

This is a repository copy of *Aggregation in Economies with Search Frictions*.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/id/eprint/173334/>

Version: Accepted Version

Article:

Kokonas, Nikolaos and Santos Monteiro, Paulo orcid.org/0000-0002-2014-4824 (2021) Aggregation in Economies with Search Frictions. *Journal of Mathematical Economics*. 102521. ISSN: 0304-4068

<https://doi.org/10.1016/j.jmateco.2021.102521>

Reuse

This article is distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs (CC BY-NC-ND) licence. This licence only allows you to download this work and share it with others as long as you credit the authors, but you can't change the article in any way or use it commercially. More information and the full terms of the licence here: <https://creativecommons.org/licenses/>

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.

Aggregation in economies with search frictions^{*}

Nikolaos Kokonas[†] and Paulo Santos Monteiro[‡]

March 8, 2021

Abstract

We derive an aggregation result in economies with indivisible labor supply choices and frictional labor markets, obtaining a tractable model of gross worker flows in aggregate labor markets with search frictions. Our result explores the fact that economies with non-convex choice sets and idiosyncratic shocks allow for sunspot equilibria à la [Kehoe et al. \(2002\)](#). We use comparative steady state analysis to demonstrate the applicability of our aggregation result. Our framework reconciles the neoclassical growth model with search frictions with a mildly procyclical participation rate and matches the gross worker flows underpinning those dynamics.

Keywords: indivisibilities, sunspots, search frictions, gross worker flows.

JEL Classification: D50, D60, D91, J22.

^{*}We thank the Associate Editor, Yiannis Vailakis, and two anonymous referees for their thoughtful and constructive comments that helped improve the paper. We are also grateful to Julio Dávila and Herakles Polemarchakis for helpful discussions, and also seminar participants at the XXVIII European Workshop on Economic Theory (EWET), Berlin, June 2019.

[†]University of Bath, n.kokonas@bath.ac.uk.

[‡]University of York, paulo.santosmonteiro@york.ac.uk.

1 Introduction

Since the seminal work of Merz (1995) and Andolfatto (1996), dynamic stochastic general equilibrium (DSGE) models with labor market search frictions have been widely used to study unemployment fluctuations. However, these two approaches place different restrictions on individual choices. In turn, Merz (1995) assumes the existence of a representative “large family”, constrained by budget sets and an employment law of motion, while Andolfatto (1996) assumes a game of “musical chairs” (exogenous shocks), that randomly allocate individuals to labor market states, with perfect insurance against idiosyncratic risk.¹ We take our cue from the latter approach and make the following contribution: we generalise the musical chairs’ approach to a model with gross worker flows and individual participation choices, using results from the literature on sunspots and lotteries, along the lines of Kehoe et al. (2002). To the best of our knowledge, ours is the first paper to offer an aggregation result in economies with three state labor markets, indivisible labor and search frictions, based on individual choices (without having to impose the assumption of the “large family”), that yields a constrained efficient competitive equilibrium.

Our approach delivers a tractable characterization of equilibrium in economies combining indivisible labor supply choices (participation margin), and labor market frictions. The literature has often restricted attention to two-state labor markets, ignoring participation and focusing on the margin between employment and unemployment.² However, recent empirical work attributes an important role to the participation margin for labor market transitions. Elsby et al. (2015) showed that the participation margin accounts for one-third of the cyclical variation in the unemployment rate. Moreover, unlike the Merz (1995) large family set-up,

¹Also, Merz (1997) used a randomisation device analogous to Andolfatto to decentralise the constrained optimum in a two-state labor market.

²Several papers consider participation in DSGE models with search frictions. Some early examples are Veracierto (2008), Ravn (2008) and Shimer (2013). This notwithstanding, the inclusion of an intertemporal labor supply margin in economies with indivisibilities and search frictions remains a difficult task. All examples above use the Merz (1995) “large family” model to achieve aggregation, and deliver stark counterfactual predictions about the labor market (for example, procyclical unemployment).

which only identifies net worker flows, our model yields a characterization of equilibrium gross worker flows. This is important, since [Krusell et al. \(2010, 2011\)](#) and [Krusell et al. \(2017\)](#) stressed the importance of gross worker flows and developed models with missing insurance markets and indivisibilities in labor supply choice to account for these transitions.

We develop a general equilibrium model of gross worker flows with complete markets, where individuals face heterogeneous employment histories and idiosyncratic risk. Following [Andolfatto \(1996\)](#), musical chairs allocate individuals to different labor market states each period; conditional on this, individuals face an indivisible participation choice, in labor market with search frictions. To overcome indivisibilities, individuals play lotteries over participation as in [Rogerson \(1988\)](#) and [Hansen \(1985\)](#). The decision of each individual is based on the joint outcome of public (“musical chairs”) and contrived randomness (lotteries) and the realisation of idiosyncratic shocks. This hybrid decision process may seem unusual, but we argue it can be microfounded as follows. We demonstrate that one can mimic the joint effects of musical chairs and lotteries by indexing on the basis of two naturally occurring random variables (sunspots) prior and after the realisation of the idiosyncratic shocks. Such an arrangement is consistent with the existence of the usual Arrow-Debreu contingent commodities.

Subsequently, similarly to [Christiano et al. \(2020\)](#) we use comparative steady state analysis as a short-cut for analyzing model dynamics. Two main insights emerge from this analysis. First, our model reconciles the neoclassical growth model with search frictions with a mildly procyclical participation rate. This result is particularly important given the tendency for models featuring intertemporal substitution in frictional labor markets to deliver excessively procyclical participation and, thus, procyclical unemployment (a problem stressed by [Ravn, 2008](#); [Veracierto, 2008](#); [Shimer, 2013](#), for example). Second, we show using a calibrated example that the model accounts well for the observed flows. In particular, it is able to match the high transition rate from unemployment to inactivity, which early papers by [Garibaldi and Wasmer \(2005\)](#) and [Krusell et al. \(2010, 2011\)](#) have shown to be challenging for equilibrium models of gross worker flows, under either complete or incomplete markets.

The literature on sunspots and lotteries in economies with non-convexities and complete markets includes, among others, Prescott and Townsend (1984), Shell and Wright (1993), Garratt (1995), Garratt et al. (2002), Kehoe et al. (2002), and Garratt et al. (2004). Our results generalise models with indivisibilities to include idiosyncratic risk arising from frictional labor markets. To achieve that, we introduce the distinction between public randomisations prior and after the realisation of idiosyncratic shocks in each period—although this distinction is already discussed by Kehoe et al. (2002), it is not important for their analysis.

Our paper contributes to a recent literature that combines indivisible labor supply choices in models with intertemporal substitution (what Krusell et al., 2008, call “non-trivial labor supply choices”), together with search frictions in the labor market. Krusell et al. (2008) show that in a set-up with indivisibility and incomplete markets (similar to Chang and Kim, 2006, 2007), search frictions avoids indeterminacy in labor supply choices.³ Building on this framework, Krusell et al. (2010, 2011) study individual transitions across employment, unemployment and non-participation, in a three-states labor market model, with incomplete markets, search frictions and non-trivial labor supply choices.⁴ They show that whilst the benchmark model is unable to match the persistence of the employment and non-participation states found in the data, a version of the same model with persistent idiosyncratic productivity shocks affecting the individual value of work is able to match the transition flows well.

Further, Krusell et al. (2011) study a version of their model with complete markets (with insurable idiosyncratic shocks), but do not discuss decentralization and, instead, consider the solution to the social planner’s problem, in which each individual receives equal weight. The

³Krusell et al. (2008) show how an economy with indivisible labor and incomplete markets, current labor supply is indeterminate for individuals with intermediate levels of wealth. They subsequently suggest labor market frictions, à la Lucas and Prescott (1974) as a mechanism to break this indeterminacy.

⁴There are, of course, several empirical papers studying gross worker flows in three-state frictional labor markets, but without modelling optimal intertemporal labor supply choices. These studies extend the matching function model (Mortensen and Pissarides, 1994), and employ a stock-flow accounting framework to model unemployment duration dependence, long-term unemployment, and non-participation. Recent examples, establishing the importance of workers’ heterogeneity and the participation margin include Barnichon and Figura (2015), Elsby et al. (2015), and Kroft et al. (2016).

86 resulting equilibrium allocations imply labor market gross flows that are comparable to those
87 obtained in the incomplete market economy. Thus, they conclude that uninsurable risk is not
88 a necessary ingredient to obtain a satisfactory representation of labor market transitions.⁵ In
89 our paper, we show how the decentralized competitive equilibrium with complete markets can
90 be obtained using either lotteries or sunspots, to produce constrained efficient allocations. At
91 the same time, the resulting model is as successful at matching empirical gross worker flows as
92 the incomplete markets model. In particular, using sunspots as the randomization mechanism
93 generates heterogeneous employment histories across individuals that are conforming with
94 realistic transitions across labor market states (comparable to what is achieved by [Krusell](#)
95 [et al.](#), 2010, 2011, using idiosyncratic productivity shocks).

96 Finally, [Krusell et al.](#) (2017) augment the set-up developed in [Krusell et al.](#) (2010, 2011) with
97 job-to-job transitions and aggregate shocks to labor market frictions, in order to study gross
98 worker flows over the business cycle. We consider shocks to the job finding rate, and show
99 how the model with indivisible labor supply, complete markets, and extrinsic randomization,
100 can deliver either a countercyclical or a procyclical participation rate. Thus, the neoclassical
101 growth model with search frictions can be reconciled with a mildly procyclical participation
102 rate, that is supported by empirically realistic gross worket flows.

103 The remainder of the paper is organized as follows: Section 2 explains the environment;
104 Section 3 establishes that an equilibrium with musical chairs and lotteries corresponds to
105 a sunspot equilibrium; Section 4 presents the comparative steady state analysis; Section 5
106 concludes.

⁵In a model with indivisible labor supply choices but without frictional unemployment, [Ljungqvist and Sargent](#) (2008) obtain a similar result.

2 Model

2.1 Agents and markets

Time is discrete, indexed $t = 0, 1, \dots$. The economy is populated by a continuum of infinitely-lived individuals, $i \in [0, 1]$. There is a single good, produced with capital and labor. Individuals buy consumption, c and invest in capital, k , depreciating at rate $\delta \in (0, 1)$, and face an indivisible participation choice: labor market participation imposes a utility loss, $\xi \geq 0$. Conditional on participation, individuals may be employed or unemployed. If employed, they incur an additional utility cost $-\ln(1 - \underline{h}) > 0$, as they sacrifice $\underline{h} \in (0, 1)$ units of their endowment of time (with \underline{h} an exogenous parameter). Workers transition between three states: employment (e), unemployment (u), and non-participation (o). We denote labor market states by $\iota \in \mathcal{L} \equiv \{e, u, o\}$. Individuals have flow utility, $U(c) - \psi(\iota)$, with $U'(c) > 0$ and $U''(c) < 0$, and with $\psi(e) \equiv \xi - \ln(1 - \underline{h})$, $\psi(u) \equiv \xi$ and $\psi(o) \equiv 0$.

Competitive firms have (identical) constant returns to scale technology which turns labor and capital into output, $F(k, n)$, that satisfy standard Inada conditions and (k, n) denote the demand for capital and labor. Firms pay wages w to hire workers, r to rent capital, and maximize profits, $F(k, n) - wn - rk$.

The economy consists of three islands, which we refer to as employment island, unemployment island and leisure island. Individuals that were unemployed (non-participants) in the previous period, start at the beginning of date t in the unemployment (leisure) island. If they decide not to participate, they relocate (remain) to the leisure island; and, if they decide to participate, they relocate to the employment island with probability f , or they remain (relocate) to the unemployment island with probability $1 - f$. New jobs become immediately productive.

Individuals previously employed, start date t in the employment island. An existing job is destroyed with probability λ , and upon destruction, previously employed individuals are allowed to search for another job and remain to the employment island with probability

132 f . With probability $1 - \lambda$ the job is not destroyed and they continue with the existing
133 employment relationship.

134 Labor frictions restrict access to the employment island; however, conditional on access, the
135 labor market is competitive and wages reflect the marginal product of labor.⁶ Goods markets
136 are competitive, with capital moving freely across islands. There is no aggregate uncertainty,
137 but frictional labor markets generate idiosyncratic risk.

138 2.2 Institutions

139 We consider two institutional trading arrangements. In the first, as in [Andolfatto \(1996\)](#), at
140 the start of date t a game of musical chairs allocates individuals to different labor market
141 states $\iota \in \mathcal{L}$. Subsequently, individuals buy lotteries over labor force participation and
142 idiosyncratic shocks realise. Each period, insurance markets open before the realization
143 of musical chairs and lotteries, with contracts traded at actuarial fair prices. At the end
144 of each date, spots market open where individuals receive income, execute contracts, buy
145 consumption and invest.

146 In the second market structure (the sunspot economy), we assume markets open only once, at
147 date -1 . Individuals trade contracts contingent on “sunspot” activity and idiosyncratic risk.
148 Sunspots act as a coordination device much like the musical chairs and affect welfare because
149 of indivisibilities in labor supply choices. This structure yields a competitive equilibrium
150 with voluntary trade in contingent commodities, where sunspots coordinate actions among
151 individuals. We label the first model “musical chairs” and the second “sunspots”, and we
152 study each in turn.

⁶The assumption of competitive markets in coexistence with search frictions has a long tradition and follows, for example, [Lucas and Prescott \(1974\)](#), [Alvarez and Veracierto \(1999\)](#) and [Krusell et al. \(2008, 2010, 2011\)](#). This approach is fruitful because, as we show in Proposition 2, it yields a constrained efficient equilibrium despite the search frictions. However, having competitive factor prices is not essential for the success of our model to match labor market transitions rates. In fact, in the steady state equilibrium, factor prices are constant and, thus, assuming non-competitive factor prices would not alter our analysis.

2.2.1 Musical chairs

At the beginning of date t a game of musical chairs assigns individuals to a labor market state $\iota \in \mathcal{L} \in \{e, u, o\}$, with probability $\alpha_t(\iota)$. Figures 1a and 1b show the sequence of events conditional on the musical chairs randomisation. Specifically, individuals assigned to the employment island ($\iota = e$) observe the realisation of the idiosyncratic shock $\kappa \in \{e_d, e_{nd}\}$, where $\kappa = e_d$ denotes destruction (d) with probability λ , and $\kappa = e_{nd}$ denotes no destruction (nd) with $1 - \lambda$; subsequently they buy lotteries over labor force participation, and conditional on the lottery outcome engage (or not) in search activity. Individuals assigned to the unemployment or leisure island ($\iota \in \{u, o\}$) buy lotteries over labor force participation and then engage (or not) in search activity. We denote by $\tilde{\iota} \in \{e_d, e_{nd}, u, o\}$ the consolidated set of states prior to the participation lottery stage, by $j \in \mathcal{L} \in \{e, u, o\}$ the labor market state at the end of the period and by $\pi(\tilde{\iota})$ the lottery over labor force participation. The pair $(\tilde{\iota}, j)$ denotes the labor market transitions of individuals during each period. Individuals discount the future with $\beta \in (0, 1)$.

The Bellman equation characterising each individual's decision is

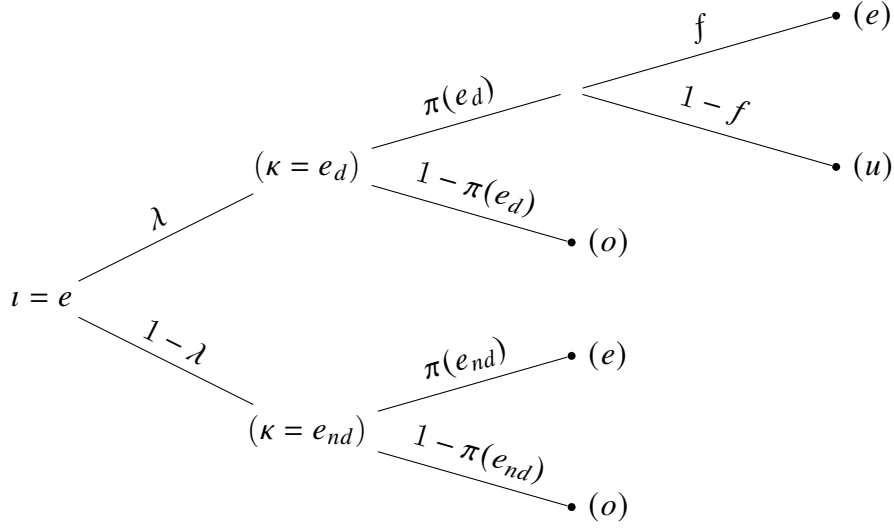
$$V_t(k_t, \bar{k}_t) = \max_{\{c, k, y, \pi\}} \left\{ \alpha_t(e) [(1 - \lambda_t)v_t(e_{nd}) + \lambda_t v_t(e_d)] + \alpha_t(u)v_t(u) + \alpha(o)v_t(o) \right\}$$

with

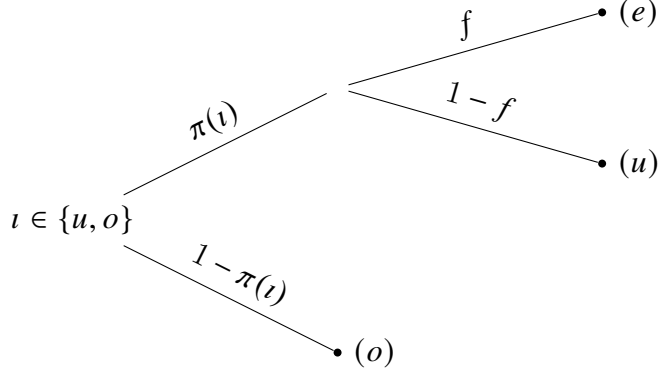
$$\begin{aligned} v_t(e_{nd}) &= \pi_t(e_{nd}) [U(c_t(e_{nd}, e)) - \psi(e) + \beta V_{t+1}(k_{t+1}(e_{nd}, e), \bar{k}_{t+1})] + \\ &\quad (1 - \pi_t(e_{nd})) [U(c_t(e_{nd}, o)) + \beta V_{t+1}(k_{t+1}(e_{nd}, o), \bar{k}_{t+1})], \end{aligned}$$

and, for $\tilde{\iota} \in \{e_d, u, o\}$,

$$\begin{aligned} v_t(\tilde{\iota}) &= \pi_t(\tilde{\iota}) f_t(U(c_t(\tilde{\iota}, e)) - \psi(e) + \beta V_{t+1}(k_{t+1}(\tilde{\iota}, e), \bar{k}_{t+1})) + \\ &\quad \pi_t(\tilde{\iota})(1 - f_t)(U(c_t(\tilde{\iota}, u)) - \psi(u) + \beta V_{t+1}(k_{t+1}(\tilde{\iota}, u), \bar{k}_{t+1})) + \\ &\quad (1 - \pi_t(\tilde{\iota}))(U(c_t(\tilde{\iota}, o)) + \beta V_{t+1}(k_{t+1}(\tilde{\iota}, o), \bar{k}_{t+1})), \end{aligned}$$



(a) Sequence of events conditional on $\iota = e$



(b) Sequence of events conditional on $\iota \in \{u, o\}$

Figure 1: Sequence of events conditional on musical chairs' randomisation

subject to the budget constraint for each pair (\tilde{i}, j) ,

$$c_t(\tilde{i}, j) + k_{t+1}(\tilde{i}, j) + \sum_{\tilde{i}} \sum_j q_t(\tilde{i}, j) y_t(\tilde{i}, j) = y_t(\tilde{i}, j) + (r_t + 1 - \delta) k_t + w_t h \mathbb{1}_j,$$

where $\mathbb{1}_j$ is an indicator function that is equal to unity if $j = e$ and zero otherwise. The relevant state space for individual optimisation consists of predetermined individual and aggregate capital stock, k and \bar{k} , respectively, and is independent of the previous period individual labor market state. At the end of date t , individuals buy consumption $c_t(\tilde{i}, j)$,

invest in capital stock $k_{t+1}(\tilde{i}, j)$, execute contracts $y_t(\tilde{i}, j)$ that are purchased at the beginning of date t (ex-ante) at price $q_t(\tilde{i}, j)$, receive capital income and, if employed, labor income. Actuarially fair insurance implies that marginal utilities of consumption $U_c[c(\tilde{i}, j)]$ are equalised across all labor market states, which implies that $c(\tilde{i}, j) = c$ for all pairs (\tilde{i}, j) . In turn, it follows that the marginal return of one additional unit of capital $V_k(k(\tilde{i}, j), \bar{k})$ is equalised across labor market states, which implies $k(\tilde{i}, j) = k$ for all pairs (\tilde{i}, j) . The individual's decision is consolidated as follows:

$$V_t(k_t, \bar{k}_t) = \max \left\{ U(c_t) - \alpha_t(e)(1 - \lambda_t)\pi_t(e_{nd})(\xi - \ln(1 - \underline{h})) - (\xi - f_t \ln(1 - \underline{h})) [\alpha_t(e)\lambda_t\pi_t(e_d) + \alpha_t(u)\pi_t(u) + \alpha(o)\pi_t(o)] + \beta V_{t+1}(k_{t+1}, \bar{k}_{t+1}) \right\} \quad (1)$$

subject to

$$\begin{aligned} c_t + k_{t+1} &= \\ \left[\alpha_t(e)\pi_t(e_{nd})(1 - \lambda_t) + \left(\alpha_t(e)\lambda_t\pi_t(e_d) + \alpha_t(u)\pi_t(u) + \alpha(o)\pi_t(o) \right) f_t \right] w_t \underline{h} + (r_t + 1 - \delta)k_t, \\ 0 &\leq \pi_t(\tilde{i}) \leq 1. \end{aligned} \quad (2)$$

This represents the decision of a stand-in agent who chooses consumption, investment and lotteries over participation to maximise (1) subject to (2).

Wages and rental prices earn their respective marginal products. Insurance markets are segmented, in the sense that there exist separate markets for each contingency (\tilde{i}, j) . Insurers in each market offer contracts $y(\tilde{i}, j)$ at actuarially fair prices $q(\tilde{i}, j)$, and free entry drives profits to zero.

Equilibrium is defined as follows:

Definition 1 (*Musical chairs*) A full insurance equilibrium is a price system $\{w, r, q\}$ and probability measures $\alpha(\iota)$ for $\iota \in \mathcal{L}$, a law of motion for aggregate capital \bar{k} , a collection of

188 individual choices $\{c, k, \pi, y\}$, an individual value function $V(k, \bar{k})$ and a collection of firm
 189 choices $\{k, n\}$ such that:

- 190 1. At given prices and $\alpha(\iota)$, $\{c, k, \pi, y\}$ and $V(k, \bar{k})$ solve the individual's problem;
- 191 2. At given prices, all firms maximise profits;
- 192 3. Good's market clears, $c + k_{+1} = f(k, n) + (1 - \delta)k$;
- 193 4. Capital market clears, $k = \bar{k}$;
5. Labor market clears,

$$n = \left[\alpha(e)\pi(e_{nd})(1 - \lambda) + \left(\alpha(e)\lambda\pi(e_d) + \alpha(u)\pi(u) + \alpha(o)\pi(o) \right) f \right] \underline{h};$$

- 194 6. $\alpha(\iota)$ is equal to the previous period measure of individuals in labor market state ι .

195 The following remarks are in order. First, we show that any interior equilibrium satisfies
 196 $\pi(e_{nd}) = 1$ (corner solution) and $\pi(\tilde{i}) \in (0, 1)$, $\tilde{i} \in \{e_d, u, o\}$ (see Appendix A for details).
 197 Second, probabilities $\alpha(\iota)$ are determined by the measures of individuals in state $\iota \in \{e, u, o\}$
 198 at the end of the previous period. Hence, although $\alpha(\iota)$ are taken as given by individuals,
 199 they are determined endogenously in equilibrium. Third, in Section 4 we offer a detailed
 200 characterisation of the equilibrium and discuss various comparative static exercises.

201 The hybrid model that we have analysed so far includes the musical chairs' framework
 202 of Andolfatto (1996) as a special case.

203 **Lemma 1** *If $\xi = 0$, then our framework reduces to the musical chairs' model of Andolfatto*
 204 *(1996).*

Proof. Suppose $\xi = 0$. Set

$$N_t = \alpha_t(e)\pi_t(e_{nd})(1 - \lambda_t) + \left(\alpha_t(e)\lambda_t\pi_t(e_d) + \alpha_t(u)\pi_t(u) + \alpha_t(o)\pi_t(o) \right) f_t,$$

205 with $N \equiv n/\underline{h}$ (see Section 4 for full details).

206 Then, (1)–(2), reduce to:

$$V_t(k_t, \bar{k}_t) = \max_{\{c, k\}} \left\{ U(c_t) + N_t \ln(1 - \underline{h}) + \beta V_{t+1}(k_{t+1}, \bar{k}_{t+1}) \right\}, \quad (3)$$

207 subject to

$$c_t + k_{t+1} = w_t N_t \underline{h} + (r_t + 1 - \delta)k_t. \quad (4)$$

208 This corresponds to Andolfatto’s model with N_t denoting the probability that an individual
 209 is allocated to employment and $1 - N_t$ the probability that is allocated to nonemployment. ■

210 This result requires that if the opportunity cost of participation is negligible, $\xi = 0$, then the
 211 randomisation devices prior and after the realisation of idiosyncratic shocks (see figures 1a
 212 nd 1b) reduce to the simple musical chair’s randomisation in Andolfatto (1996).

213 2.2.2 Sunspots

214 In this Section we abstract from sequential trading and assume Arrow-Debreu (AD) markets,
 215 with trade occurring at date -1 . Individuals trade contracts contingent on the publicly
 216 observed sunspot activity and idiosyncratic risk. Sunspot activity is constructed so that each
 217 period it induces a distribution of individuals across labor market states (islands) $\mathcal{L} = \{e, o, s\}$.
 218 We employ the distinction between ex-ante and ex-post public randomisations (sunspots)
 219 within a given period that is discussed in Kehoe et al. (2002).⁷ Figure 2 shows the sequence
 220 of events at date t . We denote ex-ante sunspot shocks with a superscript “0”, and ex-post
 221 shocks with a superscript “1”. At the beginning of date t individuals observe the ex-ante

⁷In their framework, only ex-post randomisations are important to overcome non-convexities arising from private information; and in fact, they show that the model with ex-ante and ex-post sunspots is equivalent in terms of allocations to the model with only ex-post sunspots. However, Cole (1989) showed that in a set-up with ex-ante sunspots and convex set of feasible allocations, the introduction of ex-post sunspots is still welfare improving because lotteries conditional on private information separate individuals with different risk profiles.

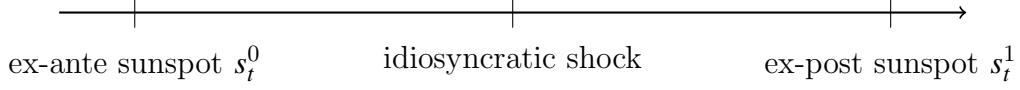


Figure 2: Sequence of events at date t

222 sunspot shock s_t^0 , subsequently the idiosyncratic shock is realised and at the end of the period
 223 the ex-post sunspot shock s_t^1 is realised and transactions take place. For example, individuals
 224 induced by the ex-ante sunspot to start date t in the employment island, observe if the job is
 225 destroyed or not and the ex-post sunspot realisation allocates them to a labor market state
 226 at the end of the period; similarly, individuals induced by the ex-ante sunspot to start date t
 227 in the unemployment island, find employment with probability f and remain unemployed
 228 with probability $1 - f$, and at the end of the period observe the realisation of the ex-post
 229 sunspot and execute all their obligations.

230 The distinction between ex-ante and ex-post sunspots is important. It serves the following
 231 purpose. Ex-ante public randomisations replicate the distribution of musical chairs, overcome
 232 non-convexities arising from indivisibilities in labor supply, and influence the distribution of
 233 idiosyncratic risk (see below); while ex-post randomisation separate those individuals whose
 234 pre-existing jobs have been destroyed and need to be assigned into a labor market state,
 235 from those individuals whose jobs have survived (see Figure 1a). Hence, in the absence of
 236 idiosyncratic risk arising from job destruction, the ex-post public randomisation device is
 237 irrelevant.

238 Time and the resolution of uncertainty are described by an event-tree, a countable set.
 239 Denote the history of ex-ante and ex-post sunspot realisations up and until date t by $s^{0t} =$
 240 $[s_0^0, s_1^0, \dots, s_t^0]$, $s^{1t} = [s_0^1, s_1^1, \dots, s_t^1]$, the joint history by $s^t = [(s_0^0, s_0^1), (s_1^0, s_1^1), \dots, (s_t^0, s_t^1)]$
 241 and the history of idiosyncratic shocks up and until date t by $\phi^t = [\phi_0, \phi_1, \dots, \phi_t]$. Let σ_t be
 242 the date-event consisting of ex-ante and ex-post sunspot realisations, s_t , and idiosyncratic
 243 shocks, ϕ_t , with history up to and including date t , $\sigma^t = [\sigma_1, \sigma_2, \dots, \sigma_t]$. We require the

244 probability distribution of ex-post shocks to have a continuous density. We assume that s_t^1
 245 is distributed uniformly on $[0, 1]$ and let $\mu_t^1(s^{0t}, \phi^t, s^{1t-1})$ be the measure of date t ex-post
 246 sunspots states conditional on history $\{s^{0t}, \phi^t, s^{1t-1}\}$. The probability distributions of ex-ante
 247 and idiosyncratic shocks are obtained by appropriate construction as we demonstrate below.
 248 Let the unconditional probability of s^{0t} be $\mu_t^0(s^{0t})$ and the probability of ϕ^t conditional on
 249 s^{0t} be $\gamma_t(\phi^t | s^{0t})$. Let $\mu_t(s^{0t}, \phi^t) = \gamma_t(\phi^t | s^{0t}) \mu_t^0(s^{0t})$. We assume that histories of ex-post
 250 shocks do not influence the distributions of ex-ante and idiosyncratic shocks.⁸ Individuals
 251 trade contingent claims against future events σ^t at price $p_t(\sigma^t)$ and firms buy inputs and
 252 sell output against s^t at $p_t(s^t)$. Prices $p_t(s^t)$ are derived from $p_t(\sigma^t)$ by summing over ϕ^t .
 253 The decision of an individual is

$$\max_{c,k} \sum_t \beta^t \sum_{\{s^{0t}, \phi^t\}} \mu_t(s^{0t}, \phi^t) \int_{s^{1t}} \left[U(c_t(\sigma^t)) - \psi(\sigma^t) \right] ds^{1t}, \quad (5)$$

254 subject to

$$\begin{aligned} \sum_t \sum_{\{s^{0t}, \phi^t\}} \int_{s^{1t}} p_t(\sigma^t) \left[c_t(\sigma^t) + k_{t+1}(\sigma^t) \right] ds^{1t} = \\ \sum_t \sum_{\{s^{0t}, \phi^t\}} \int_{s^{1t}} p_t(\sigma^t) \left[(r_t(s^t) + 1 - \delta) k_t(\sigma^{t-1}) + \underline{h} w_t(s^t) \mathbb{1}(\sigma^t) \right] ds^{1t}, \end{aligned} \quad (6)$$

255 where the indicator function $\mathbb{1}(\sigma^t)$ is equal to unity at date-events where individuals work
 256 and zero otherwise. We define multiple integrals $\int_{s^{1t}} \equiv \int_{s_0^1} \cdots \int_{s_t^1}$ up to and including date t
 257 and differentials $ds^{1t} \equiv ds_t^1 \cdots ds_0^1$ back to date zero.

258 Firms choose capital and labor to maximise profits:

$$\max_{k,n} \sum_t \sum_{s^{0t}} \int_{s^{1t}} p_t(s^t) \left[F(k_t(s^{t-1}), n_t(s^t)) - r_t(s^t) k_t(s^{t-1}) - w(s^t) n_t(s^t) \right] ds^{1t}. \quad (7)$$

259 Consider the following definition of a sunspot equilibrium.

⁸This assumption follows [Prescott and Townsend \(1984\)](#), who assume that histories of lottery outcomes do not influence the distribution of types across the population.

Definition 2 (*Sunspots*) A sunspot equilibrium is a price system $\{w, r, p\}$, a collection of individual choices $\{c, k\}$, and a collection of firm choices $\{k, n\}$ such that:

1. At given prices, $\{c, k\}$ solve the individual's problem (5)-(6);

2. At given prices, firms maximise profits (7);

3. Good's market clears,

$$\int \left(c_t(\sigma^t) + k_{t+1}(\sigma^t) - (1 - \delta)k_t(\sigma^{t-1}) \right) di = f \left(k_t(s^{t-1}), n_t(s^t) \right);$$

4. Capital market clears, $\int k_t(\sigma^t) di = k(s^t)$;

5. Labor market clears, $n_t(s^t) = \int \underline{h} \mathbb{1}(\sigma^t) di$.

3 Equivalence

The purpose of this Section is to demonstrate that the equilibrium allocations achieved by the sunspot economy and the musical chairs economy are equivalent. This equivalence result microfound the hybrid model of musical chairs, idiosyncratic risk and lotteries over labor force participation. Subsequently, we demonstrate that the sunspot allocation is constrained Pareto optimal.

To demonstrate equivalence of equilibria between the two economies, we proceed in two steps. First, we establish in Proposition 1 that the same equilibrium allocations obtained with musical chairs and lotteries can be implemented as sunspot-equilibrium allocations and, thus, the lottery equilibrium corresponds to an equilibrium with sunspots. Specifically, in the proof of Proposition 1 we present a detailed construction of the sunspot probability distribution, so that the sunspot economy achieves the same equilibrium allocation as the target lottery equilibrium allocation. Second, using well-known results in the literature (see Garratt et al., 2002; Kehoe et al., 2002), we establish that the converse of Proposition 1 is also true if the

280 sunspot state-space is sufficiently rich.

281 **Proposition 1** *An equilibrium with musical chairs and lotteries corresponds to a sunspot*
 282 *equilibrium.*

283 **Proof.** The proof is constructive. Suppose an equilibrium with musical chairs exists. Then,
 284 we construct an equilibrium with sunspots supporting the same allocations as the musical
 285 chairs equilibrium.

Consider the stand-in agent's problem in the musical chairs economy, given by (1)–(2),
 implying the first-order conditions

$$U_c(c_t) = \beta R_{t+1} U_c(c_{t+1}), \quad (8)$$

$$\xi - f_t \ln(1 - \underline{h}) = f_t w_t \underline{h} U_c(c_t), \quad (9)$$

$$\xi - \ln(1 - \underline{h}) < w_t \underline{h} U_c(c_t), \quad (10)$$

286 where $R_{t+1} \equiv r_{t+1} + 1 - \delta$. Condition (8) is the Euler equation and (9), (10) are optimality
 287 conditions with respect to $\pi(\tilde{i}) \in (0, 1)$, $\tilde{i} \in \{e_d, u, o\}$ and $\pi(e_{nd}) = 1$ (corner solution). Firm's
 288 optimality requires $r_t = F_k(k, n)$ and $w_t = F_n(k, n)$. We denote the equilibrium allocation
 289 under musical chairs with superscript “*”.

Next, we construct a sunspot equilibrium where agent decisions are identical to those in the
 musical chairs equilibrium. To that end, we set the wage rate and the return on capital in
 the sunspot equilibrium to be equal to (R_t^*, w_t^*) . Define AD prices as follows:

$$p_t(\sigma^t) \equiv \mu_t(s^{0t}, \phi^t) \times \left(\prod_{\tau=0}^t (R_\tau^*)^{-1} \right), \quad (11)$$

290 with $R_0 \equiv 1$. Define the investment portfolio x_{t+1} as follows:

$$x_{t+1} \equiv \sum_{\{s^{0t}, \phi^t\}} \mu_t(s^{0t}, \phi^t) \int_{s^{1t}} k_{t+1}(\sigma^t) d^{s^{1t}}, \quad (12)$$

291 The investment portfolio x_{t+1} , and not its composition, is the relevant choice variable. In
 292 particular, individuals buy x_{t+1} at price $\prod_{\tau=0}^t (R_\tau^*)^{-1}$, and receive return $\left[\prod_{\tau=0}^{t+1} (R_\tau^*)^{-1} \right] R_{t+1}^* x_{t+1}$.
 293 Individual optimality with respect to x_{t+1} is satisfied at prices given by (11). Moreover, (11)
 294 implies that individual marginal utilities are equal across date-events, implying $c(\sigma^t) = c_t$,
 295 for all histories σ^t . Finally, under the given price system, the decisions of all the agents in the
 296 economy are well defined since $\lim_{t \rightarrow \infty} \sum_t \left(\prod_{\tau=0}^t (R_\tau^*)^{-1} \right)$ converges (equivalently, the musical
 297 chairs equilibrium is dynamically efficient).

298 Thus, problem (5)–(6) reduces to

$$\max \sum_t \beta^t \left(U(c_t) - \sum_{\{s^{0t}, \phi^t\}} \mu_t(s^{0t}, \phi^t) \int_{s^{1t}} \psi(\sigma^t) ds^{1t} \right), \quad (13)$$

299 subject to

$$\sum_t \left(\prod_{\tau=0}^t (R_\tau^*)^{-1} \right) (c_t + x_{t+1}) = \sum_t \left(\prod_{\tau=0}^t (R_\tau^*)^{-1} \right) \left(R_t^* x_t + w_t^* \underline{h} \sum_{\{s^{0t}, \phi^t\}} \mu_t(s^{0t}, \phi^t) \int_{s^{1t}} \mathbb{1}(\sigma^t) ds^{1t} \right). \quad (14)$$

300 Optimality with respect to consumption between two consecutive dates yields (8), so that
 301 $c_t = c_t^*$ is consistent with optimality. Moreover, we set $x_{t+1} = k_{t+1}^*$. To complete the argument
 302 we need to show that optimal allocations satisfy conditions (9)–(10) as well. From the
 303 consolidated problem (13)–(14) we can observe that keeping track the histories of ex-post
 304 realisations s^{1t-1} is not relevant anymore. To that end, we construct the conditional measure
 305 of ex-post states $\mu_t^1(s^{0t}, \phi^t)$ —dropping histories s^{1t-1} —and the probability measure $\mu_t^0(s^{0t})$.
 306 Let, in turn, $S^1(s^{0t})$ denote the set of individuals who, following history s^{0t} , are allocated to
 307 a pre-existing job that is destroyed with probability λ and survives with probability $1 - \lambda$, as
 308 if starting from the employment island; $S^2(s^{0t})$ the set of individuals who purchase lottery
 309 profile yielding employment with probability f , and unemployed with probability $1 - f$, as
 310 if they started from the unemployment island; $S^3(s^{0t})$ the set of individuals who purchase

lottery profile yielding employment with probability f , and unemployed with probability $1 - f$, as if they started from the leisure island; and, $S^4(s^{0t})$ the set of individuals who choose not to participate upon observing s^{0t} .

Consider the following equilibrium conditions at history s^{0t} :

$$\begin{aligned} \int_{i \in S^1(s^{0t})} di &= \alpha_t^*(e), \\ \int_{i \in S^2(s^{0t})} di &= \alpha_t^*(u) \pi_t^*(u), \\ \int_{i \in S^3(s^{0t})} di &= \alpha_t^*(o) \pi_t^*(o), \end{aligned} \tag{15}$$

and for each individual i

$$\begin{aligned} \sum_{s^{0t}: i \in S^1(s^{0t})} \mu_t(s^{0t}) &= \alpha_t^*(e), \\ \sum_{s^{0t}: i \in S^2(s^{0t})} \mu_t(s^{0t}) &= \alpha_t^*(u) \pi_t^*(u), \\ \sum_{s^{0t}: i \in S^3(s^{0t})} \mu_t(s^{0t}) &= \alpha_t^*(o) \pi_t^*(o), \end{aligned} \tag{16}$$

where the pair (α^*, π^*) denotes the musical chairs' and participation probability measures evaluated at the musical chairs equilibrium. Conditions (15) are equilibrium conditions so that the measure of individuals at history node s^{0t} who face the prospect of job destruction or purchase each lottery profile after the sunspot realisation, is equal to the corresponding measure in the musical chairs equilibrium. Conditions (16) are consistency conditions so that the measures across history nodes where each individual faces the prospect of job destruction or purchases each lottery profile is equal to the measure of individuals at each history node s^{0t} who faces job destruction or purchase each lottery profile. Finally, construction of set $S^4(s^{0t})$ follows residually.

325 Let, in turn, $Q^1(s_t^1|s^{0t}, \phi^t)$ denote the fraction of individuals, among the measure of individuals
 326 who start from the employment island with a job that is destroyed following history $\{s^{0t}, \phi^t\}$,
 327 who end up being employed at s_t^1 ; $Q^2(s_t^1|s^{0t}, \phi^t)$ denote the fraction of individuals, among
 328 the measure of individuals who start from the employment island with a pre-existing job that
 329 is destroyed, who end up being unemployed; $Q^3(s_t^1|s^{0t}, \phi^t)$ denote the fraction of individuals,
 330 among the measure of individuals who start from the employment island with a pre-existing
 331 job that is destroyed, who end up out of the labor force; $Q^4(s_t^1|s^{0t}, \phi^t)$ the fraction of
 332 individuals, among the measure of individuals who start from the employment island with a
 333 job that is not destroyed, who end up employed; and $Q^5(s_t^1|s^{0t}, \phi^t)$ the fraction of individuals,
 334 among the measure of individuals who start from the employment island with a job that is
 335 not destroyed, who end up out of the labor force.

336 Consider the following equilibrium conditions:

$$\begin{aligned}
 \int_{i \in Q^1(s_t^1|s^{0t}, \phi^t)} di &= \pi_t^*(e_d) f_t, & \int_{i \in Q^4(s_t^1|s^{0t}, \phi^t)} di &= \pi_t^*(e_{nd}) \\
 \int_{i \in Q^2(s_t^1|s^{0t}, \phi^t)} di &= \pi_t^*(e_d) (1 - f_t), & \int_{i \in Q^5(s_t^1|s^{0t}, \phi^t)} di &= 1 - \pi_t^*(e_{nd}) \\
 \int_{i \in Q^3(s_t^1|s^{0t}, \phi^t)} di &= 1 - \pi_t^*(e_d), & &
 \end{aligned} \tag{17}$$

337 and for each individual i

$$\begin{aligned}
 \mu_t^1(s^{0t}, \phi^t) &= \pi_t^*(e_d) f_t, & \text{for each } i &\in Q^1(s_t^1|s^{0t}, \phi^t) \\
 \mu_t^1(s^{0t}, \phi^t) &= \pi_t^*(e_d) (1 - f_t), & \text{for each } i &\in Q^2(s_t^1|s^{0t}, \phi^t) \\
 \mu_t^1(s^{0t}, \phi^t) &= 1 - \pi_t^*(e_d), & \text{for each } i &\in Q^3(s_t^1|s^{0t}, \phi^t) \\
 \mu_t^1(s^{0t}, \phi^t) &= \pi_t^*(e_{nd}), & \text{for each } i &\in Q^4(s_t^1|s^{0t}, \phi^t) \\
 \mu_t^1(s^{0t}, \phi^t) &= 1 - \pi_t^*(e_{nd}), & \text{for each } i &\in Q^5(s_t^1|s^{0t}, \phi^t)
 \end{aligned} \tag{18}$$

338 where as before π^* denotes participation probability measures evaluated at the musical

339 chairs' equilibrium. Conditions (17)–(18) are equilibrium and consistency conditions similar
 340 to (15)–(16). Finally, for individuals not belonging to the set $S^1(s^{0t})$, the realisation of
 341 ex-post sunspots are irrelevant, so that the conditional measure of ex-post states is degenerate
 342 and equal to $\mu_t^1(s^{0t}, \phi^t) = 1$.

343 We require that idiosyncratic shocks and public signals are not independent events so that
 344 $\gamma_t(\phi^t | s^{0t})$ depends on histories s^{0t} . In particular, we construct a dependence structure between
 345 shocks and signals consistent with summations over histories $\{s^{0t}, \phi^t\}$ in (13)–(14) which
 346 yields the problem

$$\begin{aligned} \max \sum_t \beta^t & \left[U(c_t) - \alpha_t^*(e)(1 - \lambda_t)\pi_t(e_{nd})(\xi - \ln(1 - \underline{h})) - \right. \\ & \left. (\xi - f_t \ln(1 - \underline{h})) [\alpha_t^*(e)\lambda_t\pi_t(e_d) + \alpha_t^*(u)\pi_t(u) + \alpha_t^*(o)\pi_t(o)] \right] \end{aligned} \quad (19)$$

347 subject to

$$\begin{aligned} \sum_t \left(\prod_t (R_t^*)^{-1} \right) (c_t + x_{t+1}) &= \sum_t \left(\prod_t (R_t^*)^{-1} \right) R_t^* x_t + \\ \sum_t \left(\prod_t (R_t^*)^{-1} \right) & \left[\alpha_t^*(e)\pi_t(e_{nd})(1 - \lambda_t) + \left(\alpha_t^*(e)\lambda_t\pi_t(e_d) + \alpha_t^*(u)\pi_t(u) + \alpha_t^*(o)\pi_t(o) \right) f_t \right] w_t^* \underline{h}. \end{aligned} \quad (20)$$

348 Optimality with respect to consumption and probability measures π satisfy (8)–(10). Thus,
 349 $c_t = c_t^*$, $x_{t+1} = x_{t+1}^*$, $\pi_t(e_d) = \pi_t^*(e_d)$, $\pi_t(e_{nd}) = \pi_t^*(e_{nd})$, $\pi_t(u) = \pi_t^*(u)$, $\pi_t(o) = \pi_t^*(o)$ satisfy
 350 optimality. Feasibility at the given prices follows by multiplying (2) with $\prod_{\tau=0}^t (R_\tau^*)^{-1}$, and
 351 adding across time to obtain (20). Finally, this allocation is consistent with firm's optimality
 352 and market clearing conditions. ■

353 The construction of the proof in Proposition 1 establishes that for any lottery-equilibrium,
 354 there is an associated sunspot-equilibrium. The proof is based on the property that sunspot
 355 prices are collinear with probabilities.⁹ Moreover, we have assumed that the sunspot space is

⁹Garratt et al. (2002) call these prices constant probability adjusted prices. Furthermore, they show that in economies with complete markets all sunspot allocations can be supported by price functions that are collinear with probabilities if the sunspot variable is continuous.

rich, allowing even for continuous ex-ante sunspot variables, so that the stochastic allocations induced by the coordination on the sunspot mimics the set of gambles available to an individual in the lottery economy.¹⁰ Taken together, these two points imply that the converse of Proposition 1 is also true, completing our equivalence result. Below we elaborate on this, building on results in Garratt et al. (2002) and Kehoe et al. (2002).

In Proposition 1, we start with a target musical chairs equilibrium allocation, and show that it can be implemented as a sunspot equilibrium allocation, unique up to a relabelling of states, by using the construction (15)–(18). Conversely, a sunspot equilibrium allocation with prices collinear to probabilities as in expression (11) (which Garratt et al., 2002, call probability adjusted constant prices), corresponds to a musical chairs allocation with lotteries being pinned down by expressions (16) and (18). By construction, this candidate allocation is feasible in the musical chairs economy, yields the same factor prices, and provides the same utility level as the sunspot equilibrium allocation.¹¹ To complete the argument, we show that it is an equilibrium in the musical chairs economy. To that end, first, we show that any alternative lottery allocation that yields higher utility is not affordable; and second, that the candidate musical chairs allocation is affordable. The proof of the first part follows directly from the proof of Theorem 6.2 in Kehoe et al. (2002). A sketch of the argument is as follows. Suppose there exists an alternative lottery allocation that yields higher utility and is affordable. Then, from Proposition 1 we can use this alternative allocation to construct a sunspot allocation that is affordable at the given sunspot equilibrium prices yielding the same utility as the target allocation; hence, we arrive at our desired contradiction. Finally, the candidate allocation is affordable, since it induces the same factor prices as in the sunspot economy and, hence, satisfies budget constraints.

¹⁰Garratt (1995) shows that for any lottery equilibrium there is an associated sunspot equilibrium, but the converse is not necessarily true. The equivalence fails when the sunspot variable is restricted. He provided an example where a sunspot equilibrium exists when trade is restricted to three equiprobable states, but the same allocation is not an equilibrium in the lottery economy.

¹¹In the terminology of Kehoe et al. (2002) these two allocations are equivalent.

Proposition 1 has welfare implications. The sunspot allocation is Pareto efficient, given labor market frictions, if there is no alternative feasible allocation in which almost all households have no less utility and a positive measure of households have strictly more utility.

The following result applies.

Proposition 2 *The sunspot equilibrium allocation is Pareto efficient.*

Proof. Proposition 2 is a direct consequence of non-satiation of utility, and the first welfare theorem. To see this, consider (13)–(14) and rewrite it as an AD equilibrium under certainty so that the first welfare theorem applies. To this end, consider the following definitions:

$$\psi_t = \sum_{\{s^{0t}, \phi^t\}} \mu_t(s^{0t}, \phi^t) \int_{s^{1t}} \psi(\sigma^t) ds^{1t}, \quad h_t = \underline{h} \sum_{\{s^{0t}, \phi^t\}} \mu_t(s^{0t}, \phi^t) \int_{s^{1t}} \mathbb{1}(\sigma^t) ds^{1t}, \quad p_t = \prod_{\tau=0}^t (R_\tau^*)^{-1}. \quad (21)$$

Problem (13)–(14) modify as follows

$$\max \sum_t \beta^t [U(c_t) - \psi_t], \quad (22)$$

subject to

$$\sum_t p_t (c_t + x_{t+1}) = \sum_t p_t (R_t^* x_t + w_t^* h_t), \quad (23)$$

where ψ_t denotes the time-varying disutility cost at date t ; h_t denote the time-varying endowment of time at date t ; and, p_t denotes AD prices with $\sum_t p_t < \infty$. This is equivalent to the neoclassical growth model with time-varying endowments and preferences, so that the first welfare theorem applies. ■

4 Steady state analysis

In this Section we restrict attention to the steady state of the model and discuss comparative statics. We assume $U(c) = \ln(c)$ and $F(k, n) = k^\theta n^{1-\theta}$ with $0 < \theta < 1$. The system of

equilibrium conditions consists of two blocks. The first block includes conditions (8)–(10) and market clearing conditions. The second block consists of motion equations for the aggregate labor market variables, as follows

$$n_t/\underline{h} \equiv N_t = (1 - u_t)\Pi_t, \quad (24)$$

$$N_t = \pi_t(e_{nd})(1 - \lambda_t)N_{t-1} + H_t f_t, \quad (25)$$

$$\Pi_t = \pi_t(e_{nd})(1 - \lambda_t)N_{t-1} + H_t, \quad (26)$$

$$H_t = \pi_t(u)U_{t-1} + \pi_t(o)O_{t-1} + \pi_t(e_d)\lambda_t N_{t-1}, \quad (27)$$

where N_t , U_t and O_t , denote measures of individuals, in turn, in the employment island, the unemployment island, and the leisure island (non-participants), at the end of date t ; $\Pi_t \equiv U_t + N_t$, is the labor force measure, H_t denotes the measure of individuals searching for jobs, and $u_t \equiv U_t/\Pi_t$ is the unemployment rate.

The equilibrium is described by the following two systems of equations

$$\left\{ \begin{array}{l} c_t^{-1} = \beta R_{t+1} c_{t+1}^{-1}, \\ \xi/f_t - \ln(1 - \underline{h}) = w_t \underline{h} c_t^{-1}, \\ c_t + k_{t+1} = k_t^\theta (\underline{h} N_t)^{1-\theta} + (1 - \delta)k_t, \\ w_t = (1 - \theta) \left(\frac{k_t}{\underline{h} N_t} \right)^\theta, \\ R_{t+1} = 1 - \delta + \theta \left(\frac{\underline{h} N_t}{k_t} \right)^\theta, \end{array} \right. \quad (28)$$

$$\left\{ \begin{array}{l} N_t = (1 - u_t)\Pi_t, \\ N_t = (1 - \lambda_t)N_{t-1} + H_t f_t, \\ \Pi_t = (1 - \lambda_t)N_{t-1} + H_t, \\ H_t = \pi_t(u)U_{t-1} + \pi_t(o)O_{t-1} + \pi_t(e_d)\lambda_t N_{t-1}. \end{array} \right. \quad (29)$$

System (28) corresponds to the neoclassical growth model, with an endogenous labor market

399 wedge in the second equation of the system, given by

$$\begin{aligned}
 w_t \underline{h} &= \text{labor wedge} \times \text{MRS} \\
 &= \left[\frac{(\xi/f_t) - \ln(1 - \underline{h})}{-\ln(1 - \underline{h})} \right] \left[\frac{-\ln(1 - \underline{h})}{1/c_t} \right], \tag{30}
 \end{aligned}$$

400 where $\text{MRS} = -\ln(1 - \underline{h}) c_t$, corresponds to the marginal rate of substitution between leisure
 401 and consumption in the absence of an opportunity cost of participation. The labor wedge is
 402 an outcome of search frictions, because the opportunity cost of participation is different from
 403 zero.¹²

404 It follows from system (29) that the composition of H is indeterminate (see [Ljungqvist and](#)
 405 [Sargent, 2008](#), for a related result). In the sunspot equilibrium, an equilibrium composition for
 406 H is selected through sunspots. Specifically, any restriction on parameters $[\pi(u), \pi(o), \pi(e_d)]$,
 407 maps into a sunspot equilibrium via conditions (16) and (18).

408 Next, we focus on the steady state of (28) and (29), and study how changes in search frictions
 409 affect the equilibrium level of employment, unemployment and participation. Moreover, using
 410 the same example, we examine the ability of the model to match gross worker flows, since an
 411 advantage of our model is that it identifies individual labor market transitions (in contrast to
 412 the [Merz, 1995](#), large family model). Finally, we look at dynamics away from the steady state

¹²Our analysis follows much of the literature (for example, [Chang and Kim, 2007](#); [Shimer, 2013](#); [Krusell et al., 2011, 2017](#)) by only considering the extensive margin of adjustment in hours (hence, \bar{h} is held constant). As we are considering steady state gross flows for a given labor market, relaxing this assumption would have no implications for the analysis of the competitive equilibrium that follows and, in particular, the results reported in Table 1. However, allowing for an intensive and an extensive margin is interesting if one wants to compare outcomes across economies with different sets of labor market policies. For example, different countries with different institutions (including unemployment insurance, employment protection and other tax and transfer policies) might have a different split of total hours between the intensive and extensive margins, and this would affect gross worker flows too. [Fang and Rogerson \(2009\)](#) offer a detailed treatment of how such policies may affect the split between employment and hours per worker across countries with different labor market institutions, in a canonical labor supply model with search frictions. If one extends our analysis beyond steady state comparisons, allowing for cyclical fluctuations in the amount of hours per worker is also interesting because it implies fluctuations in the opportunity cost of employment. In an influential paper, [Chodorow-Reich and Karabarbounis \(2016\)](#) explore this channel and show empirically that procyclical hours per worker contribute to making the opportunity cost of employment procyclical.

equilibrium by changing the job finding rate to mimic a typical recession and characterise the transition back to the steady state.

4.1 Search frictions and aggregate participation

The steady state of (28) and (29) is presented in Appendix B. In particular, the steady state labor market allocations are determined by the cost of participation, ξ , and parameters describing the labor market frictions, (λ, f) , the Ins and Outs of unemployment. Following Krusell et al. (2010, 2011, 2017), we analyse how a reduction in the job-finding rate affects steady state labor market outcomes.

We establish the following Proposition:

Proposition 3 *A fall in the job finding rate f has the following impact on steady state labor market outcomes:*

1. *lowers aggregate employment, N ;*
2. *raises the unemployment rate, u ;*
3. *has an ambiguous effect on the labor force participation, Π .*

The proof of Proposition 3 is in Appendix B.

The reduction in the job finding rate lowers aggregate employment through the increase in the labor wedge, which from (30) implies that the MRS must fall (since the real wage is pinned down by technology and preferences, and is not affected by search frictions in steady state). Thus, consumption must fall, requiring a lower level of capital and employment.

The unemployment rate must increase since the steady state unemployment rate is determined only by the balancing between the inflow rate into unemployment and the outflow rate. As the inflow rate is constant, determined by the destruction rate λ , a reduction in the outflow rate, determined by f , must raise the unemployment rate in steady state.

The ambiguous effect on aggregate participation lead us to the following proposition.

Proposition 4 *There exists a threshold level of ξ , denoted $\widehat{\xi}$, such that at $\xi > \widehat{\xi}$ an increase in the finding rate, f , raises participation, and at $\xi < \widehat{\xi}$ an increase in f lowers participation. At $\xi = \widehat{\xi}$, participation is acyclical. The threshold level is equal to*

$$\widehat{\xi} = - \left[\frac{\lambda \ln(1 - \underline{h})}{1 - \lambda} \right]. \quad (31)$$

The proof of Proposition 4 is in the Appendix B.

A (permanent) change in the finding rate, f , affects the aggregate level of participation via three channels. The first channel is through returns to market work via the “effective” real wage rate, wf (the substitution effect). The second channel is through the opportunity cost of employment, the left hand side of the intratemporal condition in (28), so that increases in the finding rate increase the opportunity cost, which, in turn, discourages participation in the labor market. The net substitution effect, taking into account the opportunity cost channel, affects labor supply decisions via the labor wedge in expression (30). The third channel is the income effect from a permanent change in the finding rate, which affects the budget set of the stand-in agent through the effective real wage.

At $\xi = \widehat{\xi}$, the net substitution and income effects cancel out and aggregate participation is acyclical; while at $\xi > \widehat{\xi}$, the net substitution dominates and participation is procyclical, and at $\xi < \widehat{\xi}$, the income effect dominates and participation is countercyclical.

It follows from Proposition 4 that the model can deliver both a countercyclical or a procyclical participation rate, depending on the elasticity of the labor wedge to changes in f , controlled by the parameter ξ , the opportunity cost of participation. Thus, the neoclassical growth model with search frictions can be reconciled with a mildly procyclical participation rate. In turn, this result is particularly important given the tendency for models featuring intertemporal substitution in frictional labor markets to deliver excessively procyclical participation and,

thus, procyclical unemployment (a problem stressed by Ravn, 2008; Shimer, 2013, for example).

4.2 Gross worker flows

Unlike the Merz (1995) large family set-up which only identifies net worker flows, our model yields equilibrium outcomes for gross worker flows. To illustrate this point, we present a simple quantitative exercise to evaluate the model's ability to explain the average gross flows in the data. In particular, despite its parsimony the model is able to account well for the transitions between unemployment and inactivity, which previous literature has shown to be challenging.

The model yields gross flows across the three states of employment, unemployment, and non-participation, resulting from individual optimal behaviour, given by

$$\begin{aligned} \phi_{ee,t} &= (1 - \lambda_t) + \pi_t(e_d)\lambda_t f_t, & \phi_{ue,t} &= \pi_t(u) f_t, & \phi_{oe,t} &= \pi_t(o) f_t, \\ \phi_{eu,t} &= \pi_t(e_d)\lambda_t (1 - f_t), & \phi_{uu,t} &= \pi_t(u) (1 - f_t), & \phi_{ou,t} &= \pi_t(o) (1 - f_t), \\ \phi_{eo,t} &= \lambda_t (1 - \pi_t(e_d)), & \phi_{uo,t} &= 1 - \pi_t(u), & \phi_{oo,t} &= 1 - \pi_t(o), \end{aligned} \quad (32)$$

with $\phi_{ss',t}$ the transition rate from state s to state s' , for $s, s' \in \{e, u, o\}$, in period t , implied by the labor market parameters, f and λ and the randomisation induced by the optimal choices for the lotteries over labor force participation. The latter may induce in equilibrium different participation probabilities chosen by the individuals starting in employment but who lose their jobs, those unemployment and those out of the labor force, in turn, $\pi_t(e_d)$, $\pi_t(u)$ and $\pi_t(o)$. We argue below that this feature is important for the success of the model to match the gross worker flows across the unemployment and non-participation states.

In the sequel we focus on steady state transition probabilities. For the US, we measure gross worker flows empirically from the longitudinal monthly Current Population Survey (CPS), as explained for example in Elsby et al. (2015) and Krusell et al. (2017). We test the

Table 1: Gross worker flows (model and data)

$\phi_{s,s'}$	to s' :		
	e	u	o
from s :			
e	0.977 (0.972)	0.017 (0.014)	0.006 (0.014)
u	0.229 (0.228)	0.637 (0.637)	0.134 (0.135)
o	0.010 (0.022)	0.027 (0.021)	0.963 (0.957)
<u>calibrated values</u>			
λ	0.0290		
f	0.2645		
$\pi(o)$	0.0373		
$\pi(e_d)$	0.7848		
$\pi(u)$	0.8660		

Notes: In the first panel, values outside parenthesis are obtained from the model and the values in parenthesis are the empirical counterpart, obtained from [Krusell et al. \(2017\)](#), and used as targets. The lower panel reports the calibrated value for each parameter.

model's ability to explain labor market transitions with a simple calibration experiment. We select values for the parameters determining labor market transitions, $[\lambda, f, \pi(o), \pi(e_d)]$, to minimise a distance criterion function of the deviations of the gross transitions from their empirical counterparts, given the equilibrium conditions (29), and for an employment rate set to $N = 65\%$.

Table 1 compares the gross flows implied by the calibrated example economy to their empirical counterparts (as reported in [Krusell et al., 2017](#), based on the CPS longitudinal micro data), and reports the implied calibrated values for the vector vector of parameters. Despite the parsimonious set of parameters to match nine targets, the model does a relatively good job at matching gross flows, comparable to the results in [Krusell et al. \(2017\)](#), who develop a richer incomplete market model with heterogeneous agents. The model is particularly successful at

matching the high transition rate from unemployment to inactivity, ϕ_{uo} , which the literature has found challenging.¹³

Key to the success of the model to match the transition from unemployment to inactivity is the indeterminacy in the composition of the stock of job searchers, H . This indeterminacy is resolved by the sunspot mechanism which yields different participation probabilities chosen by the individuals starting in employment but who lose their jobs, those unemployment and those out of the labor force, in turn, $\pi_t(e_d)$, $\pi_t(u)$ and $\pi_t(o)$. From equation (32) we see that $\phi_{uo} = 1 - \pi_t(u)$, the transition rate from unemployment to inactivity is entirely determined by $\pi_t(u)$. Thus, it is possible to construct a sunspot equilibrium from (15) and (16), to match successfully the ϕ_{uo} transition rate.

5 Conclusion

This paper shows that the same aggregation as in Andolfatto (1996) can be obtained without either lotteries or additional exogenous randomization (the game of musical chairs), when individual choices over contingent commodities are coordinated by sunspots. We show that this aggregation approach offers a tractable method to construct a general equilibrium model of gross worker flows. The upshot is that the economy with sunspots yields testable predictions about gross workers flows, which may be confronted with micro level data on labor market transitions.

Although lotteries are socially optimal in economies with indivisibilities, they have been repudiated by some as an employment allocation mechanism, on the grounds that such ideal device is not empirically plausible (Browning et al., 1999; Ljungqvist and Sargent, 2011).

¹³Garibaldi and Wasmer (2005), in a model with linear utility, and Krusell et al. (2011), in a model with concave utility and incomplete markets, both show that the transition rates from unemployment to inactivity are difficult to account for in three-state equilibrium models of the labor market, without additional heterogeneity across individuals to achieve the calibration target. Garibaldi and Wasmer (2005) experiment with permanent heterogeneity across workers, while Krusell et al. (2011) consider transitory productivity shocks to match the transition from unemployment to inactivity.

512 Previous work by [Shell and Wright \(1993\)](#), [Garratt et al. \(2002\)](#) and [Kehoe et al. \(2002\)](#)
513 shows how to avoid the need for such implausible randomization mechanisms, by establishing
514 the close connection between lottery economies and sunspot economies. We extend their
515 approach to accommodate economies with labor market search frictions. The resulting model
516 can obtain plausible individual employment histories, as illustrated by the empirically realistic
517 gross worker flows in a calibrated example.

518 Turning to future work, the fact that an equilibrium with musical chairs can be decentralized
519 with sunspots, opens the possibility to study adverse selection and moral hazard in labor
520 markets with search frictions, using results for sunspot equilibria in incentive constrained
521 economies ([Kehoe et al., 2002](#)).

Appendix

A Lottery equilibrium

Proposition 5 *An equilibrium with musical chairs and lotteries is characterised by $\pi_t(\tilde{i}) \in (0, 1)$, for $\tilde{i} \in \{e_d, u, o\}$, and $\pi_t(e_{nd}) = 1$.*

Proof. Let us argue by contradiction. Suppose $\pi_t(e_{nd}) \in [0, 1)$ and $\pi_t(\tilde{i}) \in [0, 1]$. Then, $\pi_t(e_{nd}) \in [0, 1)$ requires (10) to modify as follows:

$$\xi - \ln(1 - \underline{h}) \geq w_t \underline{h} U_c(c_t) \quad (\text{A.1})$$

Multiplying both sides of (A.1) by f_t , yields

$$\xi - f_t \ln(1 - \underline{h}) > f_t \xi - f_t \ln(1 - \underline{h}) \geq f_t w_t \underline{h} U_c(c_t), \quad (\text{A.2})$$

which in turn, requires $\pi_t(\tilde{i}) = 0$, for $\tilde{i} \in \{e_d, u, o\}$. Subsequently, the employment law of motion in (25) requires $H = 0$ and imposing the steady state restriction, it follows that $\pi(e_{nd}) = 1/(1 - \lambda) > 1$, which is a contradiction. ■

B Steady state and comparative statics

In this Section we compute the steady state allocation and the comparative statics for the example economy presented in Section 4.

The steady state of the first block, system (28), after imposing steady state, reduces to

$$\frac{y}{k} = \left(\frac{1/\beta - 1 + \delta}{\theta} \right) \quad (\text{B.1})$$

$$\frac{n}{k} = \left(\frac{1/\beta - 1 + \delta}{\theta} \right)^{1/(1-\theta)}, \quad (\text{B.2})$$

$$\frac{c}{k} = \left(\frac{1/\beta - 1 + \delta}{\theta} \right) - \delta \quad (\text{B.3})$$

$$n = \frac{f}{\xi - f \ln(1 - \underline{h})} \frac{1 - \theta}{1 - \delta(k/y)}. \quad (\text{B.4})$$

In turn, the steady state of the second block yields

$$H = \frac{\lambda}{\xi - f \ln(1 - \underline{h})} \frac{1 - \theta}{1 - \delta(k/y)} \frac{1}{\underline{h}}, \quad (\text{B.5})$$

$$\Pi = \left(\frac{\lambda}{f} + 1 - \lambda \right) \frac{n}{\underline{h}}, \quad (\text{B.6})$$

$$O = 1 - \Pi, \quad (\text{B.7})$$

$$U = \Pi - \frac{n}{\underline{h}}, \quad (\text{B.8})$$

$$u = \frac{\lambda(1 - f)}{\lambda + f(1 - \lambda)}. \quad (\text{B.9})$$

It follows from (B.4), (B.6), (B.9) that the elasticity of employment, unemployment rate and participation, respectively, with respect to f is equal to

$$\epsilon_{N,f} = \frac{\xi}{\xi - f \ln(1 - \underline{h})} > 0, \quad (\text{B.10})$$

$$\epsilon_{u,f} = - \frac{1}{1 - \lambda + \lambda/f} \frac{1}{1 - f} < 0, \quad (\text{B.11})$$

$$\epsilon_{\Pi,f} = \frac{\xi}{\xi - f \ln(1 - \underline{h})} - \frac{\lambda/f}{\lambda/f + 1 - \lambda}, \quad (\text{B.12})$$

532 where $\epsilon_{X,Y} \equiv (dX/dY)(Y/X)$ denotes the elasticity of X with respect to Y . The result below
533 follows directly from (B.12).

Corollary 1 *There exist $\widehat{\xi}$ such that $\epsilon_{\Pi,f} = 0$. For $\xi > \widehat{\xi}$ it follows that $\epsilon_{\Pi,f} > 0$ whereas for*

$\xi < \widehat{\xi}$ it follows that $\epsilon_{\Pi, f} < 0$. The threshold level is equal to

$$\widehat{\xi} = - \left\lfloor \frac{\lambda \ln(1 - \underline{h})}{1 - \lambda} \right\rfloor. \quad (\text{B.13})$$

References

- Alvarez, F. and M. Veracierto (1999). Labor-market policies in an equilibrium search model. *NBER macroeconomics annual* 14, 265–304.
- Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review* 86(1), 112–132.
- Barnichon, R. and A. Figura (2015). Labor market heterogeneity and the aggregate matching function. *American Economic Journal: Macroeconomics* 7(4), 222–49.
- Browning, M., L. P. Hansen, and J. J. Heckman (1999). Micro data and general equilibrium models. *Handbook of macroeconomics* 1, 543–633.
- Chang, Y. and S.-B. Kim (2006). From individual to aggregate labor supply: A quantitative analysis based on a heterogeneous agent macroeconomy. *International Economic Review* 47(1), 1–27.
- Chang, Y. and S.-B. Kim (2007). Heterogeneity and aggregation: Implications for labor-market fluctuations. *American Economic Review* 97(5), 1939–1956.
- Chodorow-Reich, G. and L. Karabarbounis (2016). The cyclicity of the opportunity cost of employment. *Journal of Political Economy* 124(6), 1563–1618.
- Christiano, L. J., M. S. Eichenbaum, and M. Trabandt (2020). Why is unemployment so countercyclical? Technical report, National Bureau of Economic Research.
- Cole, H. L. (1989). General Competitive Analysis in an Economy with Private Information: Comment. *International Economic Review* 30(1), 249–252.
- Elsby, M. W. L., B. Hobijn, and A. Şahin (2015). On the importance of the participation margin for labor market fluctuations. *Journal of Monetary Economics* 72, 64–82.

- 556 Fang, L. and R. Rogerson (2009). Policy analysis in a matching model with intensive and
557 extensive margins. *International Economic Review* 50(4), 1153–1168.
- 558 Garibaldi, P. and E. Wasmer (2005). Equilibrium search unemployment, endogenous par-
559 ticipation, and labor market flows. *Journal of the European Economic Association* 3(4),
560 851–882.
- 561 Garratt, R. (1995). Decentralizing lottery allocations in markets with indivisible commodities.
562 *Economic Theory* 5(2), 295–313.
- 563 Garratt, R., T. Keister, C.-Z. Qin, and K. Shell (2002). Equilibrium prices when the sunspot
564 variable is continuous. *Journal of Economic Theory* 107(1), 11–38.
- 565 Garratt, R., T. Keister, and K. Shell (2004). Comparing sunspot equilibrium and lottery
566 equilibrium allocations: the finite case. *International Economic Review* 45(2), 351–386.
- 567 Hansen, G. D. (1985). Indivisible labor and the business cycle. *Journal of Monetary*
568 *Economics* 16(3), 309–327.
- 569 Kehoe, T. J., D. K. Levine, and E. C. Prescott (2002). Lotteries, sunspots, and incentive
570 constraints. *Journal of Economic Theory* 107(1), 39–69.
- 571 Kroft, K., F. Lange, M. J. Notowidigdo, and L. F. Katz (2016). Long-term unemployment and
572 the great recession: the role of composition, duration dependence, and nonparticipation.
573 *Journal of Labor Economics* 34(S1), S7–S54.
- 574 Krusell, P., T. Mukoyama, R. Rogerson, and A. Şahin (2008). Aggregate implications of
575 indivisible labor, incomplete markets, and labor market frictions. *Journal of Monetary*
576 *Economics* 55(5), 961–979.
- 577 Krusell, P., T. Mukoyama, R. Rogerson, and A. Şahin (2010). Aggregate labor market
578 outcomes: the roles of choice and chance. *Quantitative Economics* 1(1), 97–127.

- 579 Krusell, P., T. Mukoyama, R. Rogerson, and A. Sahin (2011). A three state model of worker
580 flows in general equilibrium. *Journal of Economic Theory* 146(3), 1107–1133.
- 581 Krusell, P., T. Mukoyama, R. Rogerson, and A. Şahin (2017). Gross worker flows over the
582 business cycle. *American Economic Review* 107(11), 3447–76.
- 583 Ljungqvist, L. and T. J. Sargent (2008). Taxes, benefits, and careers: Complete versus
584 incomplete markets. *Journal of Monetary Economics* 55(1), 98–125.
- 585 Ljungqvist, L. and T. J. Sargent (2011). A labor supply elasticity accord? *American*
586 *Economic Review* 101(3), 487–91.
- 587 Lucas, R. E. and E. C. Prescott (1974). Equilibrium search and unemployment. *Journal of*
588 *Economic Theory* 7(2), 188–209.
- 589 Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary*
590 *Economics* 36(2), 269–300.
- 591 Merz, M. (1997, May). A market structure for an environment with heterogeneous job-
592 matches, indivisible labor and persistent unemployment. *Journal of Economic Dynamics*
593 *and Control* 21(4-5), 853–872.
- 594 Mortensen, D. T. and C. A. Pissarides (1994). Job creation and job destruction in the theory
595 of unemployment. *The review of economic studies* 61(3), 397–415.
- 596 Prescott, E. C. and R. M. Townsend (1984). General competitive analysis in an economy
597 with private information. *International Economic Review* 25(1), 1–20.
- 598 Ravn, M. O. (2008). The consumption-tightness puzzle. In *NBER International Seminar on*
599 *Macroeconomics 2006*, pp. 9–63. University of Chicago Press.
- 600 Rogerson, R. (1988). Indivisible labor, lotteries and equilibrium. *Journal of Monetary*
601 *Economics* 21(1), 3–16.

- 602 Shell, K. and R. Wright (1993). Indivisibilities, lotteries, and sunspot equilibria. *Economic*
603 *Theory* 3(1), 1–17.
- 604 Shimer, R. (2013). Job search, labor force participation, and wage rigidities. In *Advances*
605 *in Economics and Econometrics: Volume 2, Applied Economics: Tenth World Congress*,
606 Volume 50, pp. 197. Cambridge University Press.
- 607 Veracierto, M. (2008). On the cyclical behavior of employment, unemployment and labor
608 force participation. *Journal of Monetary Economics* 55(6), 1143–1157.