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Photonic Direction-of-Arrival Estimation Based on Compressive Sensing

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Abstract: The optical approach to estimate the direction-of-arrival (DOA) estimation of microwave signals has attracted a lot of attention recently. Most of existing methods are based on a one-sensor array system, which converts the DOA estimation problem into an optical power estimation problem. Their main disadvantage is that additional work is needed to represent the relationship between the phase shifts and the optical powers before estimation. The algorithm proposed in this paper is based on an N-sensor array (N>2), where the optical power vector is obtained first and then a compressive sensing based approach is applied to directly estimate the phase shift. As demonstrated by simulation results, the proposed algorithm can achieve much better estimation accuracy and robustness than existing optical DOA estimation algorithms considered.

1. Introduction

Direction of arrival (DOA) information of microwave signals is of great importance in practice for source localization and target tracking; however, it is an extremely challenging problem when the dynamic range of the center frequency of a narrow band microwave signal is very large. For example, it is now common to have signals with a frequency varying from 2*GHz* to 100*GHz*. As the bandwidth of the electronic receiver is rather limited, optical microwave technology has been proposed, which can overcome the bandwidth bottleneck of electronic systems, is immune to electromagnetic interference and can achieve a better performance [1-5].

So far, a series of optical microwave DOA estimation methods have been proposed [6-24]. Among them, a classic approach is to transform the DOA estimation problem into a phase shift estimation problem by measuring the optical power. In such an algorithm, a one-sensor array system is constructed, the received signal of the first antenna is modulated with the signal of the continuous-wave (CW) laser at the first Mach-Zehnder modulator (MZM), and then the output is further modulated by the phase-delayed microwave signal of the second antenna at the second MZM. Since output optical power of the second MZM is concerned with the phase shift, the DOA information is then obtained. There are mainly three disadvantages associated with such an approach. Firstly, additional processing is required before the phase shift estimation, in order to digitize the relationship between the optical power and the test phase shift; secondly, the step size of the test phase shift directly influences the accuracy of the DOA estimation result; thirdly, the performance of the optical system cannot be improved when the step size of the test phase shift is fixed. Then, an optical power ratio based algorithm using two-sensor array is proposed in [25], which solves the first problem of the optical power based algorithm.

In this paper, an optical microwave DOA estimation algorithm based on an N-sensor (N>2) array using the compressed sensing method is proposed [26], which also converts DOA estimation into the phase shift estimation problem. Firstly, the optical system with N sensors is constructed, and the optical power vector with N elements can then be obtained. The compressive sensing based method is applied to the optical power vector to directly estimate the phase shift. Compared to the work in [25], where no optimization is performed in the process, a convex optimization problem is formulated and solved successfully in the current work and at least three sensors are required for its operation. The proposed algorithm overcomes the three limits of the one-sensor algorithm by directly estimating the phase shift, and the performance of the optical system can be improved by adding more sensors. Simulations are provided to show that the proposed algorithm has a better estimation accuracy and higher robustness than the optical power based algorithm and the optical power ratio based algorithm.

2. Principle of operation

The proposed optical system is schematically shown in Fig. 1. The system consists of one CW laser, (N+1) antennas (Antenna-0, ..., Antenna-N), (N+1) MZMs (MZM-0, ..., MZM-N), N optical filters (Filter-1, ..., Filter-N), and N power meters (Power-1, ..., Power-N), respectively. The distance between adjacent antennas is the same as d, while the length of the optical link between MZM-0 and any other MZMs is L. The angular frequency of the optical carrier is ω, while the angular frequency of the microwave signal is Ω. All the MZMs are biased to suppress the optical carrier, and set with the same modulation index β. All the filters are set with a fixed central wavelength and passband, which can remove all the other frequency components except for the ones with frequency ω.

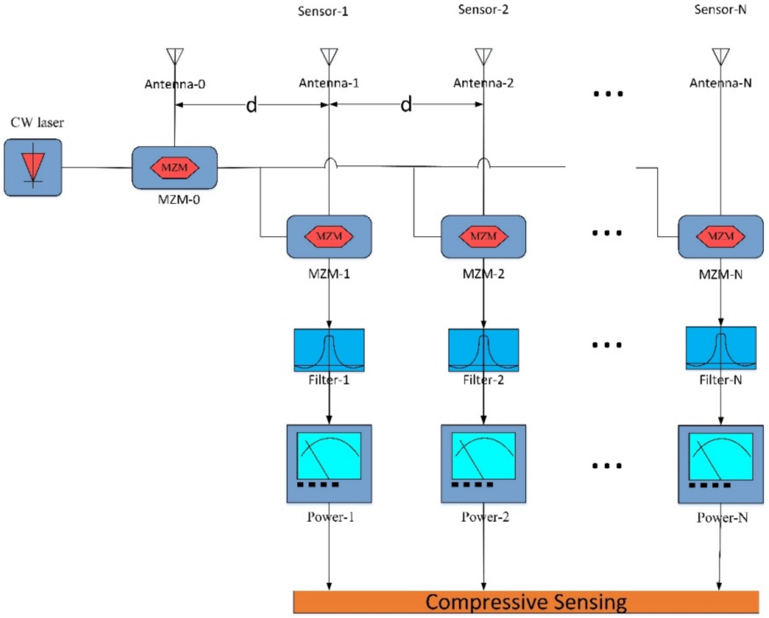

Fig. 1. A microwave photonic system.

Assume the DOA of the microwave signal is θ, which cause a phase shift Φ. If Φ is known, then θ can be estimated by [10]

$$\theta = \arccos(\Delta t c / d) \tag{1}$$

with

$$\Delta t = L/c - (\Phi + 2k\pi)/\Omega \tag{2}$$

where c is the vacuum light velocity, and k is an integer that can be determined from the physical parameters of the proposed system. The phase shift Φ is obtained in the system before estimating θ.

Firstly, the microwave signal of Antenna-0 is modulated with the optical carrier by MZM-0, and the field at the output of MZM-0 can be written as

$$\begin{aligned} E_0(\mathrm{t}) &\propto \sqrt{I_0}\,E(\mathrm{t})\exp[\beta\cos(\Omega\mathrm{t})] \\ &= jJ_1^2(\beta)[\exp j(\omega - \Omega)\mathrm{t} + \exp j(\omega + \Omega)\mathrm{t}] \end{aligned} \quad \textbf{(3)}$$

It can be seen that two sidebands are generated in the field, which are then modulated with the microwave signal of MZM-1. The field at the output of MZM-1 is

$$\begin{aligned} E_1(\mathrm{t}) &\propto \frac{j}{2}\bullet J_1(\beta)\left\{\begin{array}{l} \exp[j(\omega - \Omega)\mathrm{t} + \beta\cos(\Omega\mathrm{t} + \Phi)] + \\ \exp[j(\omega - \Omega)\mathrm{t} - \beta\cos(\Omega\mathrm{t} + \Phi)] + \\ \exp[j(\omega + \Omega)\mathrm{t} + \beta\cos(\Omega\mathrm{t} + \Phi)] + \\ \exp[j(\omega + \Omega)\mathrm{t} - \beta\cos(\Omega\mathrm{t} + \Phi)] \end{array}\right\} \\ &= -J_1^2(\beta)\left\{\begin{array}{l} \exp[j(\omega - 2\Omega)\mathrm{t} - \Phi] + \\ \exp[j(\omega + 2\Omega)\mathrm{t} + \Phi] + \\ \exp(j\omega\mathrm{t} + \Phi) + \exp(j\omega\mathrm{t} - \Phi) \end{array}\right\} \end{aligned} \quad \textbf{(4)}$$

In the field, four optical components are generated, but only two components with frequency ω are present at the output of Filter-1. The optical power of Power-1 is related to the phase shift as follows

$$P_1 \propto 2J_1^2(\beta)(1 + \cos(2\Phi)) \quad \textbf{(5)}$$

Then, one can obtain the field at the output of MZM-2 and the optical power at Power-2 as

$$\begin{aligned} E_2(\mathrm{t}) &\propto -J_1^2(\beta)\left\{\begin{array}{l} \exp[j(\omega - 2\Omega)\mathrm{t} - 2\Phi] + \\ \exp[j(\omega + 2\Omega)\mathrm{t} + 2\Phi] + \\ \exp(j\omega\mathrm{t} + 2\Phi) + \exp(j\omega\mathrm{t} - 2\Phi) \end{array}\right\} \\ P_2 &\propto 2J_1^2(\beta)(1 + \cos(4\Phi)) \end{aligned} \quad \textbf{(6)}$$

It's clear that, by using N sensors, an optical power vector $\mathbf{P}=[P_1,\ldots, P_N]^{\mathrm{T}}$ can be constructed. In fact, better accuracy can be achieved by employing the compressive sensing technique, as detailed in the following.

Take $\mathbf{P}$ as the observation vector and it is related to the phase shift Φ as

$$\mathbf{P} = 2J_1^2(\beta)Q[1+\cos(2\Phi),\ldots,1+\cos(2N\Phi)]^{\mathrm{T}} \quad \textbf{(7)}$$

where Q is an unknown constant. In the above model, $2J_1^2(\beta)Q$ is constant and can be considered as a virtual source, while $\mathbf{P}$ is determined by the source's phase shift through the vector $[1+\cos(2\Phi),\ldots,1+\cos(2N\Phi)]$.

Then, the dictionary of the compressive sensing algorithm $\mathbf{Z}(\hat{\Phi})$ can be constructed like this

$$\mathbf{Z}(\hat{\mathbf{\Phi}}) = [\mathbf{p}(\hat{\Phi}_1), ..., \mathbf{p}(\hat{\Phi}_K)](K \gg N) \tag{8}$$

with

$$\begin{aligned} \mathbf{p}(\hat{\Phi}_k) &= [p_1(\hat{\Phi}_k), ..., p_N(\hat{\Phi}_k)]^{\mathrm{T}} \\ p_k(\hat{\Phi}_k) &= 1 + \cos(2n\hat{\Phi}_k) \end{aligned} \tag{9}$$

where $\hat{\mathbf{\Phi}} = [\hat{\Phi}_1, \hat{\Phi}_2, ..., \hat{\Phi}_K]$ is the set of equally spaced angle samples within the spatial domain of interest $[0^{\circ}, 90^{\circ}]$; for example, with 1° spacing, K=91. Define $\hat{\mathbf{u}}$ as the sparse representation coefficient vector, whose nonzero element corresponds to the unknown source value $2J_1^{2}(\beta)Q$, while the corresponding vector in $\mathbf{Z}(\hat{\Phi})$ indicates the desired phase shift. Then, the phase shift estimation problem can be formulated in a constrained sparsity maximization form [25]

$$\min_{\hat{\mathbf{u}}} \left\|\hat{\mathbf{u}}\right\|_1 \text{ s.t. } \left\|\mathbf{P} - \mathbf{Z}(\hat{\mathbf{\Phi}})\hat{\mathbf{u}}\right\|_2 \leq \varepsilon \tag{10}$$

where $\|\bullet\|_1$ and $\|\bullet\|_2$ denote the l_1-*norm* for sparsity maximization and the l_2-*norm* for the overall reconstruction error, respectively, and ε is the upper limit for the reconstruction error. This problem can be solved using convex optimization software packages such as CVX [27]. The phase shift estimation results are finally obtained by examining $\hat{\mathbf{u}}$ and find its non-zero coefficients.

Note that the CS-based DOA estimation methods are mainly based on the sparsity of the problem, the mismatch in this context is the off-grid problem, i.e. the source directions do not fall exactly onto the sampled angle points. For solving such a problem, there are mainly two approaches: one is straightforward by using a denser grid, which increases the computational complexity of the algorithm significantly; the other one is to estimate this off-grid term for each source in an alternate optimization process [28]. Further improvement to the proposed method could be achieved by following the same approach as in [28].

3. Experimental results and analysis

In this section, simulations are performed using the optical power based algorithm with one sensor in [10], the optical power ratio based algorithm with two sensors in [25] and the proposed compressive sensing based algorithm with three sensors, separately. Assume the spacing of adjacent antennas is set to be 0.01*m*, the wavelength of the optical carrier is 1552*nm*, the laser RIN noise is -145dBc/Hz, the extinction ratio of the modulator is 30dB, the half-wave voltage is 5V, the bandwidth of the receivers (Antenna-0~Antenna-3) is 300GHz, and the bandwidth of the filters (Filter-1~Filter-3) is 10GHz. Take the absolute value of the estimated phase shift error as the phase shift error and 5° is the maximum phase shift error, with errors greater than it ignored.

It can be predicted that the DOA estimation error mainly comes from the drift of the working point and the limited extinction ratio of the MZM. The drift of the working point will affect the powers of the carrier and the first order sidebands, and the limited extinction ratio will affect carrier suppression [13]. These effects will introduce measurement errors into the system. To reduce this effect, a bias controller has been employed with the MZM to reduce the drift of the working point.

Experiment 1: Set the frequency and the phase shift of the microwave signal to be 18*GHz* and 65°, separately, and the signal-noise-ratio (SNR) varies from -8*dB* to 10*dB* with a step of 2*dB*. The relationship between the SNR and the phase shift error is shown in Fig. 2.

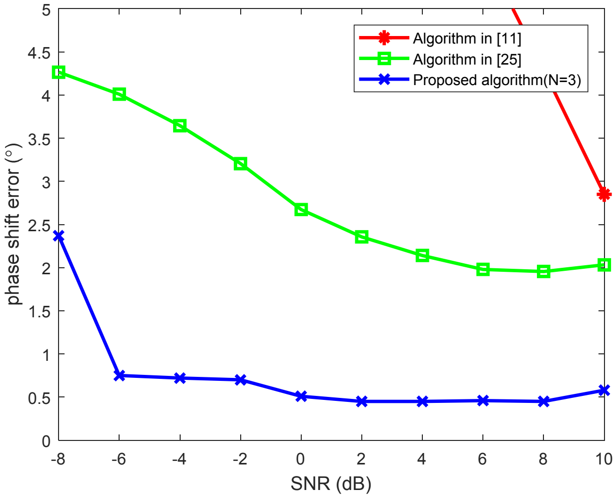

Fig. 2. The phase shift estimation error varying with the SNR of the microwave signal.

It can be seen that the phase shift error of the proposed algorithm and the algorithm in [25] is much lower than that of the algorithm in [10], and the phase shift error of the proposed algorithm is lower than that of the algorithm in [25]. Furthermore, the phase shift error of the proposed algorithm tends to be around 0.5° in most of the considered SNR region.

Experiment 2: Set the frequency of the microwave signal and the SNR to be 18*GHz* and 10*dB*, separately, and the phase shift from 10° to 80° with a step size of 5°. The relationship between the phase shift and the phase shift error is shown in Fig. 3.

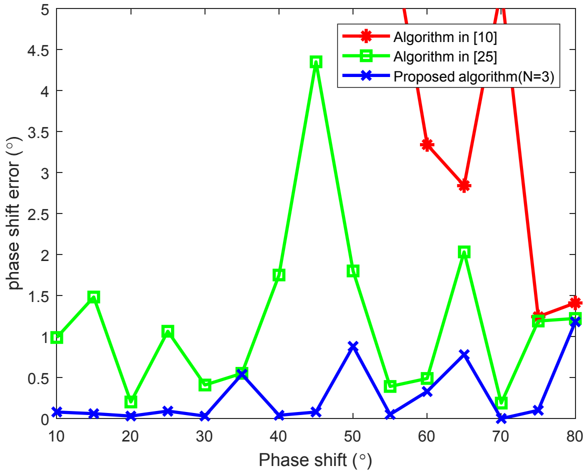

Fig. 3. The phase shift estimation error varying with the phase shift of the microwave signal.

It can be seen that the phase angle accuracy of the proposed algorithm is higher and more stable than that of the other two algorithms, which indicates that the proposed algorithm is less affected by the phase shift than the other algorithms.

Experiment 3: Set the DOA of the microwave signal and the SNR to be 65° and $10dB$, and the frequency of the microwave signal varies from $4GHz$ to $20GHz$ with a step size of $2GHz$. The relationship between the frequency of the microwave signal and the phase shift error is shown in Fig. 4.

It can be seen that the phase shift accuracy of the proposed algorithm is also higher and more stable than that of the other two algorithms. It's clear that the proposed algorithm is also less affected by the frequency than the other algorithms.

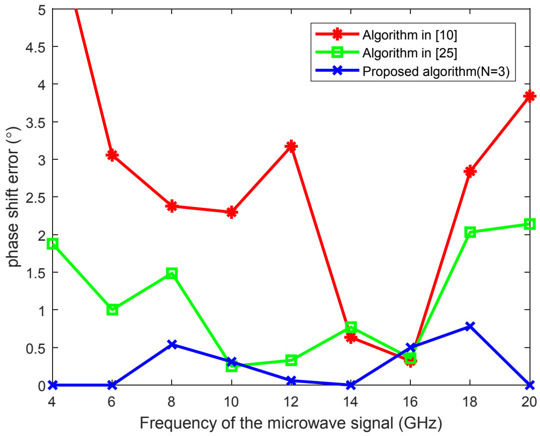

Fig. 4. The phase shift estimation error varying with the frequency of the microwave signal.

4. Conclusions

An optical microwave DOA estimation algorithm based on the compressive sensing framework is proposed, which is based on an N-sensor (N>2) array system. By examining the relationship between the optical power vector and the phase shift, a formulation based on sparsity maximization was derived. As demonstrated by simulation results, the proposed solution can improve not only the DOA estimation accuracy, but also the robustness against various varying parameters.

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Disclosures

The authors declare no conflicts of interest.

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