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Online Supplement to "Estimation and Inference in Heterogeneous Spatial Panels with a Multifactor Error Structure"

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In this Online Supplement, we provide discussions on the extension of the estimation method, the proofs of Theorems 1 to 3 with lemmas used in proving theorems, the complete and additional simulation results, as well as the data descriptions and additional empirical results. Section S1 develops the GMM estimation when the rank condition in Assumption 6 (ii) fails. Section S2 contains the proofs of main theorems. Section S3 provides the additional lemmas used in the proof of theorems. Section S4 reports the complete and additional Monte Carlo simulation results. Section S5 provides the data descriptions and the additional empirical results.

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S1 GMM Estimation with Linear and Quadratic Moment Conditions

As pointed out by one of the referees, if $\mathcal{B} = 0$ in (3), then the CCEX-IV and CCE-IV estimators are not identifiable since the rank condition in Assumption 6(ii) fails. If $\boldsymbol{\mathcal{B}} = \mathbf{0}$, we have a heterogeneous spatial autoregressive model with common factors, which is different from what we are studying in the current paper. This heterogeneous spatial model with unobserved factors has not been studied in the literature and deserves separate research. Hence, in this section, we only provide a brief discussion on how to construct GMM estimation of this model.

We follow Lee and Yu (2014) and Yang (2021), and develop the GMM estimation using both linear and quadratic moments conditions. If factors were observable, then we would have the following heterogeneous autoregressive spatial panel data (HSPD) model for the de-factored data (see (6) in the main text):

$$\tilde{\boldsymbol{y}}_0 = (\boldsymbol{I}_T \otimes \boldsymbol{\rho W})\tilde{\boldsymbol{y}}_0 + (\boldsymbol{I}_T \otimes \boldsymbol{\mathcal{B}})\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{\varepsilon}}_0, \tag{S.1}$$

where $\tilde{\boldsymbol{y}}_0 = (\boldsymbol{M}_{\boldsymbol{F}_y} \otimes \boldsymbol{I}_N)\boldsymbol{y}$ is the de-factored $\boldsymbol{y}$, and $\tilde{\boldsymbol{x}}$ and $\tilde{\boldsymbol{\varepsilon}}_0$ are defined similarly.

Let $\boldsymbol{P}_l$ be a $N \times N$ real non-random matrix with zero diagonal elements. Then, under Assumption 2, we can obtain the following quadratic moment conditions:

$$E(\tilde{\boldsymbol{\varepsilon}}_0'(\boldsymbol{I}_T \otimes \boldsymbol{P}_l)\tilde{\boldsymbol{\varepsilon}}_0) = \sum_{i=1}^{N}\sum_{j=1}^{N} p_{l,ji} E(\tilde{\boldsymbol{\varepsilon}}_{0i.}'\tilde{\boldsymbol{\varepsilon}}_{0j.}) = \sum_{i=1}^{N} p_{l,ii} E(\tilde{\boldsymbol{\varepsilon}}_{0i.}'\tilde{\boldsymbol{\varepsilon}}_{0i.}) = 0, \tag{S.2}$$

where $p_{l,ji}$ is the (j,i)-th element of $\boldsymbol{P}_l$, $\tilde{\boldsymbol{\varepsilon}}_{0i.} = (\tilde{\varepsilon}_{0i1}, \dots, \tilde{\varepsilon}_{0iT})'$ and $\tilde{\varepsilon}_{0it}$ is the $((i-1)N+t)$-th element of $\tilde{\boldsymbol{\varepsilon}}_0$. The first equality is obtained by definition, the second is by Assumption 2 and the final one is due to $p_{l,ii} = 0$ for all i. Suppose that we de-factor equation (1) using $\boldsymbol{M}_{\boldsymbol{F}_x}$. Although this leaves $\boldsymbol{f}_{2t}$ unaccounted for, the quadratic moment conditions in (S.2) still hold because the loadings associated with $\boldsymbol{f}_{2t}$ are cross-sectionally independent with zero mean and independent of idiosyncratic errors under Assumption 3.

We now collect the m non-random matrices, $\boldsymbol{P}_1, \dots, \boldsymbol{P}_m$, and the ι linear IVs contained in the matrix $\tilde{\boldsymbol{Q}}$, which is defined in the main text. The number of $\boldsymbol{P}_l$ matrices and the number of IVs should satisfy $(m+\iota) \geq N(k+1)$. We then construct the sample counterparts of the quadratic and the linear moment conditions as follows:

$$\underset{(m+\iota)\times 1}{\boldsymbol{M}_{NT}} = [\hat{\tilde{\boldsymbol{\varepsilon}}}_x'(\boldsymbol{I}_T \otimes \boldsymbol{P}_1)\hat{\tilde{\boldsymbol{\varepsilon}}}_x, \dots, \hat{\tilde{\boldsymbol{\varepsilon}}}_x'(\boldsymbol{I}_T \otimes \boldsymbol{P}_m)\hat{\tilde{\boldsymbol{\varepsilon}}}_x,\ \hat{\tilde{\boldsymbol{\varepsilon}}}_x'\tilde{\boldsymbol{Q}}]',$$

where $\hat{\tilde{\boldsymbol{\varepsilon}}}_x = (\boldsymbol{I}_T \otimes (\boldsymbol{I}_N - \hat{\boldsymbol{\rho}}\boldsymbol{W}))\tilde{\boldsymbol{M}}_{\bar{\boldsymbol{X}}}\boldsymbol{y} - \tilde{\boldsymbol{M}}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T \otimes \hat{\boldsymbol{\mathcal{B}}})\boldsymbol{x}$, and $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\mathcal{B}}}$ are some initial estimates.

Then, the GMM estimator for $\boldsymbol{\rho}$ and $\boldsymbol{\mathcal{B}}$ can be obtained by

$$(\hat{\rho}_1, \dots, \hat{\rho}_N, \hat{\boldsymbol{\beta}}_1', \dots, \hat{\boldsymbol{\beta}}_N')' = \underset{(\rho_1,\dots,\rho_N,\boldsymbol{\beta}_1',\dots,\boldsymbol{\beta}_N')' \in \boldsymbol{\Theta}}{\arg\min} \boldsymbol{M}_{NT}'\boldsymbol{W}_g\boldsymbol{M}_{NT}, \tag{S.3}$$

where $\mathbf{\Theta}$ is a compact parameter space in $\mathcal{R}^{(k+1)N}$ and $\boldsymbol{W}_g$ is an $(m+\iota)\times(m+\iota)$ positive definite weighting matrix.

Notice that the construction of quadratic moments for each individual requires us to obtain consistent estimation of idiosyncratic errors and thus the parameters from all other individuals. This implies that we cannot solve (S.3) sequentially on an individual basis. Instead, all the parameters need to be estimated simultaneously by solving the minimisation problem (S.3). This will pose a heavy computational burden, especially if N is large. Moreover, in this context, the optimal choice of the matrices $\boldsymbol{P}_l$, $1 \le l \le m$, is not a simple task for heterogeneous spatial models. These matrices are usually chosen by minimising the (asymptotic) variance of the GMM estimator, which requires further research. Due to space constraints, we leave it for future studies.

We also note in passing that the above technical issues do not occur if the parameters are homogeneous, in which case the popular choice of quadratic moments matrix is given by $\boldsymbol{P}^* = \boldsymbol{G} - diag(\boldsymbol{G})$ and the asymptotic properties can be established using the standard M-estimation techniques, see Yang (2021).

S2 Proofs of Theorems

First, we introduce the following notations:

$$\underset{T\times r_1}{\boldsymbol{F}_1} = (\boldsymbol{f}_{11},\dots,\boldsymbol{f}_{1T})', \quad \underset{T\times r_2}{\boldsymbol{F}_2} = (\boldsymbol{f}_{21},\dots,\boldsymbol{f}_{2T})', \quad \underset{T\times r_3}{\boldsymbol{F}_3} = (\boldsymbol{f}_{31},\dots,\boldsymbol{f}_{3T})',$$
$$\underset{Tr_1\times 1}{\boldsymbol{f}_1} = (\boldsymbol{f}'_{11},\dots,\boldsymbol{f}'_{1T})', \quad \underset{Tr_2\times 1}{\boldsymbol{f}_2} = (\boldsymbol{f}'_{21},\dots,\boldsymbol{f}'_{2T})', \quad \underset{Tr_3\times 1}{\boldsymbol{f}_3} = (\boldsymbol{f}'_{31},\dots,\boldsymbol{f}'_{3T})',$$
$$\underset{r_x\times 1}{\boldsymbol{f}_{xt}} = (\boldsymbol{f}'_{1t},\boldsymbol{f}'_{3t})', \quad \underset{r_y\times 1}{\boldsymbol{f}_{yt}} = (\boldsymbol{f}'_{1t},\boldsymbol{f}'_{2t})', \quad \underset{r_a\times 1}{\boldsymbol{f}_{at}} = (\boldsymbol{f}'_{1t},\boldsymbol{f}'_{2t},\boldsymbol{f}'_{3t})',$$
$$\underset{T\times r_x}{\boldsymbol{F}_x} = (\boldsymbol{f}_{x1},\dots,\boldsymbol{f}_{xT})', \quad \underset{T\times r_y}{\boldsymbol{F}_y} = (\boldsymbol{f}_{y1},\dots,\boldsymbol{f}_{yT})', \quad \underset{T\times r_a}{\boldsymbol{F}_a} = (\boldsymbol{f}_{a1},\dots,\boldsymbol{f}_{aT})',$$
$$\underset{Tr_x\times 1}{\boldsymbol{f}_x} = (\boldsymbol{f}'_{x1},\dots,\boldsymbol{f}'_{xT})', \quad \underset{Tr_y\times 1}{\boldsymbol{f}_y} = (\boldsymbol{f}'_{y1},\dots,\boldsymbol{f}'_{yT})',$$
$$\underset{N\times r_1}{\boldsymbol{\gamma}_1} = (\boldsymbol{\gamma}_{11},\dots,\boldsymbol{\gamma}_{1N})', \quad \underset{N\times r_2}{\boldsymbol{\gamma}_2} = (\boldsymbol{\gamma}_{21},\dots,\boldsymbol{\gamma}_{2N})', \quad \underset{N\times r_y}{\boldsymbol{\gamma}_y} = (\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2).$$

To prove Theorems 1 – 3, we will use Lemmas 1 to 8 in Sections S3.

S2.1 Proof of Theorem 1

S2.1.1 Consistency of the Individual Estimator

We establish the consistency of the individual estimator, $\hat{\boldsymbol{\theta}}_i$ by proving that the RHS of (20) converges in probability to $\mathbf{0}$ as $(N,T)\to\infty$. First, by the definition of $\mathbf{\Pi}_i = \tilde{\boldsymbol{Q}}_i(\tilde{\boldsymbol{Q}}'_i\tilde{\boldsymbol{Q}}_i)^{-1}\tilde{\boldsymbol{Q}}'_i$, the property of $\boldsymbol{Z}'_{i.}\mathbf{\Pi}\boldsymbol{Z}_{i.}/T$ is determined by $\tilde{\boldsymbol{Q}}'_i\tilde{\boldsymbol{Q}}_i/T$ and $\tilde{\boldsymbol{Q}}'_i\boldsymbol{Z}_{i.}/T$. By Lemma 5(c) and Assumption 6(i)&(ii), it is easily seen that as $(N,T)\to\infty$, the former is dominated by $\tilde{\boldsymbol{Q}}'_{i0}\tilde{\boldsymbol{Q}}_{i0}/T$ and the latter by $\tilde{\boldsymbol{Q}}'_{i0}\boldsymbol{Z}_{i0.}/T$. Then, by Lemma 5(c), Assumption 6(i)&(ii) and the Continuous Mapping Theorem, we have:

$$\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{Z}_{i.}}{T} = \frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{T}}\right) \ , \tag{S.4}$$

where $\boldsymbol{\Pi}_{i0} = \tilde{\boldsymbol{Q}}_{i0}(\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0})^{-1}\tilde{\boldsymbol{Q}}_{i0}'$. For the remaining terms on the RHS of (20), we use Lemma 5 and obtain:

$$\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i[\boldsymbol{F}_1\boldsymbol{\gamma}_{1i} + \boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.}]}{T} = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{T}}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right), \tag{S.5}$$

which converges to $\mathbf{0}$ in probability as $(N,T)\to\infty$. In view of (20), (S.4) and (S.5), it is easily seen that $\hat{\boldsymbol{\theta}}_i$ is a consistent estimator for $\boldsymbol{\theta}_i$ as $(N,T)\to\infty$.

S2.1.2 Asymptotic Normality of the Individual Estimator

To derive the asymptotic distribution of $\hat{\boldsymbol{\theta}}_i$, we multiply (20) by $\sqrt{T}$:

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) = \left[\frac{\boldsymbol{Z}_{i.}'\tilde{\boldsymbol{Q}}_i}{T}\left(\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T}\right)^{-1}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i.}}{T}\right]^{-1}\frac{\boldsymbol{Z}_{i.}'\tilde{\boldsymbol{Q}}_i}{T}\left(\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T}\right)^{-1}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{F}_1\boldsymbol{\gamma}_{1i} + \boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}}.$$

Using Lemma 5 and Assumption 6(i)–(ii), we have:

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) = \left[\frac{\boldsymbol{Z}_{i0.}'\tilde{\boldsymbol{Q}}_{i0}}{T}\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}\right)^{-1}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0.}}{T}\right]^{-1}\frac{\boldsymbol{Z}_{i0.}'\tilde{\boldsymbol{Q}}_{i0}}{T}\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}\right)^{-1}\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}}$$

$$+O_p\left(\frac{\sqrt{T}}{N}\right) + O_p\left(\frac{1}{\sqrt{N}}\right). \tag{S.6}$$

To establish the asymptotic normality, we need to investigate the property of $\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})/\sqrt{T}$. Under Assumptions 1-3, it is easily seen that it has zero mean. By the mutual independence among $\boldsymbol{f}_{2t}$, ε_{it} and $\boldsymbol{\gamma}_{2i}$ and under Assumptions 2 and 6(i)&(iii), its variance,

$$\text{var}\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}}\right) = \text{E}\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{F}_2\boldsymbol{\Omega}_{\gamma_2}\boldsymbol{F}_2{}'\tilde{\boldsymbol{Q}}_{i0}}{T} + \frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\Omega}_{\varepsilon,i}\tilde{\boldsymbol{Q}}_{i0}}{T}\right),$$

is finite for all i. Then, by the Central Limit Theorem (CLT), as $(N,T)\to\infty$, we have

$$\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}} \xrightarrow{d} N(\mathbf{0},\ \boldsymbol{\Sigma}_i) \text{ for each } i, \tag{S.7}$$

where $\boldsymbol{\Sigma}_i$ is defined in (16). Using (S.6), (S.7), and Assumption 6(iv), we obtain the required asymptotic distribution in Theorem 1 if $T/N^2 \to 0$ as $(N,T)\to\infty$.

S2.2 Proof of Theorem 2

S2.2.1 Consistency of the Mean Group Estimator

Under Assumption 5, we have $\sum_{i=1}^{N} \boldsymbol{\xi}_i / N \xrightarrow{p} \mathbf{0}$ as $N \to \infty$ by the weak law of large numbers (WLLN). The consistency of the Mean Group estimator for either fixed T or $T \to \infty$ follows directly from (23) and Lemma 7.

S2.2.2 Asymptotic Normality of the Mean Group Estimator

We multiply both sides of (23) by $\sqrt{N}$ and obtain:

$$\begin{aligned}
\sqrt{N}(\hat{\boldsymbol{\theta}}_{MG} - \boldsymbol{\theta}) &= \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \boldsymbol{\xi}_i + \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(\frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{Z}_{i.}}{T} \right)^{-1} \frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i (\boldsymbol{F}_1 \boldsymbol{\gamma}_{1i} + \boldsymbol{F}_2 \boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})}{T} \\
&= \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \boldsymbol{\xi}_i + \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(\frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{Z}_{i.}}{T} \right)^{-1} \frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{F}_1 \boldsymbol{\gamma}_{1i}}{T} \\
&\quad + \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(\frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{Z}_{i.}}{T} \right)^{-1} \frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{F}_2 \boldsymbol{\gamma}_{2i}}{T} + \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(\frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{Z}_{i.}}{T} \right)^{-1} \frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{\varepsilon}_{i.}}{T}.
\end{aligned} \tag{S.8}$$

By Lemma 7, the convergence rates for the last three terms on the RHS of (S.8) are:

$$\begin{aligned}
\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(\frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{Z}_{i.}}{T} \right)^{-1} \frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{F}_1 \boldsymbol{\gamma}_{1i}}{T} &= O_p\left(\frac{1}{\sqrt{T}} \right) + O_p\left(\frac{1}{\sqrt{N}} \right), \\
\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(\frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{Z}_{i.}}{T} \right)^{-1} \frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{F}_2 \boldsymbol{\gamma}_{2i}}{T} &= O_p\left(\frac{1}{\sqrt{T}} \right) + O_p\left(\frac{1}{\sqrt{N}} \right), \\
\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(\frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{Z}_{i.}}{T} \right)^{-1} \frac{\boldsymbol{Z}_{i.}' \boldsymbol{\Pi}_i \boldsymbol{\varepsilon}_{i.}}{T} &= O_p\left(\frac{1}{\sqrt{T}} \right).
\end{aligned}$$

Thus, (S.8) can be simplified to

$$\sqrt{N}(\hat{\boldsymbol{\theta}}_{MG} - \boldsymbol{\theta}) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \boldsymbol{\xi}_i + O_p\left(\frac{1}{\sqrt{T}} \right) + O_p\left(\frac{1}{\sqrt{N}} \right),$$

where $\sum_{i=1}^{N} \boldsymbol{\xi}_i / \sqrt{N} \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Omega}_{\xi})$ as $N \to \infty$ under Assumption 5. Hence, as $(N, T) \to \infty$, $\sqrt{N}(\hat{\boldsymbol{\theta}}_{MG} - \boldsymbol{\theta}) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Omega}_{\xi})$. This completes the proof of Theorem 2.

S2.3 Proof of Theorem 3

S2.3.1 Inconsistency of the Pooled Estimator under Parameter Heterogeneity

By Lemma 8(c) and under Assumption 6(i)-(ii), it is easily seen that the asymptotic property of $\hat{\boldsymbol{\theta}}_P$ in (25) is determined by

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{Z}_{i.}\boldsymbol{\xi}_i+\boldsymbol{F}_1\gamma_{1i}+\boldsymbol{F}_2\gamma_{2i}+\boldsymbol{\varepsilon}_{i.})}{T}. \tag{S.9}$$

The last three terms converge in probability to $\mathbf{0}$ as $N\to\infty$. But, this is not the case for the first term because of the correlation between $\boldsymbol{Z}_{i.}$ and $\boldsymbol{\xi}_i$. By Lemma 8(c), we have:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i.}\boldsymbol{\xi}_i}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0.}\boldsymbol{\xi}_i}{T}+o_p(1).$$

Decompose $\boldsymbol{\xi}_i$ into $(\underset{1\times1}{\xi_{\rho i}},\underset{1\times k}{\boldsymbol{\xi}_{\beta i}'})'$. Then, using the definition of $\boldsymbol{Z}_{i0.}$ and (S.43), we obtain:

$$\begin{aligned}\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0.}\boldsymbol{\xi}_i}{T}&=\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x}\ \xi_{\rho i}}{T}+\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{X}_{i.}\boldsymbol{\xi}_{\beta i}}{T}\\&=\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}g_{ij}\xi_{\rho i}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{X}_{j.}}{T}\boldsymbol{\beta}_j+\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{X}_{i.}}{T}\boldsymbol{\xi}_{\beta i},\end{aligned} \tag{S.10}$$

where g_{ij} is the (i,j)-th element of $\mathbf{G}=\boldsymbol{W}(\boldsymbol{I}_N-\boldsymbol{\rho}\boldsymbol{W})^{-1}$. Under Assumption 5, the second term in (S.10) converges to $\mathbf{0}$ in probability as $N\to\infty$. But, the first term does not converge to $\mathbf{0}$ in general. Even if $\xi_{\rho i}$ and $\boldsymbol{\beta}_j$ (or $\boldsymbol{\xi}_{\beta i}$) are uncorrelated for all i,j, $\sum_{i=1}^{N}\sum_{j=1}^{N}E\big(g_{ij}\xi_{\rho i}\big)/N$ does not converge to 0 due the correlation between g_{ij} and $\xi_{\rho i}$. Thus, the Pooled estimator becomes inconsistent under parameter heterogeneity.

We also note that, since $\sum_{i=1}^{N}\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{X}_{i.}\boldsymbol{\xi}_{\beta i}/NT\xrightarrow{p}0$ as $N\to\infty$, if the spatial coefficient is homogeneous (i.e., $\rho_i\equiv\rho$ and $\xi_{\rho i}\equiv0$), then the Pooled estimator can still be consistent, even if $\boldsymbol{\beta}_i$ is heterogeneous.

S2.3.2 The Pooled Estimator under Parameter Homogeneity

By Lemma 8 and Assumption 7(i), it is straightforward to establish the consistency of $\boldsymbol{\theta}_P^*$. Here we focus on its asymptotic distribution.

In view of (27), Lemma 8(c) and Assumption 7(ii), we only need to consider the asymptotic distribution of

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{F}_1\gamma_{1i}+\boldsymbol{F}_2\gamma_{2i}+\boldsymbol{\varepsilon}_{i.})}{T}.$$

Recall that $\tilde{\boldsymbol{Q}}_i = \boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i')\boldsymbol{Q}$. Using the definition of $\boldsymbol{Q}$, we will derive the asymptotic distribution of

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{F}_1\boldsymbol{\gamma}_{1i} + \boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})}{T}. \tag{S.11}$$

Using (18), we write the first term in (S.11) as

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} = \sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{\Gamma}_{xi}'\boldsymbol{F}_x'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_{1i} + \sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_{1i}. \tag{S.12}$$

Define $\bar{\boldsymbol{\Gamma}}_x^* = (\bar{\boldsymbol{\Gamma}}_x\bar{\boldsymbol{\Gamma}}_x')^{-1}\bar{\boldsymbol{\Gamma}}_x$ and $\bar{\boldsymbol{\Gamma}}_1^* = (\bar{\boldsymbol{\Gamma}}_1\bar{\boldsymbol{\Gamma}}_1')^{-1}\bar{\boldsymbol{\Gamma}}_1$, where $\bar{\boldsymbol{\Gamma}}_x = \sum_{i=1}^{N}\boldsymbol{\Gamma}_{xi}/N$ and $\bar{\boldsymbol{\Gamma}}_1 = \sum_{i=1}^{N}\boldsymbol{\Gamma}_{1i}/N$. Using (12), Lemma 2(a),(b)&(d) and Lemma 3(c)&(d) and under Assumption 3, we have (see similar arguments in Yang (2021, pp. S8-S9)):

$$\begin{aligned}\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{\Gamma}_{xi}'\boldsymbol{F}_x'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_{1i} = &\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\boldsymbol{\Gamma}_{xi}'\bar{\boldsymbol{\Gamma}}_x^*\frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T}\bar{\boldsymbol{\Gamma}}_1^{*'}\boldsymbol{\gamma}_{1i} + O_p\left(\frac{\sqrt{T}}{N^{3/2}}\right)\\ &+ O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),\end{aligned} \tag{S.13}$$

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_{1i} = -\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\bar{\boldsymbol{V}}}{T}\bar{\boldsymbol{\Gamma}}_1^{*'}\boldsymbol{\gamma}_{1i} + O_p\left(\frac{\sqrt{T}}{N}\right) + O_p\left(\frac{1}{\sqrt{N}}\right), \tag{S.14}$$

where $\bar{\boldsymbol{V}} = (\bar{\boldsymbol{v}}_{.1}, \ldots, \bar{\boldsymbol{v}}_{.T})'$. For the first term on the RHS of (S.13), by Lemma 2(a) and Assumption 3, we have: $\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}/T = O_p(1/N)$, which is independent of the rest of terms in $\boldsymbol{\Gamma}_{xi}'\bar{\boldsymbol{\Gamma}}_x^*\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}\bar{\boldsymbol{\Gamma}}_1^{*'}\boldsymbol{\gamma}_{1i}/T$. Furthermore, by Assumption 3, we have: $\bar{\boldsymbol{\Gamma}}_x^* \xrightarrow{p} \bar{\boldsymbol{\Gamma}}_{x0}^*$ and $\bar{\boldsymbol{\Gamma}}_1^* \xrightarrow{p} \bar{\boldsymbol{\Gamma}}_{10}^*$ as $N \to \infty$, where $\bar{\boldsymbol{\Gamma}}_{x0}^* = (\bar{\boldsymbol{\Gamma}}_{x0}\bar{\boldsymbol{\Gamma}}_{x0}')^{-1}\bar{\boldsymbol{\Gamma}}_{x0}$ and $\bar{\boldsymbol{\Gamma}}_{10}^* = (\bar{\boldsymbol{\Gamma}}_{10}\bar{\boldsymbol{\Gamma}}_{10}')^{-1}\bar{\boldsymbol{\Gamma}}_{10}$, and $\bar{\boldsymbol{\Gamma}}_{x0}$ and $\bar{\boldsymbol{\Gamma}}_{10}$ are defined in Assumption 3. Finally, note that $\boldsymbol{\gamma}_{1i}$ is IID with mean $\mathbf{0}$ and finite variance, and independent of $\boldsymbol{\Gamma}_{xi}$ and $\bar{\boldsymbol{V}}$.

Following the proof of (S.50), then it is straightforward to show that

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\boldsymbol{\Gamma}_{xi}'\bar{\boldsymbol{\Gamma}}_x^*\frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T}\bar{\boldsymbol{\Gamma}}_1^{*'}\boldsymbol{\gamma}_{1i} = O_p\left(\sqrt{\frac{T}{N}} \times \frac{1}{N} \times \sqrt{N}\right) = O_p\left(\frac{\sqrt{T}}{N}\right). \tag{S.15}$$

Similarly, by Lemma 2(d) and under Assumption 3, we have:

$$\begin{aligned}\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\bar{\boldsymbol{V}}}{T}\bar{\boldsymbol{\Gamma}}_1^{*'}\boldsymbol{\gamma}_{1i} =&O_p\left(\sqrt{\frac{T}{N}} \times \left(\frac{1}{N} + \frac{1}{\sqrt{NT}}\right) \times \sqrt{N}\right)\\ =&O_p\left(\frac{\sqrt{T}}{N}\right) + O_p\left(\frac{1}{\sqrt{N}}\right).\end{aligned} \tag{S.16}$$

Combining (S.12)-(S.16), we obtain

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} = O_p\left(\frac{\sqrt{T}}{N}\right) + O_p\left(\frac{1}{\sqrt{N}}\right). \tag{S.17}$$

For the second term in (S.11), note under Assumption 3 that $\boldsymbol{\gamma}_{2i}$ is IID with mean $\mathbf{0}$, finite variance and independent of $\boldsymbol{X}_{i.}$, $\boldsymbol{M}_{\bar{\boldsymbol{X}}}$, and $\boldsymbol{F}_2$. Using the second result in Lemma 4(a):

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_2}{T}\boldsymbol{\gamma}_{2i}=\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2}{T}\boldsymbol{\gamma}_{2i}+O_p\left(\frac{1}{\sqrt{N}}\right)+O_p\left(\frac{\sqrt{T}}{N}\right). \tag{S.18}$$

Furthermore,

$$\mathrm{E}\left(\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2}{T}\boldsymbol{\gamma}_{2i}\right)=\mathbf{0}, \tag{S.19}$$

and by Assumption 6(iii),

$$\mathrm{var}\left(\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2}{T}\boldsymbol{\gamma}_{2i}\right)=\frac{1}{N}\sum_{i=1}^{N}\mathrm{E}\left(\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2\boldsymbol{\Omega}_{\gamma_2}\boldsymbol{F}_2'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{X}_{i.}}{T}\right)\leq C. \tag{S.20}$$

To examine the third term in (S.11), we define $\boldsymbol{H}=\boldsymbol{F}_x\bar{\boldsymbol{\Gamma}}_x$ and $\boldsymbol{M}_{\boldsymbol{H}}=\boldsymbol{I}_T-\boldsymbol{H}\left(\boldsymbol{H}'\boldsymbol{H}\right)^{+}\boldsymbol{H}'$. Then by stacking (11) over t, we have:

$$\bar{\boldsymbol{X}}=\boldsymbol{F}_x\bar{\boldsymbol{\Gamma}}_x+\bar{\boldsymbol{V}}=\boldsymbol{H}+\bar{\boldsymbol{V}}, \tag{S.21}$$

where $\bar{\boldsymbol{X}}=(\bar{\boldsymbol{x}}_1,\ldots,\bar{\boldsymbol{x}}_T)'$. By Assumption 3 and using the property of the Moore-Penrose inverse, we have $\boldsymbol{M}_{\boldsymbol{F}_x}=\boldsymbol{M}_{\boldsymbol{H}}$. Hence,

$$\begin{aligned}
&\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T}-\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{i.}}{T}=-\frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\left[\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}-\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'\boldsymbol{\varepsilon}_{i.}\right]\\
&=-\frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\left[\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}-\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}+\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}\right.\\
&\qquad\left.-\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}+\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}-\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'\boldsymbol{\varepsilon}_{i.}\right]\\
&=-\frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\left[\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}+\boldsymbol{X}_{i.}'\boldsymbol{H}\left[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}-(\boldsymbol{H}'\boldsymbol{H})^{+}\right]\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}+\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.}\right].
\end{aligned} \tag{S.22}$$

Following the proof of Lemma 4(c), it is easily seen that

$$\frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}=O_p\left(\frac{1}{\sqrt{N}}\right)+O_p\left(\frac{1}{\sqrt{T}}\right), \tag{S.23}$$

$$\frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\boldsymbol{H}[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}-(\boldsymbol{H}'\boldsymbol{H})^{+}]\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}=O_p\left(\frac{1}{\sqrt{N}}\right)+O_p\left(\frac{1}{\sqrt{T}}\right). \tag{S.24}$$

For the third term on the RHS of the last equality in (S.22), notice that $\boldsymbol{\varepsilon}_{i.}$ is independently distributed over i with mean $\mathbf{0}$ and is independent of $\boldsymbol{X}_{i.}$, $\boldsymbol{H}$ and $\bar{\boldsymbol{V}}$. Hence, $\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.}/\sqrt{NT}$

has mean $\mathbf{0}$ and variance given by

$$\text{var}\left(\frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.}\right)=\text{var}\left(\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}}{T}\left(\frac{\boldsymbol{H}'\boldsymbol{H}}{T}\right)^{+}\frac{\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.}}{T}\right)$$
$$=\frac{T}{N}\sum_{i=1}^{N}\text{E}\left[\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}}{T}\left(\frac{\boldsymbol{H}'\boldsymbol{H}}{T}\right)^{+}\frac{\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.}\boldsymbol{\varepsilon}_{i.}'\bar{\boldsymbol{V}}}{T^2}\left(\frac{\boldsymbol{H}'\boldsymbol{H}}{T}\right)^{+}\frac{\boldsymbol{H}'\boldsymbol{X}_{i.}}{T}\right]=O\left(\frac{T}{N}\times\frac{1}{NT}\times N\right)=O\left(\frac{1}{N}\right),$$

where the second last equality follows from Lemma 3(a)&(g) and the proof of the first result in Lemma 2(d). This implies:

$$\frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.}=O_p\left(\frac{1}{\sqrt{N}}\right),$$

which, combined with (S.22)-(S.24), in turn implies

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T}=\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{i.}}{T}+O_p\left(\frac{1}{\sqrt{N}}\right)+O_p\left(\frac{1}{\sqrt{T}}\right). \tag{S.25}$$

By Assumptions 2, it is easily seen that the first term on the RHS of (S.25) has mean $\mathbf{0}$. Furthermore, following proofs in (S.37) and (S.38), we can establish that its variance

$$\text{var}\left(\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{i.}}{T}\right)=\frac{1}{N}\sum_{i=1}^{N}\text{E}\left(\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\Omega}_{\varepsilon,i}\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{X}_{i.}}{T}\right)\le C, \tag{S.26}$$

is finite. Combining (S.17), (S.18)-(S.20), (S.25) and (S.26), we have:

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{F}_1\gamma_{1i}+\boldsymbol{F}_2\gamma_{2i}+\boldsymbol{\varepsilon}_{i.})}{T}$$
$$\xrightarrow{d}N\left(\mathbf{0},\lim_{(N,T)\to\infty}\frac{1}{N}\sum_{i=1}^{N}\text{E}\left(\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2\boldsymbol{\Omega}_{\gamma_2}\boldsymbol{F}_2'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{X}_{i.}}{T}+\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\Omega}_{\varepsilon,i}\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{X}_{i.}}{T}\right)\right),$$

if $T/N^2\to 0$ as $(N,T)\to\infty$. The proof of Theorem 3 is thus complete.

S3 Additional Lemmas

Lemma 1 below shows the validity of using $\bar{\boldsymbol{x}}_t$ as factor proxies.

Lemma 1. *Under Assumption 2, we have, for each $1\le t\le T$:*

(a) $\bar{\boldsymbol{v}}_t\xrightarrow{q.m.}\mathbf{0}$ *as* $N\to\infty$,

(b) $\text{E}\,\|\bar{\boldsymbol{v}}_t\|^2=O\left(\frac{1}{N}\right)$ *and* $\text{E}\,\|\bar{\boldsymbol{v}}_t\|=O\left(\frac{1}{\sqrt{N}}\right)$ *for all* N,

where $\bar{\boldsymbol{v}}_t = \boldsymbol{\Upsilon}\,\text{vec}(\boldsymbol{V}'_{.t})$, $\boldsymbol{\Upsilon} = \boldsymbol{\iota}'_N \otimes \boldsymbol{I}_k/N$, *and* $\overset{q.m.}{\longrightarrow}$ *denotes convergence in quadratic mean.*

Proof. (a) It is easily seen under Assumption that 2,

$$\text{E}(\bar{\boldsymbol{v}}_t) = \text{E}(\boldsymbol{\Upsilon}\,\text{vec}(\boldsymbol{V}'_{.t})) = \frac{1}{N}\sum_{i=1}^{N}\text{E}(\boldsymbol{v}_{it}) = \boldsymbol{0},$$

$$\text{var}\left(\bar{\boldsymbol{v}}_t\right) = \text{var}\left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{v}_{it}\right) = \frac{1}{N^2}\sum_{i=1}^{N}\text{var}\left(\boldsymbol{v}_{it}\right) = \frac{1}{N^2}\sum_{i=1}^{N}\boldsymbol{\Omega}_{v,i} = O\left(\frac{1}{N}\right),$$

as $\|\boldsymbol{\Omega}_{v,i}\| < C$ for all i. The result follows by the definition of convergence in quadratic mean.

(b) First, under Assumption 2, it is easily seen that

$$\text{E}\left\|\bar{\boldsymbol{v}}_t\right\|^2 = \text{E}\left[\text{tr}\left(\frac{1}{N^2}\sum_{j=1}^{N}\sum_{i=1}^{N}\boldsymbol{v}_{jt}\boldsymbol{v}'_{it}\right)\right] = \text{tr}\left(\frac{1}{N^2}\sum_{i=1}^{N}\text{var}(\boldsymbol{v}_{it})\right) = O\left(\frac{1}{N}\right).$$

By the Lyapunov's inequality, we have:

$$\text{E}\left\|\bar{\boldsymbol{v}}_t\right\| \leq (\text{E}\left\|\bar{\boldsymbol{v}}_t\right\|^2)^{1/2} = O\left(\frac{1}{\sqrt{N}}\right).$$

■

Lemma 2. *Under Assumptions 1 – 2, we have, for all N and T:*

(*a*) $\dfrac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T} = O_p\left(\dfrac{1}{N}\right)$,

(*b*) $\dfrac{\boldsymbol{F}'_a\bar{\boldsymbol{V}}}{T} = O_p\left(\dfrac{1}{\sqrt{NT}}\right)$,

(*c*) $\dfrac{\boldsymbol{V}'_{i.}\boldsymbol{F}_a}{T} = O_p\left(\dfrac{1}{\sqrt{T}}\right)$, $\dfrac{\boldsymbol{\varepsilon}'_{i.}\boldsymbol{F}_a}{T} = O_p\left(\dfrac{1}{\sqrt{T}}\right)$, *uniformly for all i,*

(*d*) $\dfrac{\boldsymbol{\varepsilon}'_{i.}\bar{\boldsymbol{V}}}{T} = O_p\left(\dfrac{1}{\sqrt{NT}}\right)$, $\dfrac{\boldsymbol{V}'_{i.}\bar{\boldsymbol{V}}}{T} = O_p\left(\dfrac{1}{N}\right) + O_p\left(\dfrac{1}{\sqrt{NT}}\right)$, *uniformly for all i,*

(*e*) $\dfrac{\boldsymbol{X}'_{i.}\bar{\boldsymbol{V}}}{T} = O_p\left(\dfrac{1}{N}\right) + O_p\left(\dfrac{1}{\sqrt{NT}}\right)$, *uniformly for all i,*

where $\bar{\boldsymbol{V}} = (\bar{\boldsymbol{v}}_1, \ldots, \bar{\boldsymbol{v}}_T)'$ *is a* $T \times k$ *matrix with* $\bar{\boldsymbol{v}}_t = \boldsymbol{\Upsilon}\,\text{vec}(\boldsymbol{V}'_{.t}) = \sum_{i=1}^{N}\boldsymbol{v}_{it}/N$, $\boldsymbol{F}_a = (\boldsymbol{f}_{a1}, \ldots, \boldsymbol{f}_{aT})'$, $\boldsymbol{V}_{i.} = (\boldsymbol{v}_{i1}, \ldots, \boldsymbol{v}_{iT})'$, $\boldsymbol{\varepsilon}_{i.} = (\varepsilon_{i1}, \ldots, \varepsilon_{iT})'$ *and* $\boldsymbol{X}_{i.} = (\boldsymbol{x}_{i1}, \ldots, \boldsymbol{x}_{iT})'$.

Proof. The results in Lemma 2(a) – (c) correspond to $(A.10) - (A.12)$ in Lemma 2 of Pesaran (2006). Hence, we only provide the proofs for (d) and (e).

(d) By definition, we have

$$\frac{\boldsymbol{\varepsilon}_{i.}'\bar{\boldsymbol{V}}}{T}=\frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it}\bar{\boldsymbol{v}}_t'=\frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it}\left(\frac{1}{N}\sum_{j=1}^{N}\boldsymbol{v}_{jt}'\right)=\frac{1}{NT}\sum_{t=1}^{T}\sum_{j=1}^{N}\varepsilon_{it}\boldsymbol{v}_{jt}',$$

which is a k dimensional row vector. Each element of $\sum_{t=1}^{T}\sum_{j=1}^{N}\varepsilon_{it}v_{jt,p}/NT$ for $p=1,\ldots,k$, has zero mean and variance:

$$\begin{aligned}
\operatorname{var}\left(\frac{1}{NT}\sum_{t=1}^{T}\sum_{j=1}^{N}\varepsilon_{it}v_{jt,p}\right) &= \frac{1}{N^2T^2}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{j=1}^{N}\sum_{h=1}^{N}\mathrm{E}\left(\varepsilon_{it}\varepsilon_{is}\right)\mathrm{E}(v_{jt,p}v_{hs,p})\\
&= \frac{1}{N^2T^2}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{j=1}^{N}\mathrm{E}(\varepsilon_{it}\varepsilon_{is})\,\mathrm{E}(v_{jt,p}v_{js,p})\\
&\leq \frac{1}{2N^2T^2}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{j=1}^{N}\left|\mathrm{E}(\varepsilon_{it}\varepsilon_{is})\right|\mathrm{E}(v_{jt,p}^2+v_{js,p}^2)\\
&\leq \frac{C}{NT^2}\sum_{t=1}^{T}\sum_{s=1}^{T}\left|\mathrm{E}(\varepsilon_{it}\varepsilon_{is})\right|\leq\left(\frac{C}{NT}\right)\text{ uniformly for all } i,
\end{aligned}$$

where the first equality follows from the independence of ε_{it} and $v_{js,p}$ for all i,j,t,s, the second equality from the fact that $v_{jt,p}$ and $v_{hs,p}$ are independent for $j\neq h$, the third inequality from the fact that $\mathbf{v}_{jt}$ is stationary over t and has uniformly bounded variance over j, and the last inequality from the uniformly absolutely summable autocovariance property of ε_{it}. The second result can be proved similarly by noting that the existence of the $O_p\left(1/N\right)$ term is due to the correlation between $\boldsymbol{V}_{i.}$ and $\bar{\boldsymbol{V}}$.

(e) Following (18) we have

$$\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}}{T}=\boldsymbol{\Gamma}_{xi}'\frac{\boldsymbol{F}_x'\bar{\boldsymbol{V}}}{T}+\frac{\boldsymbol{V}_{i.}'\bar{\boldsymbol{V}}}{T},$$

where $\boldsymbol{\Gamma}_{xi}$ is uniformly bounded in probability under Assumption 3. Thus, we obtain the desired result by Lemma 2(b), (d) and the definition of $\boldsymbol{F}_x$. ■

Lemma 3. *Let* $\boldsymbol{H}=\boldsymbol{F}_x\bar{\boldsymbol{\Gamma}}_x$*. Under Assumptions 1 – 3, we have, for all* N *and* T*:*

(a) $\dfrac{\boldsymbol{H}'\boldsymbol{H}}{T}=O_p(1)$,

(b) $\dfrac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T}=O_p\left(\dfrac{1}{\sqrt{NT}}\right)$,

(c) $\dfrac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T}=O_p(1)$,

(d) $\dfrac{\bar{\boldsymbol{X}}'\boldsymbol{F}_x}{T}=O_p(1),\quad\dfrac{\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T}=O_p(1)$,

(e) $\dfrac{\bar{\boldsymbol{X}}'\mathbf{V}_{i.}}{T}=O_p\left(\dfrac{1}{N}\right)+O_p\left(\dfrac{1}{\sqrt{T}}\right)$, *uniformly for all* i,

(f) $\dfrac{\bar{\boldsymbol{X}}'\boldsymbol{X}_{i.}}{T} = O_p(1)$, *uniformly for all* i,

(g) $\dfrac{\boldsymbol{H}'\boldsymbol{X}_{i.}}{T} = O_p(1)$, *uniformly for all* i,

(h) $\dfrac{\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}}{T} = O_p\left(\dfrac{1}{\sqrt{T}}\right)$, *uniformly for all* i, .

Proof. (a) Under Assumption 1, $\boldsymbol{F}_x'\boldsymbol{F}_x/T = O_p(1)$ for all T. Furthermore, under Assumption 3, $\bar{\boldsymbol{\Gamma}}_x = \sum_{i=1}^N \boldsymbol{\Gamma}_{xi}/N = O_p(1)$ for all N. Hence, we have:

$$\frac{\boldsymbol{H}'\boldsymbol{H}}{T} = \bar{\boldsymbol{\Gamma}}_x' \frac{\boldsymbol{F}_x'\boldsymbol{F}_x}{T}\bar{\boldsymbol{\Gamma}}_x = O_p(1).$$

(b) It follows from Lemma 2(b) that

$$\frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} = \bar{\boldsymbol{\Gamma}}_x' \frac{\boldsymbol{F}_x'\bar{\boldsymbol{V}}}{T} = O_p\left(\frac{1}{\sqrt{NT}}\right).$$

(c) Notice that $\bar{\boldsymbol{X}} = \boldsymbol{H} + \bar{\boldsymbol{V}}$. By Lemma 2(a) and Lemma 3(a)&(b), we have:

$$\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T} = \frac{\boldsymbol{H}'\boldsymbol{H}}{T} + \frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T} + \frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{H}}{T} = O_p(1) + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) = O_p(1).$$

(d) Under Assumption 1, $\boldsymbol{F}_x'\boldsymbol{F}_x/T = O_p(1)$ and $\boldsymbol{F}_2'\boldsymbol{F}_x/T = O_p(1)$ by the Cauchy Schwartz's Inequality. Then, by Lemma 2(b) and the fact that $\bar{\boldsymbol{\Gamma}}_x = O_p(1)$, we have:

$$\frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_x}{T} = \bar{\boldsymbol{\Gamma}}_x' \frac{\boldsymbol{F}_x'\boldsymbol{F}_x}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{F}_x}{T} = O_p(1) + O_p\left(\frac{1}{\sqrt{NT}}\right) = O_p(1),$$
$$\frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} = \bar{\boldsymbol{\Gamma}}_x' \frac{\boldsymbol{F}_x'\boldsymbol{F}_2}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{F}_2}{T} = O_p(1) + O_p\left(\frac{1}{\sqrt{NT}}\right) = O_p(1).$$

(e) Similarly to (d), by Lemma 2(c)&(d): uniformly for all i, we have:

$$\frac{\bar{\boldsymbol{X}}'\boldsymbol{V}_{i.}}{T} = \bar{\boldsymbol{\Gamma}}_x' \frac{\boldsymbol{F}_x'\boldsymbol{V}_{i.}}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{V}_{i.}}{T} = O_p\left(\frac{1}{\sqrt{T}}\right) + \left[O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right)\right] = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{T}}\right).$$

(f) Recall that $\boldsymbol{X}_{i.} = \boldsymbol{F}_x\boldsymbol{\Gamma}_{xi} + \boldsymbol{V}_{i.}$. Then, it follows from Lemma 3(d)&(e) and the uniform boundedness of $\boldsymbol{\Gamma}_{xi}$ (in probability) that

$$\frac{\bar{\boldsymbol{X}}'\boldsymbol{X}_{i.}}{T} = \frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_x}{T}\boldsymbol{\Gamma}_{xi} + \frac{\bar{\boldsymbol{X}}'\boldsymbol{V}_{i.}}{T} = O_p(1) + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{T}}\right) = O_p(1) \text{ uniformly for all } i.$$

(g) Recall that $\|\boldsymbol{\Gamma}_{xi}\| \le C$ in probability, $\bar{\boldsymbol{\Gamma}}_x = \sum_{i=1}^N \boldsymbol{\Gamma}_{xi}/N = O_p(1)$ under Assumption 3 and $\boldsymbol{F}_x'\boldsymbol{F}_x/T = O_p(1)$ under Assumption 1. Then, by Lemma 2(c), we have, uniformly for all i:

$$\frac{\boldsymbol{H}'\boldsymbol{X}_{i.}}{T} = \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{F}_x}{T}\boldsymbol{\Gamma}_{xi} + \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{V}_{i.}}{T} = O_p(1) + O_p\left(\frac{1}{\sqrt{T}}\right) = O_p(1).$$

(h) This can be easily verified using Lemma 2(c)&(d):

$$\frac{\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}}{T} = \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{\varepsilon}_{i.}}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.}}{T} = O_p\left(\frac{1}{\sqrt{T}}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) = O_p\left(\frac{1}{\sqrt{T}}\right).$$

■

Lemma 4. *Under Assumptions 1-3, as $N \to \infty$, we have, uniformly for all i and j:*

(a) $\dfrac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_x}{T} = O_p\left(\dfrac{1}{N}\right) + O_p\left(\dfrac{1}{\sqrt{NT}}\right), \quad \dfrac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_2}{T} = \dfrac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2}{T} + O_p\left(\dfrac{1}{N}\right) + O_p\left(\dfrac{1}{\sqrt{NT}}\right),$

(b) $\dfrac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{X}_{j.}}{T} = \dfrac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{X}_{j.}}{T} + O_p\left(\dfrac{1}{N}\right) + O_p\left(\dfrac{1}{\sqrt{NT}}\right),$

(c) $\dfrac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{j.}}{T} = \dfrac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{j.}}{T} + O_p\left(\dfrac{1}{\sqrt{NT}}\right).$

Proof. Using Lemmas 2 and 3, these results can be proved in a similar way to Lemma 3 in Kapetanios et al. (2011).

(a) Let $\boldsymbol{M}_{\boldsymbol{H}} = \boldsymbol{I}_T - \boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'$, where $\boldsymbol{H} = \boldsymbol{F}_x\bar{\boldsymbol{\Gamma}}_x$ and $+$ denotes the Moore-Penrose inverse. As $r_x \le k$, the $k \times k$ matrix $\boldsymbol{H}'\boldsymbol{H}$ has rank $\text{rank}(\boldsymbol{H}'\boldsymbol{H}) = \text{rank}(\boldsymbol{H}) \le r_x \le k$. Hence, unless $r_x = k$, it is not invertible. The same applies to $\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}$. Notice that

$$\begin{aligned}
&\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_a}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{H}}\boldsymbol{F}_a}{T}\right\| = \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{F}_a}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'\boldsymbol{F}_a}{T}\right\| \\
&\le \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{F}_a}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{F}_a}{T}\right\| + \left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{F}_a}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{X}}'\boldsymbol{F}_a}{T}\right\| \\
&+ \left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{X}}'\boldsymbol{F}_a}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'\boldsymbol{F}_a}{T}\right\| =: d_1 + d_2 + d_3. \qquad \text{(S.27)}
\end{aligned}$$

By Lemma 2(e) and Lemma 3(c)&(d) and using the relation $\bar{\boldsymbol{X}} = \boldsymbol{H} + \bar{\boldsymbol{V}}$, we have

$$\begin{aligned}
d_1 &= \left\|\frac{(\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}} - \boldsymbol{X}_{i.}'\boldsymbol{H})(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{F}_a}{T}\right\| \le \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}}{T}\right\| \left\|\left(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T}\right)^{+}\frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_a}{T}\right\| \\
&= O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) \text{ uniformly for all } i. \qquad \text{(S.28)}
\end{aligned}$$

For d_2, note that

$$d_2 = \left\| \frac{\boldsymbol{X}_{i.}' \boldsymbol{H} [(\bar{\boldsymbol{X}}' \bar{\boldsymbol{X}})^{+} - (\boldsymbol{H}' \boldsymbol{H})^{+}]' \bar{\boldsymbol{X}}' \boldsymbol{F}_a}{T} \right\| \leq \left\| \frac{\boldsymbol{X}_{i.}' \boldsymbol{H}}{T} \right\| \left\| \left(\frac{\bar{\boldsymbol{X}}' \bar{\boldsymbol{X}}}{T} \right)^{+} - \left(\frac{\boldsymbol{H}' \boldsymbol{H}}{T} \right)^{+} \right\| \left\| \frac{\bar{\boldsymbol{X}}' \boldsymbol{F}_1}{T} \right\|. \tag{S.29}$$

By Lemma 2(a) and Lemma 3(b), and using $\bar{\boldsymbol{X}} = \boldsymbol{H} + \bar{\boldsymbol{V}}$, we have:

$$\frac{\bar{\boldsymbol{X}}' \bar{\boldsymbol{X}}}{T} - \frac{\boldsymbol{H}' \boldsymbol{H}}{T} = \frac{\bar{\boldsymbol{V}}' \bar{\boldsymbol{V}}}{T} + \frac{\boldsymbol{H}' \bar{\boldsymbol{V}}}{T} + \frac{\bar{\boldsymbol{V}}' \boldsymbol{H}}{T} = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right). \tag{S.30}$$

Next, as $N \to \infty$,

$$\left(\frac{\bar{\boldsymbol{X}}' \bar{\boldsymbol{X}}}{T} \right)^{+} - \left(\frac{\boldsymbol{H}' \boldsymbol{H}}{T} \right)^{+} = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right). \tag{S.31}$$

Using (S.30) and by Theorem 2 of Andrews (1987), we only need to show that as $N \to \infty$,

$$\Pr\left(\text{rank}\left(\frac{\bar{\boldsymbol{X}}' \bar{\boldsymbol{X}}}{T} \right) = \text{rank}\left(\frac{\boldsymbol{H}' \boldsymbol{H}}{T} \right) \right) \to 1. \tag{S.32}$$

Recall that $\boldsymbol{H} = \boldsymbol{F}_x \bar{\boldsymbol{\Gamma}}_x$. Hence, $\text{rank}\left(\boldsymbol{H}'\boldsymbol{H}/T\right) = \text{rank}(\boldsymbol{H}) = \text{rank}(\bar{\boldsymbol{\Gamma}}_x) = r_x$ for all T and N as well as $N \to \infty$. Together with the result, $\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}/T = \boldsymbol{H}'\boldsymbol{H}/T + o_p(1)$ (see (S.30)), this implies that (S.32) holds. Thus, combining (S.29), (S.31), and Lemma 3(d)&(g), we have

$$d_2 = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right), \text{uniformly for all } i. \tag{S.33}$$

Finally, for d_3, by Lemma 2(b) and Lemma 3(a)&(g), we have:

$$d_3 = \left\| \frac{\boldsymbol{X}_{i.}' \boldsymbol{H} (\boldsymbol{H}' \boldsymbol{H})^{+} (\bar{\boldsymbol{X}}' \boldsymbol{F}_a - \boldsymbol{H}' \boldsymbol{F}_a)}{T} \right\| \leq \left\| \frac{\boldsymbol{X}_{i.}' \boldsymbol{H}}{T} \right\| \left\| \left(\frac{\boldsymbol{H}' \boldsymbol{H}}{T} \right)^{+} \right\| \left\| \frac{\bar{\boldsymbol{V}}' \boldsymbol{F}_a}{T} \right\| = O_p\left(\frac{1}{\sqrt{NT}}\right). \tag{S.34}$$

Combining (S.27), (S.28), (S.33) and (S.34), then

$$\frac{\boldsymbol{X}_{i.}' \boldsymbol{M}_{\bar{\boldsymbol{X}}} \boldsymbol{F}_a}{T} = \frac{\boldsymbol{X}_{i.}' \boldsymbol{M}_{\boldsymbol{H}} \boldsymbol{F}_a}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) \text{ uniformly for all } i. \tag{S.35}$$

Under Assumption 3, we have $\boldsymbol{M}_{\boldsymbol{H}} = \boldsymbol{M}_{\boldsymbol{F}_x}$. The second result in (a) follows from (S.35). For the first result, note that $\boldsymbol{F}_x = \left(\boldsymbol{F}_1, \boldsymbol{F}_3\right)$ and $\boldsymbol{M}_{\boldsymbol{F}_x} \boldsymbol{F}_x = \boldsymbol{0}$. Hence, (S.35) implies

$$\frac{\boldsymbol{X}_{i.}' \boldsymbol{M}_{\bar{\boldsymbol{X}}} \boldsymbol{F}_x}{T} = \frac{\boldsymbol{X}_{i.}' \boldsymbol{M}_{\boldsymbol{F}_x} \boldsymbol{F}_x}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right).$$

(b) Similar to the proof of (a), we obtain the following expansion:

$$
\begin{aligned}
&\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{X}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{H}}\boldsymbol{X}_{j.}}{T}\right\| = \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'\boldsymbol{X}_{j.}}{T}\right\| \\
&= \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}+\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right. \\
&\quad \left.-\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}+\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'\boldsymbol{X}_{j.}}{T}\right\| \\
&\le \left\|\frac{(\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}-\boldsymbol{X}_{i.}'\boldsymbol{H})(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right\|+\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}-(\boldsymbol{H}'\boldsymbol{H})^{+}]\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right\| \\
&\quad +\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}(\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}-\boldsymbol{H}'\boldsymbol{X}_{j.})}{T}\right\| =: d_1^*+d_2^*+d_3^*.
\end{aligned}
$$

For d_1^*, by Lemma 2(e) and Lemma 3(c)&(f), we have:

$$
d_1^* \le \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}}{T}\right\|\left\|\left(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T}\right)^{+}\right\|\left\|\frac{\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right\| = O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right) \text{ uniformly for all } i \text{ and } j.
$$

For d_2^*, by Lemma 3(f)&(g) and (S.31), as $N\to\infty$, we have:

$$
d_2^* \le \left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}}{T}\right\|\left\|\left(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T}\right)^{+}-\left(\frac{(\boldsymbol{H}'\boldsymbol{H})^{+}}{T}\right)\right\|\left\|\frac{\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right\| = O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right).
$$

Finally, for d_3^*, by Lemma 2(e) and Lemma 3(a)&(g), we have:

$$
d_3^* \le \left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}}{T}\left(\frac{\boldsymbol{H}'\boldsymbol{H}}{T}\right)^{+}\right\|\left\|\frac{\bar{\boldsymbol{V}}'\boldsymbol{X}_{j.}}{T}\right\| = O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right).
$$

This implies that as $N\to\infty$,

$$
\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{X}_{j.}}{T} = \frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{H}}\boldsymbol{X}_{j.}}{T}+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right) \text{ uniformly for all } i \text{ and } j.
$$

The result in (b) follows directly since we have $\boldsymbol{M}_{\boldsymbol{H}}=\boldsymbol{M}_{\boldsymbol{F}_x}$ under Assumption 3.

(c) Similar to the proofs of (a) and (b), we have the following expansion:

$$
\begin{aligned}
&\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{H}}\boldsymbol{\varepsilon}_{j.}}{T}\right\| = \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'\boldsymbol{\varepsilon}_{j.}}{T}\right\| \\
&\le \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T}\right\|+\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T}\right\| \\
&\quad +\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T}-\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{+}\boldsymbol{H}'\boldsymbol{\varepsilon}_{j.}}{T}\right\| =: \tilde{d}_1+\tilde{d}_2+\tilde{d}_3.
\end{aligned}
$$

For $\tilde{d}_1$, by Lemma 2(e) and Lemma 3(c)&(h), we have

$$\tilde{d}_1 \leq \left\| \frac{\boldsymbol{X}_{i.}' \bar{\boldsymbol{V}}}{T} \right\| \left\| \left(\frac{\bar{\boldsymbol{X}}' \bar{\boldsymbol{X}}}{T} \right)^+ \right\| \left\| \frac{\bar{\boldsymbol{X}}' \boldsymbol{\varepsilon}_{j.}}{T} \right\| = O_p\left(\frac{1}{N\sqrt{T}} \right) + O_p\left(\frac{1}{\sqrt{NT}} \right) \text{ uniformly for all } i \text{ and } j.$$

For $\tilde{d}_2$, by (S.31) and Lemma 3(g)&(h), we have:

$$\tilde{d}_2 \leq \left\| \frac{\boldsymbol{X}_{i.}' \boldsymbol{H}}{T} \right\| \left\| \left(\frac{\bar{\boldsymbol{X}}' \bar{\boldsymbol{X}}}{T} \right)^+ - \left(\frac{(\boldsymbol{H}'\boldsymbol{H})^+}{T} \right) \right\| \left\| \frac{\bar{\boldsymbol{X}}' \boldsymbol{\varepsilon}_{j.}}{T} \right\| = O_p\left(\frac{1}{N\sqrt{T}} \right) + O_p\left(\frac{1}{\sqrt{NT}} \right).$$

Finally, for $\tilde{d}_3$, by Lemma 2(d) and Lemma 3(a)&(g), we have

$$\tilde{d}_3 \leq \left\| \frac{\boldsymbol{X}_{i.}' \boldsymbol{H}}{T} \right\| \left\| \left(\frac{\boldsymbol{H}' \boldsymbol{H}}{T} \right)^+ \right\| \left\| \frac{\bar{\boldsymbol{V}}' \boldsymbol{\varepsilon}_{j.}}{T} \right\| = O_p\left(\frac{1}{\sqrt{NT}} \right).$$

Combining the above results, we have:

$$\frac{\boldsymbol{X}_{i.}' \boldsymbol{M}_{\bar{\boldsymbol{X}}} \boldsymbol{\varepsilon}_{j.}}{T} = \frac{\boldsymbol{X}_{i.}' \boldsymbol{M}_{\boldsymbol{H}} \boldsymbol{\varepsilon}_{j.}}{T} + O_p\left(\frac{1}{\sqrt{NT}} \right) \text{ uniformly for all } i \text{ and } j.$$

We complete the proof of (c) by further noticing that $\boldsymbol{M}_{\boldsymbol{H}} = \boldsymbol{M}_{\boldsymbol{F}_x}$. ■

Lemma 5. *Under Assumptions 1–4 and 6(i)–(iii), as $N \to \infty$ we have, uniformly for all i:*

(a) $\dfrac{\tilde{\boldsymbol{Q}}_i' \boldsymbol{F}_1 \boldsymbol{\gamma}_{1i}}{T} = O_p\left(\dfrac{1}{N} \right) + O_p\left(\dfrac{1}{\sqrt{NT}} \right)$,

(b) $\dfrac{\tilde{\boldsymbol{Q}}_i' \boldsymbol{\varepsilon}_{i.}}{T} = \dfrac{\tilde{\boldsymbol{Q}}_{i0}' \boldsymbol{\varepsilon}_{i.}}{T} + O_p\left(\dfrac{1}{\sqrt{NT}} \right) = O_p\left(\dfrac{1}{\sqrt{T}} \right)$,

$\dfrac{\tilde{\boldsymbol{Q}}_i' \boldsymbol{F}_2 \boldsymbol{\gamma}_{2i}}{T} = \dfrac{\tilde{\boldsymbol{Q}}_{i0}' \boldsymbol{F}_2 \boldsymbol{\gamma}_{2i}}{T} + O_p\left(\dfrac{1}{N} \right) + O_p\left(\dfrac{1}{\sqrt{NT}} \right) = O_p\left(\dfrac{1}{\sqrt{T}} \right) + O_p\left(\dfrac{1}{N} \right)$,

(c) $\dfrac{\tilde{\boldsymbol{Q}}_i' \tilde{\boldsymbol{Q}}_i}{T} = \dfrac{\tilde{\boldsymbol{Q}}_{i0}' \tilde{\boldsymbol{Q}}_{i0}'}{T} + O_p\left(\dfrac{1}{N} \right) + O_p\left(\dfrac{1}{\sqrt{NT}} \right)$,

$\dfrac{\tilde{\boldsymbol{Q}}_i' \boldsymbol{Z}_{i.}}{T} = \dfrac{\tilde{\boldsymbol{Q}}_{i0}' \boldsymbol{Z}_{i0.}}{T} + O_p\left(\dfrac{1}{N} \right) + O_p\left(\dfrac{1}{\sqrt{T}} \right)$.

Proof. Recall that $\tilde{\boldsymbol{Q}}_i = \boldsymbol{M}_{\bar{\boldsymbol{X}}} (\boldsymbol{I}_T \otimes \boldsymbol{b}_i') \boldsymbol{Q}$ and $\tilde{\boldsymbol{Q}}_{i0} = \boldsymbol{M}_{\boldsymbol{F}_x} (\boldsymbol{I}_T \otimes \boldsymbol{b}_i') \boldsymbol{Q}$, where $\underset{NT \times \iota}{\boldsymbol{Q}} = (\boldsymbol{Q}_{.1}', \ldots, \boldsymbol{Q}_{.T}')'$, $\boldsymbol{Q}_{.t}$ is an $N \times \iota$ matrix consisting of ι columns of $(\boldsymbol{X}_{.t}, \boldsymbol{W}\boldsymbol{X}_{.t}, \ldots, \boldsymbol{W}^r \boldsymbol{X}_{.t}, \ldots)$ with r being a non-negative integer (if $r = 0$, then $\boldsymbol{W}^0 = \boldsymbol{I}$). To prove Lemma 5, we only need to show that they hold for any column of $\boldsymbol{Q}$. For this purpose, we denote a representative column of $\boldsymbol{Q}$ as

$$\boldsymbol{Q}_c = [(\boldsymbol{W}^r \boldsymbol{x}_{.1,p})', \ldots, (\boldsymbol{W}^r \boldsymbol{x}_{.T,p})']',$$

where $p \in \{1, \ldots, k\}$ and $\boldsymbol{x}_{.t,p} = (x_{1t,p}, \ldots, x_{Nt,p})'$.

(a) First note that as $N \to \infty$,

$$
\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} = \frac{(\mathrm{vec}(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} \\
&= \frac{(\mathrm{vec}(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} = \frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} \\
&= \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} = \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p},\ldots,\boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} \\
&= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_{1i} = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),
\end{aligned}
$$

where $\boldsymbol{w}_i^r$ is the i-th row of $\boldsymbol{W}^r$, w_{ij}^r is the (i,j)-th element of $\boldsymbol{W}^r$, $\boldsymbol{x}_{i.,p} = (x_{i1,p},\ldots,x_{iT,p})'$. The last equality follows from Lemma 4(a) and the fact that $\boldsymbol{W}^r$ has bounded row sum norm while $\boldsymbol{\gamma}_{1i}$ is uniformly bounded in probability.

(b) For the first result in (b), note that as $N \to \infty$,

$$
\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T} = \frac{(\mathrm{vec}(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T} \\
&= \frac{(\mathrm{vec}(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T} = \frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T} \\
&= \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T} = \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p},\ldots,\boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T} \\
&= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{\varepsilon}_{i.}}{T} = \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{i.}}{T} + O_p\left(\frac{1}{\sqrt{NT}}\right) \\
&= \frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{i.}}{T} + O_p\left(\frac{1}{\sqrt{NT}}\right),
\end{aligned} \tag{S.36}
$$

where the second last equality follows from Lemma 4(c) and the assumption that $\boldsymbol{W}^r$ has bounded row sum norm, and the last equality follows from the similar arguments above. This proves the first equality in the first result of (b). For the second equality, by (18), we have:

$$
\begin{aligned}
\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{i.}}{T} &= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{i.}}{T} = \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{\varepsilon}_{i.}}{T} \\
&= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{T} - \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_x(\boldsymbol{F}_x'\boldsymbol{F}_x)^{-1}\boldsymbol{F}_x'\boldsymbol{\varepsilon}_{i.}}{T} \\
&= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{T} + O_p\left(\frac{1}{T}\right),
\end{aligned} \tag{S.37}
$$

where the last equality follows from Lemma 2(c) and the fact that $(\boldsymbol{F}_x'\boldsymbol{F}_x/T)^{-1} = O_p(1)$ under

Assumption 1. Furthermore, under Assumptions 2 and 4, we have:

$$
\begin{aligned}
&\mathrm{E}\left(\sum_{j=1}^{N} w_{ij}^{r}\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{T}\right)=0,\\
&\mathrm{var}\left(\sum_{j=1}^{N} w_{ij}^{r}\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{T}\right)=\frac{1}{T^2}\sum_{j=1}^{N}\sum_{l=1}^{N}\mathrm{E}(w_{ij}^{r}w_{il}^{r}\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}\boldsymbol{\varepsilon}_{i.}'\boldsymbol{v}_{l.,p})\\
&=\frac{1}{T^2}\sum_{j=1}^{N}\left(w_{ij}^{r}\right)^2\mathrm{E}(\boldsymbol{v}_{j.,p}'\boldsymbol{\Omega}_{\boldsymbol{\varepsilon},i}\boldsymbol{v}_{j.,p})\le\left(\max_{j}\left|w_{ij}^{r}\right|\right)\lambda_{\max}(\boldsymbol{\Omega}_{\varepsilon,i})\frac{1}{T^2}\sum_{j=1}^{N}\left|w_{ij}^{r}\right|\left|\mathrm{E}(\boldsymbol{v}_{j.,p}'\boldsymbol{v}_{j.,p})\right|\\
&\le\frac{C}{T^2}\sum_{j=1}^{N}\left|w_{ij}^{r}\right|\left|\sum_{t=1}^{T}\mathrm{E}(v_{jt,p}^2)\right|\le\frac{C}{T}\text{ uniformly for all } i. \qquad \text{(S.38)}
\end{aligned}
$$

Hence, we have

$$
\sum_{j=1}^{N} w_{ij}^{r}\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{T}=O_p\left(\frac{1}{\sqrt{T}}\right),\ \text{uniformly for all } i. \qquad \text{(S.39)}
$$

By combining (S.39) with (S.37), we complete the proof of the first result in (b).

To prove the second result in (b), we use (S.36) and the second result in Lemma 4(a) to obtain: as $N\to\infty$,

$$
\begin{aligned}
\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}&=\sum_{j=1}^{N} w_{ij}^{r}\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right)\\
&=\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right),
\end{aligned}
$$

which proves the first equality. Furthermore, by Assumption 6(iii), we have

$$
\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}=O_p\left(\frac{1}{\sqrt{T}}\right),\ \text{uniformly for all } i.
$$

This proves the second equality.

(c) As the first result involves $\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i=\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}$, we take another representative column of $\boldsymbol{Q}$ denoted as

$$
\boldsymbol{Q}_c^*=\left[(\boldsymbol{W}^{r^*}\boldsymbol{x}_{.1,p^*})',\dots,\ (\boldsymbol{W}^{r^*}\boldsymbol{x}_{.T,p^*})'\right]',
$$

where r^* is a non-negative integer and $p^*\in\{1,\dots,k\}$. We show that the result holds with $\boldsymbol{Q}_c$ and $\boldsymbol{Q}_c^*$ in place of the two $\boldsymbol{Q}$'s. Note that as $N\to\infty$,

$$
\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}_c^*}{T}\\
=&\frac{(\mathrm{vec}(\boldsymbol{W}^{r}(\boldsymbol{x}_{.1,p},\dots,\boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\,\mathrm{vec}(\boldsymbol{W}^{r^*}(\boldsymbol{x}_{.1,p^*},\dots,\boldsymbol{x}_{.T,p^*}))}{T}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(\mathrm{vec}(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\,\mathrm{vec}(\boldsymbol{b}_i'\boldsymbol{W}^{r^*}(\boldsymbol{x}_{.1,p^*},\ldots,\boldsymbol{x}_{.T,p^*}))}{T} \\
&= \frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{x}_{.1,p^*},\ldots,\boldsymbol{x}_{.T,p^*})'\boldsymbol{W}^{r^*\prime}\boldsymbol{b}_i}{T} \\
&= \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p},\ldots,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{x}_{.1,p^*},\ldots,\boldsymbol{x}_{.T,p^*})'\boldsymbol{w}_i^{r^*\prime}}{T} \\
&= \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p},\ldots,\boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{x}_{1.,p^*},\ldots,\boldsymbol{x}_{N.,p^*})\boldsymbol{w}_i^{r^*\prime}}{T} \\
&= \sum_{j=1}^N\sum_{l=1}^N w_{ij}^r w_{il}^{r^*}\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{x}_{l.,p^*}}{T} \\
&= \sum_{j=1}^N\sum_{l=1}^N w_{ij}^r w_{il}^{r^*}\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{x}_{l.,p^*}}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) \\
&= \frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{F}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}_c^*}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),
\end{aligned}
$$

where the second last equality follows from Lemma 4(b) and the fact that $\boldsymbol{W}^r$ has bounded row sum norm for any non-negative integer r. This completes the proof of the first result in (c).

For the second result, we notice from (4) that

$$
\begin{aligned}
\boldsymbol{y} &= (\boldsymbol{I}_{NT} - \boldsymbol{I}_T\otimes\boldsymbol{\rho}\boldsymbol{W})^{-1}[(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x} + (\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}] \\
&= (\boldsymbol{I}_T\otimes(\boldsymbol{I}_N - \boldsymbol{\rho}\boldsymbol{W})^{-1})[(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x} + (\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}] \\
&= (\boldsymbol{I}_T\otimes\boldsymbol{S}^{-1})[(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x} + (\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}],
\end{aligned}
$$

which implies that $\boldsymbol{Z}_{i.} := (\boldsymbol{y}_{i.}^*, \boldsymbol{X}_{i.})$ can be written as:

$$
\begin{aligned}
\boldsymbol{Z}_{i.} &= \big((\boldsymbol{I}_T\otimes\boldsymbol{w}_i)(\boldsymbol{I}_T\otimes\boldsymbol{S}^{-1})[(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x} + (\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}],\ \boldsymbol{X}_{i.}\big) \\
&= \big((\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{W}\boldsymbol{S}^{-1})\big[(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x} + (\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}\big],\ \boldsymbol{X}_{i.}\big) \\
&= \big((\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\big[(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x} + (\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}\big], \boldsymbol{X}_{i.}\big) \\
&= \big((\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x},\ \boldsymbol{X}_{i.}\big) + \big((\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\big[(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}\big],\ \boldsymbol{0}\big) \\
&= \boldsymbol{Z}_{i0.} + \left((\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\big[(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_1)\boldsymbol{f}_1 + ((\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2 + \boldsymbol{\varepsilon}\big],\ \boldsymbol{0}\right),
\end{aligned}
$$

where $\boldsymbol{Z}_{i0.} = \big((\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\mathcal{B}})\boldsymbol{x}, \boldsymbol{X}_{i.}\big)$. Hence,

$$
\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i.}}{T} = \frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i0.}}{T} + \frac{\tilde{\boldsymbol{Q}}_i'\left((\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\big[(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_1)\boldsymbol{f}_1 + (\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2 + \boldsymbol{\varepsilon}\big], \boldsymbol{0}\right)}{T}. \tag{S.40}
$$

By Lemma 6, the rate of the second term on the RHS of (S.40) becomes:

$$
\frac{\left(\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})[(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_1)\boldsymbol{f}_1 + (\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2 + \boldsymbol{\varepsilon}],\ \boldsymbol{0}\right)}{T} = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{T}}\right),
$$

uniformly for i. We then focus on the first term, which can be written as

$$\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i0.}}{T} = \left(\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\mathcal{B}})\boldsymbol{x}}{T}, \ \frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{X}_{i.}}{T}\right). \tag{S.41}$$

Following the proof of the first result in (c), we show, uniformly for all i and j, that

$$\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{X}_{j.}}{T} = \frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{X}_{j.}}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) \tag{S.42}$$

$$\begin{aligned}
&\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\mathcal{B}})\boldsymbol{x}}{T} \\
=&\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\mathcal{B}})\operatorname{vec}(\operatorname{vec}(\boldsymbol{X}_{.1}'), \ldots, \operatorname{vec}(\boldsymbol{X}_{.T}'))}{T} \\
=&\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})\operatorname{vec}(\boldsymbol{\mathcal{B}}\operatorname{vec}(\boldsymbol{X}_{.1}'), \ldots, \boldsymbol{\mathcal{B}}\operatorname{vec}(\boldsymbol{X}_{.T}'))}{T} \\
=&\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})\operatorname{vec}((\boldsymbol{X}_{1.}\boldsymbol{\beta}_1, \ldots, \boldsymbol{X}_{N.}\boldsymbol{\beta}_N)')}{T} \\
=&\frac{\tilde{\boldsymbol{Q}}_i'\operatorname{vec}(\boldsymbol{b}_i'\boldsymbol{G}(\boldsymbol{X}_{1.}\boldsymbol{\beta}_1, \ldots, \boldsymbol{X}_{N.}\boldsymbol{\beta}_N)')}{T} \\
=&\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{X}_{1.}\boldsymbol{\beta}_1, \ldots, \boldsymbol{X}_{N.}\boldsymbol{\beta}_N)\boldsymbol{G}'\boldsymbol{b}_i}{T} = \sum_{j=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{X}_{j.}\boldsymbol{\beta}_j g_{ij}}{T} \\
=&\sum_{j=1}^{N} g_{ij}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{X}_{j.}}{T}\boldsymbol{\beta}_j + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) \\
=&\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\mathcal{B}})\boldsymbol{x}}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),
\end{aligned} \tag{S.43}$$

where g_{ij} is the ij-th element of $\boldsymbol{G} = \boldsymbol{W}\boldsymbol{S}^{-1}$. The third equality in (S.43) follows from:

$$\operatorname{vec}(\boldsymbol{\mathcal{B}}\operatorname{vec}(\boldsymbol{X}_{.1}'), \ldots, \boldsymbol{\mathcal{B}}\operatorname{vec}(\boldsymbol{X}_{.T}')) = \operatorname{vec}\left(\begin{bmatrix} \boldsymbol{\beta}_1'\boldsymbol{x}_{11} & \cdots & \boldsymbol{\beta}_1'\boldsymbol{x}_{1T} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\beta}_N'\boldsymbol{x}_{N1} & \cdots & \boldsymbol{\beta}_N'\boldsymbol{x}_{NT} \end{bmatrix}\right) = \operatorname{vec}\left(\begin{bmatrix} \boldsymbol{\beta}_1'\boldsymbol{X}_{1.}' \\ \vdots \\ \boldsymbol{\beta}_N'\boldsymbol{X}_{N.}' \end{bmatrix}\right),$$

and the second last equality follows from (S.42) and the fact that $\boldsymbol{G} = \boldsymbol{W}\boldsymbol{S}^{-1}$ has bounded row and column sum norms by Assumption 4. Combining (S.40)–(S.43), we have

$$\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i.}}{T} = \frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0.}}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{T}}\right) \text{ uniformly for all } i. \tag{S.44}$$

■

Lemma 6. *Under Assumptions 1–4 and 6(i)–(iii), as $N \to \infty$, we have, uniformly for all i,*

(a) $$\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_1)\boldsymbol{f}_1}{T} = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),$$

(b) $$\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T} = \frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{T}}\right),$$

(c) $$\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})\boldsymbol{\varepsilon}}{T} = \frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})\boldsymbol{\varepsilon}}{T} + O_p\left(\frac{1}{\sqrt{NT}}\right) = O_p\left(\frac{1}{\sqrt{T}}\right).$$

Proof. Recall that $\tilde{\boldsymbol{Q}}_i = \boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i')\boldsymbol{Q}$ and $\tilde{\boldsymbol{Q}}_{i0} = \boldsymbol{M}_{\boldsymbol{F}_x}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i')\boldsymbol{Q}$. As in the proof of Lemma 5, we will show that the results in Lemma 6 hold for any column, $\boldsymbol{Q}_c$, of $\boldsymbol{Q}$.

(a) Note that as $N \to \infty$,

$$\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_1)\boldsymbol{f}_1}{T}\\
&= \frac{(\mathrm{vec}(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \ldots, \boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G}\boldsymbol{\gamma}_1)\,\mathrm{vec}(\boldsymbol{F}_1')}{T}\\
&= \frac{(\mathrm{vec}(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \ldots, \boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\,\mathrm{vec}(\boldsymbol{b}_i'\boldsymbol{G}\boldsymbol{\gamma}_1\boldsymbol{F}_1')}{T}\\
&= \frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \ldots, \boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_1'\boldsymbol{G}'\boldsymbol{b}_i}{T}\\
&= \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p}, \ldots, \boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_1'\boldsymbol{G}'\boldsymbol{b}_i}{T}\\
&= \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p}, \ldots, \boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1\boldsymbol{\gamma}_1'\boldsymbol{G}'\boldsymbol{b}_i}{T}\\
&= \sum_{j=1}^{N} w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_1'\boldsymbol{G}'\boldsymbol{b}_i = \sum_{j=1}^{N} w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_1}{T}(\boldsymbol{G}\boldsymbol{\gamma}_1)_i' = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),
\end{aligned}$$

as $N \to \infty$, where $\boldsymbol{w}_i^r$ is the i-th row of $\boldsymbol{W}^r$ and $(\boldsymbol{G}\boldsymbol{\gamma}_1)_i$ is the ith-row of $\boldsymbol{G}\boldsymbol{\gamma}_1$. The last equality follows from Lemma 4(a) and the fact that $\boldsymbol{W}^r$ and $\boldsymbol{G}$ have bounded row and column sum norms and $\boldsymbol{\gamma}_{1i}$, $1 \le i \le N$, are uniformly bounded in probability.

(b) Following the same logic as in the proof of (a), we have: as $N \to \infty$,

$$\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T}\\
&= \frac{(\mathrm{vec}(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \ldots, \boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{\boldsymbol{X}}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G}\boldsymbol{\gamma}_2)\,\mathrm{vec}(\boldsymbol{F}_2')}{T}\\
&= \frac{(\mathrm{vec}(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \ldots, \boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\,\mathrm{vec}(\boldsymbol{b}_i'\boldsymbol{G}\boldsymbol{\gamma}_2\boldsymbol{F}_2')}{T}\\
&= \frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \ldots, \boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_2\boldsymbol{\gamma}_2'\boldsymbol{G}'\boldsymbol{b}_i}{T}\\
&= \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p}, \ldots, \boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_2\boldsymbol{\gamma}_2'\boldsymbol{G}'\boldsymbol{b}_i}{T}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p},\ldots,\boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_2\gamma_2'\boldsymbol{G}'\boldsymbol{b}_i}{T} \\
&= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_2}{T}\gamma_2'\boldsymbol{G}'\boldsymbol{b}_i = \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{\boldsymbol{X}}}\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i' \\
&= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i' + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right) \qquad \text{(S.45)} \\
&= \frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{F}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T} + O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),
\end{aligned}
$$

where $(\boldsymbol{G}\boldsymbol{\gamma}_2)_i$ is the ith-row of $\boldsymbol{G}\boldsymbol{\gamma}_2$. The second last equality follows from the second result in Lemma 4(a) and the assumptions that $\boldsymbol{W}^r$ and $\boldsymbol{G}$ have bounded row and column sum norms and that $\boldsymbol{\gamma}_{2i}$, $1 \le i \le N$, are uniformly bounded in probability. The last equality follows from the first few lines of the above equation. This completes the proof of the first equality in (b).

For the second equality, note that

$$
\begin{aligned}
\sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i' &= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i' \\
&= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i' - \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_x(\boldsymbol{F}_x'\boldsymbol{F}_x)^{-1}\boldsymbol{F}_x'\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i' \\
&= \sum_{j=1}^N w_{ij}^r \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i' + O_p\left(\frac{1}{\sqrt{T}}\right) = O_p\left(\frac{1}{\sqrt{T}}\right), \qquad \text{(S.46)}
\end{aligned}
$$

where the last two equalities follow from Lemma 2(c), the fact that $(\boldsymbol{F}_x'\boldsymbol{F}_x/T)^{-1} = O_p(1)$ and $\boldsymbol{F}_x'\boldsymbol{F}_2/T = O_p(1)$ under Assumption 1, and that $(\boldsymbol{G}\boldsymbol{\gamma}_2)_i'$ is uniformly bounded in probability over i due to $\boldsymbol{G}$ having bounded row sum norm and $\boldsymbol{\gamma}_{2i}$, $1 \le i \le N$, being uniformly bounded in probability. Then, by combining (S.45) and (S.46), we have the second equality in (b).

(c) Similarly, we can prove (c) using Lemma 4(c) and arguments used to prove (S.39). ■

Lemma 7. *Under Assumptions 1–4, 6(i)–(iii), as $N \to \infty$, we have:*

(a) $\displaystyle \frac{1}{N}\sum_{i=1}^N \left(\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{Z}_{i.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),$

(b) $\displaystyle \frac{1}{N}\sum_{i=1}^N \left(\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{Z}_{i.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T} = O_p\left(\frac{1}{N}\right) + O_p\left(\frac{1}{\sqrt{NT}}\right),$

(c) $\displaystyle \frac{1}{N}\sum_{i=1}^N \left(\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{Z}_{i.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{\varepsilon}_{i.}}{T} = O_p\left(\frac{1}{\sqrt{NT}}\right).$

Proof. (a) In view of the definition $\boldsymbol{\Pi}_i = \tilde{\boldsymbol{Q}}_i(\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i)^{-1}\tilde{\boldsymbol{Q}}_i'$, the results readily follow from Lemma 5(a)&(c), and Assumption 6(i)&(ii).

(b) Using Lemma 5(b)&(c), we have: as $N \to \infty$,

$$\begin{aligned}&\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{Z}_{i.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}\\ &=\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right),\end{aligned} \tag{S.47}$$

where $\boldsymbol{\Pi}_{i0} = \tilde{\boldsymbol{Q}}_{i0}(\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0})^{-1}\tilde{\boldsymbol{Q}}_{i0}'$. Under Assumption 3, $\boldsymbol{\gamma}_{2i}$, $1 \le i \le N$, are i.i.d. with mean $\boldsymbol{0}$, finite variance and are independent of the rest of terms in $(\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.})^{-1}\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}$. Hence,

$$\mathrm{E}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}\right]=\boldsymbol{0}, \tag{S.48}$$

and by Assumption 6(i)–(iii),

$$\begin{aligned}&\mathrm{var}\left(\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}\right)\\ =&\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{E}\left[\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{F}_2\,\mathrm{E}\left(\boldsymbol{\gamma}_{2i}\boldsymbol{\gamma}_{2j}'\right)\boldsymbol{F}_2'\boldsymbol{\Pi}_{j0}\boldsymbol{Z}_{j0.}}{T^2}\left(\frac{\boldsymbol{Z}_{j0.}'\boldsymbol{\Pi}_{j0}\boldsymbol{Z}_{j0.}}{T}\right)^{-1}\right]\\ =&\frac{1}{N^2}\sum_{i=1}^{N}\mathrm{E}\left[\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{F}_2\,\mathrm{E}\left(\boldsymbol{\gamma}_{2i}\boldsymbol{\gamma}_{2i}'\right)\boldsymbol{F}_2'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T^2}\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\right]\\ =&\frac{1}{N^2T}\sum_{i=1}^{N}\mathrm{E}\left[\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\left(\frac{\boldsymbol{Z}_{i0.}'\tilde{\boldsymbol{Q}}_{i0}}{T}\right)\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}\right)^{-1}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{F}_2\boldsymbol{\Omega}_{\gamma_2}\boldsymbol{F}_2'\tilde{\boldsymbol{Q}}_{i0}}{T}\right.\\ &\left.\times\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}\right)^{-1}\left(\frac{\boldsymbol{Z}_{i0.}'\tilde{\boldsymbol{Q}}_{i0}}{T}\right)'\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\right]\le\frac{C}{NT}.\end{aligned} \tag{S.49}$$

By (S.48) and (S.49), we have

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}=O_p\left(\frac{1}{\sqrt{NT}}\right), \tag{S.50}$$

which, together with (S.47), proves (b).

(c) Again using Lemma 5(b)&(c), we have, as $N \to \infty$,

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{Z}_{i.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i.}'\boldsymbol{\Pi}_i\boldsymbol{\varepsilon}_{i.}}{T}=\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{\varepsilon}_{i.}}{T}+O_p\left(\frac{1}{\sqrt{NT}}\right).$$

By Assumptions 1 and 2, $\boldsymbol{\varepsilon}_{i.}$ is independently distributed over i with mean $\mathbf{0}$ and independent of the rest of terms in $\left(\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}\right)^{-1}\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{\varepsilon}_{i.}$. Furthermore, from (S.37) and (S.38), it is easily seen that $\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\varepsilon}_{i.}/\sqrt{T}$ has a uniformly bounded second-order moment. Following the proof of (S.50) and by Assumption 6(i)–(ii), we can show that

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0.}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0.}'\boldsymbol{\Pi}_{i0}\boldsymbol{\varepsilon}_{i.}}{T}=O_p\left(\frac{1}{\sqrt{NT}}\right).$$

This completes the proof of (c). ∎

Lemma 8. *Under Assumptions 1–4 and 6(i)–(iii), as $N\to\infty$, we have:*

(*a*) $\displaystyle\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{F}_1\gamma_{1i}}{T}=O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right),$

(*b*) $\displaystyle\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{\varepsilon}_{i.}}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\varepsilon}_{i.}}{T}+O_p\left(\frac{1}{\sqrt{NT}}\right)=O_p\left(\frac{1}{\sqrt{NT}}\right),$

$\displaystyle\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{F}_2\gamma_{2i}}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{F}_2\gamma_{2i}}{T}+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right)=O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right),$

(*c*) $\displaystyle\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right),$

$\displaystyle\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i.}}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0.}}{T}+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right).$

Proof. (a) This result follows directly from Lemma 5(a).

(b) As in the proof of Lemma 5, we take a representative column, $\boldsymbol{Q}_c$, from $\boldsymbol{Q}$ and show that the results hold with $\boldsymbol{Q}_c$ in place of $\boldsymbol{Q}$. By Lemma 5(b), as $N\to\infty$, we have:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{\varepsilon}_{i.}}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\varepsilon}_{i.}}{T}+O_p\left(\frac{1}{\sqrt{NT}}\right).$$

Hence, to prove the first result, we only need to show that as $N\to\infty$,

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\varepsilon}_{i.}}{T}=O_p\left(\frac{1}{\sqrt{NT}}\right). \tag{S.51}$$

To this end, we notice that $\boldsymbol{\varepsilon}_{i.}$ is independently distributed over i with mean $\mathbf{0}$ and independent of $\tilde{\boldsymbol{Q}}_{i0}$. Hence, $\mathrm{E}\left(\sum_{i=1}^{N}\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\varepsilon}_{i.}/NT\right)=\mathbf{0}$, and by (S.37) and (S.38),

$$\mathrm{var}\left(\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\varepsilon}_{i.}}{T}\right)=\frac{1}{N^2}\sum_{i=1}^{N}\mathrm{var}\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\varepsilon}_{i.}}{T}\right)\le\frac{1}{N^2}\sum_{i=1}^{N}\frac{C}{T}=\frac{C}{NT}.$$

This proves (S.51).

The second result can be proved by first using Lemma 5(b) and then following the same argument as in the proof of (S.50) to obtain $\sum_{i=1}^{N}\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}/NT = O_p\left(1/\sqrt{NT}\right)$.

(c) The first result follows directly from the first result in Lemma 5(c). For the second result, note from (S.40) and (S.41)–(S.43) that we obtain:

$$\begin{aligned}\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i.}}{T} =& \frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0.}}{T} + \frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\left((\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\left[(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_1)\boldsymbol{f}_1+(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2+\boldsymbol{\varepsilon}\right],\boldsymbol{0}\right)}{T}\\ &+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right).\end{aligned} \tag{S.52}$$

By Lemma 6, we have

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_1)\boldsymbol{f}_1}{T} = O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right), \tag{S.53}$$

$$\begin{aligned}\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T} &= \frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T}\\ &\quad+O_p\left(\frac{1}{N}\right)+O_p\left(\frac{1}{\sqrt{NT}}\right),\end{aligned} \tag{S.54}$$

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\boldsymbol{\varepsilon}}{T} = \frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\boldsymbol{\varepsilon}}{T}+O_p\left(\frac{1}{\sqrt{NT}}\right). \tag{S.55}$$

Next, we will show that

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T} = O_p\left(\frac{1}{\sqrt{NT}}\right), \tag{S.56}$$

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\boldsymbol{\varepsilon}}{T} = O_p\left(\frac{1}{\sqrt{NT}}\right). \tag{S.57}$$

Recall that $\tilde{\boldsymbol{Q}}_{i0} = \boldsymbol{M}_{\boldsymbol{F}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}$. As in the proof of Lemma 5, we will show that (S.56) and (S.57) hold for any representative column, $\boldsymbol{Q}_c = [(\boldsymbol{W}^r\boldsymbol{x}_{.1,p})',\ldots,\ (\boldsymbol{W}^r\boldsymbol{x}_{.T,p})']'$, of $\boldsymbol{Q}$. To this end, we first follow the arguments in (S.45) and (S.46) and obtain:

$$\begin{aligned}&\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{F}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{F}_x}\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i'\\ =&\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i' - \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_x(\boldsymbol{F}_x'\boldsymbol{F}_x)^{-1}\boldsymbol{F}_x'\boldsymbol{F}_2}{T}(\boldsymbol{G}\boldsymbol{\gamma}_2)_i'\\ =&\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N}w_{ij}^r g_{il}\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_2\gamma_{2l}}{T} - \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N}w_{ij}^r g_{il}\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_x(\boldsymbol{F}_x'\boldsymbol{F}_x)^{-1}\boldsymbol{F}_x'\boldsymbol{F}_2\gamma_{2l}}{T}\end{aligned}$$

$$=:\Delta_1 + \Delta_2, \tag{S.58}$$

where g_{ij} is the (i,j)-th element of $\boldsymbol{G}$. Under Assumptions 1–3, $\boldsymbol{v}_{j.,p}$, $\boldsymbol{F}_2(\boldsymbol{F}_x)$ and $\boldsymbol{\gamma}_{2l}$ are mutually independent, and $\boldsymbol{v}_{j.,p}$ and $\boldsymbol{\gamma}_{2l}$ are cross-sectionally independent with a zero mean. Hence, both Δ_1 and Δ_2 have a zero mean. Furthermore, by Assumption 6(iii), we have

$$\begin{aligned}
\mathrm{var}(\Delta_1) =& \frac{1}{N^2T}\sum_{i_1=1}^{N}\sum_{i_2=1}^{N}\sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\sum_{l_1=1}^{N}\sum_{l_2=1}^{N} w^r_{i_1j_1}w^r_{i_2j_2}g_{i_1l_1}g_{i_2l_2}\frac{\mathrm{E}\left(\boldsymbol{v}'_{j_1.,p}\boldsymbol{F}_2\boldsymbol{\gamma}_{2,l_1}\boldsymbol{\gamma}'_{2,l_2}\boldsymbol{F}'_2\boldsymbol{v}_{j_2.,p}\right)}{T}\\
=&\frac{1}{N^2T}\sum_{i_1=1}^{N}\sum_{i_2=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} w^r_{i_1j}w^r_{i_2j}g_{i_1l}g_{i_2l}\frac{\mathrm{E}\left(\boldsymbol{v}'_{j.,p}\boldsymbol{F}_2\boldsymbol{\Omega}_{\gamma_2}\boldsymbol{F}'_2\boldsymbol{v}_{j.,p}\right)}{T}\\
\leq&\frac{C}{N^2T}\sum_{i_1=1}^{N}\sum_{i_2=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N}\left|w^r_{i_1j}w^r_{i_2j}g_{i_1l}g_{i_2l}\right|\\
\leq&\frac{C}{N^2T}\sum_{i_1=1}^{N}\sum_{i_2=1}^{N}\sum_{j=1}^{N}\left|w^r_{i_1j}w^r_{i_2j}\right|\left(\max_{i_1,l}|g_{i_1l}|\right)\left(\max_{i_2}\sum_{l=1}^{N}|g_{i_2l}|\right)\\
\leq&\frac{C}{N^2T}\sum_{j=1}^{N}\left(\max_{j}\sum_{i_1=1}^{N}\left|w^r_{i_1j}\right|\right)\left(\max_{j}\sum_{i_2=1}^{N}\left|w^r_{i_2j}\right|\right)\leq\frac{C}{NT},
\end{aligned}$$

where the last two inequalities follow from the fact that $\boldsymbol{W}^r$ and $\boldsymbol{G}$ have bounded row and column sum norms. Analogously, we can show that $\mathrm{var}(\Delta_2) \leq C/NT$. This implies:

$$\Delta_1 = O_p\left(\frac{1}{\sqrt{NT}}\right), \qquad \Delta_2 = O_p\left(\frac{1}{\sqrt{NT}}\right). \tag{S.59}$$

We thus prove (S.56) by combining (S.59) with (S.58).

Along similar lines of logic, we can prove (S.57) (by replacing $\boldsymbol{F}_2\boldsymbol{\gamma}_{2l}$ in (S.58) and the subsequent calculation of variances with $\boldsymbol{\varepsilon}_{l.}$). Combining (S.52) with (S.53)–(S.55), we obtain the second result in (c). ■

S4 Supplementary Monte Carlo Simulation Results

As summarised in Table 1, we consider the four experiments: Experiment 1 with homogeneous parameters and the same factor structures; Experiment 2 with heterogeneous parameters and the same factor structures; Experiment 3 with homogeneous parameters and the different factor structures; Experiment 4 with heterogeneous parameters and the different factor structures.

In the main text we report the simulation results for the individual and Mean Group estimators only under Experiment 4. Now, we provide the complete Monte Carlo simulation results for the finite sample performance of the individual, Mean Group and Pooled estimators, obtained using CCEX-IV, CCE-IV and infeasible estimation procedures. Moreover, as mentioned in footnotes 6 and 7, we also provide simulation results for additional DGP settings where $\boldsymbol{\gamma}_{1i}(\boldsymbol{\gamma}_{2i}) \sim$

$IIDN(0.5, 0.5)$ and/or $\kappa_1 = 1$ and $\kappa_2 = 2$, and results for Experiment 4 are reported here. Table S1 provides a summary of the results reported.

Table S1: List of Simulation Results

		Experiment 1 (Section S4.1)	Experiment 2 (Section S4.2)	Experiment 3 (Section S4.3)	Experiment 4 (Sections 4.1 and S4.4)
Individual	Bias & RMSE	Table S2	Table S8	Table S14	Table 2
	Size & Power	Table S3	Table S9	Table S15	Table 4
Mean Group	Bias & RMSE	Table S4	Table S10	Table S16	Table 3
	Size & Power	Table S5	Table S11	Table S17	Table 5
Pooled	Bias & RMSE	Table S6	Table S12	Table S18	Table S20
	Size & Power	Table S7	Table S13	Table S19	Table S21

		Additional Simulations Under Experiment 4: Non-Zero Mean Factor Loadings (Section S4.5)	Additional Simulations Under Experiment 4: $\kappa_1 = 1$ and $\kappa_2 = 2$ (Section S4.6)
Individual	Bias & RMSE	Table S23	Table S29
	Size & Power	Table S24	Table S30
Mean Group	Bias & RMSE	Table S25	Table S31
	Size & Power	Table S26	Table S32
Pooled	Bias & RMSE	Table S27	Table S33
	Size & Power	Table S28	Table S34

S4.1 Simulation Results under Experiment 1

In this section, we provide the simulation results under Experiment 1 in Tables S2 to S7. The results for the individual and Mean Group estimators are qualitatively similar to those reported for Experiment 4 in the main text. The biases of the CCEX-IV Mean Group estimator for (ρ, β_1, β_2) are more or less negligible and close to those of the infeasible estimator in almost all cases. On the other hand, the CCE-IV estimator tends to display non-negligible biases as h or ρ rises especially for $N = 20$. In particular, if the spatial weighing matrix remains dense with $h = 0.3N$, then the bias of the CCE-IV spatial coefficient does not disappear even for large N.

RMSEs of the individual and Mean Group estimators all decline with T. RMSEs for CCEX-IV and CCE-IV are quite similar if h is fixed, suggesting that the potential efficiency gain from using $\bar{y}_t$ as a factor proxy does not offset the higher biases. In particular, if the spatial weighing matrix remains dense with $h = 0.3N$, then RMSEs are significantly higher for CCE-IV estimator than CCEX-IV counterparts for all the sample sizes. RMSEs for ρ_i decline with N if h is fixed and N is small, while rising with N if h is varying with N whilst RMSEs for (β_{1i}, β_{2i}) demonstrate little dependence on N. RMSEs for Mean Group estimator are all decreasing sharply with N.

The finite sample performances of the Pooled estimators are quantitatively similar to those for the individual and Mean Group estimators. The CCEX-IV estimator shows quite satisfactory performance in terms of bias, while the performance of the CCE-IV Pooled estimator still relies crucially upon the different density of the spatial weighting matrix. RMSEs are decreasing sharply with both N and T now, verifying Theorem 3 that the convergence rate of the Pooled estimator is $\sqrt{NT}$.

As the sample size increases (particularly T for individual estimator), the size of the t-test for

CCEX-IV estimators tends to the 5% nominal level in most cases. But, the t-test for the CCE-IV spatial coefficient displays slight size distortion for small N, which does not disappear as T rises. The powers of t-tests for CCEX-IV and CCE-IV estimators are more or less similar, and both tend to 1 as the sample size (particularly T for individual estimator) rises if h is fixed. In the case with $h = 0.3N$, the power of the individual spatial coefficient falls with N due to RMSEs increasing with N, but rises sharply with T. Moreover, since the parameters are homogeneous now, the power for Mean Group and Pooled estimator are much larger than those for individual estimator because of a faster convergence rate.

Table S2: The Finite Sample Performance of Individual Estimators under Experiment 1

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
CCEX-IV		$\rho = 0.5,\ h = 2$																	
	20	0.05	0.04	-0.04	0.08	-0.07	0.03	0.01	0.00	0.03	10.00	14.63	14.74	7.53	10.87	10.87	5.19	7.59	7.62
	50	0.07	0.02	-0.02	0.00	-0.04	0.06	0.00	-0.01	0.07	9.21	14.22	14.22	6.79	10.49	10.63	4.74	7.17	7.37
	100	0.03	0.04	-0.02	-0.03	-0.01	0.10	-0.04	-0.03	-0.01	8.86	13.96	14.13	6.53	10.27	10.44	4.52	7.05	7.21
		$\rho = 0.5,\ h = 6$																	
	20	-0.07	-0.05	0.08	-0.08	0.01	-0.09	-0.03	0.04	0.07	21.80	14.53	14.63	16.26	10.81	10.89	11.23	7.54	7.61
	50	-0.01	-0.04	0.08	-0.06	0.07	-0.01	0.00	0.05	-0.08	15.66	14.15	13.99	11.56	10.51	10.51	7.96	7.19	7.26
	100	-0.03	-0.02	-0.05	-0.08	-0.06	-0.03	-0.04	-0.03	-0.01	14.50	13.92	14.00	10.62	10.27	10.30	4.52	7.05	7.21
		$\rho = 0.8,\ h = 6$																	
	20	-0.05	0.10	0.05	0.01	0.07	0.08	0.05	0.00	0.08	14.86	14.79	14.86	11.18	10.96	11.02	7.43	7.59	7.62
	50	0.07	-0.09	-0.06	0.07	0.04	0.05	0.04	0.04	0.05	9.07	14.16	14.23	6.73	10.32	10.54	4.62	7.24	7.38
	100	-0.01	-0.03	0.03	-0.02	-0.01	-0.03	0.01	0.02	0.08	7.99	14.01	14.16	5.84	10.22	10.47	4.05	7.08	7.23
		$\rho = 0.8,\ h = 0.3N$																	
	20	-0.07	0.06	0.02	0.08	-0.01	-0.09	-0.07	0.03	0.03	14.91	14.71	14.82	10.73	10.95	10.89	7.54	7.63	7.65
	50	0.08	0.09	0.02	0.03	0.07	0.08	0.02	0.08	0.03	21.01	14.30	14.23	15.46	10.52	10.55	10.56	7.23	7.30
	100	-0.04	0.01	-0.05	-0.05	-0.03	-0.09	0.03	0.04	-0.09	28.43	14.00	14.06	20.99	10.31	10.30	14.07	7.03	7.11
CCE-IV		$\rho = 0.5,\ h = 2$																	
	20	-0.24	-0.14	-0.16	-0.22	-0.09	0.01	-0.17	-0.02	0.01	10.03	14.56	14.67	7.47	10.71	10.76	5.09	7.42	7.46
	50	0.04	-0.01	-0.05	-0.02	-0.03	0.03	-0.04	-0.01	0.09	9.31	14.33	14.38	6.80	10.50	10.64	4.71	7.14	7.33
	100	0.03	0.04	-0.02	-0.04	-0.02	0.12	-0.04	-0.03	0.00	9.01	14.18	14.39	6.57	10.33	10.51	4.52	7.07	7.22
		$\rho = 0.5,\ h = 6$																	
	20	-3.01	-0.21	-0.13	-2.90	-0.10	-0.38	-2.82	-0.09	-0.21	23.54	14.41	14.68	17.68	10.65	10.79	12.55	7.37	7.54
	50	-0.14	-0.06	-0.18	-0.19	0.09	-0.01	-0.14	0.06	-0.09	15.85	14.28	14.18	11.61	10.51	10.53	7.94	7.13	7.22
	100	-0.05	-0.03	-0.04	-0.13	-0.06	-0.04	-0.04	-0.03	0.00	14.74	14.14	14.21	10.68	10.37	10.32	4.52	7.07	7.22
		$\rho = 0.8,\ h = 6$																	
	20	-4.29	0.02	-0.38	-4.21	-0.13	-0.51	-4.03	-0.19	-0.33	19.24	14.52	14.77	14.61	10.76	10.85	10.90	7.44	7.63
	50	-0.13	-0.28	-0.04	-0.13	0.05	0.15	-0.15	0.04	0.04	9.23	14.32	14.41	6.79	10.34	10.56	4.65	7.23	7.37
	100	-0.04	-0.02	0.01	-0.05	-0.02	-0.04	-0.02	0.02	0.08	8.13	14.23	14.40	5.88	10.26	10.53	4.05	7.07	7.23
		$\rho = 0.8,\ h = 0.3N$																	
	20	-4.28	-0.12	-0.34	-4.07	-0.21	-0.47	-4.34	-0.17	-0.38	19.33	14.40	14.59	14.58	10.77	10.84	11.02	7.49	7.69
	50	-3.23	-0.01	-0.10	-3.15	0.01	-0.02	-3.41	0.06	-0.08	25.66	14.25	14.28	19.27	10.46	10.54	13.69	7.16	7.30
	100	-2.86	-0.04	-0.12	-3.08	-0.05	-0.16	-2.83	0.10	-0.15	34.55	14.06	14.15	25.81	10.37	10.31	17.63	7.03	7.11
Infeasible		$\rho = 0.5,\ h = 2$																	
	20	0.04	-0.06	0.00	0.02	-0.08	0.07	0.02	0.01	-0.02	8.13	13.79	14.08	6.03	10.19	10.25	4.06	6.90	7.08
	50	0.04	-0.05	-0.10	0.00	-0.05	0.03	0.03	-0.05	0.05	8.39	13.76	13.90	6.19	10.11	10.31	4.29	6.84	7.06
	100	0.04	0.06	0.03	-0.04	0.01	0.09	-0.02	-0.02	0.01	8.49	13.74	13.98	6.22	10.07	10.29	4.30	6.88	7.04
		$\rho = 0.5,\ h = 6$																	
	20	-0.08	-0.02	0.05	-0.04	0.00	-0.04	-0.05	0.03	0.05	11.42	13.75	13.87	8.46	10.13	10.19	5.81	6.85	6.94
	50	0.02	0.00	-0.01	0.04	0.08	0.00	0.03	0.01	-0.06	12.62	13.70	13.68	9.29	10.18	10.17	6.35	6.87	6.99
	100	0.02	0.01	-0.02	-0.05	-0.06	-0.01	-0.02	-0.02	0.01	13.03	13.72	13.79	9.60	10.11	10.13	4.30	6.88	7.04
		$\rho = 0.8,\ h = 6$																	
	20	-0.03	0.06	0.01	0.01	0.05	-0.05	0.04	-0.06	0.05	5.58	13.75	14.12	4.12	10.14	10.34	2.77	6.87	7.03
	50	0.07	0.04	-0.05	0.03	0.05	0.03	0.01	0.00	0.02	6.49	13.73	13.93	4.85	9.95	10.27	3.29	6.90	7.09
	100	-0.01	0.01	0.02	-0.02	0.01	0.02	0.01	0.03	0.07	6.81	13.77	13.95	5.01	10.05	10.35	3.46	6.90	7.11
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.00	0.04	0.07	0.02	-0.02	-0.08	-0.02	0.00	0.02	5.57	13.66	14.03	4.14	10.22	10.35	2.80	6.91	7.05
	50	-0.06	0.02	-0.04	0.04	0.08	0.05	-0.03	0.07	0.08	8.49	13.65	13.77	6.28	10.11	10.22	4.24	6.85	7.00
	100	-0.01	0.01	-0.03	-0.03	-0.05	-0.08	0.05	0.03	-0.06	11.87	13.62	13.75	8.72	10.10	10.12	5.88	6.85	6.92

Notes: The simulation results are based on the DGP specified in Section 4.

Table S3: The Size and Power for Individual Estimators under Experiment 1

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
CCEX-IV											$\rho = 0.5,\ h = 2$								
	20	0.079	0.090	0.091	0.073	0.077	0.073	0.065	0.073	0.066	0.526	0.261	0.269	0.720	0.437	0.453	0.941	0.729	0.739
	50	0.079	0.085	0.083	0.067	0.070	0.071	0.064	0.058	0.065	0.564	0.288	0.281	0.793	0.471	0.468	0.974	0.789	0.758
	100	0.077	0.081	0.081	0.064	0.068	0.067	0.056	0.056	0.056	0.582	0.287	0.290	0.822	0.482	0.484	0.982	0.794	0.790
											$\rho = 0.5,\ h = 6$								
	20	0.072	0.073	0.073	0.045	0.066	0.070	0.052	0.065	0.067	0.226	0.276	0.278	0.323	0.448	0.428	0.502	0.737	0.724
	50	0.066	0.082	0.078	0.057	0.074	0.070	0.057	0.061	0.062	0.286	0.281	0.287	0.437	0.467	0.485	0.725	0.799	0.785
	100	0.071	0.081	0.079	0.061	0.066	0.067	0.056	0.056	0.056	0.309	0.286	0.296	0.484	0.494	0.479	0.982	0.794	0.790
											$\rho = 0.8,\ h = 6$								
	20	0.075	0.060	0.063	0.050	0.059	0.059	0.052	0.057	0.062	0.485	0.265	0.247	0.620	0.427	0.429	0.800	0.743	0.728
	50	0.061	0.074	0.074	0.051	0.059	0.063	0.051	0.063	0.060	0.653	0.280	0.283	0.828	0.487	0.474	0.968	0.759	0.774
	100	0.068	0.075	0.074	0.054	0.063	0.066	0.049	0.055	0.058	0.709	0.294	0.291	0.886	0.481	0.468	0.990	0.790	0.780
											$\rho = 0.8,\ h = 0.3N$								
	20	0.076	0.059	0.061	0.048	0.058	0.057	0.052	0.062	0.061	0.489	0.260	0.261	0.635	0.423	0.433	0.794	0.731	0.724
	50	0.060	0.053	0.053	0.053	0.050	0.051	0.051	0.051	0.053	0.336	0.272	0.282	0.445	0.449	0.446	0.606	0.758	0.752
	100	0.064	0.045	0.047	0.047	0.044	0.044	0.051	0.047	0.044	0.248	0.269	0.273	0.327	0.460	0.459	0.482	0.791	0.780
CCE-IV											$\rho = 0.5,\ h = 2$								
	20	0.071	0.083	0.082	0.067	0.070	0.066	0.058	0.063	0.061	0.520	0.264	0.275	0.727	0.450	0.464	0.950	0.750	0.764
	50	0.076	0.081	0.079	0.064	0.068	0.068	0.058	0.058	0.061	0.556	0.288	0.277	0.792	0.464	0.470	0.976	0.788	0.762
	100	0.075	0.079	0.078	0.063	0.066	0.064	0.055	0.056	0.057	0.570	0.287	0.285	0.815	0.483	0.482	0.982	0.788	0.788
											$\rho = 0.5,\ h = 6$								
	20	0.043	0.064	0.064	0.037	0.057	0.060	0.040	0.056	0.061	0.219	0.277	0.276	0.303	0.455	0.426	0.480	0.752	0.735
	50	0.062	0.077	0.074	0.055	0.068	0.067	0.055	0.058	0.057	0.278	0.276	0.285	0.434	0.475	0.477	0.724	0.799	0.793
	100	0.068	0.079	0.077	0.061	0.064	0.064	0.055	0.056	0.057	0.305	0.283	0.286	0.480	0.497	0.478	0.982	0.788	0.788
											$\rho = 0.8,\ h = 6$								
	20	0.031	0.049	0.052	0.034	0.047	0.049	0.034	0.045	0.049	0.442	0.264	0.253	0.553	0.422	0.425	0.689	0.735	0.712
	50	0.047	0.069	0.071	0.046	0.056	0.060	0.048	0.060	0.058	0.641	0.278	0.279	0.823	0.486	0.467	0.969	0.762	0.773
	100	0.055	0.073	0.071	0.051	0.061	0.065	0.048	0.055	0.056	0.698	0.287	0.286	0.888	0.477	0.460	0.991	0.790	0.781
											$\rho = 0.8,\ h = 0.3N$								
	20	0.030	0.047	0.050	0.034	0.048	0.049	0.036	0.050	0.052	0.449	0.261	0.245	0.560	0.418	0.421	0.683	0.721	0.710
	50	0.038	0.046	0.048	0.031	0.045	0.045	0.037	0.046	0.048	0.319	0.262	0.266	0.407	0.431	0.431	0.562	0.746	0.730
	100	0.034	0.040	0.042	0.036	0.039	0.040	0.040	0.039	0.041	0.238	0.259	0.260	0.316	0.446	0.445	0.443	0.774	0.754
Infeasible											$\rho = 0.5,\ h = 2$								
	20	0.075	0.080	0.077	0.065	0.064	0.060	0.048	0.052	0.050	0.636	0.288	0.285	0.851	0.482	0.488	0.993	0.801	0.792
	50	0.074	0.077	0.074	0.058	0.062	0.061	0.053	0.050	0.054	0.613	0.298	0.297	0.851	0.484	0.480	0.988	0.815	0.792
	100	0.073	0.077	0.077	0.059	0.063	0.059	0.052	0.049	0.051	0.606	0.296	0.296	0.847	0.492	0.492	0.988	0.810	0.806
											$\rho = 0.5,\ h = 6$								
	20	0.073	0.079	0.076	0.059	0.063	0.064	0.051	0.049	0.050	0.401	0.284	0.299	0.629	0.482	0.469	0.897	0.814	0.800
	50	0.070	0.076	0.073	0.057	0.064	0.062	0.051	0.050	0.051	0.361	0.294	0.302	0.551	0.497	0.503	0.862	0.814	0.809
	100	0.071	0.077	0.076	0.058	0.066	0.062	0.052	0.049	0.051	0.351	0.294	0.298	0.547	0.503	0.493	0.988	0.810	0.806
											$\rho = 0.8,\ h = 6$								
	20	0.070	0.074	0.079	0.058	0.061	0.061	0.045	0.049	0.049	0.881	0.296	0.285	0.985	0.466	0.490	1.000	0.816	0.782
	50	0.062	0.073	0.071	0.055	0.055	0.059	0.048	0.050	0.048	0.819	0.299	0.295	0.954	0.511	0.483	0.999	0.801	0.804
	100	0.062	0.074	0.072	0.055	0.060	0.061	0.049	0.050	0.054	0.789	0.299	0.292	0.945	0.487	0.478	0.999	0.809	0.788
											$\rho = 0.8,\ h = 0.3N$								
	20	0.066	0.073	0.074	0.056	0.064	0.060	0.047	0.049	0.050	0.888	0.293	0.294	0.985	0.473	0.481	1.000	0.807	0.791
	50	0.057	0.073	0.071	0.050	0.059	0.061	0.046	0.050	0.054	0.665	0.296	0.295	0.858	0.485	0.469	0.990	0.805	0.788
	100	0.045	0.067	0.068	0.046	0.057	0.057	0.048	0.047	0.048	0.463	0.300	0.306	0.654	0.495	0.491	0.911	0.819	0.809

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}_i^a = \boldsymbol{\theta}_i + 0.2$.

Table S4: The Finite Sample Performance of Mean Group Estimators under Experiment 1

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
									$\rho = 0.5$, $h = 2$										
CCEX-IV	20	-0.07	0.05	-0.06	0.04	0.10	-0.03	-0.05	0.04	-0.09	7.61	5.21	5.05	6.16	3.87	3.86	4.13	2.70	2.66
	50	-0.07	0.04	-0.03	-0.07	-0.07	-0.03	-0.06	0.01	0.07	5.09	3.31	3.15	3.43	2.42	2.32	2.30	1.64	1.67
	100	0.07	-0.08	0.08	-0.05	-0.08	-0.03	0.09	0.04	0.01	3.26	2.27	2.16	2.35	1.72	1.70	1.57	1.13	1.11
									$\rho = 0.5$, $h = 6$										
	20	0.02	-0.09	-0.02	-0.08	0.08	0.03	-0.01	0.07	0.03	10.33	5.63	5.59	8.34	4.03	4.05	5.93	2.77	2.61
	50	0.05	0.06	-0.05	-0.02	0.06	-0.09	-0.08	-0.08	-0.07	8.66	3.29	3.25	6.22	2.37	2.32	4.03	1.67	1.64
	100	0.06	-0.05	0.04	0.06	-0.04	-0.04	-0.09	0.06	-0.07	5.51	2.24	2.24	3.74	1.61	1.72	2.72	1.11	1.17
									$\rho = 0.8$, $h = 6$										
	20	0.06	-0.06	-0.06	0.06	0.03	0.03	0.07	0.05	0.05	6.18	7.11	5.22	4.53	4.46	4.65	2.76	2.84	2.77
	50	0.05	-0.03	-0.04	0.07	-0.01	0.01	-0.02	-0.01	-0.04	5.25	3.39	3.39	3.56	2.40	2.35	2.41	1.66	1.62
	100	-0.06	0.03	0.01	-0.02	-0.05	-0.06	-0.08	0.03	0.02	3.54	2.37	2.37	2.25	1.72	1.67	1.35	1.18	1.11
									$\rho = 0.8$, $h = 0.3N$										
	20	0.02	0.10	0.03	-0.02	-0.03	-0.09	0.00	0.05	-0.04	6.14	5.09	5.31	4.47	3.84	3.67	2.60	2.67	2.76
	50	-0.05	0.01	-0.06	-0.02	-0.05	0.04	0.02	-0.08	-0.06	5.99	3.37	3.05	3.56	2.26	2.33	2.42	1.70	1.69
	100	0.06	0.05	-0.07	-0.04	-0.04	-0.01	0.00	-0.07	-0.05	5.39	2.23	2.28	3.36	1.61	1.61	2.15	1.11	1.03
									$\rho = 0.5$, $h = 2$										
CCE-IV	20	0.15	0.13	0.14	-0.18	0.09	0.00	-0.23	0.13	-0.07	8.09	5.15	5.06	6.03	3.87	3.82	4.08	2.70	2.64
	50	-0.27	0.07	0.19	-0.15	-0.07	-0.01	-0.06	0.01	0.07	4.95	3.36	3.14	3.43	2.44	2.32	2.32	1.63	1.68
	100	0.12	-0.06	0.09	-0.05	-0.09	-0.02	0.09	0.04	0.01	3.22	2.31	2.18	2.37	1.73	1.70	1.58	1.14	1.12
									$\rho = 0.5$, $h = 6$										
	20	-3.11	-0.32	-0.08	-3.00	0.08	0.05	-2.80	-0.09	-0.01	10.35	5.10	5.05	8.44	3.86	3.84	5.42	2.70	2.58
	50	-0.09	0.08	-0.06	-0.15	0.10	-0.08	-0.22	-0.08	-0.06	8.21	3.21	3.21	5.87	2.38	2.32	3.94	1.67	1.63
	100	0.03	-0.01	0.02	-0.11	-0.04	-0.04	-0.10	0.07	-0.07	5.43	2.27	2.28	3.73	1.60	1.73	2.69	1.12	1.17
									$\rho = 0.8$, $h = 6$										
	20	-4.17	-0.26	-0.09	-4.48	-0.21	0.01	-4.37	-0.07	0.06	7.15	5.05	5.26	6.61	3.76	3.79	5.51	2.67	2.64
	50	-0.22	-0.06	-0.03	-0.09	-0.02	-0.01	-0.23	-0.03	-0.05	5.33	3.33	3.26	3.45	2.37	2.32	2.33	1.63	1.60
	100	0.09	0.04	0.02	-0.10	-0.04	-0.10	-0.11	0.03	0.03	3.19	2.30	2.33	2.20	1.72	1.70	1.34	1.17	1.10
									$\rho = 0.8$, $h = 0.3N$										
	20	-4.68	-0.24	-0.38	-4.45	-0.15	-0.37	-4.30	-0.18	-0.42	8.44	4.97	5.19	6.35	3.75	3.66	5.39	2.66	2.72
	50	-3.82	-0.03	-0.15	-3.84	-0.07	-0.22	-3.92	-0.12	-0.23	6.60	3.22	3.01	5.56	2.21	2.28	4.56	1.71	1.72
	100	-3.43	0.20	-0.13	-3.50	-0.14	-0.09	-3.60	-0.11	-0.04	6.27	2.21	2.24	5.38	1.61	1.60	4.20	1.11	1.03
									$\rho = 0.5$, $h = 2$										
Infeasible	20	-0.01	0.04	0.08	-0.08	0.09	-0.06	-0.04	-0.02	-0.09	6.49	5.15	4.87	4.87	3.75	3.56	3.24	2.63	2.48
	50	-0.06	0.00	-0.06	-0.09	-0.06	0.00	-0.01	0.03	0.05	4.54	3.23	3.08	3.19	2.36	2.20	2.07	1.63	1.60
	100	0.11	-0.11	0.09	-0.08	-0.05	-0.06	0.09	0.02	0.01	3.17	2.23	2.16	2.27	1.71	1.64	1.56	1.12	1.11
									$\rho = 0.5$, $h = 6$										
	20	-0.04	-0.09	0.05	0.07	0.04	0.07	-0.01	0.07	-0.02	9.64	5.02	4.79	6.97	3.70	3.59	4.70	2.58	2.44
	50	0.03	0.01	-0.08	-0.04	0.03	0.09	-0.04	-0.03	-0.02	6.68	3.12	3.12	4.88	2.37	2.24	3.15	1.68	1.60
	100	0.05	0.02	-0.07	0.05	0.05	-0.06	-0.03	0.05	0.03	5.18	2.07	2.13	3.43	1.53	1.52	2.58	1.12	1.09
									$\rho = 0.8$, $h = 6$										
	20	0.06	-0.05	-0.10	0.01	0.09	-0.01	0.03	0.05	0.10	4.97	5.05	5.05	3.23	3.65	3.68	2.13	2.60	2.51
	50	0.03	-0.07	-0.02	0.02	0.00	0.00	-0.04	-0.02	-0.01	3.90	3.34	3.16	2.45	2.35	2.29	1.72	1.63	1.52
	100	0.04	-0.01	0.01	0.10	-0.09	-0.05	-0.06	0.02	0.03	2.79	2.32	2.29	1.90	1.72	1.63	1.22	1.16	1.05
									$\rho = 0.8$, $h = 0.3N$										
	20	-0.02	0.06	0.06	-0.01	-0.04	-0.08	-0.02	0.03	-0.07	4.92	4.65	4.87	3.35	3.56	3.50	1.99	2.54	2.63
	50	-0.02	-0.01	-0.05	-0.07	0.02	-0.04	0.03	-0.05	-0.08	3.96	3.21	2.98	2.73	2.16	2.18	1.94	1.68	1.72
	100	0.05	-0.01	-0.05	-0.03	-0.01	0.01	0.03	-0.05	-0.05	2.90	2.20	2.13	2.35	1.57	1.59	1.37	1.08	1.01

Notes: The simulation results are based on the DGP specified in Section 4.

Table S5: The Size and Power for Mean Group Estimators under Experiment 1

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
CCEX-IV	$\rho = 0.5,\ h = 2$																		
	20	0.062	0.064	0.065	0.068	0.067	0.066	0.054	0.067	0.061	0.960	0.828	0.816	1.000	0.974	0.986	1.000	1.000	1.000
	50	0.065	0.065	0.056	0.059	0.047	0.061	0.054	0.066	0.074	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.043	0.059	0.041	0.060	0.066	0.060	0.058	0.060	0.054	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.5,\ h = 6$																		
	20	0.064	0.072	0.057	0.057	0.073	0.052	0.064	0.074	0.061	0.582	0.830	0.838	0.818	0.958	0.966	0.989	1.000	1.000
	50	0.057	0.068	0.058	0.059	0.064	0.054	0.054	0.054	0.050	0.985	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.052	0.049	0.056	0.048	0.042	0.074	0.056	0.046	0.056	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.8,\ h = 6$																		
	20	0.080	0.054	0.061	0.069	0.069	0.066	0.050	0.071	0.058	0.598	0.809	0.821	0.932	0.968	0.973	0.997	1.000	1.000
	50	0.069	0.048	0.056	0.048	0.052	0.055	0.068	0.054	0.052	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.063	0.048	0.066	0.052	0.056	0.056	0.048	0.060	0.046	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.8,\ h = 0.3N$																		
	20	0.063	0.061	0.074	0.068	0.056	0.062	0.056	0.077	0.061	0.775	0.801	0.813	0.918	0.960	0.968	0.995	1.000	1.000
	50	0.056	0.066	0.050	0.048	0.044	0.062	0.060	0.055	0.060	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.052	0.060	0.062	0.046	0.044	0.050	0.055	0.050	0.040	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
CCE-IV	$\rho = 0.5,\ h = 2$																		
	20	0.062	0.070	0.064	0.072	0.061	0.076	0.069	0.070	0.083	0.958	0.808	0.818	0.970	0.988	1.000	1.000	1.000	1.000
	50	0.061	0.058	0.054	0.057	0.052	0.056	0.050	0.058	0.082	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.041	0.065	0.039	0.060	0.076	0.058	0.060	0.062	0.052	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.5,\ h = 6$																		
	20	0.068	0.070	0.076	0.064	0.078	0.073	0.075	0.068	0.072	0.556	0.824	0.788	0.688	0.956	0.954	0.965	1.000	1.000
	50	0.059	0.066	0.058	0.060	0.066	0.053	0.056	0.066	0.054	0.970	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.052	0.053	0.054	0.052	0.042	0.074	0.046	0.050	0.054	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.8,\ h = 6$																		
	20	0.087	0.063	0.066	0.085	0.065	0.073	0.095	0.072	0.066	0.604	0.734	0.785	0.848	0.970	0.972	0.970	1.000	1.000
	50	0.069	0.062	0.046	0.063	0.053	0.058	0.060	0.052	0.052	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.060	0.042	0.060	0.050	0.066	0.060	0.032	0.062	0.048	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.8,\ h = 0.3N$																		
	20	0.080	0.065	0.080	0.092	0.070	0.064	0.097	0.073	0.066	0.610	0.764	0.741	0.856	0.964	0.940	0.985	1.000	1.000
	50	0.068	0.070	0.052	0.102	0.046	0.056	0.105	0.069	0.072	0.875	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.054	0.060	0.050	0.102	0.044	0.064	0.097	0.062	0.059	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Infeasible	$\rho = 0.5,\ h = 2$																		
	20	0.057	0.074	0.057	0.061	0.065	0.067	0.052	0.065	0.067	0.998	0.876	0.871	1.000	0.999	1.000	1.000	1.000	1.000
	50	0.056	0.062	0.056	0.056	0.054	0.039	0.062	0.070	0.066	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.046	0.050	0.046	0.056	0.058	0.056	0.064	0.058	0.058	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.5,\ h = 6$																		
	20	0.068	0.062	0.055	0.082	0.070	0.059	0.067	0.072	0.069	0.946	0.838	0.857	0.999	1.000	1.000	1.000	1.000	1.000
	50	0.067	0.055	0.049	0.052	0.070	0.050	0.052	0.074	0.048	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.042	0.049	0.052	0.048	0.054	0.072	0.054	0.044	0.056	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.8,\ h = 6$																		
	20	0.071	0.059	0.067	0.059	0.066	0.071	0.051	0.065	0.067	0.996	0.856	0.843	1.000	1.000	1.000	1.000	1.000	1.000
	50	0.071	0.052	0.048	0.056	0.050	0.042	0.058	0.048	0.046	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.062	0.054	0.057	0.050	0.060	0.056	0.048	0.050	0.052	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\rho = 0.8,\ h = 0.3N$																		
	20	0.062	0.051	0.051	0.060	0.050	0.056	0.057	0.058	0.059	1.000	0.875	0.865	1.000	1.000	1,000	1.000	1.000	1.000
	50	0.052	0.056	0.054	0.042	0.044	0.058	0.050	0.065	0.060	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.048	0.054	0.052	0.048	0.044	0.054	0.050	0.055	0.050	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

Table S6: The Finite Sample Performance of Pooled Estimators under Experiment 1

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
									$\rho = 0.5,\ h = 2$										
CCEX-IV	20	-0.02	-0.01	0.03	0.04	0.01	-0.04	-0.03	0.09	-0.08	6.97	4.70	4.51	5.51	3.62	3.67	3.80	2.62	2.59
	50	-0.04	-0.01	0.08	-0.08	-0.06	-0.03	-0.04	0.00	0.06	4.13	3.02	2.85	3.23	2.31	2.20	2.19	1.58	1.63
	100	0.09	-0.05	0.07	-0.07	-0.09	-0.03	0.06	0.02	0.00	2.76	2.06	2.00	2.16	1.58	1.64	1.55	1.09	1.11
									$\rho = 0.5,\ h = 6$										
	20	0.01	-0.06	-0.10	-0.02	0.01	0.05	0.07	0.03	-0.02	14.88	4.75	4.62	10.71	3.72	3.64	7.54	2.60	2.53
	50	-0.02	-0.07	-0.08	-0.01	0.09	-0.03	-0.06	-0.09	-0.06	6.99	2.88	2.83	5.48	2.26	2.17	3.73	1.62	1.60
	100	-0.01	0.03	-0.01	0.02	-0.07	-0.02	-0.06	0.04	-0.04	4.57	2.00	2.09	3.38	1.59	1.64	2.57	1.08	1.13
									$\rho = 0.8,\ h = 6$										
	20	0.06	-0.09	-0.09	-0.06	-0.05	0.02	0.02	0.04	0.07	10.52	4.54	4.69	7.44	3.57	3.66	5.28	2.65	2.61
	50	0.07	-0.03	-0.05	-0.02	-0.07	0.02	-0.05	-0.04	-0.02	4.28	2.94	2.91	3.08	2.19	2.18	2.13	1.59	1.55
	100	0.00	-0.01	-0.04	0.01	-0.07	-0.06	-0.05	0.01	0.03	2.59	1.99	2.06	2.05	1.65	1.60	1.31	1.13	1.06
									$\rho = 0.8,\ h = 0.3N$										
	20	0.03	-0.06	0.05	-0.08	-0.07	0.01	-0.02	0.02	0.00	10.43	4.53	4.64	7.57	3.64	3.51	5.33	2.64	2.68
	50	-0.06	0.02	-0.04	-0.08	0.00	-0.05	0.03	-0.06	-0.08	9.89	2.94	2.83	5.96	2.12	2.16	3.19	1.67	1.64
	100	-0.05	0.08	-0.09	0.06	-0.10	-0.01	-0.08	-0.06	0.01	9.12	2.04	1.99	3.88	1.51	1.54	2.04	1.12	1.04
									$\rho = 0.5,\ h = 2$										
CCE-IV	20	-0.18	-0.06	0.04	-0.17	-0.01	-0.05	-0.24	0.07	-0.11	7.14	4.73	4.61	5.54	3.67	3.68	3.81	2.63	2.60
	50	-0.12	-0.04	0.05	-0.14	-0.06	-0.02	-0.03	0.00	0.06	4.18	3.10	2.92	3.22	2.33	2.20	2.20	1.57	1.65
	100	0.11	-0.03	0.06	-0.06	-0.10	-0.04	0.05	0.02	0.00	2.82	2.09	2.03	2.18	1.59	1.66	1.55	1.09	1.12
									$\rho = 0.5,\ h = 6$										
	20	-1.46	-0.13	-0.14	-1.27	-0.05	0.06	-1.21	-0.06	-0.03	15.57	4.82	4.64	10.97	3.73	3.67	7.80	2.60	2.55
	50	-0.14	-0.09	-0.09	-0.09	0.08	-0.03	-0.24	-0.09	-0.06	7.22	2.93	2.82	5.45	2.27	2.21	3.70	1.62	1.60
	100	-0.03	0.02	-0.04	-0.08	-0.08	-0.03	-0.08	0.05	-0.04	4.67	2.04	2.12	3.42	1.60	1.65	2.57	1.09	1.13
									$\rho = 0.8,\ h = 6$										
	20	-1.60	-0.23	-0.09	-1.57	-0.15	0.01	-1.70	-0.06	0.10	11.28	4.58	4.75	8.08	3.59	3.67	5.93	2.66	2.61
	50	-0.13	-0.05	-0.06	0.11	-0.09	0.03	-0.17	-0.06	-0.04	4.40	2.97	2.92	3.15	2.21	2.18	2.16	1.59	1.54
	100	0.03	-0.02	-0.04	-0.08	-0.07	-0.08	-0.07	0.01	0.03	2.62	2.02	2.09	2.05	1.66	1.63	1.31	1.13	1.07
									$\rho = 0.8,\ h = 0.3N$										
	20	-1.81	-0.22	-0.32	-1.43	-0.36	-0.17	-1.84	-0.08	-0.22	10.29	4.64	4.78	7.59	3.68	3.52	5.95	2.64	2.70
	50	-1.82	0.07	-0.07	-1.44	-0.11	-0.17	-1.47	-0.08	-0.21	9.92	2.95	2.86	5.92	2.15	2.17	3.65	1.68	1.66
	100	-1.18	0.05	-0.12	-1.24	-0.08	-0.06	-1.10	-0.08	-0.02	9.45	2.07	2.06	3.90	1.53	1.54	2.47	1.12	1.05
									$\rho = 0.5,\ h = 2$										
Infeasible	20	0.08	-0.05	0.02	0.00	0.02	-0.05	-0.01	0.07	-0.13	5.66	4.69	4.43	4.55	3.55	3.50	3.04	2.55	2.41
	50	-0.04	-0.04	0.07	-0.07	-0.07	-0.04	0.03	0.03	0.06	3.83	2.96	2.81	3.00	2.27	2.08	1.99	1.56	1.56
	100	0.06	-0.06	0.06	-0.10	-0.06	-0.06	0.06	0.02	0.01	2.65	2.05	1.99	2.08	1.57	1.59	1.54	1.08	1.10
									$\rho = 0.5,\ h = 6$										
	20	0.00	-0.08	0.06	0.06	-0.06	0.07	0.08	0.04	-0.04	9.10	4.60	4.39	6.37	3.55	3.42	4.52	2.54	2.36
	50	0.03	-0.07	-0.13	-0.01	0.02	-0.08	-0.01	-0.06	0.01	5.75	2.84	2.81	4.51	2.24	2.10	2.98	1.62	1.58
	100	0.03	0.03	-0.04	0.07	-0.05	-0.01	-0.04	0.03	-0.02	4.15	2.01	2.06	3.05	1.58	1.61	2.31	1.06	1.10
									$\rho = 0.8,\ h = 6$										
	20	-0.01	-0.10	-0.07	-0.03	0.01	-0.09	0.07	0.03	0.07	4.36	4.45	4.54	3.12	3.51	3.50	2.10	2.52	2.46
	50	0.04	-0.06	-0.07	0.05	-0.06	0.01	-0.04	-0.06	0.00	3.05	2.92	2.84	2.26	2.19	2.15	1.60	1.59	1.47
	100	0.00	-0.03	-0.02	0.05	-0.09	-0.07	-0.04	0.00	0.04	2.25	1.99	2.03	1.74	1.65	1.58	1.14	1.11	1.05
									$\rho = 0.8,\ h = 0.3N$										
	20	0.07	-0.08	0.06	-0.03	-0.04	-0.03	-0.03	0.06	-0.07	4.85	4.37	4.52	3.49	3.41	3.38	1.99	2.53	2.59
	50	-0.06	-0.01	0.05	-0.02	0.02	0.04	0.02	-0.09	0.04	3.87	2.94	2.76	2.26	2.06	2.11	1.64	1.70	1.66
	100	-0.02	0.04	-0.02	-0.04	-0.09	0.01	0.07	-0.03	0.03	2.81	2.07	1.99	1.35	1.50	1.53	1.05	1.10	1.03

Notes: The simulation results are based on the DGP specified in Section 4.

Table S7: The Size and Power for Pooled Estimators under Experiment 1

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		$\rho = 0.5,\ h = 2$																	
CCEX-IV	20	0.068	0.075	0.077	0.079	0.072	0.074	0.065	0.054	0.055	0.986	0.892	0.901	1.000	0.993	0.995	1.000	1.000	1.000
	50	0.062	0.063	0.060	0.071	0.059	0.062	0.052	0.058	0.066	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.046	0.050	0.041	0.050	0.058	0.054	0.054	0.056	0.056	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.5,\ h = 6$																	
	20	0.073	0.074	0.076	0.068	0.072	0.074	0.061	0.058	0.058	0.682	0.916	0.899	0.813	0.975	0.975	1.000	1.000	1.000
	50	0.060	0.065	0.057	0.071	0.067	0.045	0.050	0.060	0.052	0.975	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.051	0.053	0.062	0.052	0.052	0.072	0.050	0.056	0.054	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.8,\ h = 6$																	
	20	0.076	0.067	0.072	0.062	0.080	0.081	0.065	0.060	0.066	0.890	0.872	0.902	0.973	0.955	0.983	1.000	1.000	1.000
	50	0.073	0.060	0.064	0.053	0.049	0.056	0.062	0.056	0.052	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.064	0.041	0.056	0.072	0.064	0.072	0.052	0.068	0.046	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.067	0.067	0.074	0.076	0.078	0.064	0.067	0.058	0.062	0.875	0.875	0.885	0.981	0.962	0.975	1.000	1.000	1.000
	50	0.058	0.066	0.068	0.052	0.058	0.074	0.065	0.060	0.070	0.913	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.054	0.058	0.060	0.036	0.052	0.050	0.050	0.065	0.050	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.5,\ h = 2$																	
CCE-IV	20	0.089	0.087	0.086	0.085	0.088	0.078	0.077	0.078	0.092	0.980	0.863	0.887	1.000	0.991	0.985	1.000	1.000	1.000
	50	0.078	0.065	0.070	0.072	0.060	0.062	0.062	0.072	0.082	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.052	0.054	0.045	0.058	0.064	0.058	0.078	0.048	0.050	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.5,\ h = 6$																	
	20	0.082	0.092	0.081	0.073	0.082	0.082	0.083	0.071	0.083	0.616	0.852	0.893	0.765	0.955	0.960	1.000	1.000	1.000
	50	0.062	0.060	0.056	0.063	0.067	0.054	0.050	0.074	0.052	0.965	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.052	0.051	0.060	0.052	0.050	0.062	0.056	0.052	0.058	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.8,\ h = 6$																	
	20	0.087	0.065	0.078	0.070	0.089	0.098	0.084	0.087	0.086	0.862	0.843	0.640	0.935	0.940	0.968	1.000	1.000	1.000
	50	0.080	0.064	0.059	0.065	0.056	0.056	0.056	0.052	0.048	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.066	0.048	0.057	0.074	0.080	0.074	0.056	0.064	0.050	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.082	0.079	0.082	0.086	0.092	0.076	0.103	0.087	0.089	0.831	0.832	0.861	0.941	0.953	0.969	1.000	1.000	1.000
	50	0.070	0.070	0.070	0.074	0.064	0.068	0.115	0.075	0.060	0.887	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.060	0.052	0.066	0.076	0.056	0.054	0.065	0.050	0.050	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.5,\ h = 2$																	
Infeasible	20	0.065	0.083	0.080	0.076	0.072	0.074	0.056	0.053	0.050	0.998	0.914	0.921	1.000	1.000	1.000	1.000	1.000	1.000
	50	0.063	0.061	0.054	0.061	0.055	0.044	0.050	0.058	0.048	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.047	0.051	0.046	0.048	0.056	0.054	0.050	0.048	0.046	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.5,\ h = 6$																	
	20	0.074	0.073	0.065	0.079	0.065	0.065	0.060	0.060	0.065	0.974	0.908	0.886	1.000	0.998	1.000	1.000	1.000	1.000
	50	0.066	0.059	0.058	0.055	0.060	0.051	0.048	0.062	0.058	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.052	0.048	0.053	0.050	0.054	0.062	0.056	0.046	0.054	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.8,\ h = 6$																	
	20	0.078	0.071	0.077	0.061	0.076	0.086	0.060	0.061	0.051	0.993	0.908	0.894	1.000	1.000	1.000	1.000	1.000	1.000
	50	0.062	0.056	0.064	0.055	0.047	0.054	0.056	0.056	0.046	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.065	0.047	0.059	0.064	0.054	0.060	0.046	0.052	0.048	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.066	0.065	0.073	0.086	0.064	0.068	0.067	0.056	0.060	0.999	0.903	0.931	1.000	1.000	1.000	1.000	1.000	1.000
	50	0.060	0.070	0.054	0.046	0.044	0.064	0.060	0.054	0.065	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.044	0.060	0.054	0.050	0.050	0.054	0.060	0.065	0.055	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

S4.2 Simulation Results under Experiment 2

In this section, we provide the simulation results under Experiment 2 in Tables S8 to S13. The results for the individual and Mean Group estimators are qualitatively similar to those reported for Experiment 4 in the main text as well as those reported for Experiment 1 in Section S4.1.

We now focus on analysing the performance of the Pooled estimator under parameter heterogeneity. From Table S12 we find that the Pooled estimators for (ρ, β_1, β_2) all exhibit large biases, which do not disappear even as N or T rises. This verifies Theorem 3 that the Pooled estimator is inconsistent if the parameters are heterogeneous.

As analysed in the proof of Theorem 3, the bias is caused by the interplay between ξ_{ρ_i} and $\tilde{y}^*_{it}$ because the latter contains ρ_i and thus ξ_{ρ_i}. Therefore, we expect that the bias will increase with the variance of ξ_{ρ_i}. To show this, we provide the bias results of the CCEX-IV Pooled estimator under the different values of $var(\xi_{\rho_i})$ by generating ξ_{ρ_i} from Uniform distributions with different upper/lower bounds, and $\rho = 0.5$ and $h = 2$ in Table S22 in Section S4.4 under Experiment 4. It shows that the bias of the Pooled estimator rises with $var(\xi_{\rho_i})$ in particular for the estimation of ρ.

As a result of the biased estimation, the Pooled estimators suffer from severe size distortions which do not disappear even as the sample size increases.

Table S8: The Finite Sample Performance of Individual Estimators under Experiment 2

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
CCEX-IV		$\rho = 0.5,\ h = 2$																	
	20	0.02	-0.06	0.06	0.00	-0.03	-0.02	-0.07	-0.01	0.00	10.04	14.61	14.63	7.51	10.95	10.93	5.20	7.69	7.68
	50	-0.02	0.01	-0.06	0.01	-0.03	0.06	0.01	0.05	-0.04	9.19	14.16	14.44	6.76	10.57	10.77	4.56	7.31	7.29
	100	-0.04	0.01	-0.06	0.01	-0.08	-0.03	-0.02	0.03	0.06	8.60	14.10	14.43	6.68	10.41	10.44	4.36	6.98	7.26
		$\rho = 0.5,\ h = 6$																	
	20	-0.07	0.08	-0.10	0.03	-0.06	-0.04	0.05	0.07	-0.03	20.70	14.49	14.61	16.52	11.00	10.83	11.62	7.53	7.64
	50	-0.07	-0.02	-0.07	-0.06	-0.05	-0.01	0.05	-0.08	0.01	15.30	14.23	14.14	11.68	10.48	10.56	8.06	7.14	7.18
	100	0.01	0.03	-0.02	0.01	-0.05	0.04	0.01	0.01	0.03	14.50	13.99	13.93	10.41	10.35	10.25	7.42	7.02	7.04
		$\rho = 0.8,\ h = 6$																	
	20	0.07	0.02	0.10	0.02	-0.02	-0.02	-0.03	0.02	-0.07	17.94	15.02	14.91	11.35	10.96	10.98	7.80	7.76	7.69
	50	0.03	-0.01	-0.05	-0.02	0.02	-0.09	0.01	-0.01	-0.07	9.29	14.26	14.36	6.24	10.42	10.61	4.41	7.31	7.43
	100	0.02	0.01	-0.08	-0.01	0.04	-0.03	0.01	-0.03	-0.04	7.79	14.00	14.21	6.01	10.25	10.61	3.79	7.14	7.21
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.06	-0.06	-0.09	-0.09	-0.01	-0.02	-0.02	0.08	0.06	15.57	14.98	14.80	10.88	11.07	11.03	6.92	7.68	7.63
	50	0.07	-0.05	-0.05	-0.08	0.04	0.04	-0.01	0.04	-0.08	21.55	14.41	14.44	15.89	10.60	10.60	10.62	7.31	7.29
	100	0.08	0.06	0.03	-0.07	0.03	-0.04	0.11	0.06	0.01	25.48	14.07	14.18	19.59	10.44	10.33	13.95	7.12	7.04
CCE-IV		$\rho = 0.5,\ h = 2$																	
	20	-0.21	-0.21	0.04	-0.29	-0.04	-0.03	-0.23	-0.09	-0.03	10.37	14.73	14.61	7.57	10.97	10.90	5.21	7.92	7.65
	50	-0.04	-0.01	-0.06	-0.06	0.00	0.06	0.05	0.04	-0.02	9.30	14.17	14.50	6.87	10.57	10.78	4.49	7.33	7.28
	100	-0.02	-0.02	-0.05	-0.02	-0.06	-0.03	-0.04	0.05	0.07	8.82	14.04	14.42	6.77	10.35	10.48	4.32	7.07	7.25
		$\rho = 0.5,\ h = 6$																	
	20	-1.88	0.09	-0.28	-2.07	-0.27	-0.23	-1.98	-0.03	-0.20	22.89	14.54	14.44	16.87	11.12	10.83	12.46	7.51	7.48
	50	-0.19	-0.02	-0.07	-0.16	-0.03	-0.01	-0.14	-0.08	-0.02	15.43	14.24	14.37	11.87	10.46	10.49	8.07	7.17	7.22
	100	-0.01	-0.02	-0.03	-0.06	-0.05	0.04	-0.04	0.00	0.05	14.97	13.94	13.97	10.60	10.26	10.28	7.44	7.11	7.06
		$\rho = 0.8,\ h = 6$																	
	20	-3.02	-0.12	-0.27	-2.95	-0.14	-0.35	-3.15	-0.05	-0.33	19.32	14.92	14.99	13.85	10.91	10.83	10.44	7.69	7.62
	50	-0.21	0.05	-0.06	-0.18	-0.01	-0.12	-0.21	-0.03	-0.06	9.63	14.29	14.28	6.35	10.53	10.58	4.58	7.34	7.36
	100	0.06	0.00	-0.12	0.05	0.03	-0.07	-0.03	-0.03	-0.03	8.11	14.19	14.13	6.04	10.48	10.27	3.75	7.10	7.22
		$\rho = 0.8,\ h = 0.3N$																	
	20	-2.85	-0.25	-0.17	-3.07	-0.07	-0.32	-3.10	0.04	-0.11	17.51	14.76	14.68	14.06	10.96	11.05	8.92	7.63	7.69
	50	-2.73	-0.05	-0.21	-2.75	-0.04	-0.03	-2.65	0.08	-0.17	24.84	14.41	14.51	20.18	10.60	10.54	14.57	7.41	7.33
	100	-2.49	0.01	0.04	-2.42	-0.01	-0.08	-2.45	0.03	-0.05	29.53	14.08	14.27	23.16	10.42	10.35	16.86	7.02	7.02
Infeasible		$\rho = 0.5,\ h = 2$																	
	20	-0.01	-0.03	0.19	-0.04	0.01	-0.05	-0.03	-0.02	0.01	8.27	13.92	13.94	5.91	10.30	10.29	3.99	6.98	6.92
	50	0.01	0.01	-0.06	-0.02	0.01	0.09	0.03	0.02	-0.03	8.39	13.81	13.99	6.31	10.26	10.42	4.11	6.99	7.01
	100	-0.02	0.01	-0.06	0.00	-0.09	-0.02	0.00	0.03	0.08	8.29	13.81	14.05	6.32	10.10	10.38	4.18	6.88	6.94
		$\rho = 0.5,\ h = 6$																	
	20	-0.05	0.12	-0.07	0.08	-0.03	-0.01	0.02	0.04	-0.04	11.09	13.72	13.60	8.91	10.14	10.18	5.83	6.85	6.92
	50	0.01	0.02	-0.06	-0.01	-0.08	0.02	0.05	-0.06	0.00	12.17	13.68	13.81	9.47	10.14	10.25	6.35	6.81	6.91
	100	0.01	0.01	-0.04	0.01	-0.01	0.03	0.03	0.01	0.03	13.11	13.83	13.84	9.28	10.07	10.04	6.67	6.90	7.03
		$\rho = 0.8,\ h = 6$																	
	20	0.04	0.06	-0.02	0.01	0.01	-0.03	0.02	0.07	-0.02	5.60	13.71	14.05	4.32	10.12	10.37	2.88	6.88	7.09
	50	0.01	0.04	-0.03	0.00	0.00	-0.06	-0.02	-0.04	-0.01	6.62	13.81	14.04	4.39	10.12	10.23	3.09	6.84	7.14
	100	0.02	0.02	-0.06	0.03	0.03	-0.04	0.01	-0.01	-0.06	6.73	13.80	14.01	5.08	10.20	10.39	3.21	6.84	7.12
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.03	0.00	0.07	-0.01	-0.08	0.03	0.01	0.06	0.07	5.76	13.93	14.09	4.02	10.16	10.42	2.57	6.91	7.17
	50	-0.04	0.02	-0.05	-0.06	0.04	0.08	-0.01	0.04	-0.05	9.35	13.68	13.86	6.66	10.13	10.25	4.34	6.86	6.89
	100	0.03	0.04	0.08	-0.03	0.02	-0.06	0.06	0.06	-0.03	11.68	13.56	13.81	8.22	10.08	10.17	5.88	6.81	6.95

Notes: The simulation results are based on the DGP specified in Section 4.

Table S9: The Size and Power for Individual Estimators under Experiment 2

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
		$\rho = 0.5,\ h = 2$																	
CCEX-IV	20	0.078	0.072	0.074	0.073	0.067	0.075	0.063	0.070	0.042	0.501	0.260	0.280	0.729	0.424	0.436	0.935	0.735	0.746
	50	0.080	0.075	0.077	0.068	0.069	0.058	0.050	0.055	0.062	0.567	0.302	0.270	0.757	0.482	0.467	0.983	0.777	0.784
	100	0.072	0.076	0.079	0.053	0.062	0.058	0.064	0.061	0.054	0.610	0.284	0.275	0.809	0.491	0.467	0.975	0.805	0.791
		$\rho = 0.5,\ h = 6$																	
	20	0.078	0.077	0.070	0.044	0.074	0.075	0.046	0.063	0.055	0.251	0.270	0.276	0.328	0.430	0.419	0.501	0.723	0.744
	50	0.069	0.074	0.078	0.063	0.071	0.057	0.067	0.062	0.058	0.320	0.264	0.282	0.432	0.473	0.457	0.708	0.760	0.749
	100	0.065	0.064	0.067	0.051	0.072	0.064	0.047	0.051	0.048	0.312	0.268	0.274	0.483	0.460	0.499	0.762	0.790	0.794
		$\rho = 0.8,\ h = 6$																	
	20	0.065	0.051	0.053	0.055	0.061	0.054	0.056	0.053	0.063	0.492	0.277	0.242	0.593	0.434	0.423	0.768	0.732	0.705
	50	0.055	0.070	0.076	0.050	0.055	0.054	0.052	0.058	0.060	0.651	0.301	0.267	0.852	0.453	0.472	0.963	0.768	0.755
	100	0.069	0.052	0.064	0.046	0.056	0.057	0.059	0.055	0.066	0.712	0.284	0.300	0.885	0.466	0.457	1.000	0.779	0.795
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.065	0.050	0.056	0.044	0.061	0.056	0.052	0.049	0.060	0.497	0.262	0.262	0.634	0.403	0.427	0.831	0.711	0.730
	50	0.065	0.053	0.061	0.047	0.047	0.061	0.045	0.050	0.050	0.321	0.267	0.262	0.393	0.466	0.451	0.607	0.737	0.750
	100	0.044	0.048	0.050	0.042	0.049	0.048	0.055	0.044	0.043	0.234	0.256	0.285	0.358	0.475	0.482	0.491	0.798	0.773
		$\rho = 0.5,\ h = 2$																	
CCE-IV	20	0.080	0.078	0.075	0.074	0.071	0.074	0.058	0.075	0.060	0.509	0.280	0.273	0.729	0.446	0.445	0.939	0.749	0.743
	50	0.085	0.078	0.078	0.072	0.076	0.064	0.053	0.051	0.057	0.572	0.279	0.291	0.780	0.448	0.461	0.976	0.770	0.749
	100	0.069	0.076	0.075	0.053	0.060	0.059	0.061	0.055	0.045	0.607	0.270	0.292	0.795	0.461	0.449	0.971	0.791	0.760
		$\rho = 0.5,\ h = 6$																	
	20	0.043	0.076	0.075	0.044	0.058	0.059	0.053	0.075	0.062	0.223	0.280	0.253	0.325	0.425	0.425	0.489	0.725	0.747
	50	0.055	0.073	0.072	0.048	0.070	0.057	0.052	0.065	0.053	0.311	0.265	0.283	0.436	0.482	0.467	0.715	0.756	0.783
	100	0.064	0.068	0.067	0.053	0.061	0.054	0.055	0.056	0.061	0.299	0.289	0.267	0.471	0.453	0.463	0.767	0.774	0.762
		$\rho = 0.8,\ h = 6$																	
	20	0.032	0.040	0.061	0.035	0.046	0.046	0.036	0.053	0.058	0.430	0.267	0.234	0.534	0.425	0.418	0.702	0.708	0.707
	50	0.051	0.075	0.081	0.047	0.072	0.052	0.058	0.065	0.069	0.596	0.274	0.260	0.858	0.440	0.443	0.956	0.750	0.769
	100	0.045	0.065	0.080	0.046	0.063	0.063	0.056	0.059	0.049	0.684	0.287	0.262	0.875	0.466	0.471	0.978	0.798	0.788
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.031	0.061	0.053	0.029	0.058	0.043	0.032	0.056	0.058	0.475	0.273	0.272	0.567	0.416	0.400	0.755	0.695	0.704
	50	0.038	0.045	0.047	0.033	0.042	0.047	0.036	0.051	0.054	0.286	0.235	0.260	0.381	0.434	0.438	0.567	0.744	0.772
	100	0.034	0.045	0.038	0.038	0.047	0.048	0.041	0.042	0.036	0.221	0.270	0.268	0.333	0.477	0.474	0.434	0.777	0.771
		$\rho = 0.5,\ h = 2$																	
Infeasible	20	0.073	0.079	0.076	0.057	0.058	0.068	0.044	0.050	0.042	0.642	0.313	0.287	0.860	0.469	0.463	1.000	0.829	0.789
	50	0.064	0.074	0.073	0.065	0.063	0.068	0.057	0.049	0.045	0.633	0.311	0.266	0.820	0.496	0.466	0.971	0.795	0.808
	100	0.078	0.079	0.074	0.064	0.055	0.070	0.055	0.060	0.042	0.633	0.286	0.289	0.846	0.512	0.459	0.978	0.825	0.795
		$\rho = 0.5,\ h = 6$																	
	20	0.069	0.079	0.077	0.058	0.051	0.059	0.051	0.058	0.049	0.411	0.287	0.285	0.601	0.490	0.483	0.893	0.801	0.819
	50	0.067	0.078	0.071	0.053	0.055	0.061	0.037	0.049	0.045	0.388	0.279	0.288	0.527	0.480	0.504	0.850	0.807	0.796
	100	0.069	0.085	0.071	0.067	0.057	0.059	0.053	0.048	0.053	0.354	0.284	0.277	0.552	0.473	0.487	0.826	0.803	0.792
		$\rho = 0.8,\ h = 6$																	
	20	0.063	0.072	0.071	0.065	0.054	0.050	0.048	0.042	0.045	0.878	0.298	0.293	0.983	0.466	0.498	1.000	0.810	0.787
	50	0.056	0.067	0.082	0.043	0.053	0.055	0.045	0.054	0.058	0.786	0.289	0.294	0.990	0.464	0.456	0.986	0.822	0.777
	100	0.068	0.068	0.078	0.060	0.068	0.058	0.049	0.040	0.053	0.782	0.278	0.272	0.918	0.479	0.466	0.997	0.796	0.811
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.073	0.069	0.080	0.063	0.065	0.052	0.044	0.053	0.047	0.882	0.311	0.315	0.988	0.475	0.441	0.991	0.809	0.767
	50	0.054	0.064	0.072	0.047	0.056	0.053	0.047	0.051	0.050	0.599	0.292	0.277	0.798	0.483	0.464	1.000	0.814	0.832
	100	0.053	0.076	0.064	0.045	0.063	0.063	0.044	0.056	0.052	0.472	0.294	0.311	0.676	0.478	0.507	0.906	0.807	0.817

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}_i^a = \boldsymbol{\theta}_i + 0.2$.

Table S10: The Finite Sample Performance of Mean Group Estimators under Experiment 2

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		$\rho = 0.5$, $h = 2$																	
CCEX-IV	20	-0.04	0.04	0.05	0.00	-0.08	-0.08	0.07	-0.09	0.01	4.03	12.55	8.21	3.14	11.71	7.83	2.33	11.44	6.92
	50	-0.01	0.07	0.08	-0.08	0.01	0.04	0.02	0.02	0.01	2.43	7.56	5.04	1.96	7.36	4.56	1.54	7.35	4.22
	100	-0.01	0.00	0.05	0.06	0.01	-0.07	-0.01	-0.05	0.06	1.44	5.26	3.75	1.16	5.25	3.28	1.16	5.19	2.78
		$\rho = 0.5$, $h = 6$																	
	20	0.06	0.05	0.03	-0.03	-0.05	-0.06	0.08	-0.05	0.06	8.39	11.93	8.22	6.07	11.71	8.05	4.32	11.69	6.87
	50	-0.03	-0.06	-0.03	0.09	0.03	0.03	-0.08	-0.07	0.01	3.38	8.14	5.28	2.81	7.28	4.47	2.13	7.10	4.28
	100	-0.04	0.00	0.06	0.00	0.03	0.03	-0.01	-0.07	-0.07	2.07	5.60	3.84	1.65	5.05	3.45	1.38	5.01	3.00
		$\rho = 0.8$, $h = 6$																	
	20	0.03	0.04	-0.01	0.02	-0.10	-0.04	-0.02	-0.07	0.00	6.34	11.78	8.42	4.50	11.27	7.99	3.37	11.24	6.91
	50	-0.07	0.07	-0.06	-0.02	0.05	0.11	0.02	0.00	0.02	2.25	8.13	5.11	1.87	8.01	4.31	1.60	7.51	4.16
	100	0.00	0.10	-0.01	0.01	-0.03	-0.03	-0.09	0.05	0.06	1.28	5.84	3.80	1.10	5.23	3.30	1.05	5.12	2.97
		$\rho = 0.8$, $h = 0.3N$																	
	20	0.06	-0.06	-0.06	-0.07	-0.01	0.02	0.01	0.04	0.05	6.63	12.34	8.53	4.60	11.72	7.77	3.47	11.18	6.92
	50	0.06	0.04	-0.02	0.02	0.05	-0.07	0.02	0.08	-0.08	5.95	7.97	5.56	3.57	7.45	4.81	2.46	7.09	4.56
	100	-0.03	-0.01	0.03	0.10	0.05	0.10	-0.06	-0.04	-0.06	5.65	5.24	4.07	3.46	5.14	3.33	2.41	4.92	2.99
		$\rho = 0.5$, $h = 2$																	
CCE-IV	20	-0.29	-0.02	0.05	-0.26	-0.07	-0.13	0.26	-0.19	-0.02	4.07	12.48	8.08	3.16	11.57	7.94	2.78	11.45	6.96
	50	0.05	0.12	0.10	-0.13	-0.04	0.02	-0.07	0.02	0.03	2.40	7.57	5.09	1.94	7.43	4.52	1.60	7.29	4.40
	100	0.07	0.03	0.07	0.01	0.06	-0.07	-0.04	-0.08	0.05	1.45	5.35	3.97	1.15	5.24	3.57	1.11	5.09	2.91
		$\rho = 0.5$, $h = 6$																	
	20	-2.12	-0.07	-0.14	-2.15	-0.15	-0.27	-2.29	-0.15	-0.17	8.79	12.07	8.19	6.61	11.85	7.99	5.03	11.58	6.99
	50	-0.20	-0.11	-0.04	-0.09	0.02	0.04	-0.22	-0.10	0.02	3.57	8.20	5.25	2.74	7.42	4.38	2.13	6.95	4.35
	100	-0.06	0.04	0.05	0.02	0.04	0.05	-0.10	-0.08	-0.08	2.28	5.57	3.73	1.55	5.17	3.36	1.31	5.11	2.90
		$\rho = 0.8$, $h = 6$																	
	20	-4.15	-0.04	-0.16	-4.21	-0.03	-0.23	-4.12	0.10	-0.04	8.08	11.73	8.45	6.56	11.26	7.93	5.53	11.18	6.83
	50	-0.26	0.05	-0.11	-0.29	0.04	0.11	-0.25	-0.02	0.02	2.35	8.03	5.15	1.94	7.96	4.31	1.57	7.52	4.21
	100	-0.05	0.13	-0.03	0.08	-0.07	-0.03	-0.11	0.05	0.05	1.24	5.94	3.81	1.19	5.37	3.40	1.01	5.12	2.98
		$\rho = 0.8$, $h = 0.3N$																	
	20	-4.07	-0.06	-0.26	-4.02	-0.79	-0.30	-4.36	0.06	0.04	7.75	12.33	8.40	6.29	11.77	7.63	5.22	11.20	7.06
	50	-3.49	0.16	-0.06	-3.53	0.09	0.37	-3.48	-0.01	0.08	6.21	8.00	5.39	4.86	7.40	4.90	3.57	7.05	4.52
	100	-3.03	-0.08	0.06	-2.95	0.04	0.09	-3.05	0.21	-0.21	6.03	5.33	3.97	4.74	5.22	3.32	3.66	4.78	3.09
		$\rho = 0.5$, $h = 2$																	
Infeasible	20	0.03	0.04	-0.01	-0.03	0.04	-0.08	0.03	-0.05	-0.04	3.48	12.46	8.19	2.82	11.56	7.88	2.32	11.46	6.89
	50	0.01	0.05	0.06	-0.03	-0.15	0.00	0.00	0.05	0.03	2.13	7.54	5.22	1.73	7.28	4.57	1.48	7.01	4.10
	100	0.04	0.02	0.05	0.05	0.04	-0.04	-0.02	-0.05	0.05	1.47	5.33	3.77	1.19	5.33	3.29	1.11	5.21	2.66
		$\rho = 0.5$, $h = 6$																	
	20	-0.01	0.05	0.08	-0.03	-0.05	-0.03	-0.03	-0.05	0.04	4.43	11.75	7.96	3.69	11.66	7.73	2.93	11.60	6.76
	50	-0.04	-0.10	0.01	0.05	0.07	0.02	-0.06	-0.08	-0.04	2.90	7.28	5.10	2.31	7.21	4.36	1.93	7.07	4.13
	100	-0.10	-0.03	0.01	0.05	0.00	0.01	-0.02	-0.07	-0.06	2.04	5.58	3.65	1.46	5.06	3.24	1.18	4.99	2.60
		$\rho = 0.8$, $h = 6$																	
	20	0.01	0.00	0.01	0.07	-0.02	0.02	-0.02	0.07	0.04	2.72	12.46	8.00	2.50	11.59	7.84	2.30	11.20	6.74
	50	-0.05	0.05	-0.07	0.00	0.08	0.04	0.02	-0.02	0.04	1.84	8.05	5.17	1.60	8.03	4.41	1.36	7.49	4.08
	100	-0.04	0.06	0.01	0.10	-0.07	-0.06	-0.05	0.02	0.04	1.30	5.86	3.81	1.00	5.21	3.26	0.79	5.03	2.78
		$\rho = 0.8$, $h = 0.3N$																	
	20	0.06	-0.02	-0.06	-0.03	-0.03	0.07	0.03	0.01	0.06	2.71	12.11	8.20	2.63	11.56	7.62	2.27	11.10	6.97
	50	0.08	-0.05	0.06	0.01	0.10	0.06	-0.05	-0.06	-0.05	2.14	7.91	5.42	1.86	7.38	4.87	1.42	6.97	4.53
	100	0.04	-0.08	0.03	-0.08	0.07	0.07	-0.09	-0.02	0.00	1.84	5.39	3.88	1.46	5.13	3.40	1.24	4.85	2.96

Notes: The simulation results are based on the DGP specified in Section 4.

Table S11: The Size and Power for Mean Group Estimators under Experiment 2

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
CCEX-IV										$\rho=0.5,\ h=2$									
	20	0.072	0.072	0.074	0.050	0.055	0.056	0.062	0.046	0.060	0.596	0.235	0.427	0.771	0.350	0.689	0.845	0.423	0.892
	50	0.054	0.054	0.048	0.057	0.055	0.060	0.054	0.055	0.049	0.945	0.782	0.877	0.983	0.826	0.971	0.999	0.884	1.000
	100	0.045	0.054	0.057	0.050	0.040	0.055	0.049	0.058	0.060	1.000	0.951	0.979	0.990	1.000	1.000	1.000	1.000	1.000
										$\rho=0.5,\ h=6$									
	20	0.071	0.066	0.067	0.064	0.057	0.077	0.057	0.052	0.053	0.228	0.171	0.447	0.319	0.350	0.682	0.540	0.401	0.890
	50	0.048	0.063	0.055	0.064	0.052	0.062	0.050	0.047	0.058	0.749	0.660	0.878	0.869	0.747	0.990	0.985	0.802	1.000
	100	0.052	0.065	0.038	0.054	0.057	0.044	0.059	0.047	0.062	0.999	0.966	1.000	1.000	1.000	1.000	1.000	1.000	0.995
										$\rho=0.8,\ h=6$									
	20	0.073	0.078	0.075	0.057	0.066	0.056	0.057	0.060	0.050	0.363	0.319	0.410	0.574	0.426	0.695	0.768	0.533	0.930
	50	0.052	0.045	0.043	0.049	0.052	0.067	0.044	0.063	0.059	0.968	0.714	0.844	1.000	0.816	0.983	1.000	0.914	0.991
	100	0.052	0.060	0.06	0.046	0.027	0.049	0.055	0.053	0.056	1.000	0.994	0.979	0.981	0.999	1.000	1.000	1.000	1.000
										$\rho=0.8,\ h=0.3N$									
	20	0.057	0.059	0.060	0.058	0.053	0.050	0.056	0.056	0.058	0.358	0.314	0.523	0.531	0.421	0.576	0.730	0.545	0.584
	50	0.052	0.065	0.050	0.043	0.063	0.065	0.055	0.053	0.057	0.528	0.526	0.839	0.667	0.618	0.862	0.950	0.755	0.932
	100	0.055	0.046	0.058	0.053	0.064	0.042	0.051	0.053	0.045	0.568	0.908	0.993	0.728	0.931	1.000	0.965	0.976	1.000
CCE-IV										$\rho=0.5,\ h=2$									
	20	0.080	0.092	0.077	0.056	0.066	0.057	0.079	0.065	0.082	0.608	0.215	0.437	0.763	0.347	0.694	0.799	0.424	0.907
	50	0.071	0.052	0.063	0.055	0.060	0.063	0.052	0.058	0.075	0.937	0.776	0.874	0.981	0.838	0.968	0.997	0.862	1.000
	100	0.047	0.046	0.068	0.043	0.054	0.061	0.053	0.059	0.070	1.000	0.962	0.979	0.998	1.000	0.984	0.985	1.000	0.981
										$\rho=0.5,\ h=6$									
	20	0.073	0.068	0.073	0.055	0.078	0.072	0.072	0.078	0.088	0.244	0.187	0.447	0.354	0.332	0.683	0.522	0.388	0.901
	50	0.059	0.054	0.056	0.072	0.048	0.049	0.051	0.063	0.074	0.811	0.684	0.888	0.879	0.753	0.987	0.989	0.796	1.000
	100	0.053	0.056	0.042	0.044	0.062	0.055	0.051	0.071	0.073	0.973	0.955	0.994	1.000	1.000	1.000	0.993	1.000	1.000
										$\rho=0.8,\ h=6$									
	20	0.086	0.080	0.082	0.112	0.082	0.073	0.127	0.071	0.077	0.295	0.343	0.443	0.415	0.439	0.692	0.663	0.516	0.935
	50	0.048	0.046	0.049	0.058	0.059	0.052	0.050	0.063	0.067	0.945	0.716	0.867	0.992	0.828	0.989	1.000	0.911	1.000
	100	0.054	0.055	0.057	0.049	0.019	0.060	0.053	0.030	0.057	1.000	0.987	1.000	1.000	0.981	1.000	0.980	1.000	0.997
										$\rho=0.8,\ h=0.3N$									
	20	0.086	0.069	0.068	0.086	0.075	0.079	0.110	0.072	0.051	0.299	0.336	0.475	0.428	0.425	0.546	0.636	0.539	0.603
	50	0.063	0.084	0.045	0.075	0.055	0.062	0.092	0.058	0.055	0.427	0.516	0.885	0.623	0.572	0.883	0.879	0.749	0.932
	100	0.065	0.047	0.051	0.085	0.078	0.050	0.076	0.037	0.053	0.487	0.911	0.976	0.684	0.928	1.000	0.910	0.964	0.999
Infeasible										$\rho=0.5,\ h=2$									
	20	0.074	0.071	0.068	0.058	0.052	0.063	0.058	0.058	0.063	0.708	0.231	0.479	0.856	0.364	0.695	0.983	0.430	0.912
	50	0.063	0.053	0.062	0.057	0.060	0.061	0.055	0.061	0.046	0.981	0.786	0.889	1.000	0.837	0.975	1.000	0.893	1.000
	100	0.051	0.056	0.053	0.054	0.053	0.051	0.048	0.062	0.052	0.999	0.963	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho=0.5,\ h=6$									
	20	0.056	0.048	0.059	0.062	0.061	0.064	0.050	0.064	0.059	0.533	0.203	0.450	0.702	0.353	0.717	0.878	0.415	0.915
	50	0.041	0.061	0.065	0.058	0.060	0.055	0.061	0.059	0.049	0.846	0.687	0.895	0.958	0.754	0.992	1.000	0.798	1.000
	100	0.054	0.054	0.049	0.047	0.056	0.049	0.048	0.064	0.058	1.000	0.953	1.000	1.000	1.000	0.997	1.000	1.000	1.000
										$\rho=0.8,\ h=6$									
	20	0.053	0.061	0.061	0.070	0.061	0.070	0.046	0.051	0.054	0.877	0.357	0.451	0.925	0.445	0.698	0.983	0.546	0.940
	50	0.039	0.061	0.052	0.047	0.063	0.060	0.050	0.066	0.064	0.983	0.743	0.893	1.000	0.823	0.993	1.000	0.923	1.000
	100	0.054	0.064	0.048	0.051	0.019	0.047	0.048	0.054	0.054	0.991	0.993	0.979	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho=0.8,\ h=0.3N$									
	20	0.067	0.059	0.061	0.056	0.068	0.067	0.056	0.058	0.064	0.876	0.333	0.490	0.934	0.431	0.581	0.939	0.561	0.606
	50	0.050	0.059	0.058	0.051	0.060	0.056	0.053	0.050	0.043	0.969	0.574	0.877	0.996	0.604	0.928	1.000	0.762	0.937
	100	0.054	0.054	0.055	0.048	0.069	0.042	0.049	0.042	0.053	0.998	0.919	0.984	1.000	0.942	1.000	0.987	0.991	1.000

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

Table S12: The Finite Sample Performance of Pooled Estimators under Experiment 2

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
										$\rho = 0.5,\ h = 2$									
	20	0.38	0.01	-0.27	0.46	-0.46	-0.75	0.36	-0.23	-0.61	4.28	12.69	9.28	3.62	12.24	8.28	3.03	12.04	8.04
	50	0.42	0.22	0.12	0.37	0.04	-0.29	0.34	-0.14	0.31	2.55	7.80	5.69	2.19	7.36	5.39	1.73	7.26	4.54
	100	0.41	-0.03	-0.17	0.34	0.23	-0.03	0.35	0.21	-0.32	1.82	5.83	4.21	1.50	5.37	3.79	1.28	5.23	3.25
										$\rho = 0.5,\ h = 6$									
	20	0.47	-0.09	-0.23	0.45	-0.11	-0.31	-0.43	-0.99	0.41	7.91	12.72	8.96	6.18	12.12	8.15	4.83	11.68	7.41
	50	0.26	-0.29	-0.01	0.36	-0.12	-0.37	0.30	-0.24	0.31	4.21	7.64	5.77	3.53	7.54	4.87	2.64	7.61	4.84
	100	-0.37	0.29	-0.31	0.36	-0.31	0.12	0.40	0.600	-0.02	2.83	5.78	4.08	2.40	5.03	3.61	1.70	5.08	3.16
CCEX-IV										$\rho = 0.8,\ h = 6$									
	20	0.45	0.20	0.22	0.47	-0.52	-0.33	0.46	0.02	0.36	5.80	13.29	9.47	4.84	11.55	8.10	3.69	11.65	7.35
	50	0.45	-0.26	-0.16	0.59	0.17	-0.08	0.49	0.31	0.56	2.51	8.34	5.36	2.21	7.42	5.44	1.68	7.34	4.96
	100	0.46	-0.01	0.14	0.51	-0.28	-0.23	0.45	-0.06	0.12	1.50	5.97	4.10	1.21	5.74	3.30	1.11	5.44	3.43
										$\rho = 0.8,\ h = 0.3N$									
	20	0.43	-0.22	-0.49	0.46	-0.08	-0.36	0.54	0.29	-0.33	5.89	12.76	9.34	4.67	11.92	8.19	3.57	11.80	7.78
	50	0.44	0.02	0.24	0.34	-0.37	0.23	0.31	-0.01	-0.34	5.34	8.35	5.95	3.80	7.48	5.31	2.88	7.41	5.06
	100	-0.35	0.06	-0.18	0.44	0.29	0.14	0.33	0.21	-0.03	5.27	5.88	4.05	3.67	5.54	3.63	2.86	5.45	3.23
										$\rho = 0.5,\ h = 2$									
	20	0.44	0.17	-0.39	0.45	-0.35	-0.15	0.42	-0.21	-0.11	4.44	12.98	9.27	3.64	12.06	8.44	3.21	12.13	7.90
	50	0.36	0.22	0.13	0.29	0.18	-0.26	0.43	-0.34	-0.06	2.66	7.86	5.80	2.18	7.39	5.53	1.72	7.35	4.72
	100	0.52	0.01	-0.22	0.53	0.12	-0.01	0.46	0.12	-0.17	1.89	5.75	4.29	1.55	5.53	3.85	1.14	5.28	3.26
										$\rho = 0.5,\ h = 6$									
	20	-1.44	-0.32	-0.50	-2.24	-0.25	-0.43	-2.28	-1.02	0.18	8.54	12.58	8.97	6.56	12.05	8.28	5.22	11.62	7.58
	50	-0.42	-0.28	-0.12	-0.35	-0.13	-0.35	-0.36	-0.28	0.13	4.26	7.75	5.79	3.61	7.49	4.64	2.61	7.43	4.88
	100	-0.36	0.27	-0.33	0.35	-0.25	0.19	0.45	0.19	-0.05	2.81	5.89	4.20	2.43	5.17	3.61	1.76	5.21	3.02
CCE-IV										$\rho = 0.8,\ h = 6$									
	20	-2.29	0.07	0.09	-1.92	-0.63	-0.61	-1.79	-0.08	0.15	6.59	13.35	9.41	5.08	11.46	8.10	4.43	11.69	7.33
	50	0.43	-0.27	-0.50	0.44	0.12	-0.82	0.46	0.16	0.38	2.68	8.41	5.28	2.31	7.39	5.50	1.77	7.33	4.84
	100	0.52	-0.05	0.07	0.48	-0.33	-0.60	0.46	-0.11	0.11	1.59	5.88	4.18	1.28	5.85	3.21	1.02	5.29	3.43
										$\rho = 0.8,\ h = 0.3N$									
	20	-1.78	-0.68	-0.59	-1.89	-0.87	-0.63	-1.86	0.54	-0.72	6.56	12.47	9.14	5.44	12.11	8.10	4.00	11.68	7.75
	50	-1.30	0.03	0.08	-1.32	-0.60	0.32	-1.345	-0.90	-0.46	5.55	8.24	5.97	4.23	7.35	5.20	3.12	7.41	5.07
	100	-1.59	0.07	-0.16	-1.33	0.26	0.15	-1.39	0.25	-0.06	5.58	5.87	4.01	4.14	5.66	3.62	3.17	5.42	3.28
										$\rho = 0.5,\ h = 2$									
	20	0.41	0.09	-0.33	0.33	-0.02	-0.27	0.34	-0.27	-0.36	3.95	12.79	9.49	3.19	12.22	8.42	2.78	12.02	7.92
	50	0.34	0.27	0.28	0.39	0.28	-0.11	0.39	-0.34	-0.01	2.50	7.78	5.58	2.16	7.37	5.16	1.58	7.25	4.57
	100	0.41	0.01	-0.38	0.38	0.27	-0.02	0.36	0.34	-0.21	1.94	5.71	4.00	1.72	5.30	3.94	1.12	5.17	3.22
										$\rho = 0.5,\ h = 6$									
	20	0.42	-0.08	-0.25	0.37	-0.16	-0.35	-0.33	-0.27	0.36	4.95	12.64	8.76	4.15	11.96	8.23	3.34	11.57	7.46
	50	0.37	-0.15	-0.32	0.28	-0.18	-0.34	0.37	-0.28	0.02	3.55	7.79	5.92	2.87	7.65	4.76	2.30	7.56	4.86
	100	0.31	0.18	-0.29	0.36	-0.19	0.31	0.39	0.27	-0.03	2.65	5.82	4.09	2.11	5.03	3.55	1.58	5.03	3.08
Infeasible										$\rho = 0.8,\ h = 6$									
	20	0.40	0.20	0.47	0.40	-0.56	-0.24	0.44	0.11	0.47	3.10	13.37	9.78	2.79	11.49	8.20	2.57	11.61	7.46
	50	0.36	-0.48	-0.09	0.24	0.01	-0.26	0.50	0.25	0.36	2.25	8.26	5.18	1.90	7.39	5.42	1.54	7.05	4.73
	100	0.33	0.07	0.13	0.43	-0.34	-0.35	0.41	-0.30	0.07	1.43	5.99	4.19	1.26	5.95	3.36	0.94	5.31	3.59
										$\rho = 0.8,\ h = 0.3N$									
	20	0.39	-0.39	-0.63	0.50	-0.43	-0.45	0.47	0.36	-0.53	2.93	12.59	9.85	2.78	11.80	8.33	2.54	11.80	7.79
	50	0.25	-0.08	0.10	0.31	-0.05	0.24	0.29	-0.09	-0.37	2.56	8.29	5.95	1.96	7.34	5.38	1.62	7.54	5.15
	100	-0.35	0.00	-0.18	0.21	0.38	0.13	0.20	0.57	-0.06	2.28	5.88	3.91	1.84	5.49	3.53	1.36	5.54	3.44

Notes: The simulation results are based on the DGP specified in Section 4.

Table S13: The Size and Power for Pooled Estimators under Experiment 2

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
											$\rho = 0.5,\ h = 2$								
	20	0.083	0.075	0.088	0.068	0.067	0.078	0.091	0.088	0.087	0.464	0.281	0.413	0.672	0.318	0.427	0.727	0.362	0.486
	50	0.080	0.057	0.076	0.063	0.052	0.068	0.061	0.087	0.080	0.954	0.586	0.760	0.992	0.605	0.838	1.000	0.644	0.900
	100	0.088	0.052	0.047	0.043	0.088	0.090	0.066	0.049	0.054	0.987	0.830	0.981	0.997	0.915	0.987	1.000	0.954	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.079	0.054	0.070	0.094	0.067	0.096	0.049	0.066	0.077	0.278	0.247	0.393	0.317	0.357	0.462	0.570	0.387	0.593
	50	0.056	0.031	0.071	0.075	0.051	0.053	0.049	0.074	0.076	0.680	0.604	0.775	0.685	0.642	0.921	0.911	0.691	0.928
	100	0.088	0.044	0.050	0.062	0.044	0.041	0.051	0.064	0.036	0.832	0.901	0.993	0.966	0.947	1.000	1.000	0.963	1.000
CCEX-IV											$\rho = 0.8,\ h = 6$								
	20	0.116	0.100	0.098	0.056	0.054	0.093	0.092	0.079	0.071	0.425	0.233	0.363	0.602	0.352	0.494	0.639	0.396	0.556
	50	0.080	0.056	0.066	0.102	0.057	0.076	0.075	0.068	0.062	0.897	0.604	0.804	0.966	0.643	0.837	0.995	0.699	0.892
	100	0.102	0.075	0.072	0.040	0.095	0.038	0.052	0.059	0.077	1.000	0.757	0.994	1.000	0.843	1.000	1.000	0.908	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.079	0.081	0.071	0.080	0.082	0.071	0.072	0.074	0.075	0.359	0.241	0.403	0.518	0.297	0.446	0.705	0.339	0.612
	50	0.092	0.074	0.072	0.080	0.058	0.053	0.087	0.053	0.076	0.389	0.591	0.788	0.571	0.632	0.886	0.895	0.654	0.918
	100	0.047	0.086	0.068	0.067	0.060	0.072	0.056	0.084	0.074	0.484	0.873	0.968	0.734	0.899	1.000	0.919	0.900	1.000
											$\rho = 0.5,\ h = 2$								
	20	0.085	0.083	0.102	0.092	0.087	0.099	0.094	0.067	0.084	0.490	0.257	0.389	0.685	0.242	0.424	0.794	0.287	0.479
	50	0.069	0.056	0.079	0.061	0.058	0.074	0.054	0.081	0.081	0.940	0.581	0.722	0.996	0.602	0.807	1.000	0.626	0.890
	100	0.060	0.036	0.050	0.051	0.075	0.098	0.064	0.042	0.039	1.000	0.827	0.955	0.984	0.915	0.965	1.000	0.945	0.978
											$\rho = 0.5,\ h = 6$								
	20	0.073	0.082	0.095	0.061	0.081	0.076	0.081	0.083	0.089	0.257	0.242	0.399	0.377	0.311	0.478	0.544	0.387	0.599
	50	0.064	0.058	0.087	0.064	0.060	0.063	0.030	0.071	0.065	0.581	0.615	0.793	0.730	0.658	0.915	0.928	0.685	0.922
	100	0.067	0.049	0.063	0.069	0.040	0.045	0.058	0.069	0.037	0.872	0.895	0.986	0.941	0.961	1.000	0.985	0.953	1.000
CCE-IV											$\rho = 0.8,\ h = 6$								
	20	0.104	0.106	0.113	0.067	0.070	0.076	0.115	0.092	0.081	0.387	0.240	0.372	0.623	0.357	0.504	0.622	0.398	0.521
	50	0.064	0.068	0.061	0.052	0.051	0.088	0.086	0.066	0.073	0.886	0.607	0.793	0.990	0.647	0.834	1.000	0.698	0.908
	100	0.098	0.074	0.064	0.054	0.061	0.021	0.048	0.061	0.072	1.000	0.752	0.965	0.999	0.833	0.998	1.000	0.935	0.994
											$\rho = 0.8,\ h = 0.3N$								
	20	0.088	0.073	0.063	0.095	0.097	0.094	0.121	0.083	0.085	0.340	0.252	0.408	0.428	0.283	0.419	0.699	0.329	0.607
	50	0.082	0.076	0.070	0.070	0.049	0.060	0.091	0.061	0.088	0.359	0.604	0.750	0.571	0.623	0.897	0.863	0.643	0.892
	100	0.065	0.093	0.060	0.074	0.063	0.069	0.070	0.083	0.071	0.441	0.863	0.982	0.733	0.912	0.986	0.915	0.903	0.993
											$\rho = 0.5,\ h = 2$								
	20	0.055	0.080	0.098	0.080	0.062	0.098	0.088	0.091	0.094	0.553	0.290	0.423	0.797	0.304	0.436	0.880	0.378	0.509
	50	0.087	0.051	0.087	0.068	0.066	0.052	0.074	0.082	0.069	0.960	0.596	0.742	0.975	0.613	0.851	1.000	0.660	0.906
	100	0.069	0.032	0.043	0.035	0.084	0.092	0.041	0.056	0.053	0.999	0.862	0.997	0.996	0.956	0.988	1.000	0.961	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.077	0.062	0.091	0.075	0.083	0.077	0.046	0.058	0.063	0.533	0.258	0.435	0.579	0.311	0.525	0.689	0.398	0.573
	50	0.049	0.060	0.080	0.106	0.041	0.048	0.023	0.061	0.061	0.751	0.645	0.796	0.901	0.678	0.925	0.946	0.708	0.949
	100	0.053	0.060	0.069	0.062	0.036	0.037	0.048	0.072	0.036	0.949	0.905	0.986	0.959	0.956	1.000	1.000	0.977	1.000
Infeasible											$\rho = 0.8,\ h = 6$								
	20	0.082	0.071	0.101	0.063	0.051	0.063	0.098	0.075	0.072	0.830	0.255	0.376	0.957	0.359	0.509	0.940	0.411	0.537
	50	0.089	0.054	0.029	0.084	0.062	0.087	0.084	0.050	0.044	0.969	0.664	0.806	0.991	0.672	0.842	1.000	0.712	0.895
	100	0.115	0.089	0.076	0.048	0.066	0.030	0.063	0.081	0.084	1.000	0.764	0.966	0.983	0.847	0.997	1.000	0.928	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.068	0.074	0.072	0.065	0.064	0.073	0.064	0.073	0.063	0.800	0.245	0.430	0.866	0.323	0.507	0.921	0.358	0.616
	50	0.072	0.060	0.066	0.043	0.060	0.068	0.064	0.046	0.067	0.936	0.609	0.749	0.978	0.639	0.886	1.000	0.663	0.898
	100	0.048	0.060	0.064	0.073	0.059	0.054	0.062	0.082	0.066	0.954	0.888	0.968	1.000	0.918	1.000	1.000	0.905	1.000

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

S4.3 Simulation Results under Experiment 3

In this section, we provide the simulation results under Experiment 3 in Tables S14 to S19. All the simulation results are qualitatively similar to those reported for Experiment 1 in Section S4.1. Specifically, the biases of the CCEX-IV individual, Mean Group and Pooled estimators for (ρ, β_1, β_2) are more or less negligible and close to those of the infeasible estimator in almost all cases. On the other hand, the CCE-IV estimators display non-negligible biases as h or ρ rises, especially for $N = 20$. In particular, if the spatial weighing matrix remains relatively dense with $h = 0.3N$, then the bias of the CCE-IV spatial coefficient does not disappear even as N increases.

RMSEs of the individual, Mean Group and Pooled estimators all decrease with T and/or with N. RMSEs for CCEX-IV and CCE-IV are quite similar for fixed h. However, if the spatial weighing matrix remains dense with $h = 0.3N$, then RMSEs for the CCE-IV estimator are significantly higher than the CCEX-IV counterparts at all the sample sizes.

The sizes of the t-test for the CCEX-IV individual, Mean Group and Pooled estimators all tend to the 5% nominal level as the sample size increases (particularly T for individual estimator). But, the CCE-IV spatial coefficient tends to display slight size distortions if N is small. The powers of the t-tests for both CCEX-IV and CCE-IV estimators are more or less similar, and tend to 1 as the sample size (particularly T for individual estimator), except for the individual spatial coefficient in the case with $h = 0.3N$ in which case the power falls with N. Same as in Experiment 1, the power for Mean Group and Pooled estimator are much larger than those for individual estimator because of a faster convergence rate.

Table S14: The Finite Sample Performance of Individual Estimators under Experiment 3

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
										$\rho = 0.5,\ h = 2$									
CCEX-IV	20	0.00	0.01	-0.06	-0.03	0.07	0.04	0.06	0.08	-0.02	10.04	14.64	14.64	7.40	10.79	10.92	5.11	7.49	7.55
	50	-0.05	0.04	0.01	0.06	-0.06	-0.06	-0.04	0.05	0.09	9.27	14.40	14.43	6.88	10.63	10.79	4.70	7.34	7.39
	100	-0.06	-0.04	-0.02	0.02	0.09	-0.03	0.00	-0.03	0.03	9.12	14.24	14.47	6.63	10.46	10.57	4.59	7.20	7.33
										$\rho = 0.5,\ h = 6$									
	20	-0.02	0.07	-0.01	0.07	0.04	-0.03	0.06	0.07	0.04	21.86	14.55	14.78	16.32	10.75	10.83	11.00	7.52	7.40
	50	-0.08	-0.05	0.08	-0.05	0.06	0.05	-0.04	-0.03	0.01	16.02	14.25	14.31	11.64	10.58	10.55	8.01	7.25	7.22
	100	-0.06	0.06	-0.03	0.09	-0.05	0.06	0.04	0.01	0.09	14.64	14.19	14.21	10.83	10.51	10.55	7.44	7.19	7.13
										$\rho = 0.8,\ h = 6$									
	20	-0.01	-0.06	-0.05	0.05	0.07	-0.07	0.06	0.10	0.02	14.81	14.80	14.83	10.77	10.94	10.94	7.46	7.51	7.62
	50	-0.05	-0.03	-0.07	0.00	0.10	0.05	0.00	0.03	-0.05	9.33	14.36	14.54	6.78	10.59	10.69	4.60	7.39	7.26
	100	0.01	-0.07	0.05	0.00	0.03	0.08	-0.01	-0.01	-0.01	8.16	14.28	14.35	6.07	10.63	10.81	4.11	7.17	7.36
										$\rho = 0.8,\ h = 0.3N$									
	20	0.09	0.09	-0.06	0.08	-0.07	0.05	0.06	-0.04	-0.03	14.66	14.94	14.93	10.69	10.87	10.93	7.43	7.53	7.50
	50	0.08	-0.09	-0.03	-0.04	0.08	-0.10	-0.09	0.04	-0.05	21.15	14.36	14.40	15.55	10.52	10.72	10.60	7.33	7.33
	100	0.01	0.02	0.02	-0.07	0.04	0.04	0.04	0.02	0.05	29.31	14.26	14.45	21.11	10.53	10.57	14.22	7.21	7.24
										$\rho = 0.5,\ h = 2$									
CCE-IV	20	-0.17	0.00	-0.20	-0.25	0.07	0.01	-0.13	0.06	-0.05	10.07	14.60	14.54	7.31	10.64	10.76	5.03	7.34	7.42
	50	-0.08	0.08	0.03	0.04	-0.08	-0.05	-0.07	0.04	0.13	9.36	14.56	14.62	6.84	10.66	10.76	4.67	7.27	7.32
	100	-0.08	-0.05	-0.01	0.02	0.09	-0.05	0.00	-0.04	0.04	9.25	14.45	14.67	6.66	10.52	10.65	4.59	7.19	7.31
										$\rho = 0.5,\ h = 6$									
	20	-2.84	0.00	-0.30	-2.72	-0.11	-0.31	-2.70	-0.07	-0.21	23.54	14.41	14.73	17.73	10.59	10.77	12.28	7.37	7.37
	50	-0.20	-0.12	0.04	-0.23	0.07	0.04	-0.17	-0.05	0.00	16.19	14.34	14.40	11.68	10.58	10.55	8.00	7.21	7.25
	100	0.07	0.07	0.00	0.06	-0.05	0.07	0.05	0.02	0.09	14.85	14.38	14.41	10.89	10.55	10.63	7.43	7.15	7.12
										$\rho = 0.8,\ h = 6$									
	20	-4.30	-0.37	-0.48	-3.95	-0.10	-0.45	-4.10	-0.11	-0.40	19.47	14.59	14.68	14.58	10.80	10.88	10.81	7.40	7.61
	50	-0.33	-0.03	-0.05	-0.20	0.08	0.14	-0.19	0.02	-0.15	9.52	14.51	14.68	6.88	10.62	10.71	4.64	7.36	7.21
	100	-0.01	-0.07	-0.17	-0.04	0.03	0.08	-0.05	-0.01	-0.01	8.26	14.51	14.50	6.10	10.69	10.87	4.11	7.17	7.35
										$\rho = 0.8,\ h = 0.3N$									
	20	-3.98	-0.07	-0.47	-4.13	-0.32	-0.35	-4.06	-0.24	-0.43	19.18	14.68	14.74	14.60	10.71	10.85	10.82	7.43	7.53
	50	-3.19	-0.22	-0.18	-3.25	0.03	-0.25	-3.31	-0.02	-0.15	26.11	14.35	14.46	19.50	10.46	10.67	13.70	7.28	7.30
	100	-2.87	-0.01	0.00	-3.17	0.04	-0.01	-2.82	-0.01	-0.01	35.25	14.36	14.53	26.07	10.53	10.53	17.84	7.19	7.21
										$\rho = 0.5,\ h = 2$									
Infeasible	20	-0.07	-0.09	-0.05	-0.03	0.07	-0.01	0.02	0.08	-0.02	7.66	13.64	13.80	5.59	10.06	10.19	3.83	6.85	7.01
	50	-0.03	0.04	0.03	0.08	-0.07	-0.02	-0.01	0.00	0.09	7.84	13.55	13.75	5.78	10.00	10.22	3.96	6.91	6.96
	100	-0.06	-0.02	-0.03	0.03	0.09	-0.04	-0.01	-0.01	0.03	7.95	13.62	13.82	5.85	9.93	10.13	4.02	6.85	6.97
										$\rho = 0.5,\ h = 6$									
	20	-0.08	-0.05	-0.02	0.06	0.01	0.03	-0.04	-0.01	0.05	10.26	13.45	13.77	7.56	9.94	10.05	5.11	6.81	6.76
	50	0.03	-0.02	0.07	-0.03	0.05	0.07	0.02	-0.02	0.03	11.00	13.56	13.66	8.05	10.07	10.00	5.41	6.81	6.74
	100	0.04	0.05	-0.03	0.04	-0.05	0.05	-0.06	0.02	0.01	11.19	13.55	13.57	8.30	9.99	10.06	5.65	6.81	6.86
										$\rho = 0.8,\ h = 6$									
	20	0.02	-0.03	-0.05	0.00	0.01	-0.05	0.03	0.08	0.02	4.69	13.54	13.82	3.54	10.05	10.13	2.39	6.91	7.03
	50	-0.06	-0.02	-0.01	0.00	0.10	0.04	0.01	-0.02	-0.07	5.32	13.63	13.90	3.92	9.98	10.22	2.68	6.92	6.89
	100	0.02	-0.07	0.02	0.00	0.02	0.09	-0.01	-0.02	-0.02	5.47	13.58	13.74	4.13	10.15	10.33	2.78	6.82	7.04
										$\rho = 0.8,\ h = 0.3N$									
	20	0.00	0.09	-0.08	-0.02	-0.03	-0.04	0.02	-0.08	-0.04	4.85	13.69	14.00	3.51	9.97	10.20	2.43	6.89	7.02
	50	0.03	-0.08	-0.07	-0.01	0.08	0.04	-0.06	0.09	0.02	6.11	13.45	13.63	4.63	9.91	10.10	3.15	6.84	6.99
	100	-0.01	0.02	0.06	-0.01	0.04	0.02	0.01	0.02	0.06	7.04	13.53	13.69	5.27	10.07	10.05	3.55	6.91	6.95

Notes: The simulation results are based on the DGP specified in Section 4.

Table S15: The Size and Power for Individual Estimators under Experiment 3

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
CCEX-IV											$\rho = 0.5,\ h = 2$								
	20	0.078	0.079	0.080	0.069	0.077	0.077	0.062	0.066	0.067	0.530	0.269	0.276	0.740	0.437	0.432	0.948	0.748	0.743
	50	0.084	0.078	0.076	0.070	0.073	0.072	0.065	0.065	0.065	0.556	0.286	0.275	0.788	0.478	0.463	0.974	0.762	0.769
	100	0.082	0.075	0.079	0.067	0.070	0.070	0.060	0.059	0.060	0.562	0.286	0.277	0.805	0.476	0.475	0.978	0.773	0.772
											$\rho = 0.5,\ h = 6$								
	20	0.071	0.073	0.073	0.046	0.067	0.067	0.050	0.060	0.061	0.242	0.266	0.265	0.323	0.428	0.445	0.525	0.743	0.751
	50	0.070	0.083	0.082	0.061	0.070	0.071	0.055	0.063	0.058	0.279	0.282	0.287	0.431	0.464	0.447	0.718	0.778	0.793
	100	0.073	0.084	0.083	0.063	0.069	0.072	0.057	0.059	0.061	0.307	0.284	0.290	0.483	0.484	0.466	0.756	0.784	0.783
											$\rho = 0.8,\ h = 6$								
	20	0.073	0.058	0.059	0.046	0.057	0.054	0.050	0.056	0.060	0.488	0.271	0.267	0.635	0.440	0.447	0.799	0.748	0.739
	50	0.070	0.078	0.077	0.049	0.067	0.066	0.049	0.063	0.058	0.653	0.273	0.279	0.822	0.463	0.440	0.973	0.759	0.775
	100	0.062	0.079	0.078	0.056	0.070	0.071	0.054	0.056	0.061	0.712	0.301	0.298	0.875	0.464	0.465	0.989	0.790	0.765
											$\rho = 0.8,\ h = 0.3N$								
	20	0.075	0.061	0.059	0.055	0.055	0.056	0.058	0.057	0.054	0.493	0.255	0.263	0.639	0.434	0.414	0.802	0.730	0.749
	50	0.068	0.053	0.053	0.052	0.050	0.052	0.059	0.049	0.053	0.342	0.279	0.272	0.445	0.451	0.448	0.624	0.770	0.759
	100	0.074	0.046	0.047	0.054	0.047	0.047	0.059	0.045	0.046	0.251	0.267	0.265	0.344	0.448	0.445	0.484	0.768	0.758
CCE-IV											$\rho = 0.5,\ h = 2$								
	20	0.069	0.081	0.074	0.061	0.070	0.069	0.057	0.058	0.061	0.524	0.267	0.285	0.744	0.453	0.442	0.955	0.760	0.764
	50	0.079	0.084	0.083	0.064	0.070	0.069	0.059	0.062	0.062	0.548	0.276	0.271	0.794	0.488	0.464	0.976	0.761	0.772
	100	0.080	0.083	0.084	0.063	0.069	0.069	0.059	0.060	0.059	0.552	0.281	0.274	0.806	0.472	0.475	0.979	0.772	0.777
											$\rho = 0.5,\ h = 6$								
	20	0.041	0.063	0.066	0.043	0.059	0.060	0.042	0.054	0.055	0.226	0.275	0.264	0.307	0.441	0.457	0.505	0.754	0.755
	50	0.066	0.077	0.079	0.056	0.068	0.068	0.052	0.060	0.053	0.273	0.282	0.286	0.439	0.461	0.452	0.715	0.779	0.798
	100	0.069	0.081	0.079	0.061	0.067	0.070	0.055	0.057	0.059	0.305	0.279	0.285	0.477	0.478	0.465	0.759	0.786	0.781
											$\rho = 0.8,\ h = 6$								
	20	0.038	0.048	0.048	0.031	0.047	0.044	0.037	0.047	0.050	0.453	0.262	0.254	0.575	0.437	0.427	0.700	0.726	0.726
	50	0.046	0.073	0.073	0.047	0.063	0.064	0.044	0.061	0.055	0.641	0.271	0.272	0.817	0.462	0.436	0.971	0.766	0.781
	100	0.058	0.077	0.073	0.055	0.067	0.071	0.052	0.055	0.060	0.703	0.294	0.294	0.874	0.465	0.456	0.990	0.790	0.766
											$\rho = 0.8,\ h = 0.3N$								
	20	0.310	0.051	0.050	0.031	0.046	0.046	0.036	0.046	0.046	0.447	0.250	0.249	0.567	0.420	0.410	0.694	0.716	0.732
	50	0.037	0.045	0.046	0.039	0.046	0.046	0.036	0.045	0.045	0.326	0.265	0.265	0.415	0.440	0.435	0.560	0.756	0.744
	100	0.033	0.042	0.043	0.034	0.043	0.042	0.038	0.040	0.042	0.241	0.257	0.252	0.328	0.432	0.423	0.447	0.746	0.745
Infeasible											$\rho = 0.5,\ h = 2$								
	20	0.077	0.079	0.076	0.063	0.065	0.063	0.050	0.051	0.051	0.686	0.294	0.298	0.895	0.494	0.487	0.995	0.809	0.801
	50	0.078	0.078	0.078	0.063	0.061	0.060	0.049	0.053	0.053	0.649	0.311	0.293	0.882	0.520	0.500	0.995	0.805	0.813
	100	0.076	0.077	0.077	0.062	0.060	0.063	0.050	0.052	0.050	0.650	0.305	0.293	0.880	0.506	0.498	0.993	0.813	0.812
											$\rho = 0.5,\ h = 6$								
	20	0.080	0.079	0.082	0.064	0.062	0.063	0.050	0.049	0.051	0.478	0.299	0.290	0.699	0.478	0.493	0.953	0.815	0.810
	50	0.077	0.076	0.078	0.062	0.065	0.061	0.048	0.052	0.048	0.430	0.306	0.302	0.675	0.487	0.497	0.931	0.824	0.829
	100	0.077	0.079	0.076	0.062	0.062	0.065	0.050	0.050	0.053	0.420	0.297	0.314	0.651	0.514	0.495	0.920	0.825	0.814
											$\rho = 0.8,\ h = 6$								
	20	0.073	0.073	0.078	0.059	0.064	0.058	0.047	0.052	0.052	0.944	0.306	0.292	0.997	0.481	0.497	1.000	0.806	0.806
	50	0.073	0.077	0.078	0.058	0.061	0.063	0.052	0.051	0.047	0.904	0.296	0.289	0.989	0.504	0.466	1.000	0.812	0.816
	100	0.074	0.079	0.073	0.061	0.063	0.067	0.049	0.050	0.052	0.899	0.313	0.318	0.983	0.496	0.485	1.000	0.814	0.794
											$\rho = 0.8,\ h = 0.3N$								
	20	0.077	0.077	0.077	0.059	0.061	0.063	0.052	0.053	0.052	0.937	0.303	0.290	0.996	0.502	0.461	1.000	0.805	0.801
	50	0.073	0.074	0.076	0.063	0.060	0.063	0.053	0.051	0.055	0.847	0.315	0.311	0.968	0.496	0.494	1.000	0.825	0.797
	100	0.074	0.075	0.077	0.062	0.063	0.064	0.049	0.050	0.053	0.765	0.309	0.301	0.937	0.500	0.496	0.998	0.810	0.797

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}_i^a = \boldsymbol{\theta}_i + 0.2$.

Table S16: The Finite Sample Performance of Mean Group Estimators under Experiment 3

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
										$\rho = 0.5,\ h = 2$									
CCEX-IV	20	-0.06	-0.05	0.08	0.02	0.02	0.04	-0.07	0.02	-0.07	8.27	5.34	5.48	5.95	3.86	3.71	3.95	2.72	2.53
	50	-0.08	0.00	-0.04	0.03	0.09	0.01	0.00	-0.03	0.03	4.93	3.50	3.21	3.44	2.33	2.30	2.16	1.62	1.67
	100	-0.05	-0.03	0.08	0.03	-0.04	-0.08	-0.01	0.01	-0.02	3.31	2.28	2.31	2.74	1.69	1.76	1.74	1.14	1.15
										$\rho = 0.5,\ h = 6$									
	20	0.08	0.06	-0.26	0.03	0.04	-0.09	-0.03	-0.06	0.01	11.04	5.16	5.09	8.83	4.03	3.96	5.73	2.60	2.68
	50	0.03	0.02	-0.08	0.05	-0.02	0.01	0.01	0.00	0.01	9.03	3.34	3.26	6.63	2.36	2.42	4.55	1.62	1.64
	100	-0.06	-0.02	-0.06	-0.01	0.08	-0.10	-0.07	0.08	0.03	6.30	2.24	2.25	4.44	1.67	1.64	3.11	1.19	1.11
										$\rho = 0.8,\ h = 6$									
	20	0.04	-0.03	-0.06	-0.04	0.05	-0.03	0.04	0.09	0.02	6.23	5.52	5.75	4.06	4.01	3.99	2.67	2.71	2.78
	50	-0.02	-0.08	-0.02	-0.03	0.03	-0.04	0.00	0.10	-0.02	4.81	3.49	3.58	3.15	2.45	2.41	2.09	1.68	1.56
	100	-0.02	0.09	-0.03	0.09	0.07	-0.02	0.02	0.07	0.09	3.39	2.41	2.33	2.32	1.69	1.65	1.53	1.15	1.16
										$\rho = 0.8,\ h = 0.3N$									
	20	0.03	-0.02	0.10	-0.08	-0.09	-0.08	-0.02	-0.09	-0.02	6.40	5.20	5.22	4.02	3.74	3.75	2.70	2.54	2.66
	50	0.08	-0.02	0.06	-0.09	-0.04	0.06	0.07	0.03	0.06	6.27	3.20	3.34	3.56	2.30	2.36	2.35	1.56	1.63
	100	-0.09	-0.01	-0.08	-0.04	0.00	0.08	-0.03	0.03	0.03	6.14	2.34	2.47	3.47	1.63	1.76	2.26	1.17	1.16
										$\rho = 0.5,\ h = 2$									
CCE-IV	20	-0.34	-0.09	0.20	-0.19	0.14	0.03	-0.31	0.00	-0.11	8.22	5.21	5.29	5.93	3.82	3.69	3.97	2.71	2.55
	50	0.25	-0.04	0.04	-0.09	0.12	0.01	-0.04	-0.01	0.02	5.02	3.51	3.25	3.44	2.33	2.33	2.15	1.62	1.68
	100	0.04	-0.02	0.13	0.06	-0.04	-0.06	-0.01	0.01	-0.02	3.31	2.32	2.35	2.74	1.71	1.77	1.73	1.14	1.15
										$\rho = 0.5,\ h = 6$									
	20	-2.39	0.00	-0.07	-2.25	-0.02	-0.03	-2.28	-0.21	-0.06	11.37	5.19	5.19	8.03	3.83	3.81	5.86	2.56	2.64
	50	-0.24	0.03	-0.01	-0.08	-0.02	0.00	-0.15	-0.02	0.01	8.50	3.27	3.25	6.51	2.35	2.43	4.47	1.61	1.64
	100	-0.13	-0.04	-0.09	-0.10	0.12	-0.10	-0.07	0.08	0.01	6.09	2.24	2.27	4.44	1.70	1.63	3.08	1.20	1.12
										$\rho = 0.8,\ h = 6$									
	20	-4.54	-0.16	-0.07	-4.85	-0.20	-0.03	-4.60	-0.12	0.00	7.01	5.11	5.39	6.49	3.75	3.82	5.40	2.59	2.65
	50	-0.24	-0.06	0.00	-0.15	0.05	-0.05	-0.17	0.08	-0.02	5.09	3.32	3.29	3.54	2.43	2.36	2.31	1.67	1.55
	100	-0.11	0.09	-0.05	0.05	0.09	-0.06	0.01	0.07	0.08	3.13	2.43	2.33	2.28	1.71	1.67	1.51	1.13	1.16
										$\rho = 0.8,\ h = 0.3N$									
	20	-4.36	-0.26	-0.33	-4.42	-0.25	-0.24	-4.42	-0.29	-0.42	7.94	4.94	4.89	6.63	3.67	3.76	5.62	2.51	2.64
	50	-3.50	-0.10	-0.06	-4.24	-0.18	-0.05	-4.36	-0.05	-0.04	6.87	3.08	3.22	5.28	2.31	2.32	4.53	1.57	1.60
	100	-4.36	-0.05	-0.12	-4.03	-0.03	-0.20	-3.89	0.00	-0.03	6.70	2.26	2.21	5.07	1.59	1.71	4.07	1.15	1.15
										$\rho = 0.5,\ h = 2$									
Infeasible	20	-0.06	0.03	0.05	0.03	0.03	0.03	-0.05	-0.03	-0.06	6.67	5.09	4.87	4.67	3.60	3.45	3.24	2.55	2.38
	50	-0.07	0.01	0.03	0.07	0.00	0.04	-0.04	-0.04	0.09	4.42	3.31	3.02	3.09	2.25	2.22	2.01	1.54	1.58
	100	-0.03	-0.05	0.08	0.09	0.01	-0.02	0.00	-0.02	0.01	3.25	2.24	2.23	2.56	1.63	1.69	1.64	1.08	1.09
										$\rho = 0.5,\ h = 6$									
	20	0.04	0.06	-0.01	0.04	0.02	-0.02	0.01	-0.09	-0.06	10.85	4.82	4.79	7.85	3.55	3.60	5.21	2.36	2.46
	50	-0.08	0.02	0.00	-0.01	-0.02	0.01	0.09	0.04	-0.02	6.74	3.13	3.05	5.42	2.29	2.23	3.53	1.56	1.56
	100	0.02	0.01	-0.04	0.03	0.05	-0.08	-0.03	0.09	0.03	5.39	2.17	2.16	3.86	1.59	1.56	2.77	1.14	1.09
										$\rho = 0.8,\ h = 6$									
	20	-0.05	0.04	-0.01	0.02	0.02	-0.05	0.02	0.05	0.01	5.04	4.99	4.93	3.33	3.50	3.50	2.18	2.50	2.50
	50	-0.03	0.05	0.02	-0.04	0.08	0.00	0.04	0.09	-0.01	3.42	3.22	3.08	2.46	2.36	2.24	1.56	1.60	1.53
	100	-0.04	0.07	-0.03	0.03	0.05	0.01	0.02	0.05	0.09	3.08	2.26	2.18	1.76	1.65	1.62	1.23	1.13	1.08
										$\rho = 0.8,\ h = 0.3N$									
	20	0.03	0.02	0.07	-0.01	-0.10	0.05	-0.03	-0.09	-0.03	4.75	4.73	4.87	3.26	3.50	3.70	2.58	2.39	2.55
	50	-0.09	0.06	0.17	-0.01	-0.05	0.04	0.03	0.02	0.06	3.37	2.98	3.05	2.99	2.19	2.22	1.72	1.50	1.55
	100	0.01	-0.06	0.09	0.05	-0.04	-0.07	-0.02	0.03	0.00	3.13	2.23	2.15	1.84	1.56	1.65	1.39	1.12	1.10

Notes: The simulation results are based on the DGP specified in Section 4.

Table S17: The Size and Power for Mean Group Estimators under Experiment 3

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
CCEX-IV										$\rho = 0.5,\ h = 2$									
	20	0.061	0.069	0.076	0.066	0.064	0.060	0.062	0.069	0.053	0.958	0.828	0.813	0.998	0.952	0.960	1.000	1.000	1.000
	50	0.053	0.077	0.062	0.058	0.055	0.046	0.048	0.054	0.064	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.049	0.046	0.055	0.062	0.054	0.056	0.046	0.058	0.054	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.5,\ h = 6$									
	20	0.058	0.065	0.067	0.065	0.076	0.061	0.061	0.065	0.068	0.598	0.806	0.814	0.802	0.970	0.978	0.935	1.000	1.000
	50	0.067	0.066	0.061	0.054	0.058	0.062	0.062	0.052	0.052	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.058	0.046	0.051	0.054	0.052	0.044	0.058	0.056	0.044	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 6$									
	20	0.066	0.067	0.063	0.069	0.059	0.059	0.067	0.061	0.065	0.756	0.846	0.876	0.948	0.954	0.950	1.000	1.000	1.000
	50	0.064	0.058	0.062	0.063	0.054	0.063	0.050	0.056	0.032	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.047	0.051	0.049	0.064	0.046	0.050	0.058	0.050	0.060	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 0.3N$									
	20	0.058	0.066	0.053	0.066	0.052	0.064	0.069	0.067	0.062	0.735	0.823	0.867	0.926	0.966	0.956	0.999	1.000	1.000
	50	0.071	0.048	0.057	0.048	0.044	0.051	0.046	0.044	0.054	0.875	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.048	0.056	0.051	0.054	0.044	0.068	0.045	0.064	0.052	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
CCE-IV										$\rho = 0.5,\ h = 2$									
	20	0.062	0.069	0.073	0.071	0.066	0.066	0.073	0.077	0.064	0.960	0.808	0.805	0.998	0.944	0.948	1.000	1.000	1.000
	50	0.058	0.079	0.065	0.059	0.051	0.056	0.048	0.058	0.064	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.053	0.045	0.053	0.062	0.062	0.052	0.038	0.048	0.064	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.5,\ h = 6$									
	20	0.073	0.069	0.085	0.074	0.079	0.070	0.070	0.069	0.075	0.528	0.788	0.796	0.664	0.966	0.970	0.915	1.000	1.000
	50	0.056	0.056	0.051	0.057	0.065	0.067	0.066	0.058	0.068	0.990	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.057	0.047	0.045	0.058	0.054	0.046	0.062	0.072	0.044	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 6$									
	20	0.077	0.065	0.092	0.075	0.053	0.069	0.080	0.061	0.079	0.651	0.824	0.830	0.896	0.954	0.942	1.000	1.000	1.000
	50	0.056	0.051	0.059	0.060	0.057	0.061	0.060	0.058	0.044	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.041	0.052	0.049	0.052	0.044	0.060	0.056	0.050	0.058	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 0.3N$									
	20	0.087	0.073	0.066	0.100	0.061	0.077	0.105	0.060	0.073	0.598	0.803	0.833	0.859	0.940	0.954	0.999	1.000	1.000
	50	0.065	0.052	0.052	0.085	0.047	0.056	0.102	0.056	0.056	0.820	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.079	0.051	0.046	0.084	0.044	0.064	0.092	0.066	0.054	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Infeasible										$\rho = 0.5,\ h = 2$									
	20	0.070	0.075	0.067	0.062	0.059	0.063	0.062	0.069	0.066	0.991	0.874	0.889	1.000	1.000	0.996	1.000	1.000	1.000
	50	0.062	0.064	0.051	0.051	0.053	0.060	0.046	0.062	0.070	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.049	0.054	0.058	0.076	0.048	0.052	0.046	0.040	0.058	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.5,\ h = 6$									
	20	0.052	0.063	0.063	0.070	0.058	0.077	0.068	0.056	0.067	0.960	0.852	0.863	1.000	1.000	0.999	1.000	1.000	1.000
	50	0.065	0.062	0.054	0.049	0.070	0.063	0.052	0.054	0.060	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.042	0.055	0.051	0.052	0.052	0.056	0.074	0.062	0.056	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 6$									
	20	0.069	0.061	0.076	0.067	0.046	0.053	0.058	0.055	0.069	0.994	0.882	0.868	1.000	0.998	0.996	1.000	1.000	1.000
	50	0.062	0.059	0.050	0.060	0.060	0.065	0.048	0.054	0.042	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.045	0.053	0.040	0.056	0.048	0.056	0.056	0.074	0.050	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 0.3N$									
	20	0.062	0.061	0.072	0.071	0.046	0.064	0.057	0.065	0.054	0.989	0.893	0.874	1.000	0.999	1.000	1.000	1.000	1.000
	50	0.059	0.061	0.041	0.032	0.059	0.056	0.060	0.044	0.056	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.057	0.064	0.051	0.046	0.044	0.070	0.054	0.052	0.064	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

Table S18: The Finite Sample Performance of Pooled Estimators under Experiment 3

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		$\rho = 0.5,\ h = 2$																	
	20	0.00	-0.04	0.07	0.08	-0.08	0.09	-0.06	-0.03	-0.08	6.84	4.72	4.81	5.34	3.70	3.54	3.70	2.66	2.49
	50	-0.09	-0.07	0.03	0.02	0.02	0.07	0.02	-0.05	-0.01	4.05	3.06	2.90	3.16	2.19	2.22	2.15	1.59	1.62
	100	-0.01	-0.06	0.01	0.03	-0.06	-0.06	-0.01	0.00	-0.02	2.78	2.10	2.12	2.41	1.65	1.71	1.70	1.13	1.13
		$\rho = 0.5,\ h = 6$																	
	20	-0.09	-0.08	-0.07	-0.07	-0.04	-0.02	-0.03	-0.02	-0.04	16.27	4.78	4.60	11.95	3.66	3.66	8.32	2.49	2.59
	50	-0.06	0.03	-0.07	0.03	-0.09	0.03	0.00	-0.04	0.03	7.20	2.97	2.90	6.02	2.22	2.26	4.25	1.56	1.61
	100	0.01	-0.08	-0.03	-0.03	0.01	-0.07	-0.07	0.08	0.02	5.23	2.00	2.00	4.03	1.59	1.60	2.96	1.16	1.11
CCEX-IV		$\rho = 0.8,\ h = 6$																	
	20	-0.03	0.04	-0.03	-0.07	-0.09	-0.01	-0.08	0.05	0.04	9.91	4.70	4.71	7.67	3.51	3.62	5.33	2.51	2.61
	50	-0.06	-0.10	-0.02	-0.04	-0.03	-0.06	0.05	0.07	0.02	4.16	2.98	2.90	3.13	2.32	2.21	2.10	1.59	1.51
	100	-0.04	0.07	-0.03	0.04	0.01	-0.08	0.02	0.04	0.06	2.54	2.11	2.02	1.88	1.60	1.56	1.41	1.11	1.11
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.03	-0.02	0.10	-0.08	-0.09	-0.08	-0.02	-0.09	-0.02	9.42	4.67	4.64	7.46	3.58	3.63	5.48	2.47	2.57
	50	0.08	-0.07	0.04	-0.09	-0.04	0.06	0.07	0.03	0.06	8.16	2.86	2.95	6.04	2.19	2.20	4.19	1.56	1.55
	100	-0.09	-0.01	-0.08	-0.04	0.00	0.08	-0.03	0.03	0.03	7.19	2.10	2.04	5.96	1.55	1.64	3.01	1.11	1.14
		$\rho = 0.5,\ h = 2$																	
	20	-0.24	-0.07	0.09	-0.13	-0.08	0.12	-0.26	-0.04	-0.08	7.01	4.78	4.86	5.45	3.73	3.54	3.76	2.67	2.50
	50	-0.16	-0.07	0.07	-0.03	0.01	0.07	0.00	-0.05	-0.01	4.16	3.07	2.92	3.18	2.20	2.23	2.14	1.59	1.62
	100	0.00	-0.06	0.04	0.05	-0.07	-0.07	-0.01	0.00	-0.02	2.84	2.12	2.14	2.42	1.67	1.72	1.68	1.13	1.13
		$\rho = 0.5,\ h = 6$																	
	20	-1.07	-0.19	-0.11	-1.01	-0.09	-0.02	-1.10	-0.07	-0.09	16.31	4.81	4.66	11.88	3.66	3.68	8.52	2.49	2.60
	50	-0.20	0.03	-0.05	-0.11	-0.08	0.03	-0.10	-0.04	0.03	7.27	2.99	2.94	6.07	2.22	2.29	4.27	1.58	1.61
	100	-0.06	-0.11	-0.03	-0.06	0.03	-0.06	-0.06	0.08	0.04	5.29	2.02	2.06	4.09	1.61	1.61	2.96	1.18	1.12
CCE-IV		$\rho = 0.8,\ h = 6$																	
	20	-1.91	-0.09	-0.09	-1.92	-0.19	-0.02	-1.67	-0.05	0.04	10.95	4.73	4.80	8.60	3.54	3.65	5.96	2.52	2.61
	50	-0.21	-0.15	-0.02	-0.14	-0.01	-0.07	-0.09	0.05	0.01	4.20	3.01	2.94	3.11	2.34	2.23	2.12	1.59	1.51
	100	-0.09	0.08	-0.04	0.09	0.04	-0.09	0.04	0.05	0.06	2.56	2.15	2.05	1.90	1.63	1.59	1.42	1.10	1.12
		$\rho = 0.8,\ h = 0.3N$																	
	20	-1.90	-0.25	-0.22	-1.86	-0.12	-0.08	-1.91	-0.21	-0.22	10.11	4.67	4.67	8.07	3.60	3.68	6.07	2.49	2.58
	50	-1.55	-0.11	-0.04	-1.48	-0.12	-0.03	-1.50	-0.02	0.03	8.70	2.88	2.97	6.38	2.19	2.24	4.52	1.58	1.54
	100	-1.17	-0.06	-0.15	-1.09	0.00	-0.13	-1.04	0.00	-0.01	7.62	2.12	2.06	5.32	1.56	1.64	3.38	1.12	1.15
		$\rho = 0.5,\ h = 2$																	
	20	0.09	-0.02	0.04	0.08	-0.12	0.05	-0.02	-0.05	-0.05	5.54	4.60	4.42	4.35	3.45	3.32	3.10	2.50	2.31
	50	-0.04	-0.12	-0.01	0.05	0.03	0.08	0.00	-0.05	0.07	3.74	2.96	2.70	2.88	2.15	2.13	1.96	1.49	1.52
	100	-0.04	-0.10	0.05	0.04	-0.02	-0.02	0.01	-0.03	0.03	2.65	2.02	2.02	2.32	1.57	1.59	1.57	1.07	1.06
		$\rho = 0.5,\ h = 6$																	
	20	0.08	-0.06	0.00	-0.07	-0.11	-0.02	-0.09	-0.01	-0.10	9.17	4.39	4.31	7.14	3.37	3.45	5.05	2.29	2.40
	50	-0.07	0.05	-0.01	0.01	-0.10	0.02	0.08	0.00	0.01	5.88	2.84	2.74	4.89	2.15	2.11	3.31	1.51	1.50
	100	-0.01	-0.05	0.00	0.03	-0.03	-0.06	-0.03	0.08	0.02	4.55	1.96	1.93	3.53	1.54	1.53	2.66	1.11	1.08
Infeasible		$\rho = 0.8,\ h = 6$																	
	20	-0.13	-0.08	-0.05	0.01	-0.09	-0.05	0.03	-0.03	0.03	4.43	4.58	4.32	3.13	3.27	3.38	2.14	2.42	2.44
	50	0.03	-0.07	0.02	0.00	0.02	-0.02	0.08	0.04	0.01	2.92	2.89	2.71	2.21	2.27	2.10	1.47	1.53	1.46
	100	-0.03	0.07	-0.02	0.02	-0.01	-0.07	0.04	0.03	0.06	2.08	2.04	1.91	1.57	1.57	1.52	1.15	1.07	1.05
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.03	0.02	0.07	-0.01	-0.10	0.05	-0.03	-0.09	-0.03	4.67	4.36	4.56	3.26	3.37	3.55	2.16	2.34	2.49
	50	-0.09	-0.03	-0.06	-0.01	-0.05	0.04	0.03	0.02	0.06	2.81	2.79	2.83	2.17	2.10	2.14	1.71	1.50	1.52
	100	0.01	-0.06	0.09	0.05	-0.04	-0.07	-0.02	0.03	0.00	2.07	2.08	2.00	1.79	1.51	1.55	1.27	1.08	1.08

Notes: The simulation results are based on the DGP specified in Section 4.

Table S19: The Size and Power for Pooled Estimators under Experiment 3

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
										$\rho = 0.5,\ h = 2$									
	20	0.074	0.073	0.078	0.071	0.072	0.075	0.062	0.051	0.048	0.985	0.890	0.898	1.000	0.991	0.995	1.000	1.000	1.000
	50	0.061	0.074	0.058	0.065	0.049	0.064	0.056	0.068	0.074	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.047	0.054	0.069	0.054	0.060	0.066	0.048	0.056	0.060	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.5,\ h = 6$									
	20	0.081	0.076	0.070	0.077	0.072	0.078	0.061	0.056	0.060	0.635	0.898	0.902	0.815	0.974	0.979	1.000	1.000	1.000
	50	0.065	0.070	0.065	0.064	0.056	0.065	0.054	0.052	0.058	0.983	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.054	0.054	0.051	0.066	0.052	0.048	0.052	0.056	0.054	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
CCEX-IV										$\rho = 0.8,\ h = 6$									
	20	0.069	0.087	0.078	0.069	0.056	0.064	0.057	0.057	0.061	0.875	0.888	0.912	0.965	0.980	0.980	1.000	1.000	1.000
	50	0.057	0.068	0.053	0.061	0.070	0.064	0.050	0.056	0.042	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.049	0.061	0.055	0.046	0.056	0.058	0.056	0.052	0.052	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 0.3N$									
	20	0.070	0.075	0.081	0.063	0.067	0.067	0.059	0.064	0.060	0.895	0.900	0.883	0.974	0.984	0.981	1.000	1.000	1.000
	50	0.060	0.057	0.066	0.047	0.055	0.056	0.056	0.052	0.062	0.961	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.062	0.061	0.048	0.044	0.046	0.058	0.046	0.066	0.064	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.5,\ h = 2$									
	20	0.080	0.080	0.084	0.093	0.096	0.074	0.091	0.098	0.072	0.985	0.889	0.895	1.000	0.993	0.994	1.000	1.000	1.000
	50	0.067	0.073	0.053	0.057	0.052	0.072	0.056	0.064	0.070	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.048	0.048	0.059	0.050	0.064	0.064	0.052	0.058	0.064	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.5,\ h = 6$									
	20	0.086	0.092	0.080	0.091	0.086	0.086	0.089	0.073	0.081	0.625	0.869	0.871	0.763	0.954	0.969	1.000	1.000	1.000
	50	0.067	0.064	0.061	0.066	0.057	0.076	0.074	0.056	0.066	0.975	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.050	0.052	0.058	0.068	0.054	0.054	0.068	0.068	0.056	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
CCE-IV										$\rho = 0.8,\ h = 6$									
	20	0.065	0.086	0.089	0.083	0.072	0.071	0.096	0.067	0.077	0.853	0.865	0.871	0.905	0.979	0.980	1.000	1.000	1.000
	50	0.061	0.062	0.064	0.063	0.069	0.067	0.048	0.068	0.040	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.049	0.061	0.054	0.042	0.060	0.060	0.054	0.052	0.058	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 0.3N$									
	20	0.094	0.076	0.085	0.094	0.073	0.079	0.116	0.078	0.085	0.861	0.875	0.880	0.954	0.969	0.981	1.000	1.000	1.000
	50	0.072	0.058	0.065	0.068	0.055	0.061	0.074	0.062	0.050	0.918	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.065	0.055	0.044	0.068	0.042	0.062	0.068	0.066	0.066	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.5,\ h = 2$									
	20	0.069	0.080	0.080	0.072	0.076	0.081	0.066	0.059	0.069	0.995	0.909	0.915	1.000	1.000	1.000	1.000	1.000	1.000
	50	0.056	0.073	0.048	0.060	0.056	0.067	0.058	0.052	0.066	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.059	0.051	0.063	0.064	0.052	0.048	0.048	0.056	0.070	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.5,\ h = 6$									
	20	0.063	0.073	0.075	0.071	0.068	0.088	0.046	0.059	0.058	0.955	0.903	0.911	1.000	1.000	0.989	1.000	1.000	1.000
	50	0.072	0.069	0.060	0.056	0.057	0.064	0.042	0.058	0.062	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.043	0.054	0.055	0.062	0.062	0.054	0.050	0.064	0.070	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Infeasible										$\rho = 0.8,\ h = 6$									
	20	0.070	0.074	0.071	0.071	0.044	0.061	0.049	0.064	0.055	0.994	0.915	0.921	1.000	0.999	1.000	1.000	1.000	1.000
	50	0.046	0.069	0.051	0.057	0.065	0.063	0.046	0.052	0.046	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.050	0.059	0.043	0.044	0.054	0.064	0.058	0.064	0.046	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
										$\rho = 0.8,\ h = 0.3N$									
	20	0.070	0.074	0.076	0.059	0.064	0.077	0.055	0.061	0.055	0.991	0.903	0.919	1.000	1.000	1.000	1.000	1.000	1.000
	50	0.058	0.053	0.054	0.046	0.060	0.052	0.054	0.060	0.064	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	100	0.064	0.069	0.050	0.056	0.042	0.052	0.052	0.066	0.058	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

S4.4 Simulation Results for Pooled Estimators under Experiment 4

In the main text we report the simulation results for the individual and Mean Group estimators under Experiment 4. Here we provide the additional simulation results for the Pooled estimator under Experiment 4 in Tables S20 and S21.

From Table S20 we observe that the Pooled estimators for (ρ, β_1, β_2) all exhibit large biases which do not disappear as either N or T increases, and they also suffer from severe size distortions in all the sample sizes, see Table S21. These results are in line with the results reported for Experiment 2 in Section S4.2, and verify Theorem 3 that the Pooled estimator is inconsistent if the parameters are heterogeneous.

Moreover, as mentioned in Section S4.2, we also provide the bias results of the CCEX-IV Pooled estimator under the different values of variance of ξ_{ρ_i} with $\rho = 0.5$ and $h = 2$ in Table S22 at the end of this Section. It clear shows that the bias of the Pooled estimator rises with $var(\xi_{\rho_i})$ in particular for the estimation of ρ.

Table S20: The Finite Sample Performance of Pooled Estimators under Experiment 4

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		$\rho = 0.5,\ h = 2$																	
	20	0.38	-0.17	-0.27	0.32	-0.14	-0.33	0.32	0.15	0.26	4.34	12.58	9.22	3.38	12.00	8.22	2.78	11.79	7.51
	50	0.36	-0.44	0.07	0.38	0.13	-0.34	0.33	0.07	-0.17	2.51	7.69	5.59	2.06	7.60	5.53	1.80	7.21	4.54
	100	0.32	0.08	0.03	0.28	0.10	0.13	0.17	0.31	0.00	1.70	5.62	4.08	1.35	5.31	3.99	1.18	5.33	3.34
		$\rho = 0.5,\ h = 6$																	
	20	0.35	0.03	0.05	-0.38	-1.13	0.17	0.43	0.69	-0.01	7.77	12.69	9.23	6.00	12.45	8.08	4.79	11.50	7.56
	50	0.34	-0.63	-0.09	0.28	-0.15	-0.29	0.24	0.660	0.36	3.73	7.51	5.70	3.14	7.40	4.82	2.25	6.90	4.96
	100	0.32	0.33	-0.01	0.21	0.23	0.75	0.20	-0.21	0.07	2.53	5.82	3.92	2.07	5.39	3.82	1.40	5.23	3.35
CCEX-IV		$\rho = 0.8,\ h = 6$																	
	20	0.37	-0.31	0.11	0.44	0.94	0.08	0.46	-0.03	0.25	5.79	12.61	9.07	4.15	12.28	8.13	3.22	12.06	7.33
	50	0.39	0.05	-0.32	0.43	0.51	0.80	0.43	0.24	0.05	2.38	8.13	5.60	2.02	7.52	4.97	1.63	7.10	4.49
	100	0.33	0.32	0.39	0.36	-0.36	-0.18	0.41	0.05	-0.04	1.42	5.83	4.08	1.33	5.67	3.36	1.14	5.09	3.48
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.42	0.22	-0.22	0.39	-0.36	0.34	0.40	-0.24	0.06	5.05	12.31	9.42	4.82	12.42	7.80	3.58	11.55	7.36
	50	0.43	0.17	-0.17	0.39	0.01	-0.03	0.33	0.11	-0.03	4.73	8.52	5.33	3.98	7.99	5.09	2.84	7.20	4.49
	100	-0.39	-0.08	0.30	-0.23	-0.76	0.21	0.33	0.29	0.33	4.60	6.16	3.93	3.67	5.67	3.60	2.82	5.35	3.38
		$\rho = 0.5,\ h = 2$																	
	20	-0.39	-0.38	-0.22	-0.46	-0.37	-0.04	-0.48	0.26	0.02	4.44	12.61	9.24	3.54	12.01	8.24	2.81	11.83	7.54
	50	0.42	-0.26	0.26	0.40	0.26	-0.29	0.41	0.31	-0.30	2.56	7.70	5.72	2.08	7.56	5.50	1.71	7.28	4.58
	100	0.44	0.06	-0.02	0.35	0.08	0.07	0.36	0.23	-0.03	1.67	5.66	3.98	1.45	5.31	3.91	1.30	5.22	3.45
		$\rho = 0.5,\ h = 6$																	
	20	-1.01	-0.13	-0.22	-1.25	-1.12	-0.03	-1.08	0.19	-0.17	8.09	12.74	9.15	6.48	12.48	8.10	4.99	11.50	7.58
	50	0.38	-0.79	-0.15	-0.62	-0.48	-0.16	0.25	0.28	0.35	3.85	7.54	5.65	3.08	7.40	4.64	2.23	6.96	4.95
	100	0.25	0.23	-0.03	-0.10	0.24	0.39	0.22	-0.17	0.08	2.58	5.83	3.93	2.05	5.46	3.81	1.45	5.29	3.21
CCE-IV		$\rho = 0.8,\ h = 6$																	
	20	-1.70	-0.49	0.10	-1.71	0.93	-0.10	-1.78	-0.16	-0.03	6.58	12.66	9.19	4.84	12.20	8.03	3.90	12.08	7.34
	50	0.41	0.88	-0.16	0.44	0.24	0.16	0.49	0.23	-0.06	2.36	8.15	5.54	2.01	7.55	4.84	1.79	7.09	4.31
	100	0.26	0.40	0.20	0.40	-0.32	-0.21	0.50	0.06	-0.07	1.50	5.84	4.05	1.34	5.48	3.46	0.95	5.23	3.50
		$\rho = 0.8,\ h = 0.3N$																	
	20	-1.77	0.13	-0.28	-1.89	-0.25	0.14	-1.74	-0.13	-0.12	5.98	12.35	9.23	5.29	12.36	7.90	4.14	11.39	7.44
	50	-1.28	0.19	-0.30	-1.38	-0.05	-0.09	-0.91	0.09	-0.09	5.24	8.43	5.42	4.38	8.04	5.18	3.35	7.25	4.49
	100	-1.68	-0.59	0.24	-1.48	-0.47	0.19	-0.83	0.54	0.24	4.99	6.18	3.89	3.96	5.78	3.66	3.07	5.35	3.32
		$\rho = 0.5,\ h = 2$																	
	20	0.37	-0.37	-0.29	0.40	-0.05	-0.18	0.39	0.32	0.25	3.60	12.41	9.12	2.92	11.87	8.22	2.52	11.75	7.54
	50	0.41	-0.29	0.25	0.33	0.23	-0.22	0.43	0.14	-0.20	2.24	7.59	5.65	1.97	7.53	5.60	1.78	7.26	4.51
	100	0.42	0.18	-0.06	0.44	0.10	0.17	0.29	0.23	-0.06	1.64	5.57	3.91	1.37	5.37	3.78	1.14	5.32	3.02
		$\rho = 0.5,\ h = 6$																	
	20	0.48	0.06	-0.11	-0.43	-1.18	-0.08	0.23	0.51	-0.17	4.65	12.68	9.22	3.72	12.42	8.03	2.97	11.46	7.61
	50	0.34	-0.41	-0.24	0.26	-0.37	-0.25	0.15	0.31	0.20	2.89	7.59	5.68	2.25	7.31	4.63	1.83	6.82	4.91
	100	0.29	0.24	-0.05	0.39	0.28	0.68	0.29	-0.29	0.03	2.17	5.61	3.85	1.71	5.42	3.75	1.19	5.15	3.28
Infeasible		$\rho = 0.8,\ h = 6$																	
	20	0.28	-0.17	0.15	0.29	1.04	0.00	0.29	-0.19	0.09	3.00	12.59	9.57	2.35	12.30	8.07	2.15	12.01	7.27
	50	0.53	0.02	-0.14	0.40	0.26	0.48	0.47	0.25	0.05	1.73	8.13	5.58	1.63	7.63	4.91	1.54	7.13	4.43
	100	0.40	0.26	0.21	0.43	-0.41	-0.29	0.39	0.05	0.01	1.29	5.82	4.04	0.99	5.61	3.34	1.03	5.16	3.45
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.36	0.04	-0.43	0.30	-0.62	0.14	-0.36	-0.47	0.06	2.73	12.29	9.25	2.73	12.08	7.76	2.45	11.52	7.28
	50	0.41	-0.18	-0.12	0.41	0.22	-0.02	0.31	0.22	-0.06	2.01	8.50	5.29	1.79	7.88	5.05	1.43	7.30	4.50
	100	-0.43	-0.24	0.26	-0.28	-0.16	0.26	0.33	0.24	0.27	1.33	6.16	3.93	1.27	5.61	3.73	1.16	5.31	3.45

Notes: The simulation results are based on the DGP specified in Section 4.

Table S21: The Size and Power for Pooled Estimators under Experiment 4

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
CCEX-IV	$\rho=0.5,\ h=2$																		
	20	0.081	0.061	0.091	0.089	0.050	0.077	0.087	0.071	0.076	0.562	0.261	0.377	0.717	0.290	0.534	0.874	0.333	0.579
	50	0.042	0.073	0.046	0.101	0.060	0.085	0.058	0.062	0.075	0.946	0.598	0.868	0.960	0.615	0.874	1.000	0.693	0.953
	100	0.078	0.050	0.072	0.071	0.061	0.061	0.074	0.057	0.061	1.000	0.843	0.971	1.000	0.969	0.992	1.000	0.987	1.000
	$\rho=0.5,\ h=6$																		
	20	0.073	0.082	0.067	0.108	0.056	0.074	0.068	0.062	0.078	0.274	0.255	0.455	0.347	0.303	0.448	0.477	0.366	0.528
	50	0.070	0.063	0.071	0.088	0.075	0.045	0.050	0.072	0.066	0.695	0.640	0.801	0.825	0.680	0.903	0.950	0.727	0.937
	100	0.066	0.078	0.053	0.070	0.049	0.063	0.075	0.055	0.045	0.921	0.926	0.992	1.000	0.969	1.000	0.999	0.981	1.000
	$\rho=0.8,\ h=6$																		
	20	0.091	0.050	0.083	0.103	0.073	0.064	0.081	0.101	0.056	0.407	0.278	0.444	0.522	0.298	0.514	0.740	0.311	0.567
	50	0.091	0.079	0.048	0.050	0.058	0.060	0.049	0.047	0.057	0.967	0.627	0.872	0.964	0.631	0.880	0.983	0.720	0.931
	100	0.056	0.050	0.066	0.101	0.089	0.058	0.071	0.040	0.052	1.000	0.840	0.988	1.000	0.895	0.987	0.986	0.920	1.000
	$\rho=0.8,\ h=0.3N$																		
	20	0.106	0.050	0.094	0.075	0.070	0.066	0.075	0.068	0.062	0.479	0.270	0.433	0.506	0.322	0.539	0.577	0.378	0.581
	50	0.051	0.068	0.059	0.067	0.075	0.082	0.076	0.065	0.041	0.480	0.479	0.835	0.647	0.521	0.862	0.888	0.534	0.931
	100	0.040	0.083	0.049	0.068	0.066	0.055	0.083	0.097	0.081	0.584	0.838	0.982	0.752	0.902	0.989	0.918	0.918	0.996
CCE-IV	$\rho=0.5,\ h=2$																		
	20	0.063	0.098	0.098	0.085	0.081	0.082	0.094	0.081	0.091	0.535	0.272	0.371	0.730	0.296	0.554	0.891	0.332	0.547
	50	0.049	0.074	0.074	0.081	0.084	0.082	0.064	0.074	0.084	0.933	0.589	0.834	0.962	0.637	0.866	0.980	0.668	0.936
	100	0.066	0.062	0.077	0.065	0.045	0.058	0.080	0.050	0.038	0.987	0.840	0.961	1.000	0.925	0.979	0.987	0.980	1.000
	$\rho=0.5,\ h=6$																		
	20	0.084	0.096	0.099	0.13	0.081	0.078	0.089	0.064	0.105	0.283	0.251	0.469	0.334	0.299	0.510	0.419	0.364	0.504
	50	0.069	0.051	0.084	0.110	0.063	0.045	0.041	0.044	0.084	0.682	0.631	0.803	0.848	0.658	0.938	0.947	0.728	0.918
	100	0.054	0.029	0.047	0.030	0.035	0.078	0.061	0.060	0.063	0.918	0.955	1.000	1.000	0.986	0.972	0.987	0.984	1.000
	$\rho=0.8,\ h=6$																		
	20	0.110	0.083	0.086	0.106	0.092	0.082	0.101	0.091	0.077	0.411	0.316	0.425	0.478	0.347	0.513	0.717	0.318	0.602
	50	0.068	0.042	0.042	0.047	0.046	0.064	0.057	0.056	0.068	0.959	0.618	0.868	0.984	0.623	0.879	0.998	0.712	0.910
	100	0.067	0.048	0.078	0.112	0.087	0.074	0.040	0.040	0.063	0.985	0.868	0.990	1.000	0.883	0.989	0.986	0.947	0.995
	$\rho=0.8,\ h=0.3N$																		
	20	0.093	0.076	0.081	0.094	0.082	0.097	0.090	0.092	0.062	0.431	0.274	0.420	0.462	0.281	0.513	0.537	0.383	0.562
	50	0.075	0.094	0.058	0.022	0.104	0.078	0.095	0.063	0.064	0.470	0.472	0.835	0.614	0.484	0.871	0.866	0.514	0.957
	100	0.037	0.063	0.047	0.051	0.073	0.048	0.058	0.090	0.074	0.572	0.818	0.981	0.730	0.915	0.976	0.888	0.916	0.993
Infeasible	$\rho=0.5,\ h=2$																		
	20	0.064	0.070	0.096	0.074	0.063	0.076	0.077	0.078	0.084	0.698	0.279	0.381	0.840	0.295	0.573	0.918	0.346	0.573
	50	0.031	0.058	0.055	0.093	0.078	0.076	0.080	0.059	0.070	0.968	0.586	0.856	0.987	0.629	0.883	0.979	0.763	0.993
	100	0.065	0.063	0.063	0.045	0.067	0.054	0.078	0.053	0.032	1.000	0.903	0.972	1.000	0.964	1.000	1.000	0.992	1.000
	$\rho=0.5,\ h=6$																		
	20	0.085	0.070	0.10	0.099	0.061	0.091	0.068	0.053	0.071	0.503	0.257	0.462	0.670	0.315	0.511	0.774	0.368	0.548
	50	0.063	0.067	0.062	0.094	0.065	0.042	0.059	0.022	0.081	0.894	0.655	0.815	0.944	0.683	0.911	0.996	0.733	0.939
	100	0.037	0.038	0.053	0.051	0.051	0.084	0.045	0.055	0.059	0.979	0.965	0.978	0.994	0.983	0.980	1.000	0.997	1.000
	$\rho=0.8,\ h=6$																		
	20	0.089	0.076	0.077	0.070	0.089	0.055	0.054	0.087	0.066	0.811	0.274	0.385	0.922	0.314	0.521	0.969	0.353	0.605
	50	0.061	0.056	0.046	0.041	0.058	0.085	0.073	0.052	0.064	0.989	0.667	0.879	1.000	0.636	0.898	1.000	0.745	0.926
	100	0.062	0.082	0.060	0.103	0.078	0.063	0.055	0.041	0.058	1.000	0.855	0.976	1.000	0.914	0.996	1.000	0.944	1.000
	$\rho=0.8,\ h=0.3N$																		
	20	0.076	0.069	0.096	0.079	0.073	0.060	0.065	0.058	0.067	0.856	0.261	0.438	0.889	0.327	0.555	0.944	0.416	0.595
	50	0.056	0.074	0.053	0.077	0.075	0.088	0.073	0.068	0.045	0.978	0.485	0.847	0.988	0.520	0.865	1.000	0.664	0.960
	100	0.045	0.083	0.044	0.084	0.060	0.079	0.060	0.075	0.074	1.000	0.815	0.989	1.000	0.884	0.984	1.000	0.918	0.997

Notes: The simulation results are based on the DGP specified in Section 4. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

Table S22: Biases (×100) of the CCEX-IV Pooled Estimator under Experiment 4

(N,T)	20			50			100		
	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
				$\rho_i \sim U(0.3, 0.7)$, $h = 2$					
20	0.33	0.49	0.30	0.41	0.19	0.19	0.30	0.32	-0.33
50	0.31	0.06	0.11	0.35	0.34	-0.16	0.32	-0.92	0.17
100	0.23	0.34	0.08	0.37	-0.20	-0.04	0.29	0.45	0.18
				$\rho_i \sim U(0.15, 0.85)$, $h = 2$					
20	0.58	-0.32	-0.09	0.71	-0.53	-0.30	0.55	-0.64	0.52
50	0.66	-0.19	0.06	0.76	0.25	-0.08	0.51	0.31	-0.5
100	0.73	-0.12	-0.36	0.77	-0.22	-0.16	0.60	-0.09	-0.10
				$\rho_i \sim U(0, 1)$, $h = 2$					
20	1.47	-0.42	0.02	1.49	0.46	-0.06	1.35	-0.10	-0.11
50	1.30	0.43	-0.11	1.40	-0.13	0.19	1.34	0.15	-0.05
100	1.75	-0.23	0.37	1.70	0.19	-0.36	1.29	0.75	-0.07

S4.5 Additional Simulations for Non-Zero Mean Factor Loadings

In this section, we provide the simulation results for non-zero mean factor loadings with $\gamma_{1i}(\gamma_{2i}) \sim IIDN(0.5, 0.5)$ under Experiment 4 in Tables S23 to S28.

The biases of the individual and Mean Group estimators are qualitatively similar to those reported in the main text. The biases of the CCEX-IV estimators for (ρ, β_1, β_2) are more or less negligible and close to those of the infeasible estimator in almost all cases. On the other hand, the CCE-IV estimator tends to display non-negligible biases as h or ρ rises, especially for $N = 20$. If the spatial weighing matrix is dense with $h = 0.3N$, then the bias of the CCE-IV estimator for the spatial coefficient does not disappear even for large N. Meanwhile, the Pooled estimators for (ρ, β_1, β_2) all exhibit large biases, which do not disappear even as N or T rises.

We find that RMSEs display some interesting patterns for the individual and Mean Group estimators. In terms of RMSEs CCE-IV outperform CCEX-IV for $\rho = 0.5, h = 2$ and $\rho = 0.5, h = 6$. This is not surprising since the biases of CCE-IV are now relatively small such that the efficiency gain of using $\bar{y}_t$ as an additional factor proxy, quickly offsets the biases, leading to the smaller RMSEs. However, for $\rho = 0.8, h = 6$, CCEX-IV outperforms CCE-IV for the estimation of ρ due to the higher biases of CCE-IV if N is small, whilst CCE-IV outperforms CCEX-IV only for large N. When $\rho = 0.8, h = 0.3N$, CCEX-IV outperforms CCE-IV for the estimation of ρ for all N.

The sizes of the t-test for the CCEX-IV individual and Mean Group estimators all tend to the 5% nominal level as the sample size increases. But, the test for CCE-IV spatial coefficient tends to display slight size distortions if N is small and for all N when $h = 0.3N$. Due to its biases, the Pooled estimator suffers from severe size distortion, which does not disappear even as the sample size rises.

The powers of the t-tests for CCEX-IV and CCE-IV estimators exhibit similar pattern to the RMSEs of the estimators; that is, CCE-IV tends to have higher power when $\rho = 0.5, h = 2$ and $\rho = 0.5, h = 6$, but they both tend to 1 as the sample size rises.

Table S23: The Finite Sample Performance of Individual Estimators under Experiment 4 with Non-Zero Mean Loadings

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
		$\rho = 0.5,\ h = 2$																	
	20	-0.03	-0.03	-0.07	0.03	-0.09	-0.08	-0.04	-0.01	0.02	13.01	15.44	15.30	9.13	11.44	11.50	6.06	7.98	8.05
	50	-0.04	0.04	-0.04	0.08	-0.09	-0.08	0.04	-0.02	-0.07	11.95	14.96	15.05	8.28	11.06	11.14	5.78	7.61	7.67
	100	-0.02	-0.05	0.00	0.02	-0.06	0.05	-0.01	-0.03	0.00	11.04	14.79	14.85	8.26	10.85	10.97	5.77	7.50	7.52
		$\rho = 0.5,\ h = 6$																	
	20	0.08	-0.08	-0.04	0.05	0.06	-0.06	0.08	-0.02	-0.04	28.47	14.96	15.12	19.44	11.03	11.02	14.02	7.91	7.76
	50	-0.01	-0.08	-0.03	-0.09	0.05	0.05	0.03	0.09	-0.04	20.09	14.61	14.62	14.65	10.87	10.83	10.15	7.67	7.61
	100	0.09	-0.09	0.07	0.07	-0.09	0.04	0.04	-0.08	-0.03	18.69	14.45	14.37	13.96	10.64	10.69	9.30	7.41	7.47
CCEX-IV		$\rho = 0.8,\ h = 6$																	
	20	-0.07	0.08	-0.05	-0.05	0.09	0.03	0.03	-0.01	0.05	21.61	16.00	15.99	15.21	11.39	11.47	10.64	7.99	8.11
	50	-0.07	0.02	-0.04	-0.03	0.03	0.03	-0.07	0.05	0.03	14.49	14.71	14.62	10.83	10.96	10.93	7.51	7.38	7.48
	100	0.02	-0.04	0.04	-0.01	0.05	-0.09	-0.08	0.09	-0.02	13.45	14.45	14.70	9.53	10.75	10.83	6.58	7.33	7.29
		$\rho = 0.8,\ h = 0.3N$																	
	20	-0.02	0.03	-0.06	0.00	-0.07	-0.06	-0.01	0.02	0.01	21.23	15.87	15.93	15.26	11.52	11.49	10.69	8.02	8.01
	50	0.02	0.08	-0.03	-0.03	-0.04	0.06	-0.03	-0.04	-0.06	29.74	14.82	14.94	23.01	10.84	10.95	16.80	7.49	7.52
	100	0.07	0.08	-0.02	-0.08	-0.05	0.00	0.05	-0.01	-0.10	39.85	14.26	14.19	31.67	10.33	10.34	21.60	7.13	7.17
		$\rho = 0.5,\ h = 2$																	
	20	-0.22	-0.08	-0.09	-0.22	-0.15	-0.09	-0.19	-0.04	0.03	12.58	14.94	14.52	8.79	10.82	10.85	5.69	7.71	7.44
	50	-0.11	0.03	-0.02	0.02	-0.11	-0.05	0.05	-0.02	-0.08	11.46	14.38	14.41	7.89	10.55	10.53	5.42	7.17	7.18
	100	-0.01	-0.02	0.00	0.01	-0.06	0.04	-0.03	-0.03	-0.01	10.59	14.22	14.26	7.79	10.37	10.46	5.47	7.08	7.12
		$\rho = 0.5,\ h = 6$																	
	20	-0.57	-0.21	-0.53	-0.55	0.01	-0.13	-0.63	-0.02	-0.11	28.36	14.86	14.83	19.23	10.68	10.81	13.84	7.89	7.51
	50	-0.11	-0.08	-0.03	-0.40	0.06	0.02	0.18	0.10	-0.06	19.68	14.27	14.25	14.17	10.54	10.46	9.54	7.28	7.18
	100	0.09	-0.08	0.10	0.04	-0.10	0.03	0.11	-0.08	-0.06	18.26	14.20	14.17	13.27	10.33	10.38	8.86	7.06	7.07
CCE-IV		$\rho = 0.8,\ h = 6$																	
	20	-2.48	0.16	-0.28	-3.37	0.01	-0.20	-3.75	-0.18	-0.22	22.93	15.56	15.83	16.98	10.90	10.92	12.52	7.69	7.58
	50	-0.27	0.01	-0.08	-0.26	0.06	0.01	-0.20	0.06	0.01	13.95	14.44	14.44	10.41	10.60	10.56	7.00	7.13	7.19
	100	-0.05	-0.03	0.04	-0.06	0.14	-0.07	-0.14	0.08	0.01	12.99	14.20	14.30	9.04	10.43	10.47	6.22	7.10	7.09
		$\rho = 0.8,\ h = 0.3N$																	
	20	-2.79	-0.03	-0.28	-3.08	-0.19	-0.29	-3.59	-0.05	-0.19	22.88	15.61	15.52	16.33	10.91	10.94	12.36	7.55	7.56
	50	-1.47	0.08	-0.07	-2.88	-0.08	0.02	-3.27	-0.10	-0.15	34.18	14.71	14.65	25.11	10.71	10.83	15.36	7.26	7.38
	100	-1.93	0.04	0.00	-2.53	-0.11	-0.06	-1.64	-0.04	-0.08	42.62	14.11	14.18	36.14	10.19	10.21	21.84	7.03	7.04
		$\rho = 0.5,\ h = 2$																	
	20	-0.02	-0.05	-0.03	0.02	-0.03	-0.07	-0.01	-0.06	0.01	9.74	13.62	13.48	6.89	10.01	10.15	4.40	6.87	6.84
	50	-0.03	0.04	0.03	0.04	-0.09	-0.03	0.04	-0.03	-0.04	9.92	13.58	13.60	6.80	10.02	9.96	4.67	6.77	6.84
	100	-0.03	-0.05	0.00	0.02	-0.03	0.06	0.02	-0.03	0.00	9.47	13.57	13.55	7.01	9.96	10.02	4.92	6.80	6.84
		$\rho = 0.5,\ h = 6$																	
	20	0.07	-0.03	-0.07	0.04	0.03	-0.06	0.01	0.00	-0.08	13.65	13.62	13.56	9.78	9.89	9.93	6.95	6.82	6.79
	50	-0.04	-0.06	-0.05	-0.01	0.06	0.05	0.04	0.02	-0.06	14.09	13.46	13.41	10.24	10.03	9.94	6.92	6.87	6.78
	100	0.03	-0.07	0.07	0.08	-0.05	0.03	0.12	-0.08	-0.05	14.34	13.52	13.46	10.47	9.92	9.96	7.06	6.78	6.81
Infeasible		$\rho = 0.8,\ h = 6$																	
	20	-0.03	0.07	-0.08	-0.04	0.70	-0.01	0.00	0.03	0.02	8.24	13.64	13.76	6.16	9.99	10.16	4.21	6.76	6.91
	50	-0.04	-0.01	-0.04	-0.09	-0.01	-0.01	-0.04	0.06	-0.01	9.20	13.60	13.70	6.79	10.02	10.12	4.68	6.78	6.90
	100	0.01	-0.04	0.04	-0.03	0.04	-0.06	-0.04	0.08	0.02	9.52	13.52	13.68	6.84	10.01	10.08	4.68	6.84	6.87
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.05	-0.04	-0.08	0.02	0.00	-0.05	0.01	0.02	0.00	8.23	13.48	13.64	5.82	9.93	10.18	4.14	6.86	6.86
	50	-0.03	0.10	0.00	-0.04	-0.01	0.08	-0.08	0.00	-0.02	10.83	13.58	13.66	8.02	9.97	9.97	5.50	6.88	6.85
	100	0.06	0.02	0.02	-0.02	-0.08	-0.06	0.04	-0.02	-0.05	13.02	13.63	13.56	9.67	9.98	9.99	6.63	6.86	6.93

Notes: The simulation results are based on the DGP specified in Section 4, but with γ_{1i} and γ_{2i} follow $IIDN(0.5, 0.5)$.

Table S24: The Size and Power of Individual Estimators under Experiment 4 with Non-Zero Mean Loadings

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
											$\rho=0.5,\ h=2$								
CCEX-IV	20	0.071	0.077	0.077	0.060	0.066	0.069	0.058	0.059	0.062	0.366	0.259	0.249	0.553	0.403	0.412	0.857	0.713	0.707
	50	0.069	0.076	0.076	0.054	0.061	0.060	0.050	0.051	0.052	0.387	0.257	0.259	0.629	0.433	0.424	0.889	0.728	0.730
	100	0.066	0.073	0.072	0.046	0.057	0.059	0.046	0.048	0.048	0.426	0.266	0.267	0.643	0.441	0.434	0.895	0.752	0.753
											$\rho=0.5,\ h=6$								
	20	0.050	0.067	0.064	0.048	0.066	0.066	0.054	0.062	0.055	0.158	0.250	0.249	0.232	0.432	0.415	0.344	0.691	0.695
	50	0.068	0.070	0.069	0.052	0.070	0.068	0.058	0.052	0.060	0.191	0.254	0.265	0.294	0.425	0.436	0.503	0.745	0.735
	100	0.069	0.068	0.068	0.060	0.065	0.067	0.051	0.057	0.047	0.201	0.265	0.269	0.312	0.439	0.442	0.575	0.754	0.765
											$\rho=0.8,\ h=6$								
	20	0.054	0.065	0.064	0.057	0.059	0.062	0.054	0.060	0.054	0.285	0.261	0.254	0.364	0.434	0.410	0.539	0.731	0.727
	50	0.062	0.078	0.077	0.055	0.067	0.065	0.053	0.055	0.060	0.325	0.257	0.259	0.486	0.426	0.439	0.716	0.743	0.734
	100	0.064	0.075	0.076	0.048	0.064	0.064	0.053	0.055	0.055	0.346	0.267	0.269	0.539	0.442	0.439	0.817	0.752	0.762
											$\rho=0.8,\ h=0.3N$								
	20	0.046	0.063	0.064	0.047	0.059	0.062	0.046	0.053	0.059	0.280	0.263	0.264	0.399	0.416	0.415	0.575	0.723	0.727
	50	0.056	0.056	0.059	0.056	0.053	0.053	0.053	0.053	0.051	0.199	0.253	0.246	0.251	0.434	0.413	0.359	0.728	0.736
	100	0.055	0.048	0.047	0.058	0.042	0.044	0.051	0.046	0.047	0.134	0.251	0.258	0.181	0.426	0.426	0.250	0.731	0.724
											$\rho=0.5,\ h=2$								
CCE-IV	20	0.073	0.071	0.069	0.064	0.058	0.059	0.070	0.059	0.054	0.365	0.267	0.261	0.585	0.448	0.456	0.904	0.751	0.748
	50	0.063	0.069	0.069	0.053	0.057	0.055	0.047	0.046	0.046	0.403	0.275	0.274	0.668	0.458	0.464	0.922	0.777	0.775
	100	0.063	0.068	0.067	0.054	0.054	0.056	0.044	0.046	0.046	0.443	0.275	0.282	0.671	0.466	0.461	0.921	0.785	0.789
											$\rho=0.5,\ h=6$								
	20	0.042	0.060	0.057	0.046	0.048	0.051	0.043	0.059	0.048	0.156	0.258	0.265	0.236	0.447	0.452	0.371	0.721	0.725
	50	0.063	0.066	0.064	0.047	0.056	0.054	0.045	0.050	0.047	0.192	0.267	0.277	0.305	0.445	0.468	0.535	0.769	0.775
	100	0.065	0.065	0.065	0.048	0.054	0.054	0.041	0.045	0.046	0.204	0.279	0.280	0.328	0.461	0.470	0.604	0.794	0.790
											$\rho=0.8,\ h=6$								
	20	0.032	0.054	0.052	0.320	0.047	0.048	0.033	0.047	0.045	0.276	0.254	0.258	0.361	0.444	0.406	0.511	0.732	0.719
	50	0.056	0.065	0.063	0.041	0.055	0.053	0.039	0.045	0.046	0.337	0.271	0.274	0.514	0.444	0.463	0.737	0.764	0.768
	100	0.060	0.062	0.062	0.046	0.052	0.052	0.052	0.045	0.045	0.359	0.279	0.280	0.562	0.464	0.469	0.849	0.783	0.787
											$\rho=0.8,\ h=0.3N$								
	20	0.320	0.052	0.052	0.033	0.045	0.045	0.034	0.046	0.044	0.267	0.262	0.268	0.382	0.418	0.410	0.560	0.725	0.729
	50	0.033	0.049	0.049	0.035	0.043	0.042	0.031	0.041	0.038	0.187	0.261	0.263	0.248	0.446	0.439	0.356	0.744	0.753
	100	0.037	0.042	0.041	0.038	0.037	0.036	0.034	0.038	0.038	0.133	0.256	0.262	0.180	0.438	0.445	0.256	0.748	0.747
											$\rho=0.5,\ h=2$								
Infeasible	20	0.064	0.066	0.063	0.051	0.054	0.057	0.051	0.053	0.052	0.501	0.293	0.307	0.753	0.475	0.489	0.980	0.800	0.808
	50	0.064	0.066	0.066	0.051	0.054	0.052	0.050	0.051	0.052	0.481	0.292	0.296	0.772	0.493	0.497	0.968	0.808	0.806
	100	0.064	0.067	0.066	0.051	0.052	0.053	0.051	0.051	0.053	0.508	0.293	0.301	0.754	0.482	0.493	0.956	0.820	0.813
											$\rho=0.5,\ h=6$								
	20	0.066	0.069	0.068	0.052	0.051	0.052	0.049	0.052	0.051	0.293	0.290	0.296	0.487	0.494	0.497	0.776	0.802	0.819
	50	0.063	0.066	0.065	0.050	0.053	0.051	0.052	0.053	0.049	0.278	0.294	0.305	0.467	0.475	0.500	0.772	0.804	0.814
	100	0.063	0.065	0.066	0.049	0.051	0.052	0.052	0.052	0.052	0.268	0.293	0.299	0.449	0.490	0.498	0.770	0.812	0.823
											$\rho=0.8,\ h=6$								
	20	0.064	0.065	0.065	0.049	0.051	0.055	0.050	0.048	0.052	0.596	0.291	0.297	0.827	0.493	0.478	0.985	0.817	0.795
	50	0.063	0.065	0.064	0.049	0.055	0.051	0.052	0.050	0.053	0.529	0.298	0.296	0.770	0.477	0.490	0.962	0.810	0.804
	100	0.061	0.064	0.064	0.051	0.051	0.053	0.051	0.051	0.053	0.499	0.294	0.300	0.751	0.490	0.494	0.969	0.815	0.818
											$\rho=0.8,\ h=0.3N$								
	20	0.062	0.067	0.067	0.050	0.049	0.054	0.052	0.053	0.051	0.594	0.289	0.291	0.858	0.497	0.484	0.987	0.794	0.811
	50	0.061	0.066	0.066	0.049	0.051	0.050	0.050	0.052	0.051	0.418	0.293	0.298	0.631	0.489	0.497	0.912	0.799	0.814
	100	0.059	0.066	0.064	0.048	0.051	0.051	0.051	0.052	0.053	0.318	0.288	0.298	0.499	0.479	0.492	0.805	0.809	0.810

Notes: The simulation results are based on the DGP specified in Section 4, but with γ_{1i} and γ_{2i} follow $IIDN(0.5, 0.5)$. The alternative for the power test is $\boldsymbol{\theta}_i^a = \boldsymbol{\theta}_i + 0.2$.

Table S25: The Finite Sample Performance of Mean Group Estimators under Experiment 4 with Non-Zero Mean Loadings

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
CCEX-IV		$\rho = 0.5,\ h = 2$																	
	20	0.07	-0.03	-0.09	0.08	-0.03	-0.02	0.02	0.05	-0.09	4.42	12.19	8.11	3.69	12.15	7.89	2.90	11.43	7.15
	50	-0.02	0.05	-0.01	-0.07	0.08	0.06	0.06	0.03	0.07	2.66	7.82	5.27	2.25	7.58	4.67	1.78	7.27	4.25
	100	-0.03	-0.07	0.09	0.02	-0.02	-0.04	-0.07	0.02	0.04	1.79	5.27	3.72	1.40	5.19	3.55	1.34	4.82	3.18
		$\rho = 0.5,\ h = 6$																	
	20	-0.05	-0.02	-0.04	-0.03	0.00	0.07	0.02	0.07	0.02	9.34	12.36	8.35	7.29	12.20	7.81	4.89	11.91	6.96
	50	0.07	0.05	-0.09	-0.09	-0.03	0.09	0.05	-0.07	-0.07	4.09	7.58	5.43	2.78	7.55	4.95	2.42	7.47	4.32
	100	0.07	-0.07	-0.02	-0.01	-0.03	0.06	0.05	-0.02	-0.08	2.76	5.35	3.65	2.05	5.44	3.57	1.67	5.22	3.18
		$\rho = 0.8,\ h = 6$																	
	20	0.03	-0.07	0.04	0.09	0.02	-0.04	-0.06	0.03	-0.03	8.25	12.41	8.55	6.75	12.14	7.67	4.28	11.62	6.94
	50	-0.04	-0.05	-0.08	-0.01	0.05	0.07	0.01	-0.03	-0.06	2.73	7.96	5.45	2.36	7.43	4.95	1.91	7.35	4.72
	100	0.07	0.02	-0.09	0.01	0.06	0.03	0.01	-0.02	0.03	1.74	5.73	3.77	1.50	5.41	3.43	1.29	5.33	3.24
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.05	0.06	0.01	0.02	0.03	-0.03	0.00	0.05	-0.02	7.80	12.34	8.32	6.59	12.09	7.82	4.23	11.10	7.14
	50	-0.08	-0.05	0.09	0.07	-0.08	0.08	0.04	0.07	0.03	6.71	7.73	5.25	5.24	7.42	5.10	4.05	7.09	4.53
	100	0.03	-0.03	-0.09	0.08	0.04	0.05	-0.01	0.06	-0.02	6.36	5.57	3.78	5.29	5.33	3.18	3.29	5.09	3.12
CCE-IV		$\rho = 0.5,\ h = 2$																	
	20	-0.20	-0.02	-0.10	-0.19	-0.05	-0.07	-0.26	0.09	-0.07	4.38	12.16	8.01	3.66	12.12	7.89	2.87	11.39	7.13
	50	-0.09	0.05	-0.01	0.08	0.11	0.08	0.03	0.02	0.09	2.62	7.80	5.27	2.23	7.56	4.67	1.77	7.27	4.24
	100	-0.05	-0.09	0.05	-0.01	-0.12	-0.40	-0.12	0.11	0.06	1.79	5.24	3.72	1.38	5.20	3.56	1.31	4.80	3.17
		$\rho = 0.5,\ h = 6$																	
	20	-2.38	-0.19	0.03	-2.33	-0.08	-0.06	-2.33	0.37	0.05	9.31	12.22	8.05	7.10	12.11	7.74	4.85	11.88	6.90
	50	-0.18	0.05	-0.06	-0.21	0.02	0.04	-0.07	-0.06	-0.13	3.84	7.52	5.32	2.83	7.48	4.95	2.40	7.46	4.34
	100	0.04	-0.12	-0.02	-0.06	-0.05	0.07	0.02	-0.02	-0.08	2.63	5.30	3.65	2.05	5.40	3.55	1.64	5.23	3.16
		$\rho = 0.8,\ h = 6$																	
	20	-3.95	-0.18	-0.12	-3.77	0.02	-0.03	-3.54	-0.07	-0.28	9.34	12.31	8.33	7.24	12.02	7.66	4.64	11.55	6.95
	50	-0.27	-0.11	-0.09	-0.19	0.04	-0.12	-0.16	0.24	0.02	2.68	7.94	5.41	2.28	7.39	4.92	1.87	7.34	4.72
	100	0.03	0.03	-0.08	0.01	0.07	0.04	0.00	-0.01	0.02	1.73	5.80	3.74	1.46	5.43	3.43	1.26	5.34	3.22
		$\rho = 0.8,\ h = 0.3N$																	
	20	-3.81	-0.04	-0.16	-3.43	-0.16	-0.46	-3.61	-0.07	-0.27	8.86	12.36	8.21	7.40	12.03	7.76	4.73	11.13	7.13
	50	-2.77	-0.15	0.05	-2.62	-0.15	0.09	-2.90	0.01	0.04	9.26	7.72	5.31	6.56	7.50	5.09	4.44	7.04	4.49
	100	-2.76	-0.33	0.01	-3.04	0.13	-0.04	-2.85	0.06	-0.02	10.11	5.52	3.72	6.43	5.32	3.19	4.00	5.12	3.15
Infeasible		$\rho = 0.5,\ h = 2$																	
	20	0.04	-0.08	-0.02	0.05	-0.03	-0.02	-0.04	0.04	-0.05	3.72	12.07	7.92	3.16	12.14	7.80	2.64	11.39	7.04
	50	-0.02	0.02	-0.04	-0.07	0.08	0.07	0.07	0.01	0.05	2.33	7.80	5.22	2.09	7.52	4.62	1.75	7.24	4.25
	100	0.02	-0.06	0.02	0.02	-0.01	-0.04	-0.11	0.16	0.04	1.59	5.20	3.67	1.31	5.18	3.45	1.29	4.76	3.12
		$\rho = 0.5,\ h = 6$																	
	20	-0.05	-0.09	-0.02	0.01	-0.03	0.07	-0.02	0.06	0.02	4.42	12.26	8.01	3.47	12.14	7.64	2.68	11.84	6.83
	50	-0.05	0.13	-0.08	-0.06	0.03	0.02	0.08	-0.04	-0.03	2.96	7.49	5.26	2.28	7.43	4.92	1.94	7.47	4.31
	100	0.00	-0.05	-0.05	0.03	-0.02	0.05	-0.02	-0.02	-0.08	2.11	5.24	3.59	1.65	5.39	3.52	1.40	5.26	3.14
		$\rho = 0.8,\ h = 6$																	
	20	-0.04	-0.08	0.01	-0.10	0.01	-0.03	-0.02	0.09	-0.03	2.98	12.23	8.31	2.85	11.73	7.35	2.58	11.54	6.89
	50	-0.06	-0.03	-0.03	0.00	-0.03	0.09	-0.01	-0.03	-0.02	1.96	7.92	5.40	1.78	7.37	4.81	1.64	7.32	4.74
	100	0.02	0.07	-0.04	0.04	0.00	0.03	0.00	-0.01	0.03	1.36	5.73	3.74	1.28	5.08	3.41	1.18	5.33	3.19
		$\rho = 0.8,\ h = 0.3N$																	
	20	-0.03	0.08	0.03	0.03	0.01	-0.04	0.06	0.04	-0.02	2.95	12.41	8.33	2.75	10.99	7.71	2.78	11.07	7.02
	50	-0.02	0.07	0.09	0.02	-0.04	0.03	0.07	0.06	0.09	2.12	7.65	5.18	1.78	7.38	5.08	1.75	7.00	4.50
	100	-0.01	-0.03	0.08	0.07	0.05	-0.03	-0.02	0.06	-0.02	1.59	5.54	3.70	1.38	5.34	3.14	1.14	5.12	3.17

Notes: The simulation results are based on the DGP specified in Section 4, but with γ_{1i} and γ_{2i} follow $IIDN(0.5, 0.5)$.

Table S26: The Size and Power of Mean Group Estimators under Experiment 4 with Non-Zero Mean Loadings

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
CCEX-IV											$\rho = 0.5,\ h = 2$								
	20	0.064	0.057	0.062	0.061	0.064	0.061	0.047	0.049	0.060	0.632	0.251	0.407	0.682	0.294	0.495	0.868	0.361	0.612
	50	0.049	0.060	0.049	0.050	0.052	0.046	0.056	0.056	0.048	0.955	0.578	0.862	0.986	0.622	0.934	1.000	0.662	0.952
	100	0.048	0.053	0.046	0.054	0.054	0.056	0.050	0.045	0.055	1.000	0.866	0.994	1.000	0.920	0.994	1.000	0.955	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.072	0.061	0.072	0.062	0.073	0.063	0.063	0.055	0.063	0.193	0.187	0.475	0.294	0.265	0.529	0.553	0.312	0.661
	50	0.052	0.044	0.062	0.058	0.050	0.05	0.052	0.046	0.056	0.658	0.608	0.825	0.856	0.618	0.914	0.978	0.690	0.960
	100	0.048	0.045	0.047	0.058	0.066	0.064	0.055	0.050	0.055	0.925	0.853	0.996	0.992	0.926	0.998	1.000	0.940	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.062	0.072	0.054	0.064	0.052	0.058	0.052	0.054	0.054	0.348	0.199	0.494	0.478	0.273	0.550	0.666	0.354	0.605
	50	0.055	0.073	0.055	0.064	0.052	0.046	0.044	0.058	0.067	0.897	0.507	0.862	0.954	0.662	0.916	1.000	0.672	0.916
	100	0.050	0.048	0.050	0.052	0.046	0.056	0.045	0.055	0.055	0.998	0.828	0.986	1.000	0.913	0.998	1.000	0.946	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.054	0.062	0.062	0.067	0.059	0.066	0.053	0.056	0.061	0.361	0.206	0.481	0.432	0.313	0.548	0.679	0.329	0.592
	50	0.065	0.069	0.059	0.052	0.052	0.058	0.054	0.046	0.058	0.412	0.518	0.888	0.612	0.608	0.896	0.938	0.762	0.942
	100	0.065	0.060	0.063	0.056	0.050	0.054	0.055	0.060	0.045	0.381	0.806	0.939	0.662	0.922	0.996	0.940	0.945	1.000
CCE-IV											$\rho = 0.5,\ h = 2$								
	20	0.066	0.077	0.078	0.080	0.072	0.078	0.087	0.074	0.072	0.666	0.252	0.405	0.694	0.288	0.491	0.899	0.341	0.628
	50	0.053	0.068	0.056	0.048	0.068	0.046	0.054	0.056	0.042	0.965	0.556	0.877	0.992	0.630	0.950	1.000	0.662	0.964
	100	0.052	0.042	0.046	0.048	0.058	0.052	0.054	0.045	0.040	1.000	0.876	0.994	1.000	0.916	0.994	1.000	0.970	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.075	0.075	0.066	0.078	0.080	0.067	0.091	0.095	0.070	0.208	0.206	0.493	0.306	0.231	0.543	0.576	0.308	0.673
	50	0.053	0.049	0.060	0.054	0.046	0.064	0.054	0.062	0.056	0.723	0.620	0.853	0.892	0.638	0.912	0.976	0.700	0.956
	100	0.052	0.044	0.048	0.062	0.072	0.068	0.050	0.045	0.065	0.955	0.867	0.996	0.994	0.920	0.998	1.000	0.940	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.068	0.077	0.074	0.087	0.065	0.066	0.114	0.067	0.061	0.279	0.224	0.487	0.394	0.273	0.569	0.638	0.363	0.608
	50	0.048	0.081	0.061	0.060	0.054	0.060	0.054	0.058	0.080	0.951	0.492	0.863	0.980	0.670	0.912	1.000	0.686	0.920
	100	0.056	0.062	0.048	0.044	0.040	0.060	0.046	0.065	0.055	0.998	0.828	0.992	1.000	0.916	0.998	1.000	0.947	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.075	0.074	0.070	0.079	0.070	0.077	0.107	0.066	0.065	0.266	0.196	0.486	0.400	0.314	0.553	0.644	0.327	0.594
	50	0.070	0.063	0.051	0.080	0.062	0.068	0.116	0.052	0.074	0.346	0.501	0.887	0.464	0.600	0.898	0.744	0.782	0.950
	100	0.067	0.057	0.059	0.096	0.060	0.044	0.105	0.055	0.055	0.284	0.792	0.941	0.528	0.912	0.992	0.830	0.945	1.000
Infeasible											$\rho = 0.5,\ h = 2$								
	20	0.067	0.054	0.063	0.066	0.065	0.064	0.056	0.060	0.066	0.715	0.286	0.425	0.814	0.318	0.498	0.937	0.376	0.633
	50	0.051	0.067	0.058	0.052	0.056	0.052	0.044	0.050	0.042	0.985	0.589	0.884	0.992	0.656	0.930	1.000	0.662	0.966
	100	0.054	0.040	0.050	0.046	0.054	0.046	0.045	0.045	0.040	1.000	0.874	0.994	1.000	0.922	0.996	1.000	0.970	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.060	0.062	0.067	0.053	0.048	0.057	0.056	0.083	0.062	0.595	0.215	0.493	0.795	0.269	0.552	0.930	0.315	0.67
	50	0.062	0.045	0.053	0.040	0.050	0.064	0.048	0.052	0.056	0.897	0.626	0.854	0.982	0.652	0.9222	0.998	0.708	0.970
	100	0.051	0.045	0.040	0.046	0.070	0.064	0.050	0.050	0.065	0.996	0.869	0.998	1.000	0.922	0.998	1.000	0.950	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.061	0.057	0.061	0.052	0.052	0.057	0.052	0.058	0.054	0.873	0.306	0.506	0.891	0.326	0.596	0.954	0.378	0.629
	50	0.052	0.078	0.054	0.060	0.054	0.044	0.046	0.056	0.054	0.995	0.493	0.897	1.000	0.682	0.936	1.000	0.690	0.950
	100	0.056	0.058	0.052	0.050	0.036	0.054	0.045	0.065	0.045	1.000	0.830	0.992	1.000	0.925	0.998	1.000	0.965	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.064	0.062	0.067	0.056	0.060	0.062	0.059	0.056	0.058	0.895	0.294	0.504	0.940	0.385	0.561	0.934	0.392	0.617
	50	0.059	0.061	0.054	0.040	0.048	0.066	0.053	0.054	0.062	0.980	0.554	0.888	1.000	0.626	0.902	1.000	0.797	0.958
	100	0.052	0.053	0.050	0.054	0.062	0.044	0.053	0.057	0.060	1.000	0.804	0.991	1.000	0.918	0.998	1.000	0.949	1.000

Notes: The simulation results are based on the DGP specified in Section 4, but with γ_{1i} and γ_{2i} follow $IIDN(0.5, 0.5)$. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

Table S27: The Finite Sample Performance of Pooled Estimators under Experiment 4 with Non-Zero Mean Loadings

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
										$\rho = 0.5,\ h = 2$									
	20	0.26	-0.07	-0.22	0.35	0.06	-0.11	0.15	-0.60	-0.16	4.32	12.87	8.75	4.05	12.54	8.31	3.12	11.78	7.40
	50	0.17	0.46	-0.03	0.33	0.07	0.07	0.24	0.11	0.21	2.87	8.12	5.65	2.40	7.52	5.08	1.91	7.31	4.44
	100	0.24	-0.24	0.02	0.24	-0.10	-0.45	0.19	0.11	0.05	1.93	5.51	4.12	1.57	5.37	3.71	1.46	4.82	3.33
										$\rho = 0.5,\ h = 6$									
	20	-0.19	-0.22	0.26	0.12	-0.29	0.02	0.20	0.44	0.13	8.75	12.79	8.46	6.96	12.41	8.17	4.72	12.11	7.14
	50	0.12	0.02	-0.06	0.14	-0.41	-0.42	0.22	-0.07	-0.14	4.26	7.96	5.77	3.27	7.75	5.16	2.59	7.66	4.51
	100	0.14	-0.11	0.00	-0.21	-0.02	0.02	0.17	-0.23	-0.15	2.85	5.48	3.88	2.28	5.50	3.71	1.82	5.37	3.19
CCEX-IV										$\rho = 0.8,\ h = 6$									
	20	-0.36	-0.12	-0.04	-0.28	0.03	-0.44	0.23	0.34	-0.09	6.29	13.04	8.88	5.11	12.18	7.96	4.08	11.75	7.17
	50	0.25	-0.03	-0.13	0.21	-0.23	-0.32	0.36	0.31	-0.09	2.80	8.34	5.90	2.37	7.69	5.29	2.09	7.54	4.92
	100	0.25	0.12	-0.16	0.27	0.15	-0.01	0.24	-0.13	0.00	1.84	5.90	3.95	1.62	5.29	3.55	1.41	5.44	3.34
										$\rho = 0.8,\ h = 0.3N$									
	20	0.34	-0.20	-0.10	0.27	-0.04	-0.41	0.20	-0.01	-0.21	6.17	13.09	8.82	5.14	11.58	8.01	4.23	11.29	7.35
	50	0.50	-0.19	0.06	0.43	-0.24	0.06	0.27	0.15	0.08	5.31	8.16	5.59	4.63	7.83	5.28	2.89	7.24	4.65
	100	0.12	-0.20	0.08	-0.28	0.16	-0.02	0.19	0.27	0.10	5.09	5.70	3.98	4.33	5.58	3.33	2.96	5.26	3.25
										$\rho = 0.5,\ h = 2$									
	20	-0.16	-0.05	-0.25	-0.21	0.06	-0.14	-0.22	-0.53	-0.19	4.26	12.82	8.70	4.07	12.53	8.31	3.11	11.74	7.38
	50	0.19	0.48	0.00	0.25	0.01	0.08	0.14	0.09	0.20	2.88	8.13	5.67	2.41	7.49	5.05	1.90	7.32	4.43
	100	0.19	-0.19	-0.07	0.19	-0.07	-0.47	0.14	0.11	0.06	1.96	5.59	4.13	1.56	5.34	3.73	1.44	4.84	3.29
										$\rho = 0.5,\ h = 6$									
	20	-1.45	-0.26	0.10	-1.16	-0.36	-0.08	-1.19	0.38	0.04	8.73	12.83	8.43	6.92	12.37	8.16	4.74	12.09	7.12
	50	-0.10	0.06	-0.04	-0.11	-0.39	-0.42	0.15	-0.06	-0.15	4.23	8.03	5.74	3.30	7.71	5.12	2.56	7.69	4.53
	100	-0.21	-0.10	-0.01	-0.24	-0.05	0.04	0.17	-0.23	-0.16	2.86	5.48	3.89	2.29	5.47	3.70	1.81	5.39	3.16
CCE-IV										$\rho = 0.8,\ h = 6$									
	20	-1.96	-0.12	-0.10	-1.90	0.00	-0.48	-1.61	0.26	-0.16	6.96	13.01	8.91	5.65	12.22	7.92	4.49	11.74	7.15
	50	-0.21	0.01	-0.16	-0.18	-0.25	-0.33	0.16	0.32	-0.04	2.82	8.35	5.89	2.39	7.65	5.31	2.03	7.52	4.91
	100	0.17	0.14	-0.17	0.19	0.16	0.01	0.13	-0.13	-0.02	1.86	5.99	3.97	1.61	5.33	3.53	1.39	5.40	3.34
										$\rho = 0.8,\ h = 0.3N$									
	20	-1.82	-0.29	-0.25	-1.54	-0.12	-0.53	-1.58	-0.02	-0.31	6.57	13.06	8.86	5.55	11.54	8.04	4.73	11.25	7.37
	50	-0.89	-0.17	-0.02	-0.65	-0.24	0.07	-1.07	0.14	0.11	5.83	8.13	5.62	4.56	7.83	5.33	3.14	7.26	4.63
	100	-0.87	-0.19	0.07	-1.04	0.20	-0.04	-0.91	0.25	0.10	5.17	5.66	3.99	4.47	5.58	3.33	3.18	5.27	3.26
										$\rho = 0.5,\ h = 2$									
	20	0.20	-0.01	-0.23	0.33	0.11	-0.12	0.10	-0.51	-0.18	3.76	12.59	8.58	3.53	12.49	8.19	2.97	11.84	7.31
	50	0.20	0.45	-0.11	0.29	0.07	0.04	0.24	0.06	0.22	2.57	8.02	5.65	2.28	7.53	4.97	1.93	7.31	4.44
	100	0.27	-0.19	0.01	0.24	-0.05	-0.45	0.15	0.15	0.01	1.85	5.50	4.12	1.52	5.39	3.69	1.42	4.77	3.26
										$\rho = 0.5,\ h = 6$									
	20	-0.43	-0.34	0.21	0.18	-0.26	0.05	-0.22	0.33	0.03	4.92	12.95	8.31	3.89	12.41	8.08	3.19	12.10	7.12
	50	0.13	0.09	-0.01	0.13	-0.32	-0.51	0.22	-0.11	-0.08	3.33	7.95	5.76	2.63	7.60	5.12	2.20	7.66	4.55
	100	-0.11	-0.13	-0.03	0.25	0.00	0.12	0.11	-0.20	-0.16	2.32	5.46	3.83	1.94	5.54	3.70	1.55	5.39	3.20
Infeasible										$\rho = 0.8,\ h = 6$									
	20	-0.16	0.07	-0.05	-0.18	0.11	-0.46	0.28	0.41	-0.06	3.17	13.12	9.04	3.04	12.21	8.02	2.77	11.88	7.30
	50	0.12	-0.11	-0.13	0.16	-0.36	-0.23	0.25	0.32	-0.02	2.13	8.32	5.97	1.97	7.76	5.29	1.82	7.55	4.87
	100	0.27	0.16	-0.17	0.23	0.20	0.01	0.21	-0.10	0.04	1.53	5.88	4.02	1.43	5.31	3.55	1.31	5.34	3.42
										$\rho = 0.8,\ h = 0.3N$									
	20	0.20	-0.15	-0.07	0.22	-0.04	-0.44	0.13	0.07	-0.26	3.19	13.17	8.89	2.91	11.80	8.12	2.90	11.39	7.42
	50	0.10	-0.25	-0.02	0.14	-0.27	-0.03	0.21	0.14	0.14	2.34	8.21	5.58	1.98	7.86	5.27	1.83	7.39	4.87
	100	0.21	-0.28	0.08	0.26	0.17	-0.03	-0.24	0.30	0.09	1.74	5.71	3.98	1.52	5.62	3.31	1.28	5.28	3.30

Notes: The simulation results are based on the DGP specified in Section 4, but with γ_{1i} and γ_{2i} follow $IIDN(0.5, 0.5)$.

Table S28: The Size and Power of Pooled Estimators under Experiment 4 with Non-Zero Mean Loadings

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
											$\rho = 0.5,\ h = 2$								
CCEX-IV	20	0.082	0.074	0.067	0.078	0.084	0.079	0.070	0.074	0.079	0.574	0.254	0.508	0.650	0.270	0.539	0.856	0.309	0.563
	50	0.065	0.059	0.061	0.064	0.050	0.052	0.058	0.064	0.040	0.905	0.593	0.795	0.968	0.634	0.896	0.998	0.722	0.944
	100	0.048	0.044	0.056	0.050	0.058	0.050	0.045	0.040	0.045	1.000	0.916	0.988	1.000	0.926	0.990	1.000	0.955	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.086	0.079	0.062	0.086	0.072	0.083	0.068	0.082	0.068	0.236	0.253	0.423	0.258	0.253	0.452	0.519	0.286	0.632
	50	0.072	0.054	0.071	0.046	0.060	0.068	0.044	0.068	0.056	0.597	0.582	0.764	0.846	0.614	0.860	0.964	0.660	0.942
	100	0.061	0.047	0.052	0.054	0.068	0.062	0.070	0.055	0.040	0.917	0.906	0.981	0.988	0.924	0.994	1.000	0.940	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.080	0.075	0.073	0.085	0.066	0.068	0.085	0.061	0.060	0.346	0.325	0.472	0.463	0.347	0.534	0.632	0.349	0.599
	50	0.056	0.072	0.057	0.060	0.050	0.070	0.066	0.066	0.070	0.907	0.592	0.811	0.962	0.662	0.838	1.000	0.696	0.898
	100	0.052	0.052	0.036	0.054	0.026	0.058	0.085	0.060	0.055	0.996	0.868	0.982	1.000	0.928	0.996	1.000	0.895	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.075	0.074	0.072	0.085	0.070	0.073	0.084	0.063	0.058	0.354	0.251	0.464	0.472	0.311	0.494	0.629	0.338	0.579
	50	0.069	0.055	0.064	0.072	0.070	0.070	0.076	0.048	0.066	0.437	0.538	0.813	0.683	0.676	0.852	0.920	0.707	0.942
	100	0.061	0.055	0.056	0.074	0.058	0.040	0.070	0.050	0.045	0.467	0.868	0.986	0.672	0.870	0.990	0.895	0.935	1.000
											$\rho = 0.5,\ h = 2$								
CCE-IV	20	0.096	0.093	0.098	0.084	0.094	0.088	0.085	0.098	0.092	0.586	0.256	0.469	0.638	0.283	0.546	0.868	0.331	0.565
	50	0.068	0.065	0.059	0.074	0.048	0.060	0.062	0.066	0.044	0.916	0.596	0.797	0.972	0.646	0.892	0.998	0.730	0.956
	100	0.058	0.050	0.064	0.050	0.052	0.050	0.070	0.040	0.04	1.000	0.918	0.978	1.000	0.936	0.990	1.000	0.960	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.098	0.099	0.073	0.096	0.091	0.094	0.084	0.106	0.079	0.237	0.248	0.425	0.284	0.264	0.485	0.528	0.296	0.654
	50	0.068	0.061	0.080	0.052	0.068	0.058	0.058	0.068	0.062	0.636	0.590	0.796	0.816	0.598	0.869	0.968	0.646	0.938
	100	0.070	0.050	0.053	0.064	0.064	0.068	0.075	0.055	0.035	0.921	0.904	0.980	0.988	0.928	0.992	1.000	0.935	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.092	0.089	0.088	0.104	0.082	0.095	0.107	0.077	0.065	0.343	0.319	0.436	0.453	0.339	0.521	0.621	0.342	0.604
	50	0.061	0.085	0.064	0.068	0.052	0.074	0.066	0.074	0.074	0.905	0.571	0.806	0.962	0.690	0.820	0.998	0.706	0.892
	100	0.062	0.070	0.048	0.062	0.034	0.064	0.080	0.055	0.055	0.996	0.864	0.982	1.000	0.926	0.994	1.000	0.910	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.076	0.098	0.088	0.091	0.088	0.088	0.116	0.079	0.071	0.345	0.246	0.456	0.430	0.315	0.472	0.613	0.323	0.559
	50	0.079	0.057	0.071	0.064	0.080	0.086	0.084	0.056	0.070	0.402	0.532	0.810	0.646	0.664	0.836	0.874	0.706	0.944
	100	0.046	0.065	0.058	0.082	0.058	0.038	0.085	0.050	0.050	0.454	0.855	0.987	0.638	0.868	0.990	0.870	0.935	0.995
											$\rho = 0.5,\ h = 2$								
Infeasible	20	0.087	0.082	0.084	0.086	0.079	0.081	0.085	0.082	0.074	0.629	0.294	0.509	0.749	0.317	0.548	0.877	0.341	0.586
	50	0.057	0.062	0.062	0.066	0.050	0.048	0.060	0.056	0.044	0.958	0.619	0.801	0.982	0.646	0.898	1.000	0.748	0.968
	100	0.052	0.046	0.062	0.052	0.062	0.058	0.060	0.035	0.040	1.000	0.920	0.992	1.000	0.942	1.000	1.000	0.970	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.077	0.084	0.063	0.069	0.086	0.078	0.063	0.086	0.069	0.542	0.231	0.493	0.644	0.262	0.486	0.887	0.295	0.654
	50	0.072	0.062	0.066	0.054	0.058	0.058	0.056	0.062	0.052	0.824	0.591	0.798	0.950	0.617	0.884	0.994	0.702	0.942
	100	0.052	0.049	0.049	0.046	0.068	0.066	0.050	0.050	0.040	0.982	0.896	0.988	1.000	0.931	1.000	1.000	0.935	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.081	0.075	0.073	0.089	0.065	0.075	0.089	0.067	0.063	0.796	0.328	0.486	0.824	0.344	0.545	0.912	0.350	0.611
	50	0.064	0.073	0.070	0.072	0.056	0.058	0.068	0.064	0.064	0.985	0.604	0.812	0.998	0.693	0.838	1.000	0.707	0.898
	100	0.060	0.052	0.050	0.066	0.036	0.062	0.065	0.065	0.060	1.000	0.876	0.984	1.000	0.934	1.000	1.000	0.900	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.066	0.080	0.080	0.063	0.080	0.074	0.087	0.062	0.065	0.863	0.266	0.483	0.880	0.319	0.481	0.892	0.344	0.586
	50	0.067	0.053	0.062	0.062	0.068	0.070	0.062	0.054	0.066	0.961	0.547	0.817	0.996	0.684	0.861	1.000	0.708	0.956
	100	0.060	0.059	0.059	0.062	0.074	0.038	0.070	0.050	0.050	0.997	0.864	0.992	1.000	0.880	1.000	1.000	0.935	1.000

Notes: The simulation results are based on the DGP specified in Section 4, but with γ_{1i} and γ_{2i} follow $IIDN(0.5, 0.5)$. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

S4.6 Additional Simulation for $\kappa_1 = 1$ and $\kappa_2 = 2$

In this section, we provide the simulation results for $\kappa_1 = 1$ and $\kappa_2 = 2$ under Experiment 4 in Tables S29 to S34.

Overall the results here are quantatitively similar to the results for $\kappa_1 = 2$ and $\kappa_2 = 3$ under Experiment 4 reported in Sections 4.1 and S4.4. Specifically, the biases of the CCEX-IV individual estimator of $(\rho_i, \beta_{1i}, \beta_{2i})$ and their Mean Group counterparts are more or less negligible, and close to those of the infeasible estimator in almost all cases. On the other hand, the CCE-IV estimators display non-negligible biases as h or ρ rises, especially for $N = 20$. The Pooled estimators of (ρ, β_1, β_2) exhibit large biases, which do not disappear even as N or T rises.

RMSEs for the CCE-IV estimator is no smaller than the CCEX-IV counterparts in most cases. In particular, if the spatial weighing matrix is dense with $h = 0.3N$, then RMSEs of CCE-IV estimator for ρ are significantly higher than the CCEX-IV counterparts for all the sample sizes.

As the sample size increases, the size of the t-test for CCEX-IV estimators tends to the 5% nominal level in most cases. But, the t-test for the CCE-IV spatial coefficient displays slight size distortion for small N and for all N when $h = 0.3N$, which does not disappear with T. The powers of t-tests for CCEX-IV and CCE-IV estimators are more or less similar, and both tend to 1 as the sample size rises if h is fixed. In the case with $h = 0.3N$, the power of the t-test for the individual spatial coefficient falls with N due to RMSEs increasing with N, whilst rising sharply with T.

Table S29: The Finite Sample Performance of Individual Estimators under Experiment 4 with $\kappa_1 = 1$ and $\kappa_2 = 2$

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
										$\rho = 0.5,\ h = 2$									
CCEX-IV	20	0.07	-0.07	-0.02	-0.03	-0.06	-0.03	0.01	0.07	-0.05	9.57	12.30	12.12	7.33	9.20	9.14	5.32	6.56	6.51
	50	0.03	0.00	0.04	-0.02	0.00	0.00	0.00	-0.04	0.08	9.04	11.88	11.85	6.52	8.75	8.82	4.56	6.05	6.07
	100	-0.02	-0.03	0.01	0.00	0.06	-0.01	-0.04	-0.04	0.08	8.90	11.61	11.63	6.54	8.56	8.63	4.45	5.89	5.94
										$\rho = 0.5,\ h = 6$									
	20	-0.03	-0.09	0.09	0.09	-0.08	-0.03	-0.09	0.00	-0.03	21.05	12.35	12.41	17.09	9.27	9.36	11.49	6.63	6.63
	50	0.00	-0.01	0.03	-0.01	0.03	-0.03	-0.06	-0.03	0.06	15.59	11.86	11.85	11.25	8.69	8.71	8.13	6.04	6.03
	100	0.03	-0.03	0.05	0.00	-0.06	-0.02	0.03	0.04	-0.04	14.62	11.71	11.61	11.06	8.61	8.60	7.51	5.91	5.74
										$\rho = 0.8,\ h = 6$									
	20	0.04	0.01	0.04	0.05	0.04	-0.02	-0.06	-0.03	0.00	17.55	12.30	12.47	12.60	9.25	9.22	8.39	6.53	6.62
	50	-0.06	-0.07	0.04	0.06	-0.01	-0.02	-0.01	-0.04	0.02	10.94	11.86	11.92	8.48	8.76	8.80	5.61	6.03	6.11
	100	0.03	0.01	-0.02	-0.05	0.05	0.03	-0.01	-0.03	0.00	10.38	11.68	11.71	7.58	8.63	8.64	5.05	5.92	5.91
										$\rho = 0.8,\ h = 0.3N$									
	20	-0.09	-0.08	-0.03	0.08	-0.04	-0.02	0.06	-0.03	0.05	16.11	12.36	12.29	12.28	9.33	9.18	9.16	6.56	6.56
	50	-0.05	-0.04	-0.01	0.07	0.01	-0.10	-0.02	0.01	-0.03	23.86	11.88	11.95	17.45	8.74	8.83	12.70	6.09	6.04
	100	-0.01	-0.04	0.01	-0.08	-0.01	0.01	0.07	0.05	0.08	32.17	11.69	11.74	24.04	8.63	8.66	16.64	5.94	5.89
										$\rho = 0.5,\ h = 2$									
CCE-IV	20	-0.23	-0.08	-0.11	-0.17	-0.11	-0.06	-0.16	0.07	-0.05	9.60	12.41	12.09	7.36	9.24	9.04	5.41	6.65	6.53
	50	0.03	0.01	0.02	-0.03	0.00	-0.01	-0.04	-0.04	0.09	9.07	11.85	11.81	6.53	8.76	8.82	4.56	6.09	6.11
	100	-0.03	-0.02	0.00	-0.01	0.06	-0.01	-0.04	-0.03	0.08	8.98	11.74	11.74	6.55	8.59	8.64	4.43	5.88	5.99
										$\rho = 0.5,\ h = 6$									
	20	-0.57	-0.04	0.00	-1.06	-0.08	-0.24	-0.41	-0.01	-0.05	21.31	12.47	12.43	17.76	9.44	9.33	11.56	6.75	6.58
	50	-0.11	-0.02	0.02	-0.05	0.02	-0.05	-0.14	-0.04	0.04	15.79	11.84	11.87	11.22	8.71	8.71	8.13	6.05	6.05
	100	0.01	-0.04	0.06	-0.05	-0.08	-0.01	-0.01	0.05	-0.03	14.79	11.75	11.73	11.09	8.63	8.61	7.50	5.90	5.81
										$\rho = 0.8,\ h = 6$									
	20	-1.95	0.00	-0.11	-1.46	0.08	-0.12	-2.96	-0.17	-0.33	18.31	12.31	12.49	13.05	9.26	9.13	11.13	6.58	6.79
	50	-0.21	-0.11	0.04	-0.09	0.00	-0.02	-0.18	-0.04	0.02	11.13	11.89	12.00	8.59	8.80	8.82	5.68	6.01	6.07
	100	0.01	0.01	-0.02	-0.07	0.06	0.03	-0.02	-0.04	0.00	10.49	11.63	11.75	7.59	8.66	8.65	5.10	5.92	5.89
										$\rho = 0.8,\ h = 0.3N$									
	20	-1.61	-0.18	-0.13	-1.61	-0.13	-0.17	-2.24	-0.22	-0.17	17.63	12.39	12.34	13.82	9.32	9.25	11.34	6.82	6.59
	50	-1.23	-0.10	-0.05	-1.27	-0.02	-0.15	-1.22	-0.03	-0.07	25.84	11.93	11.96	19.40	8.77	8.83	13.88	6.09	6.09
	100	-1.20	-0.05	-0.01	-1.27	-0.01	-0.02	-1.06	0.04	0.04	34.7	11.77	11.81	26.24	8.65	8.64	18.28	5.91	5.87
										$\rho = 0.5,\ h = 2$									
Infeasible	20	0.04	-0.01	-0.01	0.01	-0.09	-0.02	0.01	0.02	-0.07	6.45	10.14	9.92	4.88	7.38	7.38	3.37	5.06	5.07
	50	0.06	0.01	0.01	-0.04	-0.02	0.00	-0.01	-0.02	0.03	6.82	10.09	10.02	4.86	7.36	7.43	3.38	5.08	5.06
	100	0.00	-0.03	0.02	0.01	0.04	0.01	-0.05	-0.04	0.07	6.92	9.99	9.96	5.09	7.41	7.38	3.46	5.08	5.10
										$\rho = 0.5,\ h = 6$									
	20	-0.05	-0.01	0.02	0.08	-0.04	-0.10	0.01	0.00	-0.04	8.72	9.98	9.95	7.02	7.41	7.34	4.52	5.06	5.07
	50	0.05	0.05	0.01	0.02	0.02	-0.05	-0.05	-0.03	0.04	9.36	10.07	10.04	6.68	7.41	7.40	4.84	5.07	5.07
	100	-0.01	-0.02	0.04	0.01	-0.07	-0.03	-0.01	0.02	-0.01	9.57	10.10	10.01	7.23	7.45	7.41	4.84	5.11	5.00
										$\rho = 0.8,\ h = 6$									
	20	0.01	-0.02	-0.01	-0.02	0.04	0.02	-0.01	-0.04	0.01	5.88	10.11	10.16	4.12	7.44	7.52	2.82	5.12	5.16
	50	-0.02	-0.05	0.04	0.03	0.01	-0.01	0.00	0.00	0.03	5.91	10.13	10.20	4.47	7.47	7.53	3.02	5.11	5.14
	100	0.00	-0.01	0.01	-0.02	0.06	0.03	-0.02	-0.03	-0.01	6.21	10.10	10.20	4.54	7.48	7.50	3.08	5.09	5.15
										$\rho = 0.8,\ h = 0.3N$									
	20	0.00	-0.07	-0.01	0.03	0.01	0.01	-0.01	-0.02	0.03	5.54	9.99	10.11	4.22	7.42	7.50	3.01	5.05	5.13
	50	-0.01	0.08	-0.02	-0.06	-0.01	-0.04	0.01	0.04	-0.03	6.89	10.13	10.22	5.18	7.45	7.52	3.73	5.09	5.13
	100	-0.03	0.00	-0.01	0.04	-0.01	0.02	0.01	0.04	0.05	7.50	10.13	10.18	5.69	7.50	7.52	3.95	5.18	5.14

Notes: The simulation results are based on the DGP specified in Section 4, but with $\kappa_1 = 1$ and $\kappa_2 = 2$.

Table S30: The Size and Power of Individual Estimators under Experiment 4 with $\kappa_1 = 1$ and $\kappa_2 = 2$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}	ρ_i	β_{1i}	β_{2i}
CCEX-IV		$\rho = 0.5,\ h = 2$																	
	20	0.079	0.078	0.081	0.069	0.064	0.064	0.060	0.053	0.058	0.554	0.386	0.383	0.755	0.585	0.591	0.931	0.853	0.860
	50	0.074	0.078	0.078	0.060	0.065	0.064	0.055	0.057	0.056	0.584	0.384	0.383	0.827	0.615	0.616	0.978	0.890	0.893
	100	0.067	0.073	0.073	0.058	0.062	0.060	0.047	0.052	0.054	0.595	0.392	0.397	0.819	0.620	0.616	0.979	0.907	0.901
		$\rho = 0.5,\ h = 6$																	
	20	0.058	0.072	0.071	0.043	0.067	0.067	0.056	0.061	0.049	0.268	0.358	0.348	0.331	0.576	0.564	0.524	0.845	0.847
	50	0.055	0.073	0.073	0.051	0.060	0.062	0.049	0.056	0.055	0.305	0.393	0.381	0.470	0.611	0.608	0.697	0.894	0.889
	100	0.057	0.070	0.070	0.052	0.057	0.058	0.044	0.054	0.048	0.321	0.393	0.390	0.468	0.637	0.622	0.755	0.893	0.908
		$\rho = 0.8,\ h = 6$																	
	20	0.056	0.059	0.063	0.053	0.059	0.060	0.042	0.064	0.064	0.383	0.371	0.367	0.545	0.567	0.563	0.733	0.845	0.841
	50	0.048	0.068	0.070	0.045	0.059	0.058	0.049	0.056	0.056	0.525	0.387	0.383	0.700	0.616	0.613	0.914	0.892	0.891
	100	0.053	0.066	0.069	0.048	0.058	0.056	0.043	0.052	0.047	0.526	0.401	0.390	0.745	0.619	0.623	0.955	0.899	0.906
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.055	0.061	0.063	0.049	0.063	0.061	0.044	0.062	0.063	0.413	0.363	0.358	0.528	0.561	0.567	0.688	0.845	0.840
	50	0.052	0.051	0.052	0.055	0.048	0.049	0.052	0.047	0.049	0.270	0.371	0.373	0.370	0.592	0.591	0.494	0.883	0.879
	100	0.054	0.043	0.043	0.053	0.049	0.041	0.051	0.041	0.051	0.203	0.366	0.366	0.273	0.591	0.596	0.394	0.887	0.888
CCE-IV		$\rho = 0.5,\ h = 2$																	
	20	0.070	0.077	0.073	0.064	0.075	0.068	0.072	0.071	0.068	0.554	0.381	0.386	0.749	0.576	0.582	0.927	0.850	0.856
	50	0.068	0.074	0.073	0.057	0.062	0.062	0.054	0.057	0.053	0.576	0.389	0.380	0.829	0.613	0.609	0.976	0.888	0.891
	100	0.063	0.069	0.069	0.056	0.059	0.057	0.046	0.050	0.051	0.586	0.399	0.392	0.816	0.616	0.613	0.980	0.907	0.900
		$\rho = 0.5,\ h = 6$																	
	20	0.042	0.067	0.065	0.044	0.064	0.061	0.052	0.070	0.064	0.265	0.354	0.346	0.321	0.559	0.566	0.522	0.841	0.851
	50	0.051	0.069	0.069	0.050	0.058	0.060	0.046	0.053	0.053	0.302	0.388	0.380	0.466	0.607	0.610	0.693	0.891	0.885
	100	0.054	0.067	0.067	0.049	0.055	0.056	0.043	0.053	0.045	0.316	0.385	0.388	0.464	0.633	0.626	0.755	0.891	0.912
		$\rho = 0.8,\ h = 6$																	
	20	0.032	0.053	0.058	0.024	0.058	0.055	0.032	0.058	0.056	0.374	0.362	0.364	0.536	0.558	0.570	0.649	0.835	0.825
	50	0.043	0.064	0.066	0.043	0.056	0.055	0.047	0.052	0.053	0.516	0.389	0.382	0.688	0.612	0.607	0.907	0.889	0.890
	100	0.049	0.062	0.065	0.045	0.055	0.054	0.041	0.052	0.047	0.524	0.396	0.385	0.743	0.618	0.621	0.952	0.902	0.903
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.029	0.053	0.054	0.024	0.058	0.054	0.034	0.060	0.056	0.404	0.356	0.355	0.510	0.550	0.564	0.623	0.831	0.834
	50	0.031	0.046	0.048	0.033	0.045	0.044	0.038	0.044	0.045	0.261	0.358	0.360	0.351	0.574	0.582	0.476	0.882	0.880
	100	0.034	0.039	0.040	0.035	0.037	0.037	0.031	0.037	0.038	0.198	0.359	0.356	0.266	0.582	0.589	0.379	0.884	0.884
Infeasible		$\rho = 0.5,\ h = 2$																	
	20	0.057	0.054	0.052	0.051	0.049	0.049	0.052	0.054	0.051	0.763	0.457	0.466	0.939	0.701	0.719	0.998	0.952	0.956
	50	0.054	0.053	0.054	0.049	0.049	0.051	0.053	0.052	0.052	0.739	0.455	0.457	0.944	0.716	0.703	0.998	0.952	0.954
	100	0.051	0.053	0.052	0.051	0.051	0.051	0.052	0.054	0.053	0.737	0.464	0.464	0.928	0.712	0.709	0.997	0.956	0.953
		$\rho = 0.5,\ h = 6$																	
	20	0.054	0.053	0.054	0.051	0.049	0.050	0.052	0.050	0.051	0.573	0.464	0.454	0.743	0.707	0.711	0.971	0.956	0.955
	50	0.053	0.054	0.054	0.050	0.050	0.051	0.050	0.051	0.053	0.533	0.462	0.453	0.790	0.714	0.712	0.958	0.952	0.953
	100	0.053	0.053	0.053	0.051	0.051	0.051	0.050	0.054	0.048	0.513	0.459	0.457	0.728	0.707	0.709	0.958	0.947	0.960
		$\rho = 0.8,\ h = 6$																	
	20	0.050	0.050	0.051	0.050	0.051	0.050	0.049	0.053	0.051	0.837	0.462	0.464	0.977	0.709	0.699	1.000	0.949	0.948
	50	0.049	0.051	0.052	0.048	0.051	0.048	0.054	0.054	0.053	0.844	0.459	0.448	0.967	0.697	0.703	1.000	0.952	0.949
	100	0.052	0.052	0.053	0.050	0.051	0.049	0.053	0.053	0.052	0.809	0.456	0.456	0.963	0.702	0.707	0.999	0.953	0.953
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.048	0.049	0.051	0.051	0.050	0.051	0.051	0.051	0.051	0.886	0.480	0.456	0.973	0.705	0.695	1.000	0.951	0.949
	50	0.050	0.053	0.053	0.051	0.052	0.050	0.051	0.052	0.051	0.755	0.445	0.444	0.926	0.699	0.693	0.994	0.954	0.953
	100	0.052	0.053	0.054	0.050	0.050	0.051	0.052	0.054	0.052	0.685	0.447	0.443	0.886	0.694	0.699	0.990	0.943	0.949

Notes: The simulation results are based on the DGP specified in Section 4, but with $\kappa_1 = 1$ and $\kappa_2 = 2$. The alternative for the power test is $\boldsymbol{\theta}_i^a = \boldsymbol{\theta}_i + 0.2$.

Table S31: The Finite Sample Performance of Mean Group Estimators under Experiment 4 with $\kappa_1 = 1$ and $\kappa_2 = 2$

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
CCEX-IV										$\rho = 0.5,\ h = 2$									
	20	0.08	-0.05	-0.04	0.00	-0.01	-0.01	0.05	-0.04	-0.05	3.66	11.98	7.95	3.25	11.46	7.28	2.81	11.36	6.83
	50	-0.02	-0.02	-0.06	0.02	-0.05	0.07	-0.04	0.06	0.02	2.31	7.23	4.93	1.92	7.07	4.59	1.74	6.92	4.36
	100	0.02	0.07	0.05	-0.01	-0.06	-0.04	0.08	-0.06	-0.05	1.57	5.43	3.39	1.41	5.21	3.32	1.26	5.45	3.09
										$\rho = 0.5,\ h = 6$									
	20	0.03	-0.04	-0.09	0.05	0.05	0.03	0.03	-0.09	-0.02	6.82	11.95	7.89	5.28	11.81	7.47	4.15	11.24	7.04
	50	-0.02	0.03	-0.05	0.02	-0.05	0.02	-0.04	-0.05	0.01	3.19	7.41	4.81	1.92	7.37	4.59	2.09	7.19	4.52
	100	-0.05	-0.06	-0.08	-0.03	0.04	0.04	-0.02	0.03	-0.04	2.06	5.21	3.41	1.72	5.29	3.19	1.40	5.03	3.25
										$\rho = 0.8,\ h = 6$									
	20	0.05	-0.05	0.07	-0.02	0.08	-0.05	0.00	0.04	0.03	5.99	12.12	7.92	4.07	11.36	7.34	3.30	11.32	6.84
	50	0.00	0.01	-0.02	-0.07	0.05	0.06	-0.02	0.03	0.06	2.23	7.48	5.10	1.97	7.39	4.73	1.71	7.23	4.45
	100	0.01	0.04	0.07	0.05	0.06	0.03	-0.09	0.05	0.02	1.51	5.54	3.58	1.36	5.08	3.20	1.28	4.71	3.17
										$\rho = 0.8,\ h = 0.3N$									
	20	0.06	-0.03	0.03	-0.02	0.01	-0.02	0.02	0.04	-0.03	5.39	12.13	8.09	4.22	11.48	7.82	3.38	11.12	7.16
	50	-0.02	-0.06	0.09	0.05	0.00	0.00	0.03	-0.02	-0.02	5.29	7.62	4.72	3.20	7.28	4.82	2.50	6.92	4.42
	100	-0.02	-0.03	-0.01	0.06	-0.03	0.02	0.09	0.05	0.03	4.47	5.43	3.37	3.10	5.32	3.27	2.24	4.88	2.92
CCE-IV										$\rho = 0.5,\ h = 2$									
	20	-0.18	0.12	-0.08	-0.18	-0.02	-0.06	-0.13	-0.08	-0.05	3.74	11.97	7.96	3.25	11.46	7.26	2.83	11.37	6.83
	50	-0.08	-0.04	-0.08	-0.03	-0.06	0.08	-0.14	0.07	0.00	2.31	7.25	4.96	1.93	7.06	4.58	1.73	6.92	4.36
	100	0.01	0.08	0.06	-0.02	-0.06	-0.06	0.07	-0.06	-0.06	1.58	5.45	3.39	1.41	5.20	3.34	1.26	5.45	3.10
										$\rho = 0.5,\ h = 6$									
	20	-1.62	-0.17	-0.28	-1.37	-0.07	-0.17	-1.75	-0.15	-0.04	7.48	11.93	7.89	5.62	11.87	7.45	4.62	11.25	7.03
	50	-0.14	0.10	0.04	-0.03	-0.06	0.00	-0.16	-0.05	0.00	3.24	7.41	4.84	1.93	7.36	4.58	2.09	7.19	4.53
	100	-0.08	-0.06	-0.09	-0.06	0.05	0.04	-0.05	0.03	-0.04	2.07	5.24	3.43	1.73	5.26	3.19	1.40	5.03	3.25
										$\rho = 0.8,\ h = 6$									
	20	-2.80	-0.12	0.08	-2.85	-0.09	-0.34	-2.73	-0.11	-0.24	6.86	12.02	8.04	5.64	11.42	7.38	4.63	11.38	6.83
	50	-0.15	0.05	-0.03	-0.20	0.06	0.06	-0.14	0.02	0.11	2.26	7.48	5.10	1.97	7.41	4.69	1.75	7.24	4.46
	100	-0.02	0.04	0.07	0.02	0.03	0.03	-0.13	0.17	0.02	1.52	5.55	3.59	1.35	5.06	3.21	1.30	4.72	3.18
										$\rho = 0.8,\ h = 0.3N$									
	20	-2.78	-0.05	0.05	-2.89	-0.01	-0.05	-2.56	0.20	-0.05	6.67	12.14	8.15	5.76	11.53	7.79	4.62	11.13	7.19
	50	-2.03	-0.10	0.07	-1.82	-0.04	-0.07	-1.97	-0.09	-0.09	5.33	7.65	4.73	4.03	7.28	4.86	3.52	6.95	4.43
	100	-1.69	-0.08	-0.07	-1.69	-0.03	0.02	-1.61	0.05	0.02	5.11	5.41	3.36	3.86	5.33	3.25	3.00	4.87	2.92
Infeasible										$\rho = 0.5,\ h = 2$									
	20	0.03	0.02	0.02	-0.03	-0.01	-0.02	-0.01	-0.06	-0.05	3.12	11.91	7.61	2.82	11.33	7.18	2.63	11.32	6.74
	50	0.01	-0.06	-0.08	0.03	-0.04	0.02	-0.08	0.04	0.04	2.04	7.12	4.92	1.81	6.98	4.49	1.64	6.88	4.34
	100	0.03	0.08	0.03	0.00	-0.05	-0.06	0.03	0.01	-0.03	1.44	5.38	3.29	1.26	5.21	3.27	1.19	5.41	3.08
										$\rho = 0.5,\ h = 6$									
	20	0.01	0.02	-0.16	0.07	-0.02	0.04	-0.02	-0.09	-0.02	3.47	11.74	7.55	3.15	11.66	7.32	2.76	11.19	6.91
	50	-0.01	0.02	0.02	0.03	-0.04	0.04	0.03	-0.04	0.01	2.24	7.33	4.70	1.81	7.28	4.49	1.75	7.17	4.53
	100	0.01	-0.07	-0.08	-0.03	0.05	0.06	0.00	0.03	-0.02	1.60	5.17	3.32	1.32	5.28	3.16	1.19	4.99	3.26
										$\rho = 0.8,\ h = 6$									
	20	-0.06	-0.06	0.05	0.02	0.02	-0.02	0.10	0.04	0.06	2.72	11.81	7.34	2.63	11.36	7.17	2.40	11.33	6.73
	50	0.02	0.02	0.08	-0.05	0.01	0.05	-0.02	0.01	0.04	1.74	7.41	4.96	1.66	7.35	4.59	1.56	7.20	4.39
	100	0.01	0.04	0.11	0.06	0.07	0.02	-0.09	0.04	0.00	1.22	5.40	3.55	1.23	5.03	3.16	1.22	4.63	3.17
										$\rho = 0.8,\ h = 0.3N$									
	20	0.06	-0.04	0.03	0.02	0.02	-0.01	0.02	0.04	-0.03	2.67	11.82	7.85	2.63	11.46	7.66	2.57	11.09	7.09
	50	0.02	-0.04	0.10	0.00	-0.04	0.01	0.01	-0.01	-0.03	1.82	7.54	4.61	1.72	7.23	4.73	1.61	6.95	4.43
	100	0.04	-0.06	0.01	-0.01	-0.03	0.03	-0.02	0.05	0.03	1.31	5.36	3.30	1.18	5.28	3.26	1.16	4.88	2.94

Notes: The simulation results are based on the DGP specified in Section 4, but with $\kappa_1 = 1$ and $\kappa_2 = 2$.

Table S32: The Size and Power of Mean Group Estimators under Experiment 4 with $\kappa_1 = 1$ and $\kappa_2 = 2$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		$\rho = 0.5,\ h = 2$																	
CCEX-IV	20	0.063	0.056	0.070	0.068	0.056	0.067	0.057	0.062	0.054	0.728	0.315	0.479	0.841	0.356	0.566	0.920	0.358	0.646
	50	0.064	0.046	0.056	0.047	0.062	0.060	0.054	0.056	0.052	0.982	0.617	0.906	0.998	0.642	0.925	1.000	0.662	0.962
	100	0.060	0.055	0.043	0.054	0.050	0.066	0.052	0.065	0.055	1.000	0.896	0.998	1.000	0.930	1.000	1.000	1.000	1.000
		$\rho = 0.5,\ h = 6$																	
	20	0.060	0.064	0.061	0.062	0.061	0.063	0.064	0.057	0.055	0.311	0.312	0.480	0.442	0.322	0.568	0.614	0.378	0.626
	50	0.056	0.053	0.047	0.047	0.062	0.060	0.054	0.056	0.052	0.855	0.636	0.946	0.998	0.642	0.920	1.000	0.716	0.952
	100	0.047	0.048	0.044	0.054	0.044	0.054	0.052	0.045	0.060	0.992	0.920	0.998	1.000	0.934	1.000	1.000	0.995	1.000
		$\rho = 0.8,\ h = 6$																	
	20	0.063	0.066	0.070	0.072	0.058	0.063	0.063	0.058	0.058	0.536	0.308	0.475	0.601	0.357	0.550	0.780	0.360	0.645
	50	0.057	0.048	0.067	0.060	0.054	0.076	0.050	0.057	0.058	0.984	0.688	0.869	1.000	0.720	0.934	1.000	0.730	0.952
	100	0.047	0.055	0.055	0.053	0.046	0.054	0.055	0.054	0.050	1.000	0.913	0.998	1.000	0.930	0.998	1.000	0.980	1.000
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.062	0.060	0.071	0.063	0.062	0.056	0.057	0.062	0.050	0.501	0.295	0.501	0.631	0.363	0.543	0.749	0.381	0.556
	50	0.058	0.057	0.050	0.058	0.056	0.062	0.054	0.056	0.058	0.624	0.669	0.929	0.852	0.684	0.908	0.962	0.711	0.968
	100	0.049	0.056	0.052	0.054	0.068	0.050	0.060	0.065	0.045	0.709	0.920	0.997	0.880	0.932	1.000	0.980	0.955	1.000
		$\rho = 0.5,\ h = 2$																	
CCE-IV	20	0.065	0.062	0.071	0.070	0.064	0.071	0.076	0.075	0.061	0.726	0.311	0.476	0.835	0.353	0.568	0.920	0.356	0.647
	50	0.069	0.050	0.059	0.050	0.065	0.061	0.056	0.060	0.054	0.975	0.613	0.906	0.998	0.634	0.924	1.000	0.658	0.958
	100	0.060	0.058	0.041	0.054	0.052	0.052	0.060	0.070	0.055	1.000	0.891	0.999	1.000	0.938	1.000	1.000	0.995	1.000
		$\rho = 0.5,\ h = 6$																	
	20	0.070	0.073	0.066	0.062	0.078	0.083	0.073	0.065	0.064	0.307	0.296	0.500	0.401	0.302	0.562	0.583	0.389	0.616
	50	0.057	0.054	0.050	0.050	0.065	0.061	0.056	0.064	0.052	0.853	0.631	0.938	0.998	0.634	0.924	0.999	0.712	0.950
	100	0.055	0.049	0.050	0.046	0.044	0.054	0.055	0.045	0.065	0.993	0.918	0.997	1.000	0.940	1.000	1.000	0.995	1.000
		$\rho = 0.8,\ h = 6$																	
	20	0.074	0.071	0.074	0.098	0.070	0.063	0.100	0.064	0.070	0.387	0.309	0.464	0.526	0.353	0.537	0.659	0.361	0.641
	50	0.059	0.050	0.070	0.060	0.06	0.072	0.055	0.058	0.064	0.982	0.691	0.866	1.000	0.704	0.924	0.999	0.727	0.953
	100	0.049	0.056	0.058	0.054	0.052	0.054	0.052	0.040	0.060	1.000	0.894	0.998	1.000	0.926	0.998	1.000	0.980	1.000
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.064	0.075	0.081	0.095	0.069	0.093	0.104	0.073	0.077	0.436	0.291	0.464	0.472	0.329	0.542	0.666	0.376	0.562
	50	0.068	0.059	0.051	0.062	0.054	0.062	0.070	0.058	0.066	0.510	0.692	0.931	0.798	0.700	0.906	0.918	0.708	0.964
	100	0.065	0.053	0.047	0.064	0.066	0.058	0.070	0.060	0.055	0.540	0.922	0.997	0.830	0.928	1.000	0.985	0.955	1.000
		$\rho = 0.5,\ h = 2$																	
Infeasible	20	0.053	0.058	0.062	0.066	0.055	0.060	0.062	0.063	0.053	0.858	0.321	0.519	0.904	0.369	0.601	0.948	0.371	0.661
	50	0.057	0.046	0.069	0.060	0.062	0.065	0.060	0.058	0.048	0.996	0.626	0.906	1.000	0.657	0.944	1.000	0.674	0.964
	100	0.051	0.053	0.037	0.044	0.056	0.050	0.050	0.055	0.060	1.000	0.902	1.000	1.000	0.944	1.000	1.000	1.000	1.000
		$\rho = 0.5,\ h = 6$																	
	20	0.061	0.060	0.062	0.053	0.066	0.052	0.063	0.049	0.057	0.780	0.311	0.526	0.855	0.331	0.533	0.932	0.405	0.65
	50	0.049	0.049	0.050	0.060	0.062	0.065	0.054	0.064	0.052	0.989	0.705	0.938	1.000	0.717	0.944	1.000	0.726	0.958
	100	0.058	0.053	0.045	0.046	0.042	0.052	0.055	0.045	0.055	1.000	0.923	1.000	1.000	0.956	1.000	1.000	1.000	1.000
		$\rho = 0.8,\ h = 6$																	
	20	0.061	0.059	0.055	0.066	0.052	0.060	0.051	0.056	0.057	0.931	0.320	0.561	0.938	0.351	0.588	0.977	0.374	0.649
	50	0.053	0.051	0.067	0.050	0.060	0.062	0.048	0.060	0.057	1.000	0.698	0.882	1.000	0.708	0.928	1.000	0.731	0.967
	100	0.049	0.054	0.065	0.054	0.048	0.044	0.047	0.055	0.050	1.000	0.927	1.000	1.000	0.956	1.000	1.000	0.995	1.000
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.063	0.065	0.068	0.067	0.068	0.061	0.054	0.059	0.050	0.946	0.313	0.549	0.950	0.363	0.565	0.977	0.391	0.585
	50	0.063	0.052	0.053	0.050	0.066	0.062	0.058	0.050	0.058	0.998	0.695	0.938	1.000	0.718	0.932	1.000	0.724	0.970
	100	0.053	0.054	0.051	0.048	0.062	0.056	0.055	0.050	0.055	1.000	0.935	1.000	1.000	0.932	1.000	1.000	0.978	1.000

Notes: The simulation results are based on the DGP specified in Section 4, but with $\kappa_1 = 1$ and $\kappa_2 = 2$. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

Table S33: The Finite Sample Performance of Pooled Estimators under Experiment 4 with $\kappa_1 = 1$ and $\kappa_2 = 2$

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		$\rho = 0.5,\ h = 2$																	
	20	0.30	0.50	-0.28	0.27	-0.09	-0.55	0.17	-0.20	-0.12	4.25	12.54	8.56	3.86	12.01	7.83	3.24	11.61	7.11
	50	0.16	-0.17	-0.15	0.23	-0.36	-0.03	0.19	0.09	-0.15	2.60	7.60	5.36	2.17	7.59	4.91	1.97	7.48	4.63
	100	0.26	0.05	0.12	0.20	-0.34	-0.09	0.30	-0.31	-0.12	1.83	5.74	3.67	1.55	5.50	3.51	1.44	5.55	3.19
		$\rho = 0.5,\ h = 6$																	
	20	-0.11	-0.13	-0.06	0.41	0.06	-0.32	0.22	-0.09	-0.23	7.63	12.41	8.53	5.74	12.35	7.95	4.60	11.45	7.28
	50	0.15	0.08	0.11	0.23	-0.36	-0.03	-0.21	-0.64	0.03	3.80	7.74	5.21	2.17	7.59	4.91	1.99	7.44	4.82
	100	0.15	0.08	-0.09	0.22	0.07	0.16	0.20	0.22	0.03	2.48	5.67	3.70	1.89	5.43	3.40	1.61	5.13	3.38
CCEX-IV		$\rho = 0.8,\ h = 6$																	
	20	-0.31	-0.17	0.23	0.25	-0.16	0.23	0.22	0.00	-0.09	5.38	12.52	8.26	4.47	11.81	7.75	3.64	11.54	7.13
	50	0.16	0.02	0.01	0.19	0.02	-0.04	0.13	0.06	0.25	2.57	7.94	5.51	2.25	7.76	4.99	1.88	7.47	4.65
	100	0.22	0.42	0.06	0.27	0.16	0.03	0.13	0.05	0.16	1.70	5.80	3.97	1.54	5.32	3.43	1.38	4.84	3.21
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.34	-0.20	-0.10	0.27	-0.04	-0.41	0.20	-0.01	-0.21	6.17	13.06	8.82	5.14	11.58	8.01	4.23	11.25	7.37
	50	0.50	-0.19	0.06	0.43	-0.24	0.06	0.27	0.15	0.08	5.31	8.19	5.59	4.53	7.83	5.28	2.89	7.26	4.53
	100	0.12	-0.20	0.08	-0.18	0.16	-0.02	0.19	0.27	0.10	5.09	5.66	4.00	4.33	5.58	3.33	2.96	5.26	3.25
		$\rho = 0.5,\ h = 2$																	
	20	-0.27	0.42	-0.35	-0.17	-0.22	-0.68	-0.23	-0.29	-0.21	4.31	12.60	8.57	3.84	12.01	7.84	3.24	11.59	7.13
	50	0.24	-0.19	-0.14	0.11	-0.35	-0.05	-0.21	0.11	-0.16	2.62	7.60	5.38	2.17	7.59	4.89	1.96	7.48	4.63
	100	0.21	0.08	0.14	0.15	-0.36	-0.09	0.24	-0.33	-0.15	1.82	5.75	3.69	1.55	5.51	3.54	1.44	5.55	3.20
		$\rho = 0.5,\ h = 6$																	
	20	-1.64	-0.14	-0.23	-0.98	-0.06	-0.48	-1.34	-0.14	-0.27	7.98	12.41	8.53	5.96	12.37	7.91	4.94	11.45	7.29
	50	-0.16	0.08	0.10	0.11	-0.35	-0.05	-0.20	-0.64	0.00	3.83	7.72	5.24	2.17	7.59	4.89	1.98	7.44	4.80
	100	-0.23	0.06	-0.07	-0.07	0.09	0.14	-0.07	0.24	0.02	2.49	5.68	3.71	1.90	5.41	3.40	1.61	5.15	3.39
CCE-IV		$\rho = 0.8,\ h = 6$																	
	20	-1.73	-0.45	0.09	-1.65	-0.23	-0.02	-1.58	-0.14	-0.24	6.07	12.53	8.29	5.10	11.81	7.77	4.19	11.59	7.13
	50	-0.17	0.04	-0.01	-0.13	0.01	-0.05	-0.19	0.04	0.23	2.62	7.92	5.50	2.26	7.79	4.95	1.91	7.46	4.66
	100	0.18	0.41	0.04	0.15	0.16	0.04	0.01	0.07	0.16	1.69	5.82	3.97	1.51	5.31	3.44	1.39	4.86	3.22
		$\rho = 0.8,\ h = 0.3N$																	
	20	-1.82	-0.29	-0.25	-1.54	-0.12	-0.53	-1.58	-0.02	-0.31	6.57	13.09	8.86	5.55	11.54	8.04	4.73	11.29	7.35
	50	-0.89	-0.17	-0.02	-0.65	-0.24	0.07	-1.07	0.14	0.11	5.83	8.16	5.62	4.56	7.83	5.33	3.14	7.24	4.59
	100	-0.87	-0.19	0.07	-1.04	0.20	-0.04	-0.91	0.25	0.10	5.17	5.70	3.99	4.47	5.58	3.33	3.18	5.27	3.26
		$\rho = 0.5,\ h = 2$																	
	20	0.22	0.52	-0.32	0.16	-0.16	-0.72	0.12	-0.26	-0.07	3.63	12.44	8.47	3.36	11.95	7.74	2.98	11.44	7.07
	50	0.14	-0.30	-0.12	0.23	-0.30	0.00	0.14	0.10	-0.23	2.34	7.46	5.32	2.10	7.39	4.78	1.91	7.36	4.62
	100	0.29	0.04	0.11	0.24	-0.36	-0.02	0.30	-0.23	-0.15	1.75	5.68	3.67	1.52	5.48	3.51	1.37	5.54	3.23
		$\rho = 0.5,\ h = 6$																	
	20	-0.16	-0.03	-0.06	0.18	-0.03	-0.18	-0.27	-0.12	-0.23	4.20	12.26	8.51	3.60	12.33	7.94	3.32	11.41	7.19
	50	0.21	0.08	0.02	0.23	-0.30	0.00	0.28	-0.67	0.05	2.69	7.70	5.21	2.10	7.59	4.78	2.04	7.42	4.78
	100	0.12	0.11	-0.02	0.12	0.05	0.25	0.16	0.24	0.03	2.00	5.57	3.65	1.61	5.36	3.36	1.55	5.09	3.33
Infeasible		$\rho = 0.8,\ h = 6$																	
	20	0.22	-0.25	0.19	0.18	-0.08	0.29	0.17	0.04	-0.13	2.92	12.50	8.27	2.85	11.73	7.69	2.55	11.48	7.07
	50	0.15	0.10	0.01	0.15	0.06	-0.01	0.14	-0.07	0.31	2.00	7.89	5.49	1.88	7.71	4.94	1.69	7.37	4.64
	100	0.21	0.39	0.19	0.25	0.10	0.02	0.09	0.10	0.22	1.38	5.81	3.96	1.38	5.28	3.38	1.28	4.85	3.16
		$\rho = 0.8,\ h = 0.3N$																	
	20	0.12	-0.15	-0.07	0.22	-0.04	-0.44	0.13	0.07	-0.26	3.19	13.07	8.81	2.91	11.38	8.02	2.90	11.09	7.32
	50	0.10	-0.25	-0.02	0.14	-0.27	-0.03	0.17	0.14	0.14	2.34	8.11	5.58	1.98	7.76	5.27	1.83	7.19	4.47
	100	0.21	-0.28	0.08	0.26	0.17	-0.03	-0.24	0.30	0.09	1.74	5.71	3.98	1.52	5.55	3.21	1.28	5.18	3.18

Notes: The simulation results are based on the DGP specified in Section 4, but with $\kappa_1 = 1$ and $\kappa_2 = 2$.

Table S34: The Size and Power of Mean Group Estimators under Experiment 4 with $\kappa_1 = 1$ and $\kappa_2 = 2$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
CCEX-IV											$\rho = 0.5,\ h = 2$								
	20	0.086	0.075	0.081	0.099	0.078	0.070	0.079	0.072	0.068	0.647	0.246	0.376	0.676	0.286	0.514	0.793	0.352	0.582
	50	0.072	0.051	0.058	0.056	0.064	0.068	0.078	0.072	0.062	0.967	0.612	0.859	0.988	0.632	0.895	0.998	0.644	0.936
	100	0.061	0.062	0.043	0.068	0.054	0.054	0.065	0.080	0.050	1.000	0.855	0.994	1.000	0.900	1.000	1.000	0.912	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.087	0.077	0.078	0.078	0.083	0.088	0.077	0.069	0.074	0.254	0.281	0.454	0.380	0.293	0.514	0.495	0.339	0.577
	50	0.059	0.065	0.053	0.056	0.064	0.068	0.050	0.064	0.072	0.735	0.605	0.894	0.988	0.632	0.895	0.972	0.682	0.936
	100	0.051	0.062	0.042	0.032	0.050	0.052	0.050	0.05	0.060	0.978	0.892	0.997	1.000	0.912	0.998	1.000	0.915	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.076	0.077	0.077	0.080	0.077	0.081	0.076	0.067	0.069	0.456	0.256	0.483	0.559	0.301	0.534	0.738	0.377	0.544
	50	0.070	0.060	0.068	0.078	0.072	0.062	0.056	0.067	0.062	0.940	0.577	0.848	0.984	0.624	0.908	1.000	0.660	0.939
	100	0.055	0.064	0.065	0.068	0.056	0.046	0.065	0.030	0.070	1.000	0.869	0.988	1.000	0.900	0.994	1.000	0.960	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.075	0.074	0.072	0.085	0.070	0.073	0.076	0.063	0.058	0.354	0.246	0.456	0.472	0.315	0.492	0.629	0.328	0.579
	50	0.069	0.055	0.064	0.072	0.070	0.07	0.076	0.048	0.066	0.437	0.629	0.823	0.676	0.656	0.850	0.920	0.700	0.942
	100	0.061	0.055	0.056	0.074	0.058	0.040	0.060	0.050	0.045	0.504	0.868	0.986	0.638	0.870	0.990	0.870	0.935	1.000
CCE-IV											$\rho = 0.5,\ h = 2$								
	20	0.080	0.093	0.091	0.101	0.093	0.081	0.098	0.085	0.079	0.639	0.242	0.396	0.662	0.279	0.514	0.772	0.359	0.579
	50	0.081	0.059	0.066	0.063	0.073	0.066	0.076	0.072	0.072	0.952	0.600	0.851	0.989	0.609	0.891	0.998	0.624	0.940
	100	0.059	0.065	0.047	0.060	0.054	0.062	0.075	0.085	0.050	1.000	0.833	0.994	1.000	0.898	1.000	1.000	0.908	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.083	0.083	0.094	0.083	0.103	0.098	0.094	0.080	0.096	0.246	0.255	0.455	0.361	0.272	0.490	0.477	0.332	0.594
	50	0.067	0.067	0.058	0.063	0.073	0.066	0.050	0.068	0.084	0.734	0.597	0.881	0.989	0.609	0.891	0.972	0.668	0.930
	100	0.045	0.063	0.046	0.028	0.048	0.052	0.075	0.055	0.060	0.978	0.879	0.997	1.000	0.900	1.000	1.000	0.915	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.086	0.087	0.088	0.10	0.095	0.098	0.096	0.079	0.079	0.441	0.244	0.459	0.557	0.303	0.477	0.708	0.390	0.53
	50	0.063	0.062	0.068	0.064	0.080	0.064	0.049	0.073	0.069	0.929	0.575	0.836	0.978	0.616	0.906	1.000	0.650	0.935
	100	0.048	0.068	0.065	0.052	0.056	0.060	0.070	0.030	0.070	1.000	0.862	0.982	1.000	0.906	0.994	1.000	0.960	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.076	0.0980	0.088	0.091	0.088	0.088	0.116	0.079	0.071	0.345	0.231	0.424	0.430	0.311	0.444	0.613	0.323	0.559
	50	0.079	0.057	0.071	0.064	0.080	0.086	0.074	0.056	0.070	0.402	0.632	0.810	0.630	0.654	0.846	0.864	0.706	0.944
	100	0.046	0.065	0.058	0.082	0.058	0.038	0.065	0.050	0.05	0.499	0.855	0.987	0.672	0.868	0.990	0.865	0.935	0.995
Infeasible											$\rho = 0.5,\ h = 2$								
	20	0.083	0.068	0.082	0.095	0.074	0.088	0.083	0.068	0.067	0.768	0.281	0.403	0.756	0.317	0.517	0.896	0.359	0.603
	50	0.071	0.050	0.062	0.072	0.056	0.055	0.064	0.064	0.062	0.983	0.646	0.858	0.995	0.647	0.910	1.000	0.652	0.940
	100	0.079	0.061	0.042	0.072	0.054	0.046	0.065	0.070	0.055	1.000	0.856	0.994	1.000	0.918	1.000	1.000	0.925	1.000
											$\rho = 0.5,\ h = 6$								
	20	0.075	0.069	0.078	0.064	0.087	0.095	0.088	0.069	0.068	0.659	0.300	0.465	0.714	0.303	0.537	0.832	0.343	0.605
	50	0.050	0.063	0.056	0.072	0.056	0.055	0.066	0.066	0.084	0.943	0.616	0.896	0.995	0.647	0.910	0.996	0.694	0.942
	100	0.064	0.058	0.046	0.030	0.056	0.058	0.070	0.055	0.055	1.000	0.886	1.000	1.000	0.908	1.000	1.000	0.905	1.000
											$\rho = 0.8,\ h = 6$								
	20	0.071	0.082	0.077	0.091	0.081	0.076	0.061	0.066	0.065	0.853	0.280	0.467	0.888	0.293	0.477	0.949	0.346	0.533
	50	0.054	0.060	0.064	0.066	0.076	0.068	0.050	0.071	0.065	0.998	0.580	0.852	1.000	0.632	0.938	1.000	0.675	0.967
	100	0.054	0.062	0.066	0.078	0.056	0.066	0.050	0.030	0.060	1.000	0.873	0.974	1.000	0.918	1.000	1.000	0.965	1.000
											$\rho = 0.8,\ h = 0.3N$								
	20	0.066	0.080	0.080	0.063	0.080	0.074	0.087	0.062	0.065	0.863	0.236	0.453	0.880	0.309	0.491	0.892	0.334	0.596
	50	0.067	0.053	0.062	0.062	0.068	0.070	0.062	0.054	0.066	0.961	0.637	0.817	0.996	0.674	0.853	1.000	0.718	0.986
	100	0.060	0.059	0.059	0.062	0.074	0.038	0.040	0.070	0.050	1.000	0.864	0.982	1.000	0.880	1.000	1.000	0.965	1.000

Notes: The simulation results are based on the DGP specified in Section 4, but with $\kappa_1 = 1$ and $\kappa_2 = 2$. The alternative for the power test is $\boldsymbol{\theta}^a = \boldsymbol{\theta} + 0.2$.

S5 Supplements to the Empirical Application

S5.1 The Data

All the data are collected from the UK Office of National Statistics (ONS) website. The nominal quarterly house price data for all LADs are collected from https://www.ons.gov.uk/peoplepopulationandcommunity/housing. At the individual LAD level, only the annual data for population and nominal per capita personal income are available at https://www.ons.gov.uk/economy/regionalaccounts/grossdisposablehouseholdincome. To construct the quarterly counterparts, we use the interpolation method in Denton (1971) and Dagum and Cholette (2006).[1] Quarterly data of Consumer Price Index (CPI), available at the national level, are collected from https://www.bls.gov/cpi/data.htm, which is used in deflating the nominal house price and personal income data.

The variables hp_{it}, pop_{it} and inc_{it} used in the empirical analysis, are constructed by taking the log-difference of each series and removing seasonality by regressing each of the series on seasonal dummies.

S5.2 The Spatial Weighting Matrices

In the empirical application, we consider both distance-based and contiguity-based spatial weighting matrices. For the distance-based weighting matrix, denoted $\boldsymbol{W}_{dist}$, we collect latitude/longitude coordinates corresponding to the center of the polygon of each LAD from https://geoportal.statistics.gov.uk/datasets. The geodesic distance between two LADs is then calculated using the Haversine formula:

$$d = 2R \cdot arcsin\sqrt{sin^2(\frac{long_2 - long_1}{2}) + cos(long_1)cos(long_2)sin^2(\frac{lat_2 - lat_1}{2})},$$

where R is the earth's radius, and $long_i$ and lat_i are the corresponding longitude and latitude for the ith LAD. Then, each element of the distance based weighting matrix is obtained by taking the inverse of the squared distance.

For the contiguity based weighting matrix, denoted $\boldsymbol{W}_d$, we follow Yang (2021) and Aquaro et al. (2021), and specify a specific threshold, d (miles) to determine which LADs are considered to be neighbours. The element in $\boldsymbol{W}_d$ will take 1 if the distance between two LADs are less than d and 0 otherwise. We set $d = 25$ and each LAD has around 12 neighbours on average.

Both spatial weighting matrices are finally row-sum normalised which is standard in the spatial literature.

[1] There are various methods of data interpolation. For example, we may use the alternative method proposed in Appendix B.3 of the Global Vector Auto-Regressive (GVAR) Toolbox User Guide, which is available at https://sites.google.com/site/gvarmodelling/gvar-toolbox/download. We find that the results obtained using different interpolation methods are qualitatively similar.

S5.3 The Urban and Rural Classification of LADs

According to the UK government's "2011 Rural Urban Classification", the LADs in England have been categorised into 6 urban/rural classifications:

1) Urban with major conurbation: districts with either 100,000 people or 50% of their population residing in urban areas with a population over 750,000;
2) Urban with minor conurbation: districts with either 50,000 people or 50% of their population in urban areas with a population ranging from 250,000 to 750,000;
3) Urban with city and town: districts with fewer than 37,000 people or less than 26% of their population in rural settlements and larger market towns;
4) Urban with significant rural: districts with more than 37,000 people or more than 26% of their population in rural settlements and larger market towns;
5) Largely rural: districts with at least 50% but less than 80% of their population in rural settlements and larger market towns;
6) Mainly rural: districts with at least 80% of their population in rural settlements and larger market towns.

We also apply the same criteria to Wales where there is no official classification. Their distributions across England and Wales are plotted in the following choropleth map;

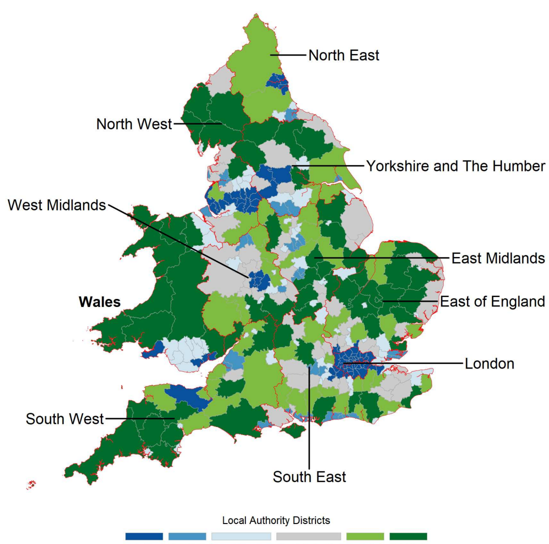


Figure S1: Rural-Urban Classification of LADs in the UK

S5.4 Individual Direct, Spill-in and Spill-out Effects at LAD Level

Here we provide the individual estimation results for HDE, HSI and HSO at each LAD. Their spatial patterns are closely related to those of individual coefficients. For all LADs, the signs of

HDE with respect to population and income growths are the same as the corresponding individual coefficients. For example, if population (income) growth in the ith LAD increases its own house price, then this will induce positive spill-outs from the ith LAD as well as the positive spill-ins from neighbourhood LADs, because the house price growths are mostly positively spatially correlated.

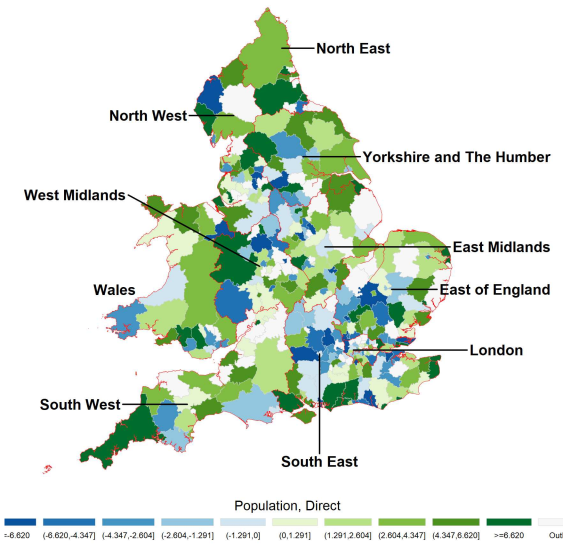
We plot the individual estimates of HDE, HSI and HSO with respect to population and income growths on the choropleth map in Figures S2a–S2f. Overall, the spatial patterns for HDE, HSI and HSO are closely related to those of individual coefficients reported in Figure 1c and 1e.

First, by comparing Figures S2a and S2b with Figures 1c and 1e, we find that the signs of HDE with respect to both pop_{it} and inc_{it} are the same as those of the individual coefficients, $\hat{\beta}_i^{pop}$ and $\hat{\beta}_i^{inc}$. Overall, we observe that the direct effects of population and income growths on house price changes are generally positive. We still find that negative direct effects of population growth mainly occur in LADs with high immigration rates (such as those located in London, the Great Manchester and their neighbourhood areas), while negative direct effects mostly appear in LADs that are located in the rural area or suffer from the high income inequality.

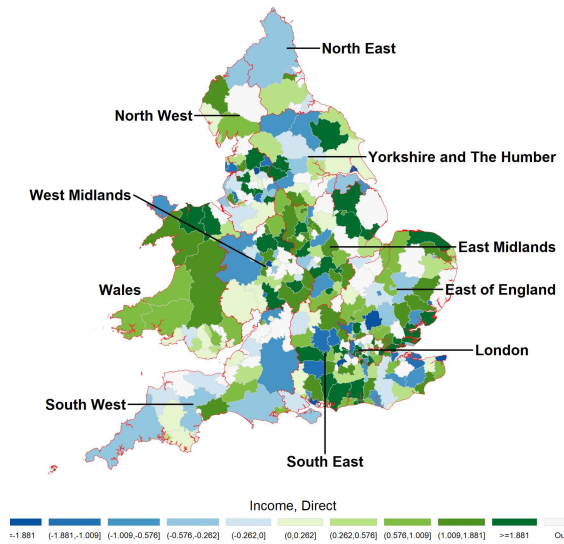
Next, house price growths are positively correlated in most LADs (due to the pervasive positive spatial coefficients), the spatial patterns of HSI and HSO are also closely related to the individual coefficients on population and income growth.

The individual HSIs are also mostly positive. But, negative HSIs occur mostly in LADs that are located close to LADs with negative individual coefficients. In particular, most negative HSIs with respect to population growths are observed in LADs located in London or the rural areas in the East Midlands (located between the Great London and the Great Manchester area). Most negative HSIs with respect to income growths are found in LADs that are located close to specific rural areas (mainly in South East) and that experienced the high income inequality.

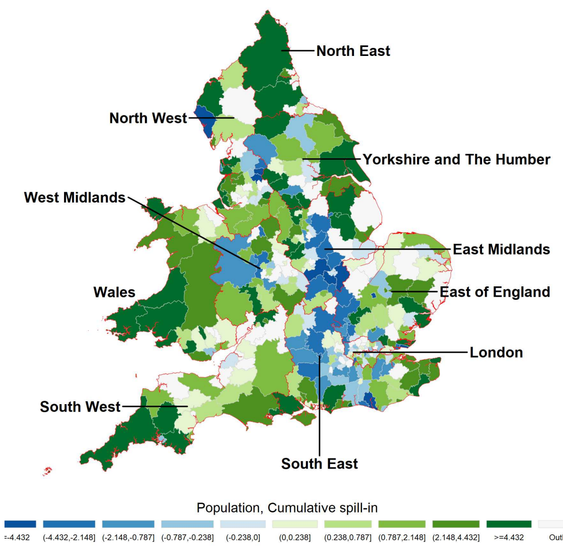
Finally, the spatial patterns of HSOs with respect to both population and income growth, are qualitatively similar to those of the individual coefficients, as evident from comparing Figures S2e and S2f with Figures 1c and 1e.



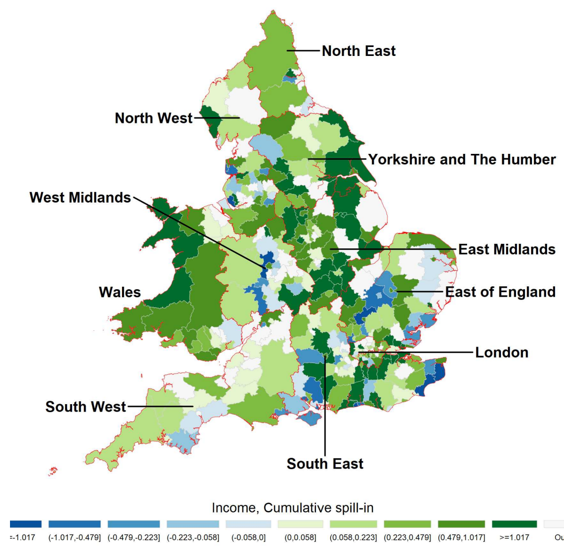
(a) HDE of pop_{it} for Each LAD



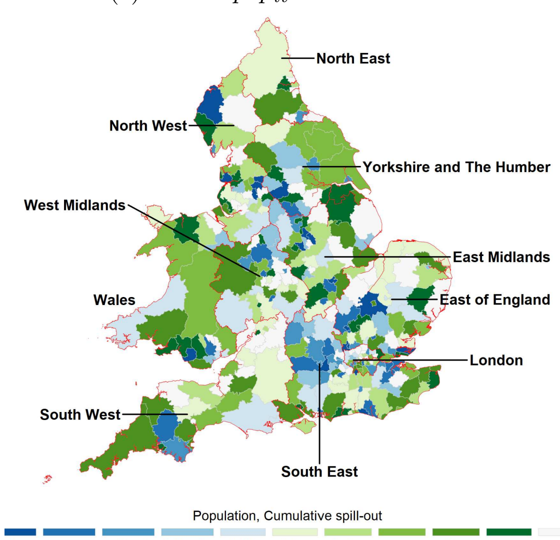
(b) HDE of inc_{it} for Each LAD



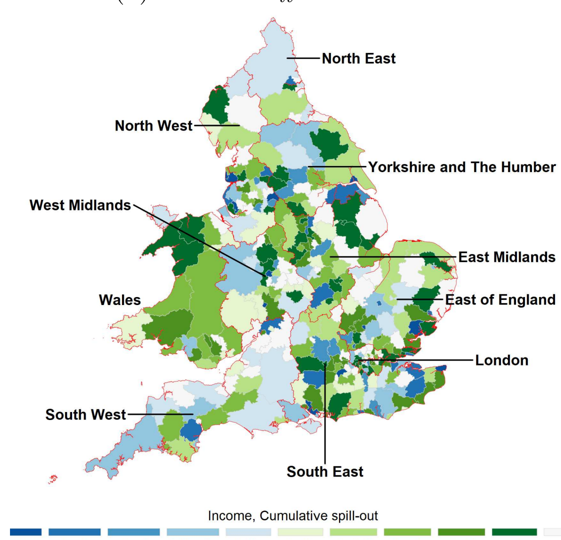
(c) HSI of pop_{it} for Each LAD



(d) HSI of inc_{it} for Each LAD



(e) HSO of pop_{it} for Each LAD



(f) HSO of inc_{it} for Each LAD

Figure S2: HDE, HSI and HSO of pop_{it} and inc_{it} for Each LAD

S5.5 Estimation Results Using $\boldsymbol{W}_{25}$ at the Regional and National Level

In this section, we provide the estimation results using the contiguity-based weighting matrix, $\boldsymbol{W}_{25}$ at the Regional and National Level, in Table S35.

We first analyse the Mean Group estimation results for spatial coefficients. Their spatial patterns are more or less similar to those estimated by using $\boldsymbol{W}_{dist}$, though the magnitudes of the coefficients are different. The spatial coefficients are mostly positive for all the regions, confirming the positive spatial linkage of house price growths across regions. Still, London has the lowest spatial coefficient, but slightly higher than that obtained using $\boldsymbol{W}_{dist}$. South East and East of England display the higher spatial coefficients. The national average is also slightly higher at 0.6. On the other hand, the spatial coefficients by CCE-IV are still less satisfactory, but they also change substantially in many cases. The national average is also higher at 0.31.

Next, the CCEX-IV estimation on population and personal income growths display more or less similar spatial patterns: $\hat{\beta}^{pop}_{MG,r}$ is large and significant for North East & York, North West and South West, whilst $\hat{\beta}^{inc}_{MG,r}$ is large and significant in the rest of regions. By contrast, it is still difficult to interpret the spatial patterns of the CCE-IV estimation results, as they are substantially different in many cases (even the signs are changed).

Still, we find that the CCEX-IV estimation results are more robust than the CCE-IV counterparts, because we find more or less similar spatial patterns of the impacts of population and income growths for CCEX-IV, whilst the CCE-IV results change substantially under the different weighting schemes. Furthermore, they are mostly difficult to interpret.

In sum, we find that the estimation results obtained using the contiguity-based spatial weighting matrix, are less satisfactory. In particular, the spatial coefficients fall outside (-1,1) for 92 LADS such that the total number of LADs in Table S35 is only 240 (implying that there are 99 blank LADs). This also makes their GCM analysis hard to interpret, for example, the higher RSI with respect to income growth in West Midlands.

Finally, we find that the CD test results are more favorable here in the sense that the null of weak CSD is rejected less with the smaller exponent of the cross section dependence. But, this may be due to the fat that the we excluded more LADs with extreme spatial coefficients.

Table S35: Mean Group Estimation Results Using $\boldsymbol{W}_{25}$ at the Regional and National Level

Region	N_r	$\hat{\rho}_{MG,r}$	$\hat{\beta}^{pop}_{MG,r}$	$\hat{\beta}^{inc}_{MG,r}$	CD test	$\hat{\alpha}$	N_r	$\hat{\rho}_{MG,r}$	$\hat{\beta}^{pop}_{MG,r}$	$\hat{\beta}^{inc}_{MG,r}$	CD test	$\hat{\alpha}$
	CCEX-IV Estimation						CCE-IV Estimation					
North East&York	23	$0.663^{\ddagger}$	$2.811^{\ddagger}$	0.047	-0.27	0.58	23	$0.553^{\ddagger}$	$1.898^{\dagger}$	0.078	-0.83	0.54
		(0.079)	(1.100)	(0.173)	[0.79]	(0.04)		(0.065)	(0.904)	(0.173)	[0.41]	(0.07)
North West	25	$0.543^{\ddagger}$	1.711^{*}	0.279	2.32	0.70	27	$0.325^{\ddagger}$	0.466	$0.459^{\ddagger}$	0.19	0.64
		(0.079)	(0.960)	(0.361)	[0.02]	(0.03)		(0.101)	(1.041)	(0.193)	[0.85]	(0.04)
East Midlands	35	$0.648^{\ddagger}$	0.584	$0.480^{\ddagger}$	7.20	0.76	30	0.020	0.363	$0.440^{\ddagger}$	1.63	0.54
		(0.048)	(0.650)	(0.177)	[0.00]	(0.04)		(0.110)	(0.587)	(0.138)	[0.10]	(0.04)
West Midlands	24	$0.585^{\ddagger}$	0.547	$0.936^{\ddagger}$	4.38	0.77	20	0.197	1.275	$0.565^{\ddagger}$	-0.23	0.62
		(0.085)	(1.185)	(0.387)	[0.00]	(0.05)		(0.131)	(0.935)	(0.228)	[0.82]	(0.03)
East of England	27	$0.648^{\ddagger}$	0.643	0.627^{*}	2.57	0.68	34	$0.380^{\ddagger}$	0.753	0.516^{*}	0.07	0.62
		(0.053)	(0.738)	(0.344)	[0.01]	(0.04)		(0.097)	(0.648)	(0.270)	[0.94]	(0.03)
London	17	$0.490^{\ddagger}$	0.178	$1.242^{\ddagger}$	3.34	0.77	23	$0.494^{\ddagger}$	$1.784^{\ddagger}$	0.409	1.08	0.65
		(0.077)	(1.007)	(0.329)	[0.00]	(0.03)		(0.091)	(0.561)	(0.322)	[0.28]	(0.03)
South East	48	$0.629^{\ddagger}$	0.062	$0.463^{\dagger}$	4.71	0.67	48	$0.370^{\ddagger}$	0.245	$0.411^{\dagger}$	3.60	0.75
		(0.046)	(0.737)	(0.219)	[0.00]	(0.02)		(0.062)	(0.521)	(0.204)	[0.00]	(0.03)
South West	22	$0.601^{\ddagger}$	$1.829^{\dagger}$	0.69	2.57	0.80	24	0.051	0.188	0.078	4.11	0.77
		(0.079)	(0.830)	(0.513)	[0.01]	(0.04)		(0.095)	(0.751)	(0.190)	[0.00]	(0.03)
Wales	19	$0.525^{\ddagger}$	1.218	$0.665^{\ddagger}$	0.41	0.68	17	$0.448^{\ddagger}$	1.223	$0.375^{\dagger}$	-1.37	0.58
		(0.087)	(1.082)	(0.185)	[0.68]	(0.05)		(0.119)	(0.947)	(0.190)	[0.17]	(0.05)
England &Walse	240	$0.603^{\ddagger}$	$0.949^{\ddagger}$	$0.564^{\ddagger}$	2.10	0.54	246	$0.313^{\ddagger}$	$0.798^{\ddagger}$	$0.381^{\ddagger}$	-1.40	0.37
		(0.022)	(0.302)	(0.101)	[0.04]	(0.02)		(0.033)	(0.248)	(0.075)	[0.16]	(0.01)

Notes: All the Mean Ggroup estimates are calculated as simple averages from district level parameter estimates and the standard errors in () are calculated according to the formula given in (24). The superscripts ‡,† and * denote that the coefficients are significant at 1, 5 and 10% level. See also footnote to Table 6.

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