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Odolinski, K orcid.org/0000-0001-7852-403X, Nilsson, J-E, Yarmukhamedov, S et al. (1 more author) (2020) The marginal cost of track renewals in the Swedish railway network: Using data to compare methods. Economics of Transportation, 22. 100170. ISSN 2212-0122

https://doi.org/10.1016/j.ecotra.2020.100170

The marginal cost of track renewals in the Swedish railway network: Using data to compare methods

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Abstract

We analyze the differences between corner solution and survival models in estimating the marginal cost of track renewals. Both approaches describe the renewal process in intuitively similar ways but have several methodological distinctions. Using Swedish data for the 1999-2016 period, results suggest the median marginal costs per gross ton-km from corner solution and survival models are SEK 0.0066 and SEK 0.0031, respectively. Since several European countries use information about marginal costs as a basis for track user charges, the choice of estimation method is obviously important. Our conclusion is that the corner solution model is more appropriate in this case, as this method considers the impact traffic has both on the probability of renewal and on the size of the renewal cost. The survival approach does not consider the latter as part of the estimations, which is problematic when we have systematic cost variations due to traffic and infrastructure characteristics.

Keywords: railways; renewal; survival model; corner solution model; two-part model; marginal cost.

JEL codes: C41; H54; L92; R48

Declarations of interest: none

1. Introduction

The vertical separation between rail infrastructure management and train operations in Europe during the 1990s generated a need to set track access charges. The rules for these charges are laid down by The Single European Railway Area (SERA) Directive (2012/34/EU), which concerns the management of railway infrastructure and transport activities of railway undertakings in EU's Member States. The Directive establishes that charges for access to infrastructure facilities shall be set at the cost that is directly incurred as a result of operating the train service (Article 31.3), and is based on economic theory that advocates marginal cost pricing for an efficient use of assets. Hence, the marginal cost of infrastructure use needs to be estimated, including both maintenance and renewal costs.

Starting with Johansson and Nilsson (2004), there is now a series of econometric analyses of how railway traffic affects the costs for day-to-day infrastructure maintenance (Andersson, 2008; Link et al., 2008; Wheat and Smith, 2008; Wheat et al., 2009; Odolinski and Nilsson, 2017). This literature has generated estimates of cost elasticities for traffic (which are used to calculate marginal costs) that are relatively stable, both within countries as data accumulates over time and between countries (Nash, 2018).

Assets must at some point of time be replaced as costs for maintenance increase with age and use, including the increased risk for technical failures affecting traffic. This is part of any cost minimizing strategy for infrastructure services and a basis of life cycle asset management in general, where a renewal marks the end of an asset's life cycle. Moreover, replacements of infrastructure assets are costly: Renewal of the Swedish railway infrastructure (defined as major replacements, excluding upgrading) accounted for about SEK 2.4 billion[1] in 2016, which is about 25 per cent of the total maintenance and renewal expenditure at SEK 10 billion – a share that has been around 20 to 30 per cent during the 1999-2016 period. Grimes and Barkan (2006) report even higher shares of renewal expenditures (around 20 to 60 per cent) for US railroads in 1978-2002, and Walker et al. (2015) report shares around 40 per cent for the Swiss railway network during 2003-2012.

There are relatively few empirical papers that address railway renewal from the marginal cost perspective, despite its large share of the railway infrastructure managers' expenditures. One reason may be that renewals on a specific part of the railway is a rare feature, often performed with around 30 years in-between[2], which implies that a long time series is required to capture how changes in traffic affect renewal costs. One solution has been to add renewal to maintenance costs when estimating the

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[1] SEK 1 ≈ EUR 0.01

[2] The appraisal guidelines in HEATCO (2006) states service lifetimes between 20 to 40 years, while the Swedish appraisal guidelines (Trafikverket, 2018) states a maximum service lifetime at 60 years for new railway.

marginal cost of rail infrastructure use (see Andersson, 2006; Tervonen and Pekkarinen, 2007; Marti et al., 2009; Wheat and Smith, 2009). But these models do not provide direct estimates on renewal cost elasticities, and any inference on such elasticities are therefore uncertain. Andersson et al. (2012) and Andersson et al. (2016) are however two studies that have access to a long time series and provide estimates of the marginal cost for track renewals using disaggregate data. These papers use different modelling approaches: a corner solution model and a parametric survival model.[3] The papers establish a significant difference between the corner solution and survival models in that marginal cost estimates were SEK 0.009 and 0.002 per gross ton-km, respectively.

The purpose of this paper is to consider the qualities of these modelling approaches and – using information about infrastructure renewals in Sweden from 1999 to 2016 – estimate the marginal costs using the respective models. This makes it possible to recommend the best method or indeed to establish that either approach can be used for the measurement of marginal costs. The choice of method is policy relevant considering the difference in marginal cost levels and since several European countries use econometric information about marginal costs for charging track users.

In the literature, the two modelling approaches are linked via censored regression models. For example, a textbook treatment of the survival model usually describes issues with censored data (missing information on time to an event) which is common in any duration analysis (see Kiefer (1988) for an accessible description). The corner solution model is also connected to censored regression models as the econometric techniques are similar (cf. Tobit regression and section 2.1 below) but is applied to a different situation which formally does not concern censored data. The reason is that the individuals/agents are solving an optimization problem with a corner solution that provides a lower bound which is (often) a zero value and a higher bound that is a continuous (observable) variable that can take any positive value (Wooldridge, 2002). The focus of the corner solution model is thus on the various outcomes of an event, while the survival approach focuses on the time to event.

In our case, we are interested in both. That is, we want to analyze the time to (or risk of) multiple kinds of events that is represented by a continuous variable taking positive values. To the authors' knowledge, the differences and similarities between these two modelling approaches has not been investigated empirically in the context of infrastructure cost analysis. We thus consider the present paper to contribute to the existing literature by spelling out the similarities and differences in their application to infrastructure costing. Indeed, the impact of covariates on time-to-event, as well

[3] Yet another approach is used by Odolinski and Wheat (2018), who apply a vector autoregressive model to rail infrastructure renewals and maintenance. As the present paper focuses on renewals only, a thorough comparison with the vector autoregressive model is beyond the scope of this paper.

as on the cost of these events, is relevant for infrastructure assets in general and not only railways. For example, models for road deterioration due to traffic and costs of resurfacing have been used to analyze pricing and investment policies, often building on the seminal work by Small et al. (1989). See Bruzelius (2004) for a survey of the different methods used to measure the (marginal) cost of road use.

The rest of the paper is organized as follows. Section 2 presents corner solution and survival models. Data and descriptive statistics are described in section 3. Results are presented in section 4. The last section of the paper comprises a discussion and conclusion.

2. The modelling approaches

The timing and frequency of renewal activities depend on the intensity of traffic, and an increase in traffic may therefore cause a deviation from the original cost minimizing plan for renewals (and maintenance[4]). Specifically, a renewal can be rescheduled to account for more traffic than originally planned and this affects the present value of renewal costs. This is the marginal renewal cost.[5] The analytical challenge for the estimation is that railway renewals are rare events. Most years, most parts of the network are *not* renovated generating an inventory of renewal activities indicating zero cost values. These are true zeros since the infrastructure manager has decided whether to make a renewal (observation is positive) or not (zero) on each section of the railway network.

With few cost observations that have a positive and continuous value and a large share of the observations being zero, the use of a linear model may result in negative probabilities. Models that can properly handle these situations have been developed (see for example Wooldridge, 2002) and are used by Andersson et al. (2012) to estimate the impact of traffic on track renewal costs (y). Since $y = 0$ in these situations, this is referred to as a corner solution. In short, this approach considers the relationship between different covariates and the renewal decision (timing and size).

Survival analysis is an alternative to the corner solution model and is used by Andersson et al. (2016). This approach considers the *time to an event*, which in our case is the time until the asset in its current condition “dies” (is renewed). Specifically, it describes the relationship between the survival time of the assets and different covariates, making it possible to assess the (marginal) effect on survival

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[4] See Odolinski and Wheat (2018) for an exploration of the dynamics between maintenance and renewal activities when traffic changes.

[5] Nilsson et al. (2015) provides a detailed description of the analytical logic for including costs for road surface renewal as part of the marginal cost for using infrastructure.

time due to an increase in traffic. The next and final step of the model is to link the variation in survival time to costs.

Indeed, the purpose of both approaches is to connect traffic to costs, where the corner solution model resembles a top-down approach that tries to establish a (more or less) direct relationship between traffic and costs, while the use of survival (duration) analysis in for example Haraldsson (2007) and Andersson et al. (2016) is more in line with a bottom-up (mechanistic) approach that focuses on how traffic impacts the deterioration of the infrastructure and then links this measure to some unit cost.

A more thorough description of the differences and similarities between these modelling approaches are described below in section 2.3. But first, a short formal presentation of the corner solution and the survival model used by Andersson et al. (2012 and 2016) as well as in the present paper, is provided in sections 2.1 and 2.2, respectively.

2.1 The corner solution model

Within the corner solution framework, there are three approaches that can be used: the Tobit, the two-part, and the Heckit models. Previous studies in the railway context (Andersson et al., 2012; Yarmukhamedov et al., 2016) provide indications in favor of the two-part model. Our formal testing also provides support for this choice. The comparison of the three approaches is suppressed for expositional convenience.

The two-part model explains the renewal decision in the first part and the renewal's size in the second. The renewal decision for track section i in year t is specified as a probit model:

$$z_{it}^{*} = \alpha_{i1} + \beta_{k1}' x_{kit1} + u_{it1} \tag{1}$$

where $I_{it} = 1$ if $z_{it}^{*} > 0, I_i = 0$ otherwise. z^{*} is a latent variable, which describes the decision whether to renew or not, I takes the value 1 (or zero) when a decision is taken to implement a renewal (or not), x_{kit1} is a vector of $k = 1, .., K$ explanatory variables that includes traffic, railway network characteristics, and geographical location and β_{k1}' its parameters. $\alpha_{i1}$are unobserved track section specific random effects and u_{it1} is the error term $\sim N(0,1)$.

The size of the renewal y is specified as a truncated regression model in the second part:

$$y_{it}|(I_{it} = 1) = \alpha_2 + \beta'_{k2}x_{kit2} + u_{it2} \tag{2}$$

where α_2 is a constant term and the expected value of the error term u_{it2} is zero for positive values of renewal costs. The error term is not necessarily normally distributed.

The two-part model allows the process for the decision to undertake renewal activities to be different from the process for the decision on the size of the renewal. Thus, the explanatory variables used in the renewal decision process can be different from the ones used in determining the size of the renewal. Even if these predictors are the same, their coefficient estimates in terms of sign, magnitude and significance level can differ.

The equation for estimating the marginal cost for track section i in year t with respect to traffic volume ($k = q$) is:

$$MC_{qit} = \gamma_{qit}\widehat{AC}_{qit} \tag{3}$$

Here, γ_{qit} is the cost elasticity:

$$\gamma_{qit} = \partial E[y]/\partial x_q \cdot x_q/E[y] = \beta_{q2} + \beta_{q1}\lambda(\hat{\beta}'_{k1}x_{k1}) \tag{4}$$

β_{q1} and β_{q2} are coefficient estimates for the traffic volume variable from the first and the second parts, respectively; $\lambda(\hat{\beta}'_{k1}x_{k1})$ is the inverse Mills ratio, $\lambda(\hat{\beta}'_{k1}x_{k1}) = \phi(\hat{\beta}'_{k1}x_{k1})/\Phi(\hat{\beta}'_{k1}x_{k1})$, where $\phi(.)$ and $\Phi(.)$ are the probability density and cumulative distribution functions derived from the probit model (Eq. 1); $\widehat{AC}_{qit}$ is the predicted average renewal cost per gross ton-km obtained from the truncated regression model (Eq.2). Specifically, predicted costs are $E[y] = \Phi(\hat{\beta}'_{k1}x_{k1})\, exp(\hat{\beta}'_{k2}x_{k2})\, exp\,(\frac{1}{2}\hat{\sigma}^2)$, assuming normally distributed and homoscedastic error terms (Dow and Norton, 2003), which are then divided by gross ton-km to get average renewal costs $\widehat{AC}_{qit}$.

Like previous studies on marginal cost of rail infrastructure usage (see for example Munduch et al., 2002; Johansson and Nilsson, 2004; Andersson, 2008; Wheat et al., 2009; Andersson et al., 2012; Andersson et al., 2016; Odolinski and Nilsson, 2017), a weighted marginal cost is computed using gross ton kilometer for each track section ($GTKM_{it}$):

$$MC^W = \sum_{it} MC_{it} \cdot GTKM_{it} / (\sum_{it} GTKM_{it}) \tag{5}$$

The weighted marginal cost measure has primarily been used to provide one single charge that can be levied for the entire railway network; it would recover the same revenue to the IM as if track-section specific marginal costs (MC_{it}) were used. However, as noted by Wheat et al. (2009), putting a higher weight on the estimates for observations with high traffic volumes can be problematic when comparing results between countries (in our case, between modelling approaches), as the tails of the distribution are likely to be (more) imprecisely estimated by the model. We therefore also consider the median marginal cost when comparing the model results.

2.2 The parametric survival model

Andersson et al. (2016) defines rail life T as the time from traffic opening to the occurrence of rail renewal. Its distribution is represented by a survival function, stating the probability of the rail surviving beyond a certain time t, that is

$$S(t) = Pr(T > t) \tag{6}$$

The cumulative lifetime distribution is defined as $F(t) = Pr(T \leq t) = 1 - S(t)$, and the unconditional probability of a track segment being renewed in time t (that is, the probability density function, $f(t)$) is then $\frac{-dS(t)}{dt}$. These measures are related to the hazard function, which can be stated as the probability of a rail being renewed directly after time t, conditioned on that it has survived until that time. For a continuous variable, the hazard function is

$$h(t) = \lim_{\Delta t \to 0} \frac{Pr[t \leq T < t + \Delta t | T \geq t]}{\Delta t} = \frac{f(t)}{1 - F(t)} \tag{7}$$

When the parametric approach is used, it is necessary to choose a functional form for analyzing the hazard rate, linking the probability of a track renewal to explanatory variables, such as traffic. Andersson et al. (2016) considers the proportional hazard (PH) form

$$h(t) = h_0(t) exp(\beta_k' x_k) \quad (8)$$

where $h_0(t)$ is a baseline hazard, that is, the change in the risk of renewal over time when covariates are zero. There are several parametric models in survival analysis, which differ in terms of the assumption on the functional form of the baseline hazard (in eq. 8), which is analogous to specifying the probability distribution for T (Allison, 1982). In the road and railway context, baseline hazard is often assumed to follow a Weibull distribution (Link and Nilsson, 2005; Nilsson et al., 2015; Andersson et al., 2016), which includes a shape parameter ($p > 0$) that allows the hazard to be constant ($p = 1$), increase ($p > 1$) or decrease ($p < 1$):

$$h_0(t) = pt^{p-1} exp(\beta_0) \quad (9)$$

The Weibull regression model in PH metric is then

$$h(t) = pt^{p-1} exp(\beta_0 + \beta_k' x_k) \quad (10)$$

where β_k' is the change in the risk of renewal due to a one-unit change in x_k. Note that with $p = 1$, the hazard is "memoryless" and follows an exponential distribution. In the case of infrastructure deterioration, the hazard is usually increasing ($p > 1$).

The Weibull PH model can also be described as an accelerated failure time (AFT) model, in which β_k' measures the change in the survival time due to an increase in x_k. Andersson et al. (2016) use this type of (Weibull) model to estimate a deterioration elasticity with respect to traffic, that is, the change in the rail's lifetime (rather than risk of renewal) due to a traffic increase. Introducing the subscripts for track section i and a random effect β_i, the Weibull PH model (eq. 10) written in log-time metric is:

$$lnt_i = \beta_i + \beta_k' x_{ki} + \varepsilon_i \quad (11)$$

where t_i is the observed survival time of track section i, and $\varepsilon_i \sim$Weibull (β_0, p). Like the corner solution model presented above, we estimate random effects survival models. In the case of the

Weibull distribution, the PH model results are easily reparametrized as an AFT counterpart by $\beta_{AFT} = -\beta_{PH}/p$.

The parameter estimates are obtained using maximum likelihood, where the parameter for traffic (β_{GT}) needs to be linked to renewal costs in the present application. The equation for estimating the marginal cost per gross ton-km in the survival analysis is in Haraldsson (2007) and Andersson et al. (2016):

$$MC_i = -\beta_{GT} \frac{c}{\bar{q}_1 \mu_i} \frac{r}{[1-exp(-r\bar{T})]} \int_0^\infty exp(-r\omega - \varphi_i \omega^p)\, d\omega \tag{12}$$

where β_{GT} is a survival time elasticity estimate (AFT) with respect to traffic, i.e. the parameter estimate for the natural logarithm of tonnage density (the gross ton-km per track-km) from eq. (11); c is average track renewal cost per renewed track length; $\bar{q}_1$ is a constant average annual traffic volume of the first renewal interval; μ is the expected value of the renewal interval. Specifically, $\mu = E(T) = \Gamma(1 + 1/p)/\lambda^{1/p}$, where Γ is the Gamma function, and T is time to renewal. r is the social discount rate; $\bar{T}$ is the constant renewal interval; ω is the remaining life time of a track segment, $\omega = T - \tilde{t}$, where $\tilde{t}$ is the renewal time; φ is the scale parameter, $\varphi = \exp{(\beta_0)}$. As in the corner solution model, we consider the median marginal cost as well as the weighted marginal cost per gross ton-km using eq. 5.

2.3. Similarities and differences between the modelling approaches

The previous description establishes that the survival model is used to estimate how an increase in traffic leads to shorter rail life time (or on the probability of experiencing a renewal in $t + \Delta t$, given that is has survived until t), while the corner solution (two-part) model considers the impact of traffic on the probability and size of renewals. A similarity between the approaches emerge when we consider the first part of the corner solution model – that is, the probit regression. First, we can note that the continuous time (as specified in Andersson et al., 2016), can be represented by discrete time units. This allows us to consider discrete-time hazard rate models, which can be estimated using models for a binary dependent variable.[6] We define the discrete time hazard rate for track i in time t as

[6] See for example Brown (1975), Alisson (1982) and Jenkins (1995). Moreover, Doksum and Gasko (1990) consider the correspondence between binary regression analysis and survival analysis.

$$h_{it} = Pr[T_i = t | T_i \geq t, x_k] \tag{13}$$

where T_i is the time to the occurrence of rail renewal. Again, this hazard rate states the probability of renewal at time t, given that the asset has not already been renewed. We then define a dummy variable indicating when rail is uncensored ($\delta_i = 1$) and censored ($\delta_i = 0$) – that is, whether we observe a track renewal during our observation period or not. Following Allison (1982) and Jenkins (1995), the likelihood equation for this type of censored data can be written

$$L = \prod_{i=1}^{n} [\, Pr(T_i = t_i)]^{\delta_i} [Pr(T_i = t_i)]^{1-\delta_i} \tag{14}$$

which can be shown to have the following log-likelihood function

$$logL = \sum_{i=1}^{n} \delta_i \log \left[\frac{h_{it_i}}{1-h_{it_i}} \right] + \sum_{i=1}^{n} \sum_{j=1}^{t_i} \log\,(1 - h_{ij}) \tag{15}$$

Replacing δ_i with a dummy variable indicating if a rail is renewed at time t ($y_{it} = 1$) or not ($y_{it} = 0$), eq. 15 is the log-likelihood for a regression analysis of a binary dependent variable, where it is necessary to specify a functional form for the hazard rate that links the probabilities of rail renewal to time and other explanatory variables (for example, using link functions such as logit, complementary log-log or probit). The first part of the corner solution model in Andersson et al. (2012) is thus a version of the discrete-time duration (survival) model. There is a possibility to include time (rail age) variables so that the underlying hazard can vary with time. A polynomial function of time can for example allow some flexibility in the effect of time on the hazard rate.

Hence, the PH estimates from a survival analysis, as in Andersson et al. (2016), should in theory be close (or similar) to the first part of the corner-solution approach in Andersson et al. (2012), depending on how the hazard rate function is specified. It should be noted that the survival analysis in Andersson et al. (2016) is based on more disaggregate data than Andersson et al. (2012), and that the latter study did not explicitly include variables for rail age and did therefore not allow the underlying hazard rate to vary with time.[7] Such a probit model is similar to an exponential distribution with a constant hazard rate (that is, the shape parameter in the Weibull regression model in eq. 10 is equal to 1). See for example Jones and Branton (2005) who demonstrate this property using the logit model.

Still, the Weibull survival regression model has an AFT counterpart, which the binary regression models do not have, and the deterioration elasticity from this model (the impact on survival time) is used in Andersson et al. (2016) together with estimates on the expected survival rail times in the marginal cost calculations (see eq. 12). The elasticity with respect to traffic in the probit model also imply shorter survival times of rail, yet it does not provide an estimate of the expected survival times for the rails that are still alive when our observation period ends.[8]

A more striking difference between the estimation approaches concerns the second part of the corner solution model: the impact of traffic on the size of the renewal cost, given that the infrastructure manager has decided to perform a renewal on a certain part of the track in a certain time period. The survival approach in Andersson et al. (2016) only concerns the impact on renewal intervals or on the probability of a renewal (using the AFT or PH estimate, respectively), and not if an increase in traffic will, for example, result in more parts of the tracks being replaced and/or a change in the cost of performing the renewal.

To conclude, it is reason to expect a difference in the marginal cost estimates generated by these two modelling approaches since the corner solution (two-part) model considers the (potential) impact traffic has on costs in addition to its impact on the decision to renew (modeled in both approaches). To test this empirically, we estimate survival regression models and corner solution models on Swedish railway data and calculate the marginal cost of track renewals.

[7] They included a variable for rail weight, which to some extent is correlated with age of rails (older rails are usually lighter) and a variable for age of switches, which also correlates with rail age.

[8] Note however that it is possible to control for the age of these rails in the estimation of the renewal elasticity with respect to traffic in the probit model and that the marginal effect of traffic can be evaluated with respect to different rail ages.

3. Data

3.1 The structure of available information

The Swedish Transport Administration (Trafikverket) is responsible for the maintenance and renewal of 14 100 km of railway infrastructure in Sweden. This network is divided into 260 sections, which provide disaggregate information about maintenance and renewal costs. These sections are the track individuals' in our model estimations. In addition, renewal costs are reported for different asset categories on each section, such as track superstructure (rail, sleepers, fastening system, switches), track substructure (bar, intersection, culvert, bridge, tunnel), marshalling yards (lights, train warming etc.), electrical wiring (overhead line, traction power network, transformer etc.), signaling (positioning system, balise, train control system etc.), telecommunications (detector, radio, tele transmission system etc.) and other equipment (property, canalization, drainage and pump systems etc.). As in Andersson et al. (2012; 2016), we focus on renewals in track superstructure since information on the age of all other constructions is not available. Moreover, we exclude switches since the timing of their renewals may differ from other parts of the track superstructure.

Maintenance activities are carried out to maintain proper performance (reliability), and can include grinding of the rail, tamping, inspections, as well as minor replacements. Renewals, which are the focus of our analysis, are major replacements that typically marks the end of one and start of another life cycle of the tracks. While this excludes asset modernization, Trafikverket (2015) indicates that it may include a degree of upgrading if the original equipment and material is no longer available. Track renewal is typically implemented on parts of the track sections. Moreover, a track renewal does not necessarily include a complete track superstructure refurbishment. For example, sleepers can be replaced on a part of the section, while the rail is not. Here it should be noted that track section renewal costs are the event indicators in the estimations.

Information on other characteristics of the railway network has also been collected, such as number of joints, rail weight, quality class (linked to maximum line speed allowed) and regional indicators. These characteristics may influence the decision to renew and the cost of the renewals and may therefore be important control variables when estimating the impact traffic has on track renewals. Missing information worth mentioning are data on recycled rails: While we have complete data on when and where rail has been installed, information about when the rails were originally constructed is not available until year 2016. The observation of a significant difference between the construction year and the installation year indicates that the rail is reused. If we consider a 10-year difference or longer between the construction year and installation year to be an indication of reused rail, then about 3 per cent of the tracks in 2016 have been reused.

The annual gross tons of traffic passing over track sections is used for estimating the marginal track renewal cost with respect to traffic. Since information about both infrastructure characteristics and traffic is available at a more disaggregate level than sections, these observations have been aggregated to the section level which is the most disaggregate level with information about renewal costs.

3.2 Descriptive statistics

Data on track sections with sparse traffic, industrial tracks, marshalling yards and privately-owned sections are excluded from the analysis due to non-available data. This leaves 216 track section observations observed over the period 1999-2016 to be used in the analysis. It would be relevant to include marshalling yards in the analysis, since the yards can be expected to have a different cost structure compared to other track sections. Too few observations make this impossible. In total, the combination of track sections and years generates 3 385 observations, including 354 observations of track renewals.

The descriptive statistics of the data are presented in Table 1. The mean renewal cost includes zero-value observations, resulting in a rather low average cost. The traffic information is gross ton-km and gross ton density, where the latter is defined as gross ton-km divided by track-km.[9] A dummy variable indicates whether the track section is in a station. The cost and deterioration structure on stations may *ceteris paribus* differ from other sections since tracks on three or more lines may intersect at stations and since tracks can be used for shunting and sometimes overnight parking of trains. We also have a set of dummy variables that indicate which regional unit within Trafikverket they belong to, which may capture information on differences in management of the tracks.

Renewal activities are rare events. However, as parts of the track sections can be renewed, it is not uncommon that other parts within the same track section is renewed in another year during our observation period from 1999 to 2016. We therefore have recurrent events in our sample. Moreover, as is often the case with this type of duration statistics, the information is right-censored. That is, for many sections, we do not know when the next renewal will take place, and this right-censoring is a so-

[9] Information freight and passenger gross ton-km is available, that is we can distinguish between these traffic types in the estimations. However, this distinction did not result in different estimates for the traffic types, which was also the case in Odolinski and Nilsson (2017). Moreover, we can note that Andersson et al. (2016) also estimated survival models with freight and passenger traffic but preferred the gross ton model.

called Type I censoring caused by a stop in our data collection. There are in total 1869 right-censored observations.

Table 1. Descriptive statistics, track sections during 1999-2016 (3385 observations)

Variable	Mean	Std. Dev.	Min	Max
Renewal cost, million SEK in 2016 prices	1.95	16.10	0.00	375.00
Renewal dummy (event indicator)	0.10	0.31	0.00	1.00
Gross ton-km, million	358	514	3.22E-04	4220
Gross ton density, thousand	4516	4092	1.47E-02	29800
Track length, km	67.3	51.7	1.9	305.5
Renewed track length, km	0.4	3.0	0.0	44.0
Rail age, years	21.6	11.3	1.0	96.0
Rail weight, kg/m	51.2	5.0	32.0	60.0
Quality class, 0 (high line speed) to 5 (low line speed)	2.2	0.2	0.0	5.0
No. of joints	161	134	1	1221
Station section, dummy variable	0.1	0.3	0.0	1.0
West region, dummy variable	0.2	0.4	0.0	1.0
North region, dummy variable	0.1	0.3	0.0	1.0
Central region, dummy variable	0.2	0.4	0.0	1.0
South region, dummy variable	0.3	0.4	0.0	1.0
East region, dummy variable	0.2	0.4	0.0	1.0

4. Results

The similarities between the first part of the (two-part) corner solution analysis and the (continuous time) survival analysis were described in section 2.3. To explore the empirical qualities of these similarities, we present results from the probit and survival models in section 4.1, estimated with random effects and standard errors adjusted with respect to track section clusters. The way in which the hazard rates are linked to marginal costs varies between the approaches, and the consequences of this difference is presented in section 4.2.

4.1 Probit and survival regression results

To test the importance of considering a (possibly) non-constant hazard rate in the model estimations, we estimate the probit model with and without rail age variables (including a second order effect of age to allow for more flexibility). We compare the results with survival regression results in which either the exponential distribution (with constant hazard rate) and Weibull distribution (allowing for non-constant hazard rate) is used.

The estimation results from the probit regressions in Table 2 indicate that an increase in track section length leads to higher probability of renewal, which is also the case with the number of joints and quality class. These two latter variables are lagged, representing the values prior to the year of a (possible) renewal. The reason is that the track will usually get a lower quality class (that is, higher line speed allowed) after refurbishment. The decision to renew (or not) should therefore be linked to the original higher quality class (lower line speed), which is corroborated by the results.

Controlling for the age of the rails has an impact on the parameter estimate for traffic (the natural log of gross ton density). The first and second order effect of rail age are positive and statistically significant at the one per cent level, and the gross ton density estimate increases from 0.2114 to 0.2703. In terms of the elasticity with respect to traffic ($\hat{\beta}_{q1}\lambda(\hat{\beta}'_{k1}x_{k1})$), it increases from 0.4297 (standard error 0.1530; Model 1a) to 0.5623 (standard error 0.1683; Model 1b) when controlling for rail age.

Table 2. Probit regression results.

	Model 1a		*Model 1b*	
	Coef.	Rob. Std. Err.	Coef.	Rob. Std. Err.
Constant	-1.8613***	0.2198	-1.8813***	0.2241
ln(gross ton density)	0.2114***	0.0580	0.2703***	0.0667
ln(track length)	0.2204*	0.1278	0.2029	0.1451
ln(rail age)	-	-	0.7285***	0.1144
ln(rail age)^2	-	-	0.6644***	0.1919
ln(no. of joints)_t-1	0.5094***	0.1324	0.5228***	0.1454
quality class_t-1	0.1868***	0.0618	0.1372**	0.0642
D.station section	0.0026	0.2054	0.0754	0.2353
D.north region	-0.3301*	0.1866	-0.0776	0.1957
D.central region	-0.6338***	0.1517	-0.5992***	0.1519
D.south region	0.1586	0.1500	0.2595*	0.1469
D.east region	-0.1879	0.1545	-0.1267	0.1547
Dependent variable	Renewal ($y = 1$) or not ($y = 0$)		Renewal ($y = 1$) or not ($y = 0$)	
No. of observations	3385		3385	
Log-likelihood	-958.474		-923.737	
AIC	1938.949		1873.474	
BIC	2006.347		1953.126	

***, **, * Significant at 1%, 5%, and 10%, respectively.

The PH regression results from the survival models using either the exponential distribution (Model 2a) or the Weibull distribution (Model 2b) are presented in Table 3. The estimate for traffic is 0.4322 (standard error 0.1045) in Model 2a which is like the elasticity at 0.4297 from the probit regression that excludes rail age variables (Model 1a). With a non-constant hazard rate from the Weibull regression model (2b), the estimate for traffic is 0.6008, which is rather close to the corresponding estimate in the probit regression (0.5623) that includes rail age variables (Model 1b). The shape parameter in the Weibull model is estimated to be 4.20, indicating an increasing hazard rate with rail age – that is, the need to renew the tracks accelerates over time as traffic accumulates. This is in line with what we expect and with the probit regression results showing that the probability of a renewal increases with rail age.

Table 3. Survival proportional hazard regression results.

	Model 2a – Exponential distribution		*Model 2b - Weibull distribution*	
	Coef.	Rob. Std. Err.	Coef.	Rob. Std. Err.
Constant	-6.2584***	0.3321	-16.4609***	1.0565
ln(gross ton density)	0.4322***	0.1045	0.6008***	0.1757
ln(track length)	0.3198	0.2392	0.1076	0.3375
ln(no. of joints)_t-1	0.7468***	0.2201	1.0026***	0.3204
quality class_t-1	0.2715***	0.1007	0.0376	0.1777
D.station section	-0.1105	0.3218	0.0181	0.5957
D.north region	-0.2221	0.2888	0.8711	0.5503
D.central region	-0.9172***	0.2305	-0.8823**	0.3886
D.south region	0.3250	0.2060	0.5095	0.3509
D.east region	-0.1800	0.2063	0.0116	0.3814
Dependent variable	Renewal or not (hazard rate, see eq. 13)		Renewal or not (hazard rate, see eq. 13)	
No. of observations	3385		3385	
Log-likelihood	-2086.941		-1749.079	
AIC	4195.882		3522.158	
BIC	4263.280		3595.683	
Shape parameter, *p*	1 (assumed)		4.20	

***, **, * Significant at 1%, 5%, and 10%, respectively.

4.2 Linking hazard rates to renewal costs: Marginal cost results

The corner solution (two-part) model make use of a truncated regression to estimate the impact different covariates have on the size of a renewal cost, given that the infrastructure manager has decided to renew. Estimation results from this model are presented in Table 4. The length of tracks renewed is included in the model estimations, showing that a 10 per cent increase in length results in a 4.5 per cent increase in costs. Moreover, lagged rail weight is included as proxy for track standard (heavier rails imply higher track standard), and the coefficient shows that heavier rail leads to lower renewal costs. The coefficient for traffic is positive and statistically significant (0.2900 with standard error 0.1508 and p-value 0.054) meaning that more traffic has an impact on the size of the renewal cost, even when we control for the renewed track length. This indicates that renewals of the track superstructure on a certain track length are not always complete replacements of the components and/or differ regarding the type of components installed. The latter may even be the case for the type of rails (within the same rail weight group) that are installed.

As described in section 3, about 3 per cent of the rails on the railway network were recycled in year 2016. A hypothesis is that the consequences of recycling for the estimation of costs is partly captured by our traffic variable. That is, sections with higher levels of traffic are more likely to experience track renewals comprising completely new rails, resulting in higher renewal costs. Since this information is only available for 2016, it is not feasible to test this hypothesis and analyze its potential impact on renewal costs during the whole period. Nevertheless, the results from the truncated regression show that track renewals have systematic variations in costs, and that traffic can explain some of this systematic variation.

Table 4. Truncated regression results (second part of the corner solution model).

	Model 3 – Truncated regression	
	Coef.	Rob. Std. Err.
Constant	18.2788***	2.4474
ln(gross ton density)	0.2900*	0.1508
ln(renewed track length)	0.4468***	0.0717
rail weight_t-1	-0.1260***	0.0376
ln(no. of joints)_t-1	0.1387	0.1999
quality class_t-1	0.0159	0.1577
D.station section	-1.2643***	0.3479
D.year 2000-2016	Yes	
Dependent variable	ln(renewal cost)	
No. of observations	354	

***, **, * Significant at 1%, 5%, and 10%, respectively.

To calculate the marginal cost based on the corner solution results, the estimated elasticities with respect to gross tons from the probit regression (0.5623) is combined with the truncated regression (0.2900) and multiplied with predicted average costs. This generates 354 marginal cost estimates, which is the number of renewal observations that can be used in the truncated regression. A weighted marginal cost per gross ton-km is then calculated (see eq. 5), as well as a median marginal cost.

The marginal costs from the survival model are estimated differently in Andersson et al. (2016). To follow this procedure, we first reparametrize the PH estimate to its AFT counterpart, using $\beta_{AFT} = -\beta_{PH}/p$. This results in a deterioration elasticity that is -0.1429, showing the impact gross tons have on the survival time of the tracks. We then use eq. (12) and multiply the deterioration elasticity with the average cost ($\frac{c}{\bar{q}_1\mu_i}$). In Andersson et al. (2016), the numerator c was the sum of all track renewal costs during the period under observations divided by the sum of total track length that was renewed. In the present analysis, we also calculate marginal costs using variations in this average cost between different observations – that is, we use c_i, which is the average track renewal cost for track section i (when there is no track renewal cost available for certain track section, we use the average c which is SEK 4.77 million per track-km).

The denominator ($\bar{q}_1\mu_i$) in the estimate of average cost ($\frac{c}{\bar{q}_1\mu_i}$) in eq. 12 is a constant average annual traffic volume of the first renewal interval ($\bar{q}_1$) multiplied with the estimated expected value of the renewal interval (μ_i),[10] which thus is a measure of the average cumulative traffic. The product of the deterioration elasticity and average costs are multiplied with a discount factor for an infinite cycle of estimated renewal intervals ($\frac{r}{[1-exp(-r\bar{T})]}$), where the interest rate (r) is set to 4 per cent, and an integral describing the estimated distributions of rail ages and expected rail life times that remains for these rails (see eq. 12). The integral is solved using numerical integration in Octave (version 5.1.0), where the integration area ranges from 0 to 100 years, the oldest rail in the sample being 96 years old. These two latter parts of the marginal cost calculation are thus specifically considering the impact traffic has on the future renewal intervals and costs. This calculation results in 3385 marginal cost estimates, one per observation in our sample, which is almost 10 times more than the estimates from the corner solution approach (354) which comprise one estimate per observation with a renewal cost.

For comparison, we also use the PH estimate from the survival model (0.6008) and calculate marginal costs using the same procedure as in the corner solution model, that is, by multiplying this elasticity with the average cost (observed renewal cost per gross ton-km).

The marginal cost estimates are presented in Table 5, indicating that the summary statistic used is important for the conclusions. The difference between each model's (weighted) mean and median marginal costs indicate that outliers in the material generate a relatively high (weighted) mean marginal cost. The corner solution model generates a median marginal cost at SEK 0.0066 and a weighted marginal cost at SEK 0.0132, while the corresponding estimates are SEK 0.0031 and SEK 0.0106 in the survival model when using the observations that correspond to the same 354 observations from the corner solution approach. Using the survival model's PH estimate and observed AC_{qit} (like eq. 3 for the corner solution approach) generates similar marginal costs as using the AFT estimate and eq. (12), especially when comparing the median costs. The median marginal costs are

[10] The estimated mean value is 53 years and the median value is 49 years in our preferred Model 2b.

SEK 0.0036 and SEK 0.0031 when using the PH estimate (in eq. 3) and the AFT estimate (in eq. 12), respectively.

Table 5. Marginal cost per gross ton-km, SEK in 2016 prices, corner solution model and survival model.

Model	*Marginal cost*		*Weighted marginal cost (eq. 5)*
	Mean	*Median*	
Two-part (corner solution) model (354 obs.) (eq. 3)	0.0323	0.0066	0.0132
Survival model (354 obs.) based on c_i (eq. 12)	0.0544	0.0031	0.0106
Survival model (354 obs.) based on PH estimate and observed AC_{qit} (similar to eq. 3)	0.0415	0.0036	0.0150
Survival model (3385 obs.) based on c_i (eq. 12)	0.1997	0.0039	0.0295
Survival model (3385 obs.) based on c (eq. 12)	0.1321	0.0041	0.0030

The possibility to use all 3385 observations from the survival model does not have a large impact on the median marginal cost (SEK 0.0039), while the weighted average marginal cost increases substantially (SEK 0.0295). For comparison, as in Andersson et al. (2016), we also use the overall average track renewal cost in the survival model approach (c = SEK 4.77 million), which results in a median marginal cost (SEK 0.0041) that is similar to the weighted marginal cost (SEK 0.0030)[11].

The marginal cost estimates (not weighted) from the survival model (354 obs.) and the corner solution model (354 obs., based on eq. 12) are illustrated in Figure 1 below. The two models generate the same decreasing relationship with traffic, and they are both like the marginal rail infrastructure maintenance costs for wear and tear (see for example Wheat et al. (2009)). This visual inspection also shows that the corner solution estimates are generally higher than the survival estimates (with only a few exceptions), when comparing costs for a certain traffic level (annual million gross tons on the x-axis).

[11] In fact, this weighted marginal cost is similar to the estimate in Andersson et al. (2016), which was SEK 0.0022 per gross ton-km in 2016 prices.

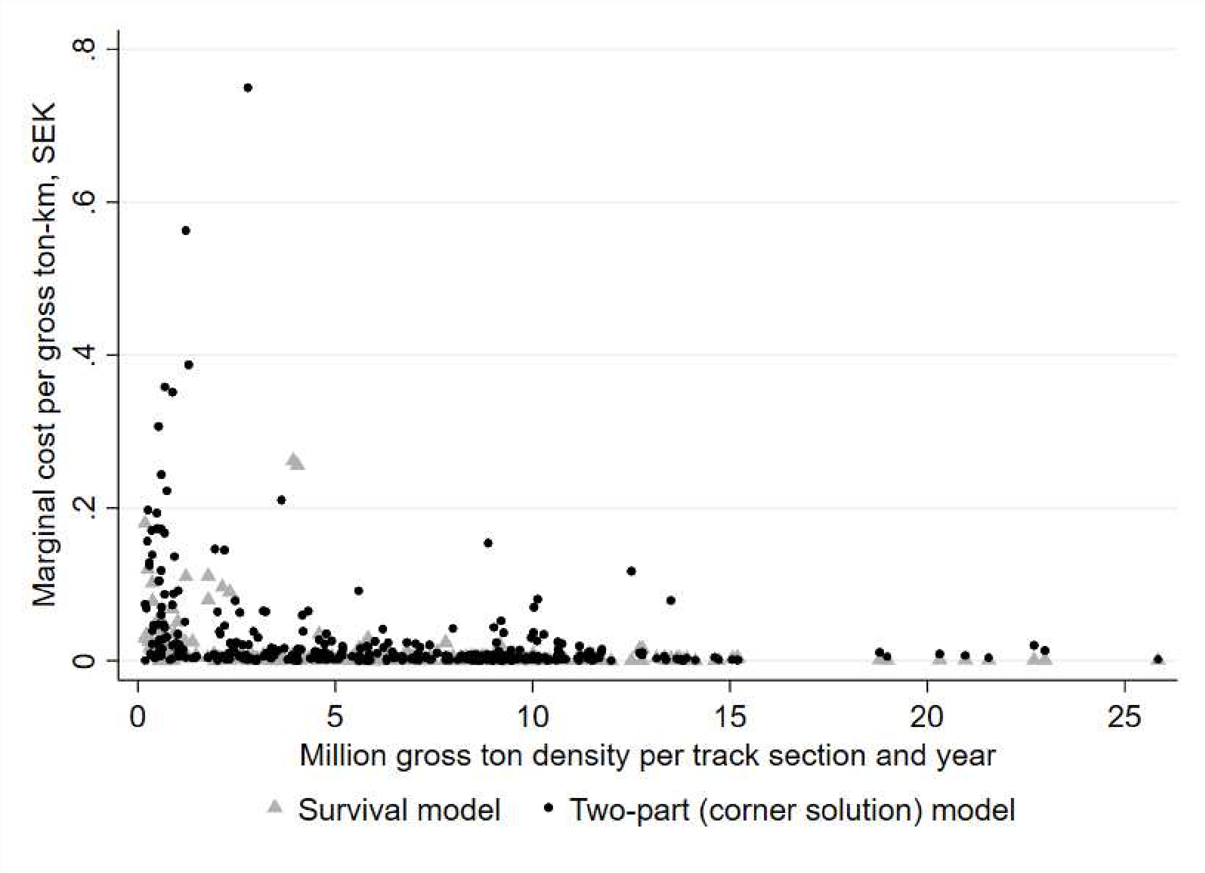

Figure 1. Marginal track renewal cost per gross ton-km, SEK in 2016 prices: Survival model (354 obs.) and Two-part (corner solution) model (354 obs.) using section specific average costs.

All in all, a comparison of the different model approaches' median estimates shows that the corner solution model results in a marginal cost (SEK 0.0066) that is significantly higher than the corresponding estimate from the survival model (SEK 0.0031). This is also the case when comparing their weighted marginal costs (based on 354 observations using eq. 12, but not using eq. 3). However, we prefer using median marginal costs when comparing modelling approaches as the weighted marginal costs put a higher weight on the tails of the distribution, which are (more) imprecisely estimated by the models.

5. Discussion and conclusion

The efficient use of railway infrastructure requires a charge that is based on the marginal cost of using it. To estimate the marginal costs for track renewal using econometric techniques, the significance of a traffic variable explains how the renewal process generates different results. Our paper has compared two different approaches for establishing a value for marginal costs.

One approach comprises the corner solution (two-part) model, which uses traffic and other covariates to explain the decision to make a renewal and the size of the renewal as two separate processes. The first part of the model uses the entire sample for estimating a probit model, while the second part uses the observations with positive renewal costs for estimating a truncated regression model. The other approach is based on a parametric survival model, where rail survival time is modelled as a function of traffic and other network characteristics. Specifically, a deterioration elasticity with respect to traffic is estimated. The traffic estimates from the different approaches are then used in the calculation of marginal costs.

The two approaches are applied on 18 years of information about the Swedish rail network to illuminate the consequences of using the different methods. The PH estimates from the survival analysis are like the probit regression estimate (first part of the corner-solution approach), depending on how the hazard rate is specified in the model. The most important difference between the two approaches relates to construction of the link between the probability of track renewal and costs. The impact traffic has on the size of the renewal cost is estimated in the second part of the corner solution approach, while the survival approach only models the impact traffic has on the probability of renewals (or alternatively, on the renewal intervals if the AFT estimate is used instead of the PH estimate) and then connects this to unit costs. That is, the impact a traffic increase has on the size of the renewal is not considered in the survival model.

The (median) marginal cost per gross ton-km when using the corner solution is almost twice the size of the survival model estimate. Similar differences can be found when comparing results from previous studies using either approach. Since both approaches have similarities with respect to the impact of traffic on the occurrence of renewals, it is reason to ask if and why one model should be preferred over the other? One crucial aspect in addressing this question is that track renewals are not homogenous. Except for variations in length, project costs differ in a non-stochastic way. Using the two-part approach makes it possible to pick up differences in costs directly related to the explanatory variables, such as traffic. Specifically, the truncated regression (the second part of the corner solution model) shows that an increase in traffic results in higher track renewal costs, while controlling for the track length of the renewal and the weight of the rails. Traffic can thus explain variations in the number and/or type of components installed when the track superstructure is renewed, resulting in cost variations.

In general, when the event (in our case, the renewal) can vary continuously in “severity” due to some covariate, the corner solution model can be preferable to survival analysis. But what if detailed cost data and complete information on the components are available; would not a survival analysis be able to consider all the variations in renewal cost triggered by traffic increases? This is only true to some extent, considering that a model estimation on disaggregate unit cost data that are not in line with the performed renewal activities might result in estimates that do no capture economies of scope or scale. There is thus reason to let the renewal production structure guide the level of detail in the cost analysis. Collecting unit costs for each component in an asset and modeling each component's survival is probably not the best way forward. As stated earlier (and indicated by the truncated regression results in the two-part corner solution model), renewals of the track superstructure are not always complete replacements of the components, one reason being differences in the traffic volume. That is, more traffic increases the probability of one component to be renewed and this needs to be modelled in relation to the renewal of other components on the same part of the section, since their renewals and costs may be linked due to the renewal production structure (which in turn is based on a cost minimizing strategy by the IM). In other words, it is better to model the systematic impact of traffic on track renewal costs and not just on the probability of track component renewals.

All in all, we consider the corner solution model to be less restrictive in capturing the behavior of the infrastructure manager compared to the survival model. This point towards a preference for the corner solution approach for estimating the marginal cost of infrastructure (track) renewals with respect to traffic.

Acknowledgements

This paper has been financed as one part of an assignment from the Government Offices of Sweden to review and develop the understanding of social marginal costs for infrastructure use. We are grateful to Phill Wheat for comments and suggestions for improvements on an earlier version of this paper, which has also been presented on a seminar at VTI, Stockholm, 25 May 2018 and on the ITEA Annual Conference and School on Transportation Economics, Hong Kong, 29 June 2018. Special thanks to the two anonymous reviewers for excellent comments and suggestions for improvements. All remaining errors are the responsibility of the authors. We also want to thank Vivianne Karlsson, Anna Willershausen, Anders Nilsson and Anne-Lie Mathiesen at the Swedish Transport Administration for providing data.

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