them by engaging in selection to improve the quality score. Furthermore, the lower the cost of selection and larger the effect of selection on the quality score, the stronger the incentive to engage in selection.

5 Welfare Analysis

5.1 Patient welfare

We begin the welfare analysis by analysing patient welfare which we label W^P. We assume patients' welfare is equal to the utility from the quality of the care they receive $\beta(q^*)^\gamma$(from Section 3.1). In period 1, the $2m-2$ low-severity patients and the $2-2s^*$ high-severity patients who are treated in equilibrium also obtain welfare equal to $\beta(q^*)^\gamma$. In period 1, $2m-2s^*$ patients are treated. For the $2s^*$ high-severity patients who are not treated, we assume that each one receives a utility u_0. In period 2 all the $2m$ patients receive the treatment. The patient welfare therefore is

$$\begin{aligned} W^P &= (2m - 2s^* + 2m)\beta(q^*)^\gamma + 2s^*u_0 \\ &= 2\beta(2m - s^*)(q^*)^\gamma + 2s^*u_0 \end{aligned}$$

We assume that $u_0 \leq 0$. That is, even if a hospital sets quality at 0, a patient will still prefer seeking the treatment at the hospital to going untreated. Patient welfare therefore is increasing in quality but decreasing in the level of selection, as fewer patients experience the quality of care that is produced.

Hospital report card policies are represented in the model by increases in the level of patient information, v. So we are particularly interested in the effect of v on patient welfare which we can express as the function $W^P(v)$. We first analyze the polar case where $\gamma = 1$. That is, the patient's utility from treatment is linear in treatment quality. Note that, when there is partial patient selection ($0 < s^* < 1$), the level of selection s^* increases in v. If u_0 is sufficiently negative, then patient welfare falls as the level of selection increases. Hence we will turn the focus to the situation where u_0 is not too low. Toward this end, for ease of notation in the following analysis, denote $\underline{u} = \frac{\beta}{\alpha\delta_q}(2\sqrt{2mp\delta_s} - 2m\delta_s - p)$. Below we will be focused on the situation where $u_0 > \underline{u}$.

Proposition 1. *Suppose the patient's utility is linear in treatment quality ($\gamma = 1$). Then the patient welfare is maximized at $\hat{v}$.*

Specifically, we have

(i) if $2m\delta_s - p \leq 0$, then $\hat{v} \leq \frac{1}{2m\alpha}$, and thus at the maximum of patient welfare there is no selection ($s^* = 0$);

(ii) if $0 < 2m\delta_s - p < 2\delta_s$, then $\frac{1}{2m\alpha} < \hat{v} < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$, and thus at the maximum of patient welfare there is partial selection ($0 < s^* < 1$);

(iii) if $2\delta_s \leq 2m\delta_s - p$, then $\frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1) < \hat{v}$, and thus at the maximum of patient welfare there is full selection ($s^* = 1$).

Proof. See Appendix C □

Point (i) of Proposition 1 shows that if the parameters are such that there is no selection, then it is patient welfare-maximising to have patient information as high as possible, as this increases quality as much as possible. Point (ii) shows that even if there is some selection, the welfare gains through increased quality to patients who are treated outweighs the welfare losses to patients who are untreated as a result of the increased selection. Therefore it is still optimal to have patient information, v, as high as possible. Point (iii) follows from points (i) and (ii): Where there is full selection, any further increase in v only leads to further increases in quality, which increases patient welfare. Since it is optimal for patient welfare to have patient information as high as possible if there is some positive selection (point (ii)), then, if parameters are such that there can be full selection (all high-severity patients are untreated in period 1), it must still be optimal for patient information to be as high as possible.

The analysis for the case $\gamma < 1$, which means patient welfare is strictly concave in the treatment quality, is much less tractable than $\gamma = 1$. But we provide numerical examples in section 5.3.4 to show that, if $\gamma < 1$, then the v that maximizes the patient welfare can be interior, that is, less than $\hat{v}$. Intuitively, as the utility becomes concave in the treatment quality, the marginal return to the patient welfare is decreasing as the treatment quality increases. Increasing v increases the incentives for hospitals to increase quality *and* select patients. Compared to when $\gamma = 1$, when $\gamma < 1$, the quantity of the patients that receive the treatment matters more for patient welfare than further increasing quality, and therefore the patient welfare-maximising level of information v may be lower than when $\gamma = 1$.

5.2 Policy implications

5.2.1 When report cards cause no risk selection

The analysis in the previous section is focused on the optimal report card policy in term of v. It makes sense to also consider the change in v caused by a report card policy to be of an arbitrary size due to, for instance, technological capacity or political constraints. In this section, we address the following question: Given an implemented report card policy in terms of v what other factors determine the change in patient welfare? To examine this question, we start from a benchmark v_0, which represents patients' information prior to the policy. Again we focus on the tractable case where $\gamma = 1$. Suppose that $v_0 < \frac{1}{2m\alpha}$, so that there is no risk selection before the policy is implemented. At v_0 the patient welfare is

$$W^p|_{v_0} = \frac{8\beta m^2 p v_0}{\delta_q} \tag{5}$$

First consider a report card policy which increases v from v_0 to a level v_1 at $\frac{1}{2m\alpha}$. At $v_1 = \frac{1}{2m\alpha}$, since there is no risk selection the change in patient welfare is simply

$$\Delta W_1^p = \frac{8\beta m^2 p}{\delta_q}\left(\frac{1}{2m\alpha} - v_0\right), \tag{6}$$

which is strictly positive. We can interpret the model parameters in ΔW_1^p fairly straightforwardly as representing the patient welfare gain from higher quality, and the incentives to raise quality as information becomes more accurate (ie as v increases). Higher β will raise the patient welfare benefit from higher quality. Higher per-patient reimbursement, p, and lower costs of quality production, δ_q, will give stronger incentives for hospitals to raise quality whereas worse risk adjustment of the quality reports, higher α, will reduce this incentive.

5.2.2 When report cards cause some risk selection

Next, suppose we define a higher level of v, v_2, such that $\frac{1}{2m\alpha} < v_2 < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$. We assume for simplicity that untreated patients receive zero utility, $u_0 = 0$. As risk selection occurs, and $0 < s^* < 1$, patient welfare is:

$$W^p|_{v_2} = \frac{4\beta m p v_2}{\delta_q}\left(2m - \frac{p(2m\alpha v_2 - 1)}{\delta_s}\right)$$

so here the change in patient welfare is

$$\Delta W_2^p = W^p|_{v_2} - W^p|_{v_0} = \frac{4\beta m p}{\delta_q}\left[\underbrace{2m\left(v_2 - v_0\right)}_{A} - \underbrace{\frac{p v_2\left(2m\alpha v_2 - 1\right)}{\delta_s}}_{B}\right] \tag{7}$$

Analysing (7) can provide several insights. Firstly, the term A in ΔW_2^p is always positive, linear in $v_2 - v_0$ and can be interpreted as the welfare gain from the quality improvement available to treated patients (all low-severity patients, and those high-severity patients who are still treated). The second term (term B) is always negative (as $2m\alpha v_2$ is always greater than 1 for $v_2 > \frac{1}{2m\alpha}$), and increasing (ly negative) in v_2, and can be interpreted as the welfare loss from untreated high-severity patients. Overall, we know from Proposition 1 that $\Delta W_2^p > 0$. In terms of the parameters, we can see from (7) that δ_s always increases ΔW_2^p, α and δ_q always decrease ΔW_2^p. Recall that

- δ_s measures the cost of reducing treatment of high-severity patients (engaging in selection);
- δ_q measures the cost of increasing quality;
- α measures the degree of risk adjustment (higher α corresponds to worse risk adjustment).

So the welfare effect of report cards ΔW_2^p is higher if risk adjustment of report cards is better (lower α), if the cost of engaging in selection is high (higher δ_s) and the cost of increasing quality is low (lower δ_s). These results indicate the directions in which policymakers can improve the report card policy when there is positive selection. These include a better risk-adjustment scheme and a mechanism such as an auditing procedure that can better track, and consequently impede, risk selection. On the other hand, as (7) indicates, the parameter p enters both terms A and B, and thus its effects on ΔW_2^p are ambiguous. This suggests that policies aimed at changing reimbursement rates for DRGs need to be implemented with caution. Higher reimbursement sharpens the incentives for increasing quality *and* for engaging in risk selection, when report cards are introduced.

5.2.3 When report cards cause complete risk selection

Next, suppose we define a higher level of v, $v_3 > \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$,and continue to assume $u_0 = 0$. Here $s^* = 1$ so patient welfare is:

$$W^p|_{v_3} = \frac{4\beta m p v_3}{\delta_q} (2m - 1)$$

so here the change in patient welfare is:

$$
\begin{aligned}
\Delta W_3^p &= \frac{8\beta m^2 p}{\delta_q}\left(v_3 - v_0\right) - \frac{4\beta m p v_3}{\delta_q} && (8) \\
&= \frac{4\beta m p}{\delta_q}\left[2m\left(v_3 - v_0\right) - 1\right] && (9)
\end{aligned}
$$

Here, the interpretation is very similar to in (6) when there is no risk selection, the welfare gain in (8) is all due to the quality gains experienced by patients who are treated. As all period 1 high-severity patients remain untreated (ie risk selection is at its maximum), there is no change in the welfare loss to these patients as v increases beyond $\frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$.

5.3 Parametric examples

We will illustrate four parametric examples, corresponding to the three possible patient-welfare maximizing information levels of v, as characterized in Proposition 1. In all cases $\beta = 1$ and $u_0 = 0$.

5.3.1 Example 1

First, consider the following parameter values which correspond to the case where the patient-welfare maximizing information is achieved at a $\hat{v} \leq \frac{1}{2m\alpha}$. We plot the equilibrium level of treatment quality q^* and selection level s^*, in Figure 5 and patient welfare in Figure 6. Here, quality is increasing in patient information v, but v is never high enough to cause any positive selection (s^*). Patient welfare increases linearly in v up to the optimum $\hat{v}$. Beyond the patient-welfare maximising level of v, $\hat{v}$, there is no equilibrium.

Example 1 parameter values

Parameter	α	p	m	δ_s	δ_q	γ
Value	0.8	1	1.5	0.5	0.5	1

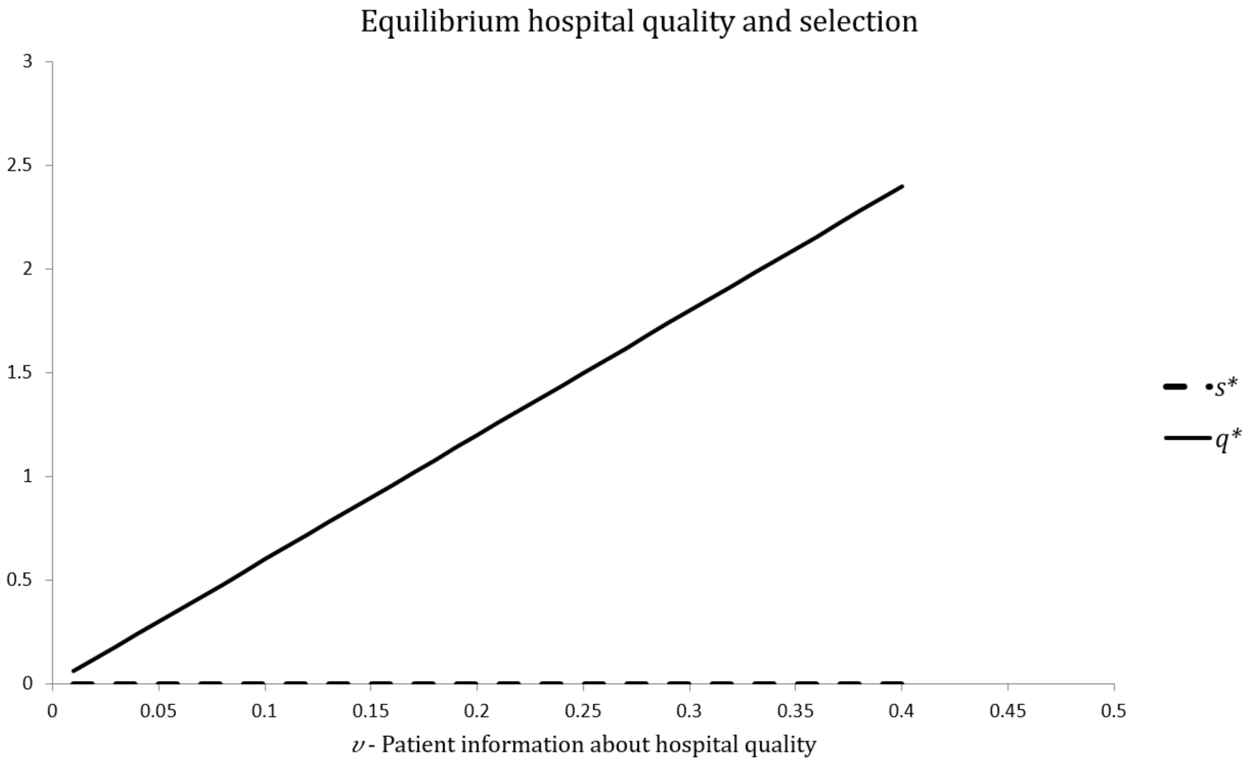

Figure 5: Equilibrium when $\hat{\upsilon} \leq \frac{1}{2m\alpha}$

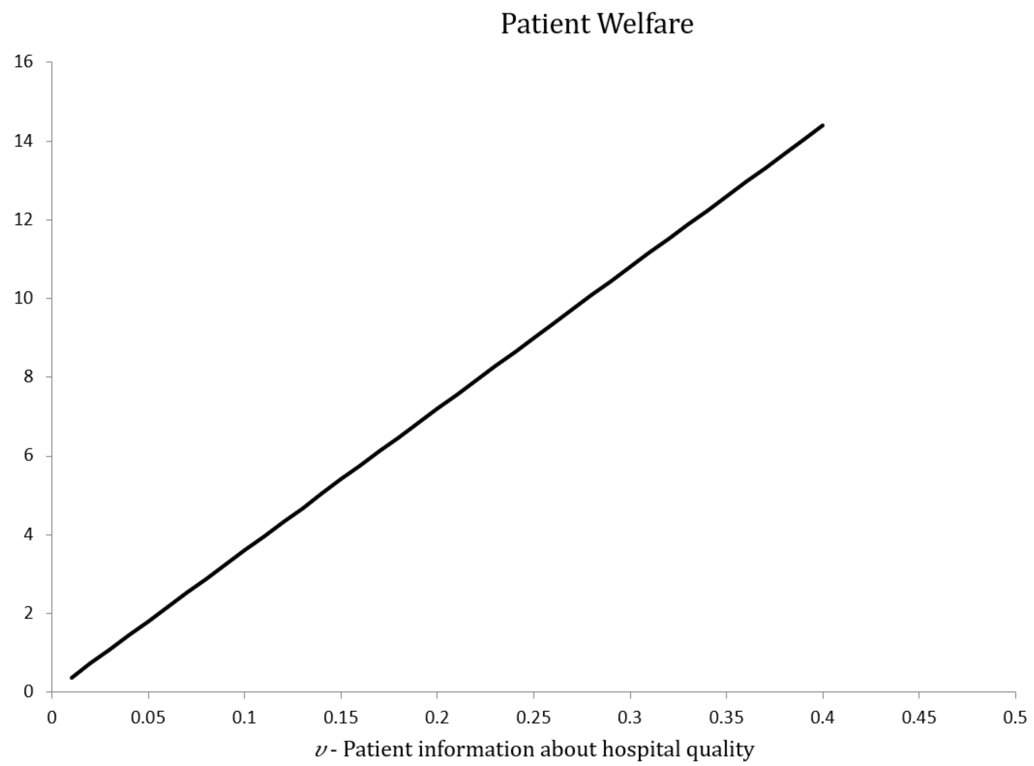

Figure 6: Patient welfare when $\hat{\upsilon} \leq \frac{1}{2m\alpha}$

5.3.2 Example 2

Secondly, increasing δ_q from 0.5 to 2, we plot the equilibrium treatment quality q^* and selection level s^* in Figures 7 and patient welfare in Figure 8 for the case where the patient-welfare maximizing information is achieved where $\frac{1}{2m\alpha} < \hat{\upsilon} \leq \frac{1}{2m\alpha}(\frac{\delta_s}{p}+1)$, and where there is partial selection, $0 < s^* < 1$. Here, quality is increasing in patient information υ, and as υ increases above $\frac{1}{2m\alpha}$, selection, s^*, begins to increase linearly from zero. Patient welfare increases linearly in υ up to $\frac{1}{2m\alpha}$, then as selection begins, consistent with Proposition 1, welfare continues to increase up to the optimum $\hat{\upsilon}$, although at a slowing rate. Beyond the patient-welfare maximising level of υ, $\hat{\upsilon}$, there is no equilibrium.

Example 2 parameter values

Parameter	α	p	m	δ_s	δ_q	γ
Value	0.8	1	1.5	0.5	**2**	1

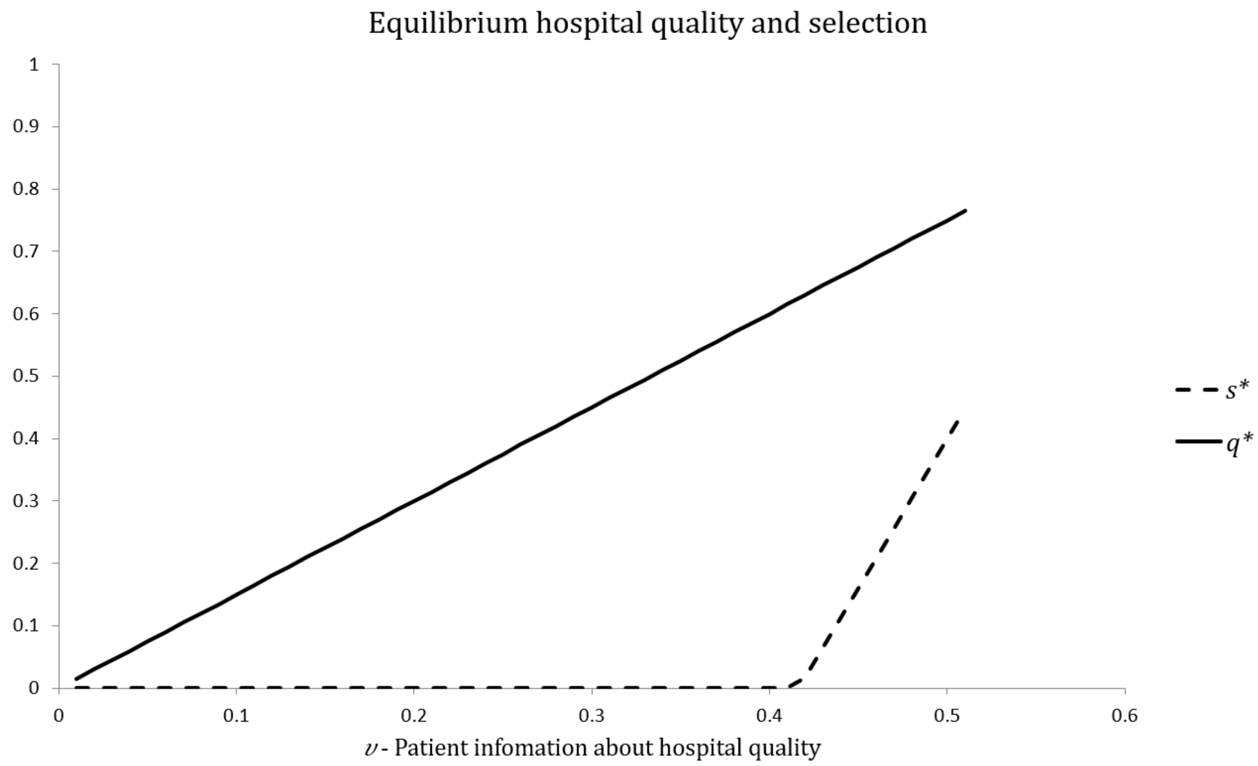

Figure 7: Equilibrium when $\frac{1}{2m\alpha} < \hat{\upsilon} \leq \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$

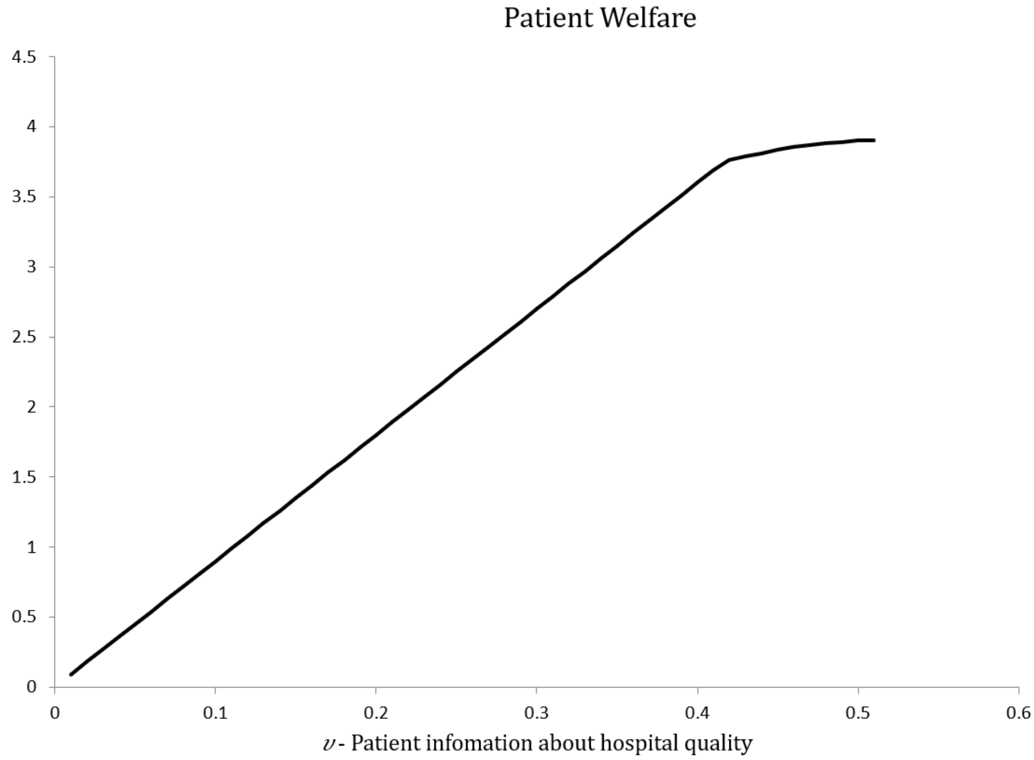

Figure 8: Patient welfare when $\frac{1}{2m\alpha} < \hat{v} \leq \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$

5.3.3 Example 3

Thirdly, increasing m to 2.5 (maintaining $\delta_q = 2$) we plot equilibrium treatment quality q^* and selection s^* in Figure 9 and patient welfare in Figure 10 for the case where the patient-welfare maximizing information is achieved where $\hat{v} > \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$, and where there is full selection, $s^* = 1$.

Example 3 parameter values

Parameter	α	p	**m**	δ_s	δ_q	γ
Value	0.8	1	**2.5**	0.5	2	1

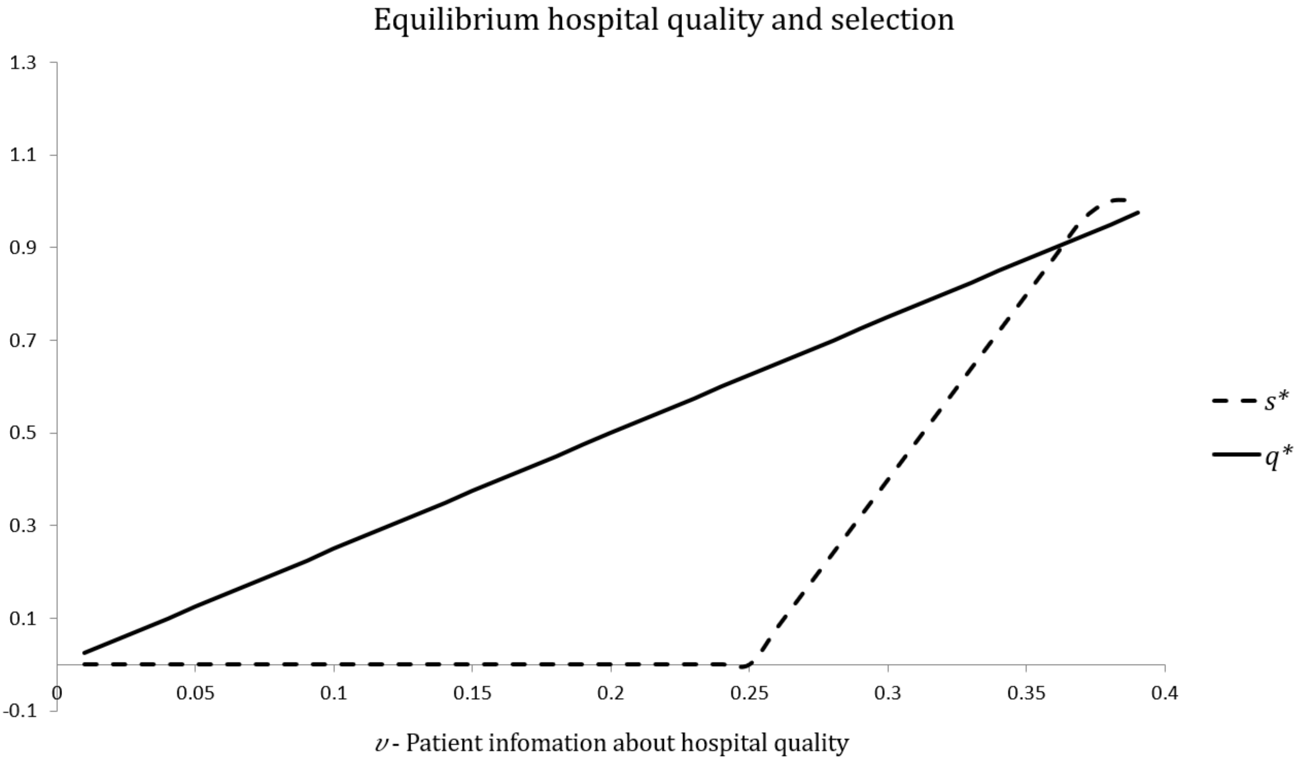

Figure 9: Equilibrium when $\hat{\upsilon} > \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$

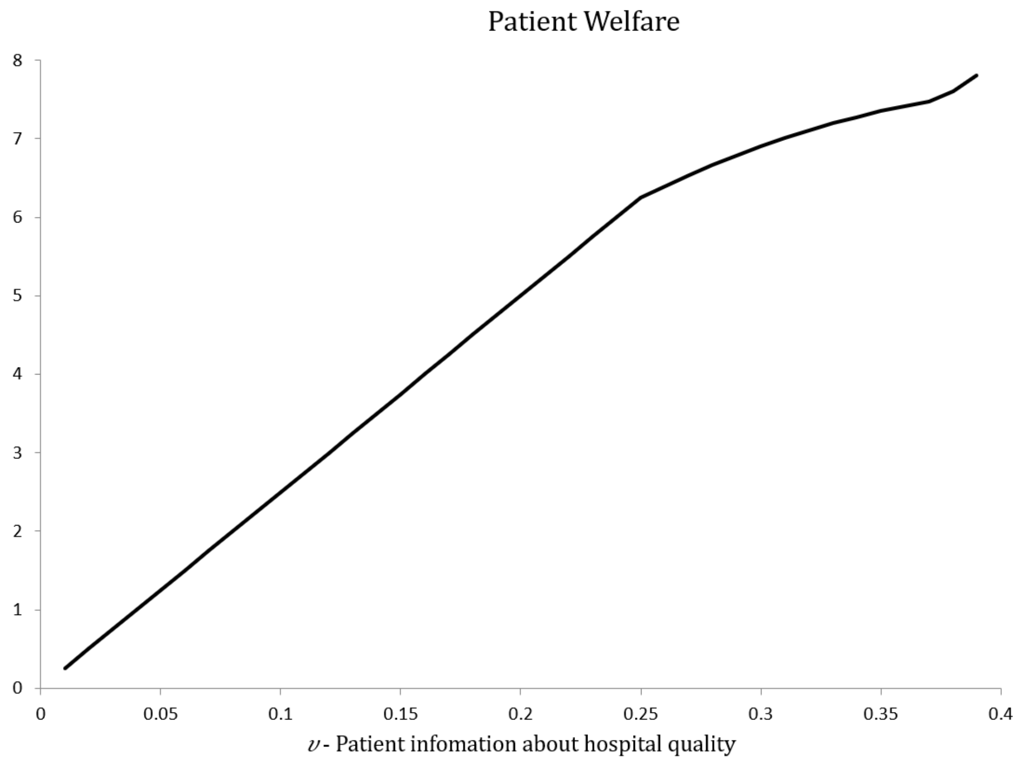

Figure 10: Patient welfare when $\hat{\upsilon} > \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$

5.3.4 Example 4

Fourthly, we show patient welfare for the same equilibrium parameters as Example 3, but with $\gamma = \frac{1}{2}$, and $\gamma = \frac{1}{3}$ such that patient utility is the square/cube root of (and therefore concave in) treatment quality. Patients benefit from higher quality, but at a diminishing rate as quality increases. Equilibrium treatment quality q^* and selection s^* is the same as in Figure 9, patient welfare for $\gamma = \frac{1}{2}$ is given by Figure 11 and for $\gamma = \frac{1}{3}$ by Figure 12.

Example 4 parameter values

Parameter	α	p	m	δ_s	δ_q	γ
Value	0.8	1	2.5	0.5	2	$\frac{1}{2}, \frac{1}{3}$

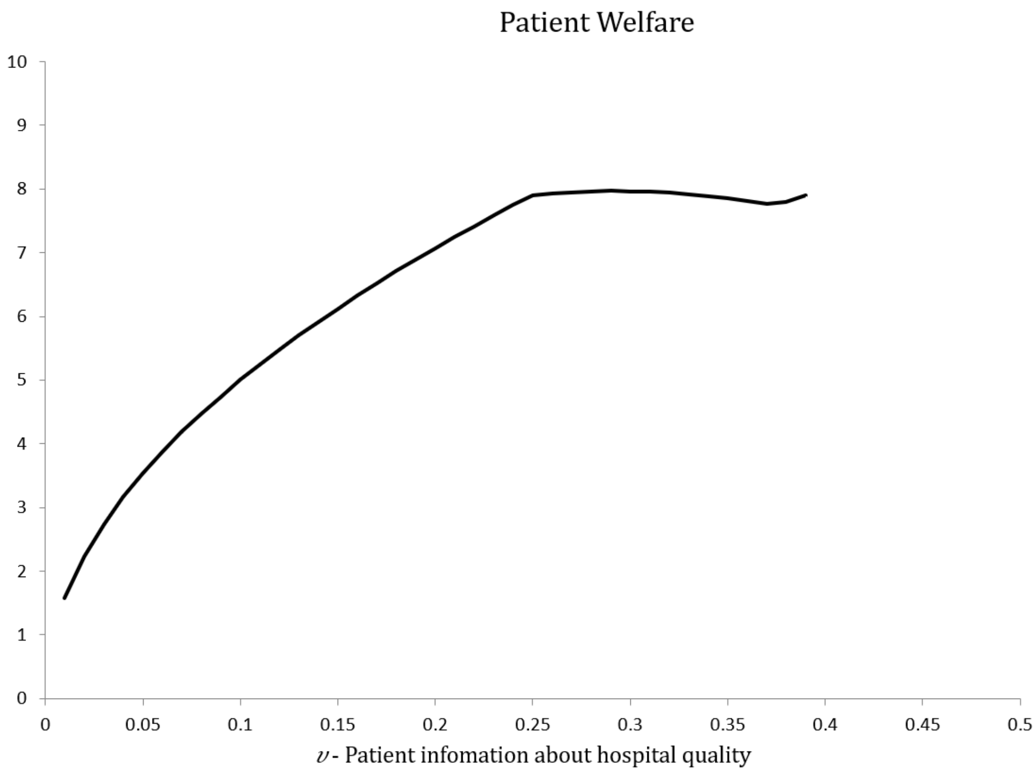

Figure 11: Patient welfare when $\hat{\upsilon} > \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$ and $\gamma = \frac{1}{2}$

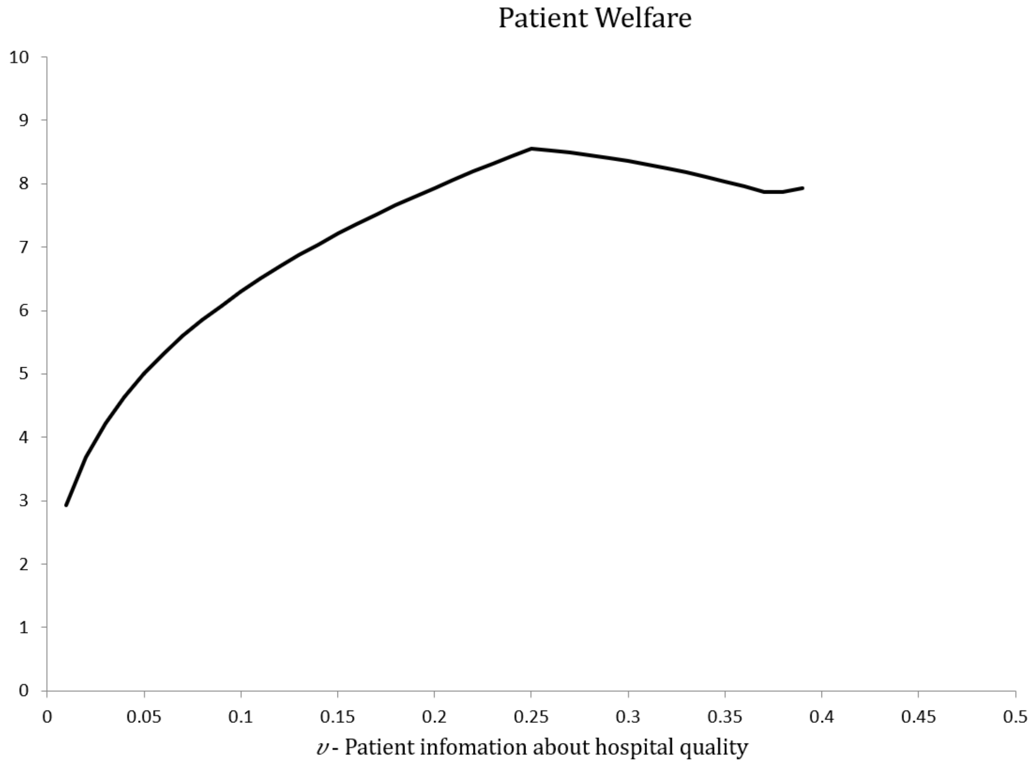

Figure 12: Patient welfare when $\hat{\upsilon} > \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$ and $\gamma = \frac{1}{3}$

In Figures 11 and 12 we can see patient welfare is concave in the level of patient information as long as $\upsilon < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$ (ie: as long as $s^* < 1$). Even when $\upsilon < \frac{1}{2m\alpha}$ (ie when $s^* = 0$) and quality is increasing with no selection, the welfare gains of increasing quality are diminishing as $\gamma < 1$. For $\frac{1}{2m\alpha} < \upsilon < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$ selection is increasing from 0 towards 1 so the patient welfare function kinks downwards at $\frac{1}{2m\alpha}$. In Figure 12 when $\gamma = \frac{1}{3}$ the kink down at $\frac{1}{2m\alpha}$ is such that $\frac{1}{2m\alpha}$ is a 'corner solution' for the patient welfare-maximising level of υ.

5.4 Total welfare

Total welfare, W, is the sum of patient welfare, hospital profits, and the transfer from the government purchaser to the hospitals multiplied by $(1+\lambda)$

where λ is the opportunity cost of public funds:

$$
\begin{aligned}
W &= W^P + 2\Pi_j^* - (1+\lambda)(2[pm + p(m - s^*)]) \\
&= W^P + 2[pm + p(m - s^*) - \frac{1}{2}\delta_s(s^*)^2 - \frac{1}{2}\delta_q(q^*)^2] - (1+\lambda)(2[pm + p(m - s^*)]) \\
&= W^P - \delta_s(s^*)^2 - \delta_q(q^*)^2 - \lambda(2[pm + p(m - s^*)]). \qquad (10)
\end{aligned}
$$

After simplifying the expression, total welfare is patient welfare, minus the costs incurred by the hospitals in producing the equilibrium quality q^* and the equilibrium level of selection s^*, minus the opportunity cost of public funds (λ) multiplied by the revenue earned by hospitals.

5.4.1 Example 5

We show the effect of accounting for total welfare by replicating Example 3 (above, for patient welfare), including the extra terms in the total welfare function (Equation 10).

Example 5 parameter values

Parameter	α	p	m	δ_s	δ_q	γ	λ
Value	0.8	1	2.5	0.5	2	1	0.05

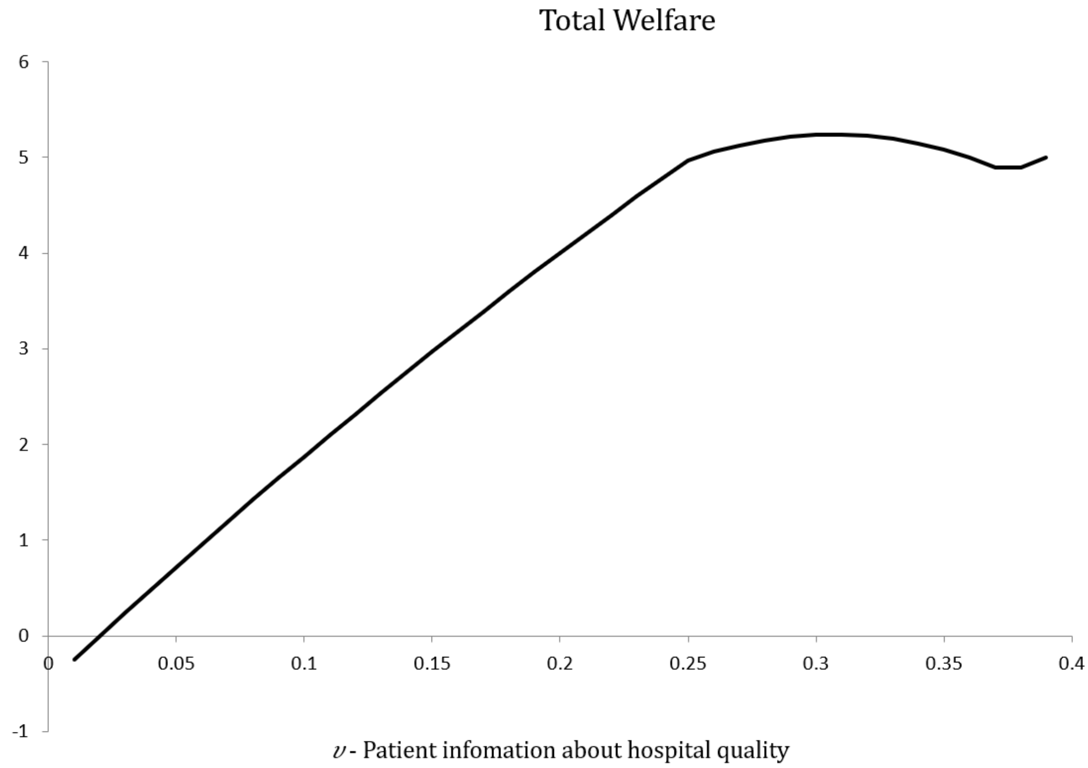

Figure 13: Total welfare when $\hat{v} > \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$ and $\lambda = 0.05$

In comparison to the patient welfare for Example 3 in Section 5.3.3, this total welfare function is concave for the section where $v < \frac{1}{2m\alpha}$ as the quadratic costs of higher quality have an increasing negative effect on total welfare. In the section where $\frac{1}{2m\alpha} < v < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$ while patient welfare is always increasing, total welfare begins to decrease at some value of v in this interval for which $s^* \in (0, 1)$, as the costs of higher quality, costs of higher selection, and welfare losses to high-severity patients who are untreated in period 1, begin to outweigh the continuing quality gains for treated patients.

6 Discussion

In this paper we analyse the role of hospital quality report cards in stimulating hospital quality improvement and incentivising risk selection. Previous studies analyse patient selection *or* quality improvement, this paper is a first attempt to combine these two features in one model. Our results show that increasing patient information about hospital quality (report cards) always increases equilibrium quality levels, exerting a positive effect on patient welfare. Report cards also increase the incentive for hospitals to engage in selection, but in equilibrium selection is zero when information is low, and selection only becomes positive when information is high relative to the share of high-severity patients and the accuracy of risk-adjustment of report cards.

Our model suggests that hospital report cards will improve aggregate patient welfare if we assume a linear relationship between hospital quality and patient utility. Some high-severity patients lose out by not being treated if selection occurs but this effect is outweighed by the increase in quality for patients who are treated. If there are diminishing returns to quality in patients' utility functions, there may be an 'optimum' level of patient information, beyond which report cards harm aggregate patient welfare as the reduction in the number of patients treated (selection) outweighs the diminishing gains from higher quality. In this case, even if just considering patient welfare, it is not always optimal to increase patient information as much as possible.

When considering total welfare, there are additional costs to hospital report cards, including the costs to hospitals of quality improvement and risk selection (if it occurs).

The results of the analysis confirm the interrelatedness of the parameters determining health system performance. In the linear welfare case, patient welfare will increase by a greater amount following report cards if other conditions are favourable: if quality scores are well risk-adjusted, hospital treatment decisions are thoroughly audited (increasing the costs of selection), and if the costs of increasing quality are low. If risk selection occurs because of a report card policy, the price paid by the government purchaser has an ambiguous effect on patient welfare: it incentivises both higher treatment quality but also more selection.

The main limitation of our approach in this paper is the specific functional forms used in the model. One example is the assumption of uniformly-distributed error terms in the quality report cards. This assumption allows the first derivatives of the demand function to be linear in quality and selection - allowing us to derive closed-form solutions of the model.

Another example is the hospital's cost function which consists of two parts: the cost of selection and the cost of quality. In the paper, we make two assumptions on the cost function. First, we assume that it is additively separable, so that the two parts are separate from each other. Secondly, we assume a quadratic function form for each part.

The quadratic function form is for tractability and ease of illustration, but it can be generalized. As can be seen from Figures 1 and 2, as long as the marginal cost of quality and the marginal cost of selection are increasing functions, we can find the equilibrium level of q and s at the point where marginal revenue equals marginal cost. However, if we relax the additive-separability assumption, then the analysis will be less tractable. This is because the two marginal costs will be entangled as s and q enter each MC function, and we need to solve a system of equations simultaneously to derive the equilibrium.

While the empirical literature on report cards has focused most on risk selection (Dranove et al 2003), the broader empirical literature on competition in healthcare has focused on quality improvement (Cooper et al 2011, Gaynor et al 2013). Report cards can potentially lead to both quality improvement and risk selection. In particular, our model suggests that accurate risk adjustment of report cards is vital to minimize the amount of risk selection and to maximise patient welfare improvement.

Our model can be validated by empirical studies showing report cards leading to quality improvement and *either* zero risk selection *or* an increase in risk selection.

Previous empirical studies (Dranove et al 2003, Chen and Meinecke 2012, Chou et al 2014) have been able to test these hypotheses using data from early report card schemes for coronary artery bypass graft (CABG) in New York and Pennsylvania. Dranove et al (2003) found evidence of selection in the form of a reduction in the number of CABGs performed on the most severely ill patients but did not consider the possibility of quality improvement. Chen and Meinecke (2012) test for both quality improvement and risk selection by exploiting the asymmetric information in the early period of report card data collection before reports were actually reported to patients. These results don't seem to validate our model as they don't show any quality improvement as a result of the report card policies. One reason may be that these papers consider only quite short-term impacts of the report card policy.

Chou et al (2014) study the effects of a later report card scheme for CABG patients in Pennsylvania where quality reports for Medicare patients were published online in 1998. This study looks at outcomes over a 10 year period and concludes that quality improved (as seen by more resources per patient and lower mortality rates), but is able to rule out any selection (which they call cream skimming) by showing no change in the severity of patients being treated with CABG. Therefore the Chou et al study validates the theoretical results in this paper for the case where report cards increase quality but cause no risk selection. This result suggests the CABG market in Pennsylvania during the time period studied by Chou et al, had report cards that were well risk-adjusted enough to allow quality improvement without risk selection. Further empirical studies may be able to shed light on whether increased risk-selection as a result of report cards is observed in contexts with more poorly risk adjusted quality reports, as implied by our model.

While empirical studies are vital to informing the policy debate, finding data and study designs that are both internally and externally valid is very challenging, so a mixture of empirical evidence, theoretical grounding, and common sense reasoning is likely to be the best approach to policy development in this area.

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8 Appendix A: Heterogeneity in costs for low and high-severity patients

In the main model we assume that the hospitals incur the same monetary cost at treating the two types of patients. In reality, however, despite the same DRG price, the cost of treating a low-severity patient may be different from that of treating a high-severity one. Our model can be extended to allow this situation, and the main results in the main model carry over.

Specifically, suppose the cost for treating a low-severity type patient is c_l, and the cost for treating a high-severity type patient is c_h. Because the regulator does not observe the patient type, it does not observe the actual treatment cost either. The regulator, therefore, pays a constant DRG price p for treating each patient. Denote $p_l = p - c_l$, and $p_h = p - c_h$. In other words, p_l (p_h) is the actual payment a hospital receives for treating a low-severity (high-severity) type patient. Everything else remains the same as the main model.

For hospital 1 in period 2, denote D^1_{2h} the demand of the high-severity type patients, and D^1_{2l} the demand of the low-severity type. It will be

$$D^1_{2h} = 2\Pr[\hat{q}_1 \geq \hat{q}_2],$$

and

$$D^1_{2l} = (2m - 2)\Pr[\hat{q}_1 \geq \hat{q}_2].$$

Similar as in the main model, we can show that the hospitals will not select low-severity type patients. Then hospital 1's profit is

$$\begin{aligned}
\pi^1 &= p_h D^1_{2h} + p_l D^1_{2l} + p_h(1-s_j) + p_l(m-1) - \frac{1}{2}\delta_s s_j^2 - \frac{1}{2}\delta_q q_1^2 \\
&= [2p_h + (2m-2)p_l] \Pr[\hat{q}_1 \geq \hat{q}_2] + p_h(1-s_j) + p_l\,(m-1) \\
&- \frac{1}{2}\delta_s s_j^2 - \frac{1}{2}\delta_q q_1^2 \\
&= P \Pr[\hat{q}_1 \geq \hat{q}_2] + p_h(1-s_j) + p_l\,(m-1) - \frac{1}{2}\delta_s s_j^2 - \frac{1}{2}\delta_q q_1^2 \\
&= \begin{cases} P\frac{1}{2}\{v[q_1 - q_2 + \alpha(s_1 - s_2)] + 1\}^2 \\ +p_h(1-s_j) + p_l\,(m-1) - \frac{1}{2}\delta_s s_j^2 - \frac{1}{2}\delta_q q_1^2 \text{ if } \Delta < 0 \\ P[1 - \frac{1}{2}\{v[q_1 - q_2 + \alpha(s_1 - s_2)] + 1\}^2] \\ +p_h(1-s_j) + p_l\,(m-1) - \frac{1}{2}\delta_s s_j^2 - \frac{1}{2}\delta_q q_1^2 \text{ if } \Delta \geq 0 \end{cases},
\end{aligned}$$

where we denote $P = [2p_h + (2m-2)p_l]$ for ease of expression.

To derive the symmetric equilibrium quality q^* and s^*, we have that, at $q_1 = q_2 = q^*$, and $s_1 = s_2 = s^*$, it will be

$$\frac{\partial \pi^1}{\partial q_1} = Pv - \delta_q q_1 = 0,$$

and thus

$$\begin{aligned}
q^* &= \frac{Pv}{\delta_q} \\
&= \frac{[2p_h + (2m-2)p_l]v}{\delta_q}.
\end{aligned}$$

Similarly, in equilibrium we will have

$$\frac{\partial \pi^1}{\partial s_1} = P\alpha v - p_h - \delta_s s_1 = 0,$$

which leads to

$$\begin{aligned}
s^* &= \frac{P\alpha v - p_h}{\delta_s} \\
&= \frac{[2p_h + (2m-2)p_l]\alpha v - p_h}{\delta_s}.
\end{aligned}$$

We can see that the results are consistent with the main model: with $p_h = p_l = p$ in the main model, we have $q^* = \frac{2mpv}{\delta_q}$ and $s^* = \frac{p}{\delta_q}(2m\alpha v - 1)$.

9 Appendix B: Equilibrium when $v \to +\infty$

Note that the equilibrium above exists on the condition that v is sufficiently low ($v < \hat{v}$). When v is sufficiently high, i.e., $v > \hat{v}$, then there does not exist a symmetric equilibrium. To see this, consider the extreme case where $v = +\infty$, that is $\varepsilon_j = 0$. Since the quality information is precise, we have $\hat{q}_j = q_j + \alpha s_j$. This means all patients in period-2 will choose the hospital with the higher $q_j + \alpha s_j$. Now suppose there is a symmetric equilibrium with $q_1^* = q_2^* = q^*$, and $s_1 = s_2 = s^*$. Then the period-2 demand is m for each hospital and the equilibrium profit is

$$\tilde{\Pi}_1 = pm + p(m - s^*) - \frac{1}{2}\delta_s(s^*)^2 - \frac{1}{2}\delta_q(q^*)^2.$$

Now suppose hospital 1 slightly increase quality by an infinitesimal amount η, then *all* period-2 patients will choose hospital 1. That is, its period-2 demand jumps from m to $2m$, and this leads to a jump of profit

$$\begin{aligned}\hat{\Pi}_1 &= 2pm + p(m - s^*) - \frac{1}{2}\delta_s(s^*)^2 - \frac{1}{2}\delta_q(q^* + \eta)^2 \\ &> \tilde{\Pi}_1.\end{aligned}$$

Thus the symmetric equilibrium cannot hold.

The situation where v is sufficiently high is similar as the extreme case where $v = +\infty$. As illustrated in Figure 4 below, a high v implies a low $\frac{1}{v}$, and thus a very narrow support of the error term ε_j's distribution. Given q_2, a q_1 that is more than $\frac{1}{v}$ above q_2, which can be achieved at a fairly low cost due to the smallness of $\frac{1}{v}$, will attract all the patients in period 2, and this causes the symmetric equilibrium to break down.

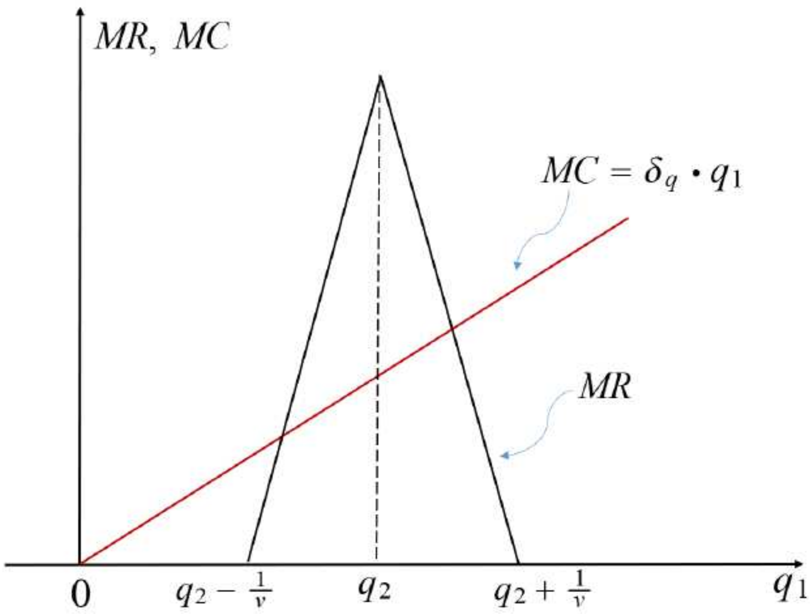

Figure 12: Symmetric equilibrium does not exist if v is sufficiently high.

To establish an equilibrium, we can add a realistic assumption about the upper limit of the quality.

Assumption: Due to the technology constraint, the upper limit of the hospital's attainable quality is $\bar{q}$.

Now suppose the government can release the precise quality information, that is, $v = +\infty$. Below we shall derive the highest possible $\bar{q}$ at which there can be a symmetric equilibrium with $q_1 = q_2 = \bar{q}$ and $s_1 = s_2 = 1$.

Proposition 2. *If the government releases the precise quality information ($v = +\infty$), the highest possible $\bar{q}$ at which there can be an equilibrium with $q_1 = q_2 = \bar{q}$ and $s_1 = s_2 = 1$ is*

$$\bar{q} = \sqrt{\frac{2p(m-1) - \delta_s}{\delta_q}}.$$

Proof. First, suppose $q_1 = 0$ and $s_1 = 0$ while $q_2 > 0$, $s_2 > 0$. Then hospital 1's period 2 demand is 0, while it treats all of its period-1 patients. Thus

$\Pi_1(q_1 = 0, s_1 = 0 \mid q_2 > 0, s_2 > 0) = pm$. Now, suppose $q_1 = q_2 = \bar{q}$, and $s_1 = s_2 = 1$, then there is

$$\begin{aligned}
&\Pi_1(q_1 = \bar{q}, s_1 = 1 | \ q_2 = \bar{q}, s_2 = 1) \\
&\quad = pm + p(m-1) - \frac{1}{2}\delta_s - \frac{1}{2}\delta_q\bar{q}^2 \\
&\quad = 2pm - p - \frac{1}{2}\delta_s - \frac{1}{2}\delta_q\bar{q}^2.
\end{aligned}$$

Since a hospital's revenue in the symmetric equilibrium is independent of q_j, the highest $\bar{q}$ is such that in equilibrium, the high $\bar{q}$ causes a high cost and consequently the firm's profit equals the profit from the least-costly strategy, i.e. $q_1 = 0, s_1 = 0$:

$$\Pi_1(q_1 = \bar{q}, s_1 = 1 | \ q_2 = \bar{q}, s_2 = 1) = \Pi_1(q_1 = 0, s_1 = 0 | \ q_2 = \bar{q}, s_2 = 1),$$

which implies that

$$2pm - p - \frac{1}{2}\delta_s - \frac{1}{2}\delta_q\bar{q}^2 = pm.$$

Hence the highest possible $\bar{q}$ is $\sqrt{\frac{2p(m-1)-\delta_s}{\delta_q}}$. To show that $q_1 = q_2 = \bar{q}$ and $s_1 = s_2 = 1$ comprise an equilibrium, note that given $q_2 = \bar{q}$ and $s_2 = 1$, if $q_1 < q_2$ or $s_1 < 1$, then the demand for hospital 1 is $D_2^1 = 0$, and thus the profit of hospital 1 will be

$$\Pi_1(q_1, s_1 | q_2 = \bar{q}, s_2 = 1) = p(m - s_1) - \frac{1}{2}\delta_s(s_1)^2 - \frac{1}{2}\delta_q(q_1)^2 \leq pm.$$

Thus Hospital 1 has no incentive to deviate from $q_1 = \bar{q}$ and $s_1 = 1$. $\square$

Note that with precise quality information, in the equilibrium with $\bar{q} =$

$\sqrt{\frac{2p(m-1)-\delta_s}{\delta_q}}$ and $s^* = 1$, the social welfare is

$$
\begin{aligned}
W &= 2(2m - s^*)\bar{q} + 2\Pi_1 \\
&= 2(2m-1)\bar{q} + 2(2pm - p - \frac{1}{2}\delta_s - \frac{1}{2}\delta_q\bar{q}^2) \\
&= 2(2m-1)\sqrt{\frac{2p(m-1)-\delta_s}{\delta_q}} + 2pm + \delta_s.
\end{aligned}
$$

10 Appendix C: Proof of Proposition 1

Proposition 1. *Suppose the patient's utility is linear in treatment quality ($\gamma = 1$). Then the patient welfare is maximized at $\hat{v}$.*

Specifically, we have

(i) if $2m\delta_s - p \le 0$, then $\hat{v} \le \frac{1}{2m\alpha}$, and thus at the maximum of patient welfare there is no selection ($s^* = 0$);

(ii) if $0 < 2m\delta_s - p < 2\delta_s$, then $\frac{1}{2m\alpha} < \hat{v} < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$, and thus at the maximum of patient welfare there is partial selection ($0 < s^* < 1$);

(iii) if $2\delta_s \le 2m\delta_s - p$, then $\frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1) < \hat{v}$, and thus at the maximum of patient welfare there is full selection ($s^* = 1$).

Proof. Suppose $\gamma = 1$. (i) Suppose $2m\delta_s - p \le 0$, then to show that $\hat{v} \le \frac{1}{2m\alpha}$, it is sufficient to show that $\sqrt{\frac{\delta_s}{2mp\alpha^2}} \le \frac{1}{2m\alpha}$. We have

$$
\begin{aligned}
&\sqrt{\frac{\delta_s}{2mp\alpha^2}} \le \frac{1}{2m\alpha}, \\
\Leftrightarrow\ &\frac{\delta_s}{2mp\alpha^2} \le \frac{1}{4m^2\alpha^2}, \\
\Leftrightarrow\ &\frac{\delta_s}{2mp\alpha^2} \le \frac{1}{4m^2\alpha^2}, \\
\Leftrightarrow\ &2m\delta_s - p \le 0.
\end{aligned}
$$

Thus if $2m\delta_s - p \le 0$, then $\hat{v} \le \frac{1}{2m\alpha}$. For $v < \frac{1}{2m\alpha}$, we have $s^* = 0$. Thus we have

$$W^P(v) = 4\beta m q^* = \frac{8\beta m^2 pv}{\delta_q}.$$

We can see that the patient welfare W^P is strictly increasing in v. Then we can conclude that the patient welfare achieves its maximum at $\hat{v}$, at which there is no patient selection. (ii) Now suppose $0 < 2m\delta_s - p < 2\delta_s$. Following the steps of inequalities in (1) and using the fact that $\min\{\sqrt{\frac{\delta_q}{2mp}}, \sqrt{\frac{\delta_s}{2mp\alpha^2}}\} \leq \sqrt{\frac{\delta_s}{2mp\alpha^2}}$ we can derive that $0 < 2m\delta_s - p$ implies that $\hat{v} > \frac{1}{2m\alpha}$. To show that $\hat{v} < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$, it is sufficient to show that $\sqrt{\frac{\delta_s}{2mp\alpha^2}} < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1)$. Note that we have

$$\sqrt{\frac{\delta_s}{2mp\alpha^2}} < \frac{1}{2m\alpha}(\frac{\delta_s}{p} + 1),$$

$$\Longleftrightarrow \frac{\delta_s}{2mp\alpha^2} < \frac{1}{4m^2\alpha^2}(\frac{\delta_s^2}{p^2} + 2\frac{\delta_s}{p} + 1),$$

$$\Longleftrightarrow 2m\delta_s < \frac{\delta_s^2}{p} + 2\delta_s + p,$$

$$\Longleftrightarrow 2m\delta_s - p < \frac{\delta_s^2}{p} + 2\delta_s,$$

which is implied by $2m\delta_s - p < 2\delta_s$. Therefore if $0 < 2m\delta_s - p < 2\delta_s$, then $\frac{1}{2m\alpha} < \hat{v} < \frac{1}{2m\alpha}(\frac{\delta_s}{p}+1)$. For $v \in [\frac{1}{2m\alpha}, \frac{1}{2m\alpha}(\frac{\delta_s}{p}+1)]$, we have $s^* = \frac{p}{\delta_s}(2m\alpha v - 1)$ and $q^* = \frac{2mpv}{\delta_q}$. Thus

$$\begin{aligned} W^P &= 2\beta(2m - s^*)q^* + 2s^* u_o \\ &= 4\beta m \frac{pv}{\delta_q}\left[2m - \frac{p(2m\alpha v - 1)}{\delta_s}\right] + 2\frac{p(2m\alpha v - 1)}{\delta_s} u_0. \end{aligned}$$

Then we have

$$\frac{\partial W^P}{\partial v} = \frac{4m\beta p}{\delta_q}\left(2m + \frac{p}{\delta_s} - \frac{4mp\alpha v}{\delta_s}\right) + \frac{4m\alpha p}{\delta_s} u_0,$$

and

$$\frac{\partial^2 W^P}{\partial v^2} = -\beta \frac{16m^2 p^2 \alpha}{\delta_s \delta_q} < 0.$$

Thus W^P is strictly concave on $[\frac{1}{2m\alpha}, \frac{1}{2m\alpha}(\frac{\delta_s}{p}+1)]$. Then at $v = \sqrt{\frac{\delta_s}{2mp\alpha^2}}$, we have $\frac{\partial W^P}{\partial v}\Big|_{v=\sqrt{\frac{\delta_s}{2mp\alpha^2}}} = \frac{4m\beta p}{\delta_q}\left(2m + \frac{p}{\delta_s} - \frac{4mp\alpha}{\delta_s}\sqrt{\frac{\delta_s}{2mp\alpha^2}}\right) + \frac{4m\alpha p}{\delta_s}u_0$. Note that for the term $\left(2m + \frac{p}{\delta_s} - \frac{4mp\alpha}{\delta_s}\sqrt{\frac{\delta_s}{2mp\alpha^2}}\right)$, it can be verified that $2m + \frac{p}{\delta_s} - \frac{4mp\alpha}{\delta_s}\sqrt{\frac{\delta_s}{2mp\alpha^2}} > 0$. This is because

$$2m + \frac{p}{\delta_s} - \frac{4mp\alpha}{\delta_s}\sqrt{\frac{\delta_s}{2mp\alpha^2}} > 0$$

$$\Leftrightarrow 2m + \frac{p}{\delta_s} - \sqrt{\frac{16m^2p^2\alpha^2}{\delta_s^2}\frac{\delta_s}{2mp\alpha^2}} > 0$$

$$\Leftrightarrow 2m + \frac{p}{\delta_s} - \sqrt{\frac{8mp}{\delta_s}} > 0$$

$$\Leftrightarrow 4m^2 + 4m\frac{p}{\delta_s} + \frac{p^2}{\delta_s^2} > \frac{8mp}{\delta_s}$$

$$\Leftrightarrow 4m^2\delta_s^2 - 4mp\delta_s + p^2 > 0$$

$$\Leftrightarrow (2m\delta_s - p)^2 > 0,$$

which is true since we are under the assumption that $0 < 2m\delta_s - p$. Then we can verify that, for $u_0 > \underline{u}$, we have

$$\frac{\partial W^P}{\partial v}\Big|_{v=\sqrt{\frac{\delta_s}{2mp\alpha^2}}} > 0.$$

Since $\hat{v} = \min\{\sqrt{\frac{\delta_q}{2mp}}, \sqrt{\frac{\delta_s}{2mp\alpha^2}}\}$ and W^P is strictly concave on $[\frac{1}{2m\alpha}, \frac{1}{2m\alpha}(\frac{\delta_s}{p}+$

1)], we can conclude that $\frac{\partial W^P}{\partial v}\Big|_{v=\hat{v}} > 0$. In summary, we have

$$\frac{\partial W^P}{\partial v}\bigg|_{v\le\frac{1}{2m\alpha}} > 0,$$
$$\frac{\partial W^P}{\partial v}\bigg|_{v=\hat{v}} > 0,$$
$$\frac{\partial^2 W^P}{\partial v^2}\bigg|_{v\in[\frac{1}{2m\alpha},\frac{1}{2m\alpha}(\frac{\delta_s}{p}+1)]} < 0.$$

Hence we can conclude that the patient welfare is maximized at $\hat{v}$. (iii) Now suppose $2m\delta_s - p > 2\delta_s$. Following (i) and (ii), it can be shown that $\hat{v} > \frac{1}{2m\alpha}(\frac{\delta_s}{p}+1)$. Moreover, there are

$$\frac{\partial W^P}{\partial v}\bigg|_{v\le\frac{1}{2m\alpha}} > 0,$$
$$\frac{\partial W^P}{\partial v}\bigg|_{v=\frac{1}{2m\alpha}(\frac{\delta_s}{p}+1)} > 0,$$
$$\frac{\partial^2 W^P}{\partial v^2}\bigg|_{v\in[\frac{1}{2m\alpha},\frac{1}{2m\alpha}(\frac{\delta_s}{p}+1)]} < 0.$$

Furthermore, for $v \in (\frac{1}{2m\alpha}(\frac{\delta_s}{p}+1), \hat{v}]$, we have $s^* = 1$ and thus

$$\begin{aligned} W^P &= 2\beta(2m-1)q^* + 2u_0 \\ &= 4\beta m(2m-1)\frac{pv}{\delta_q} + 2u_0. \end{aligned}$$

We can see that W^P is strictly increasing in v. Thus W^P achieves the maximum at $\hat{v}$. □