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# Limited-Fronthaul Cell-Free Massive MIMO with Local MMSE Receiver under Rician Fading and Phase Shifts

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**Abstract**—A cell-free Massive multiple-input multiple-output (MIMO) system is considered, where the access points (APs) are linked to a central processing unit (CPU) via the limited-capacity fronthaul links. It is assumed that only the quantized version of the weighted signals are available at the CPU. The achievable rate of a limited fronthaul cell-free massive MIMO with local minimum mean square error (MMSE) detection is studied. We study the assumption of uncorrelated quantization distortion, which is commonly used in literature. We show that this assumption will not affect the validity of the insights obtained in our work. To investigate this, we compare the uplink per-user rate with different system parameters for two different scenarios; 1) the exact uplink per-user rate and 2) the uplink per-user rate while ignoring the correlation between the inputs of the quantizers. Finally, we present the conditions which imply that the quantization distortions across APs can be assumed to be uncorrelated.

## I. INTRODUCTION

### A. Limited-fronthaul Cell-Free Massive MIMO

Cell-free massive multiple-input multiple-output (MIMO) is a promising technique, where large number of distributed access points (APs) serve a much smaller number of users [1]. Similar to the methodology in [2], we model the phase of the line-of-sight (LoS) path as a uniformly distributed random variable, which enables us to take the phase shifts due to mobility and phase noise into account. In [3], the authors investigate the effect of phase shifts in cell-free massive MIMO. An investigation of device-to-device-based cell-free massive MIMO with limited fronthaul links is presented in [4]. The effect of channel aging in cell-free massive MIMO is investigated in [5]. The authors in [6] investigate the performance of perfect-fronthaul cell-free massive MIMO over spatially correlated Rician fading channels. A limited fronthaul cell-free massive MIMO system is investigated, where the access points (APs) send the quantized versions of the received signals and the channel estimates to a central processing unit (CPU) through limited fronthaul links [7]–[11]. The limited capacity links from the APs to the CPU constitute one of the most principal challenges in the cell-free massive MIMO system. The assumption of infinite fronthaul in [1] is not realistic in practice. In the uplink transmission, the fronthaul network will carry quantized signals, which will affect the system performance. This paper therefore provides an approach for the analysis of the effect of fronthaul quantization distortion correlation on the uplink of cell-free massive MIMO.

### B. Why Local Minimum Mean Square Error (L-MMSE) Detection?

We study the case when only the quantized version of the weighted signal is available at the CPU which employs local minimum mean square error (L-MMSE) detection. The L-MMSE detection is interesting due to following reasons: 1) The L-MMSE detector has low complexity and is practically feasible. Note that inversion of the aggregate channel matrix is required for designing zero forcing (ZF) and MMSE. Hence, due to the large number of users and APs in cell-free massive MIMO, ZF and MMSE detectors

impose huge complexity burden on the CPU and they are not suitable for practical implementations. It should be noted that the current hardware cannot perform matrix inversion for a matrix with the dimension larger than  $25 \times 25$  in practical systems [12], [13]. 2) The L-MMSE detector provides good performance in cell-free massive MIMO with the use of only the local channel matrix [14]. 3) The L-MMSE detector can be implemented in a distributed manner, as each AP uses the complex conjugate of the channel estimates in a distributed approach [14]. 4) The L-MMSE detector can facilitate flexible functional splits in cell-free massive MIMO [15].

### C. Motivation and Contribution

In [16], the authors generalize the Bussgang theorem for MIMO distortions. According to [16], it is common that the quantization distortion correlation is neglected without any justifications and motivations in the literature. In [16], the authors extend the Bussgang results to be applicable hardware impairments [7]. In [16], the authors investigate the cumulative distribution function (CDF) of the absolute value of the correlation coefficient between elements of the MIMO system.

In this paper, we study the performance of a cell-free massive MIMO with the L-MMSE detector. The effect of the quantization distortion correlation on the cell-free massive MIMO system is investigated. In general, the inputs of the quantizers at different APs are correlated, the quantization distortions across APs are therefore correlated. However, according to [9]–[11], the correlation between the inputs of the quantizers at different APs renders, the closed-form achievable rate formulas overly complicated and many optimization problems (i.e., sum-rate maximization, max-min rate, energy efficiency maximization) are intractable. For this reason, in [9]–[11], [17], the quantization distortion correlation is not taken into account and an approximated achievable rate derived by ignoring the quantization distortion correlation is exploited to facilitate the capacity analysis in [9]–[11], [17]. In this paper, we present approximation and exact closed-forms for the capacity of the limited-fronthaul cell-free massive MIMO system to analyze the impact of quantization distortion correlation. However, in this paper, we present the exact definition of the power of the quantization distortion. The analytical and numerical results for typical cases imply that, for cell-free Massive MIMO, under the necessary conditions listed below, the quantization distortions are approximately uncorrelated: 1) There is a large number of users, or 2) there are a few antennas at each AP. In this paper, we show that for small number of antennas per AP, the approximation on the achievable rate while ignoring the quantization distortion correlation is very close to the exact achievable rate. However, if there are few active users and each AP is equipped with a large number of antennas, then the correlation between the outputs of the quantizers have a substantial impact on the achievable rate. The approximate rate expression that neglects the correlation should not be used in that case.

## II. SYSTEM MODEL

We consider uplink transmission in a cell-free massive MIMO system with  $M$  APs and  $K$  single-antenna users randomly dis-

tributed in a large area. Moreover, we assume each AP has  $N$  antennas.

### A. Channel Model

The uplink channel between the  $m$ th AP and the  $k$ th user is presented by  $\mathbf{g}_{mk}$  which is modeled as [2]

$$\mathbf{g}_{mk} = \sqrt{\alpha_{mk}} e^{j\phi_{mk}} + \sqrt{\beta_{mk}} \tilde{\mathbf{h}}_{mk}, \quad (1)$$

where  $\sqrt{\alpha_{mk}} e^{j\phi_{mk}}$  corresponds to the LoS component and  $\sqrt{\beta_{mk}} \tilde{\mathbf{h}}_{mk}$  accounts for the NLoS components. Note that the term  $\phi_{mk} \sim [-\pi, \pi]$  is the phase-shift of the LoS component. Moreover, we have  $\alpha_{mk} = \frac{\kappa_{mk}}{\kappa_{mk}+1} \zeta_{mk}$  and  $\beta_{mk} = \frac{1}{\kappa_{mk}+1} \zeta_{mk}$ , where  $\zeta_{mk}$  denotes the large-scale fading coefficient. In addition, the term  $\kappa_{mk}$  is modelled as [2]

$$\kappa_{mk} = \frac{P_{\text{LoS}}(d_{mk})}{1 + P_{\text{LoS}}(d_{mk})}, \quad (2)$$

where  $d_{mk}$  is the distance between the  $m$ th AP and the  $k$ th user,  $P_{\text{LoS}}(d_{mk})$  is the LoS probability depending on the distance  $d_{mk}$ , where the LoS probability is defined in Section III.

### B. Channel Estimation at the APs

All pilot sequences transmitted by the  $K$  users in the channel estimation phase are collected in a matrix  $\Phi \in \mathbb{C}^{\tau_p \times K}$ , where  $\tau_p$  is the length of the pilot sequence for each user and the  $k$ th column,  $\phi_k$ , represents the pilot sequence used by the  $k$ th user, where  $\|\phi_k\|^2 = 1$ . Let  $\sqrt{\tau} \phi_k \in \mathbb{C}^{\tau \times 1}$  be the pilot sequence assigned to the  $k$ th user. We assume the case where channel statistics are available at the APs, however, the phase shifts are completely unknown. Then, we can use the non-aware linear MMSE (LMMSE) estimator. Based on the analysis in [2, Section III-B], the LMMSE estimate of the channel  $\mathbf{g}_{mk}$  is given by

$$\hat{\mathbf{g}}_{mk} = c_{mk} \left( \sqrt{\tau_p p_p} \mathbf{g}_{mk} + \sqrt{\tau_p p_p} \sum_{k' \neq k} \mathbf{g}_{mk'} \phi_{k'}^H \phi_k \Omega_{p,m} \phi_k \right), \quad (3)$$

where  $\Omega_{p,m}$  denotes the noise vector at the  $m$ th AP whose elements are independent and identically distributed (i.i.d.)  $\mathcal{CN}(0, 1)$ ,  $p_p$  represents the normalized signal-to-noise ratio (SNR) of each pilot symbol. The  $\bar{p}_p$  denotes the power of pilot sequence where  $p_p = \frac{\bar{p}_p}{p_n}$  and  $p_n$  is the noise power [18]. Moreover, we have  $c_{mk} = \frac{\sqrt{\tau_p p_p} (\beta_{mk} + \alpha_{mk})}{\tau_p p_p \sum_{k'=1}^K (\beta_{mk'} + \alpha_{mk'}) |\phi_{k'}^H \phi_k|^2 + 1}$  and  $\gamma_{mk} = \sqrt{\tau_p p_p} c_{mk} (\beta_{mk} + \alpha_{mk})$ .

### C. Data Detection

The transmitted signal from the  $k$ th user is denoted by  $x_k = \sqrt{\rho} q_k s_k$ , where  $s_k$  with  $\mathcal{CN}(0, 1)$  and  $q_k$  denotes the transmitted symbol and the transmit power, respectively. Moreover,  $\rho$  refers to the normalized uplink SNR. The  $N \times 1$  received signal at the  $m$ th AP is given by

$$\mathbf{y}_m = \sqrt{\rho} \sum_{k=1}^K \mathbf{g}_{mk} \sqrt{q_k} s_k + \mathbf{n}_m, \quad (4)$$

where  $\mathbf{n}_m \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$  is the noise vector at the  $m$ th AP. We consider the case when each AP multiplies the received signal by the L-MMSE receiver, and sends back a quantized version of this weighted signal to the CPU. Let  $\mathbf{v}_{mk} \mathbf{C}^N$  be the L-MMSE vector that the  $m$ th AP design for the  $k$ th user. Then the local estimate of the transmitted signal  $s_k$ , i.e.,  $\hat{s}_{mk}$ , is given by

$$z_{mk} \triangleq \hat{s}_{mk} = \mathbf{v}_{mk}^H \mathbf{y}_m, \quad (5)$$

where

$$\mathbf{v}_{mk} = \left( \tilde{a}^2 \rho \sum_{k'=1}^K q_{k'} \hat{\mathbf{g}}_{mk'} \hat{\mathbf{g}}_{mk'}^H + \mathbf{R}_{mk} \right)^{-1} \hat{\mathbf{g}}_{mk}, \quad (6)$$

where  $\mathbf{R}_{mk} = \rho \sum_{k'=1}^K q_{k'} \mathbf{W}_{mk'} + \mathbf{I}_N + \mathbf{F}_m$  and

$$\mathbf{W}_{mk'} = \mathbf{S}_{mk'} - \mathbf{T}_{mk'}, \quad (7a)$$

$$\mathbf{S}_{mk'} = \left( \frac{\sigma_{\tilde{e},B}^2}{\tilde{a}^2} + 1 \right) \text{diag} [\text{rep}(\beta_{mk'} + \alpha_{mk'}, N)], \quad (7b)$$

$$\mathbf{T}_{mk'} = (1 - \sigma_{\tilde{e},B}^2) \text{diag} [\text{rep}(\gamma_{mk'}, N)], \mathbf{F}_m = \frac{\sigma_{\tilde{e},B}^2}{\tilde{a}^2} \mathbf{I}_N, \quad (7c)$$

where  $\tilde{a}$  and  $\sigma_{\tilde{e},B}^2$  are the constant term in the Bussgang decomposition and the quantization distortion power, respectively, which are defined in [9, Table 1]. The  $m$ th AP quantizes the terms  $z_{mk} = \mathbf{v}_{mk}^H \mathbf{y}_m, \forall k$ , and forwards the quantized signals in each symbol duration to the CPU, where  $\mathbf{v}_{mk}$  is the L-MMSE receiver. Hence  $z_{mk}$  is the input of the quantizer at the  $m$ th AP. Using the Bussgang decomposition, the estimate of the signal  $s_k$  at the CPU can be written as

$$\begin{aligned} \hat{s}_k &= \sum_{m=1}^M \mathcal{Q}(z_{mk}) = \sum_{m=1}^M \mathcal{Q}(\mathbf{v}_{mk}^H \mathbf{y}_m) \\ &= \sum_{m=1}^M \tilde{a} \mathbf{v}_{mk}^H \mathbf{y}_m + \underbrace{\sum_{m=1}^M d_{mk}^z}_{\text{TQD}_k}, \end{aligned} \quad (8)$$

where  $\text{TQD}_k$  refers to the total quantization distortion (TQD) at the  $k$ th user. Since the input to the quantizer is the sum of many random variables, from the central limit theorem, it has a nearly Gaussian distribution [10]. Moreover, using Bussgang decomposition the elements of the quantization distortion are uncorrelated with the input of the quantizer [19], i.e.,

$$\mathbb{E} \left\{ (\mathbf{v}_{mk}^H \mathbf{y}_m)^H d_{mk}^z \right\} = 0. \quad (9)$$

The achievable rate of the  $k$ th user is given by  $R_k = \log_2(1 + \text{SINR}_k)$ , where the  $\text{SINR}_k$  is the achievable signal-to-interference-plus-noise ratio (SINR) of the  $k$ th user and is given by

$$\text{SINR}_k = \quad (10)$$

$$\frac{q_k \mathbf{1}^T \bar{\mathbf{f}}_{kk} \bar{\mathbf{f}}_{kk}^H \mathbf{1}}{\sum_{k'=1}^K q_{k'} \mathbb{E} \left\{ \mathbf{1}^T \bar{\mathbf{f}}_{kk'} \bar{\mathbf{f}}_{kk'}^H \mathbf{1} \right\} - q_k \mathbf{1}^T \bar{\mathbf{f}}_{kk} \bar{\mathbf{f}}_{kk}^H \mathbf{1} + \frac{1}{\rho} \sum_{m=1}^M \mathbf{1}^T \mathbf{D}_k \mathbf{1} + \frac{1}{\rho} \mathbb{E} \left\{ |\text{TQD}_k|^2 \right\}},$$

where  $\bar{\mathbf{f}}_{kk'} = \left[ \mathbb{E} \{ \mathbf{v}_{1k}^H \hat{\mathbf{g}}_{1k'} \} \cdots \mathbb{E} \{ \mathbf{v}_{Mk}^H \hat{\mathbf{g}}_{Mk'} \} \right]^T$ ,  $\mathbf{D}_k = \text{diag} \left[ \mathbb{E} \{ \|\mathbf{v}_{1k}\|^2 \} \cdots \mathbb{E} \{ \|\mathbf{v}_{Mk}\|^2 \} \right]^T$  where  $\mathbf{1} = [1, \dots, 1]^T \in \mathbb{C}^N$ . The power of the quantization distortion for user  $k$  is given by

$$\begin{aligned} \mathbb{E} \left\{ |\text{TQD}_k|^2 \right\} &= \mathbb{E} \left\{ \left| \sum_{m=1}^M d_{mk}^z \right|^2 \right\} \\ &= \mathbb{E} \left\{ \left( \sum_{m=1}^M d_{mk}^z \right) \left( \sum_{m=1}^M d_{mk}^z \right)^* \right\} \\ &= \sum_{m=1}^M \mathbb{E} \left\{ |d_{mk}^z|^2 \right\} + \sum_{m=1}^M \sum_{n \neq m}^M \mathbb{E} \left\{ d_{mk}^z (d_{nk}^z)^* \right\} \\ &= \sum_{m=1}^M [\mathbf{C}_{d_k^z d_k^z}]_{mm} + \sum_{m=1}^M \sum_{n \neq m}^M [\mathbf{C}_{d_k^z d_k^z}]_{nm}, \end{aligned} \quad (11)$$

where  $\mathbf{C}_{d_k^z d_k^z} = \mathbb{E} \left\{ \mathbf{d}_k^z (\mathbf{d}_k^z)^H \right\}$  is the covariance matrix of the quantization distortion and  $\mathbf{d}_k^z = [d_{1k}^z \cdots d_{Mk}^z]^T$  is the quantization distortion vector. Note that  $[\mathbf{C}_{d_k^z d_k^z}]_{mn}$  is the  $mn$ th element of  $\mathbf{C}_{d_k^z d_k^z}$ . To calculate  $\mathbf{C}_{d_k^z d_k^z}$ , we first re-write the aggregate received signal at the CPU as follows:

$$\mathbf{r}_k = \mathcal{Q}(\mathbf{z}_k) = \mathbf{A} \mathbf{z}_k + \mathbf{d}_k^z, \quad (12)$$

where  $\mathbf{r}_k = [r_{1k} \cdots r_{Mk}]^T$  and  $\mathbf{z}_k = [z_{1k} \cdots z_{Mk}]^T$ . Moreover, based on the analysis in [20], it can be shown that the matrix  $\mathbf{A}$  is

diagonal. The matrix  $\mathbf{A}$  is determined by the LMMSE estimation of  $\mathbf{r}_k$  from  $\mathbf{z}_k$  as follows [20]:

$$\mathbf{A} = \mathbb{E} \{ \mathbf{r}_k \mathbf{z}_k^H \} \mathbb{E} \{ \mathbf{z}_k \mathbf{z}_k^H \}^{-1} = \mathbf{C}_{\mathbf{r}_k \mathbf{z}_k} \mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}^{-1}, \quad (13)$$

and the error has the following covariance matrix [20]

$$\begin{aligned} \mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z} &= \mathbb{E} \left\{ (\mathbf{r}_k - \mathbf{A} \mathbf{z}_k) (\mathbf{r}_k - \mathbf{A} \mathbf{z}_k)^H \right\} \mathbb{E} \{ \mathbf{z}_k \mathbf{z}_k^H \}^{-1} \\ &= \mathbf{C}_{\mathbf{r}_k \mathbf{r}_k} - \mathbf{C}_{\mathbf{r}_k \mathbf{z}_k} \mathbf{A}^H - \mathbf{A} \mathbf{C}_{\mathbf{z}_k \mathbf{r}_k} + \mathbf{A} \mathbf{C}_{\mathbf{z}_k \mathbf{z}_k} \mathbf{A}^H \\ &= \mathbf{C}_{\mathbf{r}_k \mathbf{r}_k} - \mathbf{C}_{\mathbf{r}_k \mathbf{z}_k} \mathbf{C}_{\mathbf{z}_k \mathbf{z}_k}^{-1} \mathbf{C}_{\mathbf{z}_k \mathbf{r}_k}. \end{aligned} \quad (14)$$

**Proposition 1.** *The covariance matrices  $\mathbf{C}_{\mathbf{r}_k \mathbf{r}_k}$ ,  $\mathbf{C}_{\mathbf{r}_k \mathbf{z}_k}$ , and  $\mathbf{C}_{\mathbf{z}_k \mathbf{r}_k}$  are obtained using the Price Theorem.*

*Proof:* To characterize the cross-correlation and auto-correlation properties of Gaussian input signals, we exploit the Price Theorem [21]. Based on the Price theorem, the correlation coefficient at the output of nonlinear functions  $f_1(x_1)$  and  $f_2(x_2)$  with correlated inputs  $x_1$  and  $x_2$  having zero-mean and the variances  $\sigma_1$  and  $\sigma_2$ , respectively, and the correlation coefficient  $\rho_{x_1 x_2} = \frac{\mathbb{E} \{ x_1 x_2^* \}}{\sigma_1 \sigma_2}$ , has the following derivatives [21]

$$\begin{aligned} \frac{\partial^k \mathbb{E} \{ f_1(x_1) f_2(x_2) \}}{\partial \rho_{x_1 x_2}^k} &= \sigma_1^k \sigma_2^k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{f_1^{(k)}(x_1) f_2^{(k)}(x_2)}{2\pi \sigma_1 \sigma_2 \sqrt{1 - \rho_{x_1 x_2}^2}} \\ &\exp \left( -\frac{1}{2(1 - \rho_{x_1 x_2}^2)} \left[ \frac{x_1^2}{\sigma_1^2} + \frac{x_2^2}{\sigma_2^2} - \frac{2\rho_{x_1 x_2} x_1 x_2}{\sigma_1 \sigma_2} \right] \right) dx_1 dx_2. \end{aligned} \quad (15)$$

Next, for the special case  $f_1(x_1) = x_1$ , then we have [20]

$$\begin{aligned} \frac{\partial \mathbb{E} \{ x_1 f_2(x_2) \}}{\partial \rho_{x_1 x_2}} &= \\ &\sigma_1 \sigma_2 \int_{-\infty}^{\infty} \frac{1}{\sigma_2 \sqrt{2\pi}} f_2'(x_2) \exp \left( \frac{x_2^2}{\sigma_2^2} \right) dx_2, \end{aligned} \quad (16)$$

resulting in

$$\begin{aligned} \mathbb{E} \{ x_1 f_2(x_2) \} &= \\ &\sigma_1 \sigma_2 \rho_{x_1 x_2} \int_{-\infty}^{\infty} \frac{1}{\sigma_2 \sqrt{2\pi}} f_2'(x_2) \exp \left( \frac{x_2^2}{\sigma_2^2} \right) dx_2. \end{aligned} \quad (17)$$

Next, we use the uniform quantizer as follows [20, Chapter 2]

$$\begin{aligned} f_2(x) = Q(x) &= \sum_{i=1}^{2^\alpha} l^i (u(x - l^{l^0, i}) - u(x - l^{\text{up}, i})) \\ &= l^1 + \sum_{i=2}^{2^\alpha} (l^i - l^{i-1}) u(x - l^{l^0, i}). \end{aligned} \quad (18)$$

Using the  $l^{\text{up}, i} = l^{l^0, i+1}$ , we have

$$\frac{\partial Q(x)}{\partial x} = l^1 + \sum_{i=2}^{2^\alpha} (l^i - l^{i-1}) \delta(x - l^{l^0, i}), \quad (19)$$

where  $\delta$  is the Dirac Delta function. Therefore

$$\begin{aligned} \mathbb{E} \{ x_1 Q(x_2) \} &= \\ &\sigma_1 \rho_{x_1 x_2} \sum_{i=1}^{2^\alpha} \frac{l^i \left( \exp \left( -\frac{(l^{l^0, i})^2}{2\sigma_2^2} \right) - \exp \left( -\frac{(l^{\text{up}, i})^2}{2\sigma_2^2} \right) \right)}{\sqrt{2\pi}}. \end{aligned} \quad (20)$$

Next, we find the covariance at the output of the quantizer as follows

$$\begin{aligned} \mathbb{E} \{ Q(x_1) Q(x_2) \} &= \sum_{i=2}^K \sum_{k=2}^K \Delta^2 \\ &\int_0^{\rho_{x_1 x_2}} \frac{\exp \left( -\frac{1}{2(1 - \rho'^2)} \left[ \frac{(l^{l^0, i})^2}{\sigma_1^2} + \frac{(l^{l^0, k})^2}{\sigma_2^2} - \frac{2\rho' l^{l^0, i} l^{l^0, k}}{\sigma_1 \sigma_2} \right] \right)}{2\pi \sqrt{1 - \rho'^2}} d\rho'. \end{aligned} \quad (21)$$

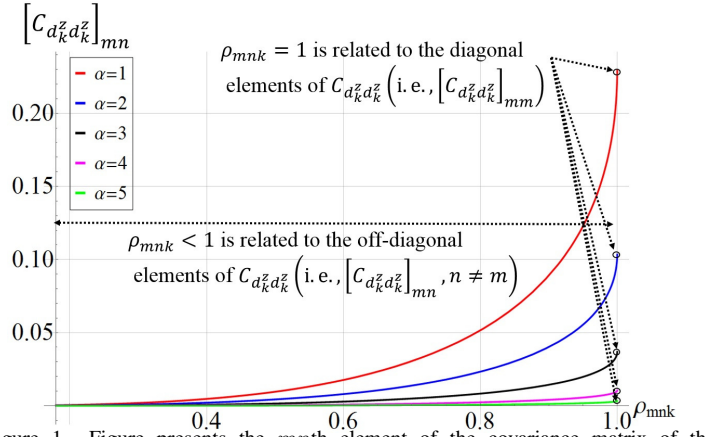


Figure 1. Figure presents the  $m$ th element of the covariance matrix of the quantization distortion (i.e.,  $[C_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{mn}$  in (11)) versus  $\rho_{mnk}$  for different number of quantization bits for a given user  $k$ .

Note that in cell-free Massive MIMO, we have  $\rho_{x_1 x_2} = \rho_{mnk}$ , where  $\rho_{mnk}$  is defined in (24). Finally,  $\mathbf{C}_{\mathbf{r}_k \mathbf{r}_k}$ ,  $\mathbf{C}_{\mathbf{r}_k \mathbf{z}_k}$  and  $\mathbf{C}_{\mathbf{z}_k \mathbf{r}_k}$  are determined using (20) and (21) and the following equalities:

$$\mathbf{C}_{\mathbf{r}_k \mathbf{r}_k} = \mathbb{E} \{ \mathbf{r}_k \mathbf{r}_k \} = \mathbb{E} \{ Q(\mathbf{z}_k) Q(\mathbf{z}_k) \}, \quad (22a)$$

$$\mathbf{C}_{\mathbf{z}_k \mathbf{r}_k} = \mathbb{E} \{ \mathbf{z}_k \mathbf{r}_k \} = \mathbb{E} \{ \mathbf{z}_k Q(\mathbf{z}_k) \}, \quad (22b)$$

$$\mathbf{C}_{\mathbf{r}_k \mathbf{z}_k} = \mathbb{E} \{ \mathbf{r}_k \mathbf{z}_k \} = \mathbb{E} \{ Q(\mathbf{z}_k) \mathbf{z}_k \}, \quad (22c)$$

which completes the proof.  $\blacksquare$

Based on the above derivations,  $[C_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{mn}$  is a function of the number of quantization bits  $\alpha$ , the step-size of the quantizer  $\Delta$ , and the correlation coefficient between the inputs of the quantizer at the  $m$ th and  $n$ th APs, i.e.,  $\rho_{mnk}$ , which is obtained numerically.

### III. NUMERICAL RESULTS AND DISCUSSION

A cell-free Massive MIMO system with  $M$  APs and  $K$  single-antenna users is considered in a  $D \times D$  numerical area, where both APs and users are uniformly distributed at random points. The path loss and noise power are the same as [1]. For the LoS probability, we use the following model from the 3GPP-UMa [2]

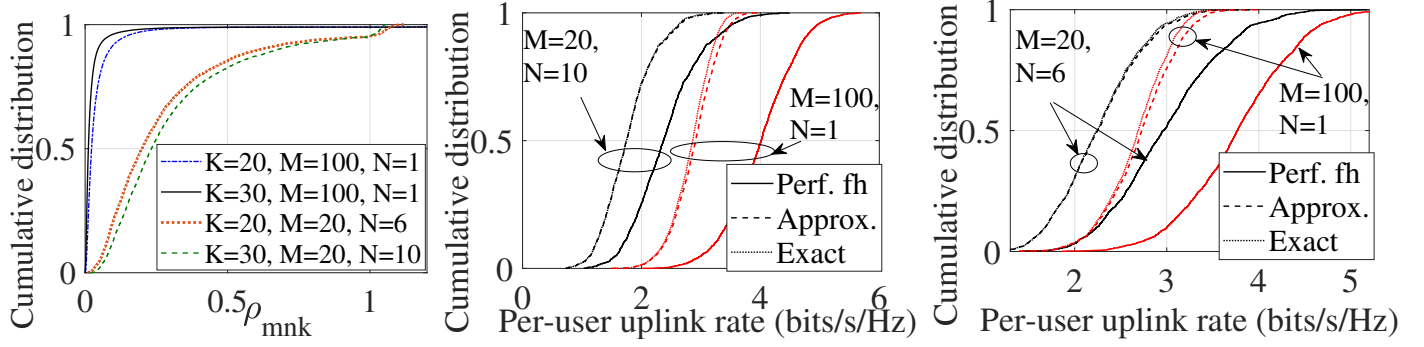
$$P_{\text{LoS}}(d_{mk}) = \min \left( \frac{18}{d_{mk}}, 1 \right) \left( 1 - e^{-\frac{d_{mk}}{63}} \right) + e^{-\frac{d_{mk}}{63}}, \quad (23)$$

where  $d_{mk}$  is in meters. In cell-free massive MIMO, the inputs of quantizer at the  $m$ th and  $n$ th APs are  $z_{mk} = \mathbf{v}_{mk}^H \mathbf{y}_m$  and  $z_{nk} = \mathbf{v}_{nk}^H \mathbf{y}_n$ , respectively. We aim to calculate the correlation coefficient between the  $m$ th and  $n$ th APs, which is given by

$$\rho_{mnk} = \frac{|\mathbb{E} \{ z_{mk} z_{nk}^* \}|}{\sigma_{z_{mk}} \sigma_{z_{nk}}}, \quad (24)$$

where we have  $\sigma_{z_{mk}}^2 = \mathbb{E} \{ |z_{mk}|^2 \}$ , and the expectation is taken over the small-scale fading coefficients.

1) *The Actual Value of Correlation Between the Quantization Distortions Versus the Correlation Between the Inputs of the Quantizers:* Fig. 1 plots  $[C_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{mn}$  versus  $\rho_{mnk}$  for different numbers of quantization bits  $\alpha$ , where the diagonal elements of the covariance matrix are obtained by setting  $\rho_{mnk} = 1$ . Fig. 1 plots  $[C_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{mn}$  versus  $\rho_{mnk}$  for different numbers of quantization bits  $\alpha$ , where the diagonal elements of the covariance matrix are obtained by setting  $\rho_{mnk} = 1$ . From Fig. 1, it can be observed that  $\rho_{mnk} \leq 0.4$  is small enough so that the off-diagonal elements of the correlation matrix can be safely ignored. Note that the performance gap between the exact uplink per-user rate and the uplink per-user rate while ignoring the correlation between the inputs of quantizers depend on total



(a) CDF of  $\rho_{mnk}$ , given in (24). (b)  $K = 20$ . (c)  $K = 30$ .  
 Figure 2. The term ‘‘Perf. fh’’ refers to the perfect fronthaul link. The term ‘‘Exact’’ is the case where we include the correlation between the quantization distortions at different APs whereas the term ‘‘Approx.’’ refers to the case where we ignore the correlations between the error at different APs.

number of users in the area  $K$  and the number of antennas per-AP  $N$ . Hence, in the next section, we investigate the effect of  $K$  and  $N$  on the correlation coefficients  $\rho_{mnk}$  and the achievable rate. Next, we provide numerical results and discussion to address the effect of correlation between the inputs of the quantizers at different APs.

2) *How Large is the Correlation Between the Input of the Quantizers at Different APs in Cell-Free Massive MIMO?*: The CDF of  $\rho_{mnk}$  in cell-free Massive MIMO for the cases of  $K = 20$  and  $K = 30$  users is plotted in Fig. 2a. The figure shows that in the case of a few antennas per AP and large number of users in the area, the correlation between the inputs of the quantizer at different APs is small. The correlation coefficient  $\rho_{mnk}$  enables us to investigate the performance gap between the exact rate and the approximated rate obtained by omitting the correlation  $\rho_{mnk}$ . Note that by increasing the number of APs, the correlation  $\rho_{mnk}$  decreases, resulting in a smaller gap between the approximated rate and the exact rate. This will be investigated in the next subsection.

3) *The Performance Gap Between the Exact Uplink Per-User Rate and the Uplink Per-User Rate While Ignoring the Correlation Between the Inputs of the Quantizers*: In this section, we present the uplink per-user rate with different system parameters for two different scenarios. To remind, the correlation between the quantization distortions at different APs is given by

$$\mathbb{E} \left\{ |\text{TQD}_k|^2 \right\} = \sum_{m=1}^M [\mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{mm} + \sum_{m=1}^M \sum_{n \neq m}^M [\mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{nm}. \quad (25)$$

The rate obtained by the exact value of  $\mathbb{E} \left\{ |\text{TQD}_k|^2 \right\}$  in (26) is referred to as ‘‘Exact’’ in Figs. 2b-2c. Next, we exploit the results in Figs. 2b-2c, the covariance matrix of the quantization distortion is approximated with a diagonal matrix as follows:

$$\mathbb{E} \left\{ |\text{TQD}_k|^2 \right\} = \underbrace{\sum_{m=1}^M [\mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{mm}}_{\text{sum of diagonal elements of } \mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}} + \underbrace{\sum_{m=1}^M \sum_{n \neq m}^M [\mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{nm}}_{\text{sum of off-diagonal elements of } \mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}} \approx \sum_{m=1}^M [\mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{mm}. \quad (26)$$

This scenario is given as ‘‘Approximation’’ in Figs. 2b-2c. As Figs. 2b-2c show, there is a negligible performance gap between the exact rate and the approximate rate obtained by ignoring the quantization

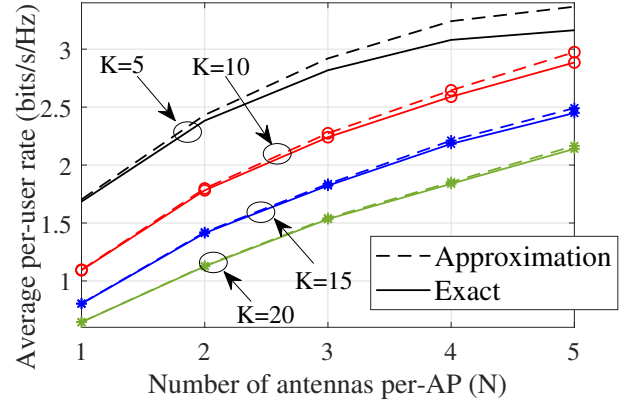


Figure 3. This figure presents the average uplink per-user rate versus number of antennas per AP with  $M = 15$  and  $\alpha = 1$ . Here the term ‘‘Exact’’ refers to the exact uplink rate whereas the term ‘‘Approximation’’ presents the uplink per-user rate while ignoring the quantization correlation.

distortion correlation. Here we consider the uncorrelated quantization distortions at different APs, and the total quantization distortion is obtained as follows:

$$\begin{aligned} \mathbb{E} \left\{ |\text{TQD}_k|^2 \right\} &= \mathbb{E} \left\{ \left| \sum_{m=1}^M d_{mk}^z \right|^2 \right\} \\ &\approx \sum_{m=1}^M \mathbb{E} \left\{ |d_{mk}^z|^2 \right\} = (\tilde{b} - \tilde{a}^2) \\ &\quad \sum_{m=1}^M \left[ N^2 \sum_{k'=1}^K \gamma_{mk'}^2 |\phi_{k'}^H \phi_k|^2 \rho q_{k'} + N \gamma_{mk} \sum_{k'=1}^K \beta_{mk'} \rho q_{k'} + N \gamma_{mk} \right]. \end{aligned} \quad (27)$$

Next, we investigate the ratio between ‘‘the sum of off-diagonal elements of the distortion covariance matrix’’ and ‘‘the sum of all elements of the distortion covariance matrix’’ in (26). We define the following ratio:

$$\text{ratio}_C = \frac{\sum_{m=1}^M \sum_{n \neq m}^M [\mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{nm}}{\sum_{m=1}^M [\mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{mm} + \sum_{m=1}^M \sum_{n \neq m}^M [\mathbf{C}_{\mathbf{d}_k^z \mathbf{d}_k^z}]_{nm}}. \quad (28)$$

Fig. 4 demonstrates the cumulative distribution of  $\text{ratio}_C$  with the same parameters as in Fig. 3. Moreover, Fig. 4 shows that the power of the off-diagonal elements of the quantization distortion matrix is very small relative to the power of all elements of the quantization distortion matrix, except for smaller  $K$  and larger  $N$ . This explains the performance gap between the systems using ‘‘Approximation’’ (obtained by ignoring off-diagonal elements of the distortion covariance matrix) and ‘‘Exact’’ in Fig. 3.

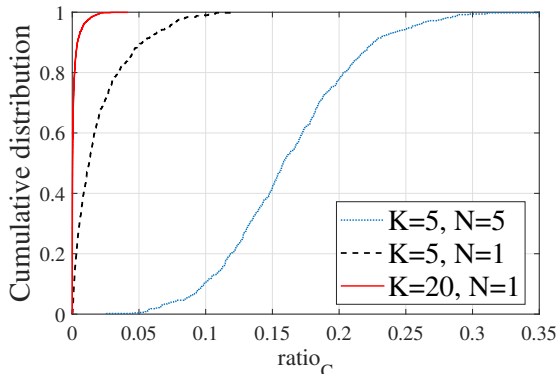


Figure 4. The cumulative distribution of  $\text{ratio}_C$  with  $M = 15$ .

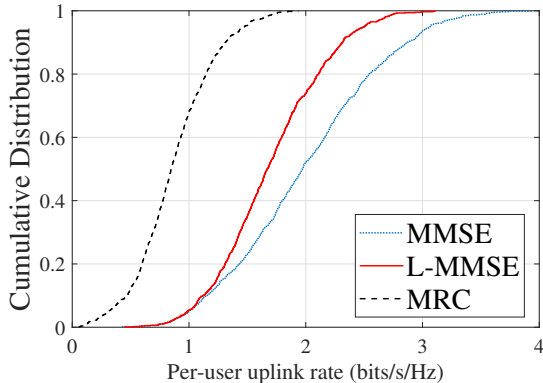


Figure 5. The average uplink per-user rate with three different detectors with  $M = 60$ ,  $N = 1$ , and  $K = 20$ .

4) *The effects of number of users and number of antennas per AP:* Finally, we investigate the effect of the total number of users and number of antennas per AP on the system performance. In Fig. 3, the average per-user uplink rate is presented versus the number of antennas per AP ( $N$ ) for different cases of total number of users in the system (namely  $K = \{5, 10, 15, 20\}$ ). As the figure shows for the case of single antenna APs, the average uplink per-user rate while omitting the quantization correlation is very close to the exact uplink per-user rate. Moreover, as the figure demonstrates, if there are  $K \leq 10$  active users in the area, the average uplink per-user rate while omitting the quantization correlation (shown as Approximation in the figure) is very close to the exact rate. As a results, if there are  $K < 10$  active users and the AP are equipped with  $N > 2$  antennas, the correlation between the inputs of the quantizers should not be ignored.

5) *The effect of different linear detectors:* This section investigates the average uplink per-user rate of cell-free massive MIMO with three different receivers, namely the MMSE, L-MMSE and MRC receivers. In Fig. 5, we assume that  $M = 40$  APs each with  $N = 2$  antennas are uniformly distributed in the area. Moreover, we consider  $K = 20$  users and  $\tau_p = 20$  as the length of pilot sequences. As the figure shows the MMSE receiver provides the greatest median uplink rate. Note that in the case of MRC and L-MMSE, the CPU does not have access to the quantized channel estimates and exploits only the statistics of the channel to decode the data whereas to design the MMSE receiver the CPU needs to have the quantized version of channel to design the receiver.

#### IV. CONCLUSIONS

We have considered cell-free massive MIMO with L-MMSE detector, when the quantized version of the weighted signals are available at the CPU. Bussgang decomposition has been used to

model the quantization effects. We have investigated the assumption of uncorrelated quantization distortion and showed that this assumption will not affect the insights obtained in our works. We presented a comparison between the exact uplink per-user rate and the uplink per-user rate while ignoring the correlation between the inputs of the quantizers. Finally, we have presented the conditions which imply that the quantization distortions across APs can be considered as uncorrelated signals.

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