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1 **Liquefaction and post-liquefaction of granular material under multi-**
2 **directional cyclic loading**

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Liquefaction and post-liquefaction of granular material under multi-directional cyclic loading

Abstract

Soil liquefaction can be induced by natural events that entail complicated loading directions and magnitudes. To investigate the liquefaction behaviour of granular material under complex loading conditions, a series of strain-controlled cyclic simple shear tests are conducted on the uniform-sized glass beads. These tests include uni-directional and multi-directional loading paths. An energy-based method is used to assist the understanding of the cyclic behaviour of the specimens. After the first liquefaction happens, the specimens are re-consolidated and subjected to monotonic undrained shearing to investigate their post-liquefaction behaviour. The test results indicate that the specimens subjected to multi-directional cyclic shearing are more prone to liquefy than the ones under uni-directional loading. Furthermore, the cyclic shear strain amplitude and cyclic loading path have significant influences on the soil liquefaction resistance, re-consolidation volumetric strain and post-liquefaction shear strength. Nevertheless, the total energy that is dissipated for liquefying a specimen is only dependent on its relative density.

Keywords: Multi-directional cyclic simple shear; liquefaction and post-liquefaction; granular material; settlement; energy dissipation

Introduction

It is well accepted that soil liquefaction induced by natural events is one of the major contributors to construction damages; thus, soil liquefaction and soil behaviour after liquefaction are of great interest to researchers. To investigate the liquefaction and post-liquefaction behaviour of soil, numerous studies have been carried out by using various experimental techniques and testing apparatuses *in the last decade* (Ishihara & Yasuda, 1975; Sivathayalan, 1994; McCarron *et al.*, 1995; Porcino & Caridi, 2007; Wang *et al.*, 2012; Bastidas *et al.*, 2017; Hubler *et al.*, 2017; Castelli *et al.*, 2019; Kumar *et al.*, 2020). On account of technical difficulties and apparatus limitations, most of these studies only focus on shear behaviour in a single direction. However, in reality, natural events, such as earthquake and tsunami, involve complicated loading directions and magnitudes. With the assistance of advanced facilities, a handful of research with the multi-directional simple shear devices has investigated shear behaviour under such condition (Ishihara & Yamazaki, 1980; Boulanger & Seed, 1995; Kammerer *et al.*, 2002; Matsuda *et al.*, 2011; Mirbaha, 2017). Kammerer *et al.* (2002) found that, compared with the pore water pressure generation under the uni-directional cyclic loading, those under the multi-directional condition were far more rapid and complicated. Matsuda *et al.* (2011) clarified that the cyclic shear direction and shear strain amplitude have significant effects on the post-earthquake settlements.

Most of the studies mentioned above use the stress-controlled method to conduct experiments. As a result, the liquefaction resistance of tested material is interpreted based on the influence of cyclic stress ratio (cyclic shear stress amplitude/ total vertical stress) within a preset number of cycles. In this method, the criterion for liquefaction is that the single amplitude of the induced shear strain reaches 3% to 3.75% (Ishihara & Yamazaki, 1980; Boulanger & Seed, 1995; Wijewickreme *et al.*, 2005). However, for loose samples, the induced shear strain can be significantly large when liquefaction initiates. In comparison to stress controlling, a few researchers believe that strain

1 controlling is more intuitive, since liquefaction is a displacement-dependent phenomenon. It is
2 clarified that shear strain is the key parameter that allows control over the ground settlements and
3 the development of pore water pressure during the cyclic loading (Dobry et al., 1982; Talaganov,
4 1992; Vucetic, 1992; Matsuda et al., 2011; Kang et al., 2015; Kumar et al., 2018; Dammala et al.,
5 2019). More importantly, unlike the stress controlling method, cyclic tests controlled by shear
6 strain do not reach uncontrolled strain condition. In other words, a full liquefaction process can be
7 obtained, which is necessary to undertake the post-liquefaction investigation.

8 Although it is reported that soils are more prone to liquefaction under multi-directional condition
9 than under uni-directional condition, most of the comparison is made based on one type of multi-
10 directional path (circular shape). The assessment of the data obtained from different multi-
11 directional loading paths are desired. Likewise, the interpretation of the data so obtained is
12 expected to be made from other aspect, for example, in the view of energy dissipation.
13 Consequently, the authors believe that our knowledge of the mechanism characterizing multi-
14 directional cyclic liquefaction is still limited. Studies that investigate post-liquefaction and
15 consider multi-directional loading history are even more limited. Therefore, the main objective of
16 this work is to provide an in-depth understanding of cyclic liquefaction and post-liquefaction
17 behaviour of granular materials under multi-directional simple shearing. To meet this end, a series
18 of strain-controlled uni- and multi-directional cyclic simple shear tests were conducted on
19 uniform-sized glass beads. Accordingly, the following influencing factors, namely shear strain
20 amplitude, relative density and loading path, on liquefaction resistance, were investigated. To
21 analyze the exact impact of these factors, an energy-based method is adopted. Lastly, the undrained
22 monotonic shearing test was conducted on liquefied and virgin samples (without experiencing
23 liquefaction) to study their post-liquefaction shear behaviours.

24 **Multi-directional cyclic simple shear tests in constant volume condition**

25 *Description of the testing apparatus*

26 The testing apparatus, known as the variable direction dynamic cyclic simple shear (VDDCSS)
27 apparatus, is shown in Fig. 1a. Three electro-mechanical actuators control the apparatus: One
28 vertical actuator (z-direction) provides the device with vertical load or displacement. Conversely,
29 the other two orthogonal actuators (x- and y-direction) allow the device to exert shear stress or
30 strain in any direction on the horizontal plane.

31 In this device, a cylindrical specimen (Fig. 1b & c), which is 70 mm in diameter and 22.6 mm in
32 height, is laterally confined by several circular Teflon coated rings. Each ring is 71 mm in diameter
33 and 1 mm in height. A latex membrane is placed between the rings and the specimen to protect the
34 rings from being damaged and to ensure the uniform shearing of the specimen. During the test, the
35 smooth surface of the rings allows the specimen to be sheared freely with minimum friction. The
36 VDDCSS can perform monotonic and cyclic shear tests, and more detailed information of this
37 apparatus can be found in Li (2016).

38 *Testing material and testing procedure*

39 It is reported that the factors, such as grading curve, angularity and fine composition of the tested
40 material can affect the cyclic behaviour (references). In this work, the aim is to investigate the
41 multi-directional loading paths on the liquefaction and post-liquefaction shear behaviour. As a

1 result, the uniform-distributed glass beads are selected as the tested material. Moreover, in discrete
2 element method (DEM), simulations are often implemented on the spherical particles, for example,
3 Zhang *et al.* (2019) employed uniform-sized glass beads to investigate static simple shear
4 behaviour in bi-direction. The testing results obtained from glass beads in this work may provide
5 a reference for future research either from the experimental aspect or the numerical aspect. In this
6 research, the glass beads consists of spherical particles that primarily contain silicon dioxide with
7 some other silicates. It is uniformly graded with a mean diameter (D_{50}) of 0.8mm (Fig. 2). Its
8 specific gravity, maximum and minimum dry void ratios were measured according to the
9 American Society for Testing and Materials (ASTM) standards D854, D4253 and D4254. The
10 physical properties of this material are summarized in Table 1.

11 Each specimen was prepared as follows: oven-dried glass beads samples with predetermined
12 weight were poured into the cylindrical shear box by employing the dry funnel method. This
13 method is believed to be a proper way to duplicate the densified soil sample in earthquake regions
14 (Li *et al.*, 2018). To obtain higher relative densities, A low-energy and high-frequency shaking
15 table was used for providing even striking (vibration), which is of 0.5mm in the amplitude and 2
16 Hz in the frequency.

17 After the sample preparation, the specimens were tested following the process shown in Fig. 3.
18 During the stage of consolidation, the specimen was K_0 consolidated for 30 mins under the vertical
19 confining stress $\sigma'_{vc} = 50$ kPa. The relative densities obtained at the end of consolidation are $D_r =$
20 45%, $D_r = 70\%$ and $D_r = 81\%$, which denote medium dense, dense and very dense categories
21 respectively. Subsequently, the undrained cyclic simple shear tests were conducted on these
22 specimens with various shear strain amplitudes, relative densities and loading paths. During the
23 cyclic shearing, the vertical displacement was fixed to keep the volume of the specimen constant
24 and obtain an equivalent undrained shearing condition. In this condition, the decrease of the
25 vertical stress that is attained in a dry sample is equal to the increase of the pore water pressure
26 that is found in a saturated sample under the true undrained shearing test (Dyvik *et al.*, 1987). The
27 details of the performed tests are summarized in Table 2. In particular, the tests employed three
28 different cyclic paths, namely the uni-directional Path-X, multi-directional Path-O and Path-8. The
29 Path-X represents the conventional cyclic loading in a single direction. Fig. 4a shows the shear
30 waveform and plan view of the strain path for this type of loading, in which the shear strain was
31 exerted on a specimen along the x-direction. The Path-O and Path-8 represent the complex cyclic
32 loadings with the same or different frequencies in two directions. Their typical shear waves and
33 shear strain movement are illustrated in Fig. 4b& c, respectively. For the multi-directional tests
34 conducted in this study, the same shear strains were simultaneously applied to the specimens with
35 the phase difference of 90 degrees in x- and y-direction. Specifically, in the Path-O, the frequencies
36 of x- and y-direction are the same, while in the Path-8, the frequency in x-direction is half of the
37 one employed in y-direction. Since liquefaction is a deformation-based phenomenon, it is
38 insensitive to loading frequency (Wong, 1971; Jong & Seed, 1988). Consequently, the loading
39 frequency was set as 0.1 Hz in this study for ease of control. The cycle number used in this paper
40 denotes the cycles in the x-direction. The liquefaction cycle number is recorded when the
41 equivalent pore water pressure reaches over 95% of the vertical confining stress.

1 After reaching the first liquefaction, the shear displacement was returned to the original point
2 before the specimens were re-consolidated under the same conditions as in the first consolidation.
3 Since the cyclic shear strain amplitudes used during the tests are relatively small, the nominal
4 relative density for each category of specimens (medium dense, dense, and very dense) after re-
5 consolidation is considered to be the same, and they are $D_r' = 50\%$, $D_r' = 73\%$ and $D_r' = 82\%$
6 respectively (The real relative density of each specimen is provided in Table 3). Finally, the
7 monotonic undrained shearing test was conducted on the specimens in the x-direction. This stage
8 is controlled in displacement with a speed of 0.01mm/min to ensure the adequate generation of
9 equivalent pore water pressure. To provide a reference, the samples with no liquefaction history
10 were also examined with the same monotonic undrained shearing test.

11 **Test results and discussion**

12 *Initial liquefaction*

13 In this study, the liquefaction resistance of tested specimens, which were tested by the strain-
14 controlled method, has been estimated based on the number of liquefaction cycles and the
15 magnitude of shear strain amplitude. Taking the tests with the shear strain amplitude of 0.1% under
16 vertical confining stress of 50kPa and relative density of 45% as examples, Fig. 5-7 illustrate the
17 cyclic shear behaviour of the specimens with different loading paths when tested with these
18 conditions. Fig. 5 shows the shear stress-strain hysteresis loops in x- and y-direction. From this
19 image, it can be seen that, as the cycle increases, the secant shear moduli (the ratio of cyclic shear
20 stress and strain amplitude) decrease gradually to nearly zero. To compare the variation of shear
21 stress under different loading paths more easily, Fig. 6 shows the development of shear stress
22 against the number of loading cycles in x- and y-direction. It proves that at the first few cycles, the
23 cyclic loading paths have little impact on the sample stiffness since their respective shear stress
24 magnitudes are almost the same. However, as the tests continue, the decreasing rates of shear stress
25 for Path-X, Path-O and Path-8 in the x-direction are in ascending order. Similar results are also
26 found in the y-direction, where the specimen under the Path-8 loses its shear stress faster than
27 under the Path-O.

28 From a more direct point of view, as shown in Fig. 7, the pore water pressure, which is one of the
29 key indicators of cyclic liquefaction, is plotted against the number of cycles for different cyclic
30 loading paths. During the cyclic shearing, the specimens tend to decrease the volume, and in a
31 drained condition, this is called densification. However, the decreasing of this volume is prevented
32 by the undrained condition. Similarly, the tendency of the volume to change throughout successive
33 cycles determines the magnitude and rate of pore water pressure generation in the specimens. From
34 Fig. 7 it could also be observed that the specimen under uni-directional cyclic path develops pore
35 water pressure most slowly and the specimen under multi-directional cyclic Path-8 develops pore
36 water pressure most quickly.

37 To closely compare the liquefaction behaviour of tested material under uni- and multi-directional
38 loading paths, Fig. 8 displays the effects that any given cyclic loading path and relative density
39 had on the liquefaction resistance. In this figure, the number of cycles that is necessary to reach
40 liquefaction N is plotted against the cyclic shear strain amplitude A_γ in a logarithmic format. Fig.

1 8a presents the result obtained under the medium dense state and Fig. 8b & c show the
2 corresponding results for the dense and very dense states.

3 Firstly, it can be seen that under the same relative density and shear strain amplitude, tests that
4 were conducted under Path-X require far more cycles to reach liquefaction than the tests sheared
5 multi-directionally. At the same time, the tests run under the multi-directional Path-O only require
6 slightly more cycles than the tests performed under the Path-8. The trend of the curves is the same
7 as the results previously presented in Fig. 5-7. The difference of liquefaction cycle numbers is
8 resulted by the different tendencies of volume change, which may be related to the rearrangement
9 of particles due to different loading paths. Secondly, as each graph of Fig. 8 shows, the specimens
10 are sheared under different relative densities but by the same loading path. It is consistent with all
11 loading paths, where increasing the relative density will increase the liquefaction resistance. It is
12 because the specimens tend to contract less under the higher relative density, and a less contractive
13 specimen is more difficult to liquefy.

14 Lastly, in this logarithm-format figure Fig. 8, it can be observed that the curves are almost parallel
15 to each other. Moreover, any increase in the shear strain amplitude reduces the number of cycles
16 to reach liquefaction. When the shear strain amplitude reduces significantly (close to the threshold
17 value), the effects of cyclic loading path and relative density on the liquefaction resistance are
18 noticeable. However, when the shear strain amplitude increases, the differences previously found
19 between liquefaction cycle numbers for different cyclic loading paths or relative densities decline.

20 *Energy dissipation*

21 Nemat-Nasser and Shokooh (1979) first established an energy approach to understanding the
22 liquefaction potential of sand under cyclic shearing. Their work indicates that the rearrangement
23 of sand particles engendered by cyclic loading dissipate a certain amount of energy. Verified by
24 experiments and models only considering single-directional shearing, the energy dissipated inside
25 of the specimen is proved to be directly related to the rate of liquefaction (Figuroa et al., 1994;
26 Liang, 1995; Baziar & Jafarian, 2007; Sonmezer, 2019). In this approach, the energy per unit
27 volume (kJ/m^3) dissipated in one cycle is denoted by the area in the corresponding shear stress-
28 strain loop (for example, in Fig. 5a).

29 Because of considering multi-directional cyclic loading, in this study, the total energy per unit
30 volume (δW) necessary to liquefy a specimen is given by:

$$31 \quad \delta W = \sum_{i=1}^{n-1} \frac{1}{2} (\tau_{x_i} + \tau_{x_{i+1}}) (\gamma_{x_{i+1}} - \gamma_{x_i}) + \frac{1}{2} (\tau_{y_i} + \tau_{y_{i+1}}) (\gamma_{y_{i+1}} - \gamma_{y_i}) \quad \text{Equation (1)}$$

32 Where n is the total number of recorded data before liquefaction, τ_x and τ_y are the shear stresses
33 in x- and y-direction, γ_x and γ_y are the shear strains in x- and y-direction.

34 To investigate the mechanism of liquefaction under uni- and multi-directionally cyclic conditions,
35 the total energy per unit volume calculated from Equation (1) is shown in Fig. 9. It is presented
36 against the shear strain amplitude of different cyclic loading paths. Fig. 9a shows the curves
37 obtained under the medium dense state. Similarly, Fig. 9b & c show the curves found under the
38 dense and very dense state, respectively. It is found that in Fig. 9a & b, the curves of the unit energy
39 dissipated for liquefaction are extremely flat, while Fig. 9c shows the similar trend except for the

1 one obtained from the Path-X and strain amplitude of 0.1%. Refer to Fig. 8c, the number of
2 liquefaction cycles is more than 100 for this case, and the friction error induced inside of the
3 apparatus may be enlarged significantly to affect the energy calculation. However, in the big picture,
4 Figure 9 illustrates that under a specific loading path and a certain relative density, the total energy
5 per unit volume for liquefaction is generally a constant value, despite the shear strain amplitude
6 that is exerted on the specimen. Moreover, this amount of energy grows in accordance to the
7 increase registered in the relative density. This result is consistent with previous research findings
8 (Figueroa *et al.*, 1994; Baziar & Jafarian, 2007; Jafarian *et al.*, 2012), which claimed that the total
9 dissipated energy per unit volume up to liquefaction rises in concomitance with the increase of
10 relative density. However, it is less dependent on the shear strain amplitude. More importantly,
11 from this figure, it can also be observed that when the relative density is the same, the total energy
12 dissipated per unit volume under different loading paths is, by and large, the same. A similar study
13 conducted by Polito *et al.* (2013) with stress-controlled cyclic triaxial tests showed that the
14 accumulated energy per unit volume at the onset of liquefaction is not affected by the loading
15 shape (sinusoidal, square, triangular, irregular symmetric and irregular asymmetric).

16 To discuss the difference of liquefaction resistance that were found between uni- and multi-
17 directional simple shearing, the set of tests with the shear strain amplitude of 0.1% under vertical
18 confining stress of 50kPa and relative density of 45% is chosen as an example. Fig. 10, whose data
19 is calculated based on Equation (1), illustrates the development of the energy dissipated per cycle
20 as well as the accumulated energy. This figure shows that the energy dissipated drops quickly in
21 the last few cycles, which, in turn, indicates the occurrence of liquefaction.

22 Since the total energy dissipated per unit volume for liquefaction is a constant value independent
23 of the cyclic loading path, Fig. 10 evidently shows that the energy dissipated in each cycle of the
24 specimen shearing under the Path-X is considerably lower than that under the multi-directional
25 paths. The primary reason for this can be found in Fig. 10a, which shows that the energy dissipated
26 per cycle in the Path-X is contributed by a single direction. While Fig. 10b & c show that the energy
27 dissipated per cycle in the Path-O and Path-8 is composed of the x- and y-direction simultaneously.
28 The energy dissipated per cycle in one direction is much lower than the energy dissipated in two
29 directions. Although both in the Path-O and Path-8 energy is dissipated in two directions, the
30 number of cycles necessary to achieve liquefaction in the Path-8 is slightly lower than in the Path-
31 O as compared in Fig. 10b and Fig. 10c. This is the case in that the frequency of the Path-8 in the
32 y-direction is twice that the one of the Path-O in the y-direction. In conclusion, more energy is
33 dissipated in the Path-8 than in the Path-O in the y-direction.

34 ***Post-liquefaction***

35 After the initial liquefaction, the specimens were re-consolidated under the same conditions as the
36 first consolidation. Then, they were sheared in the x-direction monotonically in an undrained
37 manner. In this study, this process is called post-liquefaction. Since the Path-O is a representative
38 of multi-directional loading path, the set of tests, which have experienced liquefaction in the Path-
39 O, is taken as the main example to investigate the impact that the history of the first liquefaction
40 on the post-liquefaction stage. Fig. 11 and Fig. 12 display the volume changes that occur
41 throughout the re-consolidation stage. Fig. 11 shows the volumetric strain development of the

1 specimens with a liquefaction history in the Path-O under the relative densities of 50%, 73% and
2 82%. It can be seen from the figure that the amount of volume reduction so observed is lower
3 under the higher relative density. More importantly, the volumetric strain increases in accordance
4 to the amount of the first liquefaction shear strain amplitude. Fig. 12 demonstrates the volumetric
5 strain development of specimens with the shear strain amplitude of 0.1% under different initial
6 liquefaction loading paths. By doing so, it could be observed that the multi-directionally liquefied
7 specimens generate more volumetric strain than the uni-directionally ones. Same results are
8 presented by Matsuda *et al.* (2004) by employing a multi-directional simple shear apparatus. Their
9 tests, which were conducted with limited initial cyclic conditions, showed that the settlements
10 obtained from the multi-directionally liquefied cases in the re-consolidation stage are larger than
11 the ones acquired from the uni-directionally liquefied cases.

12 To study the static simple shear behaviour of the tested material in the post-liquefaction stage, the
13 results of the specimens with different initial liquefaction histories are presented here. To facilitate
14 the analysis, the data of the virgin samples are also provided as references. Fig. 13a shows the
15 development of shear stress and stress path of specimens that liquefied in the Path-O with different
16 strain amplitudes under the relative density of 50%. Fig. 13b & c show the specimens that liquefied
17 under relative densities of 73% and 82% respectively. From these data, it can be observed that the
18 specimens with liquefaction history, despite the amount of strain amplitude, have greater peak
19 strength than the virgin samples. This phenomenon is most obvious for the medium dense
20 specimens with $D_r' = 50\%$. As shown in Fig. 13a, the specimens with liquefaction history shift
21 shear behaviour from strain-softening to strain-hardening. Similar conclusions have been reported
22 in the previous studies (Porcino *et al.*, 2009; Bastidas *et al.*, 2017) that were conducted through
23 post-liquefaction undrained cyclic tests of sand and by using uni-directional simple shear devices.
24 They showed that, although there is no significant change in the relative density, the sand samples
25 manifest higher post-liquefaction resistance if they have experienced small cyclic pre-shearings.
26 This concept of small cyclic pre-shearing was first introduced by Ishihara and Okada (1978), who
27 conducted undrained cyclic triaxial compression tests on sands. They defined the small pre-
28 shearing as a stress history where the stress path is restricted to the domain bounded by two PT-
29 lines (lines of phase transformation). According to this definition, the initial liquefaction loading
30 histories used in this study in the post-liquefaction stage are all confirmed to be the small pre-
31 shearing. The stress paths of specimens in the x- and y-direction under the vertical stress of 50kPa,
32 relative density of 45% and strain amplitude of 0.1%, are shown in Fig. 14 as an example. Thus it
33 is reasonable that the rest of the tests agree with the trend shown in Fig. 13.

34 Apart from the above observations, there is another important finding in Fig. 13. It indicates that
35 for each category of relative density, as the initial liquefaction shear strain amplitude increases,
36 the peak strength in the post-liquefaction stage raises as well. These results are also seen in the
37 specimens liquefied in the Path-X and Path-8. Zhou and Chen (2005), who investigated the
38 influence of previous strain histories on liquefaction resistance, have obtained similar results. It is
39 found that, within the range of small pre-shearing, the liquefaction resistance is larger whenever
40 the shear strain is larger too. This may be the case in that larger shear strains can eliminate more
41 local instabilities in the sample structure than smaller ones.

1 Fig. 15 shows the undrained shear behaviour of specimens with different loading paths in the initial
2 liquefaction stage. Data presented in this figure are extracted from the tests conducted with a shear
3 strain amplitude of 0.1% and the relative density of 73%. The results so obtained confirm the
4 above-discussed conclusion that the specimens with a liquefaction history (no matter which
5 loading path caused the liquefaction) present a greater peak strength than the specimens which do
6 not have a liquefaction history. Moreover, they prove that, the specimens that are liquefied multi-
7 directionally, in general, exhibit a higher peak strength than those liquefied uni-directionally. For
8 the multi-directionally samples, the result of the specimen liquefied with the Path-8 history is
9 slightly higher than the one liquefied with the Path-O history. The same results are also observed
10 in the tests which were run with the relative densities of 50% and 82%. This observation proves
11 that specimens undergoing liquefaction under multi-directional loading paths are stronger in the
12 post-liquefaction stage than those subjected to liquefaction in the uni-directional loading path.

13 **Conclusions**

14 To study the liquefaction behaviour of granular material under multi-directional conditions, a
15 series of cyclic simple shear tests have been conducted on the uniform-sized glass beads. Two
16 types of multi-directional loading paths, namely the Path-O and Path-8, are employed and
17 compared with the uni-directional loading Path-X. After initial liquefaction, the monotonic
18 undrained simple shearing was conducted on the re-consolidated specimens to investigate their
19 post-liquefaction behaviour. The effects of relative density and strain amplitude have also been
20 considered. The main conclusions are summarized below:

- 21 1) In the initial liquefaction stage, the liquefaction resistance of specimen is higher under the
22 multi-directional path. Moreover, the number of cycles necessary to reach liquefaction are
23 increased by an increasing relative density or a decreasing shear strain amplitude.
- 24 2) The total energy dissipated to liquefy a specimen is dependent on the relative density but
25 independent of the shear strain amplitude and loading path. Nevertheless, during one cycle
26 of loading, the multi-directional Path-8 dissipates energy most quickly. In contrast, the uni-
27 directional Path-X dissipates energy most slowly. Therefore, this finding explains the
28 difference in liquefaction resistance that a specimen exhibits under different loading paths.
- 29 3) In the re-consolidation stage, the settlements obtained with multi-directionally liquefied
30 specimens are higher than those accrued with uni-directionally liquefied specimens.
31 Likewise, the settlements gained with higher initial shear strain amplitudes are larger than
32 those attained with lower initial shear strain amplitudes.
- 33 4) In the post-liquefaction stage, the initially liquefied specimens are generally stronger than
34 the virgin specimens. The larger the initial shear strain amplitude is, the stronger the
35 specimen becomes. Furthermore, the specimens liquefied with the Path-8 shows the highest
36 peak strength. In contrast, the specimens liquefied with the Path-X shows the lowest one.

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