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Employing non-contact sensing techniques for improving efficiency and automation in numerical modelling of existing masonry structures: A critical literature review

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Abstract

This paper presents approaches for the employment of non-contact sensing to enhance both the efficiency and reliability of numerical modelling of historic masonry. It commences with a thorough review of the high-level numerical modelling approaches of historic masonry. Following, the accuracy and cost-effectivity of available non-contact sensing techniques are reviewed for surveying masonry structures. These are: a) the total station; b) the laser tracker; c) Structure-from-Motion (SfM) photogrammetry; and d) terrestrial laser scanning (TLS). Then, strategies of automatically developing geometric models (i.e., numerical models before structural analysis) from geospatial data are reviewed, considering their potential for automation and usage. These were based on the employment of: a) point clouds; b) meshes; c) non-uniform rational basis splines (NURBSs); d) building information models (BIMs); e) orthoimages; and f) discrete points. Primarily, the review found that high-level numerical modelling approaches such as the continuum and block-based models are highly effective, but necessitate accurate geometric data for reliable results. To bridge this gap, the potential of emerging technologies such as SfM photogrammetry was found to significantly improve the efficiency and robustness of high-level structural analysis, through providing geometric data accurately and with a low cost. Moreover, the cloud-based (i.e., with a point cloud) and image-based (i.e., with an orthoimage) approaches of converting geospatial data into numerical models were also found the most effective, for continuum and block-based modelling respectively. This contribution demonstrates the potential to employ novel digital technologies such as non-contact sensing techniques to improve the efficiency and robustness of high-level numerical modelling approaches.

Keywords: masonry, numerical modelling, non-contact sensing techniques, SfM photogrammetry, BIM, terrestrial laser scanning, laser tracker

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1 Introduction

Historic masonry structures, such as retaining walls, domestic property, bridges, viaducts, tunnels etc form a significant part of our building stock. Most of our masonry stock is ageing, often is well beyond 100 years old, and showing significant signs of deterioration and distress. Weathering, changing loading demands, vibrations due to earthquakes and anthropogenic events, plus factors such as increased frequency of flood events due to climate change have introduced extreme uncertainty in the long-term performance of such assets. Also, much of our masonry structures have significant heritage and cultural value (e.g., the Grade II-listed Hungerford Canal Bridge, in Berkshire, England (Garrity, 2013)) and according to UN Sustainable Development Goal 11 (United Nations, 2016), efforts should be placed on "retain and repair", rather than "demolish and replace". Failure of such structures could lead to direct and indirect costs to the economy and society. Therefore, there is an urgent need to better understand the in-service performance of our ageing historic masonry structures and to provide detailed and accurate data that will better inform maintenance programmes and asset management decisions.

However, assessing the structural performance of old and deteriorated historic masonry structures is a complex task. Previous research has clearly demonstrated that the assessment methods currently used by the industry are antiquated and/or over-simplistic. For example, for the assessment of masonry arch bridges, the Military Engineering Experimental Establishment (MEXE) method of assessment is still in use (Highway Agency, 2001). This method dates back to the 1940s, has a very limited predictive capability, and offers little scope for future enhancement. Also, although the primary focus of past research has been into the prediction of structural failure of ageing historic masonry structures, prediction of the service load above which incremental damage occurs is now a key priority for owners. Over the last three decades, significant efforts have been devoted to the development of numerical models to represent the complex and non-linear behaviour of masonry structures subjected to external loads. Such models range from considering masonry as a continuum (macro-models) to the more detailed ones that consider masonry as an assemblage of units and mortar joints (block-based models). However, a vital aspect when modelling masonry structures is the accuracy in which the geometry and material performance characteristics of the masonry constituents (i.e. masonry units, mortar joints) are transferred in the numerical model. Geometric model development (or otherwise termed solid model development), is the procedure of developing the geometry of the structure in an appropriately digital format so that it can be inputted into a structural analysis numerical model. According to various investigations (Brenner, 2005; Hinks *et al.*, 2012), the geometric models can be described by: a) a boundary representation, in which the geometric model represents the masonry structure explicitly; b) constructive solid geometry (CSG), in which the geometric model represents the masonry structure from Boolean operations of simpler geometric objects; and c) spatial enumeration, in which the geometric model of the masonry structure is represented as a composition of smaller geometric models occupying the domain of the masonry structure, e.g. voxels.

Evidence from past studies (Heyman, 1969) has shown that geometric changes in masonry structures can greatly influence their mechanical response. Following this intuition of Heyman, numerous investigations, it has been shown that the variation in the geometry on a block-based level (i.e. joint inclination, block size and bond pattern) causes significant differences in the predicted structural behaviour of a masonry structure to be analysed, see

Table 1. Notably, Szakály *et al.* (2016) demonstrated that the variance in masonry bonding pattern yielded significant influence on both failure mode and collapse load of the masonry wall panels. In another study (Godio *et al.*, 2018), found that the arrangement and size of blocks are significantly influential on the out-of-plane capacity of wall panels subjected to seismic loading. The same year, Forgács *et al.* (2018), also found that the construction method of masonry arches with the same span and same angle of skew significantly influences the collapse load and failure mode of the arches. Finally, in a more recent study (Napolitano and Glisic, 2019), found that the block pattern significantly influences the structural capacity of wall panels with the same height and length.

Table 1. The effect of geometric uncertainty of ad-hoc geometric models on structural behaviour.

Study	Geometrical properties investigated	Structural behaviour indices investigated	Main findings
(Szakály *et al.*, 2016)	Masonry wall patterns	Shear resistance of masonry wall due to horizontal point load	Vertical bricks affect shear resistance for low confining vertical loads
(Godio *et al.*, 2018)	Block size, bed joint orientation	Collapse load due to horizontal gravitational load	The larger the blocks, the higher the structural capacity. The bed joint angle also influences structural capacity
(Forgacs *et al.*, 2017)	Block size, block length-to-width ratio (L/H)	Stabilitydue to self-load	The larger the blocks (the larger the L/W for constant W), the more stable the arch is
(Forgács *et al.*, 2018)	Masonry arch construction methods	Collapse load due to vertical point load	The method of construction (false skew, helicoidal or logarithmic) influenced the structural capacity of masonry arches
(Pulatsu *et al.*, 2018)	Masonry arch block bond pattern and layer number (ring number)	Collapse load due to vertical point load	Double-layer arch had a lower structural capacity of the respective single. Bond pattern of arch voussoirs was not influential on structural capacity
(Napolitano and Glisic, 2019)	Bonding course of masonry walls	Maximum vertical displacement and principal stress due to settlement	The consideration of bonding courses increased structural capacity for settlement

Although from the above studies (see Table 1) it is evident that the employment of simplified (i.e. ad-hoc or idealised) geometric models can significantly compromise the robustness of the structural analysis, the reasons why the majority of numerical modelling studies focusing on employ simplified geometric models are: a) difficulties in geometric data acquisition, i.e. obtaining the exact location of each block and joint (geometry on a block-based level) manually is an extremely laborious procedure, and potentially prohibitive for the case of a large-scale structure; b) developing accurate geometric model development is extremely complex (i.e. the block-by-block procedure (Sarhosis *et al.*, 2016c) in comparison with the user-friendly continuum-based models as will be discussed in the forthcoming sections), while methodical and automated frameworks for such task lack; and c) comprehensive investigation still lacks on the automation of the process from surveying to geometric model development for the numerical modelling of masonry structures. Of major importance to the geometric model development process is the numerical method employed i.e. continuum or discontinuum approach (Bobet *et al.*, 2009). Within continuum methods, the discontinuities may be implemented either implicitly or explicitly. Conversely, in discontinuum methods, discontinuities in masonry are explicitly incorporated.

Over the last two decades, advances in non-contact sensing techniques such as terrestrial laser scanning (TLS) and Structure-from-Motion (SfM) photogrammetry have started to drastically change the building industry due to such techniques rapidly and remotely harvesting digital geometric records of objects and features in a point cloud format. SfM photogrammetry is a passive non-contact sensing technique in which, interest points (IPs) are detected in overlapping images of a structure and used to reconstruct a point cloud using common feature matching and triangulation (Westoby *et al.*, 2012). Compared to SfM photogrammetry, TLS is an active non-contact sensing technique. Time-of-flight scanners, which are more relevant to applications of masonry structures, measure distance by timing the emission of a pulse of laser energy to the detection of the reflected signal (Mills and Barber, 2004). Both SfM photogrammetry and TLS have a demonstrated suitability for accurately and rapidly obtaining the complex geometry of masonry structures (Altuntas *et al.*, 2017; D'Altri *et al.*, 2018b).

The aim of this paper is to review approaches for the employment of non-contact sensing to enhance both the efficiency and reliability of numerical modelling of historic masonry. Firstly, the numerical modelling approaches of historic masonry structures are reviewed, with emphasis on those with the potential to perform high-level structural analysis. Following, non-contact sensing techniques approaches are reviewed for surveying masonry structures, concerning their accuracy and cost-effectivity. Finally, approaches of developing geometric models from the data of structural surveying are reviewed with respect to their degree of automation and facility in implementation. The organization of this paper is as follows: Section 2 introduces masonry as a construction material; Section 3 presents numerical modelling strategies of masonry; Section 4 reviews current high-level numerical modelling approaches; Section 5 reviews non-contact sensing techniques for capturing geometric data for numerical modelling; Section 6 reviews geometric model development strategies for automating numerical modelling; and Section 7 reports the paper's conclusions.

2 Masonry as a construction material

Masonry is one of the oldest construction materials, still widely adopted in a similar manner to that of thousands of years ago (Lourenco, 1996). In the 19th century, the appearance of building materials such as steel and reinforced concrete have led to its reduction to either load-bearing walls of small-scale buildings or mere infill walls (Hendry, 2001) yet historic masonry structures encompass a considerable proportion of the built environment. While masonry is highly effective for sustaining vertical compressive (Heyman, 1997; Como, 2013), it is susceptible to sustain tension and shear loads. Masonry is generally a highly durable form of construction. However, the materials used, the quality of the mortar and workmanship, and the pattern in which the units are assembled can substantially affect the durability of the overall masonry construction.

The art of construction by assembling units of stone, clay, brick, or concrete blocks defines masonry. Masonry is classified based on the spatial organisation of its units, the so-called masonry typology, which also significantly influences masonry's structural behaviour (Zhang *et al.*, 2018a). Brick masonry, for instance, can be distinguished into periodic (similar to stone ashlar, in which the patter is termed bond) and rubble. On the other hand, stone masonry has been classified (Vanin *et al.*, 2017) by various levels of regularity, such as: a) irregular stone masonry, with pebbles, irregular stone units (Figure 1a); b) uncut stone masonry (Figure 1b); c) cut stone masonry with good bonds (Figure 1c); d) soft stone regular masonry (Figure 1d); e) ashlar masonry, built with sufficiently resistant blocks (Figure 1e); and ashlar masonry, built with sufficiently resistant blocks and the blocks are perfectly rectangular and all blocks of one row have the same height (Figure 1f).

Of major importance to the structural behaviour of masonry is also the cross-section morphology (Binda *et al.*, 2009). For masonry walls, for instance, this may be: a) single leave; b) double leaves without connection; c) double leaves with connection; and treble leaf stone masonry walls. This aspect is important due to its influence on the monolithic behaviour of the masonry (Binda *et al.*, 2009). Finally, the type of structure masonry also significantly affects its structural behaviour.

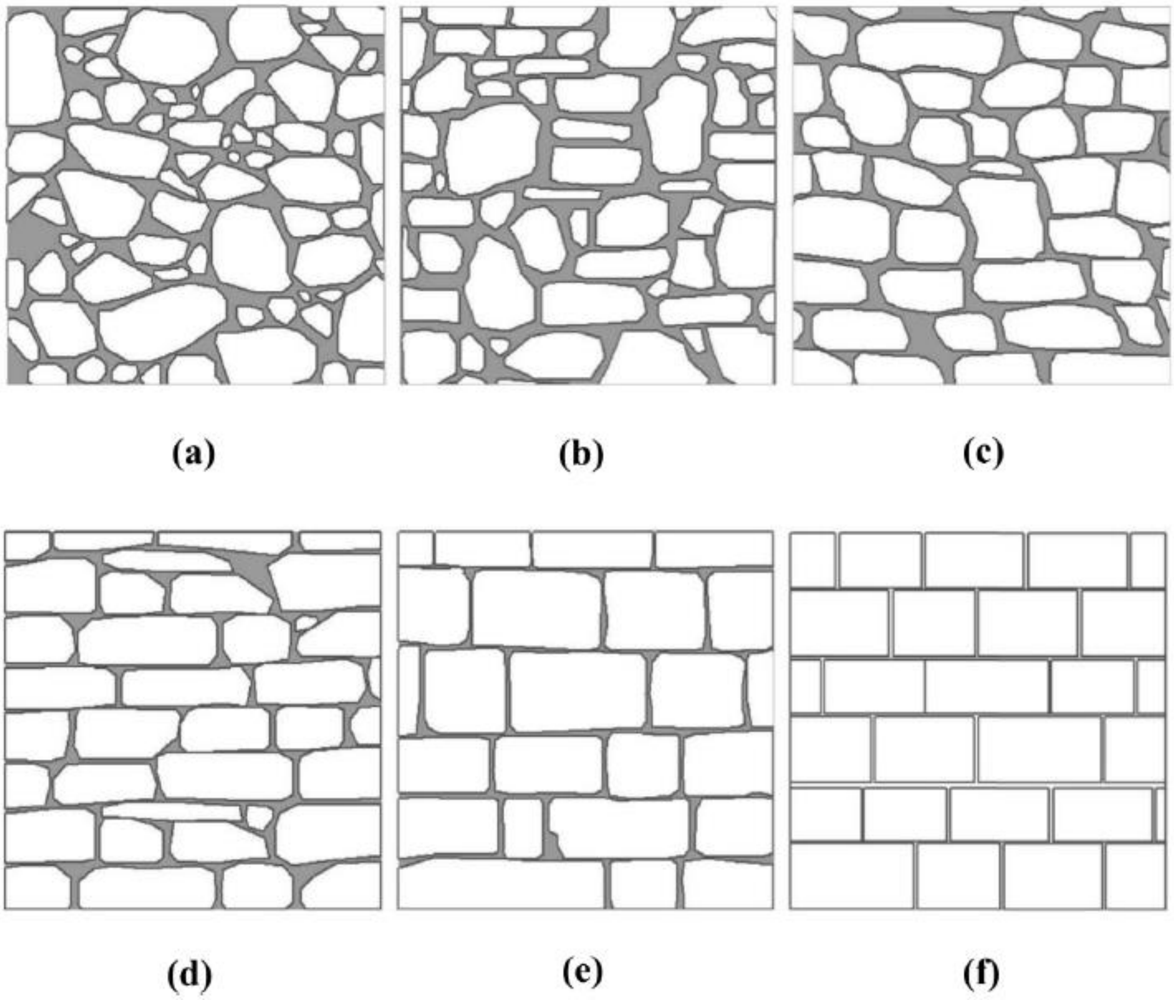


Figure 1: Typologies of stone masonry (Vanin et al., 2017): (a) class A; (b) class B; (c) class C; (d) class D; (e) Class E1; and (f) Class E2.

3 Numerical modelling strategies

Depending on the required accuracy and detail required of the structural analysis, multiple modelling strategies have been proposed by various investigators (Lourenco, 2002; Asteris *et al.*, 2015; D'Altri *et al.*, 2019). Modelling strategies are distinguished according to how heterogeneity of masonry is simulated in a given numerical model. According to a recent classification (D'Altri *et al.*, 2019), the following modelling strategies can be employed: a) continuum models; b) block-based models; c) macro-element models; and e) geometry-based models.

3.1 Continuum models

In this strategy, the structure is considered as a homogenous anisotropic continuum which is effective for instances where the anisotropy of masonry is less significant (D'Altri *et al.*, 2019) such as the case of rubble masonry structures. The main advantages of continuum modelling, in comparison with block-based modelling are: a) the anisotropy of the masonry is not explicitly represented with a block-by-block manner, which significantly simplifies geometric model development (as will be further detailed in Section 4.1); and b) since the geometric refinement is reduced, the computational burden is significantly reduced accordingly. However, a major limitation of this strategy is that the determination of material properties is an extremely complicated task; which is accomplished through either the employment of experimentally derived constitutive laws or homogenisation processes. Furthermore, another limitation is that only knowledge of global behaviour is emphasised whilst the local behaviour (i.e. interaction between blocks) is neglected.

3.2 Block-based models

In the block-based modelling strategy, the masonry blocks and joints are explicitly described within the geometric model as blocks and mortar elements. Due to this very detail, a simplified variation of the block-based modelling is usually employed in which the blocks are expanded to maintain the initial geometry and mortar is replaced with a zero-thickness element. This is particularly advantageous for regular masonry structures (and generally low bond strength masonry), where the anisotropy of the masonry is well-defined (D'Altri *et al.*, 2019) and failure predominantly occurs due to sliding and cracking between blocks. Furthermore, the Young's modulus, Poisson's ratio, and inelastic properties of both the masonry block and mortar are accounted for. The main advantages of the block-based modelling strategy are: a) it enables a realistic description of a given masonry structure, with an explicit representation of the masonry's anisotropy and structure's geometric details; b) it facilitates the capturing of accurate structural behaviour and failure modes; and c) mechanical properties can be derived straightforwardly, directly from small-scale experiments. However, its limitations are also notable, such as that: a) it can be computationally burdensome, especially for the case of large-scale masonry structures; b) the development of geometric models is extremely cumbersome which can also severely delay the given numerical modelling approach (as will be further detailed in Section 4.2); and c) it is only employed in research except for few high-level practising engineers (D'Altri *et al.*, 2019).

3.3 Macro-element models

In this strategy, the masonry structure is considered as an assembly of structural components, typically piers and spandrels (vertical and horizontal load-bearing elements respectively). Within the geometric model, spandrels and piers are assigned based on experimental knowledge of the modelled structure. Regarding material properties, a constitutive law is employed that governs the structural behaviour on a spandrel and pier level. Owing to this simplicity, the main advantages of the macro-element modelling strategy is its low computational burden which makes it of the most employed for the dynamic analysis of unreinforced masonry (URM) (D'Altri *et al.*, 2019). However, it has significant limitations, such as:

a) the initial predefinition of the spandrels and piers appears to be oversimplified which necessitated judicious application; and b) the local behaviour associated with the anisotropy of masonry cannot be accounted for.

3.4 Geometry-based models

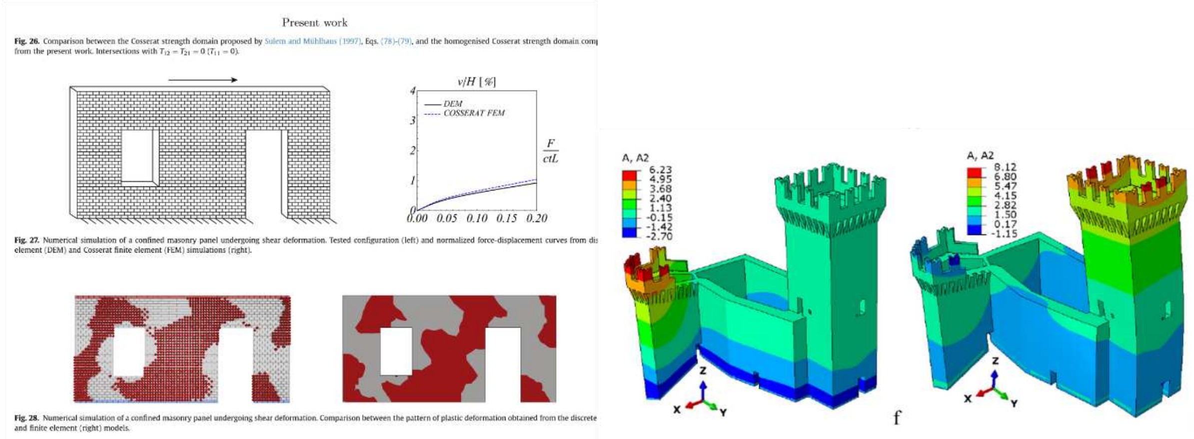
In this strategy, further, than the loading scenario, the only other variable is the geometric model which is described as a rigid medium. Such models typically employ limit analysis (LA) approaches and lead to the estimation of the collapse or equilibrium load through the lower and upper boundary theorems respectively. The main advantage of this strategy is its simplicity in implementation and low computational burden (D'Altri *et al.*, 2019). However, the limitation of such models is associated with their simplicity (i.e. the only variables being geometry and loading), which does not permit an in-depth understanding of the structural behaviour of masonry. Furthermore, they share the inherent limitations of the LA models which is the fact that they only provide the collapse load and failure mechanism.

4 High-fidelity numerical modelling approaches

In this Section, the current state-of-the-art of numerical modelling approaches are reviewed. Due to the large volume of existent research on the numerical modelling of masonry, a comprehensive review of all the literature would be unrealistic. Thus, the current Section is limited to the two most advanced strategies (i.e. high-level) which are the continuum and block-based, according to D'Altri *et al.* (2019). The interested reader is however referred to further literature (Roca *et al.*, 2010; Smoljanovic *et al.*, 2013b; Asteris *et al.*, 2015; Sarhosis *et al.*, 2016c; Baraldi *et al.*, 2017; Ademovic and Hadzima-Nyarko, 2019; D'Altri *et al.*, 2019).

4.1 Continuum models

Within the continuum models, the following approaches have been identified: a) LA continuum models; and b) finite element method (FEM) continuum models. The following paragraphs introduce both the approaches and relevant studies, as demonstrated in Figure 2.



LA continuum models, (Godio et al., 2017) FEM continuum models, (D'Altri et al., 2018)
(a) (b)

Figure 2: Continuum-based models.

4.1.1 LA continuum models

LA models are well-established for the structural analysis of masonry structures. The main advantage of such models is that they provide the masonry structure's collapse load and failure mode, rapidly with a relatively little computational demand in most cases. For this very reason, they are particularly attractive to practising engineers through their availability within various commercial software packages such as *Ring* (LimitState, 2019). However, a significant limitation is that only the collapse load is provided whilst failure displacements are unknown (i.e. load-displacement type responses are unattainable). Furthermore, the assumptions the LA employs, especially concerning material properties can be oversimplified.

A first class of LA models were developed within the continuum modelling strategy. In the so-called direct-continuum models, the macroscopic constitutive law ascribed to the numerical model is derived directly from experiments. For instance, Milani *et al.* (2012) developed direct continuum models of a full-scale historic masonry tower, employing a piecewise linear approximation with a Mohr-Coulomb failure criterion and tension cut-off and cap in compression for masonry interfaces. Concerning the solution, linear programming was employed. A second approach is based on the homogenisation theory; in which masonry is represented with a periodic regular texture while the macroscopic constitutive law is obtained from the solution of a boundary cell problem in at a cell level. Within this approach, Milani *et al.* (2006a) presented a pioneering investigation with homogenisation within the LA, by employing the polynomial expression of the stress field inside a representative volume element (RVE) whist the structural capacity of masonry was deduced by utilizing the strength domain. Many state-of-the-art studies have subsequently followed with advanced homogenisation approaches (Milani *et al.*, 2006b;

Cecchi *et al.*, 2007; Cecchi and Milani, 2008; Milani, 2011; Cavalagli *et al.*, 2013; Godio *et al.*, 2017), as shown in Figure 2a.

4.1.2 FEM continuum models

Whilst the LA is effective for an accurate prediction of the collapse load, it cannot provide a detailed structural analysis, which consists in describing the in-service and collapse behaviour. In the need for more sophisticated structural analysis than the LA, FEM continuum models have also been employed, which can provide load-displacement type responses. The main advantages of the FEM continuum models according to Sarhosis *et al.* (2016c), include:

- Straightforward implementation due to a multitude of commercial practice-oriented software packages;
- Facilitated geometric model development through user-friendly tools;
- Common application by both practising engineers and researchers.

The earliest class of FEM continuum models attempted to simulate masonry on a global scale through ascribing a constitutive law capable of reproducing the anisotropy of masonry. The so-called non-tension models, developed by Del Piero (1989), were built on the idealisation that masonry has a zero tensile strength. Whilst they were effective for an initiatory structural analysis, they could not be adopted for the tensile regime of masonry structures. Effectively, since actual masonry structures do possess a tensile strength (even if small), non-tension models cannot simulate the post-peak behaviour of masonry structures and lead to incorrect failure modes.

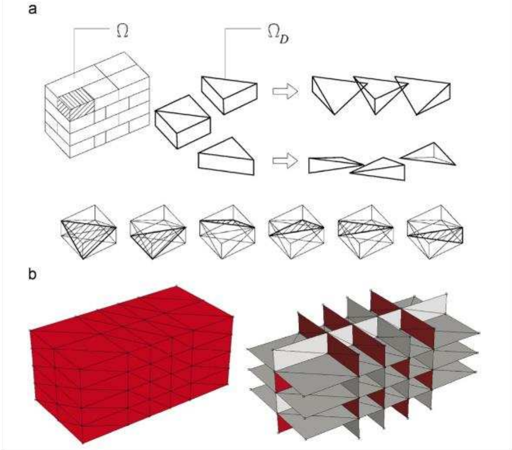
The necessity for capturing the non-linear behaviour of masonry led to the replacement of non-tension models with more advanced, non-linear models, inspired by the numerous smeared crack, orthotropic plasticity and orthotropic damage models of reinforced concrete (Hofstetter *et al.*, 2011; Jirásek, 2011). Lotfi and Shing (1991) pioneered the employment of non-linear models by evaluating the smeared crack model on masonry shear walls. Generally, smeared crack models are advantageous for historic masonry structures (such as rubble masonry structures) due to: a) the randomness of the masonry's geometry, the assumption of isotropy is well-standing; and b) their facility of implementation since are found within most commercial FEM codes (D'Altri *et al.*, 2019). For this reason, they are still employed to this day, as in the study of (D'Altri *et al.*, 2018) (shown in Figure 2b). However, for the case of regular masonry, especially of a low-bond strength, the assumption of orthotropy is not well-standing and smeared crack models cannot be employed.

In an attempt to overcome the limitations of smeared crack models, Lourenco and Rots (1997), formulated the first orthotropic plasticity models which effectively represented the tensile strength of the material in the principal directions. The specific models were validated in comparison with experimental masonry panels and proved extremely effective for capturing experimental behaviours. Recently, orthotropic damage models have been extensively employed, including many state-of-the-art studies. (Lopez *et al.*, 1999; Berto *et al.*, 2002; Reyes *et al.*, 2009; Pelà *et al.*, 2011; Pelà *et al.*, 2013; Pelà *et al.*, 2014).

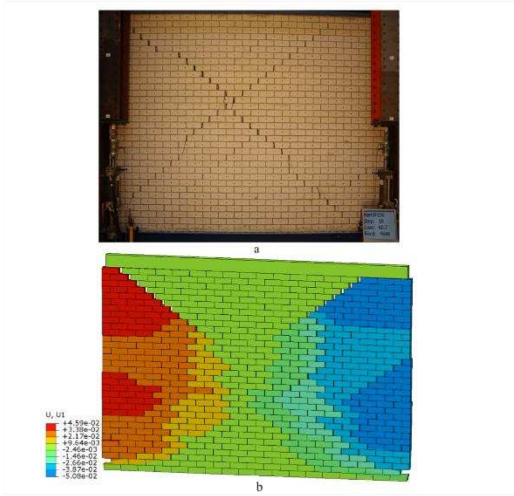
As with the LA, homogenisation processes have also been applied to the FEM. Pietruszczak and Niu (1992) presented an early homogenisation approach which was performed on masonry wall panels with the FEM in a two-stage manner, by introducing the head joints and bed joints as elastic inclusions and dispersed sets of weaknesses. However, Antoine (Anthoine, 1995), formalised the homogenisation procedure by carrying it out in one step only, introducing the actual pane thickness and the actual brick geometry. Further on, more advanced homogenisation approaches have been developed, capable of considering complex failure mechanisms such as in (Lopez *et al.*, 1999; Zucchini and Lourenço, 2002). State-of-the-art homogenisation studies have included multi-scale approaches which overcome mesh dependency whilst representing localised failure (Leonetti *et al.*, 2018). This is carried out with a so-called first-order homogenisation until a threshold of damage is reached. After reaching the threshold, the damaged region of interest is replaced with a heterogeneous material (D'Altri *et al.*, 2019).

4.2 Block-based models

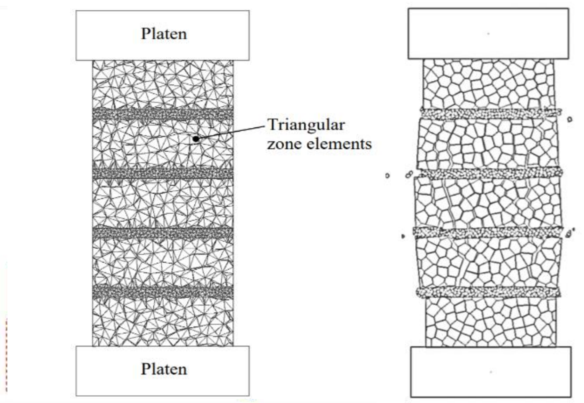
Within the block-based models, the following approaches have been identified: a) LA block-based models; b) FEM block-based models; c) distinct element method (DEM) models; d) non-smooth contact dynamics (NCSD) models; e) discontinuous deformation analysis (DDA) models; and e) finite-discrete element method (FDEM) models. The following paragraphs introduce both the approaches and relevant studies, as demonstrated in Figure 3.



LA block-based models, (Milani, 2008)
(a)

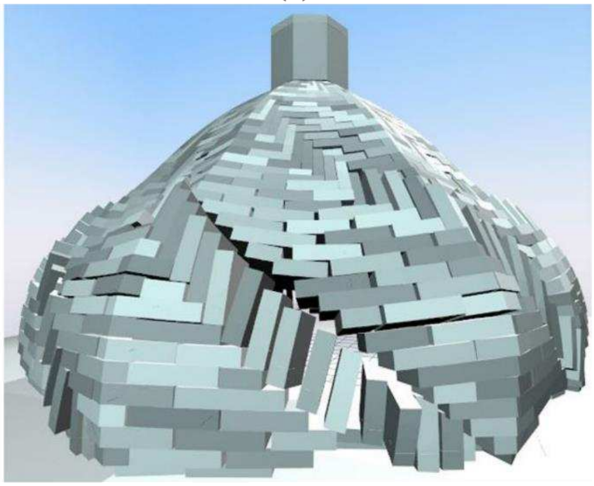


FEM block-based models, (D'Altri et al., 2019)
(b)

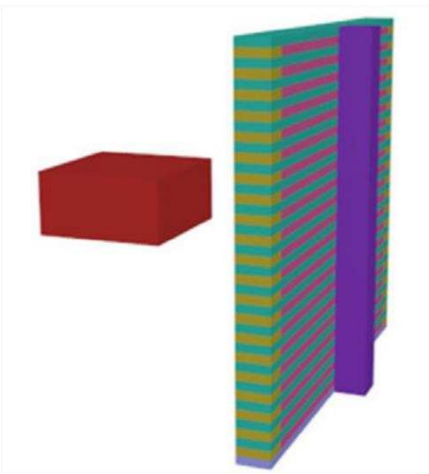


DEM models (Sarhosis & Lemos 2018)

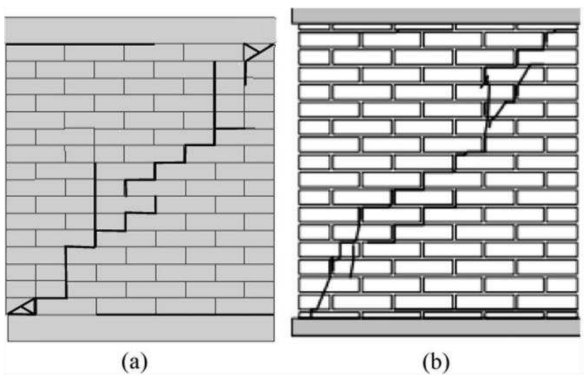
(c)



NSCD models (Beatini et al., 2018)
(d)



DDA models (Liu et al., 2018)
(e)



FDEM models (Smoljanovic et al., 2013a)
(f)

Figure 3: Block-based models.

4.2.1 Limit Analysis block-based models

A second class of LA models have been developed specifically for the block-based strategy. Livesley (1978) presented a pioneering lower bound LA solution for 2D masonry arches. However, due to the employed associated flow rule, two major shortcomings arose from this model: a) unreliable failure mode prediction; and b) overestimation of collapse load. Many investigators thus attempted to overcome these limitations, by implementing a non-associated flow rule. Notably, Fishwick (1996) developed a mixed lower and upper bound solution, to carry out non-associated LA of multiring arch bridges. This was through utilizing a mathematical program to solve an underlying mixed complementary problem (MCP) involving a system of orthogonal sign-constrained vectors. Despite its robustness for minimum collapse load calculation, it was effective for a small number of blocks only. Similarly, Baggio and Trovalusci (Baggio and Trovalusci, 1998; Baggio and Trovalusci, 2000) developed an MCP-like non-associated solution which attempted to find the minimum load factor by direct minimisation (with the so-called optimisation problem) under complementary constraints. This solution was also found unmanageable for structures of several blocks. Subsequently, Ferris and Tin-Loi (2001) proposed another approach for the collapse loads of discrete rigid block systems through a constrained optimisation problem known as a Mathematical Program with Equilibrium Constraints (MPEC). Orduña and Lourenço (2005) additionally employed a novel load path following procedure which yielded in the robust structural analysis of 3D masonry assemblies. Finally, in recent years, other investigators have employed more sophisticate techniques such as cone programming (Portioli *et al.*, 2014) to carry out the non-associated LA.

Despite the effectivity of non-associated LA models for the analysis of masonry, two disadvantages are made apparent here: a) they all assume infinitely resistant bricks which permits plastic dissipation at the interfaces; b) the combination of a non-tension and rigid block can lead miscalculation of the failure load (Milani, 2008). Thus, another group of investigators, have employed the so-called finite element limit analysis (FELA), without resorting to a non-associative flow rule. In a pioneering study, Sutcliffe *et al.* (2001) developed a novel lower bound finite element limit analysis (FELA) solution to calculate collapse loads of unreinforced masonry shear walls. The solution was derived from the imposition of equilibrium with appropriate yield and stress boundary conditions. Later on, another FELA approach was also presented, however with an upper bound solution (Milani, 2008), as shown in Figure 3a. This included interfaces with a Mohr-Coulomb failure criterion, a tension cut-off and cap in compression for mortar joints in combination with a Mohr-Coulomb failure criterion for bricks, enabling complex failure modes (such as masonry crushing) to be captured. Other studies have also followed this approach (Milani, 2008; Milani *et al.*, 2009). Finally, the upper bound FELA has also been applied to the 2D static analysis of large-scale structures such as masonry arch bridges (Cavicchi and Gambarotta, 2006).

4.2.2 FEM block-based models

A significant scientific intent has also been devoted to the development of FEM models capable of block-based modelling. Page (1978) pioneered the so-called textured continuum approach, in which the discontinuities of masonry are represented implicitly by locally altering the texture of the mesh, corresponding to the mortar. Whilst many studies have the textured continuum approach (Ali and Page, 1988; Addessi and Sacco, 2016; Petracca *et al.*, 2017; Serpieri *et al.*, 2017), a significant limitation is the implicit representation of discontinuities (based on a continuum). This can primarily make it difficult to capture specific failure modes (e.g. sliding and separation of blocks) and secondarily computationally expensive to implement.

Another group of FEM block-based models involve the explicit representation of masonry's discontinuities through zero-thickness interfaces. This so-called interface element approach was formulated by Lotfi and Shing (1994). Later on, the pioneering study of Lourenco and Rots (1997) greatly improved the interface approach with the so-called multi-surface models in which, the structure's damage was gathered at the interfaces only, permitting a notably increased efficiency of structural analysis. Recently, owing to the effectivity of the multi-surface models, they have been enhanced by other investigators, particularly for masonry wall panels (Gambarotta and Lagomarsino, 1997; Macorini and Izzuddin, 2011; Chisari *et al.*, 2018). Another confirmation of their effectivity is that they are of the few FEM block-based models which have been successfully employed for full-scale masonry structures such as bridges (Zhang *et al.*, 2016; Zhang *et al.*, 2017; Zhang *et al.*, 2018b; Tubaldi *et al.*, 2019). However, here, a setback of the approach is also made apparent in the fact that, especially for the case of full-scale structures, they appear computationally burdensome, necessitating high power computational (HPC) facilities.

Whilst the interface approach is highly-effective, the representation of complex behaviours such as the crushing of masonry is still a challenge, since numerical properties cannot be obtained with ease. Attempting to overcome such difficulties, an innovative research group (D'Altri *et al.*, 2018a; D'Altri *et al.*, 2019) introduced a novel, so-called contact-based approach within a FEM framework (within the commercial software *Abaqus* (Simulia Inc., 2017)). Specifically, contact-based interfaces were coupled with 3D non-linear-damaging textured blocks to explicitly represent the mortar and masonry, representing one of a handful of detailed block-based models available in the literature. In the initial investigation (D'Altri *et al.*, 2018a), the approach was proposed and implemented on experimental panels and validated, for quasi-static loading. In the follow-up study (D'Altri *et al.*, 2019), the approach was implemented (Figure 3b) for cyclic loading on a full-scale experimental terraced house, yielding unprecedented results for a FEM model such as large displacements (i.e. 50 mm), crushing effects and manageable computational times. Additionally, material properties were derived directly from small scale experiments.

4.2.3 DEM models

Despite the apparent suitability of the FEM block-based models to simulate the heterogeneous nature of masonry, the state-of-the-art approaches such as the aforementioned interface approaches appear to still be generally computationally expensive, and in some case necessitating HPC resources. Furthermore, the more recent, innovating contact-based approaches, are also evidently suitable for the block-based modelling of masonry, and computationally manageable, however, have only found a small application which means their employability and efficiency is still questionable. In an attempt to overcome such difficulties, researchers have been attracted to discontinuum numerical methods, which have been effectively employed for the block-based modelling of full-scale masonry structures, such as masonry arch bridges, temples and churches.

Of the most diffused discontinuum methods employed is the distinct element method (DEM), initially developed for problems of sliding and crashing rocks (Cundall, 1971). The abbreviation DEM will herein be used interchangeably for both discrete element method and distinct element method. The main advantage of the DEM, as any such discontinuum method, is that discontinuities can be explicitly implemented into the numerical model and also handled efficiently during the simulation. Another advantage is that, like the FEM, it can capture both the in-service and collapse behaviour of the masonry. However, at the present moment, the DEM also has limitations (Sarhosis *et al.*, 2016c), including:

- A high-computational cost (which is, however, lower than that of a FEM block-based model);
- Its inherent need for block-based geometric models;
- Its limited employment to academia only at the present moment.

Also, of major interest to the numerical modelling of masonry are the conditions which define the DEM (Cundall and Hart, 1992), which are:

a) Finite (e.g. large) displacements and block rotation and detachment can be followed in an evolutive analysis;
b) Formation of new contact can be accounted for;
c) Block detachment is permissible.

The first condition ensures that the complex failure mechanisms of masonry can be captured whilst the second condition that arbitrary damage and post-peak behaviours can be efficiently simulated without the need of predefinition (Sarhosis *et al.*, 2016c).

Here a significant aspect of the discontinuum methods such as the DEM is made apparent, which is the contact type. This may be either the "soft contact" approach (also termed force-displacement formulation) or "hard contact" one. Essentially, the soft contact means that for two given deformable blocks, interpenetration is permitted by employing the assumption of elasticity to derive the normal stiffness. Conversely, hard contact implies that only shear movement and opening can occur (Cundall and Hart, 1992). According to Lemos (2007), the soft contact approach is preferable for the masonry where the shear and sliding forces significantly influence contacts forces in masonry structures.

The fact that DEM models are the most diffused discontinuum method employed in masonry is most likely owed to the existence of two commercial software packages, *UDEC* and *3DEC* (Itasca, 2019a; Itasca, 2019c). It is, however, important to note that other software also implements the DEM, such as the commercial software package *PFC* (Itasca, 2019b), also developed by Cundall (Cundall and Strack, 1980) and the open-source code *YADE* (Šmilauer *et al.*, 2010). As opposed to *UDEC* and *3DEC,* both the former employ spherical elements and have seldomly been employed for masonry.

Of the earliest studies with the DEM on masonry was by Dialer (1992) to investigate the shear strength of wall panels within *UDEC*. In recent years, extensive research has followed on masonry wall panels. For instance, many studies have focused on the methodical definition of material properties employing both optimisation and stochastic methods (Sarhosis and Sheng, 2014; Sarhosis *et al.*, 2020). Another innovating contribution paved the way for the detailed block-based modelling within the DEM (Sarhosis and Lemos, 2018) by employing with Voronoi blocks to explicitly represent the mortar. It is of interest to note that this was also extended to 3D Voronoi blocks in *3DEC* (Pulatsu *et al.*, 2019), as shown in Figure 3c.

Given the DEM's effectivity and efficiency, it has also been widespread for masonry arch bridges. Lemos (1995) was of the earliest to carry out a 3D structural analysis of full-scale, masonry arch bridge in *3DEC*. In a later study (Jiang and Esaki, 2002), the influence of weakened material properties of an ancient bridge in Japan was assessed. Many more studies have followed on masonry arch bridges, however more recently, the research community has focused its concerns on addressing the complex soil-structure and spandrel wall behaviour of masonry arch bridges. For this aim, Sarhosis *et al.* (2019) employed Voronoi blocks to represent the soil, finding a good agreement with experimental studies. Finally, Forgács *et al.* (2019) also addressed the complex failure mechanisms of spandrel walls.

The DEM has also been extensively employed for the dynamic analysis behaviour of ancient temples and colonnades. Psycharis *et al.* (2003) carried out a 3D dynamic analysis of part of the ancient Acropolis in Athens with *3DEC*. Through this research, potential remediation strategies by reinforcing the temple with steel were considered. In the same spirit, many more investigations followed on ancient temples, incorporating more complex geometrical details and arbitrary loading scenarios (Psycharis *et al.*, 2013; Stefanou *et al.*, 2015; Bui *et al.*, 2017; Tavafi *et al.*, 2019). Finally, it must be noted that plenty of other studies have also focused solely on the dynamic behaviour colonnades such as in (Sarhosis *et al.*, 2016a; Sarhosis *et al.*, 2016b; Pulatsu *et al.*, 2017).

4.2.4 NSCD models

Whilst the DEM paved the way for employment of discontinuum numerical methods, others have also been developed with time. One such class of models belong to the NSCD, developed by Moreau (1988) which employs hard contacts and implicit time integration. In comparison with the DEM, the NSCD is advantageous in that it does not employ fictitious numerical damping (Moreau, 1988) while its implicit time integration method permits unconditionally stable solutions with larger time steps. The NSCD has been employed for masonry, however in significantly fewer studies than the DEM which could be attributed to the fact that solely one code, *LMC90* implements it (Dubois and Jean, 2003). Chetouane *et al.* (2005) were of the first to employ the NSCD for masonry, whilst applying the *LMC90* for the 2D structural analysis of both masonry panels and arch bridges. Amongst others, the study demonstrated the performance of the NSCD for capturing the structural behaviour of masonry in a manageable time. In another publication (Rafiee *et al.*, 2008), a 3D dynamic analysis was carried out on the Arles aqueduct in the south-east of France with *LMC90*. Further than demonstrating both the potential and efficiency of the NSCD, this pioneering study is still of a handful of full-scale dynamic analyses of masonry arch bridges in literature. Recently, the NSCD has also been particularly attractive to investigators on historic churches and domes. Beatini *et al.* (2019) implemented the NCSD within a custom-built software to assess Brunelleschi's dome in Florence, Italy. Since the actual geometry of the dome (Figure 3d) is hidden from view, various scenarios of the geometric model were considered of, octagonal and circular domes with varying bond patterns developed by a parametric function. From all the previous studies, the NCSD was demonstrated as highly-effective for efficiently capturing the complex behaviour of full-scale masonry structures.

4.2.5 DDA models

Another group of discontinuum models is the DDA, which was developed by Shi and Goodman (Shi and Goodman, 1985; Shi and Goodman, 1989), again adopting a hard contact, implicit time integration. In comparison with the DEM, the DDA is advantageous due to its implicit time integration method, and compatibility with the FEM. Thavalingam *et al.* (2001) pioneered the employment of the DDA (as well as employing *PFC*) for the 2D quasi-static analysis of an experimental arch. Within the study, the load-displacement responses of the DDA showed good agreement with the experimental, whilst outperforming a commercial FEM code, *DIANA* (TNO DIANA BV. Delft, 2020). In recent years, Perez-Aparicio *et al.* (2013) also analysed 2D arches within the DDA to examine the influence of load, voussoir number (i.e. block number) and arch embankment (i.e. the thickness of the fill above the arch crown's height). Finally, in another recent study, the DDA has also been applied to investigate the boulder impact on masonry structure in mountainous areas (Liu *et al.*, 2018). In the specific study (as shown in Figure 3e), the DDA enabled the 3D dynamic analysis of various scenarios of boulder velocity and construction type (i.e. masonry bond) to propose remediation strategies for masonry structures at risk.

4.2.6 FDEM models

A final class of discontinuum models regard the FDEM, developed by Munjiza *et al.* (1995) employing a hard contact, implicit time integration method. Here the superiority of the FDEM is made apparent in that it is only discontinuum method that permits masonry crushing (i.e. blocks that can break and separated without the use of zero-thickness interfaces) whilst also sharing the advantages of the DDA and NSCD, in comparison to the DEM. Owen and co-workers (Owen *et al.*, 1998) were of the earliest to demonstrate the potential of the FDEM with the quasi-static 2D analysis of a full-scale masonry arch bridges whilst assessing reinforcement strategies (so-called *CINTEC* system). Amongst others, one of the model's novelties included the coupling spherical and polyhedral elements. Another more recent FDEM investigation involved the modelling of arch reinforcements (Smoljanovic *et al.*, 2015). Further than masonry arches, the FDEM has also found application in the context of cultural heritage masonry structures. Indicatively, Smoljanovic *et al.* (2013a) carried out a 2D dynamic analysis of the Prothyron in Split, Croatia. The models demonstrated the capabilities of the FDEM to capture extremely complex failure modes including cracking of the stone units, something unprecedented in block-based models (as shown in Figure 3e). Finally, as for most numerical methods within the DEM, the FDEM has been particularly attractive for the research of out-of-plane seismic loading of URM, such as (Smoljanovic *et al.*, 2018).

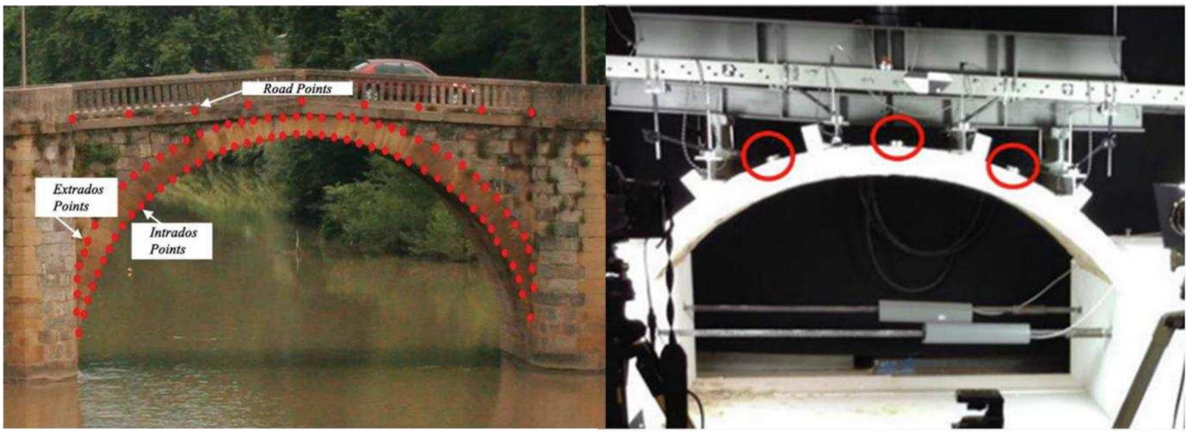
5 Structural surveying of masonry structures for numerical modelling

As found in Section 4, there is an array of approaches for high-level structural analysis of masonry structures. However as highlighted in Section 1, the majority of these studies employ simplified or ad-hoc geometric models. One reason why state-of-the-art numerical modelling approaches employ simplified geometric models is the difficulty of geometric data acquisition, owing to the employment of manual measurements (i.e. traditional geospatial techniques such as direct measurement with a tape). Nowadays, however, non-contact sensing techniques such as non-contact sensing have revolutionised many applications of civil engineering, including numerical modelling (Tang *et al.*, 2007; Chen, 2012;

Olsen and Kayen, 2012; Vosselman and Maas, 2014; Ye *et al.*, 2018). Of particular interest is the SfM photogrammetry pipeline which further than practical, is a considerably accurate and low-cost non-contact sensing technique (Dai and Lu, 2010). Following this intuition, this Section examines the suitability of emerging non-contact sensing techniques such as SfM photogrammetry for providing geometric data rapidly and reliably. It is noteworthy that the presented non-contact sensing techniques are classified by the author into: point-based techniques, which provide discrete points only; and cloud-based, which provide point clouds and orthoimages.

5.1 Point-based techniques

Within the point-based structural surveying, the following approaches have been identified as candidates for accurate geometric model development: a) total station; b) laser tracker. The following paragraphs introduce both the techniques and relevant studies, as demonstrated in Figure 4.



Total station, (Morer et al., 2011b) **(a)**

Laser tracker, (Yang and Xu, 2019) **(b)**

Figure 4: Structural surveying with point-based techniques: (a) total station; and (b) laser tracker.

5.1.1 Total station

A total station consists of an electronic theodolite combined with an electronic distance measurement. Through the recording of angles and points, the accurate 3D positions of discrete points are obtained. In the context of numerical modelling of masonry structures, total stations are employed to directly measure the structure (e.g., the block and joints positions) and develop a geometric model according to a given modelling strategy (e.g., with the point-based approaches of Section 6.2.1) as well as to provide control information for other non-contact sensing techniques. The main advantages of the total station are its simplicity of use and sub-cm accuracy in each direction (Morer *et al.*, 2013) which indeed makes it particularly attractive for providing control information (i.e. georeferencing ground control points), as will be demonstrated in this investigation of Figure 4a. However, the main limitation of the total station is owed its relatively high cost and to the nature of its observations (i.e. discrete points). This makes its employment for the numerical modelling of large-scale structures costly and time-prohibitive due to the impractical and laborious task of measuring unmanageable numbers of blocks.

5.1.2 Laser tracker

The laser tracker is a recent technology used in large scale precision manufacturing such as aerospace and the automotive industry (Estler *et al.*, 2002). It is similar to the total station in that it is placed upon a tripod, is pointed at targets in sequence, and measures the distance to each, as well as the angles between each pair. From this raw data, full 3D coordinates of each target can be calculated. Like a robotic total station, the tracker can move itself to find the centre of the target. The tracker measures the position of a retro-reflective prism, which rather than a traditional target, is mounted in nests, permanently fixed to the structure. The prism is set in a stainless-steel sphere, such that the measurement point is at the centre of the sphere with extremely high accuracy. The nests are designed as such so that the sphere sits on three points and is held in place by a magnet, ensuring repeatability of the measurements. As is surveying with a total station, it is not possible to see all the measurement points from one instrument position which requires measurements from several positions are combined into a complete survey.

While the application of laser trackers is still limited in masonry structures, some pioneering investigations do exist. In one recent study, the employment of laser trackers was carried out (Barazzetti *et al.*, 2015b) to detect the static movement of the column the Cathedral of Milan. The achieved precision which was 0.1 mm, clearly demonstrated the performance of the laser tracker in such applications. In another study (Yang and Xu, 2019), laser tracking was employed for providing control information to a TLS survey of a concrete bridge, as shown in Figure 4b. Specifically, parameters of a B-spline model developed from the TLS point cloud were calibrated and validated surface accuracy in the region of 0.1 mm. These studies both demonstrate the main advantage of the laser tracker which is its accuracy, which is invaluable for providing control information. However, as the total station, it does not appear advantageous for geometric model development of large-scale structures due to providing discrete points.

5.2 Cloud-based techniques

Within the cloud-based structural surveying techniques, the following approaches have been identified: a) TLS; and b) SfM photogrammetry. The following paragraphs introduce both the techniques and relevant studies, as demonstrated in Figure 5.

TLS models, (Chen et al., 2019)
(a)

SfM photogrammetry, (Chen et al., 2019)
(b)

Figure 5: Surveying with cloud-based techniques: (a) laser scanning; and (b) SfM photogrammetry.

5.2.1 Laser scanning

At the present moment, laser scanning is one of the most important non-contact sensing techniques for the structural surveying of masonry structures (Tobiasz *et al.*, 2019). For the so-called time-of-flight type laser scanners which are more pertinent to this investigation, pulses of light are emitted from the scanner's position to the masonry structure's surface whilst distance measurement results from recording the time interval between light emission and return (Baltsavias, 1999). Typical laser scanner components are a rotary mirror, a laser source, and a data storage module (Tobiasz *et al.*, 2019). Whilst laser scanning provides several returns, the main product concerning numerical modelling is a dense point cloud. Apart from 3D positions, this also includes a fourth parameter; the intensity of the returning signal, which is particularly useful for characterizing the scanned material (Tobiasz *et al.*, 2019).

Laser scanning can be distinguished based upon the platform in which it is employed. When the laser scanning is carried out from the ground, it is TLS. Whilst, when airborne platforms are employed (such as an unmanned aerial vehicle), it is airborne laser scanning (ALS). In the context of this investigation, TLS is more pertinent due to the scale and required accuracy of the problem. Additionally, the errors of TLS according to Tobiasz *et al.* (2019) are summarised as: a) internal, such as instrumental errors, laser beam errors (propagation, reflection, and refraction); and b) external errors, such as the case of the material colour affecting the intensity, and material translucency.

The main advantage of laser scanning, in comparison with all the aforementioned point-based techniques, is the rapid geometric data acquisition (e.g. M Pts level) and high-accuracy (e.g. sub-cm level), comparable to a total station (Vosselman and Maas, 2014). For this reason, TLS is the benchmark method of structural surveying of masonry structures, as will be demonstrated further on. However, the disadvantages of TLS are found in the high cost of equipment, the necessity of multiple scan stations when oblique incidence angles occur, and in the lack of textural information provided (Peppa, 2018).

5.2.2 Photogrammetry

Photogrammetry is another commonly employed non-contact sensing technique (Tobiasz *et al.*, 2019) that deals with extracting geometric data from imagery (Wolf *et al.*, 2014). As with TLS, photogrammetry may by terrestrial or airborne according to the platform of the employed sensors. In the past, high-quality analogue metric cameras were employed in conventional photogrammetry such as in Mills and Barber (2004). Now, photogrammetry is also employed with digital cameras in combination with low-cost SfM platforms (the formulation of which will be detailed in the forthcoming paragraphs).

In stereo photogrammetry, which is more pertinent to this investigation, two optical rays, representing conjugate image points, ideally, intersect at an object point through a so-called spatial intersection. The establishment of the camera's internal geometry is termed interior orientation carried out by defining the interior orientation parameters (IOPs). These geometric parameters, also reported in the literature as the inner, intrinsic orientation or camera intrinsic (Luhmann et al., 2006) are: a) the focal length, which is the distance between the lens centre and the lens focus point; b) the principal point (the intersection of fiducial lines); c) symmetrical radial lens; and d) the decentring distortion parameters. Relative orientation consists of the determination of the position and orientation between two images, relative to each other, resulting in the generation of a stereo model. Absolute orientation consists of defining the 3D position of control points of a stereo model in a desired coordinate system, via a 3D conformal coordinate transformation using at least two horizontal and three vertical control points. After absolute orientation, the camera's exterior orientation parameters (EOPs) are defined which are three translations and three rotations. Simultaneous multiple image orientation is determined by aerial triangulation whilst the establishment of the position and orientation of each bundle of the optical ray is termed bundle block adjustment. In the case of self-calibrating bundle adjustment such as in the software *Metashape*, re-optimisation of IOPs and EOPs are

The theoretical basis of photogrammetry is the so-called collinearity condition, as shown in equation (5.1), according to Dai and Lu (2010). According to this condition, any given optical ray (Figure 6) can be defined by three points: a) the image point; b) the camera perspective centre; and c) the object point. Moreover, any point of an image captured by a camera is the representation of the convergence of many optical rays (Historic England, 2017). Where f is the nominal focal length; (x_o, y_o) and (X_o, Y_o, Z_o) are the coordinates of the perspective centre in the image plane and object space, respectively; (x_n, y_n) and

(X_n, Y_n, Z_n) are the coordinates of the n-th target in the image plane and object space, respectively; Λ is the scale factor and M is the rotation matrix.

$$\begin{bmatrix} x_n - x_o \\ y_n - y_o \\ -f \end{bmatrix} = \Lambda \times M \times \begin{bmatrix} X_n - X_o \\ Y_n - Y_o \\ Z_n - Z_o \end{bmatrix} \tag{5.1}$$

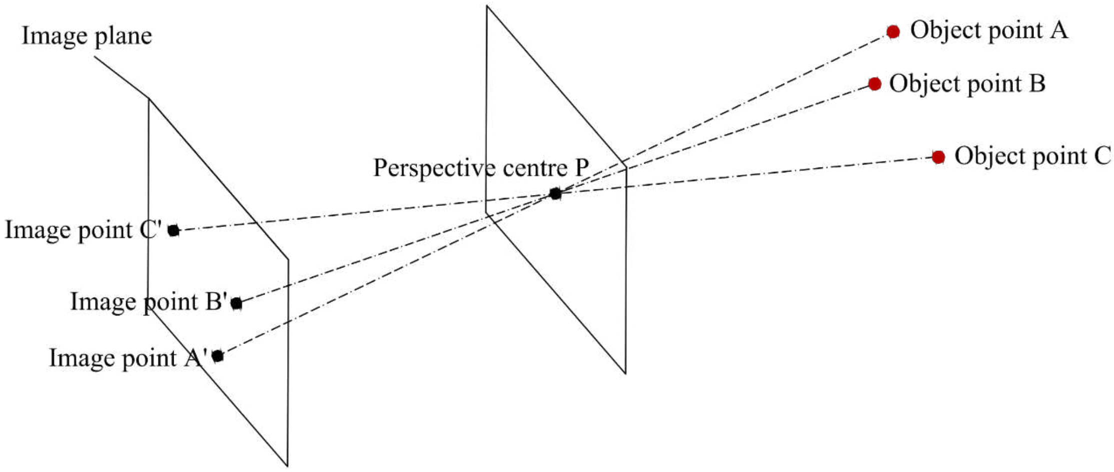


Figure 6: The optical rays for object points A, B and C.

SfM photogrammetry, which is of major interest to this investigation, is a recent addition to photogrammetry which has been widely employed for the structural surveying of masonry in the latest years. In comparison with TLS, it is advantageous due to its low-cost, facility of employment and the high quality of its returns (e.g. high-quality RGB orthoimagery and point clouds). Additionally, SfM photogrammetry consists of three main phases which are: a) sparse point cloud reconstruction; b) georeferencing; and c) dense point cloud reconstruction. Sparse point cloud reconstruction regards the process of aligning acquired images with a process of automated feature detection and correspondence until all the photogrammetric block is oriented (Golparvar-Fard *et al.*, 2011). In specific, feature matching firstly is carried out, which effectively finds distinct features on each image, allowing for the automated matching across a subset of images. For example, a well-known method of carrying this out is with the so-called Scale Invariant Feature Transform (SIFT) developed by Lowe (2004). Then, once feature detection has been carried out throughout the dataset, quantification of the detected features match in each image pair is carried out. The result of this process is a sparse point cloud which refers to a point cloud of tie points. Georeferencing regards the providing of control information for the scaling and orientation of sparse point cloud. This is commonly carried out with the use of ground control points (GCPs) in two ways: a) indirect georeferencing, in which the points are the result of surveying; and b) direct georeferencing, in which the obtained points are the actual camera positions (e.g. provided by GPS or RTK). The process of georeferencing consists of recalculation of both the camera's IOPs, EOPs resulting in a recalculated sparse point cloud coordinates which are in accordance with the control information provided. This is carried out in a least-squares bundle adjustment using the information as weighted in conjunction with the tie points. Finally, once the sparse point cloud is georeferenced, the dense cloud reconstruction follows by employing a pixel disparity calculation with area-based image matching. Thereafter, pixel back-projection and triangulation (i.e. via spatial intersection) follows, in which a 3D surface is formed via gradient-based and energy minimisation algorithms to avoid irregularities.

As a result of the aforementioned pipeline, the main product of SfM photogrammetry is a dense point cloud, which is RGB-coloured, being a significant advantage in comparison to TLS, (which normally doesn't have RGB). However, it is to be noted that SfM photogrammetry, as opposed to TLS, does not provide the intensity data of the surveyed structure. Further than a dense point cloud, of major interest to masonry structures is orthoimagery, which may be digital elevation models (DigEMs), orthophotos or orthomosaics. Specifically, a DigEM is a mathematical description of a 3D surface in which, each grid point represents a single elevation value (Aguilar *et al.*, 2005; Wolf *et al.*, 2014) whilst the data type of DigEM is a double array of square pixels with a uniform size (Wolf *et al.*, 2014). Moreover, an orthophoto is the result of the orthorectification of DigEM and represents a continuous image grid of a uniform scale (Wolf *et al.*, 2014). The so-called orthorectification regards describing an object in its true orthographic position through the collinearity condition. Finally, orthomosaics result from joining multiple orthophotos together.

Also, of major importance are the errors of SfM photogrammetry, which may be due to: a) image overlap; b) GCPs; and c) external factors. The following paragraphs detail each error type. Indeed, an important aspect of acquiring images of SfM photogrammetry is the relative overlap between consecutive images. A lack of overlap has been found to cause erroneous initial image alignments and consequent erroneous sparse point cloud reconstruction (e.g. discontinuities) according to (Harwin *et al.*, 2015; Dietrich, 2016). Whilst the obvious solution is though high overlap this increases the point determination redundancy (Haala and Rothermel, 2012), the computational burden can become unmanageable and thus requires consideration. Furthermore, GCP's can both decrease the systematic errors of the bundle adjustment and increase the photogrammetric abundancy (Wolf *et al.*, 2014; James *et al.*, 2017). The two main factors associated with effecting the accuracy of the end-product are the GPC layout and the geometrical accuracy of the measurement GPC itself. Concerning their layout, the importance of the existence of GCPs on the border of the surveyed object has been stated on many occasions (James and Robson, 2012; Eltner *et al.*, 2016). Concerning the metric accuracy of the GCPs, it has been stated that they should be measured with an accuracy three times higher than that of the expected result (Remondino *et al.*, 2014). Finally, errors can also be associated with external factors such as image surface texture, lighting, weather conditions and instability of the camera. These such factors have been attributed to affecting the SfM photogrammetry image matching algorithms (James and Robson, 2012; Remondino *et al.*, 2014) and thus causing errors.

Both terrestrial and unmanned aerial vehicles (UAV) -based SfM photogrammetry are well-established techniques of structural surveying of masonry structures, as summarised in Table 2. Notably, Bosché *et al.* (2015), surveyed walls of a historic masonry castle with the terrestrial SfM photogrammetry, yielding comparable results to TLS in terms of accuracy and point cloud density. Another study (Barrile *et al.*, 2015) investigated the performance of various SfM photogrammetry pipeline software types against a reference TLS point cloud. Of all the point clouds were generated from 219 images and three software types, *Metashape* presented the best agreement with the TLS data. In the context of the deformation analysis of historic masonry tower (Teza *et al.*, 2016), terrestrial SfM photogrammetry was also found to yield comparable results with the TLS, with errors in the range of 5-20 mm. Finally, the deformation analysis of masonry arch bridges (Soni *et al.*, 2015) also demonstrated the effectivity and accuracy of SfM photogrammetry.

As for many other applications of civil engineering, the use of UAVs has also grown immensely in the past decade, especially for the structural surveying of cultural heritage masonry structures. For instance, Bosché *et al.* (2015) compared the outputs of UAV-based SfM photogrammetry against those of TLS and terrestrial SfM photogrammetry for the survey of a historic castle. In this case, the UAV-based SfM photogrammetry was found to be disadvantageous, possibly due to the anteriority of the approach which lacked a methodically pre-defined flight path design. In recent years, Barrile *et al.* (2017) assessed the UAV-based SfM photogrammetry against TLS finding that the data acquisition faster, more flexible, and cost-effective but dependant on uncontrollable conditions such as weather and lighting.

Due to the lack of accessibility and highly irregular geometries, UAV-based SfM photogrammetry is increasingly favoured for the surveying of masonry arch bridges. For instance, one study (Bruno *et al.*, 2019), combined UAV-based SfM photogrammetry and TLS for the 3D documentation of a historic bridge in Italy, leading to highly-detailed and accurate surveying, which also included textural information due to inclusion of SfM photogrammetry. In another innovating study, Pepe *et al.* (2019) captured nadir images of the intrados of a masonry arch bridge by mounting a camera (in specific a smartphone) on top of the UAV. This low-cost approach also resulted in an accurate and detailed structural surveying. Finally, in a recent study (Chen *et al.*, 2019), structural surveying of a historic aqueduct was carried out with UAV-based SfM photogrammetry to assess the performance of consumer-grade UAVs for bridge inspection (as shown in Figure 5a-b). The study demonstrated that the UAV-based SfM photogrammetry was easier to apply and more cost-effective than TLS. However, problems arose, regarding non-covered areas (e.g. railings), high noise levels and low geometrical accuracy persisted (cm-level compared to the mm-level of the TLS).

From the summarised studies in Table 2, it is evident that SfM photogrammetry can provide accurate geometric data (cm-level) and rapidly (M-Pts) which is comparable to a benchmark geospatial technique such as a total station or laser scanner. Furthermore, SfM photogrammetry can be cost-effective due to the potential of employing of consumer-grade digital cameras. Finally, it has been demonstrated as straightforward, which can be employed by on-site engineers with available cameras (i.e. such as a smartphone) replacing the necessity to purposely carry survey-grade equipment (Kim *et al.*, 2019).

Table 2: Structural surveying of masonry structures with the terrestrial and UAV-based SfM photogrammetry.

	Study	Application	Structure	Software	Image #	Dense point cloud # (M pts)	Reported GCP error (cm)	Camera
Terrestrial SfM photogrammetry	(Soni *et al.*, 2015)	Deformation monitoring	MAB	*Visual SfM*	-	-	0.1	Nikon D3200
	(Bosché *et al.*, 2015)	3D documentation	CH	*Metashape*	260	79	3	Nikon D810
	(Barrile *et al.*, 2015)	3D documentation	CH	*Metashape*	219	28.9	2	Samsung model PL20
	(Teza *et al.*, 2016)	Deformation monitoring	CH	*Metashape*	156	14.9	0.5-2	Nikon D300S
UAV-based SfM photogrammetry	(Bosché *et al.*, 2015)	3D documentation	CH	*Metashape*	460	34	3	LC Sony Alpha-7R
	(Bruno *et al.*, 2019)	3D documentation	MAB	*Metashape*	610	-	-	DJI Phantom 4 (20 Megapixel)
	(Pepe *et al.*, 2019)	3D documentation	MAB	*Metashape*	768	10	0.7	Xiaomi Mi Drone 4K UHD WiFi FPV
	(Chen *et al.*, 2019)	3D documentation	MAB	*Metashape*	295	-	-	DJI Phantom 4 (12 Megapixel)

6 Geometric model development approaches

As found in Sections 1 and 4, another reason for which the majority of state-of-the-art numerical modelling approaches employ simplified geometric models is the complex procedure of geometric model development. Indicatively, it has been found that for FEM continuum models, (which are significantly simpler than block-based), geometric model developments consumes up to 80% of the total modelling time (Zhang, 2013). To overcome this difficulty, various approaches have been adopted to automatically convert the data of various non-contact sensing techniques into geometric models, however mainly for numerical methods such as the FEM and LA. In the following paragraphs, such approaches are detailed. It is noteworthy that since few studies (Zhang, 2013; Riveiro *et al.*, 2020) exist on this relatively novel subject, the classification is proposed by the author specifically for masonry, distinguished according to the continuum and block-based modelling strategies of Section 4.

6.1 Geometric model development for continuum modelling

Within the continuum modelling strategy, the following approaches have been adopted for accurate geometric model development: a) cloud-based; b) mesh-based; c) non-uniform rational basis spline (NURBS)-based; and d) building information model (BIM)-based. The following paragraphs introduce both the approaches and relevant studies, as demonstrated in Figure 7.

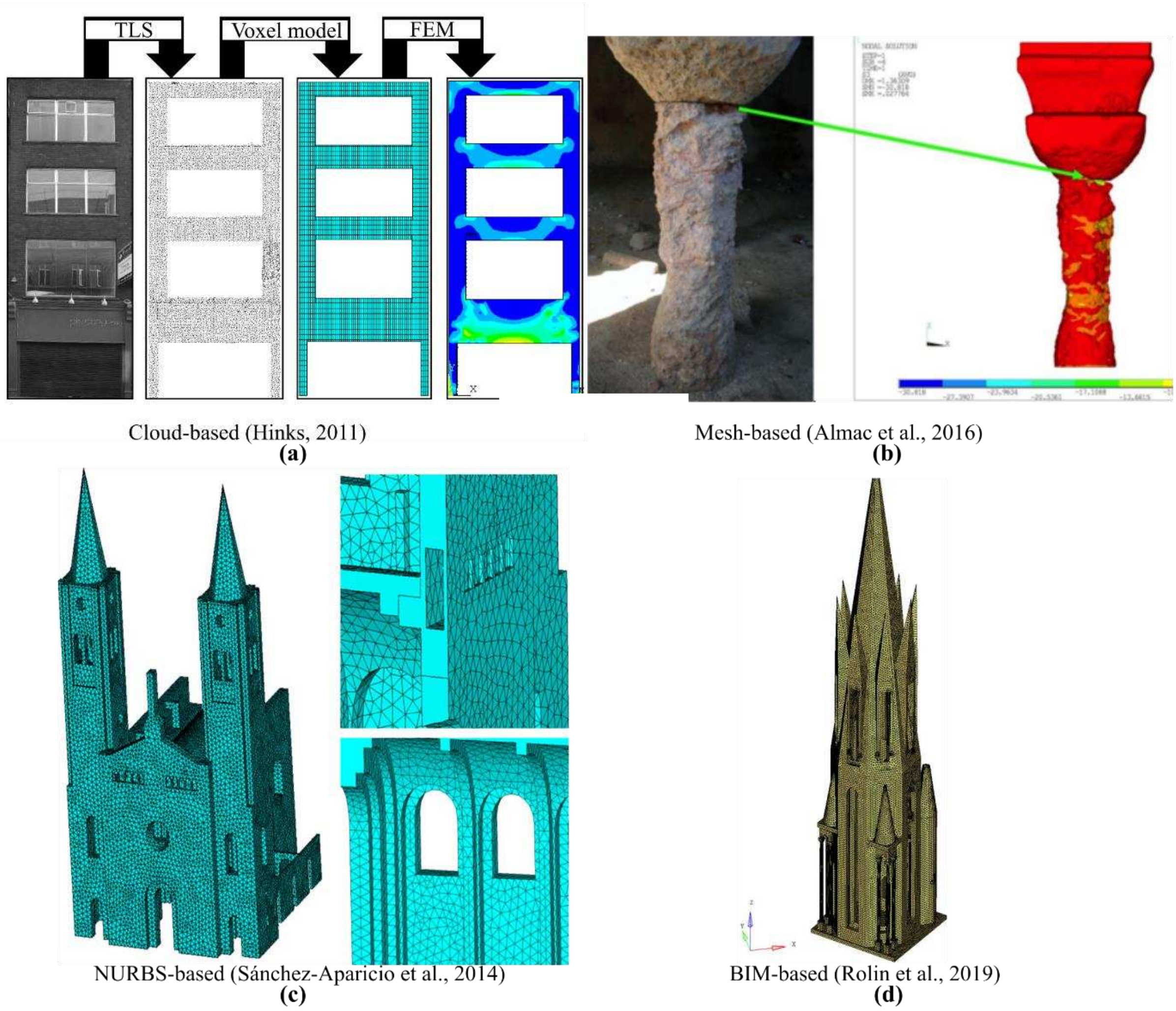


Figure 7: Geometric model development for continuum-based models.

6.1.1 Cloud-based approaches

With the cloud-based approach, a point cloud is directly converted into a geometric model, usually through spatial enumeration (Section 1), such as voxelization. The term voxelization describes the conversion of a masonry structure's geometric domain into an equivalent volumetric representation in form of cuboids (voxels). The main advantage of this approach is that the structure can be of any geometric form (i.e. non-watertight or non-convex), without the necessity of mesh generation, whilst the volumetric modelling is achieved directly with the voxels themselves.

As part of his doctoral investigation, Hinks (2011) presented a pioneering voxelization approach (shown in Figure 7a) for developing of geometric models of URM building facades. This was a novel point-based (i.e. employing point clouds) voxelization method based on volumetric subdivision rather than the previously applied methods of surface reconstruction (i.e. using meshes). Due to the anteriority of this work however, the geometric models were only two-dimensional. Based on this pioneering study, various studies employing point cloud segmentation techniques improved the 2D geometric models of the façades (Linh *et al.*, 2012; Linh and Laefer, 2013; Linh and Laefer, 2014; Truong-Hong and Laefer, 2014; Iman Zolanvari and Laefer, 2016) which were all incorporated in FEM software. Castellazzi *et al.* (2015) further advanced the cloud-based approaches by developing the first three-dimensional geometric models. This was another instance of a point-based voxelization workflow which led to the full-scale FEM structural analysis of a historic masonry castle. The same particular been further employed with the structural analysis of other historic masonry structures (Bitelli *et al.*, 2016; Castellazzi *et al.*, 2017). Finally, more recently, Selvaggi *et al.* (2018) added a simplified process of geometrical assessment of the geometric models developed to the workflow. Whilst all the aforementioned studies demonstrate the cloud-based approach as efficient, they all represent the masonry as voxels, i.e., cuboids. Future cloud-based approaches should potentially consider other methods of spatial enumeration should be employed, that better approximate the anisotropic nature of masonry (for instance with Voronoi blocks, as in (Pulatsu *et al.*, 2019)).

6.1.2 Mesh-based approaches

The mesh-based approaches refer to converting a mesh into a geometric model. Mesh herein refers to a surface mesh such as triangulated irregular network constructed from nodes of a dense point cloud and facets by Delaunay triangulation. Often, other processes precede a mesh-based approach such as watertight conversion and mesh simplification, for the structural analysis software to be able to handle a manageable amount of faces and vertices from the mesh (Riveiro *et al.*, 2020). After the surface of the structures is represented through the final mesh, it is volumetrically subdivided into either pyramidal or tetrahedral finite elements within either the structural analysis or a third-party software itself. The main advantage of the mesh-based approach is its simplicity in implication.

Due to its very simplicity, the mesh-based approach has been extensively applied within the context of masonry arch bridges. In an early study, Vatan and Arun (2005) developed a geometric model of an aqueduct with the mesh-based approach for structural analysis within the FEM. Later on, Arias *et al.* (2007) employed a mesh-based approach for geometric model development of a historic bridge for structural analysis within a FEM framework. This study innovatively combined data of ground-penetrating radar (GPR) to additionally determine the interior (fill) material of the bridge. Similarly, Lubowiecka *et al.* (2011) later on employed the mesh-based approach for a FEM structural analysis with a textured mesh from the SfM photogrammetry, which included the damaged areas of the bridge detected and marked, whilst GPR was again used to determine the fill of the bridge. A further study (Stavroulaki *et al.*, 2016) successfully added damage to the FEM mesh corresponding to cracks from the textured mesh of SfM photogrammetry. Finally, while all the previous studies regarded single-span masonry arch bridges, Conde *et al.* (2017) developed a pioneering 3D geometric model of a full-scale, multi-span bridge for FEM structural analysis.

The mesh-based approach has also been applied within the context of cultural heritage masonry structures. Notably, Pieraccini *et al.* (2014) developed FEM geometric models with the use of a mesh of a historic tower. Moreover, Almac *et al.* (2016) developed FEM models of ancient columns based on a mesh obtained by TLS (shown in Figure 7b). In another study (Meschini *et al.*, 2015), FEM analysis of a fortress was also carried out using a mesh from developing a geometric model from a TLS mesh. Barrile *et al.* (2016) also followed a similar approach, however employing terrestrial SfM photogrammetry. Furthermore, Hacıefendioğlu and Maraş (2016) were of the first to employed UAV-based SfM photogrammetry to develop geometric models (FEM) of a mosque. D'Altri *et al.* (2018b) recently employed the mesh-based approach which enabled semi-automated structural analysis of a leaning tower with both FELA geometric models. Finally, more recently, Bassier *et al.* (2018) presented a mesh-based approach of which the contribution was the additional crack introduction tool, capable of adding cracks to the geometric model with manual intervention.

6.1.3 NURBS-based approaches

The NURBS-based approach is a manner of approximating a complex geometry, to facilitate its handling within the structural analysis software, whilst retaining a high degree of geometric accuracy (Riveiro *et al.*, 2020). The basis of the NURBS is the mathematical spline, a curve defined by multiple nodes (named control nodes) and polynomial functions. The simplest form of a spline is a line joining two control points. For n control points, the general rule is for polynomial function with a degree of n-1. Base Splines (B-Splines), are the subcategory of spline curves with the mathematical property of minimal support. Minimal support means that a linear combination of B-spline can be employed to express any spline function of the same degree. NURBS curve are common to B-Splines except to that, each control point has a weight; if weights were equal to 1, then the NURBS would be a B-spline. The result of a tensor product of two NURBS curves which originates from a quadrangular patch is a NURBS surface (patch). In this way, data of non-contact sensing techniques can be used as control points for retopology in a NURBS approach. It's noteworthy that the NURBS-based approach belongs to the boundary representation method of geometric model development. The NURBS approach is particularly advantageous due to providing an accurate geometrical representation of the masonry structure while requiring less manual intervention (Riveiro *et al.*, 2020). However, as with the mesh-based approach, since it represents the masonries surface only, it must be volumetrically subdivided into either pyramidal or tetrahedral finite elements within the structural analysis software.

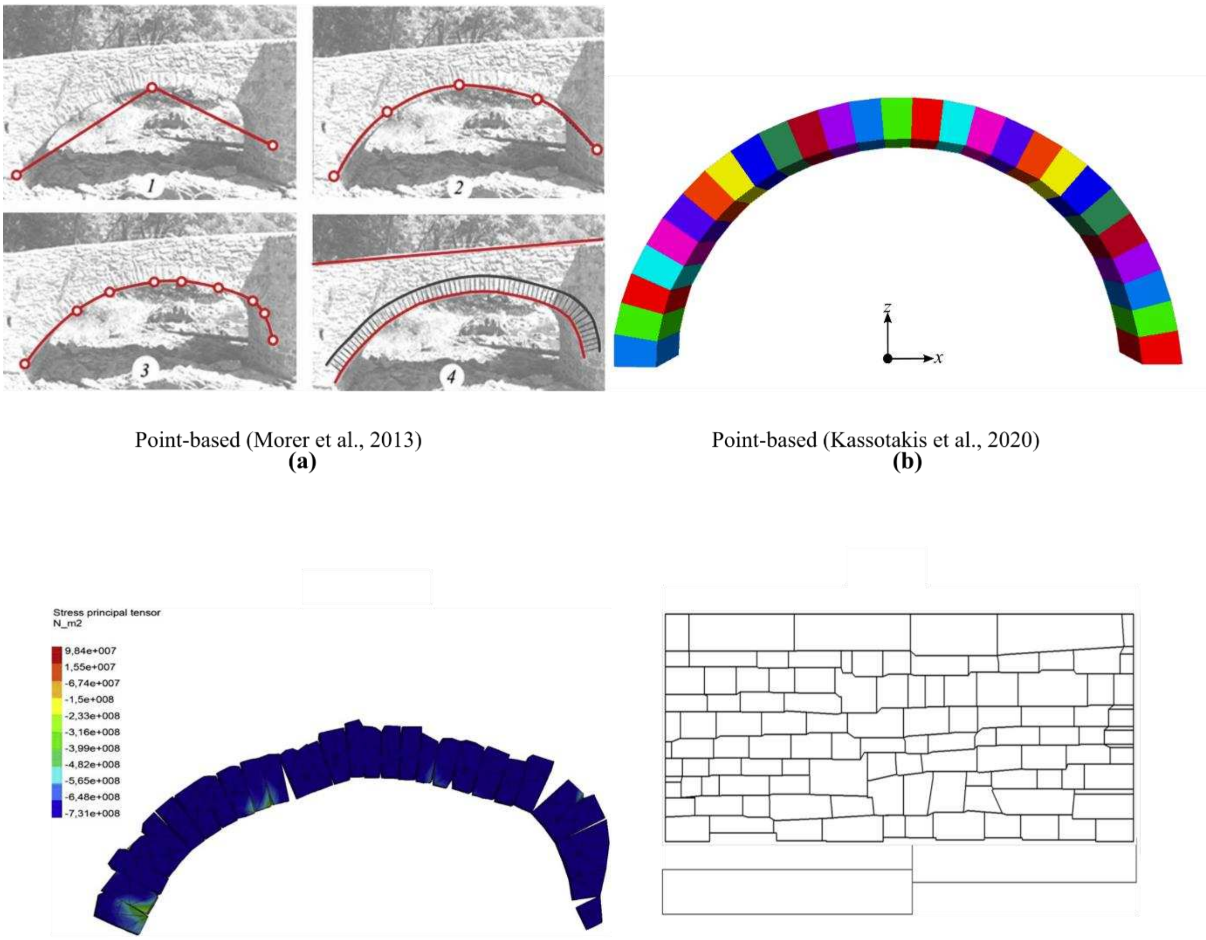
Tucci and Guardini (2014) proposed a procedure of developing geometric models using a NURBS-based approach. This was carried out by applying mesh retopology within third-party software, in which the mesh was made compliant to NURBS generation. In another pioneering study (Sánchez-Aparicio *et al.*, 2014) the NURBS-based approach was used to develop a FEM geometric model (shown in Figure 7c) of a historic masonry church from the data of UAV-based SfM photogrammetry. The same research group (Sánchez-Aparicio *et al.*, 2016) carried out another structural analysis of a historic masonry structure with geometric model development carried out with the NURBS-based approach (shown in Figure 7c). Korumaz *et al.* (2017) carried out a structural analysis of a leaning minaret with the FEM with the NURBS-based approach based on TLS data. Vincenzi *et al.* (2019) used a NURBS approach to develop the geometric model of a historic tower employing combined UAV-based SfM photogrammetry and TLS data.

6.1.4 BIM-based approaches

Another way of developing a geometric model is by employing a BIM. In the context of construction, the BIM is defined according to Volk *et al.* (2014) as a shared digital representation of physical and functional characteristics of a given built object which forms a reliable basis for decisions. Though BIMs are most often employed solely for documentation, in some cases, BIM models have been directly converted into geometric models for subsequent structural analysis, though on few occasions as the other approaches. In the context of cultural heritage masonry structures for instances, a pioneering investigation involving the conversion of points clouds to BIMs and BIMs to geometric models for FEM structural analysis was presented (Barazzetti *et al.*, 2015a). It must be noted that to develop the complex geometry of the church within the BIM, generative NURBS profiles were used to obtain a rigorous geometric representation of the vault, while the simple shapes were used for regular sections of the building. These procedures were carried out in a manual CAD-based environment. In a more recent study (Rolin *et al.*, 2019), a slicing method of developing BIMs from point cloud was used (shown in Figure 7d) and then the BIMs were automatically converted into FEM geometric models. Whilst the BIM approach is easy to implement, it is disadvantageous due to the lack of automation since it requires the manual development of a BIM. This, however, could be changed, owing to the vast amount of work regarding the automated conversion of point clouds to BIM, in the entitled "*scan-to-BIM*" approaches such as of (Andriasyan *et al.*, 2020; Bassier and Vergauwen, 2020; Bagnolo and Argiolas, 2021). Furthermore, another limitation to be mentioned is common with all the previous approaches except for the cloud-based. Effectively, even though the BIMs structural elements are explicitly represented, each element must still be volumetrically subdivided into finite elements (e.g. pyramidal or tetrahedral) within the structural analysis software.

6.2 Geometric model development for block-based modelling

Within the block-based modelling strategy, the following approaches have been adopted for geometric model development: a) point-based; and b) image-based approaches. The following paragraphs introduce the aforementioned approaches, including relevant studies as shown in Figure 8.



Point-based (Morer et al., 2013)
(a)

Point-based (Kassotakis et al., 2020)
(b)

Image-based (Riveiro et al., 2011b)
(c)

Image-based (Napolitano et al., 2019a)
(d)

Figure 8: Geometric model development for block-based models.

6.2.1 Point-based approaches

A point-based approach of geometric model development implies the employment of discrete points to develop a geometric model, in a block-by-block manner. This is carried out by use of manual CAD-based software to represent the structure using the measurements of point-based non-contact sensing techniques such as a total station, or even direct measurement with a tape or a gauge. In a notable study, Morer *et al.* (2013) employed a total station to carry out numerical modelling of a masonry arch bridge, employing the block-based modelling strategy. The total station was placed in a suitable position to be able to scan all the desired target points levelled, and measurement commenced (as shown in Figure 8a). In the specific study, the vertices of the masonry arch's voussoirs (i.e. blocks) were measured by a total station. In a recent study, (Kassotakis *et al.*, 2020) quantified the effect of geometric uncertainty between point-based and image-based approaches (as shown in Figure 8b). The study employed tape measurements to measure the block vertices and found that point-based approaches (such as direct measurement) can be effective for small-scale structures, however unreliable geometric data can induce uncertainty into the structural analysis. Whilst point-based approaches can be adequate for small-scale structures of relatively few blocks (e.g. less than one hundred), a major disadvantage is laborious nature of both measuring discrete points and developing a geometric model from them within manual CAD-based design.

6.2.2 Image-based approaches

The image-based approach implies introducing an orthoimage of a masonry structure within a manual CAD-based framework and manually tracing the blocks and joints of the structure. Acary *et al.* (1999) pioneered the image-based approach with the structural analysis of a historic masonry structure from an orthorectified image with the NCSD. The pioneering study showed that the accurate geometric model led to a realistic failure mode of a full-scale building façade.

The approach has since then primarily found widespread application for masonry arch bridges. For instance, Morer *et al.* (2011) carried out the structural analysis of masonry arches with various LA approaches. Geometric models of various arches of a multi-span arch bridge were developed by manually extracting the contours of the arches from the orthoimagery of TLS. Here a limitation to the study is made apparent in that the voussoirs were not extracted from the orthoimagery, however, obtained roughly by dividing the arch contours into arbitrarily defined segments. Later on, Riveiro *et al.* (2011) extended the approach, by accurately representing the arch blocks. Both FEM and LA models resulted from this study (as shown in Figure 8c). Subsequently, Solla *et al.* (2012) also accurately represented arches with voussoirs from both SfM photogrammetry and GPR. However, in this innovating study, the authors defined the internal profiles of the arches from GPR data and compared various scenarios. A later study also employed GPR for together with orthoimagery to develop accurate geometric models of arches within the LA to investigate the influence of geometric uncertainty. In a final study, an array of geometric models was developed, among which with the image-based approach. Through the specific study, it was shown that whilst 2D block-based models and 3D continuum-based models showed good agreement for vertical loading, the 3D models (from mesh-based approach) were advantageous for complex loading cases which include transverse loading effects.

Furthermore, apart from masonry bridges, the image-based approach has recently found application within cultural heritage masonry structures, albeit on a smaller scale. For instance, Napolitano *et al.* (2019b), developed accurate DEM geometric models of a baptistery from SfM photogrammetry. In a follow-up study (Napolitano *et al.*, 2019a) the same research team also investigated the importance of accurate geometry through comparison of simplified and accurate geometric models respectively (as shown in Figure 8d). The study effectively demonstrated that accurate geometric models were indeed advantageous for capturing structural behaviour.

As can be observed from the aforementioned studies, the main advantage of the image-based approach is the nature of the geometric data (i.e. orthoimages). Especially in the case such as SfM photogrammetry, orthoimagery (especially orthomosaics) is more straightforwardly and rapidly attainable (as demonstrated in Section 5.2.2) in comparison to discrete points whilst it also contains textural information. However, commonly with the point-based approach, a disadvantage is the dependency on manual CAD-based block segmentation. It is notable that in an attempt to overcome this difficulty, various computer vision techniques have been applied for automated-block segmentation, though not yet explicitly for numerical modelling. Another inherent limitation of image-based approaches, in general, is also made apparent here which is that they are limited to describing the structure in two-dimensions since they employ two-dimensional metric information (i.e. of the orthoimagery) and have a constant, user-assigned thickness in the transverse direction. Although evidence suggests for the structural analysis of regular masonry structure such as bridges, 2D and 3D models agree in the absence of transverse loading, two consequences are associated with this limitation: a) only one layer of masonry is described in the geometric model (i.e. only the spandrel walls and arch of a masonry arch bridge); and b) the geometric models have planar faces, due to constant transverse coordinates. Prior to concluding, one final note should be made. Specifically, it should be remembered that any given numerical model is necessarily a simplification of reality. Therefore, the transition from the acquired geometry to the numerical model will always involve some elimination of detail. In fact, keeping an excessively complex geometry may lead to unrealistic or unsafe predictions (e.g., block shapes that can easily break, excessive roughness of joints). Therefore, there are occasions where the structural engineer/ numerical modeler should maintain a degree of tolerance (especially for large-scale structures such as masonry arch bridges). This would eliminate unwanted detail, in order to assure an efficient structural analysis.

6.2.3 The potential of computer vision for automating image-based approaches

Over the last decade, various investigations have demonstrated that computer vision techniques can be employed for automating the procedure of block segmentation. For instance, (Sithole, 2008) presented the first development of a deliberate methodology for brick segmentation with point cloud processing techniques. Later on, Willis *et al.* (2010) employed image processing techniques (IPTs) for estimating the shape of masonry elements present in the facade of a Gothic building from a single image based on automatically detected radiometric variations to separate individual stones the façade of a historic masonry church. For masonry/mortar detection, the theoretical background of this method was a watershed-based binary with the segmentation of the façade image into stones (black) and mortar (white) using a merge criterion based on colour similarity. Later on, Oses *et al.* (2014) also presented an IPT-based block segmentation method, based on the detection of mortar lines independent of conventional edge detection methods (e.g. Canny, Prewitt etc.). Specifically, to delineate the mortar lines, a framework was developed using fine-grained visual categorisation within the open-source computer vision library, *OpenCV* by extracting a set of straight-line segments. With a specific focus on numerical modelling, however, without structural analysis, Riveiro *et al.* (2016) employed IPTs for block segmentation. Block segmentation was based on the intersection of the maximum intensity lines. On the other hand, Shen *et al.* (2016) also employed point cloud processing for block segmentation, through K-means clustering. Of the first investigations to successfully segment rubble masonry (Valero *et al.*, 2018) was based on the 2D continuous wavelet transform with an IPT framework. The same research team (Forster *et al.*, 2019) later extended this approach by using machine learning techniques, which make it of the most advanced and robust block segmentation methods reviewed, able to extract regular masonry or arbitrary shape without a high dependency on block-joint colour contrast such as in the case of IPTs. Finally, another recent study (Shen *et al.*, 2019) recently employed IPTs for block segmentation, entailing principal component analysis in combination with rectangle fitting.

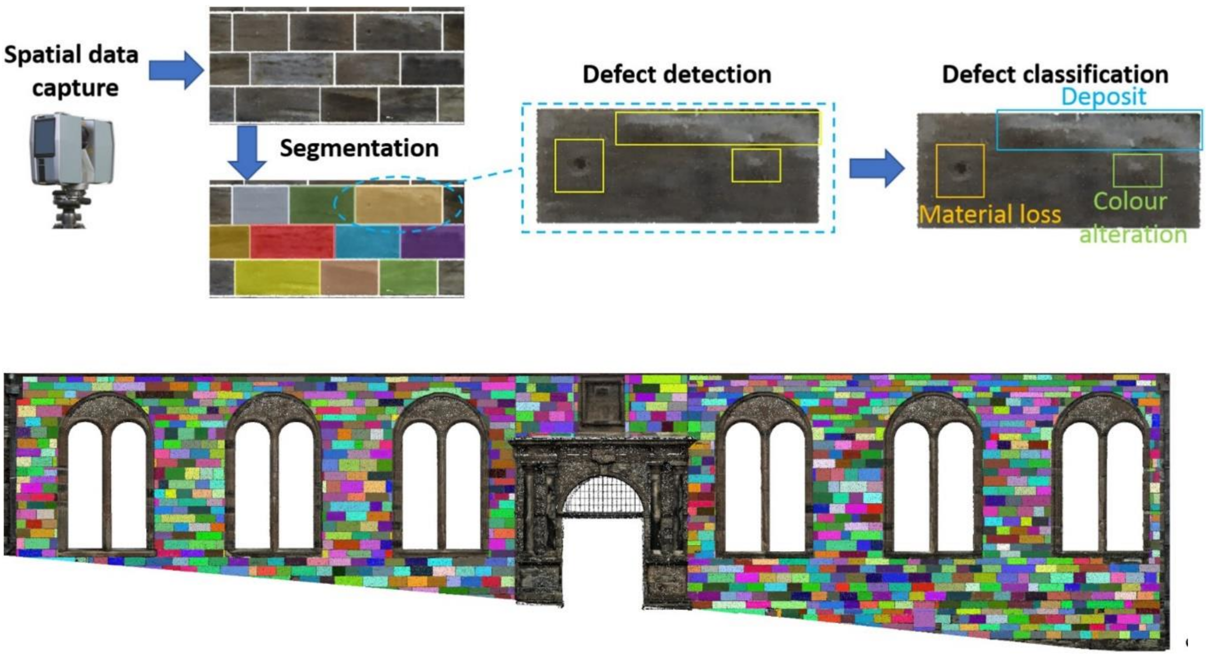


Figure 9: Automated block segmentation point clouds with machine learning techniques (Forster et al., 2019).

Despite the numerous investigations such as the previous that demonstrate the potential of employing computer vision for automated block segmentation, no study has yet evaluated the implementation of automated block segmentation specifically for structural analysis of regular masonry. Therefore, computer vision techniques have remained unexploited for the high-level numerical modelling of masonry such as the DEM.

6.2.4 The potential of GPR for defining the internal geometry of the masonry

Prior to concluding this review, it is important to make a note regarding both the aforementioned structural surveying techniques and approaches to geometric model development. This is that they either omit or approximate the definition of the internal geometry of the masonry (for example, the cloud-based approaches only approximate the internal geometry, e.g., with voxels). At the same time, it is well-known that the internal geometry can highly affect the structural response of a given structure. For instance, headers of multi-leaf stone masonry wall panels, particularly influence their out-of-plane behaviour. Additionally, cracks, unconnected wall panels, defects are well known to affect the structural response of historic masonry buildings. Currently, the experienced contribution of a structural engineer is still necessary for the manual definition of the internal geometry (e.g., cracks, headers and defects). Future research should examine a systematic and automated definition of the internal geometry masonry structure prior to numerical modelling, for example through the employment of GPR, as in Solla *et al.* (2012).

7 Conclusions

This paper presented approaches for the employment non-contact sensing to enhance both the efficiency and reliability of numerical modelling of historic masonry. It commenced with a review of the high-level numerical modelling approaches of historic masonry. Then, non-contact sensing techniques for surveying masonry structures were reviewed, concerning their accuracy and cost-effectivity. These were: a) the total station; b) the laser tracker; c) Structure-from-Motion (SfM) photogrammetry; and d) terrestrial laser scanning (TLS). After, approaches of automatically developing geometric models (i.e., numerical models prior to structural analysis) from geospatial data were reviewed, concerning their degree of automation and facility in implementation. These approaches were organized based on the employment of a: a) point cloud; b) mesh; c) NURBS; d) BIM; e) orthoimage; and f) a sum of discrete points.

Concerning numerical modelling, it was found that an array of numerical modelling approaches has potential for the high-level structural analysis of masonry structures. However, it was also found that the majority of the such numerical modelling approaches employ simplified geometric models (i.e. ad-hoc or idealised/simplified). Consequently, their efficiency is compromised due to the limitations of laborious manual measurements and procedures relating to developing a geometric model. Furthermore, the fact that the reliability of the geometrical properties is neglected also means that uncertainty is induced into the structural analysis itself. The reasons for which state-of-the-art numerical modelling approaches studies neglect the employment of accurate geometric models were attributed to: a) difficulties in geometric data acquisition; b) the complex geometric model development of the DEM (i.e. block-by-block); and c) the lack of comprehensive investigation on the effect of geometric uncertainty to justify the employment of accurate geometric models in the first place.

To overcome the difficulty of geometric model development, various non-contact sensing techniques were reviewed, in their suitability for providing geometric data for numerical modelling rapidly and reliably. Techniques as SfM photogrammetry were found to be particularly attractive for this, due to its accuracy (i.e., comparable to TLS), low operational cost and straightforwardly implementation. Furthermore, concerning the difficulty of geometric model development, various strategies were also found to greatly improve the efficiency and robustness of the structural analysis. On the one hand, for continuum-based modelling approaches, the most efficient was considered the cloud-based approach (e.g., such as through the employment of voxelization). However, it was also highlighted that the only cloud-based approach found regarded representing the masonry as voxels, i.e., cuboids. It is recommended that other methods of spatial enumeration should be employed, that better approximate the anisotropic nature of masonry (for instance with Voronoi blocks, as in (Pulatsu *et al.*, 2019)). On the other hand, for block-based modelling approaches, the image-based approach, was considered the most efficient, especially for large-scale masonry structures. However, here it was found that this approach still relies on manual CAD-based block definition. It is recommended that computer vision approaches be employed, especially novel technologies such as machine learning (as in (Forster *et al.*, 2019)) for automating this process.

Whilst more investigations are necessary on this novel topic, this contribution demonstrates various approaches for the employment of emerging non-contact sensing techniques to enhance both the efficiency and robustness of the structural analysis.

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