Proceedings Paper:

Aftab, M.S. and Rossiter, J.A. orcid.org/0000-0002-1336-0633 (2021) Pre-stabilised predictive functional control for open-loop unstable dynamic systems. In: Pannocchia, G., Johansen, T.A., Mönnigmann, M., Kvasnica, M. and Klaučo, M., (eds.) IFAC-PapersOnLine. 7th IFAC Conference on Nonlinear Model Predictive Control NMPC 2021, 11-14 Jul 2021, Virtual conference (Bratislava, Slovakia). Elsevier , pp. 147-152.

https://doi.org/10.1016/j.ifacol.2021.08.537

Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 54-6 (2021) 147–152

Pre-stabilised Predictive Functional Control for Open-loop Unstable Dynamic Systems

Muhammad Saleheen Aftab * **John Anthony Rossiter** *

* *Department of Automatic Control and Systems Engineering, University of Sheffield, Mappin Street, S1 3JD, UK. (e-mail: msaftab1@sheffield.ac.uk, j.a.rossiter@sheffield.ac.uk)*

Abstract: Predictive functional control (PFC) is the simplest model-based algorithm, equipped with the attributes of a fully fledged predictive controller but at the cost and complexity threshold of a standard PID regulator. It has proven benefits in controlling stable SISO dynamic systems, but similarly to its competitor PID, it loses efficacy when a challenging application is introduced. In this paper, we present a modified PFC approach, especially tailored for open-loop unstable processes, using pre-stabilisation to efficiently control the undesirable dynamics at hand. This is essentially a two-stage design scheme with implications for PFC tuning and constraint handling. The proposal, nevertheless, is straightforward and intuitive, and provides improved closed-loop control in the presence of external perturbations against the standard PFC, and significantly better performance overall compared to the common PID algorithm, as demonstrated in a numerical case-study.

Keywords: PFC; coincidence point; feedback compensation; pre-stabilisation.

1. INTRODUCTION

Model Predictive Control (MPC) is an advanced optimal control strategy with powerful and well-defined procedures for complex multivariate processes (Mayne, 2014). Nevertheless, its computation-heavy nature has traditionally favoured applications with slow dynamics, although the increasing availability of cheap computing resources has significantly widened its scope and utility in recent years (Fernandez-Camacho and Bordons-Alba, 1995; Qin and Badgwell, 2003). But there are areas and applications, for example industrial servo loops, where such an implementation would be logically and financially infeasible and where a cost-effective approach like PID still makes more sense. However, there are scenarios when PID falls short, for instance processes with significant dead-time or tight physical constraints; such cases require additional complexity such as Smith predictors (Skogestad, 2018) and anti-windup algorithms (Visioli, 2006) for improvement. Nonetheless, these solutions are generally ad hoc and, more often than not, degrade other performance attributes; poor robustness to uncertainties is one prominent side-effect of such post-design alterations.

Clearly, there is a need for a systematic yet simpler and cost-effective algorithm, and over the years Predictive Functional Control (PFC) has proved its efficacy as a viable alternative (Richalet et al., 1978). PFC belongs to the family of model-based predictive controllers, and exhibits similar characteristics. As a result, process dead-times and physical constraints are easily integrated in the design, with some degree of robustness owing to the use of a receding horizon (Rossiter, 2018). The main difference, however, arises from the parametrisation of the manipulated variable, which in the case of PFC, is predefined as the linear combination of simple polynomial basis functions (Maciejowski, 2002). The optimisation process is further simplified by noting that constant set-point tracking is achievable with constant control moves within the prediction horizon (Khadir and Ringwood, 2008). Although, unlike mainstream MPC, PFC's heuristic nature merely provides a sub-optimal solution, its simplistic design traits have attracted a wide spectrum of applications (Richalet and O'Donovan, 2011; Richalet and O'Donovan, 2009; Richalet, 1993).

Arguably the unique selling point of PFC is its simplicity. Nevertheless, it lacks flexibility to tackle challenging dynamics. For example, open-loop instability, where unreliable predictions cause ill-posed decision-making (Rossiter and Haber, 2015), has been difficult to control with PFC. Previous studies in this area (Rossiter, 2016; Abdullah and Rossiter, 2018a) have proposed algorithmic level modification by shaping the control input, that although they can improve the closed-loop performance, do so with increased computational complexity, thus negating the core notion of simplicity. Pre-stabilisation of dynamics (Rossiter et al., 1998; Mayne et al., 2000) is fairly common in the MPC literature as a means to modify dynamic behaviour of a difficult system to ensure reliable control performance. Surprisingly however, this concept is still relatively unexplored in PFC and largely restricted to first-order (Aftab et al., 2021) and integrator dynamics (Zhang et al., 2018; Abdullah and Rossiter, 2018b). Researchers, in this context, argue that complex pre-stabilisation may also complicate constraint handling (Rossiter, 2018; Richalet and O'Donovan, 2009) which, despite being sub-optimal, is fairly intuitive and powerful in the PFC formulation.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2021.08.537

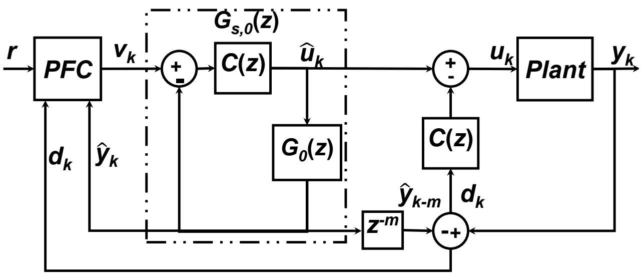

Fig. 1. Proposed PPFC Control Architecture

In this paper, we extend the idea of pre-stabilisation, presented in (Aftab et al., 2021), to higher-order unstable dynamics using the analytical method of internal feedback loop design (Ogata, 1995). It has been found that although the additional control layer burdens the constraint management to some extent, it provides better control of tuning parameters and an impressive overall closed-loop performance, verified by an industrial case study. The remainder of this paper is organised as follows: Section 2 defines the problem and sets control objectives. The main methodology is presented in Section 3 where the Pre-stabilised PFC design is discussed in detail. Next, implications of pre-stabilisation are presented in Section 4. A simulation case study follows next in Section 5 which presents performance comparisons with standard PFC and PI. Finally, the paper concludes in Section 6.

2. PROBLEM STATEMENT

Consider an n^{th}-order coprime delay-free system model

$$G_0(z) = \frac{b(z)}{a(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}} \quad (1)$$

Moreover, $a(z) = a^-(z)a^+(z)$ where the factor $a^+(z)$ contains the p_u open-loop unstable poles. The complete model including time-delay of m samples is $G(z) = z^{-m}G_0(z)$. Moreover, the dynamic plant is subject to actuation limits i.e.

$$\begin{aligned} u_{min} \leq & u_k \leq u_{max} \\ \Delta u_{min} \leq & \Delta u_k \leq \Delta u_{max} \end{aligned} \quad (2)$$

where $\Delta = 1 - z^{-1}$. The aim is to design a predictive functional controller that operates on pre-stabilised model predictions via internal feedback compensation. The Pre-stabilised PFC (PPFC) is, therefore, expected to perform in the presence of disturbances and modelling uncertainty while adhering to constraints (2).

3. PRE-STABILISED PFC

The fundamental idea behind PPFC, as shown in Fig. 1, is to first stabilise the unstable open-loop dynamics, using a simple and well understood classical approach, and then implement PFC in the standard way, as an outer loop, for improving performance and constraint management. The following two sub-sections explain the proposed design procedure.

3.1 Design of Pre-stabilising Loop

The proposal is based on the analytical approach of feedback compensator design presented in (Ogata, 1995). Assume that a $(n-1)^{th}$-order bi-proper feedback compensator $C(z) = q(z)/p(z)$ is used to modify the open-loop model $G_0(z)$, as shown in Fig. 1, resulting in the pre-stabilised transfer function $G_{s,0}(z)$, with a smooth and monotonically convergent prediction behaviour. Then one may write:

$$G_{s,0}(z) = \frac{\beta(z)}{\alpha(z)} = \frac{q(z)b(z)}{p(z)a(z) + q(z)b(z)} \quad (3)$$

where $\alpha(z)$ is the $(2n-1)^{th}$-order pre-stabilised pole polynomial, and the underlying relationship,

$$p(z)a(z) + q(z)b(z) = \alpha(z) \quad (4)$$

is called the *Diophantine Equation*. In order to design the $C(z)$, one must define the desired pre-stabilised characteristic polynomial $\alpha(z)$ and then utilise linear algebra to obtain the coefficients of $p(z)$ and $q(z)$ with,

$$\mathbf{M} = \mathbf{S}^{-1}\mathbf{D} \quad (5)$$

where $\mathbf{M} = [p_{n-1} \ldots p_0 \quad q_{n-1} \ldots q_0]^T$, $\mathbf{D} = [\alpha_{2n-1} \ldots \alpha_0]^T$ and $\mathbf{S}$ is the *Sylvester Matrix* (Goodwin, 2001) given by:

$$\mathbf{S} = \begin{bmatrix} a_n & 0 & \ldots & 0 & b_n & 0 & \ldots & 0 \\ a_{n-1} & a_n & \ldots & 0 & b_{n-1} & b_n & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \ldots & \vdots \\ 1 & a_1 & \ldots & a_{n-1} & 0 & b_1 & \ldots & b_{n-1} \\ 0 & 1 & \ldots & a_{n-2} & 0 & 0 & \ldots & b_{n-2} \\ \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & a_1 & 0 & 0 & \ldots & b_1 \\ 0 & 0 & \ldots & 1 & 0 & 0 & \ldots & 0 \end{bmatrix} \quad (6)$$

Note that $\alpha(z)$ is factorised as:

$$\alpha(z) = o(z)a^-(z)\alpha^+(z) \quad (7)$$

where $o(z)$ is the $(n-1)^{th}$-order observer and $\alpha^+(z)$ represents the p_u pre-stabilised poles. We propose if $a^+(z) = \prod_{i=1}^{p_u}(z - z_{p,i})$ with $z_{p,i} > 1$, then $\alpha^+(z) = \prod_{i=1}^{p_u}(z - 1/z_{p,i})$. In case an integrator factor $(z-1)$ is present, then one may simply replace it with $(z - 0.5)$ (Abdullah and Rossiter, 2018a). Moreover, the minimum order observer is generally selected as $o(z) = z^{n-1}$ (Ogata, 1995). This completes the internal feedback loop design.

Remark 1. For $n = 1$, the compensator reduces to simple proportional gain, i.e. $C(z) = K$. The Pre-stabilised PFC design for first-order unstable systems using proportional compensation has been investigated more extensively (Aftab et al., 2021) so will not be pursued here.

3.2 Pre-stabilised PFC Control Law

The PPFC algorithm works similarly to the original PFC but implemented on the pre-stabilised model dynamics. At each time step, the predicted output y_k is matched to the pre-defined target behaviour at only one coincidence point n_y steps ahead with constant control moves. The process is repeated at the next sample and owing to the receding horizon, an implied feedback is established that moves the plant output closer to the target. The desired behaviour is generally represented as a first-order pole ρ. The ideal n_y-step ahead prediction based on a first-order response is given as:

$$y_{k+n_y+m|k} = r - (r - E[y_{k+m|k}])\rho^{n_y} \quad (8)$$

where r is the set-point and $E[y_{k+m|k}]$ is the expected m-sample delayed plant output (Rossiter, 2018). Furthermore, $E[y_{k+m|k}] = \hat{y}_k + d_k$ with $d_k = y_k - \hat{y}_{k-m}$, where

d_k accounts for prediction bias due to modelling errors and/or disturbances and $\hat{y}_k$ is the model output. On the other hand, one may obtain the output predictions from $G_{s,0}(z)$ i.e. $\alpha(z)\hat{y}(z) = \beta(z)v(z)$:

$$\hat{y}_{k+n_y|k} = \mathbf{H}\underset{\rightarrow}{\mathbf{v}}_k + \mathbf{P}\underset{\leftarrow}{\mathbf{v}}_{k-1} + \mathbf{Q}\underset{\leftarrow}{\hat{\mathbf{y}}}_k \tag{9}$$

where $\mathbf{H}$, $\mathbf{P}$ and $\mathbf{Q}$ depend on the model parameters. For a generic N^{th} order model:

$$\underset{\rightarrow}{\mathbf{v}}_k = \begin{bmatrix} v_k \\ v_{k+1} \\ \vdots \\ v_{k+n_y-1} \end{bmatrix}; \underset{\leftarrow}{\mathbf{v}}_{k-1} = \begin{bmatrix} v_{k-1} \\ v_{k-2} \\ \vdots \\ v_{k-N+1} \end{bmatrix}; \underset{\leftarrow}{\hat{\mathbf{y}}}_k = \begin{bmatrix} \hat{y}_k \\ \hat{y}_{k-1} \\ \vdots \\ \hat{y}_{k-N+1} \end{bmatrix}$$

The delay-free prediction model $G_{s,0}(z)$ essentially provides m-step ahead estimate of the plant output, which implies:

$$y_{k+n_y+m|k} = \hat{y}_{k+n_y|k} + d_k \tag{10}$$

The decision variable remains constant throughout the horizon i.e. $v_{k+i} = v_k$, $\forall i > 0$, which results in the following PPFC control law:

$$v_k = \frac{r - (r - E[y_{k+m|k}])\rho^{n_y} - (\mathbf{P}\underset{\leftarrow}{\mathbf{v}}_{k-1} + \mathbf{Q}\underset{\leftarrow}{\hat{\mathbf{y}}}_k + d_k)}{h} \tag{11}$$

where $h = \sum_{j=1}^{n_y} H_j$ and H_j is the j^{th} element of $\mathbf{H}$. Next, we will discuss the impact of pre-stabilisation on some key aspects of the proposed PFC approach.

4. IMPLICATIONS OF PRE-STABILISATION

Clearly pre-stabilisation has transformed the decision variable from u_k to v_k, which has significant implications for parameter tuning and constraint handling. Since u_k drives the physical process and the fact that internal compensation is not hard-wired, a direct relationship between both variables must be established for control implementation. Details follow next.

4.1 Relationship between u_k and v_k

If the compensator $C(z)$ were hard-wired, we would have got $u_k = C(z)[v_k - y_{k+m}]$, where y_{k+m} shows the delayed response due to u_k at the current sample. Obviously y_{k+m} is unknown being a future value, but can be replaced with its expected value $E[y_{k+m|k}] = \hat{y}_k + d_k$. Thus,

$$u_k = C(z)[v_k - (\hat{y}_k + d_k)] \tag{12}$$

Furthermore, a similar expression can be written for the pre-stabilised model:

$$\hat{u}_k = C(z)[v_k - \hat{y}_k] \tag{13}$$

Thus after subtracting (13) from (12), the relationship between u_k and $\hat{u}_k$ is established:

$$u_k = \hat{u}_k - C(z)d_k \tag{14}$$

Finding $\hat{u}_k$ is an additional step and adds slightly to the coding complexity. Nevertheless, it is directly related to v_k as shown in the following theorem.

Theorem 1. The control variable $\hat{u}_k$ can be obtained from decision variable v_k using the following expression:

$$\hat{u}_k = \frac{q(z)}{o(z)}.\frac{a^+(z)}{\alpha^+(z)}v_k \tag{15}$$

Proof. Since $\hat{y}_k = G_0(z)\hat{u}_k = G_{s,0}(z)v_k$, eliminating $\hat{y}_k$ results in:

$$\frac{b(z)}{a^-(z)a^+(z)}\hat{u}_k = \frac{q(z)}{o(z)}.\frac{b(z)}{a^-(z)\alpha^+(z)}v_k$$

which simplifies to (15). □

Thus one may replace $\hat{u}_k$ from (15) in (14) to directly evaluate u_k from v_k and vice versa,

$$A(z)u_k = B(z)v_k + E(z)d_k \tag{16}$$

with the polynomials $A(z)$, $B(z)$ and $E(z)$ defined as:

$$\begin{aligned} A(z) &= o(z)\alpha^+(z)p(z) = 1 + A_1z^{-1} + \cdots + A_lz^{-l} \\ B(z) &= q(z)a^+(z)p(z) = B_0 + B_1z^{-1} + \cdots + B_lz^{-l} \\ E(z) &= -o(z)\alpha^+(z)q(z) = E_0 + E_1z^{-1} + \cdots + E_lz^{-l} \end{aligned} \tag{17}$$

where $l = p_u + 2n - 2$. Finally,

$$u_k = B_0v_k + f_k \tag{18}$$

where $f_k = -\mathbf{A}\underset{\leftarrow}{\mathbf{u}}_{k-1} + \mathbf{B}\underset{\leftarrow}{\mathbf{v}}_{k-1} + \mathbf{E}\underset{\leftarrow}{\mathbf{d}}_k$ and the vectors $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$ contain appropriate coefficients of the respective polynomials.

4.2 Controller Tuning

The standard procedure of PFC parameter tuning is based on the conjecture presented in (Rossiter and Haber, 2015), which is mainly applicable to stable dynamics with monotonic steady-state convergence. The recommendation is to select the coincidence horizon n_y preferably within the time window corresponding to 40%-80% rise in the step response with significant gradient. As for finding ρ, one may overlay several first-order responses on the step response to identify which target behaviour coincides within the mentioned n_y range. Evidently this method will not work well with an unstable process, for which a constant input would inevitably lead to divergent output predictions. Parameter selection for such systems is far less consistent and mostly ineffective with no concrete guidelines (Rossiter and Haber, 2015; Rossiter, 2018). Clearly pre-stabilisation makes intuitive sense here, since controller tuning with the stable $G_{s,0}(z)$ in this case will be far more meaningful than the originally unstable process. See, for instance, Fig. 3 that displays PFC parameter selection for a modified system based on the aforementioned procedure.

4.3 Constraint Handling

The standard constraint handling mechanism generally implements simple saturation of the decision variable, which is fairly straightforward with a constant control formulation. Nevertheless, the additional feedback loop in the Pre-stabilised PFC re-parametrises the degree-of-freedom such that u_k no longer remains constant. One possible solution in this case is to transfer the original constraints to the new variable v_k at every sample using a process of back calculation (Richalet and O'Donovan, 2009). Clearly back calculation is computationally intensive; this may work easily with simple feedback designs, for example see (Aftab et al., 2021; Zhang et al., 2020), but with more involved controllers, such as the one in this study, it complicates the validation process. A more efficient approach, however, is to implement constraints on (18) directly, with v_k updated

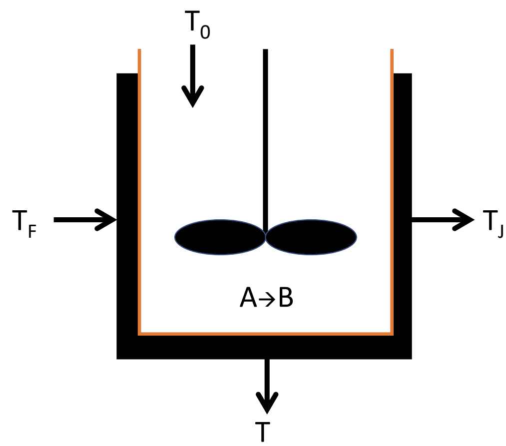

Fig. 2. The Jacketed CSTR Process

only if violation occurs. Each row of the following vector inequalities corresponds to the $(k+i)^{th}$ predicted input (Rossiter, 2018):

$$\mathbf{L}u_{min} \leq \underset{\rightarrow}{\mathbf{u}}_k \leq \mathbf{L}u_{max}$$
$$\mathbf{L}\Delta u_{min} \leq \Delta \underset{\rightarrow}{\mathbf{u}}_k \leq \mathbf{L}\Delta u_{max} \qquad (19)$$

where $i = 0, 1, \ldots, n_c$ and $\mathbf{L} = [1 \quad 1 \ldots]^T$. The validation window n_c (i.e. the length of above inequalities) must extend well beyond the point of coincidence to observe and validate long range adherence. This is crucial because any unobserved input violation could eventually lead to infeasibility, thus invalidating the current input computation. Ideally, n_c should cover the settling period of $G_{s,0}(z)$, i.e. the time to reach and stay within 95% of the implied steady-state, which roughly corresponds to three to five times n_y.

Remark 2. Constraint handling with Pre-stabilised PFC is recursively feasible as long as n_c is sufficiently large (Abdullah and Rossiter, 2018a). This, however, may not be true with open-loop unstable dynamics, for which, in truth, rigorous generic recursive feasibility properties require computations which might be considered beyond the *price range* of PFC.

5. INDUSTRIAL CASE STUDY

This section demonstrates the efficacy of the proposed PPFC algorithm with a case study involving temperature control of a Jacketed CSTR. The Continuous Stirred Tank Reactor (CSTR) is a common industrial unit that is widely employed in different chemical manufacturing processes. The reaction dynamics converting component A into component B in an ideal CSTR has a non-linear first-order behaviour. Nevertheless, many chemical reactions also require a specific temperature to be maintained within the tank for a flawless yield. Therefore, the tank is generally equipped with an outer jacket in which the temperature of a flowing fluid T_J is used as the manipulated variable to regulate the inside reaction temperature T, as shown in Fig. 2. The overall coupled model has two-state nonlinear dynamics with potential for exotic behaviour owing to multiple steady-states (Bequette, 2002). In this study, the linearised model around one operating point depicts unstable second-order dynamics given by (Rao and Chidambaram, 2008):

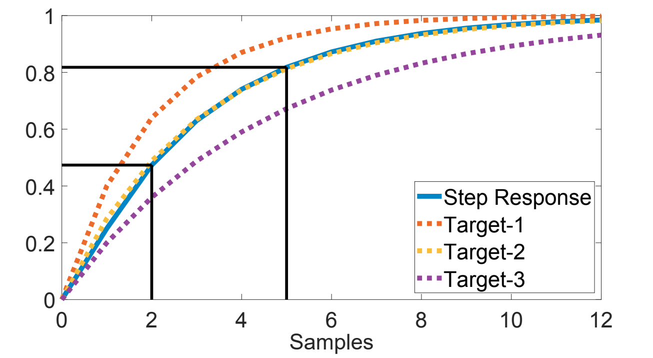

Fig. 3. Target responses with $\rho = [0.6, 0.715, 0.8]$ overlaying the normalised step response of $G_{s,0}(z)$

$$G(z) = \frac{T(z)}{T_J(z)} = \frac{2.102z + 0.4011}{z^2 - 1.465z + 0.058}.z^{-1} \qquad (20)$$

subject to $|T_J| \leq 0.21°F$ and $|\Delta T_J| \leq 0.075°F$. Note that both T and T_J are deviation variables around the steady-state values $T_{ss} = 101.1°F$ and $T_{J,ss} = 60°F$.

5.1 Pre-stabilisation and Offline Tuning

Next we pre-stabilise the Jacket CSTR model. Here, p_u=1 with one unstable pole $a^+(z) = z - 1.424$, one stable pole $a^-(z) = z - 0.041$ and a delay of $m = 1$ minute in measurement. Since $n = 2$, the pre-stabilised pole polynomial must be third-order with $\alpha(z) = z(z - 0.041)(z - 1/1.424)$. The first-order bi-proper compensator $C(z)$ is then constructed using (4)-(7) resulting in $C(z) = \dfrac{0.303z - 0.0123}{z + 0.0852}$. The following pre-stabilised delay-free model is obtained:

$$G_{s,0}(z) = \frac{T(z)}{v(z)} = \frac{0.637z^2 + 0.096z - 0.005}{z^3 - 0.743z^2 + 0.0288z} \qquad (21)$$

The next step is the controller tuning i.e. finding appropriate n_y and ρ. Fig. 3 shows the pre-stabilised step response curve overlaying various first-order target responses and suggests $2 \leq n_y \leq 5$ as the suitable coincidence horizon window. Interestingly, the target behaviour with $\rho = 0.715$ almost exactly overlaps the step response, whereas those with $\rho = 0.6$ or $\rho = 0.8$ do not match predictions within the desirable n_y range and hence would need over-actuation or under-actuation to enforce an intercept. In this study, we have selected $\rho = 0.715$ and $n_y = 3$.

In order to assess the performance of PPFC, two more controllers are implemented: the original PFC tuned with the same parameters given above, and a PI controller designed with $K_P = 0.02$ and $K_I = 0.004$. Note that the PI controller operates on the pre-stabilised plant after hardwiring the internal feedback loop with $C(z)$. However, doing so also introduces time-delay in the feedback design, hence a relatively poor PI performance is anticipated.

5.2 Nominal Unconstrained Performance

The unconstrained closed-loop performance in the absence of disturbance and modelling uncertainty is analysed first, with the results shown in Fig. 4. The temperature step response (top figure) achieved with PPFC and standard PFC is smooth and monotonically convergent. Neverthe-

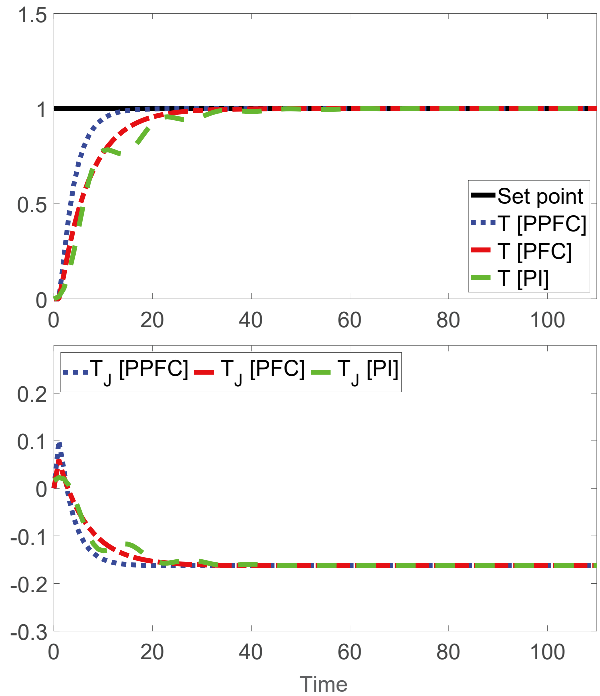

Fig. 4. Unconstrained performance without external perturbations (nominal case)

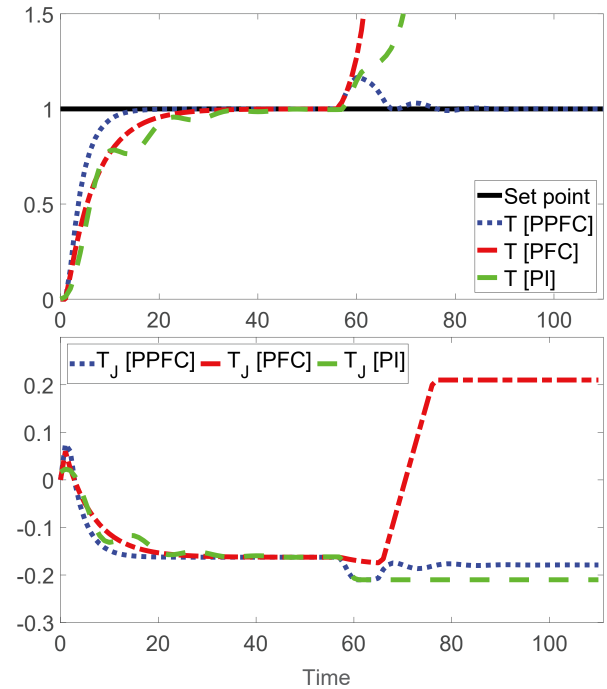

Fig. 5. Constrained performance in the presence of constant disturbance in jacket temperature

less, the tuning parameter ρ appears to have stronger linkage with the output after pre-stabilisation; that is, tuning will be easier and more intuitive in practice with PPFC. On the other hand, the response with the PI controller is rather oscillatory in the beginning, possibly due to the effect of time-delay in the error computation. Overall, the PI controller seems to have the slowest performance amongst the three choices. On the other hand, the jacket temperature control (bottom figure) corresponds to the associated step performance and demonstrates similar behaviour. Evidently, the fastest PPFC response is due to an aggressive control action, peaking at approximately $0.1°F$, as opposed to the other two with slightly lower peak values. While there is no remarkable difference in the nominal performance, it hardly portrays the true picture and the effect of external perturbations must also be considered for a more complete evaluation.

5.3 Constrained Performance in a Practical Situation

In this section, the effects of external perturbations on closed-loop performance are studied. Consider the scenario when a sudden process variation increases the jacket feed temperature by approximately 10% of the planned value. This is simulated as a constant disturbance signal introduced at the plant input around the 55^{th} minute of operation. The simulation results are depicted in Fig. 5. Clearly, both the standard PFC and the PI controllers respond poorly, immediately driving the system into instability. Moreover, the controllers appear highly sensitive as suggested by the aggressive input activity soon after the introduction of disturbance. This inevitably leads to actuator saturation, with possible equipment failure in practice. The Pre-stabilised PFC, on the other hand, displays commendable tracking with far superior disturbance rejection characteristic, providing fast and smooth normalisation of operation. Interestingly, apart from the slight deviation around the 60^{th} minute, the PPFC control performance appears very similar to the nominal behaviour displayed in Fig. 4.

Next an unmodelled pole at $z = 0.1$ is added to analyse the controllers' robustness against modelling mismatch with the results depicted in Fig. 6. While the transient performance of the PPFC is slightly affected (a slow target pole will be better in this case), the benefits of pre-stabilisation are even more pronounced as the constrained performance remains recursively feasible and stable throughout. In comparison, the PI controller clearly fails to accommodate the modelling uncertainty with immediate output divergence along with input and rate constraint violations. Interestingly, the standard PFC also destabilises, although this becomes apparent only around the 100 minute mark, owing to the use of unreliable and numerically infeasible divergent open-loop predictions in the decision making. In practice, this leads to spoiled product and financial loss to the manufacturer.

To conclude, the Pre-stabilised PFC appears to be the most reliable choice for the temperature control of Jacketed CSTR process in the presence of disturbances and modelling uncertainty.

6. CONCLUSION

This paper has presented the concept of pre-stabilised predictive functional control for unstable open-loop dynamic systems. An analytical approach to designing the pre-stabilising compensator is proposed, which is fairly simple and intuitive, and works well in combination with PFC. Specifically, it preserves the simplistic PFC parameter tuning and adds reliability, but at the cost of slightly more onerous constraint management. Nevertheless, the

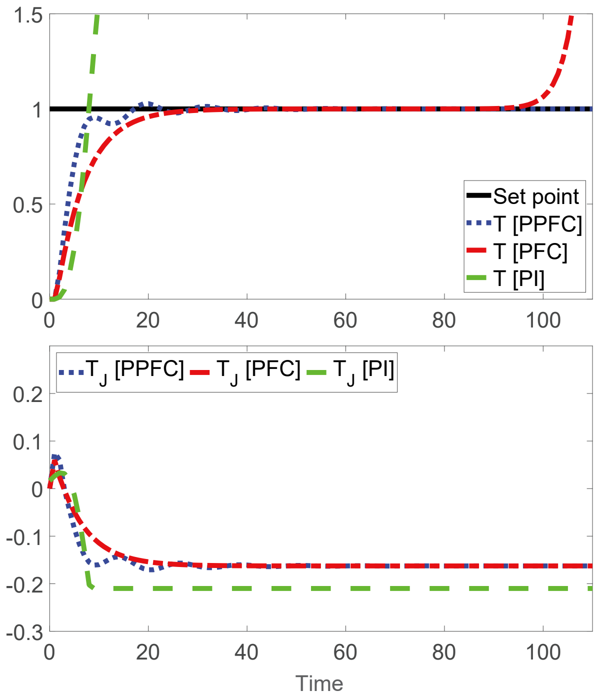

Fig. 6. Constrained performance in the presence of plant-model mismatch

overall advantage of pre-stabilisation in terms of closed-loop performance compared to a standard PFC and PI control has been observed in simulation studies, which also guarantees reliable operation in the presence of external perturbations.

While this study has highlighted the key benefits of pre-stabilisation using one application, in future, the authors plan to extend the scope of validation across a range of case-studies and real-time experiments. The future work will also focus on a more rigorous analysis of loop sensitivity to gain better understanding. Furthermore, a possible extension to accommodate a variety of challenging scenarios, including non-minimum phase and poorly damped dynamics, is also under development.

REFERENCES

Abdullah, M. and Rossiter, J. (2018a). Input shaping predictive functional control for different types of challenging dynamics processes. *Processes*, 6(8), 118.

Abdullah, M. and Rossiter, J.A. (2018b). Alternative method for predictive functional control to handle an integrating process. In *2018 UKACC 12th International Conference on Control (CONTROL)*, 26–31. IEEE.

Aftab, M.S., Rossiter, J.A., and Zhang, Z. (2021). Predictive Functional Control for Unstable First-Order Dynamic Systems. In J.A. Gonçalves, M. Braz-César, and J.P. Coelho (eds.), *CONTROLO 2020*, volume 695 of *Lecture Notes in Electrical Engineering*, 12–22. Springer International Publishing, Cham.

Bequette, B. (2002). Behavior of a CSTR with a recirculating jacket heat transfer system. In *Proceedings of the 2002 American Control Conference (IEEE Cat. No.CH37301)*. IEEE. doi:10.1109/acc.2002.1025296.

Fernandez-Camacho, E. and Bordons-Alba, C. (1995). *Model Predictive Control in the Process Industry.* Springer London. doi:10.1007/978-1-4471-3008-6.

Goodwin, G. (2001). *Control system design.* Prentice Hall, Upper Saddle River, N.J.

Khadir, M. and Ringwood, J. (2008). Extension of first order predictive functional controllers to handle higher order internal models. *International Journal of Applied Mathematics and Computer Science*, 18(2), 229–239. doi:10.2478/v10006-008-0021-z.

Maciejowski, J.M. (2002). *Predictive Control with Constraints.* Pearson Education.

Mayne, D.Q. (2014). Model predictive control: Recent developments and future promise. *Automatica*, 50(12), 2967–2986. doi:10.1016/j.automatica.2014.10.128.

Mayne, D.Q., Rawlings, J.B., Rao, C.V., and Scokaert, P.O. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, 36(6), 789–814.

Ogata, K. (1995). *Discrete-Time Control Systems.* Prentice Hall Englewood Cliffs, NJ, 2nd ed. edition.

Qin, S. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7), 733–764. doi:10.1016/s0967-0661(02)00186-7.

Rao, A.S. and Chidambaram, M. (2008). Analytical design of modified smith predictor in a two-degrees-of-freedom control scheme for second order unstable processes with time delay. *ISA Transactions*, 47(4), 407–419. doi:10.1016/j.isatra.2008.06.005.

Richalet, J. (1993). Industrial applications of model based predictive control. *Automatica*, 29(5), 1251–1274. doi:10.1016/0005-1098(93)90049-y.

Richalet, J. and O'Donovan, D. (2011). Elementary predictive functional control: A tutorial. In *2011 International Symposium on Advanced Control of Industrial Processes (ADCONIP)*, 306–313.

Richalet, J. and O'Donovan, D. (2009). *Predictive Functional Control: Principles and Industrial Applications.* Springer Science & Business Media.

Richalet, J., Rault, A., Testud, J., and Papon, J. (1978). Model predictive heuristic control. *Automatica (Journal of IFAC)*, 14(5), 413–428.

Rossiter, J.A., Kouvaritakis, B., and Rice, M. (1998). A numerically robust state-space approach to stable-predictive control strategies. *Automatica*, 34(1), 65–73.

Rossiter, J. (2016). Input shaping for pfc: how and why? *Journal of control and decision*, 3(2), 105–118.

Rossiter, J. (2018). *A First Course in Predictive Control.* CRC Press.

Rossiter, J. and Haber, R. (2015). The effect of coincidence horizon on predictive functional control. *Processes*, 3(1), 25–45.

Skogestad, C.G.S. (2018). Should we forget the smith predictor? *IFAC-PapersOnLine*, 51(4), 769–774. doi:10.1016/j.ifacol.2018.06.203.

Visioli, A. (2006). *Practical PID Control.* Springer London. doi:10.1007/1-84628-586-0.

Zhang, Z., Rossiter, J., Xie, L., and Su, H. (2018). Predictive functional control for integral systems. In *International Symposium on Process System Engineering.*

Zhang, Z., Rossiter, J.A., Xie, L., and Su, H. (2020). Predictive functional control for integrator systems. *Journal of the Franklin Institute*, 357(7), 4171–4186. doi:10.1016/j.jfranklin.2020.01.026.