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RESIDUAL FLEXURAL STRENGTH OF SFRC: A MULTIVARIATE PERSPECTIVE

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ABSTRACT

The main contribution of steel fibres to the hardened state performance of steel fibre-reinforced concrete (SFRC) is the residual flexural strength the material exhibits, which is commonly characterised by the residual flexural strength parameters (f_{R1}, f_{R2}, f_{R3}, and f_{R4}) as defined by EN 14651. A database of values of residual strength parameters corresponding to hundreds of prismatic specimens from different SFRC mix designs has been put together from previously published papers. Multiple linear regression has been applied to derive a model which relates these parameters to the steel fibres aspect ratio, length and volume fraction as well as the relative amounts of the SFRC mix constituents. The model obtained presents a very good fit to the data collected, and its relatively simple specification makes it a promising tool to optimise SFRC mix designs from the point of view of residual flexural strength. The effect of fibre dosage and dimensions and that of their interactions with other mix design parameters such as water, cement, or aggregate contents are analysed by means of response surface plots representing the average trends reproduced by the model. These modelling and analysis efforts are part of an ongoing study, and this paper focuses on the residual flexural strength parameters f_{R1} and f_{R3}. In relation to the dimensions of the fibres, the effect of fibre length on residual flexural strength has been found to be comparable to that of fibre volume fraction. This, together with the sensitivity of residual flexural strength to the fibre aspect ratio, leads to the conclusion that it is not necessary to use steel fibres in high dosages to proportion SFRC mixes with better-than-average levels of residual flexural strength. The key points emerging from the interpretation of the proposed model are presented and discussed in the context of the wide range of SFRC mixes represented by the database it is based upon.

KEYWORDS: data science, mechanical properties, residual flexural strength, steel fibres.

1. INTRODUCTION

Fibre reinforced concrete (FRC) is defined as any concrete made primarily of hydraulic cement, aggregates, and discrete reinforcing fibres [1]. Fibres are known to improve the hardened state performance of concrete, particularly in terms of mechanical properties such as tensile, flexural strength and toughness in the cracked state [2]. In fact, the residual flexural strength parameters (f_{R1}, f_{R2}, f_{R3}, f_{R4}) and the limit of proportionality (f_L), together with compressive strength (f_c), are the basis of FRC characterisation and specification. The flexural test set-up configurations to standards EN 14651:2005 [3] and ASTM C1609/1609M [4] are shown in Figure 1, together with an example of stress-strain curve that illustrates the limit of proportionality and the residual flexural strength parameters.

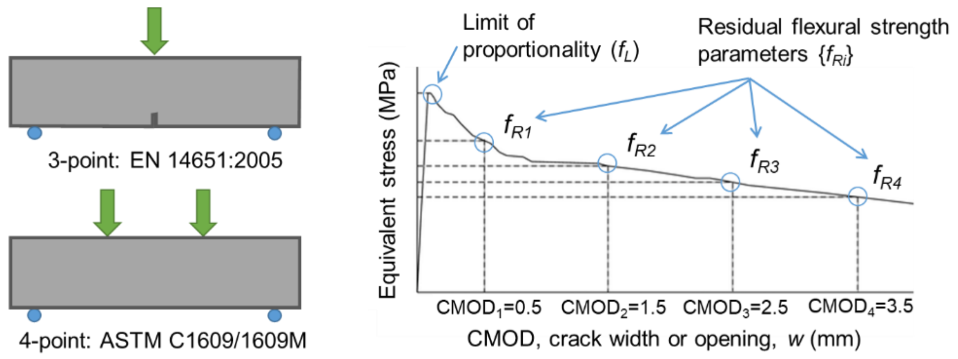

Figure 1. Definition of residual flexural strength parameters [5].

The sensitivity of the residual flexural strength to the proportioning of the concrete mix has been rarely studied, and only in relation to specific fibres considered in the context of specific mixes. Furthermore, most studies concerned with the cracked state performance of FRC look into the effect of varying fibre contents and compare different fibre types, shapes or sizes, but the synergistic effects due to the interaction of fibres with other mix constituents are often neglected.

The two abovementioned aspects define the research gap that the work presented in this paper intends to address. It is part of an ongoing study concerned with the compilation and analysis of a database of FRC mix designs and the results of their characterisation tests [5]. In particular, this paper is concerned with steel FRC (SFRC) mixes and the analysis of the residual flexural strength parameters f_{R1} and f_{R3} in relation to the mix design. The relationships that exist between f_{R1} and f_{R3} and different variables describing the SFRC mixes (that is, relative amounts of the mix constituents and their fundamental descriptors) are analysed and quantified.

2. DATASET OF SFRC MIX DESIGNS

A dataset of steel fiber reinforced concrete (SFRC) mixtures was compiled from papers published between 2000 and 2019. The sources of information considered for this study were papers published in journals indexed in ScienceDirect® since 1999, resulting from the search with the terms "fiber-reinforced concrete" or "fibre-reinforced concrete". After an initial version of the dataset was completed, a preliminary analysis was carried out to detect and discard cases where the information was either clearly misreported or not consistent with the vast majority of the rest of the mixes. Cases where most of the information regarding the mix proportions was missing were also discarded. The resulting, final dataset comprised 765 different cases, extracted from more than 100 papers. More details on the construction of this dataset can be found elsewhere [5].

The methodological approach adopted for this analysis relied on the statistical technique known as multiple linear regression [6] in order to obtain equations that could explain f_{R1} and f_{R3} as a function of mix design variables, with two (and equally important) objectives. First, to be able to use these equations as part of the mix proportioning process. And second, to use them to produce different plots that can be used to interpret how the variation of mix design variables is associated with changes in f_{R1} and f_{R3}, and to quantify such variations.

All variables concerned with the relative proportions of the mix constituents were expressed in terms of relative weight of the constituent per unit volume of concrete, in kg/m^3. The fibre content in each mix was expressed as the volume fraction (V_f), in percentage. For each of the variables relevant to this paper, Table 1 provides the median as representative average, and the 5th and 95th percentiles as representative minimum and maximum values, respectively. A detailed descriptive analysis of the information in the SFRC dataset compiled and used in this study can be found in [5].

Table 1. Descriptive statistics of the database of SFRC mixes.

Parameter	Median	5% percentile	95% percentile
Cement (kg/m^3)	400	325	678
Additions (kg/m^3)	60	20	198
Water/cement ratio	0.45	0.22	0.60
Superplasticizer (kg/m^3)	4.0	1.3	14.0
Fibre length (mm)	45	13	60
Fibre aspect ratio	65	38	85
Fibre volume fraction (%)	0.51	0.25	2.0
Fine aggregate (kg/m^3)	835	524	1071
Coarse aggregate (kg/m^3)	880	388	1157
Max. aggregate size (mm)	15	1	20
f_{R1} (MPa)	5.3	0	21.6
f_{R3} (MPa)	4.3	0	17.6

3. MODELLING OF THE RESIDUAL FLEXURAL STRENGTH

The residual flexural strength parameters f_{R1} and f_{R3} were modelled as a function of the mix design variables by means of multiple linear regression. Prior to that, the 99th percentile was calculated for each of these parameters, and the cases where they took values above this percentile were removed, thereby discarding the 1% most extreme values.

Since residual flexural strength parameters are known to be strongly correlated [5, 7], the following modelling assumption was made: the regression equations for f_{R1} and f_{R3} had to be similar and differ only in the values of their coefficients. Initial models including all pairwise interactions were considered. Statistically non-significant terms were identified and removed following the application of various model selection methods [8].

The final, refined regression equations showed very good fit to the cases in the dataset. The R-squared values were 86% and 78% for f_{R1} and f_{R3}, respectively, which are remarkably high considering that data was obtained from more than 100 different sources and that SFRC residual flexural strength parameters are known to present significant variability [9, 10].

Both regression equations had the same structure, which is shown as Eq. (1), where: f_{Ri} stands for either f_{R1} or f_{R3}, G and S are the coarse and fine aggregates contents (kg/m^3), A is the dosage of mineral additions (kg/m^3), SP is the amount of superplasticiser (kg/m^3), λ_f is the fibre aspect ratio, M is the maximum aggregate size (mm), C is the cement content (kg/m^3), L_f is the fibre length (mm), and V_f is the fibre volume fraction (percentage). The terms corresponding to the effect of fibre length, volume fraction and the total amount of aggregates were found to be dependent on other variables, which is represented by the functions noted as K_L, K_V and K_{GS} in Eq. (1). These functions are given separately in Eqs. (2) to (4). The fitted coefficients k_0 to k_6, a_0, a_1, b_0 to b_4, and c_1 to c_3 take different values for f_{R1} and f_{R3}, which are given in Table 2.

$$f_{Ri} = k_0 + k_1 \frac{G}{S} + k_2 A + k_3 SP + k_4 \lambda_f + k_5 M + k_6 C + K_L L_f + K_V V_f + K_{GS}(G + S) \quad (1)$$

$$K_L = a_0 + a_1 M \quad (2)$$

$$K_V = b_0 + b_1 C + b_2 SP + b_3 \lambda_f + b_4 L_f \quad (3)$$

$$K_{GS} = c_1 C + c_2 A + c_3 \frac{G}{S} \quad (4)$$

Table 2. Coefficients in the fitted equations.

	Coefficients in Eq. (1)						
	k_0	k_1	k_2	k_3	k_4	k_5	k_6
For f_{Ri}=f_{R1}	-1.61	0	0.0756	0.0963	-0.0679	0.385	-0.0112
For f_{Ri}=f_{R3}	1.39	-7.16	0.0207	-0.1054	-0.0668	0	0.0147
	Coefficients in Eqs. (2) and (3)						
	a_0	a_1	b_0	b_1	b_2	b_3	b_4
For f_{Ri}=f_{R1}	0.1842	-0.00807	-3.89	0.0056	-0.0984	0.1616	-0.0708
For f_{Ri}=f_{R3}	0.0894	0	-5.41	-0.0038	0.0943	0.2077	-0.0433
	Coefficients in Eq. (4)						
	c_1	c_2	c_3				
For f_{Ri}=f_{R1}	0.000005	-0.000044	0				
For f_{Ri}=f_{R3}	-0.000006	-0.000012	0.0038				

4. ANALYSIS OF THE FITTED MODEL

The fitted model can be discussed in relation to different combinations of variables. An exhaustive examination of all possible visualisations is not attainable in one single paper. The following sections focus on the most interesting aspects, including findings in relation to variables that have usually attracted less attention such as the amount of aggregates or additions in the SFRC mix.

4.1. Fibre content and aspect ratio

Figure 2 shows the response surfaces for f_{R1} and f_{R3} versus the fibre volume fraction and aspect ratio, obtained by plotting Eq. (1) and setting the rest of mix design variables to their median (Table 1).

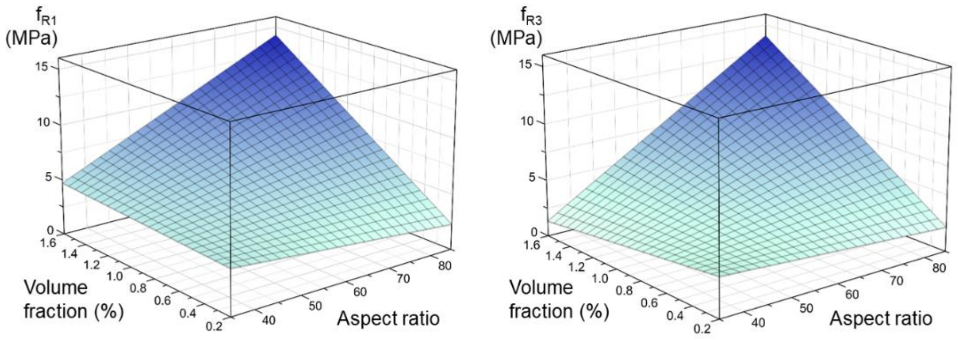

Figure 2. Residual flexural strength vs fibre volume fraction and aspect ratio.

Both residual flexural strength parameters presented very similar trends with respect to the fibre aspect ratio and volume fraction. The model showed that the effect of fibre volume fraction on f_{R1} and f_{R3} depends on the fibre aspect ratio. Increasing the aspect ratio had an important positive effect on f_{R1} and f_{R3} for moderate to high fibre dosages, and the plots in Figure 2 show that this effect becomes more noticeable the higher the fibre dosage is. However, this was not the case for very low volume fractions: when fibres were considered in very low contents, higher aspect ratios were not associated with better residual strength. There was, therefore, a certain volume fraction at which the trend with respect to aspect ratio changed, which was calculated by differentiating Eq. (1) with respect to the aspect ratio:

$$\frac{\partial f_{Ri}}{\partial \lambda_f} = k_4 + \frac{\partial K_V}{\partial \lambda_f} V_f = k_4 + b_3 V_f = 0 \;\; \rightarrow \;\; V_f = \frac{-k_4}{b_3} \tag{5}$$

Using the coefficient values from Table 2 in Eq. (5), the fibre dosage at which the trend with respect to the aspect ratio is reversed was 0.42% or 0.32%, for f_{R1} and f_{R3}, respectively. From this, it can be said that increasing aspect ratios were associated with increasing residual flexural strength as long as the fibre content was higher than 0.42%. Also, for fibre contents between 0.32% and 0.42%, the effect that varying the aspect ratio has on residual flexural strength was practically negligible.

These response surfaces were also analysed in contrast with the median values of f_{R1} and f_{R3} from the database compiled for this study, which was taken as reference of the performance of an average SFRC. As Figure 3 shows, the intersection of the response surfaces for f_{R1} and f_{R3} with their respective median planes made it possible to identify the combinations of aspect ratio and volume fraction values that were associated with better-than-average performance.

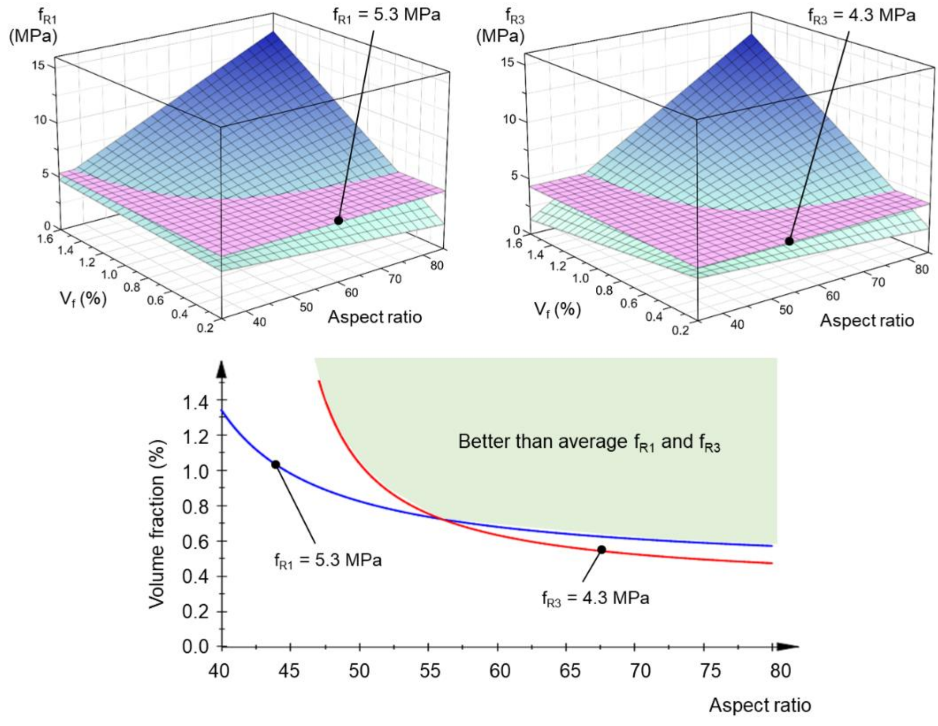

Figure 3. Fibre dosage and aspect ratio requirements to improve residual flexural strength.

The surface plots in Figure 3 show that improving the residual flexural strength was not necessarily linked with increasing the fibre content only. By projecting the intersections between each of these surfaces and their respective median planes unto the aspect ratio and volume fraction axes, two useful curves were obtained. For any fibre aspect ratio, these curves yield the minimum volume fraction requirement in order to achieve better-than-average performance. If fibres with an aspect ratio of 60 are considered, the fibre dosage requirement would be 0.69%, and this becomes 0.56% if the aspect ratio is 80. That is, better-than-average residual flexural strength was found to be achievable with fibre contents well below 1%. This was an interesting finding, especially bearing in mind that current trends in SFRC production indicate a preference for mixes with volume fractions not much higher than 0.5%.

4.2. Fibre length and maximum aggregate size

The surfaces for f_{R1} and f_{R3} against fibre length and maximum aggregate size shown in Figure 4 were obtained by plotting Eq. (1) assuming the fibre volume fraction at 0.5%, 1.0% and 1.5%, and median values for the other mix design variables as per Table 1. A significant interaction between fibre length and maximum aggregate size was observed in terms of their effect on f_{R1} (Figure 4, left). That is, the trend followed by f_{R1} with respect to the maximum aggregate size was dependent on the fibre length.

This was not the case with f_{R3} (Figure 4, right). This was not sensitive to changes in maximum aggregate size, and higher f_{R3} values were associated with longer fibres. However, the relative effect of increasing fibre length on f_{R3} was more important at low to moderate fibre dosages. Increasing the fibre length in 10 mm was associated with an average increase of 0.68 MPa when the fibre volume fraction was 0.5%, but only 0.25 MPa when the fibre volume fraction was 1.5%.

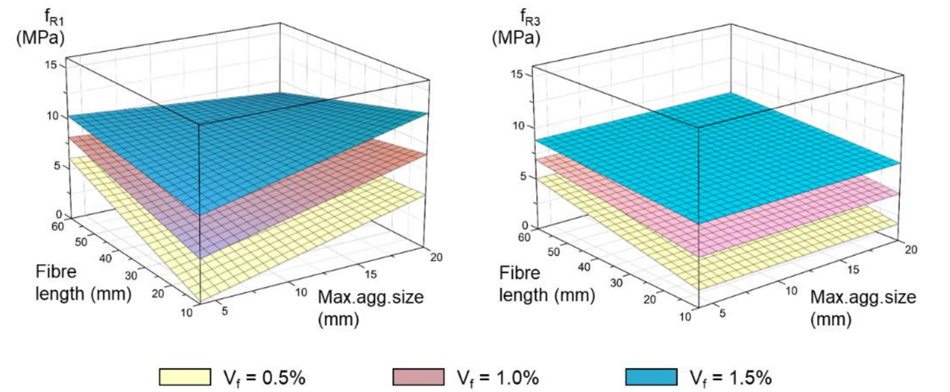

Figure 4. Effect of maximum aggregate size and fibre length on residual flexural strength.

The discussion around f_{R1} presented more complexity (Figure 4, left). For short fibre lengths, increasing the maximum aggregate size was associated with increasing f_{R1} values. However, with long fibres, the effect that increasing maximum aggregate sizes have on f_{R1} was the opposite. In consequence, there was a fibre length at which the trend followed by f_{R1} with respect to maximum aggregate size was reversed.

A similar observation can be made regarding the relationship between f_{R1} and the fibre length. For small values of the maximum aggregate size, increasing the fibre length was associated with an increase in f_{R1}. However, when the maximum aggregate size was higher than a certain value, longer fibres were found to reduce f_{R1} rather than improve it.

The fibre length at which the trend of f_{R1} with respect to maximum aggregate size is reversed was obtained by differentiating Eq. (1) with respect to the maximum aggregate size:

$$\frac{\partial f_{Ri}}{\partial M} = k_5 + \frac{\partial K_L}{\partial M} L_f = k_5 + a_1 L_f = 0 \;\; \rightarrow \;\; L_f = \frac{-k_5}{a_1} \tag{6}$$

Similarly, the maximum aggregate size at which the trend of f_{R1} with respect to fibre length is reversed was obtained by differentiating Eq. (1) with respect to fibre length:

$$\frac{\partial f_{Ri}}{\partial L_f} = K_L + \frac{\partial K_V}{\partial L_f} V_f = a_0 + a_1 M + b_4 V_f = 0 \;\; \rightarrow \;\; M = \frac{-a_0 - b_4 V_f}{a_1} \tag{7}$$

By using the coefficient values for $f_{Ri} = f_{R1}$ from Table 2 in Eq. (6), the fibre length at which the relationship between f_{R1} and maximum aggregate size is reversed was found to be 47.7 mm, very close to the commercially available fibre length of 45 mm. Therefore, it can be said that smaller maximum aggregate sizes were associated with higher f_{R1} values when fibres longer than 45 mm were used. Also, as the contour plots in Figure 5 show, when the fibre length was 45 mm, f_{R1} was found to be practically insensitive to the maximum aggregate size. Interestingly, this threshold value for the fibre length was independent of the fibre content and other mix design parameters, as Eq. (6) and the contour plots in Figure 5 show, and therefore the abovementioned considerations have general validity.

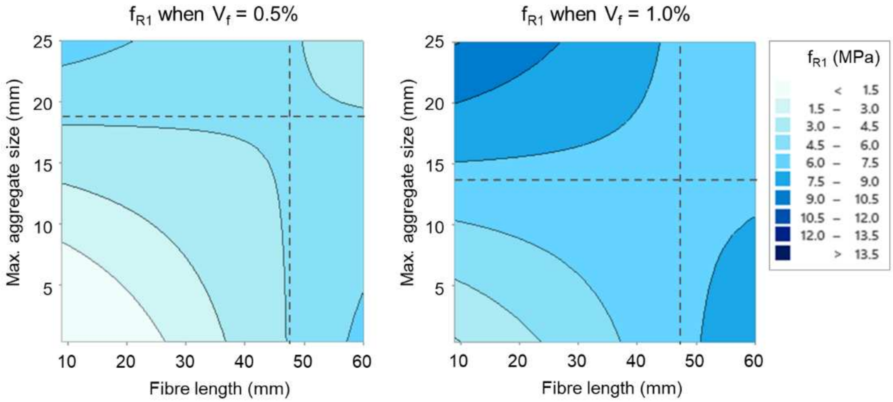

Figure 5. Effect of maximum aggregate size, fibre length and fibre content: contour plots.

That was not the case for the maximum aggregate size that makes f_{R1} insensitive to changes in fibre length, which was a function of the fibre content as per Eq. (7). As shown in Figure 5, for a fibre volume fraction of 0.5%, increasing the fibre length increased f_{R1} when the maximum aggregate size was not higher than 19 mm (Figure 5, left). However, for higher fibre dosages, this limit to the maximum aggregate size decreased linearly, being 14 mm for a fibre content of 1% (Figure 5, right), and 10 mm for a fibre content of 1.5%. Therefore, in terms of optimising f_{R1} in SFRC mixes with high fibre contents, smaller maximum aggregate sizes were found to be most effective.

4.3. Additions and total aggregate content

Figure 6 shows the surfaces for f_{R1} and f_{R3} against the total contents of aggregates and additions, obtained by plotting Eq. (1) for fibre volume fractions of 0.5%, 1.0% and 1.5%, and assuming median values for the other mix design variables (Table 1). The effects of these two variables on residual flexural strength were not independent from one another, particularly in relation to f_{R1}, as the trend followed by this parameter with respect to the amount of additions varied with the total aggregate content, and vice versa. This can be observed in Figure 6 (left). In relation to f_{R3}, however, the interaction between the total aggregate content and the amount of additions was much less relevant, as

Figure 6 (right) shows. In fact, considering the plots in Figure 4 (right) and Figure 6 (right), it can be said that f_{R3} was found to be practically insensitive to variations in maximum aggregate size, total aggregate content or dosage of additions, in contrast with the behaviour of f_{R1}.

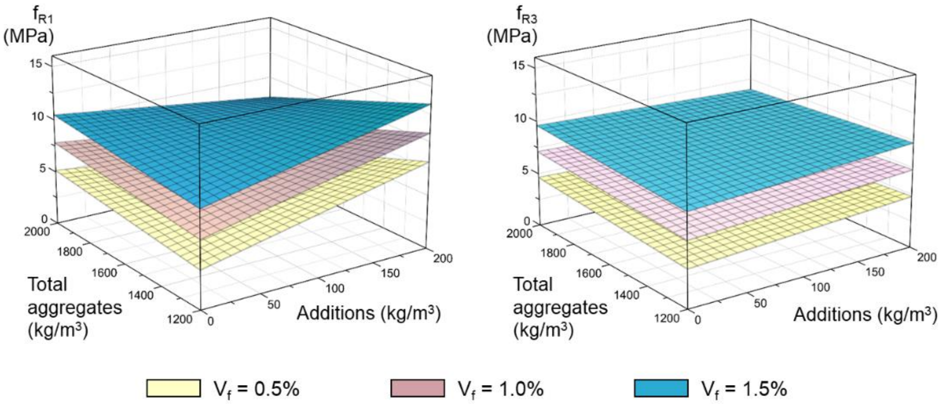

Figure 6. Effect of the total aggregate and additions contents on residual flexural strength.

Figure 6 (left) shows that, regardless of the fibre content, increasing dosages of additions were associated with higher f_{R1} values, especially when the total aggregate content was relatively low. However, the trend followed by f_{R1} with respect to the amount of additions was reversed when total aggregate contents on the higher end of the range were considered. There was, in consequence, a value of the total aggregate content at which the relationship between f_{R1} and dosage of additions changed. This was determined by differentiating Eq. (1) with respect to the amount of additions:

$$\frac{\partial f_{Ri}}{\partial A} = k_2 + \frac{\partial K_{GS}}{\partial A}(G+S) = k_2 + c_2(G+S) = 0 \;\rightarrow\; G+S = \frac{-k_2}{c_2} \qquad (8)$$

Considering the coefficient values for $f_{Ri} = f_{R1}$ from Table 2 in Eq. (8), the total aggregate content at which the relationship between f_{R1} and the amount of additions is reversed was found to be 1718 kg/m^3. It is interesting to note that this value, as per Eq. (8), is independent from any other mix design parameters and, in particular, does not change with the fibre size or volume fraction.

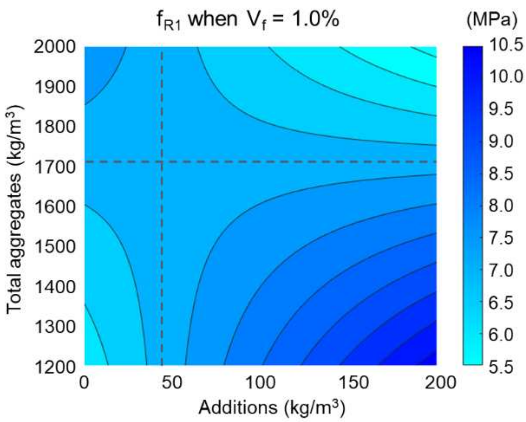

Figure 7. Contour plot for f_{R1} versus total aggregate content and dosage of additions.

The relative effect of the aggregate content and dosage of additions on f_{R1} can be better appreciated in the contour plot shown in Figure 7, which represents f_{R1} values as per Eq. (1) against these two parameters for a fibre volume fraction of 1.0%. The horizontal dashed line corresponds to a total aggregates content of 1718 kg/m^3. As the contour plot shows, f_{R1} was practically insensitive to the dosage of mineral additions in SFRC mixes with a total aggregates content around this value. However, in mixes with a total aggregate content lower than 1718 kg/m^3, increasing amounts of additions were associated with increasing f_{R1} values. In fact, the contour plot clearly shows that, in order to optimise f_{R1}, the introduction of mineral additions combined with a reduction in the total aggregate content is an advantageous strategy.

Conversely, the vertical dashed line in Figure 7 corresponds to the total amount of additions which makes f_{R1} insensitive to the total aggregate content. This value was determined by differentiating Eq. (1) with respect to the total amount of aggregates, and was a function of the relative amount of cement in the SFRC mix:

$$\frac{\partial f_{R1}}{\partial(G+S)} = K_{GS} = c_1 C + c_2 A + c_3 \frac{G}{S} = c_1 C + c_2 A = 0 \;\; \rightarrow \;\; A = \frac{-c_1 C}{c_2} = 0.114\, C \qquad (8)$$

That is, f_{R1} values were found to not be affected by changes in the total aggregate content when the amount of additions was 11.4% of the amount of cement in the SFRC mix. This corresponds to 45 kg/m^3 of additions when the cement content is considered at its median (400 kg/m^3) as per Table 1, which is the case in Figure 7. Furthermore, as Figure 6 (left) and Figure 7 show, this was the minimum amount of mineral additions to be incorporated to the SFRC mix for them to have a positive effect on f_{R1} values.

In consequence, based on the dataset of SFRC mixes analysed in this study and the proposed model for residual flexural strength values, the following two general recommendations for the optimisation of f_{R1} emerged: that the total aggregate content is maintained below 1718 kg/m^3, and that mineral additions are considered in dosages of at least 11.4% of the relative amount of cement.

5. CONCLUSIONS

Based on a dataset of SFRC mixes compiled from research papers published in the last two decades, a detailed analysis and modelling is underway. This paper presents the analysis of the relationships that exist between the residual flexural parameters f_{R1}, f_{R3}, and the relative amounts of the mix constituents and their fundamental characteristics. The main conclusions can be summarised as follows:

- A regression model relating f_{R1} and f_{R3} values to the mix design parameters has been obtained. The resulting equations show a very good level of accuracy in fitting the information in the dataset (R-squared of 86% and 78%, respectively), considering the heterogeneity of the data and the variability of residual flexural strength parameters.
- The effect of fibre volume fraction on f_{R1} and f_{R3} is dependent on the fibre aspect ratio, and vice versa. For fibre contents above 0.42%, increasing aspect ratios are unequivocally associated with increasing f_{R1} and f_{R3} values.
- Fibre contents well below 1.0% should suffice to optimise the residual flexural strength and obtain f_{R1}, f_{R3} values that are better than average. The minimum dosage requirement to achieve better-than-average residual flexural strength is 0.69% for fibres with an aspect ratio of 60, and 0.56% for an aspect ratio of 80.
- When fibres with a length of 45 mm are used, f_{R1} is practically insensitive to the maximum aggregate size. However, when longer fibres are used, reducing the maximum aggregate size leads to increasing f_{R1} values. Reducing the maximum aggregate size is particularly advantageous when fibres are considered in high volume fractions.

- The total aggregate content and the amount of additions have a significant impact on f_{R1}, and their effects on this parameter are not independent from one another. On the other hand, the effect of maximum aggregate size, total aggregate content or dosage of additions on f_{R3} values is practically negligible.
- Whatever the fibre content, increasing dosages of additions are associated with higher f_{R1} values when the total aggregate content is not higher than 1718 kg/m^3. In order to optimise f_{R1}, a general recommendation derived from this study is that the total aggregate content is maintained below that threshold and the relative amount of additions is at least 11.4% the relative amount of cement.

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