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Simulation of Railway Drainage Asset Service Condition Degradation in the UK Using a Markov Chain–Based Approach

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Abstract: UK railway drainage systems are facing increasing challenges due to poor completeness of the asset inventory, long asset life cycles, more intense use of the UK railway system, and a changing climate. It is therefore important for drainage managers to acquire a better understanding of the current and future condition of the drainage assets for which they are responsible. This study presents a Markov model for simulating the potential future service condition of various classes of UK railway drainage assets based on observed historical changes in asset condition. Linear regression analysis was performed on distinct asset groups and the influence of the characteristics of asset construction material, size, shape, and location on the rate of the degradation process was quantified. These results were incorporated with the continuous time Markov chain model to improve the accuracy of the degradation rate prediction for several drainage asset classes. The model is illustrated on a case study of the Network Rail drainage assets showing the minimum number of samples required to make a reliable estimation of the service condition degradation process. **DOI: 10.1061/(ASCE)IS.1943-555X.0000630.**

Introduction

Network Rail (NR) is the owner of the vast majority of railway infrastructure, including associated drainage systems, in England, Scotland, and Wales. The rail network is divided into nine strategic geographical routes (pre-2020), each responsible for its own day-to-day asset management decisions, with standards, assurance, and support systems provided by a technical authority from a central strategic department. Each consecutive 5-year period is referred to as a control period (CP) for NR; a strategic business plan is agreed at the beginning of a control period stating goals and objectives for the period. For drainage, asset management plans are created with the aim of developing strategies to prevent increase in risks to passengers, workers, and members of the public due to drainage asset failure, while minimizing whole life, whole system costs.

There is an ever-increasing recognition that effective and reliable drainage systems can significantly enhance the operational performance of the entire railway system (Drainage Asset Policy, unpublished report, 2017). Inadequate hydraulic capacity in railway drainage systems can cause unexpected trackside flooding, which can lead to temporary speed restrictions or temporary closures of railway lines. In the past 5 years, there were on average 450 flooding events per year, which caused 0.3 million hours of delay each year, leading to a compensation costs to Network Rail at an average of £17 million per year. Such cost is made up solely of payments to impacted train operation companies and does not include the cost of replacing damaged assets.

Adequate control of water is also crucial to the management and maintenance of other railway infrastructure, such as tracks, track beds, earthworks, and signaling (known as parent assets). This is because water can play a role in many failure mechanisms that affect parent assets, such as the long-term degradation of the stiffness of the materials that form the track support system and earthworks (Drainage Asset Policy, unpublished report, 2017). An impaired drainage system can thus result in damage to parent assets, and hence further disruption to train operation as well as higher maintenance costs and risk to human safety. It is NR's major concern to eliminate the safety consequences of drainage failure, such as derailments and injuries of passengers and NR workers.

Flooding would occur when there is a lack of local hydraulic capacity in the system, which could be caused by inadequately designed capacity, asset degradation, change in land use/land cover, or increased load due to climate change. All drainage assets are expected to be designed and built in accordance with NR's design standard with the hydraulic capacity to operate for a rainfall event of a specified return period and duration. A 1 in 10-year return period is the lowest standard; therefore, all the drainage systems are expected to withstand a 1 in 10-year rainfall event. However, results from flooding events analysis of the 2,250 cases of flooding incidents recorded during the last 5 years show that around 95% of flooding happened with precipitation less than the expected rainfall volume of a 1-day-duration, 10-year return period rainfall event. Since design standards can be expected to be followed in a regulated industry, this preliminary analysis provides evidence that this flooding could be either due to poor design or due to the degradation of installed drainage assets from their original condition.

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Degradation is reflected by changes in asset condition. At NR, the asset condition is split into two parts: the structural condition and the service condition:

- Structural condition: the fabric of the asset and the severity of structural defects that affect its integrity. Structural defects can be addressed by repairing or replacing the asset.
- Service condition: defects that affect the performance of the asset and the severity of the defects that reduce its hydraulic capacity below the original design level. These defects may be independent of the structural condition or may be linked. Service defects can be addressed by maintenance of the asset such as cleansing or vegetation clearance.

As stated in CIRIA C714 (CIRIA 2014), the effective management and maintenance of the drainage network requires knowledge of the asset inventory, its previous and current condition, hydraulic capacity, historical performance, and current status. NR is in the process of improving its drainage asset knowledge by scheduling surveys and inspections to verify the existing data record and identify unrecorded assets, achieving a 25,000 asset inventory increase in the last control period (April 1, 2014–March 31, 2019). The degradation process controls how hydraulic performance has changed since the last inspection, or will change in the future. It is therefore important to study the degradation process of the assets and develop appropriate modeling tools to enable a better understanding and estimation of the status of drainage assets.

Literature Review

There are many factors thought to influence the rate of infrastructure asset deterioration such as asset type, age, size, material, and local soil characteristics. Railway drainage systems are composed of buried drainage pipes linked at catchpits (chambers that provide inspection access and allow sediment to settle), which drain via outlets to adjacent surface water bodies; they operate by gravity and so are analogous to stormwater sewers. There is little information on the deterioration of railway drainage systems. In Ana et al. (2009), an investigation into the important factors affecting pipe deterioration in the sewer network of Leuven (Belgium) was carried out using logistic regression. It revealed that out of the 10 variables considered, age, material, and length are the only three that significantly affected the pipe service condition. However, by comparing results with similar studies in UK (Ariaratnam et al. 2001) and Canadian networks (Davies et al. 2001), they found that each of the studied networks has a slightly different set of significant variables, and thus concluded that there is no single set of variables that can explain sewer deterioration; it seemed to vary from one network to another.

Due to the many uncertainties in the deterioration process such as unobservable explanatory variables and measurement errors, deterioration is often predicted using a probabilistic model to capture its stochastic nature. Although deterioration models for railway drainage systems have not been developed, studies of other piped systems such as sewer systems and stormwater pipes have been published.

Some of the models only describe two states, deficient or nondeficient, such as the cohort survival model proposed by Herz (1998), which is based on the Herz distribution, determining the lifetime probability distribution derived from the current stock of pipes. It has been applied to drinking water distribution networks to predict the future rehabilitation need. However, these methods only provide information on when assets are expected to fail; they lack the condition information in between functioning and failure, which is critical for asset managers in formulating a planned maintenance regime.

The Markov approach is a probabilistic model widely used for simulating infrastructure deterioration that can describe systems with multiple condition states. Micevski et al. (2002) developed a Markov model for the structural deterioration of stormwater pipe infrastructure, where the Markov transition probabilities were estimated using the Metropolis-Hastings algorithm. Both Baik et al. (2006) and Wirahadikusumah et al. (2001) presented the use of a Markov chain–based deterioration model in sewer pipes. While Baik et al. (2006) used the ordered probit model to estimate the probability of deterioration, Wirahadikusumah et al. (2001) used nonlinear optimization focused only on structural deterioration. Kleiner et al. (2010) introduced the nonhomogeneous Poisson process (NHPP) for future prediction of the structural failure for an individual water main, considering both static factors (i.e., pipe intrinsic) and dynamic factors (e.g., climate, cathodic protection, breakage history). Markov models have also been applied to many infrastructures other than piped systems such as bridges and pavements. Mizutani et al. (2017) used a Markov model to predict reinforced concrete bridge elements deterioration due to chloride-induced corrosion of the reinforcement, using Bayesian statistics as an estimate for transition probabilities when there is little to no available time series inspection information. Wellalage et al. (2015) presented a Metropolis-Hasting algorithm–based Markov chain Monte Carlo simulation approach to calibrating Markovian bridge deterioration models using inspection data for 15 years on Australian railway bridges. Surendrakumar et al. (2013) provided a Markovian probability process to predict the future condition of the pavement, which can be used to design a decision support system for pavement maintenance management.

Neural networks are thought to be of relevance because they are particularly effective in dealing with data that have high volatility and nonconstant variance. Tran et al. (2006) used neural networks to predict the condition of stormwater pipes and Najafi and Kulandaivel (2005) used them on sewer networks. The probabilistic neural network (PNN) model developed by Tran et al. (2006) was tested with snapshot-based sample data and compared with a traditional parametric model using discriminant analysis. The data set is consistent of 650 data points taken by closed-circuit television (CCTV) inspections, obtained from 27 km of the total 800 km of stormwater pipes in the City of Greater Dandenong, Australia. The structural and hydraulic conditions are graded into three levels: (1) good, (2) fair, and (3) poor. Results show it slightly outperforms others in terms of prediction performance; however, the accuracy of the model is still not high because the percentage of correct prediction of PNN is only 66.9%, and also the key factors for prediction are difficult to interpret.

Markov models and neural networks are both widely used for modeling infrastructure degradation. However, neural network models essentially classify the assets into different condition groups based on some input factors mostly consisting of asset characteristics and surrounding geographical conditions; they do not actually simulate the degradation process, and hence would not be able to capture the stochasticity in the degradation process. Also, because they rely heavily on the quality and quantity of the input factors, they are not suitable for systems that have a limited amount of such information.

There currently does not exist any serviceability deterioration model for railway drainage pipe systems. Hence, this paper will be focused on predicting the service condition of drainage assets using the Markov model, so as to eventually be able to predict the impact of condition deterioration on the long-term loss of service performance. Although Network Rail records both structural condition and service condition, only service condition is used in this paper since it relates more directly to hydraulic performance and thus flood risk. However, the same approach used here could

Table 1. Description of service condition score

Service condition	Description
1	Clear
2	Superficial deposits with no loss of capacity
3	Capacity slightly reduced
4	Capacity severely reduced
5	Blocked or unsafe condition

Source: Adapted from Drainage Asset Policy, unpublished data, 2017.

be applied to the structural condition scores to model structural degradation.

Methodology

Input Data

As stated in the Drainage Asset Policy (unpublished report, 2017), the service condition is measured on a 1–5 grading system as illustrated in Table 1. The system adopted is compatible with guidance from CIRIA (2014). NR has service conditions recorded for 88% of their drainage assets. These condition scores will be used to build a model for predicting future states of drainage assets. In the "Case Study" section, this will be applied to an exemplar group of railway drainage assets.

Markov Model Framework

In this study, a Markov chain approach is used to model the degradation rate, which gives an estimation of transition probability from one state to a lower state. This decision is made under the assumption that the probability of degradation depends only on the current condition of the asset. Such an assumption is made based on expert opinion and will be verified subsequently. Since drainage assets could degrade to a worse state any time during the year, in order to correctly estimate the adverse effect of degradation on the drainage capacity throughout the year, a Markov model with continuous time steps is chosen because it is believed to better reflect the degradation process of railway drainage assets.

Because the change of condition is expected to happen at any time during the useful working life, a continuous time Markov chain (CTMC) is used, which is described by a stochastic process $X = \{X(t)|0 \leq t\}$ with discrete state space $S = \{s_1, s_2, \ldots, s_n\}$ that satisfies the following for any time $s, t \geq 0$, and $i, j \in S$:

$$P(X(s+t) = j|X(s) = i, \{X(u):0 \leq u < s\}) = P(X(s+t) = j|X(s) = i) \qquad (1)$$

In other words, CTMC is a stochastic process having the Markovian property: the conditional distribution of the future $X(s+t)$ given the present state $X(s)$ and the past states $X(u)$, $0 \leq u < s$, depends only on the present and is independent of the past (Ross 1993).

In the case of modeling railway drainage asset service condition degradation, $X(t)$ is the condition score of the modeled asset at time t, and the state space $S = \{1, 2, 3, 4, 5\}$ represents the 1–5 grading system mentioned previously. The matrix Q is the transition rate matrix, or infinitesimal generator, of the Markov chain

$$Q = \begin{pmatrix} -q_1 & q_{12} & q_{13} & q_{14} & q_{15} \\ 0 & -q_2 & q_{23} & q_{24} & q_{25} \\ 0 & 0 & -q_3 & q_{34} & q_{35} \\ 0 & 0 & 0 & -q_4 & q_{45} \\ 0 & 0 & 0 & 0 & -q_5 \end{pmatrix}$$

where $q_{ij} = \lim_{\Delta t \to 0}[P(X(\Delta t) = j|X(0) = i)]/\Delta t$, representing the transition rate from condition i to condition j given that the asset is currently in condition i. The diagonal of the matrix is defined as $-q_i$, where $q_i = \sum_{j=i+1}^{n} q_{ij}$. The holding time of an asset in rating i is exponentially distributed with parameter q_i. It is assumed that the assets' condition cannot improve without human intervention, since $q_{ij} = 0$ for $i < j$. Although degradation is a gradual process, it is not always possible to monitor the status of an asset continuously, so the condition of an asset may has degraded by more than one state before the next inspection; thus, transition from state i to state j where $j > i + 1$ is also included in the model. With the transition rate matrix Q, the probability matrix P for any arbitrary time interval s to t can be obtained by $P(s, t) = \exp((s - t)Q)$.

Verification of the Markov Property

For the Markov property to stand, it is necessary to prove that the probability of an asset degrading into score j with a given current score i is not related to its previous conditions.

This can be done by analyzing the three state transition sequence $(X_t|X_{t-1}, X_{t-2})$ of the historical data set, where $(X_t = i|X_{t-1} = j, X_{t-2} = k) = (i|j, k)$ represents an asset condition jump from j to i, given that the previous condition before j is k, i.e., the condition transfer from state k to state j and then state i. If the Markov property holds, for any given i and j, there would be no difference in the probability of the sequence $(i|j, X_{t-2})$ to exist, for all $X_{t-2} < j$.

The χ^2 test is one of the most widely used statistical hypothesis tests for independence and goodness of fit, testing whether two or more categorical variables are related in some population. Hence it is adopted here to test whether the precondition of an asset is related to its current condition. A similar method has also been used in water distribution networks (Sempewo and Kyokaali 2016) and other infrastructure such as pavements (Surendrakumar et al. 2013).

For a given current condition i and previous condition j, the null hypothesis is that the past condition X_{t-2} has no effect on the probability of the asset jump from condition j to i. The contingency table for the given current condition i and previous condition j is constructed by listing all possible sequences $(i|j, X_{t-2})$ as rows, then calculating under the given past condition j and X_{t-2} the number of occurrences that the current condition is i and the number of occurrences that the current condition is not i. The contingency table is given as Table 2.

Table 2. Contingency table for sequence $(X_t = i|X_{t-1} = j, X_{t-2})$

Sequence	Number of sequence occurrence $X_t = i$	Number of occurrence of all other sequences with same past condition $X_t \neq i$
$(i\|j, 1)$	$N(i\|j, 1)$	$\sum_{s=j,s\neq i}^{s=5} N(s\|j, 1)$
$(i\|j, 2)$	$N(i\|j, 2)$	$\sum_{s=j,s\neq i}^{s=5} N(s\|j, 2)$
⋮	⋮	⋮
$(i\|j, j-1)$	$N(\|j, j-1)$	$\sum_{s=j,s\neq i}^{s=5} N(s\|j, j-1)$

Note: $N(i, j|k)$ is the number of occurrences of the sequence $(i|j, k)$.

The test statistic for this table is

$$\chi^2 = \sum \frac{(O-E)^2}{E^2} \tag{2}$$

where O = observed value; and E = expected value for each scenario. For example, for the sequence $(i|j,1)$, the observed value $O = N(i|j,1)$, and the expected value

$$E = \sum_{x=1}^{x=j-1} N(i|j,x) \times \frac{N(i|j,1)}{\sum_{s=j+1}^{s=5} N(s|j,1)} \tag{3}$$

The null hypothesis is normally rejected at a 5% significance level, meaning that if the χ^2 statistic with $j-2$ degrees of freedom is less than 0.05, the Markov property holds because the current condition is independent of past conditions.

Development of Transition Rate Matrices

The transition rate matrix is computed using the maximum likelihood method, and for each element of the matrix

$$\hat{q}_{ij} = \frac{N_{ij}(T)}{R_i(T)} \tag{4}$$

where $R_i(T) = \int_0^T 1_{x(s)=i} ds$, which is the total value of the holding time at condition score i by the time t; and $N_{ij}(T)$ = number of times for ij transition by the time T (Inamura 2006).

After obtaining the transition rate matrix, the potential future condition score of the drainage system can be simulated using the stochastic simulation algorithm (SSA), also known as the Gillespie algorithm. The detailed procedure is described as follows:

1. Initialize the state of the system x_0 at time $t=0$, which is the current condition score of the asset;
2. For the given state $x_0 = i$, find the transition rate λ_{ij} from state i to all other states, i.e., generator matrix elements $\lambda_{ij} = \hat{q}_{ij} \quad \forall\ j \in s, j \neq i$;
3. Calculate the sum of all transition rates, $\lambda_i = \sum_{j \neq i} \lambda_{ij}$;
4. Simulate the time, τ, until the next transition by drawing from an exponential distribution with mean $1/\lambda_i$. Generate a pseudo random uniform variable u_1 from the interval [0,1], $\tau = -\ell n(u_1)/\lambda_i$;
5. Simulate the transition type by drawing from the discrete distribution with probability $\mathrm{Prob}\,(\text{transition to state}\, j) = \lambda_{ij}/\lambda_i$. Generate a pseudo random uniform variable u_2 from the interval [0,1], and choose the transition as follows: if $0 < u_2 < \lambda_{i1}/\lambda_i$, choose Transition 1; if $\lambda_{i1}/\lambda_i < u_2 < (\lambda_{i1} + \lambda_{i2})/\lambda_i$, choose Transition 2; and so on;
6. Update the new time $t = t + \tau$ and the new system state x_t; and
7. Iterate Steps 2–6 until t is larger than the designed simulation period (Banks et al. 2011).

Determine the Minimum Sample Size Required

Because drainage assets are often buried underground, they are more costly to inspect than other assets in the railway system, so it is often in the interest of asset managers to minimize such costs while obtaining sufficient data to build a robust deterioration model. Hence, a study is performed to determine the number of samples required to obtain a stable transition rate matrix that would not alter more than 5% by including more training data. The procedure is as follows:

1. Randomly select n samples from the whole asset database, and calculate transition rate matrix $Q_{n,1}$ using service condition score of these n samples;
2. Repeat the previous step m times, giving a sample of m transition rate matrices $\{Q_{n,1}, Q_{n,2}, \ldots, Q_{n,m}\}$;
3. Calculate sample mean $\overline{Q_n}$ and standard deviation σ_n, where $\overline{Q_n} = \sum_{s=1}^{s=m} Q_{n,s}/n$ and $\sigma_n = \sqrt{\sum_{s=1}^{s=n} (Q_{n,m} - \overline{Q_n})^2/n}$;
4. Increase the number of samples in steps of n, repeating Steps 2–3 to obtain matrices $\{\overline{Q_n}, \overline{Q_{2n}}, \ldots\}$ and $\{\sigma_n, \sigma_{2n}, \ldots\}$; and
5. Find the critical sample number r where $\overline{Q_r}$ is within 5% of the actual transition rate calculated using all assets.

Model Validation

To examine the performance of the model, the data set will be split into two groups: a training group and a validation group. The transition matrix Q will be calculated using the training data set then applied to the validation data set to predict the number of transitions in each condition category. Observed and expected percentages of transitions will then be compared to test the accuracy of the Markov model proposed.

Case Study

Data Analysis

To obtain a comprehensive analysis of how drainage assets may degrade, a historical condition record of 13 years was extracted from the Network Rail drainage asset database. Out of 349,678 drainage asset records, 308,465 have a service condition score.

Assets with only one condition score recorded were removed because there is no possibility of a transition from one state to another being observed; hence, these asset data are deemed as not carrying meaningful information. The number of assets in each asset group was then examined to see whether there were enough data to produce a reliable transition rate matrix. The results are given in Table 3. Because granular drain, siphon, and pond have less than 1,000 assets recorded with condition scores, it is assumed that there are not enough historical data points to produce a reliable probability prediction, so they were not considered in the following analysis.

Because the inspection guidance is usually reviewed every control period, to ensure the consistency of the condition assessment standard, analysis of this case study was performed for the duration of CP5, which is from April 1, 2014 to March 31, 2019.

After an initial check of the data, it was noted that the recorded condition scores can go both higher and lower over time. As the condition score of an asset improves (goes lower), it would be assumed that either an unknown intervention has taken place or there

Table 3. Number of assets in each asset group

Asset type	Count	Percentage (%)
Chamber	63,553	44.85
Pipe	32,410	22.87
Channel	16,532	11.67
Structure	9,851	6.95
Culvert	7,960	5.62
Outfall	4,960	3.50
Inflow	4,154	2.93
Covered channel	1,482	1.05
Granular drain	584	0.41
Pond	189	0.13
Siphon	29	0.02
Sum	141,704	—

Table 4. Chi-squared test contingency table for chamber and pipe

Sequence	Chamber $X_t = i$	Chamber $X_t \neq i$	Chamber χ^2	Pipe $X_t = i$	Pipe $X_t \neq i$	Pipe χ^2
$(4,3\|1)$	11	67	0.19	4	18	0.77
$(4,3\|2)$	76	294		22	83	
$(5,3\|1)$	4	74	0.65	1	21	0.68
$(5,3\|2)$	24	346		3	102	
$(5,4\|1)$	6	24	0.38	0	5	0.49
$(5,4\|2)$	27	54		1	6	
$(5,4\|3)$	27	68		4	14	

is an inspection error. Since this study is focused on the degradation of drainage asset condition, only the transitions where score is degraded were considered to calculate the transition matrix. However, in order to eliminate the effect of score improvement on the assets without affecting the total holding time $R_i(T)$ used to calculate the transition matrix [as stated in Eq. (4)], for all the transitions where the score is upgraded, the asset is assumed to have stayed in the starting condition score until the moment the asset's service condition is improved.

Verification of the Markov Property

As explained in the "Methodology" section, the Markov property can be tested with the χ^2 test to verify that the net transition from the current condition is independent of past conditions. Examples of this for chambers and pipes are given in Table 4.

As shown in Table 4, the test statistic χ^2 for each contingency table is above the significance level of 5%, meaning that there is no evidence to reject the null hypothesis of independence between each category. Hence, all the assets are proven to possess the Markov properties, so the Markov methods described in the "Methodology" section may therefore be applied.

Cohort Analysis

Each asset group is further divided into subclasses based on their function or characteristic; for example, pipes are divided into three subclasses based on the type of water they carry: surface water, foul water, or combined. For all assets, other characteristics such as size and material are also recorded in the database. To decide whether the transition rate matrix should be produced based on these groups and characteristics, a correlation between each of these parameters and the service condition was explored with linear regression using a least-squares approximation. Because the characteristics of inflow, outflow, and structure are difficult to quantify, they were not further divided into subgroups. Tests were therefore performed for channel, chamber, culvert, and pipe, and the resulting significance level of each characteristic is listed in Table 5, of which below the 5% (0.05) critical level would be deemed as influential.

As seen in Table 5, channel service condition is correlated with its material, shape, route, and subclass (natural ditch, artificial ditch, flume, aqueduct, cascade); chamber service condition is correlated with its material, route, and asset subclass (catchpit, manhole, interceptor, pumping); culvert service condition is correlated with its material, shape, and location; and pipe service condition is correlated with its size, material, and location.

The construction material of the assets may affect the rate of deterioration, which can lead to higher surface roughness and hence an increase in the chance of lower hydraulic capacity, lower flow velocities, and higher likelihood of sediment-derived blockage. With different sizes, assets are expected to enable different flow

Table 5. Linear regression results of the significance coefficient of different characteristics for channel, chamber, culvert, and pipe

Characteristic	Significance
Channel	
Material	3.7×10^{-14}
Shape	1.9×10^{-49}
Route	6.1×10^{-40}
Subclass	2.8×10^{-5}
Culvert	
Material	1.7×10^{-4}
Shape	1.1×10^{-11}
Route	6.2×10^{-28}
Chamber	
Material	3.3×10^{-5}
Shape	0.06
Route	7.3×10^{-24}
Subclass	3.3×10^{-35}
Pipe	
Size	3.8×10^{-64}
Material	0.04
Shape	0.72
Route	2.2×10^{-29}
Subclass	0.45

rates to pass through. Higher flow rates without an adequate slope gradient could bring more debris and cause sedimentation, which can then lead to loss of hydraulic capacity. Also, smaller pipes may more easily become blocked by large debris at lower flow velocities. The dependence of service score on location may be due to different local hydrological characteristics, because all drainage assets are designed to withstand rainfall events with a certain return period depending on route classification following the company design standards (as seen in Table 5). It may also be due to the way each route inspects and records the score, which could warrant a future study on uncertainty in condition scoring.

Estimate Future Condition State

An example of pipes with 300-mm diameter was chosen because this is the highest population among all pipe sizes: 78% of pipes that have a diameter record are 300 mm. After the data cleanse process proposed in the "Data Analysis" section, the total number of assets analyzed is 15,295. The transition rate matrix for pipes with 300-mm diameter is given by the Markov chain degradation model as

$$Q = \begin{pmatrix} \mathbf{-0.2030} & \mathbf{0.1800} & \mathbf{0.0156} & \mathbf{0.0039} & \mathbf{0.0035} \\ \mathbf{0} & \mathbf{-0.0541} & \mathbf{0.0444} & \mathbf{0.0069} & \mathbf{0.0028} \\ \mathbf{0} & \mathbf{0} & \mathbf{-0.0559} & \mathbf{0.0439} & \mathbf{0.0120} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-0.0851} & \mathbf{0.0851} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

The 1-year transition probability can be then calculated by taking the exponential of the transition rate matrix Q

$$P_1 = e^{1xQ} = \begin{pmatrix} \mathbf{81.6\%} & \mathbf{15.8\%} & \mathbf{1.7\%} & \mathbf{0.4\%} & \mathbf{0.4\%} \\ \mathbf{0} & \mathbf{94.7\%} & \mathbf{4.2\%} & \mathbf{0.7\%} & \mathbf{0.3\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{94.6\%} & \mathbf{4.1\%} & \mathbf{1.3\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{91.8\%} & \mathbf{8.2\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{100\%} \end{pmatrix}$$

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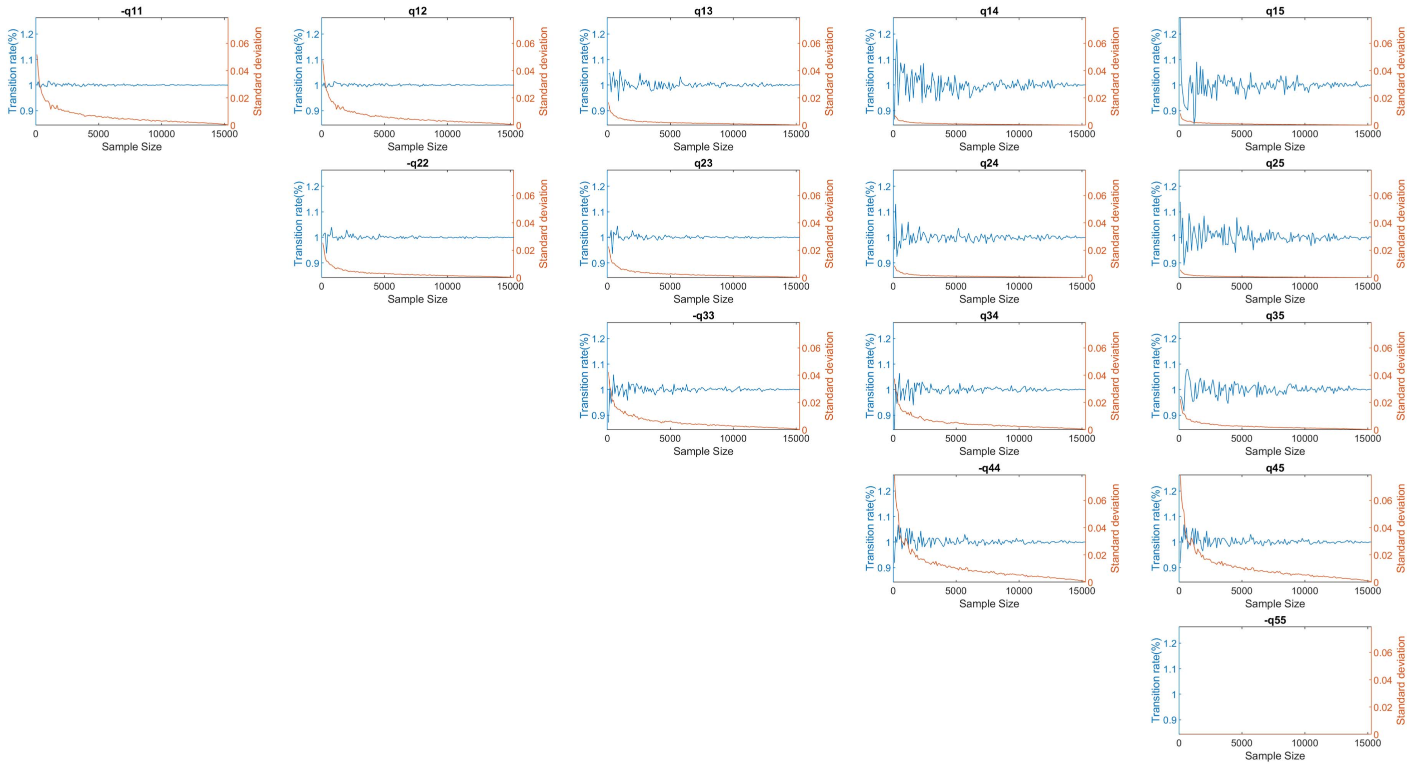

Fig. 1. Mean and standard deviation of transition rate matrix Q as sample number increases.

As seen in the matrix, for pipes in all condition states except condition one, less than 10% of the assets degrade to a worse state. Although the number does not seem large, this small number of degraded pipes can still have a large effect on individual drainage systems. Railway drainage systems consist of groups of subdrainage systems, where each subdrainage system consists of a series of drainage nodes and links that are interconnected to form several pipelines alongside tracks to bring water from an inflow to an outflow. Hence, if one of the assets fails, for example, one of the pipes is blocked or one of the catchpits is blocked or obstructed, it will not only affect its own water-carrying ability, but also diminish the upstream hydraulic capacity and cause the whole subsystem to fail.

The large percentage transitioning from Condition 1 to Condition 2 is anticipated because Condition 1 is described as no defect/clear, which is only expected to be seen in new-build assets; any new build will soon show superficial defects and have very small amounts of deposits, which has negligible effect on its capacity but warrants escalation to Condition 2.

The likelihood of transition from Conditions 2 to 3 and 3 to 4 is very similar. This may be because of how different conditions are classified. It is easy to see the transition from Condition 1, new build, to Condition 2; and it is easy to spot when a defective asset turns into completely failed. However, it is hard to objectively differentiate between Conditions 2, 3, and 4, because how much capacity is reduced is purely examined by visual inspection based on expert knowledge. A minor defect in one inspector's opinion may seem much more severe to another. Without a clear quantification method, human subjectivity is inevitable. A similar 1–5 grading system is also being used in the sewer drainage system but with more complex inspection rules. This may be adapted in the railway drainage system in the future for better condition classification.

Determination of Minimum Sample Size

In order to reduce the costs of inspection for buried assets, a method was proposed in the "Methodology" section to determine the minimum samples that could provide sufficient data to obtain a stable matrix for the deterioration model.

With the predivided cohorts from the previous linear regression test, the transition rate matrix will be calculated with various numbers of randomly selected samples. Such results are then to be compared with the transition rate matrix generated from the whole cohort in order to investigate what is the critical number of assets that can represent the whole cohort.

The cohort of pipes with 300-mm diameter was tested as an example, following the procedure listed in the "Determine the Minimum Sample Size Required" section with chosen value $n = 100$ and $m = 100$. Results are shown in Fig. 1, where the left-hand $y-$axis of each subplot represents the change of one element $\overline{q_{ij}}$ in the estimated transition rate matrix $\overline{Q_s}$ with increasing number of samples. For better clarity of the comparison of elements q_{ij} in transition rate matrix Q, all figures of $\overline{q_{ij}}$ are shown as $\overline{q_{ij}}/q_{ij}$. The right-hand $y-$axis demonstrates the change in standard deviation for each sample size.

As shown in Fig. 1, the sample mean $\overline{Q_s}$ graphs for all elements in the transition rate matrix start to flatten out after around 5,000 samples. Also, the standard deviation decreases dramatically first and then slows down after 5,000 samples, which makes sense because as sample size gets larger, there is less error in estimating the true transition rate matrix. Although for each element $\overline{q_{ij}}$ of $\overline{Q_s}$ the rate of convergence is different, they all converge within 5% of the transition matrix Q after 5,400 samples.

Hence, it can be said that for this cohort a 5,400 sample will be able to provide a stable transition rate matrix with a 5-year historical data record. Although the transition rate matrix produced with the minimum required sample size is a sound estimation of the

Table 6. Number of assets required for a stable matrix with 1–5 years' historical condition record

Number of years	Total number of assets	Number of assets required for a stable matrix	Percentage required
5	15,295	5,400	35
4	15,302	6,000	39
3	15,127	8,200	54
2	14,437	7,900	54
1	11,166	8,200	73

whole cohort, a certain degree of uncertainty will always be present, which the asset managers should consider while interpreting the results of this method.

Minimum Sample Size Required with Shorter Time Frame

Although there exists a quite sizable historical condition data record ranging from 2007 to present nationwide, there are some areas that have fewer years of past data and hence it is important to understand how that could affect the number of samples that are required to obtain an accurate matrix.

Hence, assuming when there are less than 5 years of track record provided, the same analysis is performed to investigate the number of assets required to obtain a stable matrix (Table 6). As expected, the percentage of samples required to provide a stable matrix increases as the number of years' data decreases. This is because in a shorter period, fewer condition transitions occur, and hence the behavior of asset degradation would appear to be more volatile, and hence the degradation matrix over a smaller time window may be less easy to extrapolate into the future.

Model Validation

The proposed model was validated with 300-mm-diameter pipe asset data. The data were randomly split into two groups; one is the training group, consisting of 5,400 assets, which is shown to be the minimum amount required to generate a stable transition rate matrix; the other contains the remaining assets and is the validation group. The transition matrix was calculated using the training data set assuming no intervention is performed. This matrix was then applied to the validation set to predict the number of transitions in each condition category during the period of CP5. The observed and expected percentage of transitions during CP5 are given in Table 7, while the differences between observed and expected percentages are given in Table 8. The number of expected values is on average within 1% of the observed value, which provides a sound prediction of the possible transitions in a given period. The only transition with a high difference in transition rate is that from Conditions 1 to 2. This is due to the high variability of the data for transitions from Conditions 1 to 2, which is reflected in Fig. 1, where the standard deviation of q_{11} and q_{12} is one of the highest. Hence, it is more likely that the randomly selected sample groups have a higher difference in these transition rate than the others. However, because the difference is within a 5% range, it is still deemed a reliable prediction.

Case Study for Other Asset Groups

Application to other asset types is also possible using the methods stated previously. Three additional examples are briefly summarized:
1. Pipes with 450-mm diameter.

Table 7. Observed and expected percentage of transition in validation data set during CP5

Start condition	Observed/expected	End condition 1	2	3	4	5
1	Observed	79.11%	18.45%	1.66%	0.39%	0.39%
	Expected	81.42%	15.96%	1.78%	0.48%	0.36%
2	Observed	0	94.96%	4.14%	0.63%	0.26%
	Expected	0	95.25%	3.72%	0.70%	0.32%
3	Observed	0	0	94.56%	4.27%	1.17%
	Expected	0	0	94.01%	4.44%	1.56%
4	Observed	0	0	0	91.34%	8.66%
	Expected	0	0	0	91.52%	8.48%
5	Observed	0	0	0	0	100.00%
	Expected	0	0	0	0	100.00%

Table 8. Difference between observed and expected percentage

Start condition	End condition 1	2	3	4	5
1	−2.30%	2.49%	−0.13%	−0.09%	0.03%
2	0	−0.29%	0.42%	−0.07%	0.07%
3	0	0	0.55%	−0.17%	−0.38%
4	0	0	0	−0.18%	0.18%
5	0	0	0	0	0.00%

2. Chamber with the following characteristics:
   - Subclass: Catchpit, and
   - Material: Precast concrete.
3. Inlet or outlet structure.

One-year Probability Matrix

1. Pipes with 450-mm diameter

$$P_1 = \begin{pmatrix} \mathbf{82.1\%} & \mathbf{15.2\%} & \mathbf{2.3\%} & \mathbf{0.3\%} & \mathbf{0.1\%} \\ \mathbf{0} & \mathbf{93.8\%} & \mathbf{5.2\%} & \mathbf{0.8\%} & \mathbf{0.2\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{97.4\%} & \mathbf{1.9\%} & \mathbf{0.6\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{93.1\%} & \mathbf{6.9\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{100\%} \end{pmatrix}$$

2. Precast concrete catchpit

$$P_1 = \begin{pmatrix} \mathbf{87.36\%} & \mathbf{9.74\%} & \mathbf{1.76\%} & \mathbf{0.76\%} & \mathbf{0.38\%} \\ \mathbf{0} & \mathbf{94.14\%} & \mathbf{4.07\%} & \mathbf{1.13\%} & \mathbf{0.67\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{96.17\%} & \mathbf{2.61\%} & \mathbf{1.22\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{94.94\%} & \mathbf{5.06\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{100\%} \end{pmatrix}$$

3. Inlet or outlet structure

$$P_1 = \begin{pmatrix} \mathbf{83.63\%} & \mathbf{12.61\%} & \mathbf{3.16\%} & \mathbf{0.43\%} & \mathbf{0.16\%} \\ \mathbf{0} & \mathbf{92.13\%} & \mathbf{6.31\%} & \mathbf{1.17\%} & \mathbf{0.39\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{95.92\%} & \mathbf{3.16\%} & \mathbf{0.92\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{97.45\%} & \mathbf{2.55\%} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{100\%} \end{pmatrix}$$

Table 9. Number of assets required for a stable matrix for three example cohorts

Asset type	Total number of assets	Number of assets required for a stable matrix	Percentage required
Pipes with 450-mm diameter	2,300	2,783	82
Precast concrete catchpit	4,600	32,457	14
Structures	4,500	9,501	47

Table 10. Difference between observed and expected percentage of transition in validation data set for pipes with 450-mm diameter

	End condition				
Start condition	1	2	3	4	5
1	−3.03%	3.37%	−0.29%	−0.06%	0.01%
2	0	−0.26%	0.28%	0.02%	−0.03%
3	0	0	−0.02%	0.08%	−0.06%
4	0	0	0	−0.20%	0.20%
5	0	0	0	0	0.00%

The 1-year probability matrices of the three chosen cohorts are listed. They all present similar patterns as the probabilities of transition for pipes with 300-mm diameter shown previously. For all three cohorts, assets in Condition 1 have the highest degradation rate, whereas only less than 10% of asset in other states will degrade. They also showed very similar degradation rates for assets in Conditions 2, 3, and 4. Out of the three cohort, pipes with 450-mm diameter have the highest rate of transition from Conditions 4 to 5; this indicates that this type of asset will be more likely to fail once it is already in a very bad condition.

Minimal Sample Size for Stable Matrix

Analysis has been performed for these three cohorts to determine the number of samples required to obtain a stable transition rate matrix; results are given in Table 9.

Pipes with 450 mm diameter require the highest percentage of assets to reach a stable matrix, while precast concrete catchpit requires the lowest percentage. However, looking at the actual number of assets required, pipes with 450-mm diameter require the lowest number of assets, whereas precast concrete catchpit requires the highest number. This is because the total number of assets in each cohort is different, and it is expected that the number of samples required for a stable matrix is positively correlated with the number of total assets.

Also, even though the total number of precast concrete catchpits is much higher than the total number of inlet or outlet structures, the minimal number of samples required for a stable matrix for them are quite similar. This might imply that, after reaching a certain number of samples, the size of the cohort will not affect the minimal sample required for a stable matrix. Hence, for a very large cohort, it is expected that only a certain number of samples will be required.

Model Validation

Model performance tests were carried out for three cohorts. The differences between observed and expected percentage of transition in the validation data set are given in Tables 10–12. All differences are within a 5% range. Similar to pipes with 300-mm diameter, the transition from Conditions 1 to 2 has the highest difference in transition rate.

Table 11. Difference between observed and expected percentage of transition in validation data set for precast concrete catchpit

	End condition				
Start condition	1	2	3	4	5
1	−2.57%	2.90%	−0.17%	−0.04%	−0.13%
2	0	−0.26%	0.30%	0.01%	−0.05%
3	0	0	−0.21%	0.32%	−0.11%
4	0	0	0	−0.33%	0.33%
5	0	0	0	0	0.00%

Table 12. Difference between observed and expected percentage of transition in validation data set for inlet and outlet structures

	End condition				
Start condition	1	2	3	4	5
1	−3.84%	4.35%	−0.23%	−0.20%	−0.09%
2	0.00%	−0.44%	0.53%	−0.04%	−0.05%
3	0.00%	0.00%	−0.18%	0.23%	−0.05%
4	0.00%	0.00%	0.00%	−0.10%	0.10%
5	0	0	0	0	0

Discussion

As shown, the continuous Markov chain model could provide a sensible prediction of the degradation process of the 300-mm-diameter railway drainage pipes with only a 35% randomly selected sample from the whole asset group with 5 years of historical data. This methodology may be applied to every drainage assets group to identify the minimum number of samples needed to make degradation simulations. To demonstrate this, three additional asset groups—pipes with 450-mm diameter, precast concrete catchpit, and inlet or outlet structure—were presented as examples. Results show that they require 82%, 14%, and 47% of the total number of samples in whole asset group to acquire a stable matrix for degradation prediction. Such analysis would help asset managers to justify the overall inspection costs while maintaining a sufficient understanding of degradation process for different asset classes, which would further contribute to objective budget planning of potential maintenance and renewal schemes. Also, as shown in Table 6, with a longer duration of historical record, fewer asset samples are required to simulate the whole cohort's behavior. This would provide asset managers with quantitative evidence of the advantages of maintaining a consistent and continuous inspection regime, and guide the extent of such a regime.

Moreover, by combining the degradation estimation with a hydraulic model of the drainage system, there is a possibility of estimating the frequency and scale of drainage failure under different maintenance strategies. This could allow asset managers to weigh the cost of intervention against the loss of performance quantitatively, hence bringing stronger arguments when producing budget estimations for each CP.

The Markov model may also form a cornerstone of a decision support tool that could assist route managers in prioritizing drainage works. For assets that have a detailed track record of service condition scores, by comparing the degradation rate of different asset groups and different routes, asset managers will be able to identify the type of asset and the location of the system that are more prone to degradation. Hence, they may justify decisions to increase the inspection frequency and prioritize maintenance/renewal works of these assets.

Impact of Intervention

In this study, the effect of apparent historical interventions is removed by disregarding the upgrading incidents in the historical database. Although in this way the effect of intervention is limited to be minimal, interference in the degradation rate cannot be fully eliminated. It is assumed that if an asset has been upgraded due to an intervention, it stayed in the previous condition until intervention happened. Without intervention, the particular asset may have stayed in its current condition for a further amount of time before degrading; hence, this may cause an underestimation of the possibility of remaining in the same condition state, and thus an overestimation of the possibility of degradation. This problem cannot be rectified without establishing a model that could simulate the intervention activities. However, overestimating the degradation rate may not be a shortcoming in real life, because degradation can always be accelerated due to unforeseen events such as extreme adverse weather conditions; hence, it can prepare asset managers with a worst-case scenario.

There are various intervention options that NR carries out on drainage assets in order to slow down, stop, or reset degradation level, and hence remedy unsatisfactory performance. Each type of intervention is believed to affect the degradation level as follows:

- Renew, upgrade, and new build will set the degradation to none,
- Refurbish will improve performance and reset the degradation to a certain level,
- Maintain will offset degradation for a certain time, and
- Inspect and survey will not have direct impact on the degradation level but will affect the efficiency of other interventions (Drainage Asset Policy, unpublished report, 2017).

Besides resetting the asset service condition score to 1, renewal of an asset might have other effects on the degradation rate. The degradation rate of a new-build asset might be slower than older assets in the same condition score category. Such difference in rate can only be examined if there is information about the age of drainage assets. However, almost all railway drainage assets are of unknown age; many may date from as early as Victorian times. Until additional age-related data are provided, this will remain as one of the limitations of the model.

Routine maintenance will defer the rate of degradation in service condition. If routine maintenance is applied to all assets nationwide with the same schedule, its effect will be normalized and will not cause bias in the degradation rate. However, in real life, the frequency of maintenance for a particular asset can depend on many terms such as the criticality of the asset failure, the budget allocation of the region, and the current condition of the asset. The effect of routine maintenance is to be quantified in further studies and is assumed to be negligible in this paper.

There are currently limited studies on how interventions affect the rate of degradations. The effect of intervention is to be investigated by linking the intervention records to the improvement of condition score. Such data are scarce and unorganized. It is uncertain whether the asset owner always updates asset condition data once an intervention is made. Also, drainage asset interventions can be carried out as part of the work order of other parent assets such as earthwork or track, and hence make the linkage to condition score improvement harder to locate. Since all these questions are to be investigated, it is decided to overlook the effect of interventions in this study.

Conclusion

This paper presented a continuous Markov chain model to quantify the degradation process of the service condition of railway drainage infrastructure in the UK. The model was informed by service condition data collected by the asset owner Network Rail. The characteristics influencing the degradation process were studied so that the drainage assets could be divided into homogeneous groups. Hence, the transition matrix derived from each group could predict the probability of the degradation process of individual assets in the group. Methodologies were performed on the case study with NR drainage assets to verify the Markov property of the data set, compute the transition rate matrix, and find the minimum number of samples needed for any cohort of assets in order to get a stable transition matrix that can represent the whole cohort. The model was applied to predict degradation of pipes with 300-mm diameter, pipes with 450-mm diameter, precast concrete catchpits, and inlet or outlet structures over the period of Control Period 5 and it was found that the results give an estimation within 5% of the real degradation process.

Data Availability Statement

Some or all data, models, or code used during the study were provided by a third party. Direct requests for these materials may be made to the provider as indicated in the Acknowledgments. Railway drainage asset inventory and condition score data were provided by Network Rail and are confidential. Requests for the underlying data may be made via the corresponding author.

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