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The Dependence Structure between Equity and Foreign Exchange Markets and Tail Risk Forecasts of Foreign Investments

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Abstract

Motivated by the importance of the dependence structure between equity and foreign exchange rates in international financial markets, we investigate whether modelling the dependence structure can help forecast the tail risk of foreign investments. We propose a new time-varying asymmetric copula for modelling the dependence structure and forecasting the tail risk. We conduct backtesting on our tail risk forecasts for 12 major developed and emerging markets. We find that modelling the dependence structure can improve the tail risk forecast and make risk management of foreign investments more robust.

Keywords: Foreign investments, dependence structure, TVAC model, Value-at-Risk, Expected Shortfall.

JEL Codes: G15, F21, F37

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1. Introduction

Understanding the dependence structure between equity market and foreign exchange (forex) market is of particular importance for foreign investments. It is relevant not only for academics, but also for financial practitioners. For instance, global investors trade equities in foreign countries to diversify their portfolios internationally. These investors holding foreign equities are exposed to both equity risk and exchange rate risk. Moreover, they have to consider the dependence between equity and exchange rate to control risk that the dependence creates. In this paper, we show the importance of modelling a dependence structure between equity and foreign exchange rate for the tail risk forecast of foreign investments.

This study is motivated by two main reasons. First, let us suppose that US investors put their money into a foreign equity portfolio at time t. Then its logarithmic return in US dollars at time $t+1$ is given by

$$r_{t+1} = r_{e,t+1} - r_{f,t+1} \tag{1}$$

where $r_{e,t+1}$ denotes the logarithmic return of equity portfolio denominated by the local currency and $r_{f,t+1}$ the logarithmic return of foreign exchange rate (i.e., US dollars to foreign currency) (see Solnik and McLeavey, 2008). Thus, the risk of foreign investments (in US dollars) has three components: equity risk, forex risk and risk induced by the dependence structure between equity and forex.[1] Among these three, it is of paramount importance for international investors to identify the dependence structure for hedging currency risk associated with their investments. It has become increasingly important as we have experienced a major global financial crisis over the past decade.

Second, a great deal of research studies the co-movements of international financial markets. The relationship of international equity markets is of especial interest (Christoffersen et al., 2014; Dungey and Martin, 2007; Forbes and Rigobon, 2002; Karolyi and Stulz, 1996; Longin and Solnik, 2001; Okimoto, 2008) or forex markets (Patton, 2006). Recently, Ning (2010) and Wang et al. (2013) investigate the dependence structure between equity and forex using copulas. Ning (2010) finds the significant symmetric tail dependence from the G5 countries and Wang et al. (2013) find the regime-dependent dependence structure in the six major industrial countries. However, their findings rely entirely on statistical evaluations using the Goodness-of-Fit test for copula models rather than risk management practices. It has not been clearly answered yet how the dependence structure between equity and forex is economically important in the international financial markets.

To achieve the goal of this study, it is crucial to identify and model the dependence structure as shown in Christoffersen et al. (2014), Okimoto (2008) and Patton (2006). But it is hard to uniquely determine the dependence structure because it relies on the underlying market regime and country- (or market-) specific characteristics. For example, "return chasing effects" (Hau and Ley, 2004; Wang et al., 2013) explains a negative dependence structure; a bull market attracts international investors, which leads to a currency appreciation in that country. By contrast, "portfolio rebalancing effects" (Hau and Ley, 2004; Wang et al., 2013) explains a positive one; a bull market encourages domestic investors to move their capital to other countries where stock markets offer better investment opportunities. This leads to a currency depreciation in that country. "Funding currency in the carry trade" (Galati et al., 2007; Gyntelberg and Remolona, 2007) and a "Safe Haven" currency (Ranaldo and Söderlind, 2010) also suggest the positive one. Therefore, the dependence structure is likely to be time-varying and asymmetric across market regimes. All these explanations suggest that the identification of a dependence

[1] For example, suppose that the global economic recession has led to a stock market downturn and resulted in losses on both domestic and foreign markets. However, if the foreign currency is negatively correlated with stock prices for some reasons (e.g. "Safe Haven" currency), we can partially hedge the risk of foreign investments (in US dollars).

structure is an empirical issue, rather than a theoretical one and we need flexible models for modelling the dependence structure.

To this end, we first identify the dependence structure using various diagnostic tests, such as a threshold correlation test (Hong et al., 2007), a tail dependence test (Patton, 2013) and structure break tests in a rank correlation (Andrews, 1993; Andrews and Ploberger, 1994; Patton, 2013). Next, we propose a time-varying asymmetric copula, which combines a new skewed t copula based on the multivariate skewed t distribution (Bauwens and Laurent, 2005) and a generalized autoregressive score model (Creal et al., 2013) for modelling the dependence structure. Lastly, we forecast the Value-at-Risk (VaR) and expected shortfall (ES) of foreign investment with/without modelling the dependence structure to see the improvement of tail risk forecasts.

We use a daily Morgan Stanley Capital International (MSCI) country index as a proxy for the foreign equity portfolio and a corresponding daily forex for 12 major developed and emerging countries from 2000 to 2014. First, we conduct statistical diagnostic tests to confirm the time-varying and asymmetric dependence structure between equity and forex in our sample. This finding extends the results of Ning (2010) and Wang et al. (2013) with more countries. Second, our proposed time-varying asymmetric copula model provides a better Goodness-of-Fit than constant or symmetric copulas. Third, the backtesting results of VaR and ES forecasts verify that we can conduct a more robust risk management of foreign investments by modelling the dependence structure explicitly. In particular, it becomes more important to model the dependence structure during the crisis and post-crisis periods.

The remainder of this paper is structured as follows. Section 2 describes the data, as well as the results of diagnostic tests on the dependence structure between two assets. Section 3 describes the modelling of the dependence structure identified by the diagnostic tests. Section 4 tests whether the modelling of the dependence structure helps to forecast the tail risk of the foreign investments using backtesting tools. Section 5 concludes.

2. Equities and Foreign Exchange Rates

We use two groups of countries for the sake of comparison. The developed markets sample comprises the European Union (EU), the United Kingdom (UK), Japan, Switzerland, Canada and Australia; the emerging markets sample comprises Brazil, India, Russia, Turkey, South Korea and South Africa. We use the MSCI index, which represents an equity portfolio benchmark for each country/region. It is provided by Morgan Stanley Capital International (MSCI). All the stock indices are expressed by the local currencies, and foreign exchange rates are expressed as the number of units of local currency per US dollar. We collect all the daily data from Datastream over the period January 3, 2000 to December 31, 2014.

2.1. Returns

Table 1 reports descriptive statistics of logarithmic return series for equities and forexs. The non-zero values of skewness indicate that all the return series are either positively or negatively skewed. The values of kurtosis indicate fat tails for returns. Thus, both sample skewness and kurtosis suggest that the returns are not normally distributed. The Jarque-Bera test statistics also validate the non-normality. The Ljung-Box Q-statistics show the presence of autocorrelation in 17 out of 24 return series. The Ljung-Box Q-statistics on the squared returns and the Lagrange Multiplier tests for ARCH effects are significant for all return series, implying heteroscedasticity. Overall test results support the use of AR-GARCH model with a skewed distribution for modelling the marginal distributions of return series.

Next, we report Pearson's linear correlation and Spearman's rank correlation between equity and forex. All the correlations are negative, except that of Japan and Switzerland. The negative correlation is in line with the "return chasing effect", as discussed in Hau and Ley (2004) and Wang et al. (2013). Meanwhile, the positive correlations of Japan and Switzerland could be explained through several channels like "funding currencies in the carry trade" (Galati et al., 2007; Gyntelberg and Remolona, 2007), "portfolio rebalancing effects" (Hau and Ley, 2004; Wang et al., 2013), and "Safe Heaven" properties (Ranaldo and Söderlind, 2010). In Figure 1, the scatter plots between the two return series for each country also show these relationships.

[INSERT TABLE 1 AND FIGURE 1 ABOUT HERE]

2.2. Return Dynamics

We assume that both equity and forex returns follow a stochastic process:

$$r_{k,t} = \mu_{k,t} + \epsilon_{k,t},\ \epsilon_{k,t} = \sigma_{k,t} z_{k,t},\ k = e, f \tag{2}$$

where $\mu_{k,t}$ denotes a conditional mean, $\sigma_{k,t}$ a conditional volatility, e an equity and f a forex, respectively.

Relying on our stylized facts from the descriptive statistics, we employ the AR(1) model to account for the autocorrelation of asset returns and the GJR-GARCH(1,1,1) model of Glosten, et al. (1993) to capture volatility persistence, heteroskedasticity and the leverage effect:

$$r_{k,t} = \phi_{k,0} + \phi_{k,1} r_{k,t-1} + \epsilon_{k,t}, \tag{3}$$

$$\sigma^2_{k,t} = w_k + \alpha_k \epsilon^2_{k,t-1} + \gamma_k \epsilon^2_{k,t-1} I_{(-\infty,0]}\left(\epsilon_{k,t-1}\right) + \beta_k \sigma^2_{k,t-1} \tag{4}$$

where standardized returns, $z_{k,t}$, are assumed to follow the skewed t distribution of Fernández and Steel (1998):

$$F_{\lambda_k,\nu_k}(x) = \int_{-\infty}^{x} \frac{2}{\lambda_k + \lambda_k^{-1}} \left\{ f_{\nu_k}\left(z\lambda_k^{-1}\right) I_{[0,\infty)}(z) + f_{\nu_k}(z\lambda_k) I_{(-\infty,0]}(z) \right\} dz \tag{5}$$

where I is an indicator function, λ_k a skewness parameter, ν_k a degree of freedom and f_{ν_k} a standardized Student's t distribution with ν_k degree of freedom, which is unimodal and symmetric around 0.[2]

Table 2 and 3 report the estimation results of the AR-GJR-GARCH model with the skewed t innovations. All the leverage parameters for the equity returns are significantly positive, which indicates higher volatility in the "downside" market movements. On the other hand, most of the leverage parameters for the forex are significantly negative except for Japan and Switzerland. The negative sign indicates higher volatility when the currency depreciates. We test the Goodness-of-Fit for our proposed univariate model using Kolmogorov-Smirnov (KS) and Cramer-von Mises (CvM) tests following Patton (2013). We find that our univariate model accurately fits all equity and forex return series.

[INSERT TABLE 2 AND 3 ABOUT HERE]

2.3. Asymmetric Dependence Structure

Much of the equity literature demonstrates that correlations in "downside" markets are much greater than those in "upside" ones. We thus extend the analysis in the previous literature to the dependence structure between equity and forex.

[2] It is econometrically coherent (and practically implementable) to adopt a score-driven update scheme for both univariate and multivariate specifications. However, it is a common practice to use GARCH-type models, especially the GJR-GARCH, with the skewed t distribution to model univariate marginal distributions in the GAS literature, see for instance, Oh and Patton (2018), Eckernkemper (2018) and Bernardi and Catania (2019).

First, we explore the asymmetric dependence relying on the threshold correlations (Ang and Chen, 2002; Patton, 2004):

$$\rho^{-+}(u) = corr\left(r_{e,t}, r_{f,t} | r_{e,t} < F_e^{-1}(u) \text{ and } r_{f,t} > F_f^{-1}(1-u)\right) \text{ when } u < 0.5 \quad (6)$$

$$\rho^{+-}(u) = corr\left(r_{e,t}, r_{f,t} | r_{e,t} \geq F_e^{-1}(u) \text{ and } r_{f,t} \leq F_f^{-1}(1-u)\right) \text{ when } u \geq 0.5 \quad (7)$$

where u is a threshold between 0 and 1, and $F_k^{-1}(u)$ the empirical quantile of univariate distribution for r_k. It is the correlation between equity and forex when the equity downturns (upturns) and the currency depreciates (appreciates). We apply the model free test of Hong et al. (2007) to the null of symmetric correlation:

$$H_0 : \rho^{-+}(u) = \rho^{+-}(u) \text{ for all } u \quad (8)$$

$$H_1 : \rho^{-+}(u) \neq \rho^{+-}(u) \text{ for some } u \quad (9)$$

For the m threshold levels, $\boldsymbol{\rho}^{-+} - \boldsymbol{\rho}^{+-} = \left(\rho^{-+}(u_1) - \rho^{+-}(u_1), \ldots, \rho^{-+}(u_m) - \rho^{+-}(u_m)\right)'$, a test statistics is given by

$$J_\rho = \left(\boldsymbol{\rho}^{-+} - \boldsymbol{\rho}^{+-}\right)' \hat{\boldsymbol{\Omega}}^{-1} \left(\boldsymbol{\rho}^{-+} - \boldsymbol{\rho}^{+-}\right) \xrightarrow{d} \chi_m^2 \quad (10)$$

where the consistent estimator of covariance matrix, $\hat{\boldsymbol{\Omega}}$, is well defined in Hong et al. (2007). Table 4 shows that symmetric correlations are not rejected for all countries in the developed markets while they are rejected for the four emerging markets at the 5% significance level.

Second, we test the difference between "lower-upper" tail dependence (LUTD) and "upper-lower" tail dependence (ULTD) (see Christoffersen et al., 2012; Elkamhi and Stefanova, 2015; Patton, 2009; Poon et al., 2004):

$$\lambda^{-+} = \lim_{q \to 0} \frac{P\left\{r_{e,t} < F_e^{-1}(q), r_{f,t} > F_f^{-1}(1-q)\right\}}{P\left\{r_{e,t} < F_e^{-1}(q)\right\}} = \lim_{q \to 0} \frac{q - C(q, 1-q)}{q}, \quad (11)$$

$$\lambda^{+-} = \lim_{q \to 0} \frac{P\left\{r_{e,t} > F_e^{-1}(1-q), r_{f,t} < F_f^{-1}(q)\right\}}{P\left\{r_{e,t} > F_e^{-1}(1-q)\right\}} = \lim_{q \to 0} \frac{q - C(1-q, q)}{q}. \quad (12)$$

Like the threshold correlation, the "lower-upper" indicates that equity suffers a downturn and the corresponding currency depreciates; thereby a US investor suffers a dual loss. If the copula has an analytic solution, the coefficients can be calculated easily. In our application, the Student's t copula is applied to compute tail dependence. The copula-based tail dependence test (see McNeil et al., 2005) provides the statistically significant evidence of asymmetric tail dependence for 6 out of 12 countries at the 5% significance level in Table 4. Note that, we find that the asymmetric tail dependence is more evident in the emerging markets. This asymmetry is also illustrated in Figure 2, which shows the evolution of tail dependence based on a 5-year rolling window estimates.[3]

Overall, the test results of asymmetric dependence structure extend the symmetric finding of Ning (2010) and are consistent with the empirical findings of other asset classes (see Okimoto, 2008; Patton, 2004, 2006).

[INSERT FIGURE 2 AND TABLE 4 ABOUT HERE]

[3] We use a rolling window to estimate time-varying tail dependence following Eckernkemper (2018). It is worth noting that the tail dependence significantly increased in all countries after the financial crisis in 2008, which highlights the importance of tail dependence modeling for foreign investments in recent years.

2.4. Time-varying Dependence Structure

In this section we investigate whether the dependence structure is time-varying. We conduct three tests for structural breaks in the rank correlation. Table 5 presents the test results.

[INSERT TABLE 5 ABOUT HERE]

First, we test whether breaks occur at some specified points in the sample period. We follow the naïve test as in Patton (2013) and assume three arbitrary break points, at $t^*/T \in 0.15, 0.50, 0.85$. These dates correspond to April-1-2002, June-2-2007 and September-28-2012, respectively. The results show that each country has at least one significant break point. This indicates the evidence against the constant rank correlation. We further test whether the rank correlation between equity and forex statistically changes after the financial crisis breaks out. We assume that the first breakpoint is September 15, 2008 (the collapse of Lehman Brothers). This break point is statistically significant for eight equity-currency pairs (see the result in the column "US Crisis") at the 5% significance level. The second break point considered is on January 01, 2010 (European sovereign debt crisis).[4] Not surprisingly, this break is significant for the equity-forex pair of the EU. It is also significant for several other markets. (see the result in the column "EU Crisis").

Second, we consider another test for time-varying dependence, as in Patton (2013). This test is based on Engle (1982)'s ARCH LM test of time-varying volatility. Test results, reported in columns AR(1)-(3), provide solid evidence against constant correlation.

Finally, the right-hand-side column of Table 5 shows the test results based on the generalized break test without *a priori* point proposed by Andrews and Ploberger (1994). The results indicate that all the equity-forex pairs experience at least one break point.

Overall, all the test results show solid evidence that the dependence structure is time-varying for most equity-currency pairs. As Christoffersen and Langlois (2013) show that risk management based on a constant dependence structure is dangerous, the time-varying dependence structure is of particular importance in risk management of foreign investments.

3. Modeling Dependence Structure

In this section, we model the dependence structure between equity and forex using the time-varying asymmetric copula (TVAC) which combines a new skewed t copula based on the multivariate skewed t distribution of Bauwens and Laurent (2005) and the GAS model of Creal et al. (2013). We also compare our proposed copula with alternative copulas using Goodness-of-Fit tests.

3.1. Time-varying Asymmetric Copula

First, we assume that the asymmetric dependence structure between standardized equity and forex returns in Equations (3) and (4), $\boldsymbol{z}_t = (z_{e,t}, z_{f,t})'$, can be captured by the skewed t copula based on the bivariate skewed t distribution of Bauwens and Laurent (2005).[5] Let the probability integral transform (PIT) of the standardized return be $u_{k,t} = F_{\lambda_k,\nu_k}(z_{k,t})$ for $k = e, f$. Then we can write our TVAC model as

$$C(\boldsymbol{u}_t|\boldsymbol{P}_t, \boldsymbol{\xi}, \eta) = F_{\boldsymbol{\xi},\eta}\left(F^{-1}_{\xi_e,\eta}(u_{e,t}), F^{-1}_{\xi_f,\eta}(u_{f,t})\right), \ t = 1, \ldots, T, \tag{13}$$

[4]Several EU countries were affected by the European sovereign debt crisis after 2009. Thus, we arbitrarily assume that January 1, 2010, is a breakpoint.

[5]Christoffersen et al. (2012) use a skewed t copula based on the multivariate skewed t distribution of Demarta and McNeil (2005) and Lucas et al. (2014) use one based on the multivariate generalized hyperbolic t distribution. By contrast, Wang et al. (2013) and Fei et al. (2017) use a Markov switching model to capture the asymmetric dependence structure.

where C is the skewed t copula of $\boldsymbol{u}_t = (u_{e,t}, u_{f,t})'$, $F_{\boldsymbol{\xi},\eta}$ the cumulative distribution function (CDF) of the standardized bivariate skewed t distribution (with zero mean, correlation matrix $\boldsymbol{P}_t$, skewness parameter vector $\boldsymbol{\xi} = (\xi_e, \xi_f)'$ and degree of freedom η) and $F^{-1}_{\xi_k,\eta}$ the inverse CDF of the univariate skewed t distribution (with asymmetric parameter ξ_k and degree of freedom η). Note that the correlation matrix

$$\boldsymbol{P}_t = \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix} \tag{14}$$

is the correlation of copula shocks $x_{k,t} \equiv F^{-1}_{\xi_k,\eta}(u_{k,t}) = F^{-1}_{\xi_k,\eta}(F_{\lambda_k,\nu_k}(z_{k,t}))$ and not of the standardized return $z_{k,t}$ (see Christoffersen et al., 2012). The most general assumption is that both the skewness parameter and the degree of freedom are also time-varying. However, we find that their evolution is not well fitted by standard autoregressive processes.[6] Thus, following Christoffersen et al. (2012) and Christoffersen and Langlois (2013), we allow only the correlation matrix to evolve over time.

Based on the bivariate skewed t distribution of Bauwens and Laurent (2005), the probability density function (PDF) of the skewed t copula is given by

$$c(\boldsymbol{u}_t|\boldsymbol{P}_t,\boldsymbol{\xi},\eta) = \frac{f_{\boldsymbol{P_t},\boldsymbol{\xi},\eta}\left(F^{-1}_{\xi_e,\eta}(u_{e,t}), F^{-1}_{\xi_f,\eta}(u_{f,t})\right)}{\prod_{k=e,f} f_{\xi_k,\eta}\left(F^{-1}_{\xi_k,\eta}(u_{k,t})\right)} \tag{15}$$

$$= \frac{\Gamma\left(\frac{\eta+2}{2}\right)\Gamma\left(\frac{\eta}{2}\right)}{|\boldsymbol{P}_t|^{\frac{1}{2}}\,\Gamma\left(\frac{\eta+1}{2}\right)^2} \left\{ \frac{\left(1+\frac{\boldsymbol{a}(x_{e,t},x_{f,t})'\boldsymbol{a}(x_{e,t},x_{f,t})}{\eta-2}\right)^{-\frac{\eta+2}{2}}}{\prod_{k=e,f}\left(1+\frac{a_k(x_{k,t})^2}{\eta-2}\right)^{-\frac{\eta+1}{2}}} \right\} \tag{16}$$

where

$$\boldsymbol{a}(x_{e,t},x_{f,t}) = (a_e(x_{e,t}), a_f(x_{f,t}))',\ a_k(x_{k,t}) = \left(s_k x^*_{k,t} + m_k\right)\lambda_k^{I_{k,t}},$$
$$\boldsymbol{x}_t = (x_{e,t}, x_{f,t})',\ \boldsymbol{x}^*_t = \left(x^*_{e,t}, x^*_{f,t}\right)' = \boldsymbol{P}_t^{-1/2}\boldsymbol{x}_t,$$

$$m_k = \frac{\sqrt{\eta-2}\,\Gamma\left(\frac{\eta-1}{2}\right)\left(\xi_k - \xi_k^{-1}\right)}{\sqrt{\pi}\left(\frac{\eta}{2}\right)},\ s_k = \left(\xi_k^2 + \xi_k^{-2} - 1\right) - m_k^2, \text{and } I_{k,t} = \begin{cases} 1 & \text{if } x^*_{k,t} \geq -\frac{m_k}{s_k} \\ -1 & \text{if } x^*_{k,t} < -\frac{m_k}{s_k} \end{cases}.$$

Therefore, the skewed t copula has the form of

$$C(\boldsymbol{u}_t|\boldsymbol{P}_t,\boldsymbol{\xi},\eta) = \int_{-\infty}^{F^{-1}_{\xi_e,\eta}(u_{e,t})}\int_{-\infty}^{F^{-1}_{\xi_f,\eta}(u_{f,t})} \frac{\Gamma\left(\frac{\eta+2}{2}\right)\Gamma\left(\frac{\eta}{2}\right)}{\sqrt{1-\rho_t^2}\,\Gamma\left(\frac{\eta+1}{2}\right)^2} \left\{ \frac{\left(1+\frac{\boldsymbol{a}(s_e,s_f)'\boldsymbol{a}(s_e,s_f)}{\eta-2}\right)^{-\frac{\eta+2}{2}}}{\prod_{k=e,f}\left(1+\frac{a_k(s_k)^2}{\eta-2}\right)^{-\frac{\eta+1}{2}}} \right\} ds_f ds_e. \tag{17}$$

Next, the time-varying dependence structure is implemented by the generalized autoregressive score (GAS) model of Creal et al. (2013), which is an observation-driven model based on a score function and lagged copula parameters. It has several advantages over other observation-driven models in the literature. For example, Christoffersen and Langlois (2013) rely on the dynamic conditional correlation (DCC) model of Engle (2002). However, DCC cannot properly take into account the fat-tailedness and skewness of returns; therefore time-varying volatility

[6] We have observed extremely large estimates of the skewness parameter or degrees of freedom parameter when dot.com bubble, mortgage crisis, and sovereign debt crisis occur in the global market. With these outliers, it is not easy to fit the evolution of these parameters by a standard autoregressive process.

and correlations could have biased estimates (Lucas et al., 2014). On the other hand, GAS is able to provide a coherent approach to deal with such characteristics. Thus the estimates of time-varying volatility and correlations can become more robust to abnormal observations than others.

We allow the correlation parameter (ρ_t) to vary over time according to the GAS dynamics, holding other parameters constant. We use the strictly increasing transformation function of ρ_t,

$$g_t = \log(1-\rho_t) - \log(1+\rho_t) \Leftrightarrow \rho_t = \frac{1-e^{-g_t}}{1+e^{-g_t}}, \tag{18}$$

to ensure that $\rho_t \in (-1,1)$; i.e.,

$$\lim_{g_t \to \infty} \rho_t = 1 \text{ and } \lim_{g_t \to -\infty} \rho_t = -1. \tag{19}$$

Following Creal et al. (2013) and Patton (2013), we specify the evolution of transformed copula correlation by

$$g_{t+1} = \omega + \delta\left(\frac{s_t}{q_t}\right) + \varphi g_t,\ t = 0, \dots, T-1, \tag{20}$$

where ω is a constant and s_t/q_t the standardized score of the copula log-likelihood in which $s_t \equiv \partial \log c(\boldsymbol{u}_t|0, \boldsymbol{P}_t)/\partial\rho_t$ and $q_t^2 \equiv \boldsymbol{E}_{t-1}[s_t^2]$.

We apply a two-stage maximum likelihood estimator (MLE) to estimate our proposed copula model (see Joe and Xu, 1996; Joe, 1997, 2005). First, we estimate the parameters of univariate models by maximizing the log-likelihood:

$$\hat{\boldsymbol{\theta}}_k = \underset{\boldsymbol{\theta}_k}{\operatorname{argmax}} \sum_{t=1}^{T} \log f_{\lambda_k,\nu_k}(z_{k,t}|\mathcal{F}_{k,t-1};\boldsymbol{\theta}_k),\ k = e, f \tag{21}$$

where $\boldsymbol{\theta}_k = (\phi_{0,k}, \phi_{1,k}, \alpha_k, \gamma_k, \beta_k, \lambda_k, \nu_k)'$. See Equation (3) – (5) for the univariate model. Then we compute PIT, $\hat{u}_{k,t} = F_{k,\lambda_k,\nu_k}\left(\hat{z}_{k,t}|\mathcal{F}_{k,t-1};\hat{\boldsymbol{\theta}}_k\right)$, using the estimates from the first stage, and estimate the copula parameters by maximizing the log-likelihood:

$$\hat{\boldsymbol{\Theta}} = \underset{\boldsymbol{\Theta}}{\operatorname{argmax}} \sum_{t=1}^{T} \log c(\hat{u}_{e,t}\hat{u}_{f,t}|\mathcal{F}_{t-1};\boldsymbol{\Theta}). \tag{22}$$

where $\boldsymbol{\Theta} = (\xi, \eta, \omega, \delta, \varphi)'$.

Alternatively, we apply a two-stage semiparametric estimation (also known as canonical maximum likelihood estimator) (see Chen and Fan, 2006a,b). First, we estimate $\hat{\boldsymbol{\theta}}_k$ by MLE in Equation (21) and nonparametrically estimate the empirical distribution function (EDF) of $\hat{z}_{k,t}$:

$$\hat{F}_k(z) \equiv \frac{1}{T+1}\sum_{t=1}^{T} 1\{\hat{z}_{k,t} \le z\}. \tag{23}$$

Then we compute the PIT, $\hat{u}_{k,t} = \hat{F}_k(\hat{z}_{k,t})$, and estimate the copula parameters by Equation (22).

In our empirical analysis, we report results obtained by the semiparametric estimation in the main analysis because it is recognized as a more robust estimation than the parametric one in many empirical studies. Also, Chen and Fan (2006a) show that the estimated conditional quantile functions based on semiparametric copulas are automatically monotonic across different quantiles, which is attractive for the estimation of ES. For a robustness check, we also report results obtained by the parametric estimation in Section 4.2.3.

We plot the time-varying skewed t copula correlation between equity and forex implied by the GAS model in Figure 3. It is evident that the copula correlations fluctuate greatly over time. The positive correlation during the pre-crisis period switched sign in the EU and UK, after the 2008 global financial crisis. This implies that after the crisis the Euro or the Pound no longer provided a hedging function for currency risk in foreign investments.

[INSERT FIGURE 3 ABOUT HERE]

3.2. Tests of Goodness-of-Fit

We examine how closely our proposed copula model fits the bivariate probability distribution of equity and forex using two in-sample tests of goodness-of-fit (GoF). One is the Kolmogorov-Smirnov (KS) test and the other is the Cramer-von Mises (CvM) test. We follow a testing procedure proposed by Patton (2013).

The Panel A of Table 6 reports the p-values of the GoF tests for four different copula models; constant t copula, constant skewed t copula, time-varying t copula and our proposed time-varying skewed t copula. The p-values are based on 100 simulations. We make two notable observations. First, asymmetric copula models have higher p-values than symmetric ones. Second, time-varying copula models have considerably higher p-values than those of constant ones. More specifically, the constant t copula is rejected for eight countries while the skewed one is rejected for four countries at the 5% significance level.[7] However, time-varying t and skewed t copulas are not rejected for any countries. Therefore, the overall test results show that our proposed TVAC is more appropriate for fitting the bivariate probability distribution of our sample than other copula ones.[8]

The Panel B of Table 6 reports the log likelihood values for two constant copula and two time-varying copula models, and the p-values for likelihood ratio test. Clearly, the likelihood values of time-varying copulas are significantly higher than the values of constant copulas. "LR test" reports the p-values of likelihood ratio test of model specification. We use this test to assess whether our data provides enough evidence to favor the unrestricted model (TV SkT) over the restricted model (SkT). As can be seen from Table 6, all the p-values are close to 0, which indicates that there is strong evidence suggesting that time-varying skewed t copulas fit the data better than constant skewed t copulas.

[TABLE 6 ABOUT HERE]

4. Algorithm for Forecasting Tail Risk

In this section, we investigate whether modelling the dependence structure between equity and forex helps forecast the tail risk of foreign investments. We carry out the out-of-sample forecasts of value-at-risk (VaR) and expected shortfall (ES). Then we conduct backtesting to compare predicted losses from the tail risk forecasts. In order to evaluate the coverage ability and statistical accuracy of VaR forecasts, we employ three widely used methods of backtesting; the empirical coverage probability (ECP), the conditional coverage test (CC test; Christoffersen, 1998), and the dynamic quantile test (DQ test; Engle and Manganelli, 2004). In addition, we employ a mean absolute error (MAE) test to evaluate the predictive loss from the ES forecasts. (See the Appendix for details on backtesting.)

We use a rolling window forecast which sets a window size of 250 days. The copula-based forecasting procedure is as follows:

STEP-1 We estimate the AR-GJR-GARCH model for each return series. Then, we predict one-day-ahead conditional mean and conditional volatility from the prespecified time series model on rolling window for each margins.

[7] The model is rejected if it is rejected by either the KS test or the CvM test.

[8] The time-varying model seems to have more influence on fitting the bivariate probability distribution than the asymmetric model.

STEP-2 We estimate probability integral transforms for each forecasted margin using the univariate skewed t distribution and EDF of standardized residuals over the past 250 days.

STEP-3 We parametrically and semiparametrically estimate copula models using standardized returns obtained in STEP-1. We set the estimated copula parameters as initial values and estimate time-varying copula parameters implemented by the GAS model. Then we obtain the one-day-ahead forecast of copula parameters.

STEP-4 With the forecast of copula parameters in hand, we carry out Monte Carlo simulation to generate one-day-ahead multivariate innovations. We revert them to returns using the forecast of parameters for the marginal probability distribution in STEP-1. Then we get one-day-ahead portfolio returns such that

$$r_{t+1}^{(b)} = r_{e,t+1}^{(b)} - r_{f,t+1}^{(b)}, \; b = 1, 2, \ldots, B, \tag{24}$$

where $r_{e,t+1}^{(b)}$ and $r_{f,t+1}^{(b)}$ denote simulated equity and forex returns, respectively. We iterate this sampling procedure 1000 times.

STEP-5 Given the empirical distribution of 1000 simulated portfolio returns,

$$\left\{ r_{t+1}^{(1)}, r_{t+1}^{(2)}, \ldots, r_{t+1}^{(B)} \right\}, \tag{25}$$

we obtain the VaR and ES corresponding to a nominal rate α.

4.1. Main Analysis

We focus on three particular issues. First, we compare the univariate filtered historical simulation model (FHS; Baron-Adesi et al., 2002)[9], which does not explicitly model the dependence structure between equity and forex, with our proposed copula models. From this comparison, we can see forecasting improvements by explicitly modelling the dependence structure. Second, we compare a multivariate GARCH, which models a time-varying linear dependence, with time-varying copula models.[10] From this comparison, we can see forecasting improvements when a nonlinear dependence in tails is taken into account. We employ the dynamic conditional correlation model (DCC; Engle, 2002) as the multivariate GARCH.[11] Third, we consider four copula models including constant t copula (T), constant skewed t copula (SkT), time-varying t copula (TV T) and time-varying skewed t copula (TV SkT) and compare them. This comparison verifies how modelling the time-varying dependence or the asymmetric dependence contributes to improving forecasting performance.[12] The testing period is from December 18, 2000, to

[9]We choose this model because it is the most successful univariate VaR models.

[10]We thank for reviewer's careful comment on the use of this term. It is true that the correlations we are modeling in multivariate DCC-GARCH only capture linear dependencies, but it is also true that their evolution is nonlinear. Indeed, the multivariate DCC-GARCH are nonlinear time-series models, in the sense that the innovations in their Wald decomposition are not i.i.d (Tsay, 2010). Multivariate DCC-GARCH allows a different dependence at the time dimension, e.g. normal time vs. extreme time, but not at location dimension, e.g. center of probability distribution vs. tail of probability distribution. On the other hand, time-varying copula allows both of these. Therefore, if a model does not allow the different dependence across the location, we use the expression 'linear dependence' in our paper.

[11]We also evaluate other multivariate GARCH models, such as BEEK (Engle and Kroner, 1995) and CCC (He and Teräsvirta, 2004). We find that DCC shows more stable estimation results and better forecasting performance than others.

[12]Many different copulas exist for modelling the dependence structure. However, because our study is interested in modelling dependence structure between equity and forex, the most commonly used t-copula family in financial time series modelling should be an appropriate choice for our study. The horse-race of various copulas is outside the scope of our study.

December 31, 2014, and the total number of out-of-sample forecasts is 3,663.

4.1.1. VaR Forecasts

Prior to backtesting, we investigate how closely the VaR forecasts from different approaches track with one another. To this end, we plot the time series of the VaR forecasts for the EU market, which has the greatest trading volume in our sample. This visualization can help us understand the backtesting results, which is discussed below.

Figure 4 plots the VaR forecasts of TV SkT and FHS. From the beginning of 2000 to the time before the financial crisis in 2008, FHS under-forecasts the VaR compared to TV SkT. However, this under-forecast becomes negligible in the post-crisis period. An explanation of this change is in Figure 3. Equity and forex are positively correlated before the financial crisis; when equity experiences a downturn, the currency depreciates. As a consequence, the forex creates further risk for foreign investments because of the lack of a hedging function. In this case, it is likely to underestimate the tail risk of foreign investments unless the dependence structure is explicitly modelled. After the financial crisis, the correlation changes negatively; therefore, forex can provide a hedging function. In this case, FHS would underestimate the tail risk less because the equity loss can be cancelled out by currency appreciation.

[INSERT FIGURE 4 ABOUT HERE]

Next, Figure 5 plots the VaR forecast of TV SkT and DCC. Overall, DCC under-forecasts the VaR compared to TV SkT. In particular, the gap between two models during the dot.com bubble in early 2000 or the global financial crisis in 2008-2010 is more pronounced. The reason is that modelling the dependence structure using a linear dependence cannot reflect the nonlinearity in both tails. This nonlinearity is more pronounced during the crisis period.

[INSERT FIGURE 5 ABOUT HERE]

Figure 6 plots the VaR forecasts of symmetric and asymmetric copulas and Figure 7 plots those of constant and time-varying copulas, respectively. As shown in the figures, the VaR forecasts vary slightly between symmetric and asymmetric copulas and between constant and time-varying ones. The differences do not seem to be large. Thus, after we model the dependence structure in any form, it accounts for much of the tail risk. Then asymmetric or time-varying modelling can complement the rest.

[INSERT FIGURE 6 and 7 ABOUT HERE]

For a more specific analysis, we investigate the failures of VaR forecasts in Table 7. A failure is defined as an event in which a realized loss is not covered by the VaR forecast. From the beginning of 2000 to 2009, we find that FHS fails to cover the loss more frequently than TV SkT. This result is closely related to that FHS under-forecasts the VaR, as seen in Figure 4. Analogously, DCC also fails to cover the loss more frequently than TV SkT in most of the sample period. The comparison between TV SkT and DCC is visually presented in Figure 8. When the time-varying dependence is reflected (TV T or TV SkT), the overall number of failures is slightly less than that in constant copulas (T or SkT).

[INSERT TABLE 7 and FIGUE 8 ABOUT HERE]

In summary, the overall results suggest that the explicit modelling of the dependence structure between two assets and the nonlinear dependence in both tails are of paramount importance for forecasting the tail risk of foreign investments.

4.1.2. Backtesting

Following the analysis of the VaR forecast, we conduct backtesting to measure the accuracy of tail risk forecasts.

First, Table 8 reports ECP. All the copula models have lower RMSEs than FHS and DCC. More importantly, ECPs of FHS and DCC are higher than 1% for all countries, while we cannot find such a tendency in the copula models. This implies that those two models tend to underforecast the VaR systematically. As shown in the previous analysis, this is because the dependence structure is not explicitly modelled in FHS and the nonlinear dependence in the tails is ignored in DCC.

Next, Table 9 reports the results of the CC test. The copula models are rejected for two countries at most. In particular, TV SkT is not rejected for any countries. On the other hand, DCC is rejected for all countries and FHS is rejected for three countries. In Table 10, the number of rejections for the DQ test overall exceed that for the CC test, but the statistical conclusions of the DQ test are consistent with the CC test.

Last, Table 11 reports the mean absolute error (MAE) of the ES forecasts. (See Appendix how we measure MAE.) When we explicitly model the dependence structure, the forecasting error is smaller than it is otherwise. However, the assumption of linear dependence does not help to improve the tail risk forecast as shown in DCC. Therefore, we reach the same conclusions as the VaR forecast.

[INSERT TABLE 8, 9, 10 and 11 ABOUT HERE]

We can derive the following implications from the backtesting results: First, economic losses could occur in risk management when investors ignore the dependence structure between equity and forex. Second, the modelling of dependence structure relying on a linear dependence cannot help in forecasting the tail risk of foreign investments. Third, when we switch from the symmetric copula (the constant one) into the asymmetric one (the time-varying one), forecasting might improve. Overall, our tail risk forecasting exercise demonstrates that the dependence structure between equity and forex plays an important role in robust risk management of foreign investments.

4.2. Robustness Analysis

In this section we consider a couple of sensitivity issues in the tail risk forecast of foreign investments and provide robustness analyses to validate conclusions from our main analysis.

4.2.1. Crisis and Post-crisis Analysis

We investigate two sub-testing-periods: crisis (2007 - 2010) and post-crisis (2011 - 2014). The crisis period includes major US/EU financial crises such as the collapse of Lehman Brothers in September 2008 and EU sovereign debt crisis in January 2010. This sub-period test helps us understand how the dependence structure between equity and forex changed after the crisis and how it affected the tail risk of foreign investments.

Panels A and B of Table 12 present the backtesting results for the crisis and post-crisis periods. Compared to the full sample analysis, it is not surprising that the performance of all models is found to be degraded. This is driven mainly by unforeseen extreme events during the crisis period and increased market volatility after the crisis. Nevertheless, we find that all the findings from the full sample period are still valid in both periods. This suggests that modelling the dependence structure is important regardless of whether the condition of international financial markets is normal or extreme.

4.2.2. Longer Window Size

Since market risk is sensitive to change in the underlying market regime, over- (or under-) fitting problems may occur in a longer window size. Hence, we normally choose 180 or 250 days as a window size for estimation in the market risk analysis. On the other hand, if no sudden structure break or regime change occurs in the estimation period, we can obtain more accurate parameter estimates with a longer window size; the forecasting performance can thereby be improved as well. For this reason, we conduct a robustness analysis with a window size of 500 days (two-year).

Panel C of Table 12 summarizes the backtesting results. Comparing the results with those of 250 days, we find that overall performance slightly improves in all models. In particular, the DQ test shows that TV SkT is rejected only for Russia and its number of rejection decreases dramatically compared to four countries in the 250-day window size. The overall backtesting results are consistent with the 250 days and TV SkT is the best performing model. Therefore, our main conclusion is robust to the longer window size.

4.2.3. Parametric Estimation

Our proposed copula models are semiparametrically estimated because the semiparametric conditional quantile estimator has very attractive property for the estimation of ES (Chen and Fan, 2006a). To further check the robustness of our results, we summarize the results by the parametric estimation in this section.

Panel D of Table 12 summarizes the backtesting results when we estimate the copula models parametrically. Overall, ECP or the number of rejections slightly increases in some models, but there are no significant differences from the semiparametric estimation. Overall, the conclusions drawn from the main analysis are confirmed to be very robust to the estimation strategy.

4.2.4. Nominal Probability

In the main analysis, we evaluated $\alpha = 1\%$ (i.e., 99% VaR and ES), which has been used conventionally in market risk management. However, in some cases, we use a less conservative nominal probability, such as $\alpha = 5\%$. In this case, models relying on a linear dependence, such as MGARCH, may perform reasonably because the sensitivity of VaR or ES to tail dependence tends to decrease. In order to investigate this issue, we run backtesting in the 95% VaR and ES to measure predictive losses. Panel E of Table 12 summarizes the backtesting results. As we expected, the overall performance of DCC improves compared to the 1% nominal probability. Nevertheless, TV SkT still outperforms DCC and FHS under this less conservative nominal probability.

[INSERT TABLE 12 ABOUT HERE]

5. Concluding Remarks

In this paper, we revisit the dependence structure between equity and foreign exchange markets. In particular, we investigate whether modelling the dependence structure can help forecast the tail risk of foreign investments. Studying 12 major developed and emerging markets over the sample period 2000 - 2014, we find that the lower tail dependence is significantly greater than the upper tail one in many countries, suggesting that equity and foreign exchange markets tend to move together more closely during a crash period. We find solid evidence against the constant dependence structure and further verify that the time-varying and asymmetric dependence structure has become stronger through the crisis period.

To capture the characteristics of dependence between the two financial markets, we propose a time-varying asymmetric copula (TVAC) model which combines the skewed t copula from Bauwens and Laurent (2005) and the GAS model of Creal et al. (2013). Both parametric and semiparametric methods are applied. The empirical results show that our model has consistently better fitness than others. We further demonstrate the importance of modelling the dependence structure in the risk management of foreign investments. The backtesting results show that neglecting the dependence structure causes significant predictive loss in the tail risk forecast. Our empirical results strongly suggest that international investors should model the dependence between equity and foreign exchange markets to make their risk management more robust.

In addition to the tail risk forecast of foreign investments, the dependence structure between equity and foreign exchange markets contains various implications for international finance. For example, "How does the dependence structure influence international asset pricing?" or "What is the economic cost generated by the dependence structure in the international asset allocation problem?" may be important questions for both academics and practitioners. We leave these research questions for future challenges.

Appendix: Backtesting

We first define the failure of VaR as the event that a realized return R_s is not covered by the VaR forecast. We identify it by the indicator function taking the value unity in the case of failure:

$$I_s = 1\left\{R_s < \widehat{VaR}_s\left(\alpha|\mathcal{F}_{s-1}\right)\right\},\ s = 1, \dots, N, \tag{A.1}$$

where $\widehat{VaR}_s\left(\alpha|\mathcal{F}_{s-1}\right)$ is the VaR forecast based on the information set at $s-1$, denoted by $\mathcal{F}_{s-1}$, with a nominal coverage probability α. Henceforth, we abbreviate the notation $\widehat{VaR}_s\left(\alpha|\mathcal{F}_{s-1}\right)$ to $\widehat{Var}_s\left(\alpha\right)$.

Empirical Coverage Probability (ECP) is calculated by the sample average of I_s,

$$ECP = \frac{1}{N}\sum_{s=1}^{N} I_s \tag{A.2}$$

which is a consistent estimator of the coverage probability. The VaR model for which ECP is closest to its nominal coverage probability is preferred.

Accurate VaR forecasts should satisfy the condition that the conditional expectation of the failure is the nominal coverage probability:

$$\mathbb{E}\left[I_s|\mathcal{F}_{s-1}\right] = \alpha. \tag{A.3}$$

Christoffersen (1998) shows that it is equivalent to testing if $I_s|\mathcal{F}_{s-1}$ follows an i.i.d. Bernoulli distribution with parameter α:

$$H_0 : I_s|\mathcal{F}_{s-1} \sim i.i.d.\ Bernoulli\left(\alpha\right). \tag{A.4}$$

The CC test uses the LR statistic which follows the chi-squared distribution with two degrees-of-freedom under the null hypothesis, Eq. (A.4).

$$CC \overset{d}{\sim} \chi_2^2 \tag{A.5}$$

The DQ test is a general extension of the CC test allowing for more time-dependent information of $\{I_s\}_{s=1}^{N}$. The out-of-sample DQ test is given by

$$DQ = \frac{\left(\tilde{\boldsymbol{I}}'\boldsymbol{Z}\right)\left(\boldsymbol{Z}'\boldsymbol{Z}\right)^{-1}\left(\boldsymbol{Z}'\tilde{\boldsymbol{I}}\right)}{\alpha\left(1-\alpha\right)} \overset{d}{\sim} \chi_{p+2}^2, \tag{A.6}$$

where $\tilde{\boldsymbol{I}} = \left(\tilde{I}_{p+1}, \tilde{I}_{p+2}, \dots, \tilde{I}_N\right)'$, $\tilde{I}_s = I_s - \alpha$, $\boldsymbol{Z} = \left(\boldsymbol{z}_{p+1}, \dots, \boldsymbol{z}_N\right)'$ and $\boldsymbol{z}_s = \left(1, \tilde{I}_{s-1}, \dots, \tilde{I}_{s-p}, \widehat{VaR}_s\left(\alpha\right)\right)'$. We use the first four lags for our evaluation, i.e., $\boldsymbol{z}_s = \left(1, \tilde{I}_{s-1}, \dots, \tilde{I}_{s-4}, \widehat{VaR}_s\left(\alpha\right)\right)'$.

The backtesting of ES is not a straightforward task because it fails to satisfy elicitability (see Gneiting, 2011). We consider a backtesting for the ES forecast given the sample of N ES forecasts,

$$\left\{\widehat{ES}_1\left(\alpha\right), \dots, \widehat{ES}_N\left(\alpha\right)\right\}, \tag{A.7}$$

where $\widehat{ES}_s\left(\alpha\right)$ is the ES forecast based on the information set at $s-1$. We simply evaluate the ES forecast based on a loss function which enables researchers to rank the models and specify a utility function reflecting the concern of the risk manager. We define a loss function by

$$\text{Absolute error} := \left|R_s - \widehat{ES}_s\left(\alpha\right)\right| I_s, \tag{A.8}$$

where $I_s = 1\left\{R_s < \widehat{VaR}_s\left(\alpha\right)\right\}$. In order to rank the models, we compute the mean absolute error (MAE):

$$MAE = \frac{1}{N}\sum_{s=1}^{N}\left|R_s - \widehat{ES}_s\left(\alpha\right)\right| I_s. \tag{A.9}$$

The smaller value indicates more accurate forecast.

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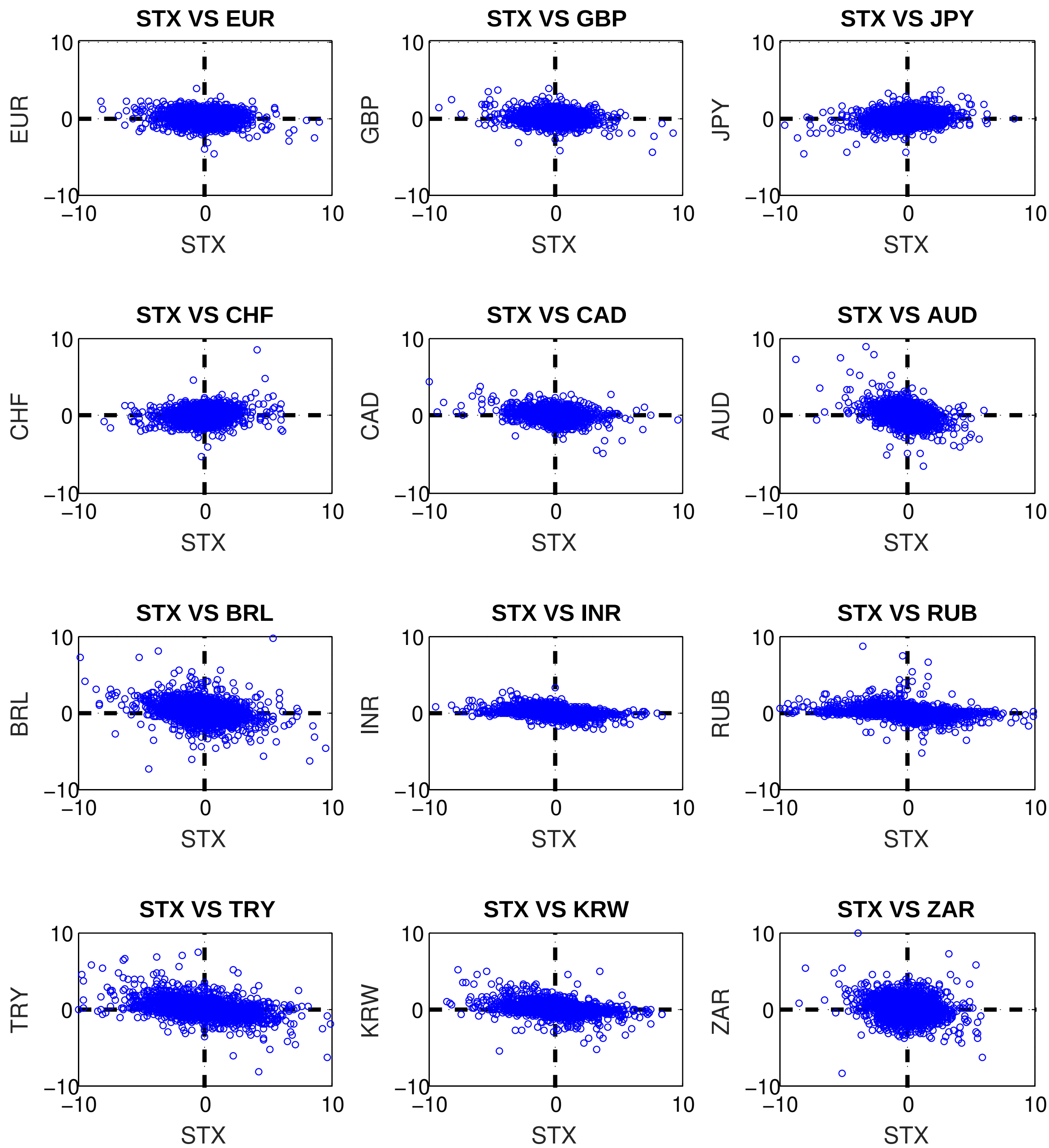

Figure 1: MSCI Country Equity Index and Corresponding Foreign Exchange Rate
Note: This figure shows the scatter plots for country equity index return and foreign exchange rate return pairs for the period from January 3, 2000 to December 31, 2014. STX indicates a MSCI equity index for each country.

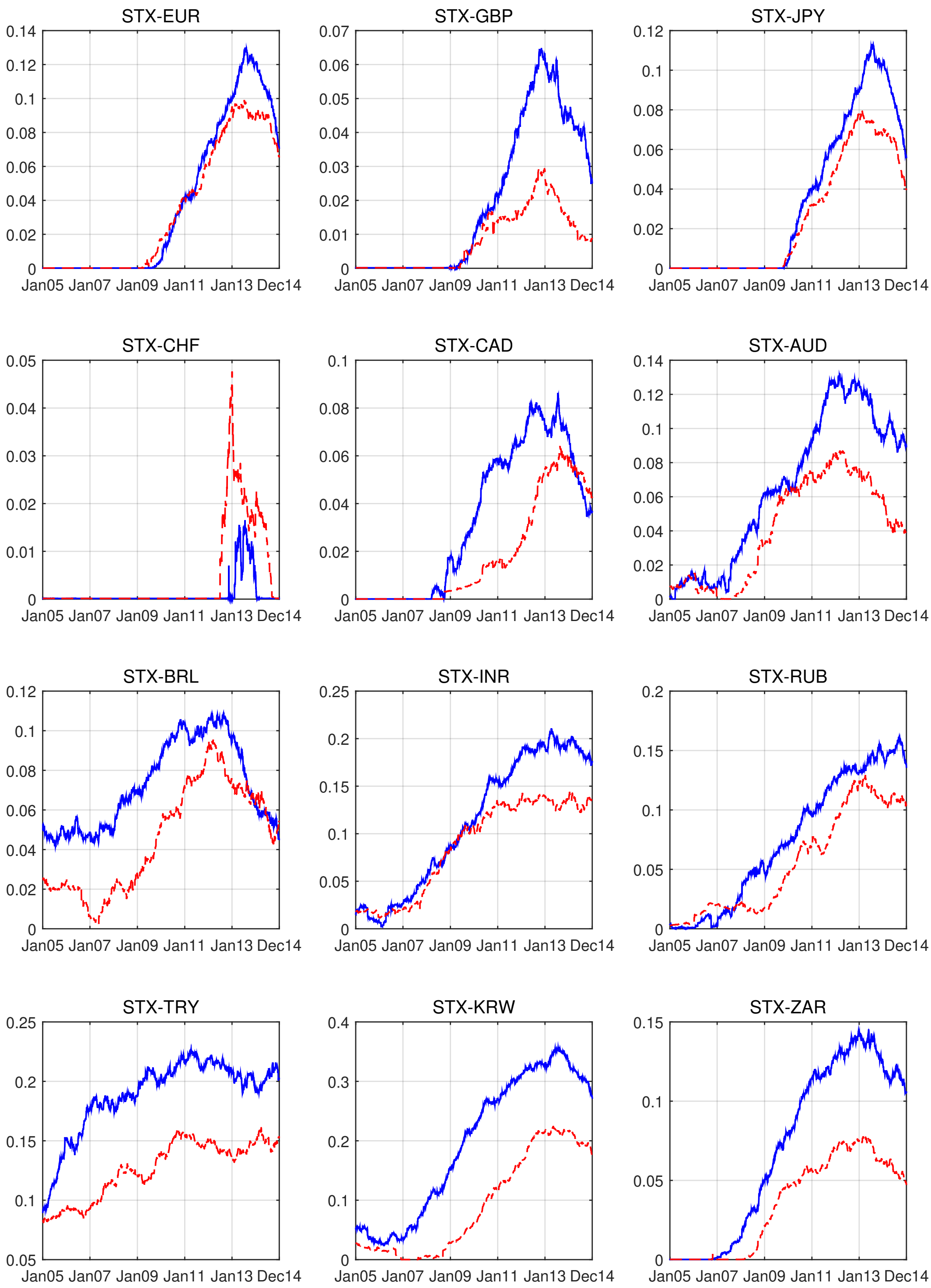

Figure 2: Dynamic Evolution of Tail Dependence

Note: This figure shows the time-varying tail dependence between country equity index return and foreign exchange rate return based on 5-year rolling window estimates of the Student's t copula. It presents the lower-upper (solid lines) and upper-lower (dashed line) tail dependence for the period from January 5, 2005 to December 31, 2014. STX indicates a MSCI equity index for each country.

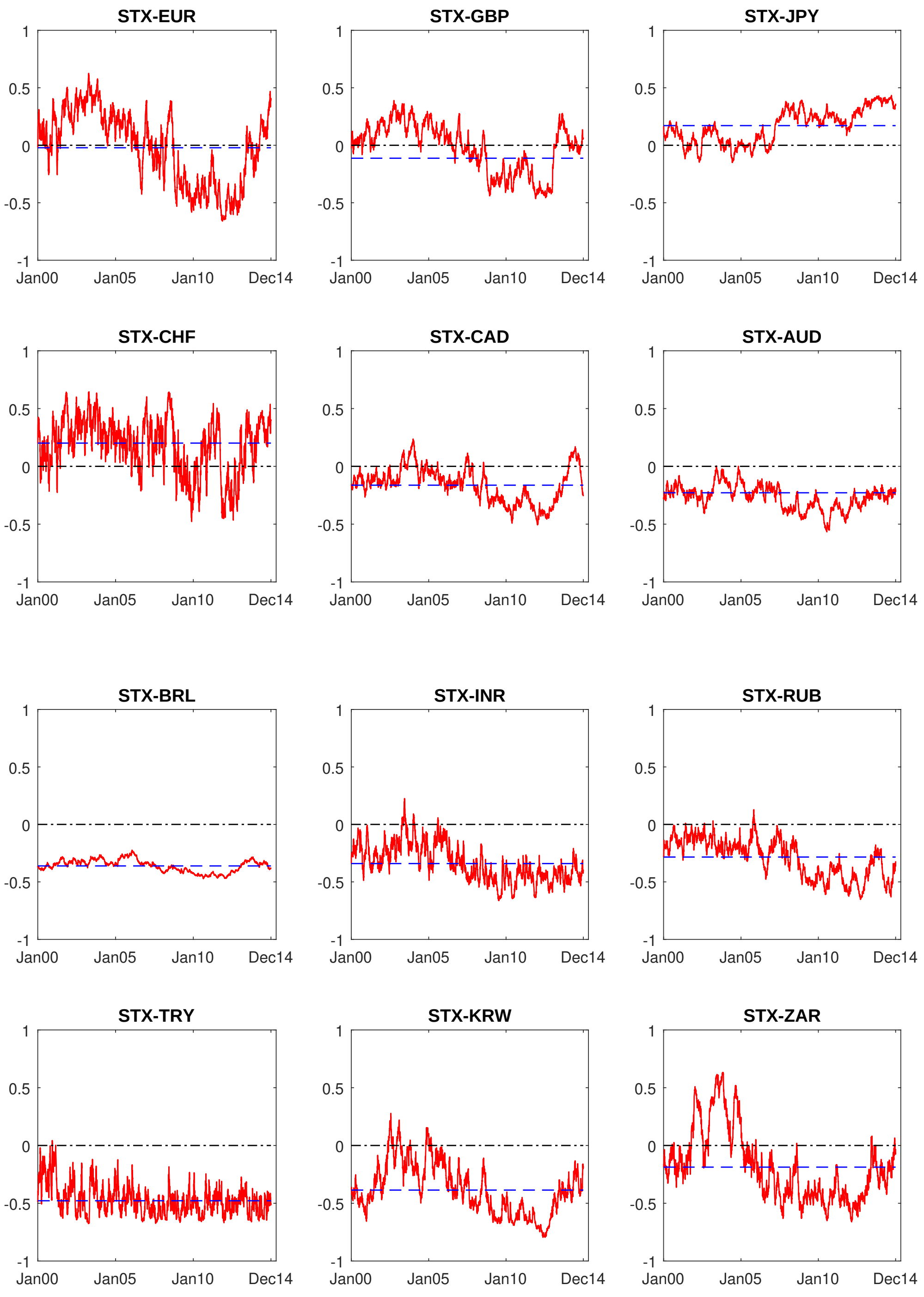

Figure 3: Dynamic Evolution of Dependence Structure

Note: This figure shows the time-varying copula correlations between country equity index return and foreign exchange rate return implied by the GAS model (solid line) and constant copula correlation (dashed line) for the period from January 3, 2000 to December 31, 2014. STX indicates a MSCI equity index for each country

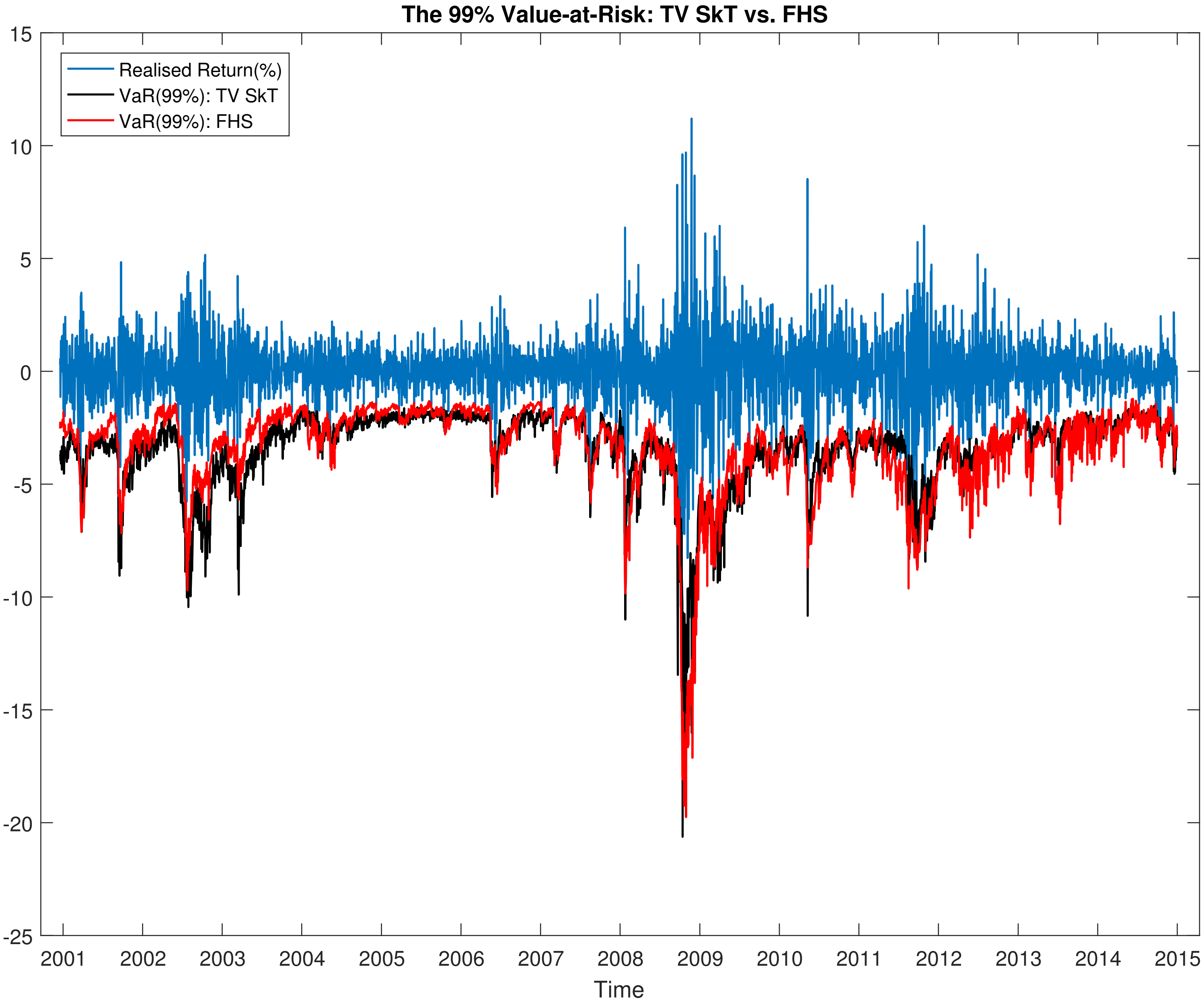

Figure 4: VaR forecasts: TV SkT and FHS (EU)

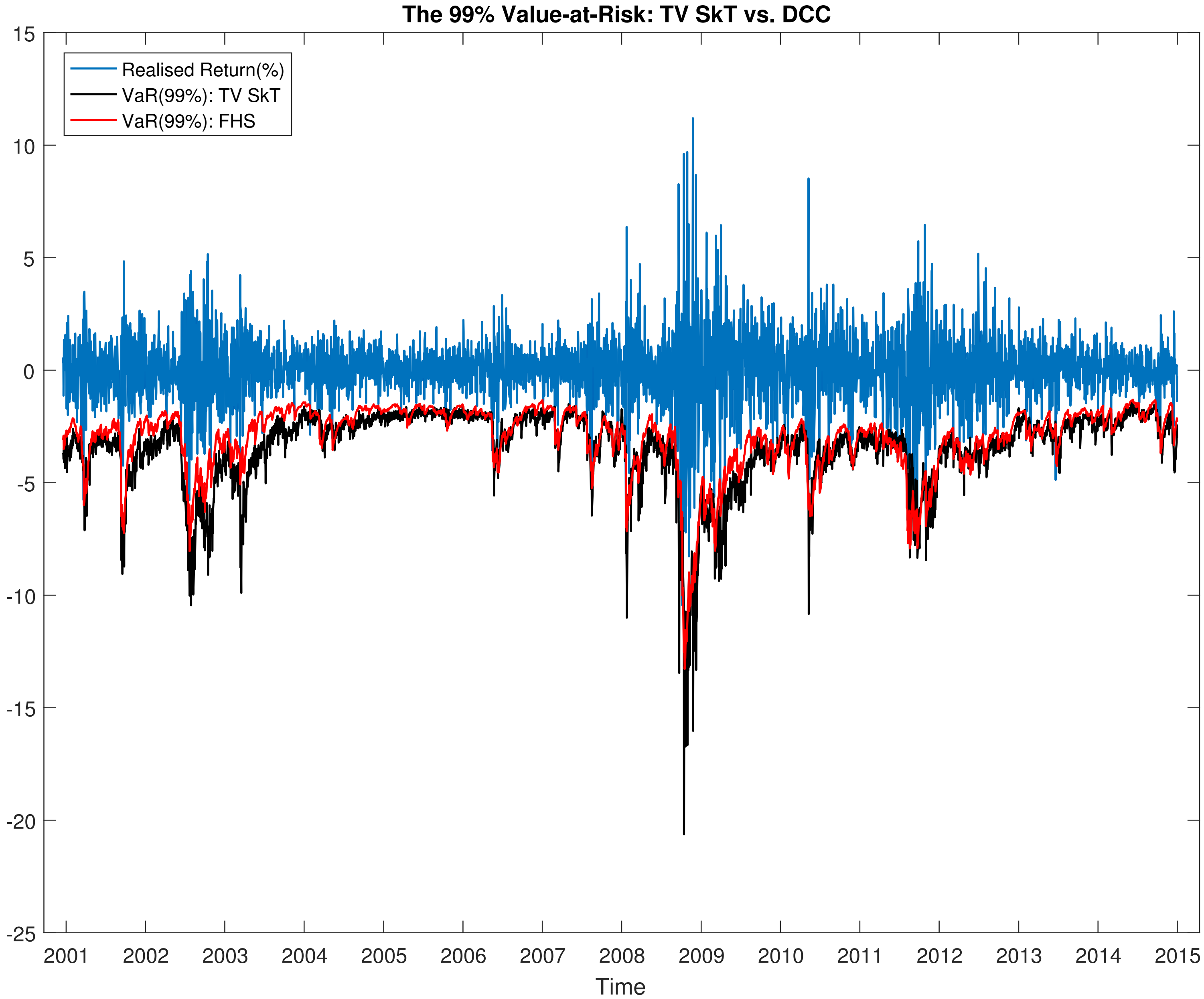

Figure 5: VaR forecasts: TV SkT and DCC (EU)

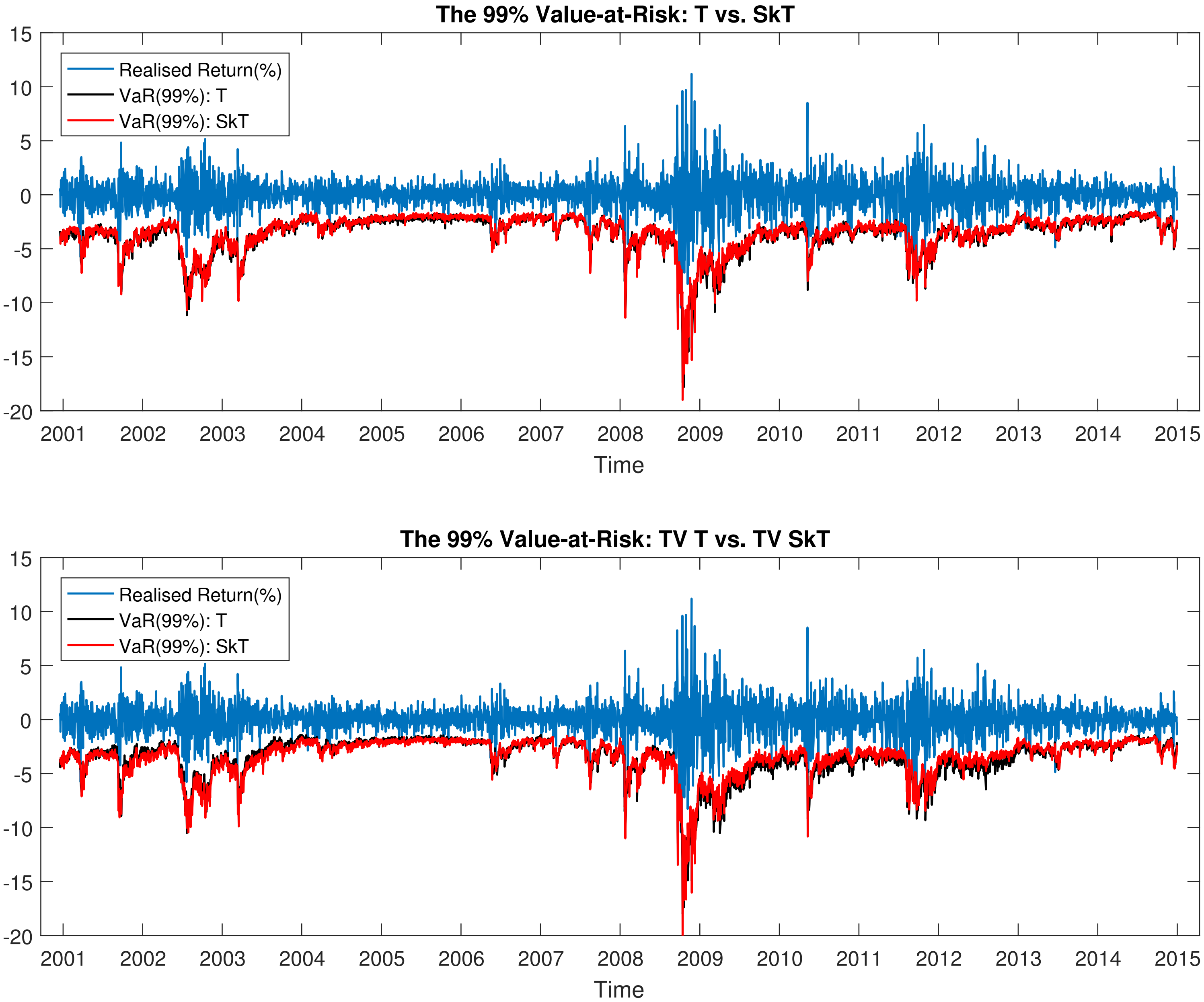

Figure 6: VaR forecasts: Symmetric copula and asymmetric copula (EU)

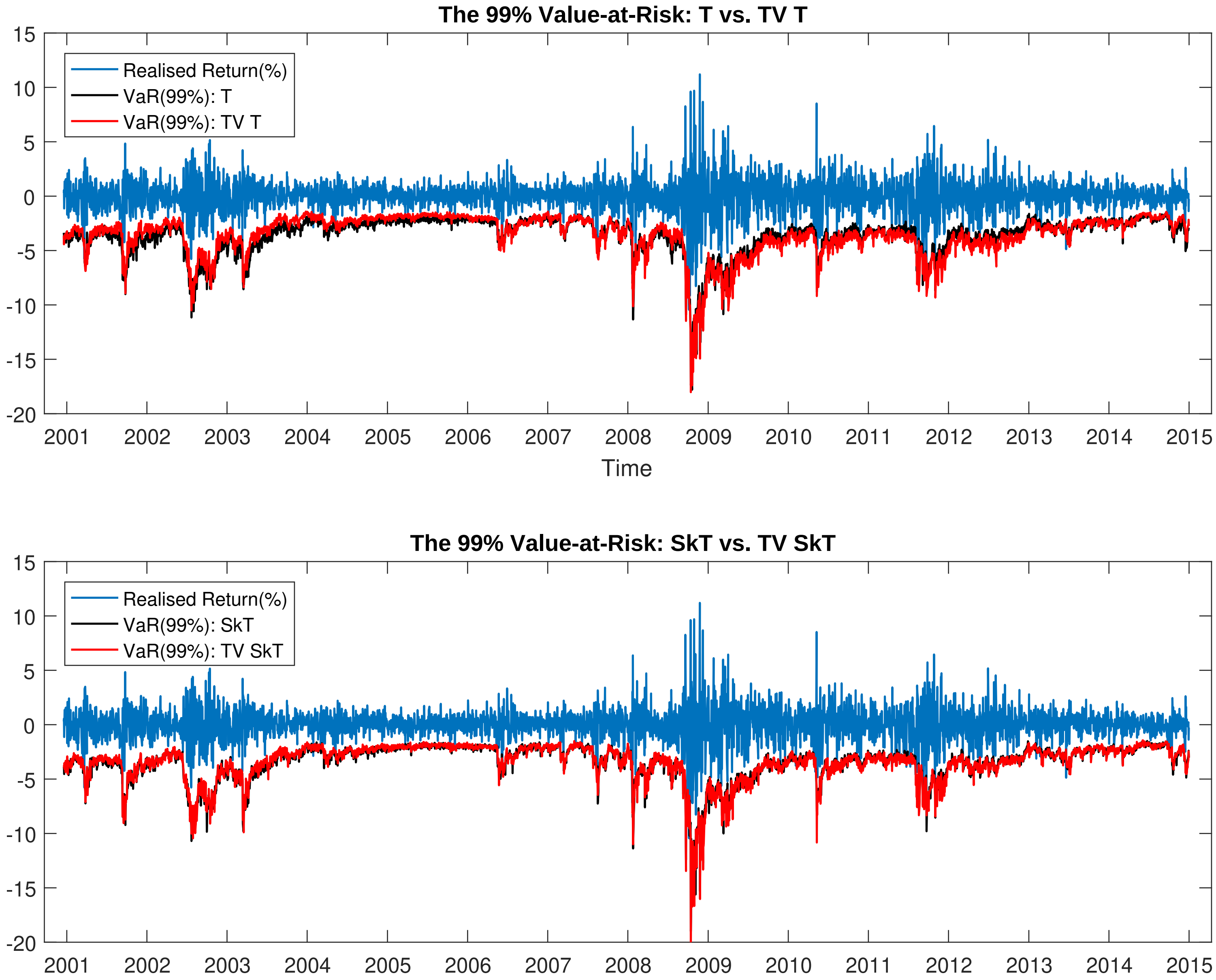

Figure 7: VaR forecasts: Constant copula and time-varying copula (EU)

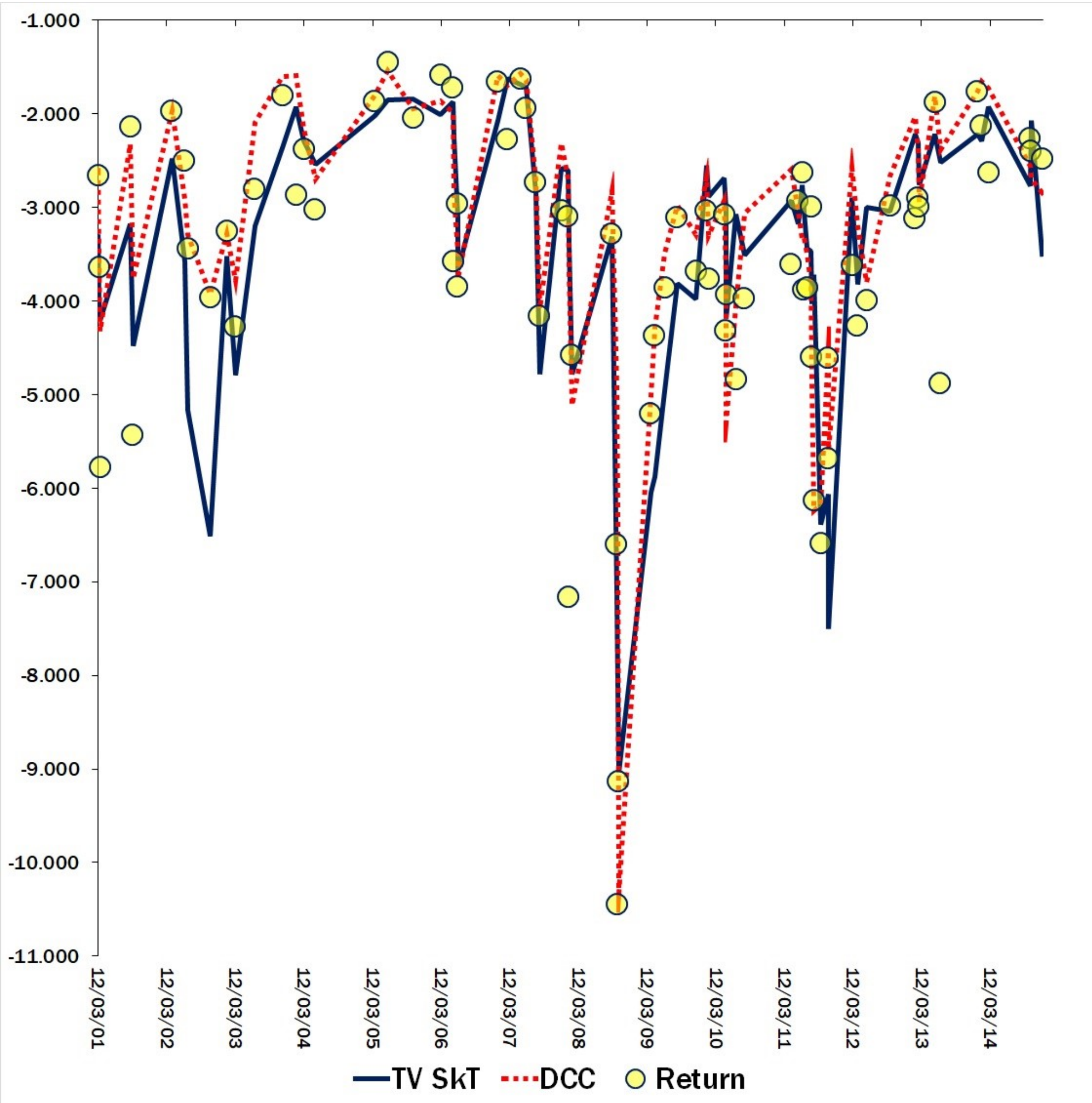

Figure 8: Failures of VaR forecasts: TV SkT vs. DCC (EU)

Table 1: Descriptive Statistics of Daily Log Return

Country	EU		UK		JAPAN		SWITZERLAND		CANADA		AUSTRALIA	
Asset	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex
Mean	-0.004	-0.005	-0.001	0.001	-0.004	0.004	0.005	-0.012	0.014	-0.006	0.015	-0.006
Std.	1.267	0.622	1.208	0.576	1.394	0.642	1.168	0.677	1.214	0.583	1.033	0.835
Skewness	-0.123	-0.145	-0.168	0.054	-0.327	-0.252	-0.071	0.358	-0.595	-0.076	-0.396	0.867
Kurtosis	8.342	5.658	9.515	7.422	9.393	6.885	9.641	12.162	12.380	8.777	8.854	15.710
Min	-8.188	-4.617	-9.158	-4.474	-10.435	-4.610	-7.871	-5.451	-10.433	-5.046	-8.679	-6.701
Max	9.021	3.844	9.265	3.919	13.062	3.710	10.506	8.475	9.723	4.338	6.101	8.828
JB test	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LB Q(12)	0.000	0.272	0.000	0.001	0.054	0.554	0.000	0.199	0.000	0.000	0.070	0.000
LB $Q^2(12)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LM	0.000	0.014	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000
Linear	-0.097		-0.118		0.192		0.174		-0.229		-0.334	
Rho	-0.041		-0.020		0.130		0.155		-0.150		-0.230	
Observations	3913	3913	3913	3913	3913	3913	3913	3913	3913	3913	3913	3913
Country	BRAZIL		INDIA		RUSSIA		TURKEY		SOUTH KOREA		SOUTH AFRICA	
Asset	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex
Mean	0.028	0.010	0.040	0.010	0.028	0.020	0.041	0.037	0.022	-0.001	0.042	0.016
Std.	1.672	0.997	1.589	0.395	2.353	0.664	2.291	1.176	1.719	0.677	1.254	1.047
Skewness	-0.132	0.111	-0.233	0.268	-0.482	0.790	0.051	8.765	-0.341	-0.702	-0.156	0.316
Kurtosis	8.921	16.843	10.487	10.627	16.349	152.776	9.925	292.104	8.336	62.744	6.119	8.573
Min	-14.068	-11.778	-12.050	-3.064	-25.279	-15.523	-19.715	-16.252	-13.097	-13.265	-8.448	-8.523
Max	13.441	9.677	16.423	3.251	23.950	14.268	17.816	37.462	11.722	10.351	5.962	9.808
JB test	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LB Q	0.007	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.268	0.000	0.000	0.108
LB $Q^2(12)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Linear	-0.328		-0.361		-0.232		-0.232		-0.383		-0.184	
Rho	-0.308		-0.326		-0.279		-0.433		-0.359		-0.162	
Observations	3913	3913	3913	3913	3913	3913	3913	3913	3913	3913	3913	3913

Note: This table reports descriptive statistics for daily logarithmic returns on country equity index (MSCI) and currencies of 6 developed markets (European Union (EU), United Kingdom (UK), Japan, Switzerland, Australia and New Zealand) and 6 emerging markets (Brazil, India, Russia, Turkey, South Korea and South Africa) over the period from January 3, 2000 to December 31, 2014, which correspond to a sample of 3,913 days for each market. LB $Q(12)$ and LB $Q^2(12)$ are the Ljung-Box statistics for serial correlation of order 12 in returns and squared returns. LM denotes the Lagrange Multiplier test for autoregressive conditional heteroskedasticity. Note that we report p-values for these four tests.

Table 2: Estimation for Univariate Distribution (Developed Markets)

Country	EU		UK		JAPAN		SWITZERLAND		CANADA		AUSTRALIA	
Asset	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex
AR												
$\phi_{k,0}$	0.023	-0.008	-0.003	-0.005	0.020	0.013	0.027	-0.009	0.048	-0.002	0.038	-0.018
	(0.014)	(0.008)	(0.011)	(0.008)	(0.018)	(0.009)	(0.013)	(0.009)	(0.013)	(0.007)	(0.012)	(0.010)
$\theta_{k,1}$	-0.022	-0.001	-0.032	0.009	0.036	-0.034	0.005	-0.026	-0.007	-0.007	-0.032	0.012
	(0.018)	(0.017)	(0.016)	(0.016)	(0.017)	(0.016)	(0.017)	(0.016)	(0.017)	(0.017)	(0.017)	(0.017)
GARCH												
ω_k	0.016	0.001	0.014	0.001	0.045	0.004	0.020	0.002	0.010	0.001	0.010	0.004
	(0.003)	(0.000)	(0.002)	(0.001)	(0.008)	(0.001)	(0.003)	(0.001)	(0.002)	(0.000)	(0.002)	(0.001)
α_k	0.000	0.035	0.000	0.050	0.023	0.032	0.000	0.024	0.016	0.054	0.003	0.064
	(0.011)	(0.006)	(0.010)	(0.007)	(0.010)	(0.007)	(0.009)	(0.006)	(0.011)	(0.008)	(0.008)	(0.008)
γ_k	0.165	-0.011	0.282	-0.025	0.116	0.009	0.173	0.013	0.098	-0.014	0.120	-0.039
	(0.016)	(0.005)	(0.024)	(0.008)	(0.016)	(0.009)	(0.017)	(0.008)	(0.015)	(0.010)	(0.013)	(0.010)
β_k	0.903	0.969	0.859	0.958	0.892	0.955	0.893	0.965	0.921	0.950	0.922	0.947
	(0.009)	(0.007)	(0.010)	(0.006)	(0.011)	(0.007)	(0.010)	(0.005)	(0.008)	(0.006)	(0.008)	(0.007)
SkT												
ν_k	12.523	8.987	11.316	11.879	9.179	5.804	8.653	7.694	9.505	12.112	11.938	8.391
λ_k	0.881	0.997	0.889	1.038	0.945	0.972	0.903	0.953	0.862	1.010	0.896	1.101
KS	0.271	0.223	0.947	0.384	0.376	0.114	0.221	0.186	0.192	0.701	0.248	0.327
CvM	0.126	0.187	0.941	0.209	0.453	0.229	0.265	0.142	0.172	0.660	0.273	0.268

Note: This table summarizes the parameter estimates of AR(1) and GJR-GARCH(1,1,1) models for conditional mean and volatility of country equity index returns and forex returns for developed markets. See Section 2.2 for the detailed model specification. We estimate all parameters using the sample from January 3, 2000 to December 31, 2014, which correspond to a sample of 3,913 observations for each series. The values in parenthesis represent the standard errors of the parameters.

Table 3: Estimation for Univariate Distribution (Emerging Markets)

Country	BRAZIL		INDIA		RUSSIA		TURKEY		SOUTH KOREA		SOUTH AFRICA	
Asset	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex	Equity	Forex
AR												
$\phi_{k,0}$	0.036	-0.010	0.070	-0.003	0.080	0.003	0.080	0.002	0.044	-0.020	0.041	0.011
	(0.021)	(0.010)	(0.017)	(0.002)	(0.023)	(0.002)	(0.027)	(0.009)	(0.019)	(0.006)	(0.016)	(0.013)
$\phi_{k,1}$	0.024	0.035	0.081	-0.013	0.038	0.026	0.006	0.038	0.005	-0.025	0.044	0.022
	(0.017)	(0.017)	(0.017)	(0.015)	(0.016)	(0.014)	(0.016)	(0.017)	(0.017)	(0.016)	(0.017)	(0.017)
GARCH												
ω_k	0.055	0.013	0.050	0.000	0.059	0.000	0.062	0.011	0.015	0.003	0.027	0.011
	(0.010)	(0.003)	(0.008)	(0.000)	(0.012)	(0.000)	(0.014)	(0.002)	(0.004)	(0.001)	(0.006)	(0.003)
α_k	0.014	0.194	0.041	0.159	0.078	0.073	0.047	0.178	0.015	0.134	0.019	0.088
	(0.009)	(0.018)	(0.011)	(0.015)	(0.015)	(0.007)	(0.010)	(0.018)	(0.007)	(0.015)	(0.008)	(0.010)
γ_k	0.096	-0.108	0.139	-0.036	0.072	-0.004	0.059	-0.067	0.077	-0.052	0.113	-0.039
	(0.014)	(0.020)	(0.019)	(0.016)	(0.019)	(0.009)	(0.013)	(0.022)	(0.011)	(0.017)	(0.014)	(0.012)
β_k	0.914	0.850	0.867	0.859	0.880	0.929	0.912	0.852	0.941	0.892	0.906	0.923
	(0.010)	(0.012)	(0.012)	(0.008)	(0.011)	(0.005)	(0.009)	(0.012)	(0.006)	(0.010)	(0.010)	(0.008)
SkT												
ν_k	8.986	7.685	7.314	4.222	5.419	4.431	6.229	5.429	6.406	4.793	11.284	8.472
λ_k	0.943	1.051	0.945	1.036	0.954	1.037	0.997	1.076	0.944	1.061	0.930	1.068
KS	0.178	0.414	0.345	0.843	0.185	0.947	0.215	0.165	0.213	0.924	0.326	0.459
CvM	0.258	0.547	0.336	0.901	0.147	0.941	0.264	0.387	0.267	0.965	0.342	0.562

Note: This table summarizes the parameter estimates of AR(1) and GJR-GARCH(1,1,1) models for conditional mean and volatility of country equity index returns and forex returns for emerging markets. See Section 2.2 for the detailed model specification. We estimate all parameters using the sample from January 3, 2000 to December 31, 2014, which correspond to a sample of 3,913 observations for each series. The values in parenthesis represent the standard errors of the parameters.

Table 4: Tests for Asymmetric Dependence between Equity and Foreign Exchange Rate

Test	Threshold correlaton		Tail dependence			
	HTZ	p-value	LUTD	ULTD	Diff	p-value
Panel A: Developed Markets						
EU	40.537	0.239	0.037	0.030	0.008	0.772
UK	31.515	0.637	0.002	0.000	0.002	0.729
Japan	24.508	0.907	0.032	0.017	0.015	0.041
Switzerland	30.619	0.680	0.037	0.047	-0.010	0.633
Canada	42.456	0.181	0.035	0.005	0.030	0.024
Australia	36.708	0.390	0.058	0.047	0.011	0.538
Panel B: Emerging Markets						
Brazil	38.065	0.332	0.068	0.030	0.038	0.022
India	130.595	0.000	0.104	0.089	0.015	0.575
Russia	60.366	0.005	0.076	0.049	0.027	0.044
Turkey	50.731	0.042	0.184	0.147	0.037	0.203
South Korea	31.339	0.646	0.198	0.067	0.131	0.000
South Africa	49.808	0.049	0.061	0.015	0.046	0.016

Note: This table presents the statistics and p-values from two asymmetric tests. "HTZ" denotes the statistic from a model-free symmetry test proposed in Hong et al. (2007) to examine whether the exceedance correlation between (foreign) stock index return and its corresponding forex return is asymmetric. "LUTD", "ULTD" and "Diff" denote the coefficients of lower-upper tail dependence and upper-lower tail dependence estimated by Student's t copula, and the difference between them for all the portfolios pairs. The copula is semiparametrically estimated (Patton, 2013). The p-values from the tests that the low tail and upper tail dependence coefficients are computed with 500 bootstrap replications.

Table 5: Tests for Time-varying Dependence between Equity and Foreign Exchange Rate

	0.15	0.5	0.85	Any	AR(1)	AR(5)	AR(10)	US crisis	EU crisis	Quandt-Andrews
Panel A: Developed Markets										
EU	0.013	0.000	0.184	0.000	0.930	0.151	0.003	0.000	0.003	0.000
UK	0.078	0.000	0.075	0.000	0.165	0.064	0.005	0.002	0.063	0.000
Japan	0.136	0.014	0.000	0.000	0.000	0.006	0.017	0.001	0.003	0.000
Switzerland	0.570	0.011	0.088	0.020	0.805	0.587	0.155	0.001	0.249	0.000
Canada	0.304	0.091	0.001	0.000	0.000	0.000	0.293	0.057	0.398	0.000
Australia	0.022	0.002	0.000	0.010	0.004	0.958	0.200	0.096	0.748	0.000
Panel B: Emerging Markets										
Brazil	0.056	0.070	0.112	0.043	0.000	0.000	0.065	0.327	0.955	0.000
India	0.000	0.000	0.507	0.000	0.742	0.007	0.000	0.000	0.001	0.000
Russia	0.000	0.000	0.890	0.000	0.403	0.684	0.005	0.000	0.001	0.000
Turkey	0.000	0.043	0.093	0.000	0.029	0.474	0.015	0.884	0.577	0.000
South Korea	0.077	0.000	0.260	0.000	0.903	0.365	0.031	0.000	0.000	0.000
South Africa	0.001	0.000	0.323	0.000	0.070	0.193	0.023	0.000	0.021	0.000

Note: This table reports the p-value from tests for time-varying dependence between forex returns and corresponding country stock index returns. Without a priori dates to consider for the timing of a break, we use naïve tests for breaks at three chosen points in sample period, at $t^*/T \in \{0.15, 0.50, 0.85\}$, which corresponds to the dates 01-Apr-2002, 02-Jun-2007, 28-Sep-2012. The "Any" column reports the results of test for dependence break of unknown timing proposed by Andrews (1993). The F-statistic in column "Quandt-Andrews" is based on a generalized break test without priori point in (Andrews and Ploberger, 1994). To detect whether the dependence structures between currency and equity significantly changed after the US and EU crisis broke out, we use 15-Sep-2008 (the collapse of Lehman Brothers) and 01-Jan-2010 (EU sovereign debt crisis) as two breakpoints and the "Crisis" panel reports the results for this test. The "AR" panel presents the results from the ARCH LM test for time-varying volatility proposed by Engle (1982). Under the null hypothesis of a constant conditional copula, we test autocorrelation in a measure of dependence (see Patton, 2013).

Table 6: Goodness-of-Fit and Likelihood Ratio Tests for Copula Models

Panel A: Goodness-of-Fit Tests

	EU		UK		Japan		Switzerland		Canada		Australia	
	KS	*CvM*	*KS*	*CvM*	*KS*	*CvM*	*KS*	*CvM*	*KS*	*CvM*	*KS*	*CvM*
T	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.07	0.08	0.00	0.00	0.00
SkT	0.00	0.21	0.03	0.00	0.05	0.07	0.17	0.06	0.17	0.06	0.08	0.09
TV T	0.42	0.14	0.36	0.19	0.47	0.53	0.38	0.25	0.45	0.78	0.15	0.33
TV SkT	0.72	0.66	0.27	0.35	1.00	1.00	0.99	1.00	1.00	1.00	0.78	0.89
	Brazil		India		Russia		Turkey		South Korea		South Africa	
	KS	*CvM*	*KS*	*CvM*	*KS*	*CvM*	*KS*	*CvM*	*KS*	*CvM*	*KS*	*CvM*
T	0.13	0.41	0.00	0.00	0.00	0.00	0.13	0.09	0.04	0.07	0.05	0.02
SkT	0.42	0.64	0.07	0.06	0.02	0.02	0.15	0.18	0.09	0.06	0.17	0.06
TV T	0.47	0.88	0.34	0.31	0.06	0.07	0.17	0.17	0.09	0.11	0.43	0.70
TV SkT	1.00	1.00	0.75	0.86	0.87	1.00	0.21	0.38	0.99	1.00	1.00	1.00

Panel B: Likelihood Ratio Test

	EU	UK	Japan	Switzerland	Canada	Australia
T	64.34	23.02	56.91	81.90	44.40	100.98
SkT	67.15	33.42	67.77	100.94	54.66	118.45
TV T	261.26	102.21	101.36	185.27	93.16	129.00
TV SkT	279.50	127.90	109.58	202.47	114.30	195.85
LR Test	0.00	0.00	0.00	0.00	0.00	0.00
	Brazil	India	Russia	Turkey	South Korea	South Africa
T	214.48	247.82	169.32	477.02	338.84	107.45
SkT	263.55	266.09	174.89	487.40	350.44	120.25
TV T	221.23	304.10	235.16	533.05	465.33	289.39
TV SkT	279.80	322.61	246.70	568.59	475.13	301.16
LR Test	0.00	0.00	0.00	0.00	0.00	0.00

Note: This table presents the test results from Goodness-of-Fit and likelihood ratio tests for four different copula models for the standardized residuals of country equity index returns and forex returns when the marginal distributions are estimated (non)parametrically. Panel A reports the p-values from Goodness-of-Fit tests for different copula specifications. "T", "SkT", "TV T" and "TV SkT" denote the "constant t copula", "constant skewed t copula", "time-varying t copula" and "time-varying skewed t copula" respectively. KS and CvM refer to the Kolmogorov-Smirnov and Cramer-von Mises tests respectively. The p-values are based on 100 simulations. Panel B reports the log likelihood values for different copula models, and likelihood ratio test results. "LR test" reports the p-values of likelihood ratio test of model specification. We use the this test to assess whether our data provide enough evidence to favor the unrestricted model (TV SkT) over the restricted model (SkT).

Table 7: Failures of VaR forecasts (EU)

Date	Return	T	SkT	TV T	TV SkT	DCC	FHS	Date	Return	T	SkT	TV T	TV SkT	DCC	FHS
12 Mar 2001	-2.655	-3.371	-3.135	-3.367	-3.316	-2.607	-2.648	30 Mar 2009	-5.205	-6.214	-5.396	-6.864	-6.045	-4.996	-4.801
16 Mar 2001	-3.640	-4.084	-4.440	-3.987	-4.101	-3.578	-4.130	20 Apr 2009	-4.366	-5.953	-6.004	-6.410	-5.878	-4.287	-5.635
22 Mar 2001	-5.778	-4.209	-4.339	-3.990	-4.206	-4.324	-4.783	15 Jun 2009	-3.860	-4.623	-4.612	-5.211	-4.960	-3.492	-3.506
30 Aug 2001	-2.143	-3.246	-3.397	-2.446	-3.179	-2.316	-2.031	17 Aug 2009	-3.111	-3.900	-3.635	-4.840	-3.814	-2.979	-3.083
11 Sep 2001	-5.429	-4.401	-4.549	-3.437	-4.478	-3.758	-3.185	26 Nov 2009	-3.687	-3.776	-4.244	-4.454	-3.978	-3.290	-3.826
08 Apr 2002	-1.973	-2.632	-2.624	-2.262	-2.482	-1.966	-1.530	20 Jan 2010	-3.033	-3.054	-2.995	-3.312	-2.547	-2.680	-2.750
14 Jun 2002	-2.507	-3.514	-3.486	-3.186	-3.581	-2.918	-2.402	04 Feb 2010	-3.765	-2.920	-2.943	-3.322	-2.890	-3.297	-3.762
02 Jul 2002	-3.445	-4.990	-5.023	-4.410	-5.165	-3.290	-2.922	27 Apr 2010	-3.079	-2.832	-2.650	-3.161	-2.684	-2.923	-3.367
29 Oct 2002	-3.966	-7.193	-6.157	-5.927	-6.517	-3.930	-4.461	04 May 2010	-4.322	-3.097	-3.081	-3.783	-3.032	-3.779	-4.186
27 Jan 2003	-3.259	-3.438	-4.077	-3.093	-3.519	-3.284	-3.031	07 May 2010	-3.936	-3.332	-4.017	-4.739	-4.191	-5.301	-6.346
12 Mar 2003	-4.278	-4.022	-4.508	-4.285	-4.795	-3.781	-3.779	29 Jun 2010	-4.836	-3.374	-3.346	-4.155	-3.075	-4.053	-4.637
23 Jun 2003	-2.803	-4.213	-3.887	-3.045	-3.195	-2.092	-2.072	11 Aug 2010	-3.978	-3.965	-3.830	-4.403	-3.507	-3.061	-3.123
17 Nov 2003	-1.809	-2.747	-2.457	-1.947	-2.366	-1.606	-2.031	18 Apr 2011	-3.604	-2.650	-2.944	-3.948	-2.924	-2.594	-2.799
29 Jan 2004	-2.871	-2.020	-1.926	-1.673	-1.927	-1.596	-1.924	23 May 2011	-2.930	-2.779	-2.941	-3.483	-3.176	-3.047	-4.470
11 Mar 2004	-2.374	-2.390	-2.109	-1.804	-2.289	-2.169	-2.313	15 Jun 2011	-2.625	-2.613	-2.400	-3.549	-2.755	-3.342	-3.463
10 May 2004	-3.024	-3.348	-2.837	-2.675	-2.543	-2.700	-3.189	23 Jun 2011	-3.875	-3.491	-3.055	-3.774	-3.056	-3.321	-4.357
23 Mar 2005	-1.870	-2.318	-2.028	-1.743	-2.014	-1.803	-1.761	11 Jul 2011	-3.862	-3.377	-3.378	-3.919	-3.437	-3.496	-4.066
31 May 2005	-1.452	-2.008	-1.872	-1.652	-1.852	-1.540	-1.400	01 Aug 2011	-2.991	-2.835	-2.998	-4.247	-3.463	-3.817	-3.901
13 Oct 2005	-2.045	-2.247	-2.096	-1.860	-1.838	-1.950	-1.884	04 Aug 2011	-4.601	-4.188	-4.030	-3.745	-4.035	-4.893	-4.535
07 Mar 2006	-1.586	-2.308	-2.053	-1.813	-2.013	-1.861	-1.471	18 Aug 2011	-6.128	-3.891	-3.556	-4.122	-3.720	-6.239	-5.963
12 May 2006	-1.719	-2.230	-2.085	-2.176	-1.869	-1.985	-1.679	22 Sep 2011	-6.593	-6.168	-6.037	-6.024	-6.391	-6.182	-7.322
17 May 2006	-3.581	-2.397	-2.001	-2.129	-2.262	-2.881	-2.386	31 Oct 2011	-4.611	-5.632	-5.811	-6.849	-6.068	-4.483	-5.160
06 Jun 2006	-2.967	-2.841	-2.724	-2.668	-3.125	-3.476	-3.255	01 Nov 2011	-5.686	-6.372	-6.454	-7.347	-7.499	-5.592	-5.651
08 Jun 2006	-3.852	-3.353	-3.228	-3.414	-3.672	-3.758	-3.416	06 Mar 2012	-3.623	-3.101	-2.687	-3.155	-2.898	-2.569	-2.491
05 Jan 2007	-1.659	-2.088	-2.163	-1.621	-2.075	-1.593	-1.685	04 Apr 2012	-4.267	-4.305	-3.477	-4.421	-3.827	-3.206	-2.916
27 Feb 2007	-2.270	-1.800	-1.804	-1.814	-1.621	-1.698	-1.702	23 May 2012	-4.000	-3.667	-3.197	-3.491	-2.995	-3.799	-4.366
08 May 2007	-1.633	-2.327	-1.902	-1.835	-1.686	-1.568	-1.529	26 Sep 2012	-2.984	-3.310	-3.596	-3.916	-3.040	-2.668	-2.645
06 Jun 2007	-1.946	-1.991	-1.967	-2.091	-1.713	-1.634	-1.530	04 Feb 2013	-3.119	-2.394	-2.208	-2.697	-2.219	-2.026	-2.036
26 Jul 2007	-2.735	-2.728	-2.495	-2.663	-2.854	-2.730	-2.418	21 Feb 2013	-2.902	-2.284	-2.565	-2.858	-2.309	-2.413	-3.024
16 Aug 2007	-4.162	-4.364	-3.839	-4.028	-4.777	-4.084	-4.432	26 Feb 2013	-2.991	-3.055	-2.596	-3.162	-2.760	-2.950	-3.772
13 Dec 2007	-3.035	-2.737	-2.921	-2.833	-2.585	-2.313	-2.342	23 May 2013	-1.881	-2.286	-2.222	-2.583	-2.222	-1.812	-2.114
15 Jan 2008	-3.100	-2.521	-2.380	-2.969	-2.598	-2.693	-3.040	20 Jun 2013	-4.876	-2.694	-2.568	-2.606	-2.520	-2.388	-2.440
21 Jan 2008	-7.160	-3.451	-3.443	-3.246	-3.396	-3.713	-4.395	02 Jan 2014	-1.761	-2.356	-2.219	-2.505	-2.220	-1.758	-3.208
05 Feb 2008	-4.578	-5.204	-4.956	-4.398	-4.784	-5.112	-7.407	24 Jan 2014	-2.134	-2.036	-2.114	-2.364	-2.294	-1.657	-1.709
04 Sep 2008	-3.288	-4.183	-3.349	-3.559	-3.311	-2.824	-4.337	03 Mar 2014	-2.630	-2.336	-2.216	-2.239	-1.926	-1.728	-1.694
29 Sep 2008	-6.600	-6.618	-6.234	-6.479	-7.119	-4.966	-5.666	10 Oct 2014	-2.266	-2.469	-2.166	-2.252	-2.766	-2.566	-3.259
06 Oct 2008	-10.448	-7.600	-7.746	-6.757	-8.122	-6.311	-9.591	15 Oct 2014	-2.404	-2.347	-2.324	-2.406	-2.067	-2.696	-2.689
10 Oct 2008	-9.136	-9.481	-9.685	-10.025	-9.035	-10.328	-14.232	15 Dec 2014	-2.481	-3.257	-3.507	-3.228	-3.504	-2.840	-2.311

Note: This table reports the failure of VaR forecast in EU. The failure is defined as the event that a realized loss is not covered by the VaR forecast. The failure is coloured by the red colour.

Table 8: Backtesting of Value-at-Risk: Empirical Coverage Probability

Country	T	SkT	TV T	TV SkT	DCC	FHS
EU	0.98%	1.09%	0.96%	0.98%	1.58%	1.31%
UK	1.15%	1.09%	1.04%	0.96%	1.64%	1.31%
Japan	0.76%	0.87%	0.85%	0.98%	1.28%	1.23%
Switzerland	0.98%	0.96%	1.06%	1.01%	1.45%	1.15%
Canada	1.09%	0.96%	1.04%	1.06%	1.56%	1.20%
Australia	1.26%	1.23%	1.34%	1.20%	1.94%	1.20%
Barzil	1.15%	1.06%	1.15%	1.15%	1.80%	1.23%
India	1.09%	1.12%	1.12%	0.98%	1.61%	1.01%
Russia	1.06%	0.93%	1.09%	1.01%	1.88%	1.15%
Turkey	1.31%	1.31%	1.17%	1.28%	1.34%	1.12%
South Korea	0.85%	1.04%	0.98%	1.01%	1.86%	0.98%
South Africa	1.06%	1.04%	0.98%	1.06%	1.53%	1.17%
Bias	0.06%	0.06%	0.06%	0.06%	0.62%	0.17%
Stdev	0.15%	0.12%	0.12%	0.10%	0.21%	0.10%
RMSE	0.17%	0.14%	0.14%	0.12%	0.66%	0.20%

Note: This table reports the empirical coverage probability (ECP) of the 99% Value-at-Risk for 12 countries. Bias is defined as the average of $ECP - 1\%$ over 12 countries, Stdev the standard deviation of ECP and RMSE the root mean square error of ECP, respectively. Smaller value is preferred. "T", "SkT", "TV T" and "TV SkT" denote the "constant t copula", "constant skewed t copula", "time-varying t copula" and "time-varying skewed t copula" respectively. All copula models are semiparametrically estimated. "DCC" denotes the multivariate GARCH model by Engle (2002) and "FHS" the filtered historical simulation by Baron-Adesi et al. (2002). We estimate the VaR models using 250 business days over the period January 3, 2000 - December 15, 2000, and compute the one-day-ahead forecast of the 99 percent VaR for December 18, 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for December 31, 2014. This generates 3,663 out-of-sample daily forecasts over the testing period, December 18, 2000 - December 31, 2014.

Table 9: Backtesting of Value-at-Risk: Conditional Coverage Test

Country	T	SkT	TV T	TV SkT	DCC	FHS
EU	0.73	1.19	0.75	0.73	10.72*	4.53
UK	1.74	1.19	0.85	0.75	14.64*	3.44
Japan	3.79	1.76	2.16	0.82	12.64*	2.11
Switzerland	0.82	0.96	0.76	0.74	8.06*	1.74
Canada	3.39	4.06	0.72	3.40	15.97*	1.75
Australia	7.55*	4.15	5.67	2.48	25.84*	2.48
Barzil	1.74	0.99	1.74	1.74	21.66*	6.71*
India	0.85	1.00	1.00	0.82	12.57*	2.51
Russia	0.99	1.16	0.85	0.74	24.55*	2.11
Turkey	11.80*	11.80*	7.07*	4.81	23.62*	11.01*
South Korea	1.45	0.85	0.73	0.76	21.74*	0.85
South Africa	3.40	3.48	0.82	0.76	10.07*	8.90*
Number of rejection	2	1	1	0	12	3

Note: This table reports the conditional coverage test of the 99% Value-at-Risk for 12 countries. It uses the LR statistic and follows the Chi-squared distribution with two degrees-of-freedom under the null hypothesis. We report a test statistic for each country and model and summarize the performance of model by the number of rejection at the 5% significance level. See Table 8 for the detailed description of models and estimations.

Table 10: Backtesting of Value-at-Risk: Dynamic Quantile Test

Country	T	SkT	TV T	TV SkT	DCC	FHS
EU	10.59*	8.83	3.19	9.65	21.73*	8.37
UK	8.23	8.39	9.42	9.06	35.33*	15.47*
Japan	4.78	4.54	13.39*	3.22	48.78*	9.99
Switzerland	22.26*	11.31*	19.53*	21.11*	12.56*	7.90
Canada	13.75*	11.25*	15.04*	8.94	32.93*	6.61
Australia	68.57*	27.39*	58.33*	64.98*	78.55*	14.56*
Barzil	8.16	8.16	8.16	8.16	31.56*	17.54*
India	24.14*	27.80*	13.35*	22.68*	29.87*	15.72*
Russia	8.21	3.74	8.64	9.43	47.72*	9.37
Turkey	33.67*	33.67*	30.98*	8.15	23.62*	25.12*
South Korea	13.11*	9.55	9.63	10.03	33.58*	3.69
South Africa	19.31*	20.11*	10.75	8.98	21.19*	26.47*
Number of rejection	8	6	6	4	12	6

Note: This table reports the dynamic quantile test of the 99% Value-at-Risk for 12 countries. It uses the Wald statistic and follows the Chi-squared distribution with 6 degrees-of-freedom under the null hypothesis. We report a test statistic for each country and model and summarize the performance of model by the number of rejection at the 5% significance level. See Table 8 for the detailed description of models and estimations.

Table 11: Backtesting of Expected Shortfall: Mean Absolute Error

Country	T	SkT	TV T	TV SkT	DCC	FHS
EU	0.006	0.006	0.003	0.003	0.007	0.008
UK	0.006	0.006	0.004	0.003	0.008	0.006
Japan	0.009	0.007	0.005	0.004	0.007	0.011
Switzerland	0.006	0.005	0.004	0.003	0.006	0.005
Canada	0.008	0.008	0.006	0.005	0.009	0.007
Australia	0.010	0.008	0.007	0.005	0.011	0.006
Barzil	0.010	0.010	0.009	0.008	0.013	0.010
India	0.010	0.007	0.006	0.004	0.013	0.009
Russia	0.018	0.011	0.013	0.013	0.023	0.023
Turkey	0.019	0.020	0.010	0.009	0.022	0.018
South Korea	0.007	0.012	0.010	0.009	0.016	0.012
South Africa	0.007	0.006	0.006	0.005	0.009	0.009
Average	0.010	0.009	0.007	0.006	0.012	0.010

This table reports the mean absolute error (MAE) of the 99% Expected Shortfall for 12 countries. We report MAE for each country and model and summarize the performance of model by the average of MAE. See Table 8 for the detailed description of models and estimations.

Table 12: Backtesting of Value-at-Risk: Robustness checks

	T	SkT	TV T	TV SkT	DCC	FHS
Panel A. Crisis period						
ECP	0.32%	0.33%	0.26%	0.21%	0.97%	0.25%
CC	2	3	1	1	9	1
DQ	8	6	4	4	9	5
MAE	0.012	0.011	0.008	0.007	0.016	0.008
Panel B. Post-crisis period						
ECP	0.37%	0.31%	0.23%	0.20%	0.56%	0.23%
CC	1	1	1	1	9	2
DQ	7	6	4	2	7	2
MAE	0.013	0.011	0.008	0.007	0.015	0.008
Panel C. Window size: 500 days						
ECP	0.17%	0.15%	0.13%	0.11%	0.62%	0.28%
CC	3	0	0	0	8	0
DQ	7	6	5	1	10	3
MAE	0.009	0.008	0.007	0.006	0.011	0.009
Panel D. Parametric estimation						
ECP	0.18%	0.16%	0.16%	0.12%	0.66%	0.20%
CC	3	2	2	0	12	3
DQ	7	5	6	4	12	6
MAE	0.010	0.009	0.007	0.007	0.012	0.010
Panel E. 95% Value-at-Risk						
ECP	0.31%	0.26%	0.20%	0.08%	0.54%	0.25%
CC	6	4	5	2	5	4
DQ	10	9	7	5	5	7
MAE	0.041	0.038	0.034	0.034	0.036	0.037

Note: This table summarizes the backtesting results for robustness checks. Panel A and B report the results for crisis period (2007 - 2010) and post-crisis period (2011 - 2014), respectively. Panel C reports the results for the 500-day window size. Panel D reports the results for the parametric estimation. Panel E reports the results for the 95% Value-at-Risk and Expected Shortfall. ECP reports the RMSE of empirical coverage probability for 12 countries. CC and DQ reports the number of rejection. MAE reports the average of mean absolute error for 12 countries.